The M(atrix) Theory at Strong Coupling

[2401.XXXXX]

With S. Komatsu, A. Martina, J. Penedones, N. Suchel, A. Vuignier

Xiang Zhao
Fields and Strings Laboratory, EPFL
2024.01.12 @ CERN

Outline

1. Motivation & Review
2. Two approaches to finding an effective description at strong coupling
    1. Born-Oppenheimer approach
    2. Path integral: the naive & the right way
3. The strong coupling limit of the BMN model
4. Conclusion & Outlook

Motivation & Review

Motivation

- M(atrix) theories are interesting models of holography:
  - Simpler than QFT; finite d.o.f..
  - More accessible to numerical methods: Monte Carlo, Bootstrap, Hamiltonian truncation, quantum computation (in the near future), etc.
- We focus on the model of Berenstein, Maldacena and Nastase (BMN) [Berenstein, Maldacena, Nastase '02]
  - The SUSY-preserving mass deformation of the model of Banks, Fischler, Shenker, Susskind (BFSS). [Banks, Fischler, Shenker, Susskind '96]

The BMN model

- The Hamiltonian reads (everything dimensionless except for μ):

$$H/\mu = \boxed{\mathrm{Tr}\left[\frac{1}{2}(P^I)^2 - \frac{g^2}{4}[X^I, X^J]^2 - \frac{g}{2}\hat{\Theta}_\alpha \gamma^I_{\alpha\beta}[X^I, \hat{\Theta}_\beta]\right]} \; H_{\mathrm{BFSS}}$$

$$+ \frac{1}{2}\,\mathrm{Tr}\left[\frac{1}{3^2}\sum_{i=1}^{3}(X^i)^2 + \frac{1}{6^2}\sum_{p=4}^{9}(X^p)^2 + i\frac{1}{4}\hat{\Theta}_\alpha \gamma^{123}_{\alpha\beta}\hat{\Theta}_\beta + i\frac{2g}{3}\epsilon_{ijk}X^i X^j X^k\right]$$

- $g^2 = \dfrac{g_{YM}^2}{\mu^3}$ is a dimensionless coupling, μ is the mass deformation parameter.

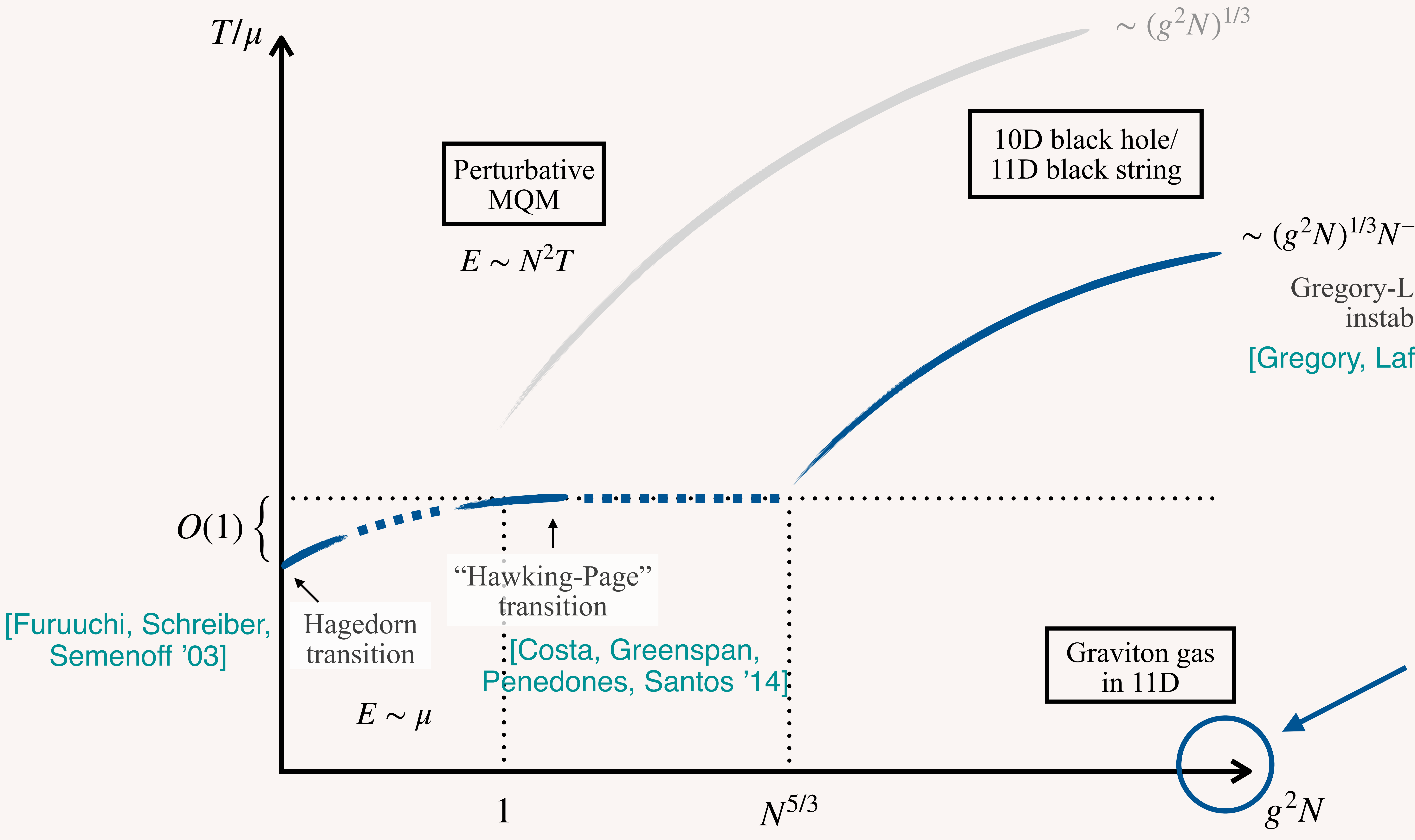
- X^I and $\hat{\Theta}_\alpha$ are $N \times N$ hermitian matrices. $I = 1,2,\ldots,9$, $\alpha = 1,2,\ldots,16$.

The BMN model

- Symmetries: $SO(3) \times SO(6) \times SU(N)$, $\mathcal{N} = 16$ supersymmetry
- Features from the mass deformation:
  - IR regulator: discrete spectrum; easier for numerics
  - Degenerate vacua labelled by integer partitions of N (size of the matrices)

    [Dasgupta et al ’02; Kim, Plefa ’02; Lin, Maldacena ’05]
  - Dimensionless tunable coupling g and a two-dimensional phase diagram

The BMN phase diagram (large N)



Perturbative MQM

$E \sim N^2T$

10D black hole/
11D black string

Gregory-Laflamme
instability

[Gregory, Laflamme '93]

Hagedorn
transition

[Furuuchi, Schreiber,
Semenoff '03]

"Hawking-Page"
transition

[Costa, Greenspan,
Penedones, Santos '14]

$E \sim \mu$

Graviton gas
in 11D

Also See [Itzhaki, Maldacena,
Sonnenschein, Yankielowicz '98]
for BFSS

Focus of this talk:
$T = 0, g \to \infty$, **fixed** N

Goal: find a EFT

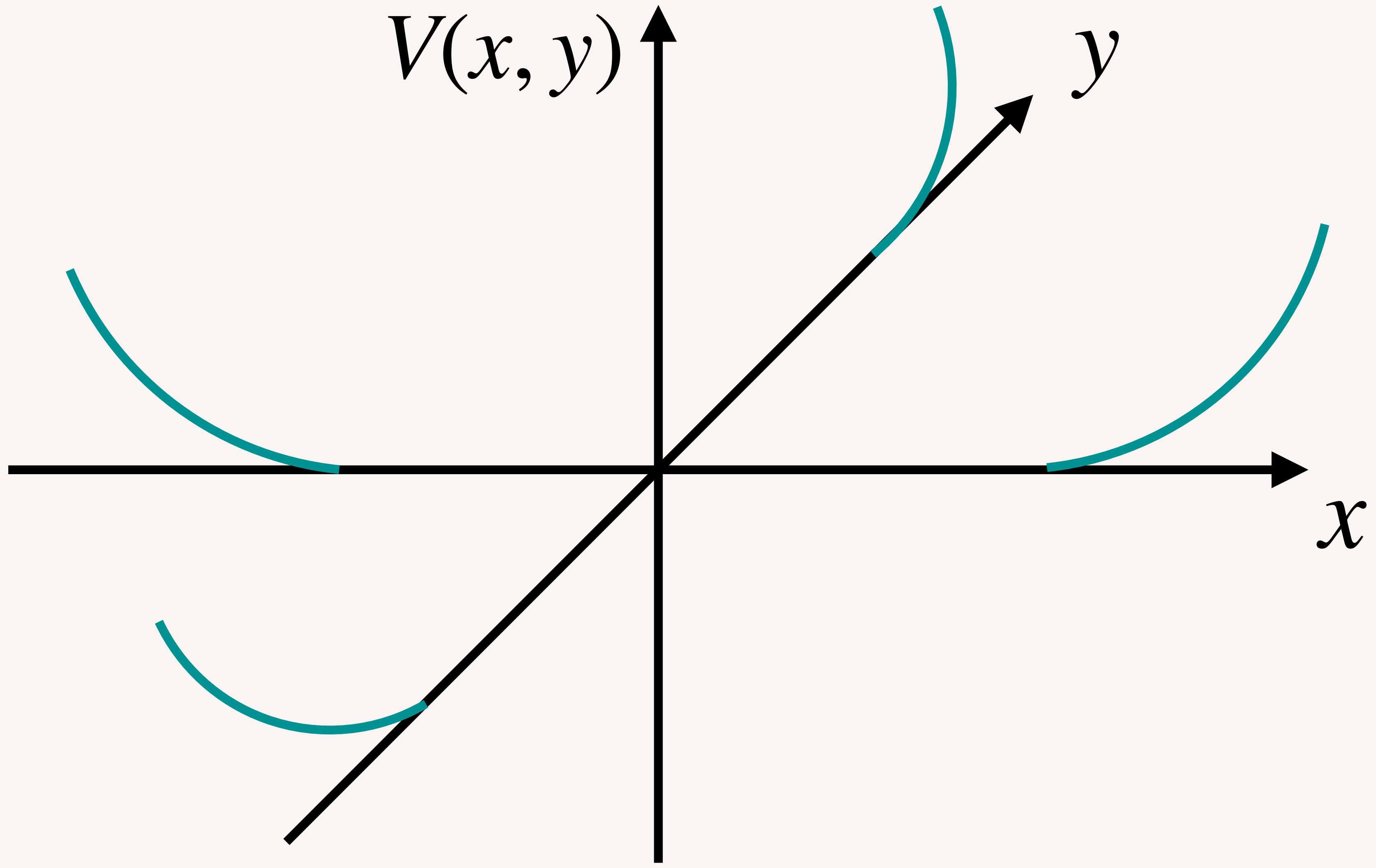
Two Approaches to EFT

The toy model

- In the strong coupling limit $g \to \infty$ the potential term in the BMN model $-g^2 \operatorname{Tr}\left([X_I, X_J]^2\right) \sim \sum_{I,J} g^2 z_A^I z_C^I Z_B^J Z_D^J f_{ABE} f_{CDE} + \dots$
  - Off-diagonal matrices are suppressed.
- Consider a toy model:

$$H = \frac{1}{2}p_x^2 + \frac{1}{2}p_y^2 + g^2 x^2 y^2$$

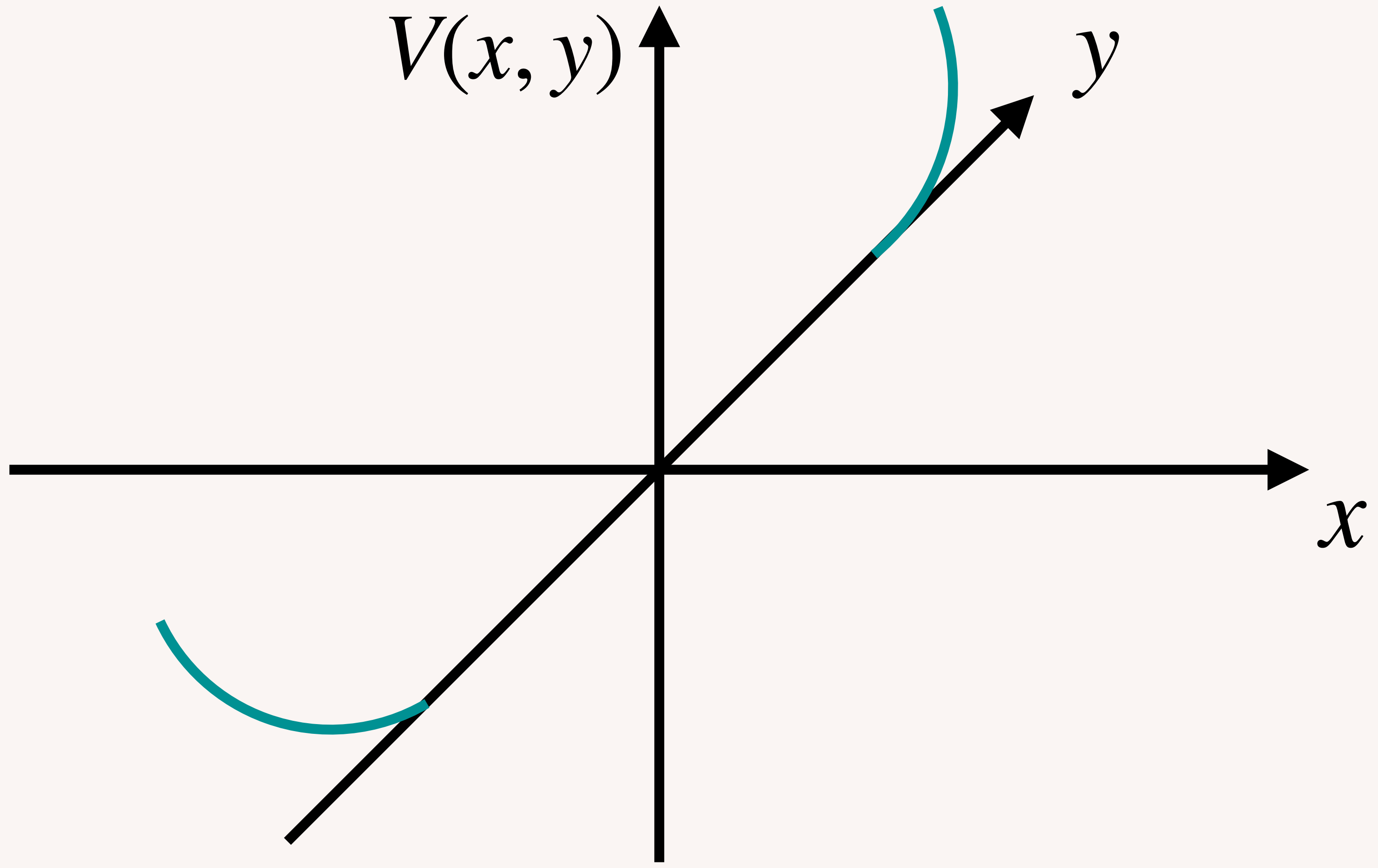
[Simon '83]



Classical flat directions uplifted by quantum effect (zero-point energy)

The toy model

- A better toy model is $H = \frac{1}{2}\left(p_x^2 + x^2 - 1\right) + \frac{1}{2}\left(p_y^2 + y^2 + g^2x^2y^2 - \sqrt{1+g^2x^2}\right)$



Flat direction along x is
(almost) restored

- At large g, y-frequency is ~$g\,|\,x\,|$, much larger than x-frequency.
- x mimics diagonal matrix elements in BMN and y mimics off-diagonal ones.
- Q: What's the effective Hamiltonian/action at $g \to \infty$?

The naive path integral approach

$$H = \frac{1}{2}\left(p_x^2 + x^2 - 1\right) + \frac{1}{2}\left(p_y^2 + y^2 + g^2x^2y^2 - \sqrt{1+g^2x^2}\right)$$

- To integrate out the fast modes, it seems natural to define the effective action (in Euclidean time τ)

$$\int \mathcal{D}x\mathcal{D}y e^{-S_E[x,y]} \equiv \int \mathcal{D}x e^{-S_{E,\text{eff}}[x]}$$

with

$$S_E[x,y] = \frac{1}{2}\int_{-\beta/2}^{\beta/2} d\tau\, y(\tau)\left(-\partial_\tau^2 + g^2x^2 + 1\right)y(\tau) + \frac{1}{2}\int_{-\beta/2}^{\beta/2} d\tau \left(\dot{x}^2 + x^2 - 1 - \sqrt{1+g^2x^2}\right)$$

- The path integral for y is Gaussian and we get

$$S_{E,\text{eff}}[x] = \frac{1}{2}\int_{-\beta/2}^{\beta/2} d\tau \left(\dot{x}^2 + x^2 - 1 + O(1/g)\right)$$

The Born-Oppenheimer approach

- Idea: solve the Schrödinger equation order by order in g

$$H\Psi(x,y) = E\Psi(x,y) \overset{g\to\infty}{\longrightarrow} H_{\text{eff}}\psi(x) = E\psi(x)$$

- Rescale $y \to y/\sqrt{g}$, so that $y = O(1)$, and the expansion of H is

$$H = gH^{(1)} + H^{(0)} + O(1/g)\,, \quad H^{(1)} = \frac{1}{2}\left(p_y^2 + x^2y^2 - |x|\right)\,, \quad H^{(0)} = \frac{1}{2}\left(p_x^2 + x^2 - 1\right)$$

- The ground state of $H^{(1)}$ is a gaussian, denoted as $\Omega_x(y)$: $H^{(1)}\Omega_x(y) = 0$

- Ansatz for the wavefunction: put y on its ground state plus corrections

$$\Psi(x,y) = \Psi^{(0)}(x,y) + g^{-1}\Psi^{(-1)}(x,y) + \ldots\,, \quad \Psi^{(0)}(x,y) \equiv \psi(x)\Omega_x(y)$$

The Born-Oppenheimer approach

- Sch. Eq. at $O(g)$: $$gH^{(1)}\Psi^{(0)}(x,y) \equiv gH^{(1)}\psi(x)\Omega_x(y) = 0$$

- At $O(1)$: $$\underbrace{\langle \Omega_x(y), H^{(1)}\Psi^{(-1)}\rangle_y}_{=0} + \underbrace{\langle \Omega_x(y), H^{(0)}\Psi^{(0)}\rangle_y}_{H^{(0)}_{\text{eff}}\psi(x)} = E\langle \Omega_x(y), \Psi^{(0)}\rangle_y = E\psi(x)$$

Inner product in y

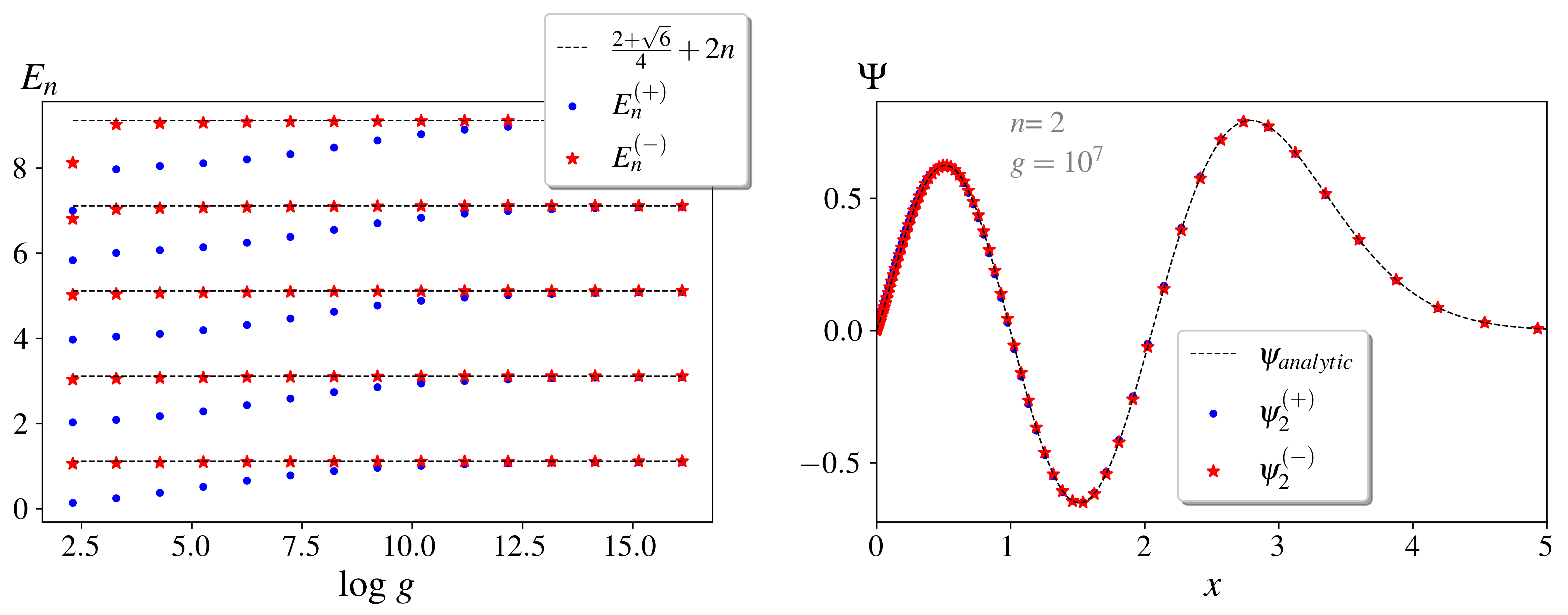
- The final result: $$H^{(0)}_{\text{eff}} = \frac{1}{2}\left(p_x^2 + x^2 - 1 + \frac{1}{8x^2}\right)$$

Comes from acting p_x^2 on $\Omega_x(y)$

- Recall from naive path integral: $H^{(0)}_{\text{eff}} = \frac{1}{2}\left(p_x^2 + x^2 - 1\right)$
- Which one is correct??

Compare B-O result with numerics

- Solving the spectrum of $H^{(0)}_{\text{eff}} = \frac{1}{2}\left(p_x^2 + x^2 - 1 + \frac{1}{8x^2}\right)$ gives $E_n^{\pm} = \frac{2+\sqrt{6}}{4} + (\text{subleading}), \quad n = 0, 1, 2, \ldots$
- Solving the original Hamiltonian numerically:



The path integral done right

- The naive path integral approach assumed $x(\tau)$ to be a smooth function, but *there are also fast modes in x.*

- The proper way is to split $x(\tau) = x_s(\tau) + x_f(\tau)$ and the effective action is defined as:

$$S_{\text{eff}}[x] \equiv -\log \int \mathcal{D}y\, e^{-S_E[x,y]} \;\rightarrow\; S_{\text{eff}}[x_s] \equiv -\log \int \mathcal{D}y \mathcal{D}x_f\, e^{-S_E[x_s,x_f,y]}$$

- One way to do the split $x(\tau) = x_s(\tau) + x_f(\tau)$ is

$$x_s(\tau) = \sum_{|n|\leq\Lambda} a_n e^{2\pi i n\tau/\beta}, \qquad x_f(\tau) = \sum_{|n|>\Lambda} a_n e^{2\pi i n\tau/\beta} \qquad (a_{-n} = a_n^*)$$

The path integral done right

- One way to do the split $x(\tau) = x_s(\tau) + x_f(\tau)$ is

$$x_s(\tau) = \sum_{|n|\leq\Lambda} a_n e^{2\pi i n\tau/\beta}\,, \qquad x_f(\tau) = \sum_{|n|>\Lambda} a_n e^{2\pi i n\tau/\beta} \qquad (a_{-n} = a_n^*)$$

- The cutoff Λ should be that x_s is much slower than y:

$$\omega_{x_s} \sim \Lambda/\beta \quad \omega_y \sim g|x_s|\,, \quad q \equiv \omega_y/\omega_{x_s} = g|x_s|\beta/\Lambda \gg 1$$

The path integral done right

- The split of the action is $S_E = S^{\text{kin}}_{\text{fast}} + S^{\text{int}}_{\text{fast}} + S_{\text{slow}}$

$$S_{\text{eff}}[x_s] \equiv -\log \int \mathcal{D}y\mathcal{D}x_f\, e^{-S_E} = \underbrace{S_{\text{slow}} - \log Z_{\text{fast}}}_{= \frac{1}{2}\int_{-\beta/2}^{\beta/2} d\tau\, \left(\dot{x}_s^2 + x_s^2 - 1 + O(1/g)\right)} - \log\langle e^{-S^{\text{int}}_{\text{fast}}}\rangle_{\text{fast}}$$

Expectation value under $S^{\text{kin}}_{\text{fast}}$, compute by perturbation

- The expansion in g gives

$$-\log\langle e^{-S^{\text{int}}_{\text{fast}}}\rangle_{\text{fast}} = \langle S^{\text{int}}_{\text{fast}}\rangle - \frac{1}{2}\left(\langle (S^{\text{int}}_{\text{fast}})^2\rangle - \langle S^{\text{int}}_{\text{fast}}\rangle^2\right) + \cdots = \frac{1}{2}\int_{-\beta/2}^{\beta/2} d\tau\, \frac{1}{8x_s^2} + \ldots$$

- The final result agrees with the Born-Oppenheimer approach exactly!

Comments on the path integral approach

- Going to subleading orders properly seems difficult even for the toy model.
- Naive path integral approach may work sometimes e.g. due to supersymmetry. But it's not clear to what extent this works...
- Ordering ambiguity from path integral. E.g. v^4/r^7 term in the BFSS model.

The BMN model at strong coupling

Two notable differences in the BMN model

- To obtain the effective description at $g \to \infty$, we will adopt the Born-Oppenheimer approach explained above.
- The procedure of finding the H_{eff} is mostly the same, with two differences:
  - $SU(N)$ symmetry. We focus on $SU(N)$ singlet states. Also possible to consider non-singlet states.
  - $\mathcal{N} = 16$ supersymmetry. This can simplify computation.

Change of coordinates

- We are interested in $SU(N)$ singlets, so we need to separate out $SU(N)$ invariant variables:

[Lin, Yin '14]

$$\vec{X} = U^{-1}\begin{pmatrix} \vec{r}_1 & \vec{y}_{12}/\sqrt{g} & & \\ (\vec{y}_{12})^*/\sqrt{g} & \vec{r}_2 & & \\ & & \ddots & \\ & & & \vec{r}_N \end{pmatrix} U \quad \text{with} \quad (\vec{r}_a - \vec{r}_b)\cdot\vec{y}_{ab} = 0 \quad a, b = 1, \ldots, N,\ a \neq b$$

- Fermions become

$$\hat{\Theta}_\alpha = U^{-1}\begin{pmatrix} (\theta_1)_\alpha & (\Theta_{12})_\alpha & & \\ (\Theta_{12})^*_\alpha & (\theta_2)_\alpha & & \\ & & \ddots & \\ & & & (\theta_N)_\alpha \end{pmatrix} U$$

- Overall $SU(N)$ rotations only act on U matrix.

Finding the H_{eff}

- Expansion in large g gives $H = gH^{(1)} + \sqrt{g}H^{(1/2)} + \dots$
  - In particular

$$gH^{(1)} = \sum_{a \neq b} \left(-\frac{1}{2} g \Pi^{IJ}_{ab} \frac{\partial}{\partial y^{I}_{ab}} \frac{\partial}{\partial y^{J}_{ba}} + \frac{1}{2} g |r_{ab}|^2 y^{I}_{ab} y^{I}_{ba} + \frac{1}{2} g r^{I}_{ab} \Theta^{\top}_{ab} \gamma^{I} \Theta_{ba} \right)$$

SUSY harmonic oscillators in off-diagonal modes

- The g.s. satisfies $H^{(1)}|\Omega\rangle = 0$ and the leading w.f. is $|\Psi^{(0)}\rangle = |\psi(r,\theta)\rangle\,|\Omega\rangle$
- Through $H^{(0)}_{\text{eff}}|\psi\rangle := \langle\Omega|H^{(1/2)}|\Psi^{(-1/2)}\rangle + \langle\Omega|H^{(0)}|\Psi^{(0)}\rangle = E|\psi(r,\theta)\rangle$ we get

$$H^{(0)}_{\text{eff}} = \sum_{a=1}^{N} \left(-\frac{1}{2} \frac{\partial^2}{\partial r^{I}_{a} \partial r^{I}_{a}} + \frac{1}{2} \frac{1}{3^2} (r^{i}_{a})^2 + \frac{1}{2} \frac{1}{6^2} (r^{p}_{a})^2 + \frac{i}{8} \theta^{\top}_{a} \gamma^{123} \theta_{a} \right)$$

The diagonal modes in the full BMN Hamiltonian,
again SUSY oscillators

Comments on H_{eff}

- Validity regime:

$$|\vec{r}_a - \vec{r}_b| \gg g^{-1/3}$$

- We also used the BMN SUSY algebra to get $Q_{\text{eff},\alpha}$. Then H_{eff} is obtained from $\{Q_{\text{eff},\alpha}, Q_{\text{eff},\alpha}\}$ and the two results agree.

Also see [Smilga '87]

M-theory dual of BMN MQM

- Consider 11D plane wave background with DLCQ ($x^- \sim x^- + 2\pi R$) implemented:

$$ds^2 = -2dtdx^- + dx^i dx^i + dx^p dx^p - \left(\frac{\mu^2}{3^2}x^i x^i + \frac{\mu^2}{6^2}x^p x^p\right) dt^2 \qquad F_4 = \mu\, dt \wedge dx^1 \wedge dx^2 \wedge dx^3$$

- The strong coupling limit in BMN corresponds to $g \to \infty$, with

$$g^2 = \frac{g_{\text{YM}}^2}{\mu^3} = \frac{R^3}{\mu^3 \ell_P^6}$$

- Conjecture: The DLCQ of M-theory in plane wave with fixed $-p_- = N/R$ is dual to $U(N)$ BMN MQM.

Comparison of the spectra

- Gravity side:
  - The total momentum $-p_{-} = N/R$ is distributed to q $(\leq N)$ gravitons, each with k units of momentum: $k_1 + k_2 + \ldots + k_q = N$.
  - The spectrum is $E_N = \varepsilon_{k_1} + \ldots + \varepsilon_{k_q}$.

$$\varepsilon = \frac{\mu}{3}\sum_{i=1}^{3} n^i + \frac{\mu}{6}\sum_{p=4}^{9} n^p + \frac{\mu}{2}\varepsilon_0$$

[Kimura, Yoshida '03]

Vacuum energy of supergraviton modes →

ε_0	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
d.o.f.	1	8	28	56	70	56	28	8	1

Comparison of the spectra

- The supergraviton spectrum: $E_N = \varepsilon_{k_1} + \ldots + \varepsilon_{k_q}$

$$\varepsilon = \frac{\mu}{3}\sum_{i=1}^{3} n^i + \frac{\mu}{6}\sum_{p=4}^{9} n^p + \frac{\mu}{2}\varepsilon_0$$

[Kimura, Yoshida '03]

Vacuum energy of supergraviton modes →

ε_0	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
d.o.f.	1	8	28	56	70	56	28	8	1

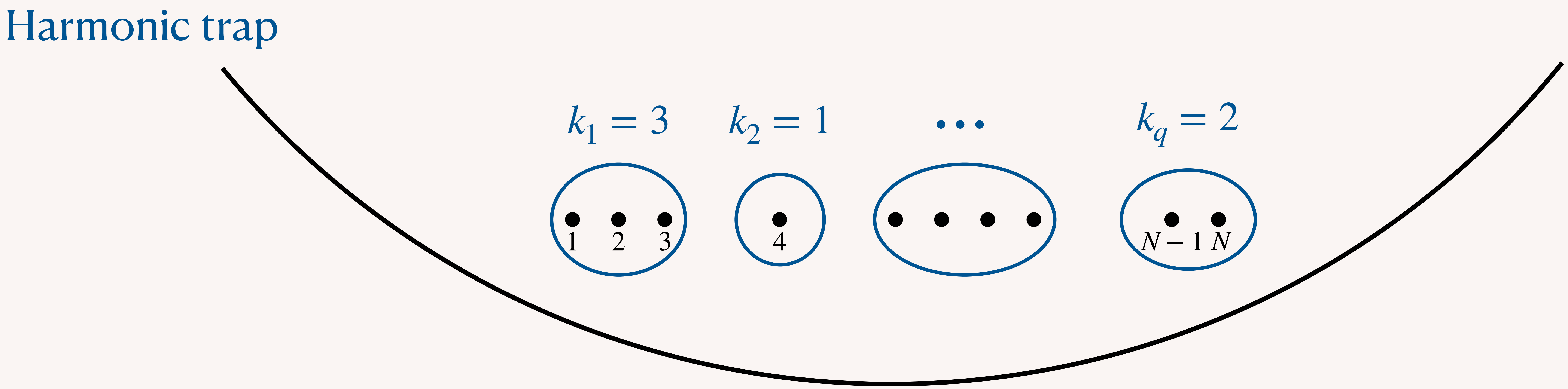
- The $H_{\text{eff}}^{(0)}$ can be written explicitly as oscillators

$$H_{\text{eff}}^{(0)} = \sum_{a=1}^{N}\left(\frac{\mu}{3}\sum_{i=1}^{3}(b_a^i)^\dagger b_a^i + \frac{\mu}{6}\sum_{p=4}^{9}(b_a^p)^\dagger b_a^p + \frac{\mu}{4}\sum_{\alpha=1}^{8}(c_a)_\alpha^\dagger (c_a)_\alpha\right)$$

  - Our H_{eff} is simply N decoupled copies of $H_{\text{eff},N=1}$, with the spectrum $E_N = \varepsilon_1 + \varepsilon_1 + \ldots + \varepsilon_1$. Matches one momentum distribution case in SUGRA.

Comparison of the spectra

- To match with the other configurations on the gravity side, we *postulate* the existence of bound states of D0 branes.
- Each bound state moves freely in the harmonic trap. They should have size $\sim g^{-1/3}$ and internal excitations $\sim g^{2/3}\mu$.



Bound state postulation

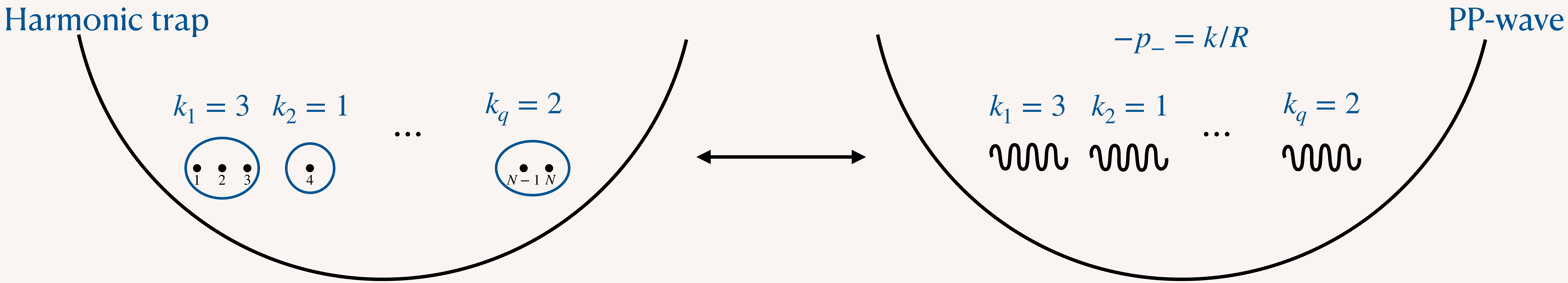
- Bound states are not controversial :)
- Take the BFSS ground state (with fixed N) and put it in the harmonic trap of BMN: slightly deformed.

More generally, multiple bound states

- For ground states this postulation makes matching work: BMN has $p(N)$ vacua; there are $p(N)$ configurations of momentum distribution for gravitons.

The Main Claim

- In the $g \to \infty$ limit, the energy spectrum of the $SU(N)$ singlet sector of the BMN MQM matches the free supergraviton spectrum on the 11D plane wave geometry with total momentum $-p_- = N/R$.
- Wavefunction also matches.



Conclusion & Outlook

Conclusion

- Path integral approach is more subtle than one might have thought.

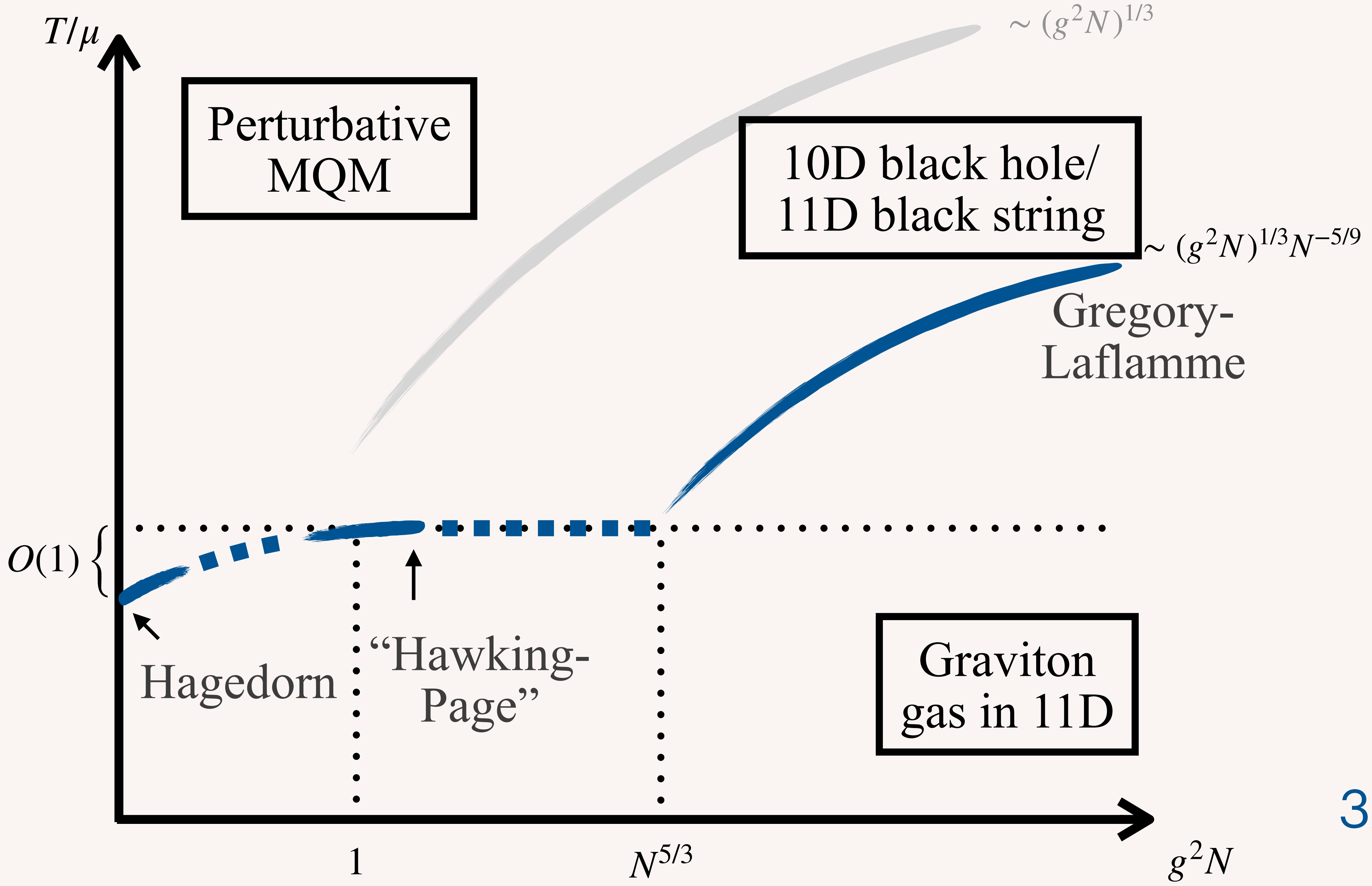
$$S_{\text{eff}}[x] \equiv -\log \int \mathcal{D}y\, e^{-S_E[x,y]} \;\rightarrow\; S_{\text{eff}}[x_s] \equiv -\log \int \mathcal{D}y \mathcal{D}x_f\, e^{-S_E[x_s,x_f,y]}$$

- We explained a scenario where finite N duality works:
  - The leading-order H_{eff} of BMN MQM in $g \rightarrow \infty$ limit is found.
  - The spectrum is consistent with linearised 11D SUGRA on the DLCQ plane wave background.

Outlook

- Subleading corrections to BMN H_{eff}
  - Match with loop expansions in DLCQ SUGRA? (Mismatches exist in BFSS)

    E.g. [Helling, Plefka, Serone, Waldron '99]
  - Turning up N:
    - $T = 0$: find Lin-Maldacena geometry from MQM? [Lin, Maldacena '05], also see [Asano '14]
    - Finite T: find black hole from MQM?
- Non-singlets? (Not special at finite N)
- Is the strong form of BMN conjecture true?



Supplementary

The path integral done right

- The split of the action is $S_E = S^{\text{kin}}_{\text{fast}} + S^{\text{int}}_{\text{fast}} + S_{\text{slow}}$

$$S_{\text{eff}}[x_s] \equiv -\log \int \mathcal{D}y\,\mathcal{D}x_f\, e^{-S_E} = \underbrace{S_{\text{slow}} - \log Z_{\text{fast}}}_{= \frac{1}{2}\int_{-\beta/2}^{\beta/2} d\tau\, \left(\dot{x}_s^2 + x_s^2 - 1 + O(1/g)\right)} - \log\langle e^{-S^{\text{int}}_{\text{fast}}}\rangle_{\text{fast}}$$

Expectation value under $S^{\text{kin}}_{\text{fast}}$, compute by perturbation

- With $x_s \sim 1$, $x_f \sim y \sim g^{-1/2} \sim \Lambda^{-1/2}$, the expansion in g gives

$$S^{\text{int}}_{\text{fast}} = \frac{1}{2}\int_{-\beta/2}^{\beta/2} d\tau \left[\underbrace{2g^2 x_s x_f y^2 - g x_s x_f/\sqrt{x_s^2}}_{O(g^{1/2})} + \underbrace{g^2 x_f^2 y^2}_{O(1)} + \dots\right]$$

$$-\log\langle e^{-S^{\text{int}}_{\text{fast}}}\rangle_{\text{fast}} = \langle S^{\text{int}}_{\text{fast}}\rangle - \frac{1}{2}\left(\langle (S^{\text{int}}_{\text{fast}})^2\rangle - \langle S^{\text{int}}_{\text{fast}}\rangle^2\right) + \cdots = \frac{1}{2}\int_{-\beta/2}^{\beta/2} d\tau\, \frac{1}{8x_s^2} + \dots$$

Two-point function for path integral approach

$$\langle x_f(\tau_1) x_f(\tau_2) \rangle = \sum_{|n|>\Lambda} e^{2\pi i n(\tau_1-\tau_2)/\beta} \frac{\beta}{(2\pi n)^2}$$

$$\langle y(\tau_1) y(\tau_2) \rangle \approx \frac{1}{2g\omega(x_s(\tau))} e^{-g\omega(x_s(\tau))|\tau_1-\tau_2|}$$

BFSS-like regime

- For $X \sim O(g^{-1/3})$, rescale $X = g^{-1/3}\tilde{X}$, $P = g^{1/3}\tilde{P}$, then the BMN Hamiltonian becomes

$$
\begin{aligned}
H/\mu =& g^{\frac{2}{3}} \underbrace{\operatorname{Tr}\left(\frac{1}{2}\left(\tilde{P}^I\right)^2 - \frac{1}{4}\left[\tilde{X}^I, \tilde{X}^J\right]^2 - \frac{1}{2}\hat{\Theta}^\top \gamma^I \left[\tilde{X}^I, \hat{\Theta}\right]\right)}_{H_{\mathrm{BFSS}}} \\
&+ \operatorname{Tr}\left(\frac{i}{3}\epsilon_{ijk}\tilde{X}^i\tilde{X}^j\tilde{X}^k + \frac{i}{8}\hat{\Theta}^\top\gamma^{123}\hat{\Theta}\right) \\
&+ g^{-\frac{2}{3}} \operatorname{Tr}\left(\frac{1}{2}\frac{1}{3^2}\left(\tilde{X}^i\right)^2 + \frac{1}{2}\frac{1}{6^2}\left(\tilde{X}^p\right)^2\right)
\end{aligned}
$$

SUSY algebra of the BMN model

- The BMN model has 16 (real) supercharges

$$Q_\alpha = \mathrm{Tr}\left[P^I\gamma^I\hat{\Theta} - \frac{i}{2}g\left[X^I, X^J\right]\gamma^{IJ}\hat{\Theta} - \frac{1}{3}X^i\gamma^{123}\gamma^i\hat{\Theta} + \frac{1}{6}X^p\gamma^{123}\gamma^p\hat{\Theta}\right]_\alpha$$

$\alpha = 1, \ldots, 16$

$I, J = 1, \ldots, 9$

$i = 1, 2, 3;\; p = 4, \ldots, 9$

- The SUSY algebra is

$$\{Q_\alpha, Q_\beta\} = 2\delta_{\alpha\beta}H - \frac{1}{3}\left(\gamma^{123}\gamma^{ij}\right)_{\alpha\beta}\underset{\substack{\uparrow \\ SO(3)\text{ generators}}}{M^{ij}} + \frac{1}{6}\left(\gamma^{123}\gamma^{pq}\right)_{\alpha\beta}\underset{\substack{\uparrow \\ SO(6)\text{ generators}}}{M^{pq}} + 2g\mathrm{Tr}(X^I\underset{\substack{\uparrow \\ SU(N)\text{ generators}}}{G})\gamma^I_{\alpha\beta}$$

Hamiltonian truncation for the minimal BMN model

The minimal BMN model

- Minimal BFSS: $2+1d$ $\mathcal{N}=1$ SYM dimensionally reduced to $0+1d$
- Minimal BMN is the mass deformation of minimal BFSS:

$$H = \mathrm{Tr}\left[\frac{1}{2}(P^i)^2 - \frac{g^2}{4}\left[X^i, X^j\right]^2 - \frac{g}{2}\hat{\Theta}^\top \gamma^i \left[X^i, \hat{\Theta}\right] + \frac{1}{2}(X^i)^2 - \frac{3i}{4}\hat{\Theta}^\top \gamma^{12}\hat{\Theta}\right], \quad i,j=1,2$$

- Why study it? — Similar to the BMN model (but has a unique vacuum, so no bound state):
  - No flat directions and has a discrete spectrum.
  - Also becomes SUSY oscillators at $g \to \infty$.
  - Simpler because of fewer matrices.

Setup with $N = 2$

- Symmetry: $SU(2) \times SO(2)$, $\mathcal{N} = 2$ SUSY

- SUSY algebra: $\{Q, Q^\dagger\} = 2(H - M)$ → BPS condition: $E = M$. M is $SO(2)$ charge.

- $[H, M] = 0\,,\ [H, Q^{(\dagger)}] = (-)\frac{1}{2}Q^{(\dagger)}\,,\ [M, Q^{(\dagger)}] = (-)\frac{1}{2}Q^{(\dagger)}$

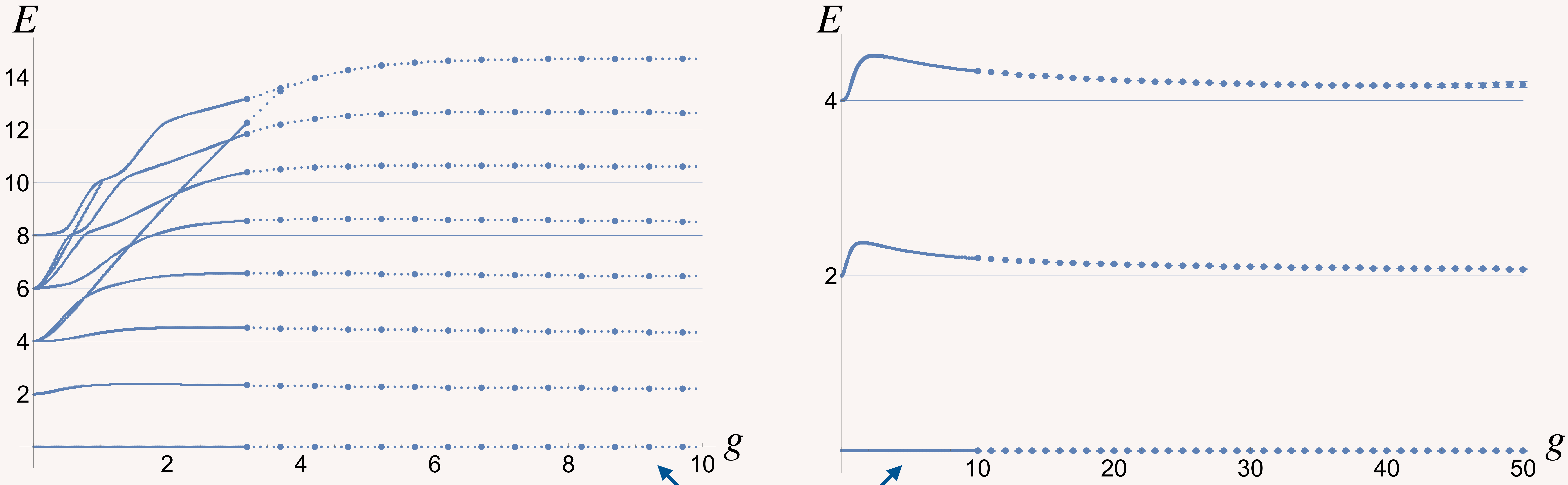
- Charge sectors: $M = \boxed{2n,}\ 2n + 1/2,\ 2n + 1,\ 2n + 3/2$ Focus of the numerics
  - $|\Psi_{E(>M),M(=2n)}\rangle \underset{Q^\dagger}{\overset{Q}{\rightleftarrows}} |\Psi_{E+1/2,M+1/2}\rangle\,,$ (for any g)

- Build the $SU(2)$ invariant oscillator basis from $H|_{g=0}$, $\Lambda/\mu = 200$

Hamiltonian truncation results

$M = 0$

straight lines: analytic prediction at $g \to \infty$

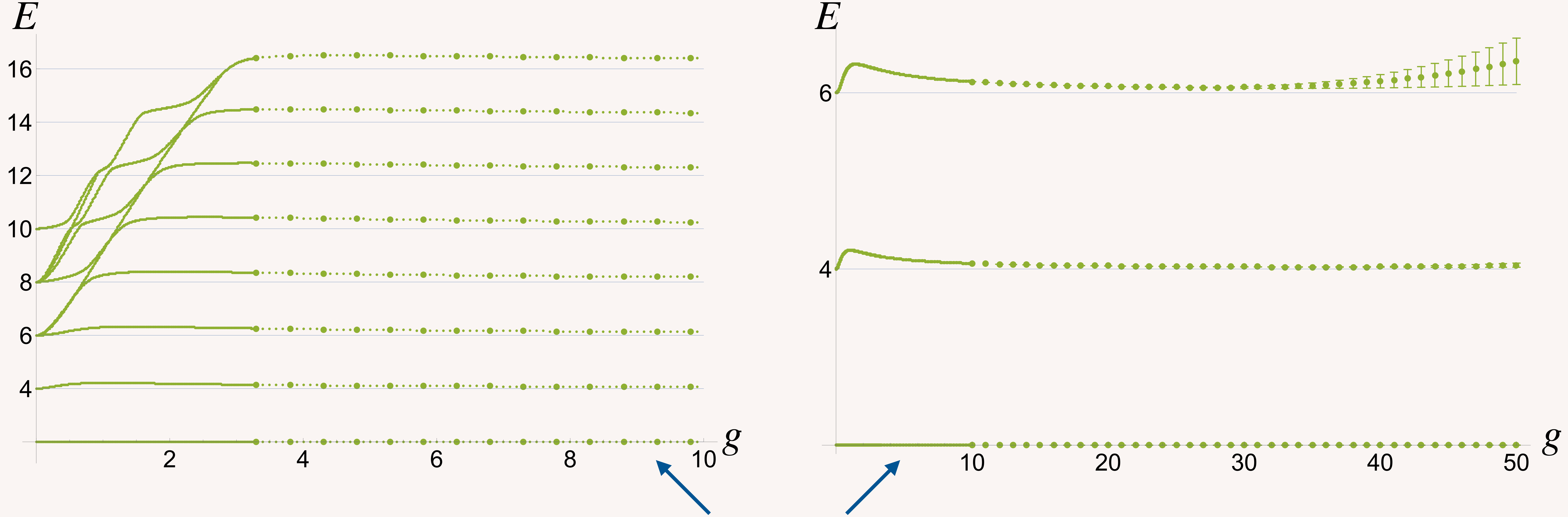


BPS state

Hamiltonian truncation results

$M = 2$

straight lines: analytic prediction at $g \to \infty$

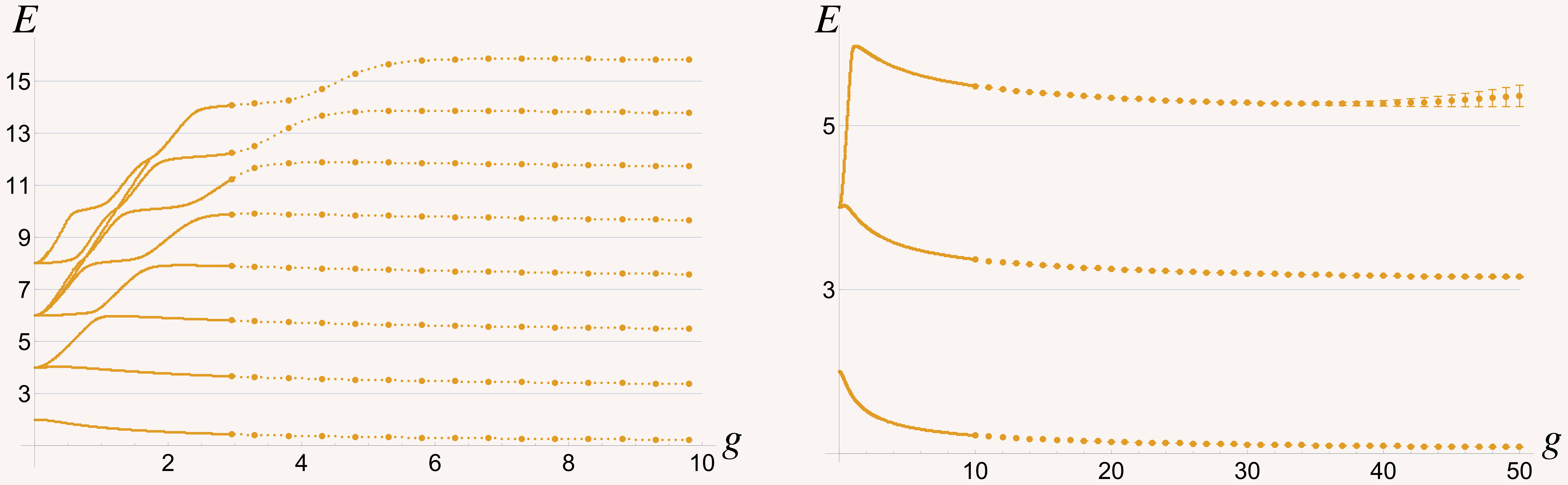


BPS state

Hamiltonian truncation results

$M = -2$

straight lines: analytic prediction at $g \to \infty$



No BPS state