Aspects of partial deconfinement

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gauge/gravity

- Gauged matrix quantum mechanics are theories of “strings”: planar diagrams.
- Most theories of strings are of theories of “quantum gravity” in extra dimensions.
- Best known example is $AdS_5 \times S^5$ being dual to N=4 SYM.

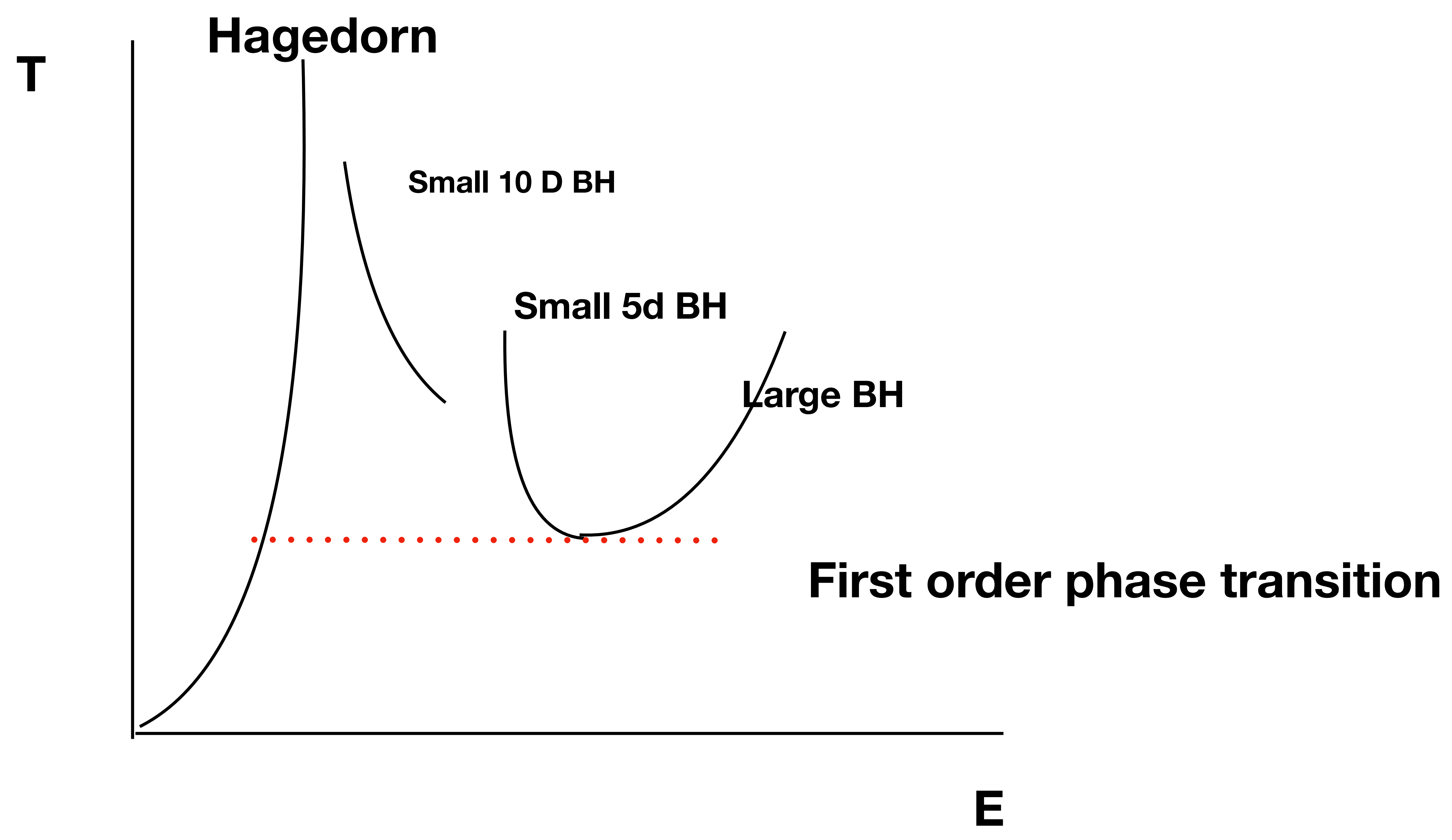
Dualities

The two dual theories are the same: must be able to go back and forth.

The “dictionary” is the lookup table for how to do this.

Our goal is to interpret aspects of phase diagram in dual YM
and say something about it.

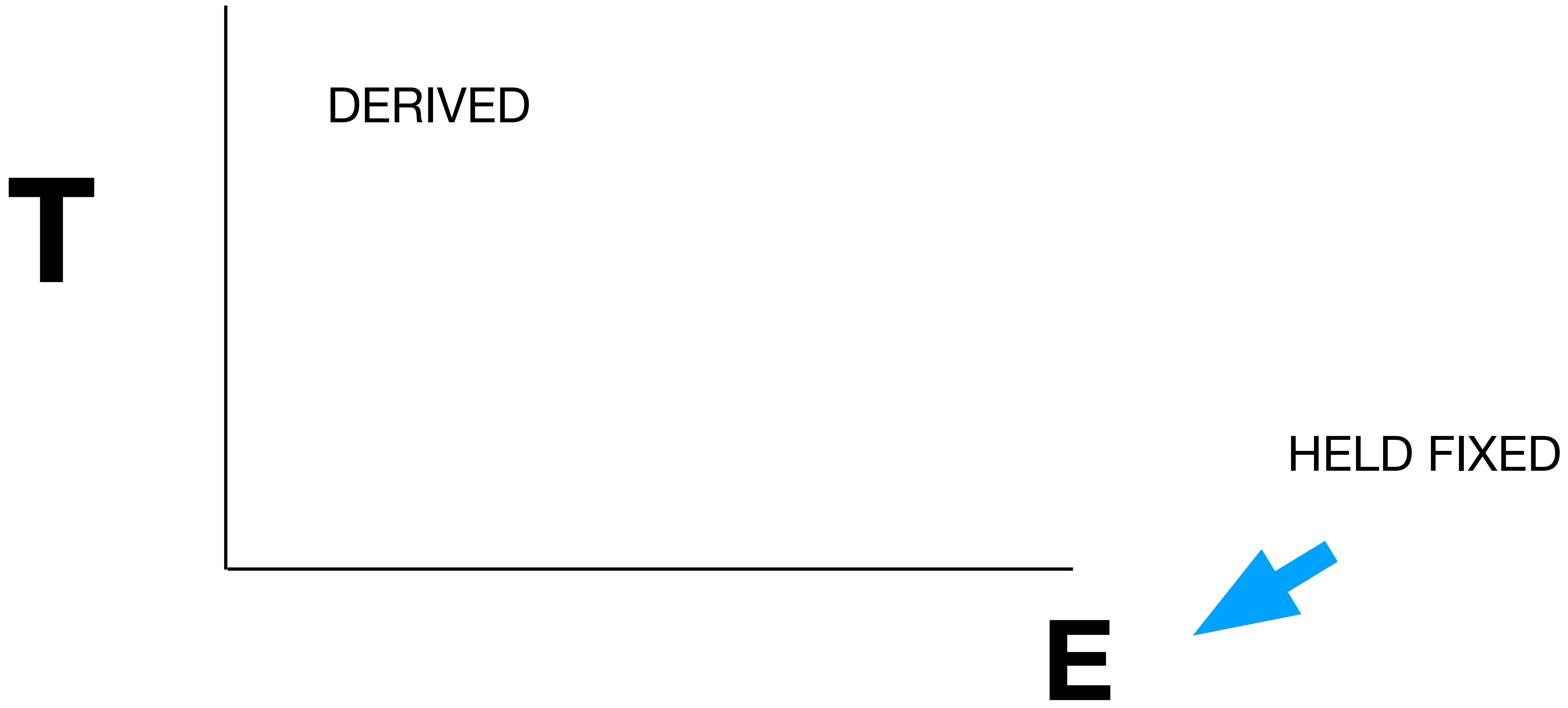
Phase diagram in AdS



Hawking-Page = confinement/deconfinement (Witten)

For global AdS transition occurs only at infinite N

Look at drawing



IT IS IN THE MICROCANONICAL ENSEMBLE

$S(E)$

NEED TO COMPUTE DENSITY OF STATES

First law of thermo:

$$TdS = dE \rightarrow T = 1/(dS/dE)$$

$T>0$ means that as energy increases
we have more states (phase space) available.

Specific heat deals with convexity properties of Entropy

$$C = 1/(dT/dE) \equiv 1/(d^2S/dE^2)$$

No singularity in gravity

- There is no phase transition (discontinuity) in the family of solutions for small black holes and large black holes.
- If large black holes are deconfined, the small black holes should also be in the same (similar) phase.
- The topology of spacetime (Black hole) is roughly the order parameter for confinement/deconfinement. The entropy of small black hole is also of order N^2 . Problem: we don't know how to think about this (too low for naive scaling field theory arguments with T).

No transition between strings and black hole.

As we increase coupling, a crumpled string becomes a black hole smoothly.

A very stringy black hole is just a long crumpled string before it eventually evaporates by string perturbative interactions.

Susskind, Horowitz, Polchinski.

“Solution:” restore naive scaling

$$N^2 \to N_{eff}^2(E)$$

Make the number of colors depend on energy and say that only a sub matrix is “deconfined”.

$$N - N_{eff} \sim O(N) \gg 0$$

History

- D.B., C. Asplund 0809.0712 (something like this was argued for small 10D BH)
- Hanada, Maltz, 1608.03276
- D.B. 1806.05729 (Calculable model, solved in more detail later on)
- Hanada, Ishiki, Watanabe 1812.05494
- …

How to count states?

Simplest gauge theory

1-matrix model quantum mechanics

$$H = tr(\dot{X}^2) + tr(V(X))$$

Invariant under U(N): take singlet sector.

$$V(X) = X^2$$

Solved by free fermions.

There is no phase transition
No Hagedorn behavior

Two matrix model (Next simplest)

$$H = Tr(\dot{X}^2) + Tr(\dot{Y}^2) + V(X,Y)$$

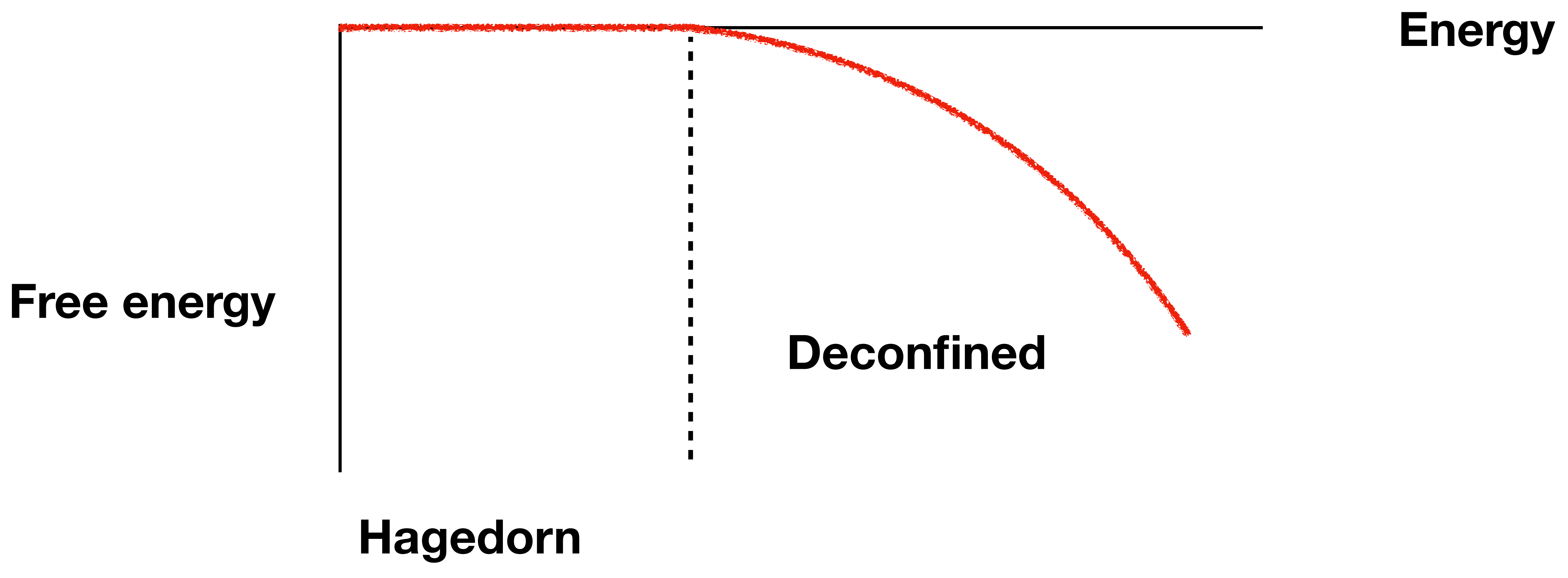
With X,Y in adjoint of U(N): a 2 matrix model.

$$V(X,Y) = Tr(X^2) + Tr(Y^2)$$

In free theory at large N, there is a
confinement/deconfinement first order phase transition

- Aharony, Minwalla, Papadodimas, van Raamsdonk: hep-th/0310285

Phase diagram



The “order parameter” is the dependence with N of the entropy.

$$S_{conf} \simeq O(1) \qquad \text{(low T)}$$

$$S_{deconf} \simeq O(N^2) \qquad \text{(high T)}$$

To get the phase transition we need to study the density of states with the energy: we need to count states.

Write states in an oscillator basis:

$$(a^\dagger)^i_j = A$$

$$(b^\dagger)^i_j = B$$

All states are produced by matrix valued raising operators.

Gauge invariance requires upper indices be contracted with lower indices

$$tr(ABA\ldots)$$

For example: traces and multitraces (strings — copied from AdS/CFT dictionary)

For single traces.

$$\ell = \#\text{Letters}$$

$$\#\ \text{States}_{1-string} \sim 2^{\ell}/\ell$$

The entropy is the log

$$S \simeq \ell \log 2$$

From first law

$$T = \frac{1}{dS/d\ell} = \frac{1}{\log 2}$$

Multi-traces only add subleading corrections to the entropy: same T.

This is the Hagedorn density of states $S \propto E$

Protocol

- Study at large energy but much less than the number of degrees of freedom of deconfined phase

$$1 << E << N^2$$

How do these excitations fit in the matrix?

In the model there is an extra $U(N)^4$ symmetry.

Another counting of states

$$(a^\dagger)^{i_1}_{j_1} (a^\dagger)^{i_2}_{j_2} \ldots (a^\dagger)^{i_k}_{j_k}$$

Transforms as tensor of U(N) x U(N) (upper and lower indices)

Decompose into irreps: Young diagrams (symmetrize/amtisymmetrize)

Same diagram on upper and lower indices: bose statistics of a oscillator.

Same works with B: we count all states this way.

Take tensor product on upper (and lower indices) and decompose again on diagonal.

$$Y_A \otimes Y_B \simeq \bigoplus Y_{A+B}$$

This still counts all states: but there might be multiplicities in products.

For fixed energy E, we need E boxes

$$E = \ell$$

To get a singlet: upper index boxes of final Young diagram need to have same shape as lower index boxes.

Counting of states

After al the Young diagrams are specified, we count degeneracies of representations.

Count degeneracy on upper index multiplicities and lower index multiplicities

$$N(n_x, n_y) = \sum_{\nu=\Upsilon(n_x),\mu=\Upsilon(n_x),\sigma=\Upsilon(n_x+n_y)} (c^{\sigma}_{\mu\nu})^2 \simeq \exp(\beta_q(n_x+n_y))$$

There are not enough young diagrams. (Scale as partitions of n). This means that the LR coefficients must be large.
The largest c can dominate the ensemble.

Asymptotics of LR coefficients

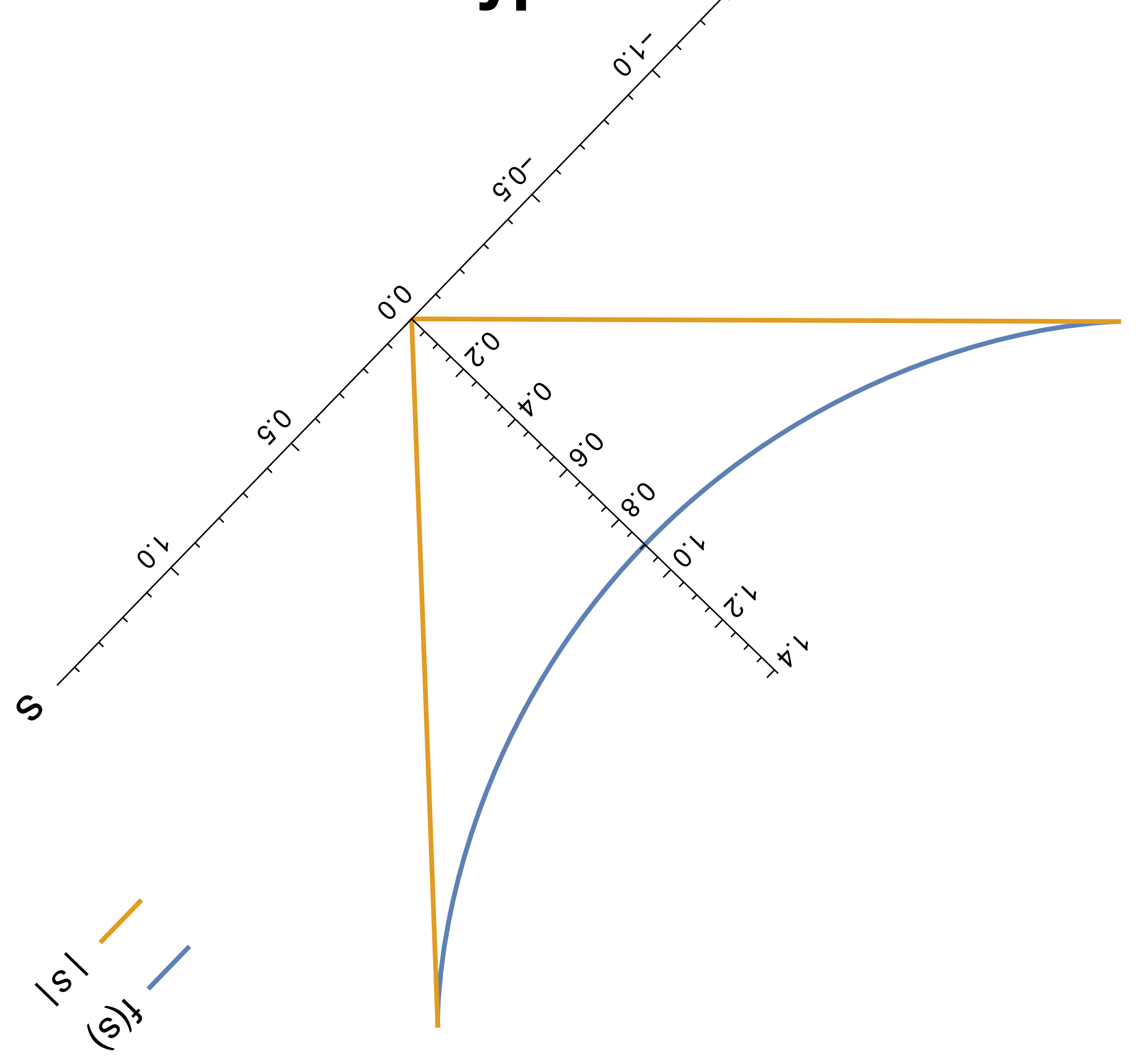
Calculating LR coefficients is a hard math problem (NP)

Estimating largest LR coefficients is done in papers by mathematicians,
there is a dominant shape of all the Young diagrams appearing in the problem.

New result

D.B., Kai Yan, 2307.06122

In the typical state all three Young tableaux have the same typical shape (VKLS shape)



$$f(s) = \frac{2\left(\sqrt{2-s^2} + s\sin^{-1}\left(\frac{s}{\sqrt{2}}\right)\right)}{\pi}$$
$$\tilde{f}(s) = |s|$$

The shape arises from minimizing the hook-length formula. Roughly, one maximizes the dimensions of the representations at large N

VKLS: Vershik, Kerov, Logan, and Shepp

Problem for the shape: minimize the hook length product formula in the large Young diagram limit

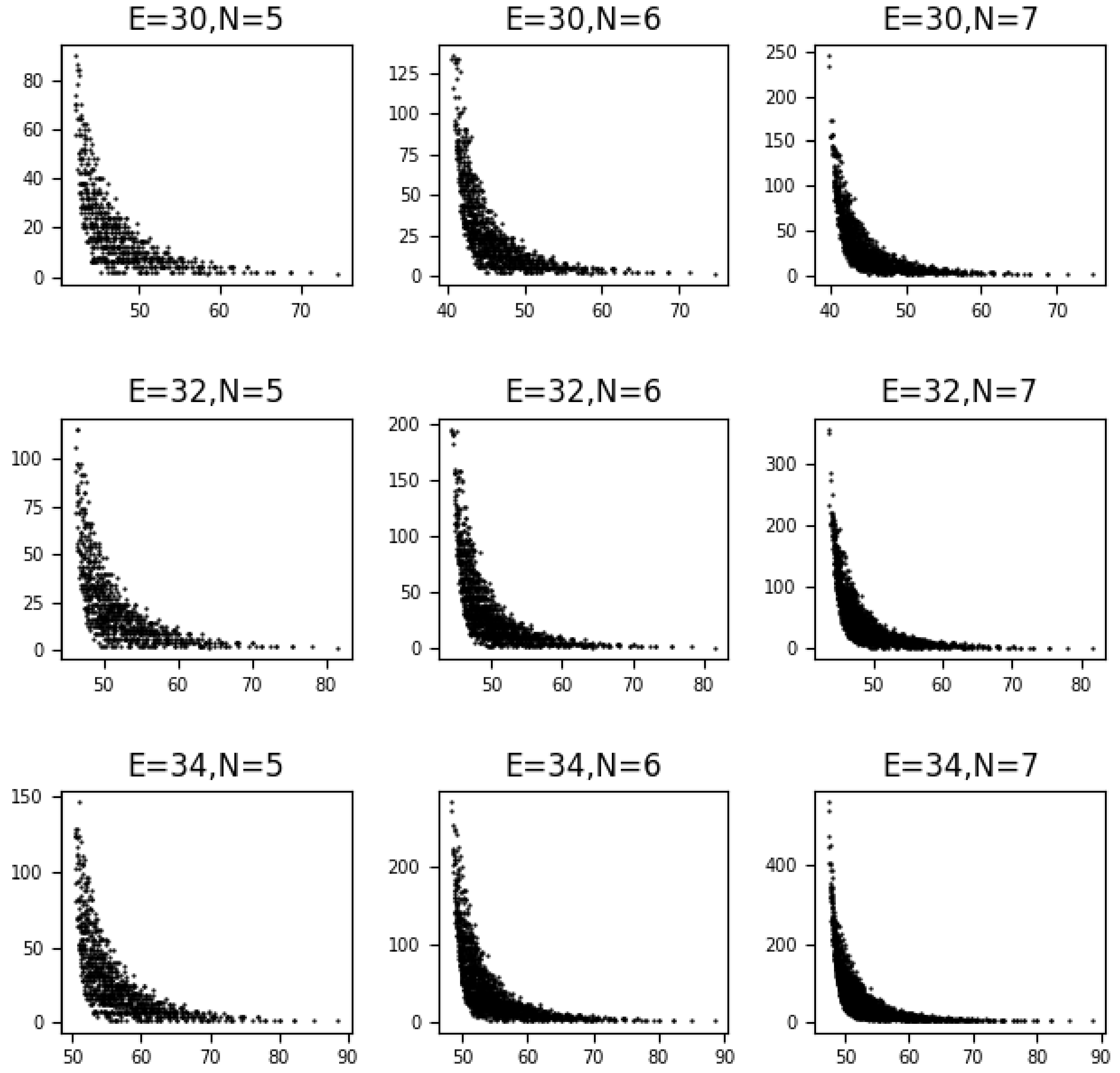
Roughly, this is maximizing the dimension of all the involved representations at large N, fixed number of boxes

LR coefficients:

Even though problem is in principle computationally hard (asymptotic statement), there are good math resources to compute them for “small” Young diagrams

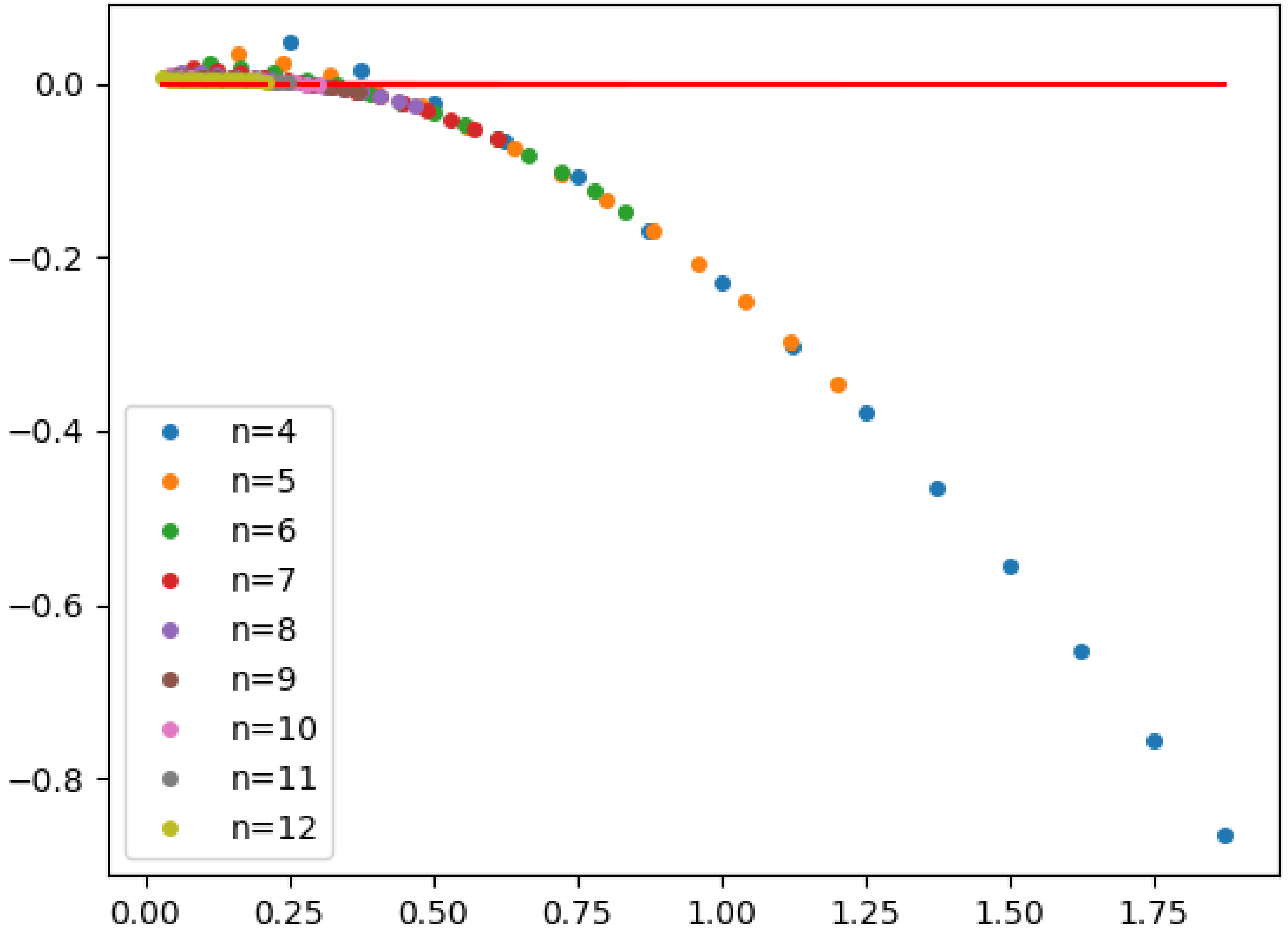
IN PICTURES

LR COEFF.



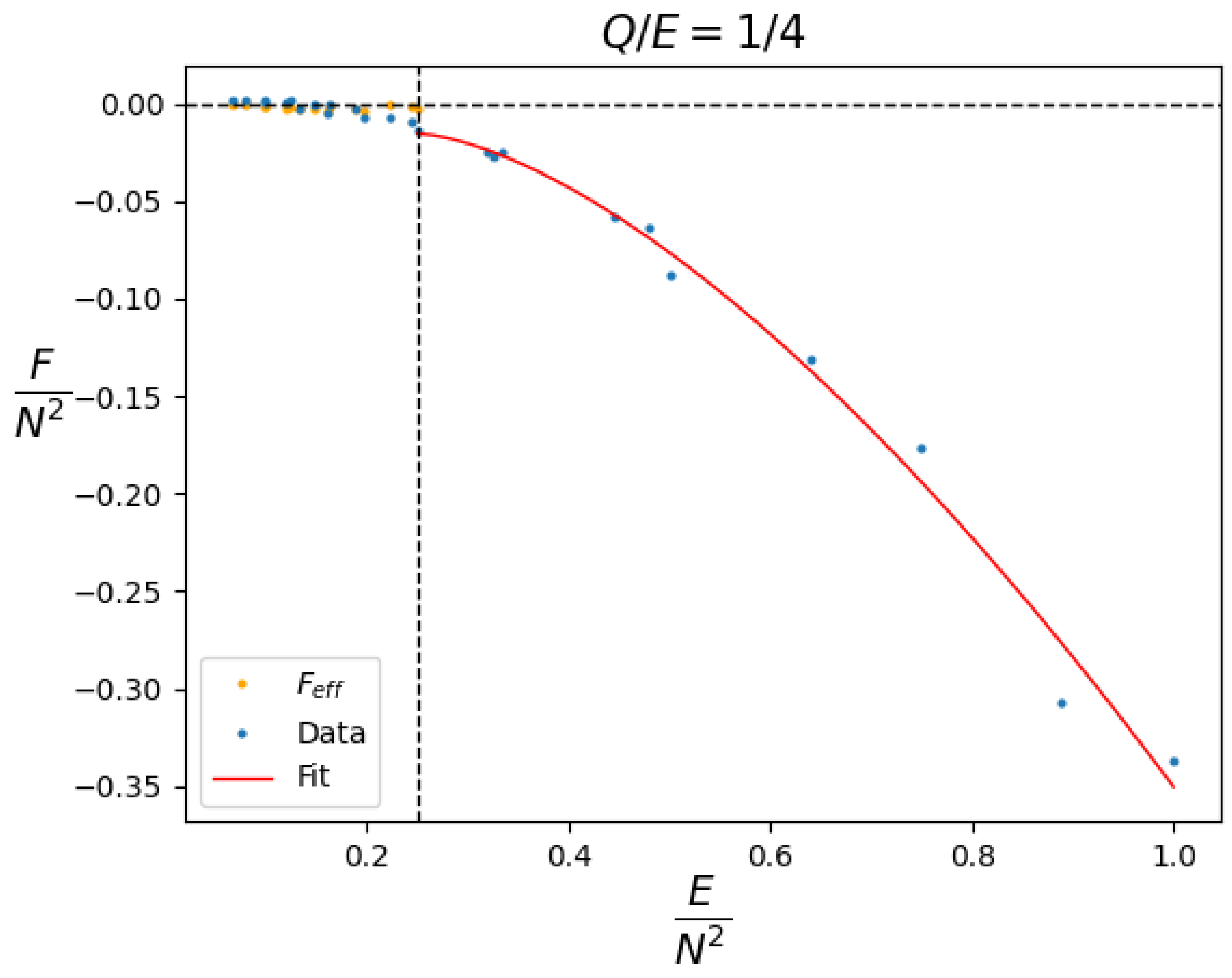
LOG(HOOK LENGTH)

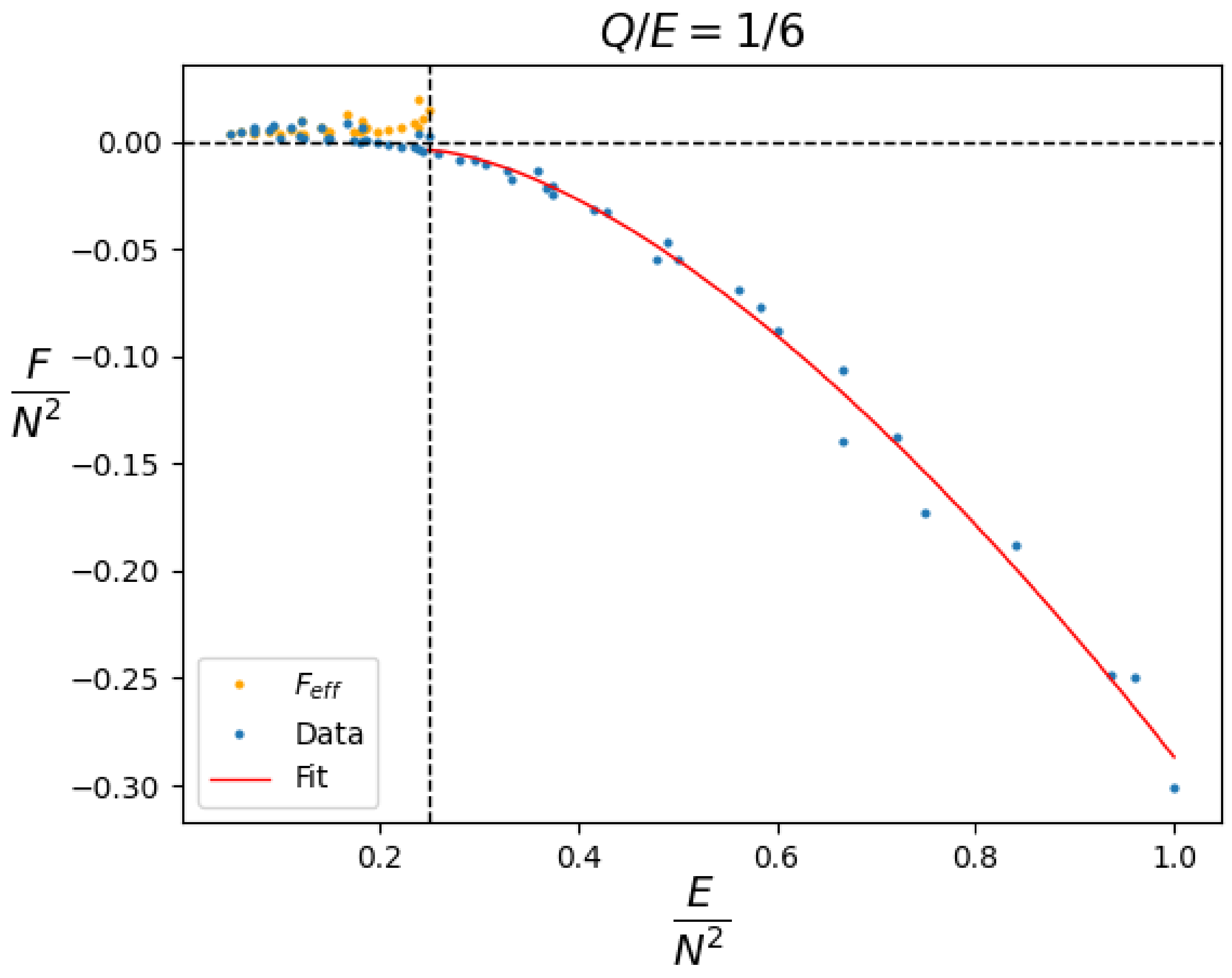
FREE ENERGY/N^2

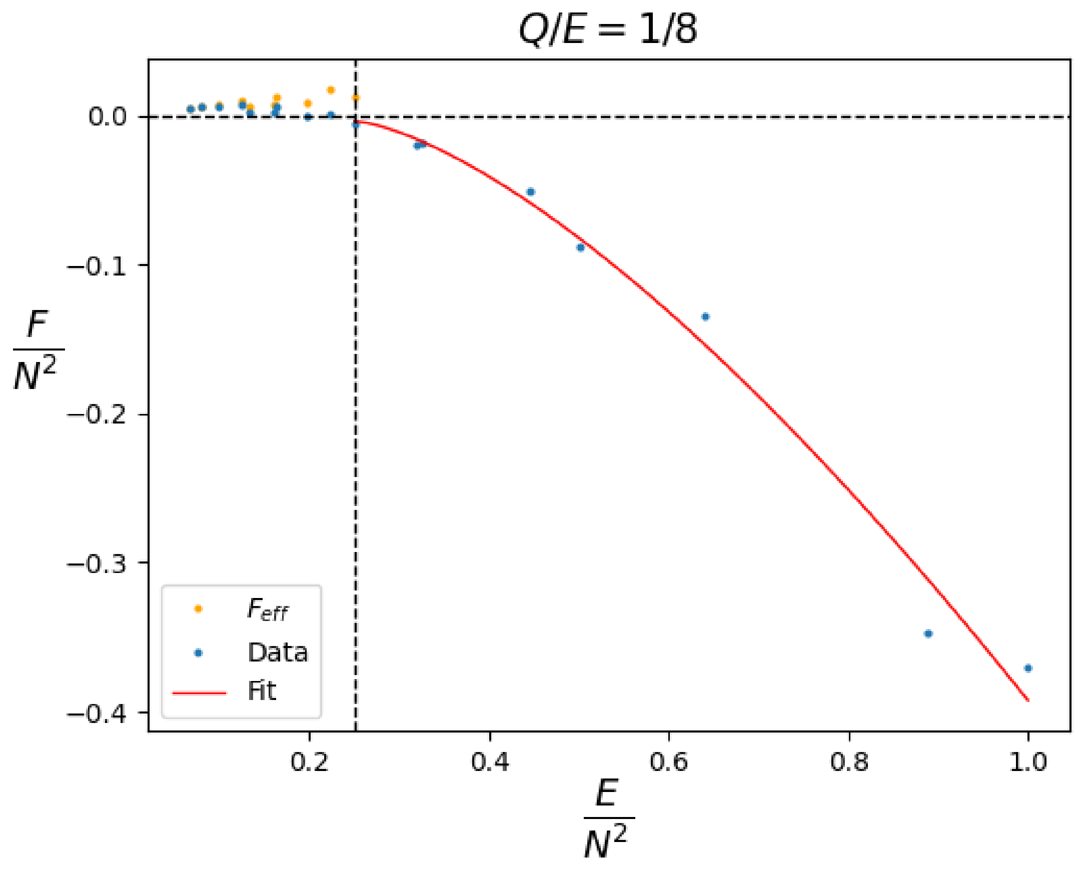


ENERGY/N^2

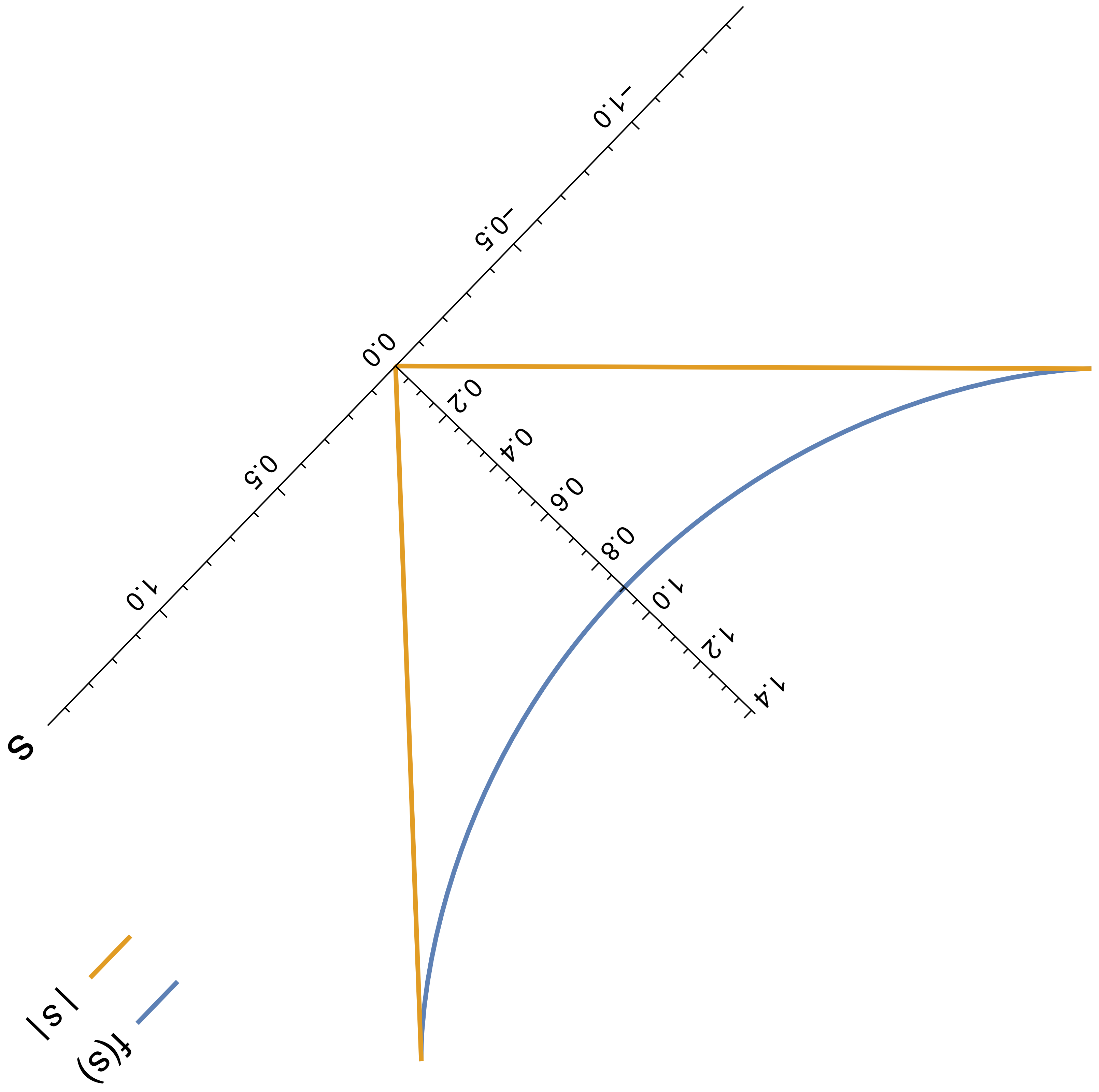
TRANSITION ENERGY IS INDEPENDENT OF THE CHARGE (different number of boxes on the sub diagrams**).**







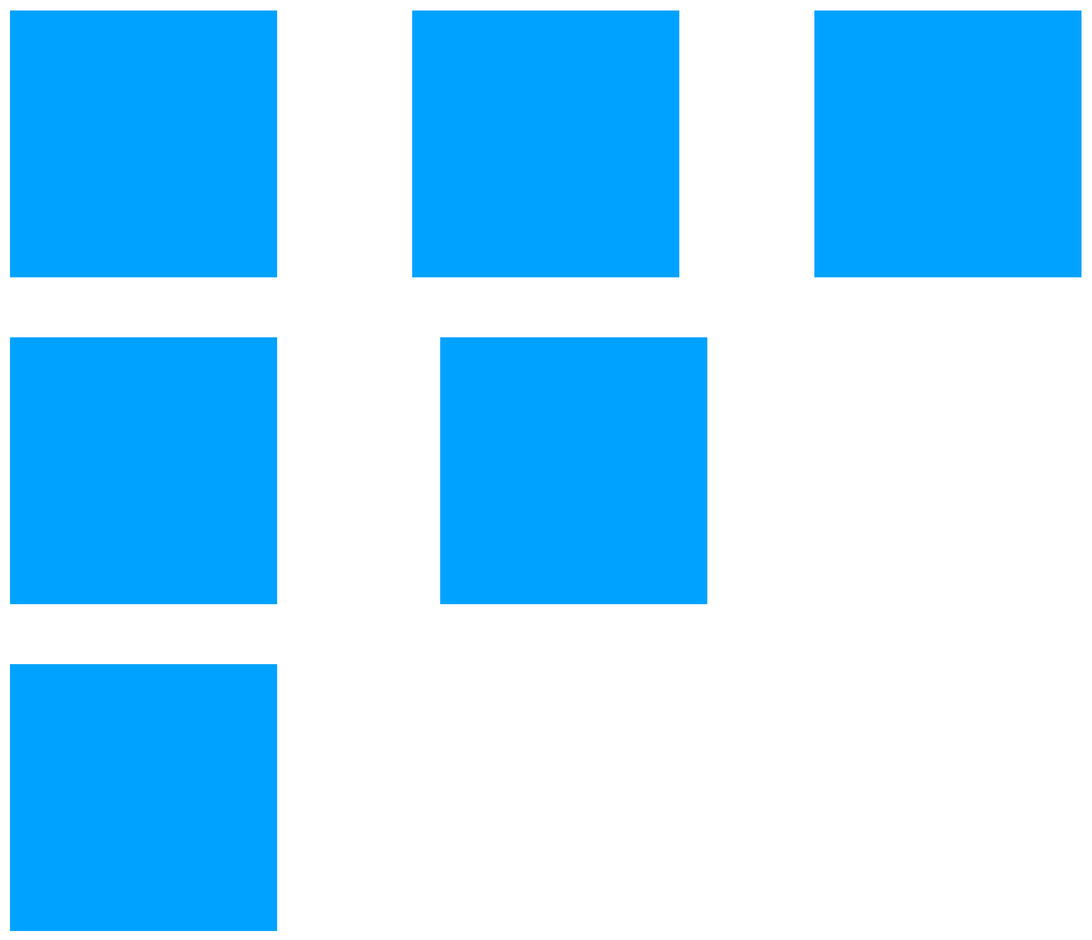
When the shape dominates.



Must be allowed Young diagram for U(N): depth less than N.

When it becomes disallowed there is a transition.

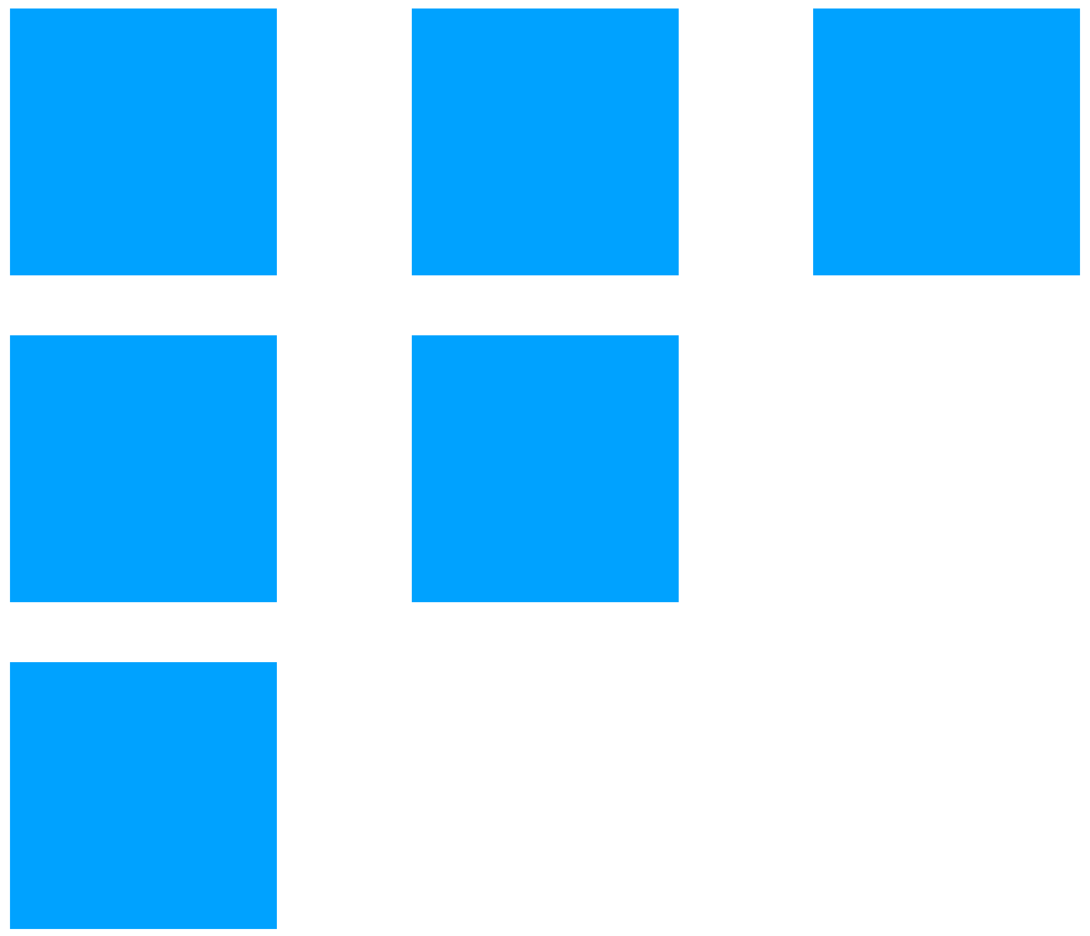
Nomenclature



Shape of Young diagram can be related to eigenvalues in a 1-matrix model.

Number of rows is number of eigenvalues excited.

Young diagram is also in correspondence with a highest weight state of the corresponding representation.



HWS leaves unbroken U(N-#rows)

We call partial deconfinement the Hagedorn phase. The U(X) with X the depth of the tableaux is deconfined (excited). The unbroken symmetry of h.w.s. is called confined (same as ground state).

Full deconfinement occurs when the typical tableaux reaches the maximal depth allowed by U(N): that is the end of the hagedorn behavior.

$$E \equiv N^2/4 + O(N)$$

Comes from area of the VKLS shape: theory prediction.

Gauge invariant characterization

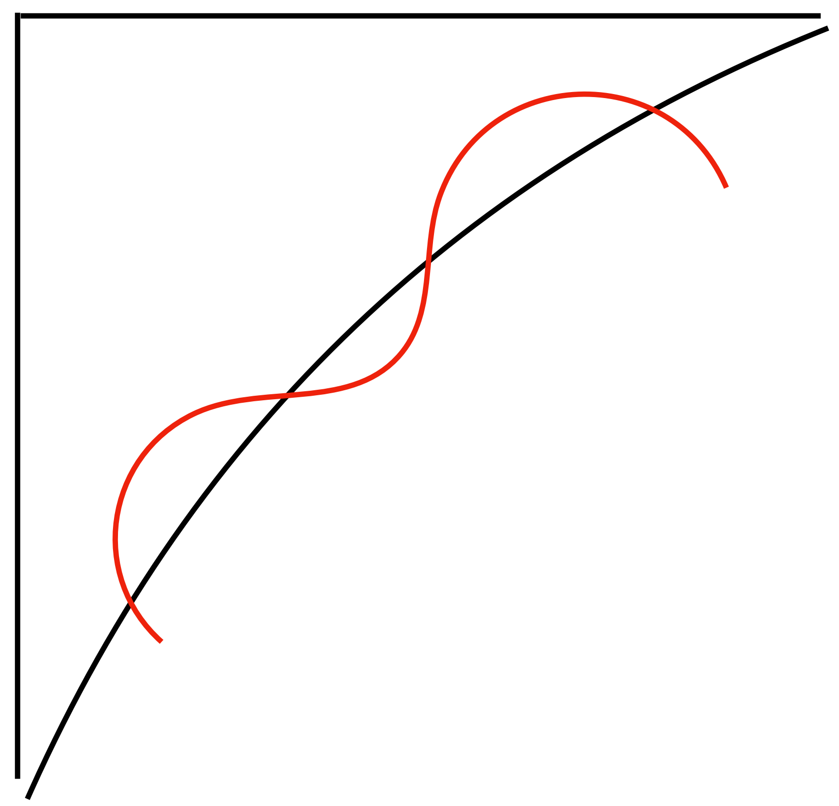
- We are actually using the shape of Young diagrams for a Global auxiliary symmetry to pin a notion of eigenvalues $U(N)^4 \neq U(N)$
- The Cassimirs of all the global symmetries commute with the U(N) that is gauged: good gauge invariant observables.
- This means that the shapes can be determined from a measurement.

Interactions

- The potential causes transitions between states. X,Y add or subtract boxes. The dynamics of all these additions and subtractions are on edge of all Young diagrams.

- Edge dynanics?

Hope: shape hints at “geometry” of “eigenvalues”



Some local edge dynamics at strong coupling?

There is a lot of entropy in these configurations, and the transitions generated by perturbations mostly keep the shape.
Hopefully there is some remnant of the dominant shape when the global symmetries that were used to anchor it are broken.

In this sense, there is a notion of partial deconfinement in the sense of “eigenvalues of the matrix” that survives the breaking of the global symmetries.

Entropy transition

The transition from Hagedorn (partial deconfinement) to deconfined phase is an entropy transition; the counting of states starts differing from infinite N

In the language that Denjoe O'Connor uses, the reduction of available phase space is represented in the basis ofbtraces as an increase in relations between them. The number of relations becomes so large that it reduces the entropy growth substantially: the specific heat becomes positive (not infinite).

$$S \simeq E T_H \to S \simeq N^2 \log(E)$$

Conclusion

- The idea of partial deconfinemnet makes sense in some models.
- Roughly this is the idea that small black holes can be traced as a deconfined phase for a subgroup (in some EFT sense)
- In the simplest model it is also the Hagedorn phase.
- Tha transition to full deconfiment occurs when partial deconfined subgroup becomes everything. This is a large N transition in the simplest model.

Extra comments

- Masanori and collaborator have extended these ideas to other gauge theories.

- They can relate the partial deconfined phase to other ideas (Polyakov loop, etc).

- It is hard to pin down N_{eff} in models where there are no extra U(N)-like global symmetries that can easily produce a notion of Young diagram (or some notion of generalized eigenvalues)