L_2 sensitivity to quantify the interplay of experimental constraints in global analyses

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for the CT collaboration & authors of PRD108, 034029

[plus L. Kotz]

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PDF4LHC meeting

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Use of L_2 sensitivity

- The L_2 sensitivity is a **fast method** to estimate pulls on PDFs by experiments in a PDF fit.
- It can be computed using **LHAPDF grids for Hessian PDFs** and χ^2 **values for the error PDFs.**
- The L_2 sensitivity streamlines comparisons among independent analyses, using the log-likelihood values for the fitted experiments and the error PDFs

- The L_2 sensitivity has been used
  - by CT (in CT18) [PRD 103, 014013 (2021)],
  - by the PDF4LHC21 benchmarking group [J.Phys.G 49, 080501 (2022)],
  - by CT—CJ to estimate deuteron corrections [Eur.Phys.J.C 81, 603 (2021)],
  - by AC & Nadolsky to study constraints on large-x PDFs [PRD 103, 054029 (2021)],
  - by CT, MSHT and ATLASpdf [PRD 108, 034029 (2023)],
  - on xFitter as well (L. Kotz, upcoming),
  - in preparation for CT2X (see M. Guzzi's talk).

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Fast and accessible tool.

L_2 sensitivity — definition

The L_2 sensitivity incorporates both the dependence on the observable on PDF and on the resolving power of the data sets: it is a way of viewing the pulls of all of the experiments used in a global PDF fit, for a particular parton flavor, as a function of a kinematic variable.

Hessian formalism

D error PDFs are used to determine the PDF uncertainty (assuming the probability distribution is approximately Gaussian)

We consider an expansion of a function X of the parameters R in the vicinity of the global χ^2 minimum

$$X(\vec{R}) = X_0 + \sum_{i=1}^{D} \frac{\partial X}{\partial R_i}\bigg|_{\vec{R}=\vec{0}} R_i + \frac{1}{2}\sum_{i,j=1}^{D} \frac{\partial^2 X}{\partial R_i \partial R_j}\bigg|_{\vec{R}=\vec{0}} R_i R_j + \cdots \quad \Rightarrow \quad \delta_{\mathrm{H}} X = |\vec{\nabla} X| = \frac{1}{2}\sqrt{\sum_{i=1}^{D} [X_{+i} - X_{-i}]^2}$$

The symmetric PDF uncertainty is the maximal variation of $X(\overrightarrow{R})$ within the tolerance hypersphere

L_2 sensitivity — definition

The L_2 sensitivity helps visualize the correlation of the χ^2 with the PDF values, for a given experiment or a given flavor (combination).

$$S^{\rm H}_{f,L2}(E) \equiv \frac{\vec{\nabla}\chi^2_E \cdot \vec{\nabla} f}{\delta_{\rm H} f}, \\ = \left(\delta_{\rm H}\chi^2_E\right)\, C_{\rm H}(f,\chi^2_E)$$

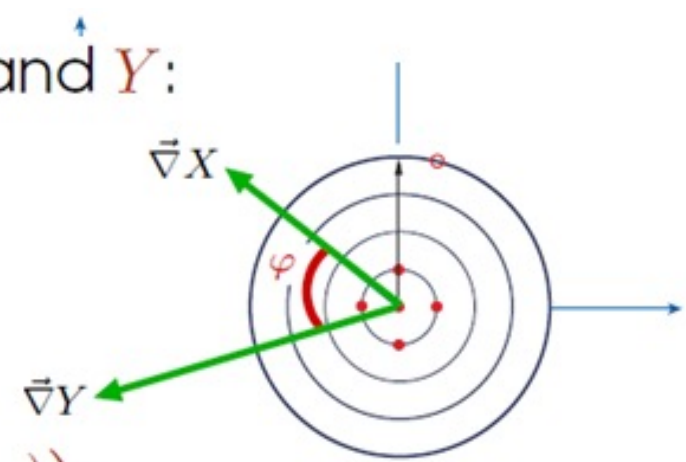
depends on the tolerance criteria

Correlation cosine for observables X and Y:

hep-ph/0110378

$$\cos\varphi = \frac{\vec{\nabla}X\cdot\vec{\nabla}Y}{\Delta X \Delta Y} =$$

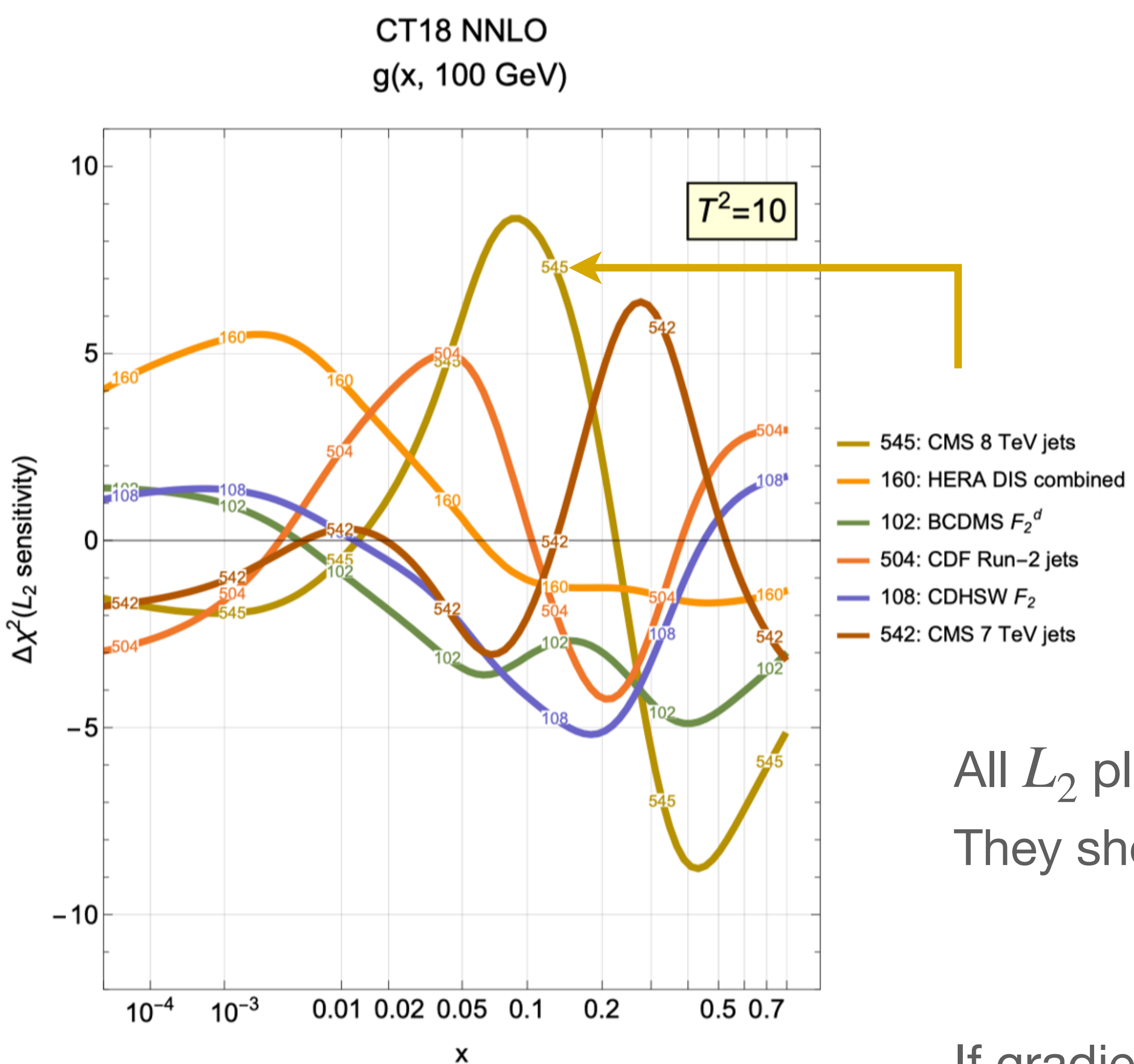
$$\frac{1}{4\Delta X\,\Delta Y}\sum_{i=1}^{N}\left(X_i^{(+)} - X_i^{(-)}\right)\left(Y_i^{(+)} - Y_i^{(-)}\right)$$



(b)
Orthonormal eigenvector basis

here with
$X = f(x, Q^2)$
$Y = \chi^2$

Comparison with Lagrange Multiplier (LM) scans



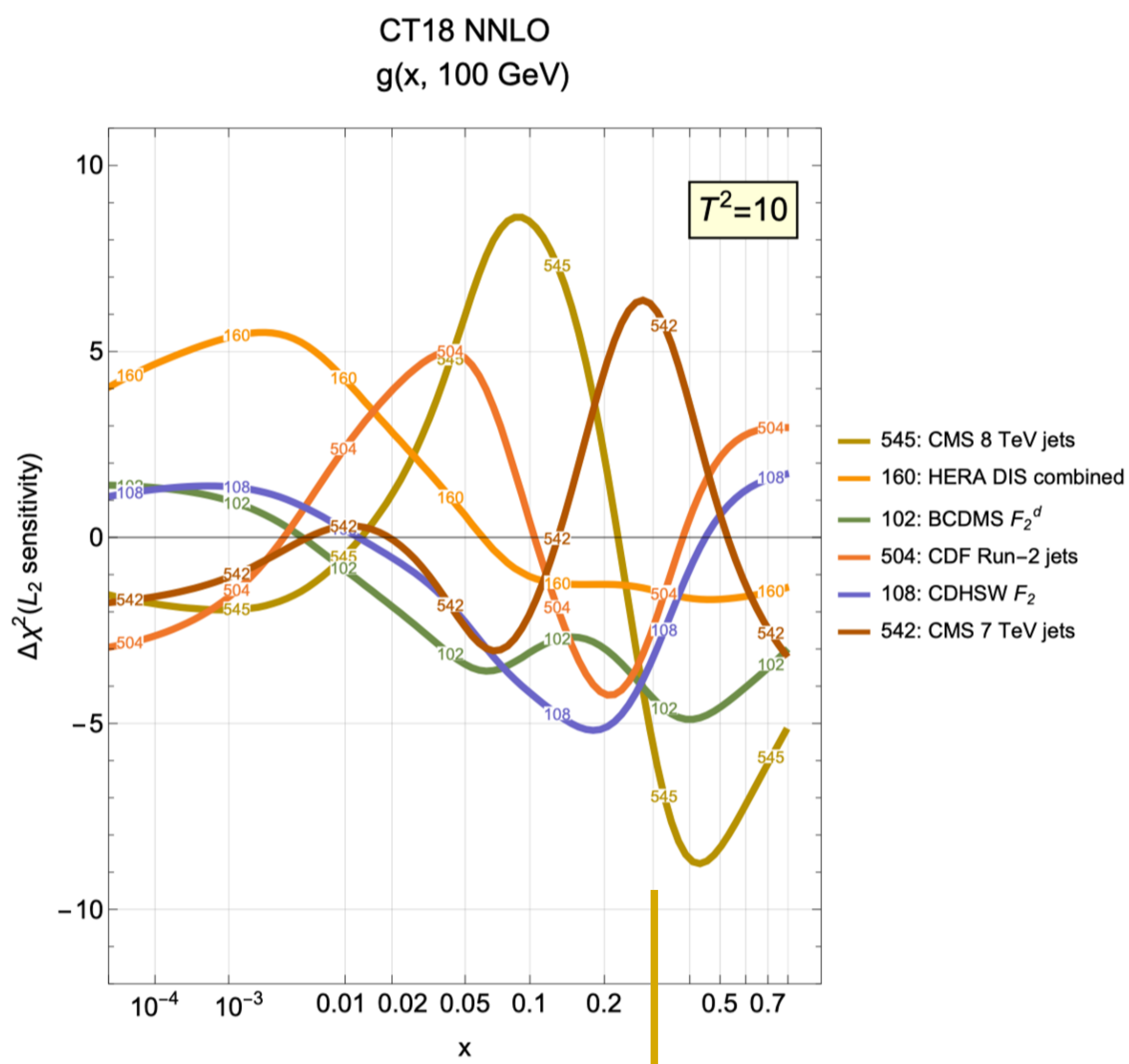
All L_2 plots made for a *global tolerance* of $T^2 = 10$.
They show only the most sensitive experiments.

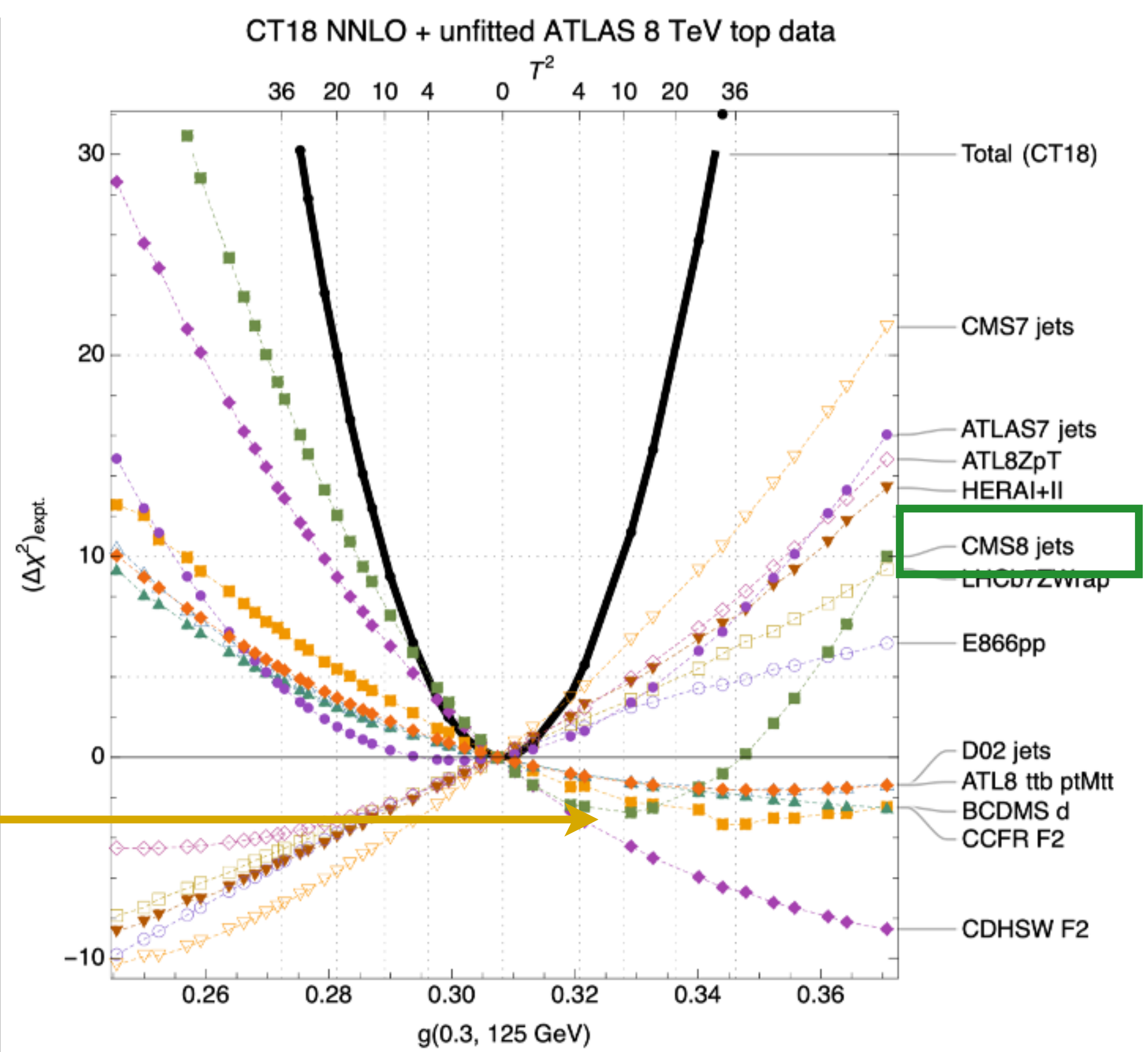
If gradient for descending chisquare is aligned with that of the descending PDF, the correlation is positive

⇒ **positive L_2 indicate a preference for lower PDFs.**

⇒ **negative L_2 indicate a preference for higher PDFs.**

Comparison with Lagrange Multiplier (LM) scans





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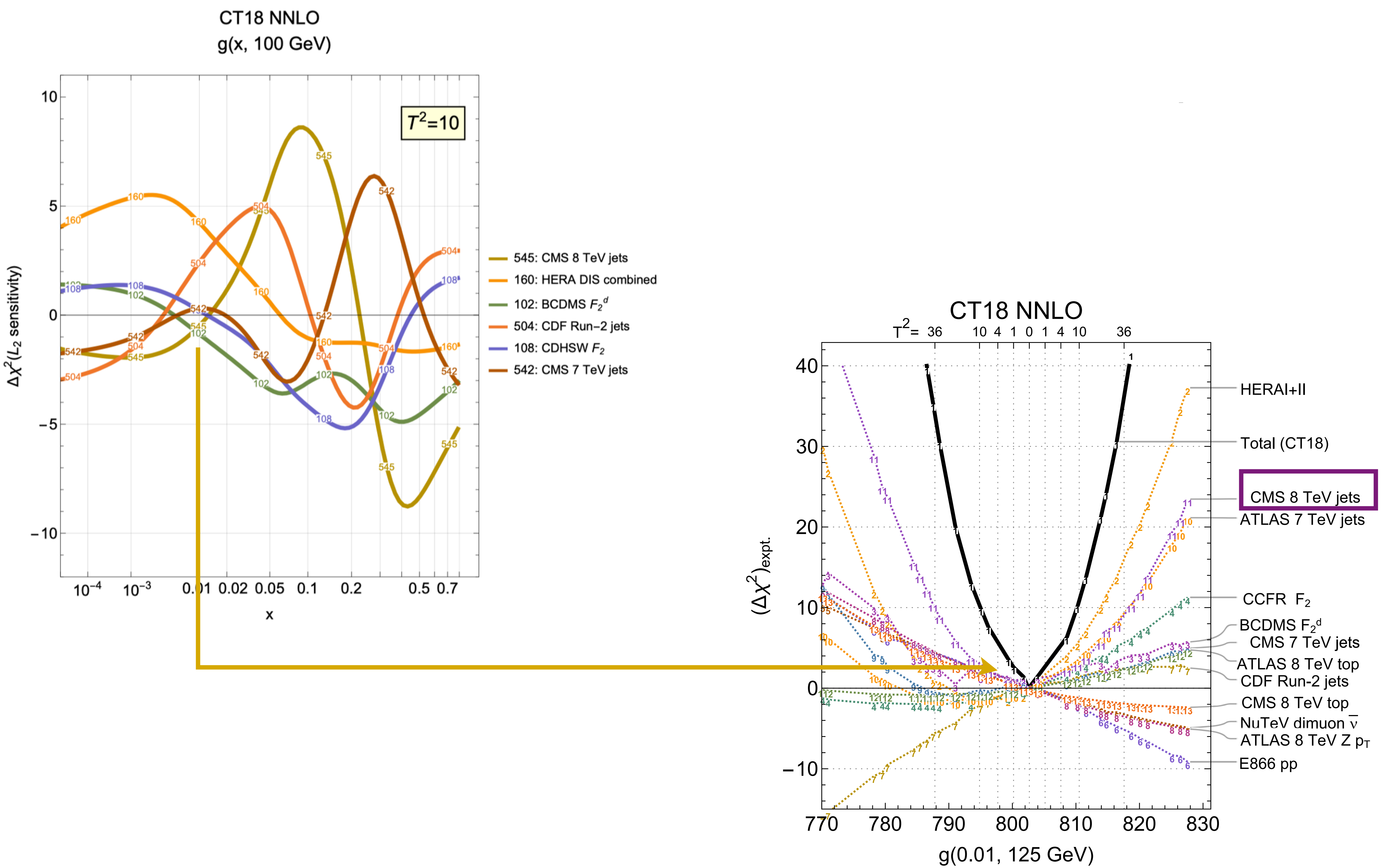
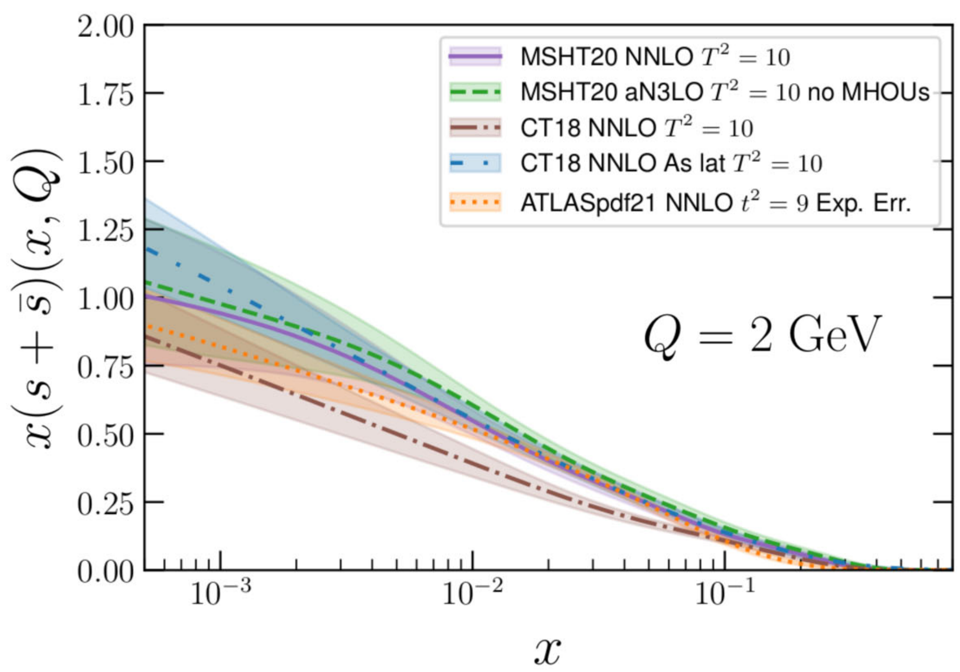


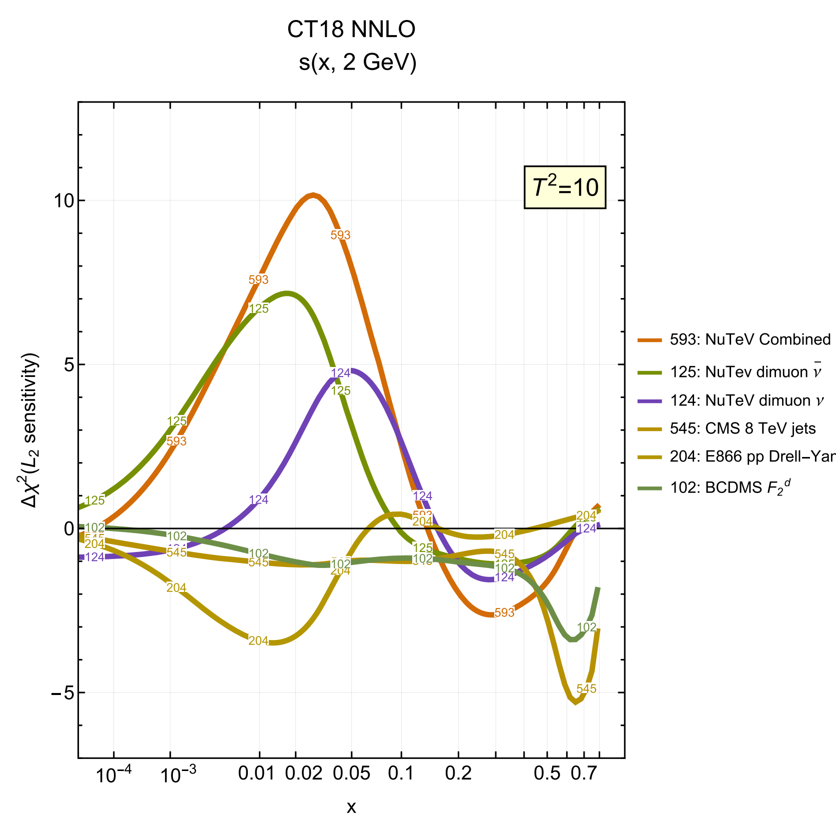
Compare CT18 with and w/o strangeness asymmetry

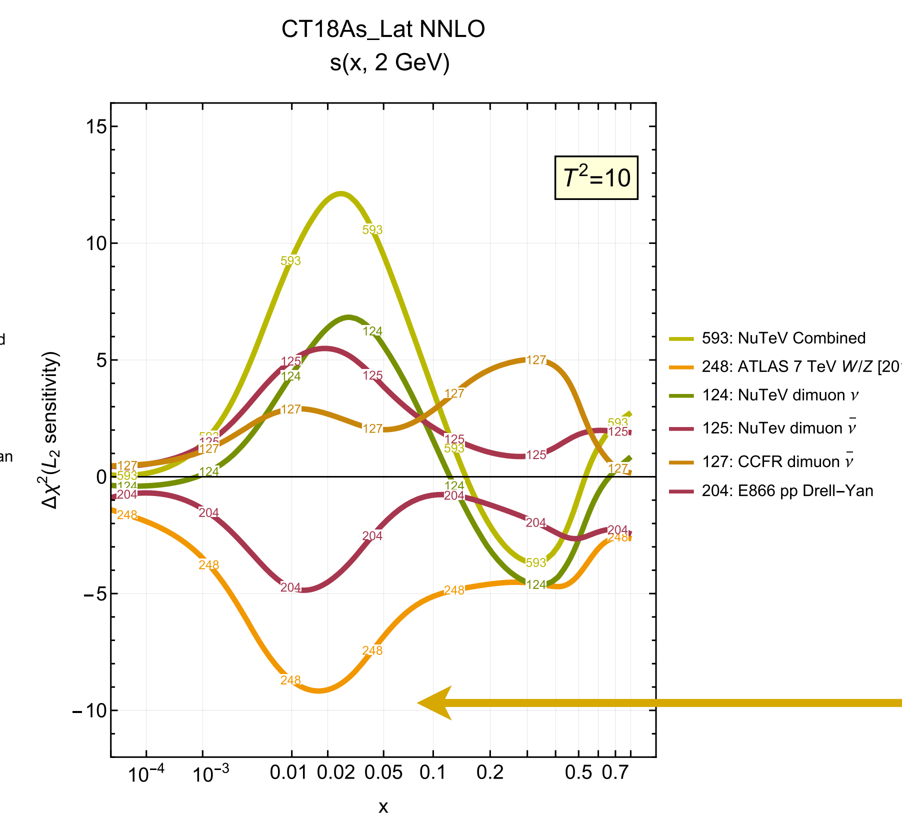


CT18A family includes the ATLAS 7 TeV high precision W, Z data [248]

CT18As subfamily releases $s = \bar{s}$

CT18As_lat includes lattice data for $s - \bar{s}$

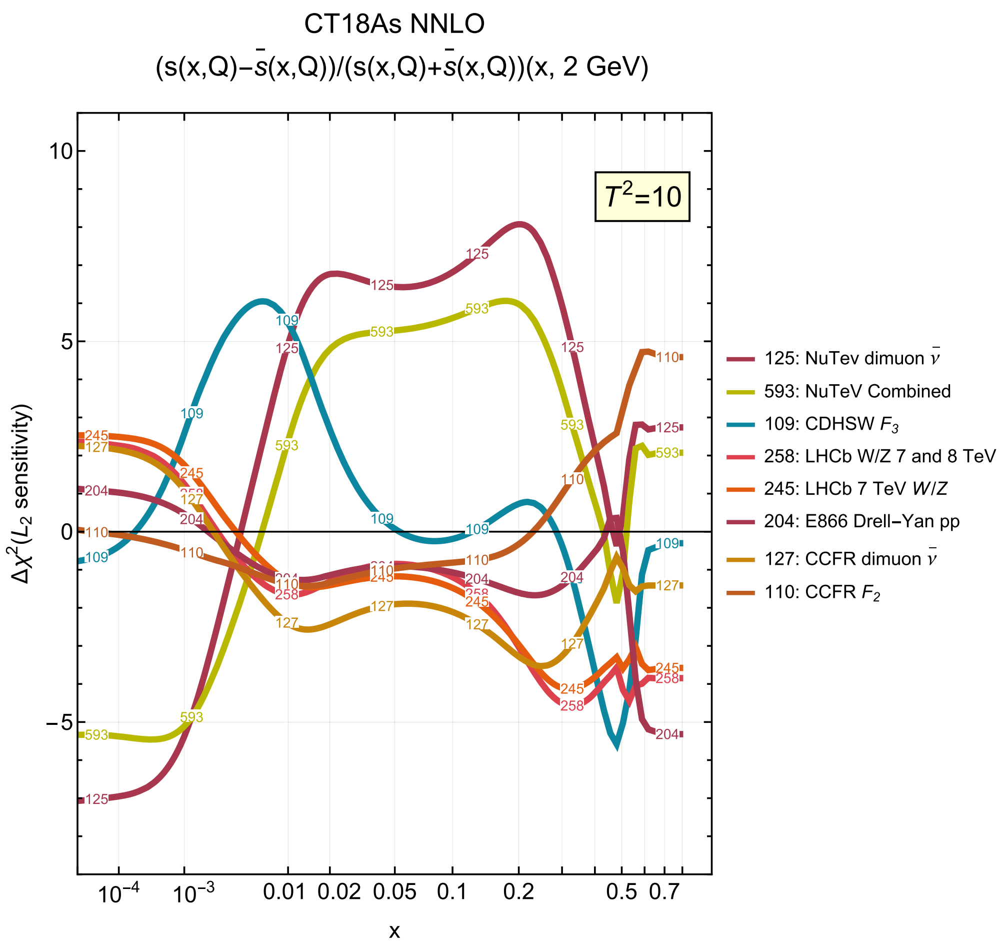


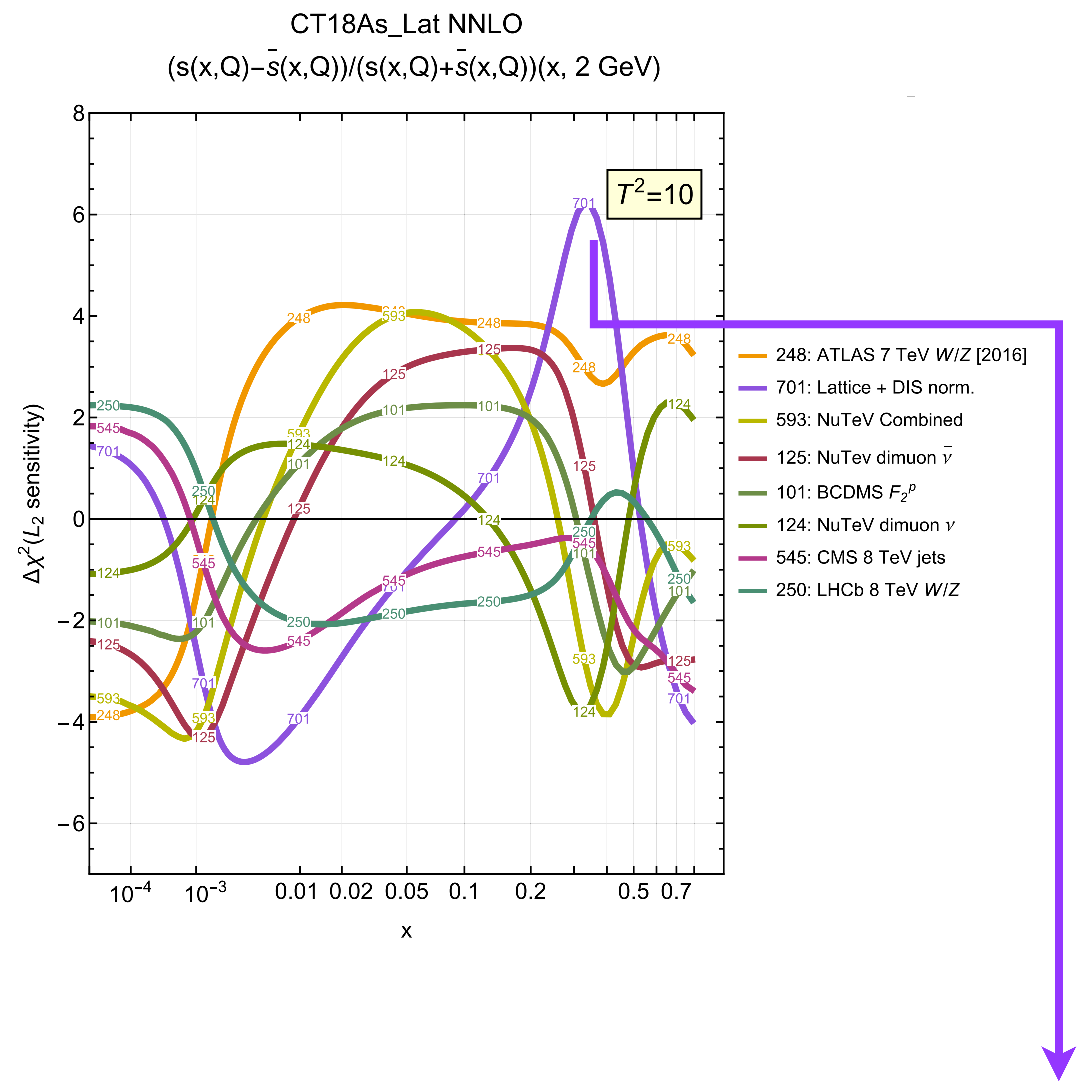


Balance between all experiments as the sum of sensistivities is bounded:

$$0 < \sum_E S_{f,L_2} \ll T^2 < \sum_E |S_{f,L_2}|$$

Strangeness with lattice input



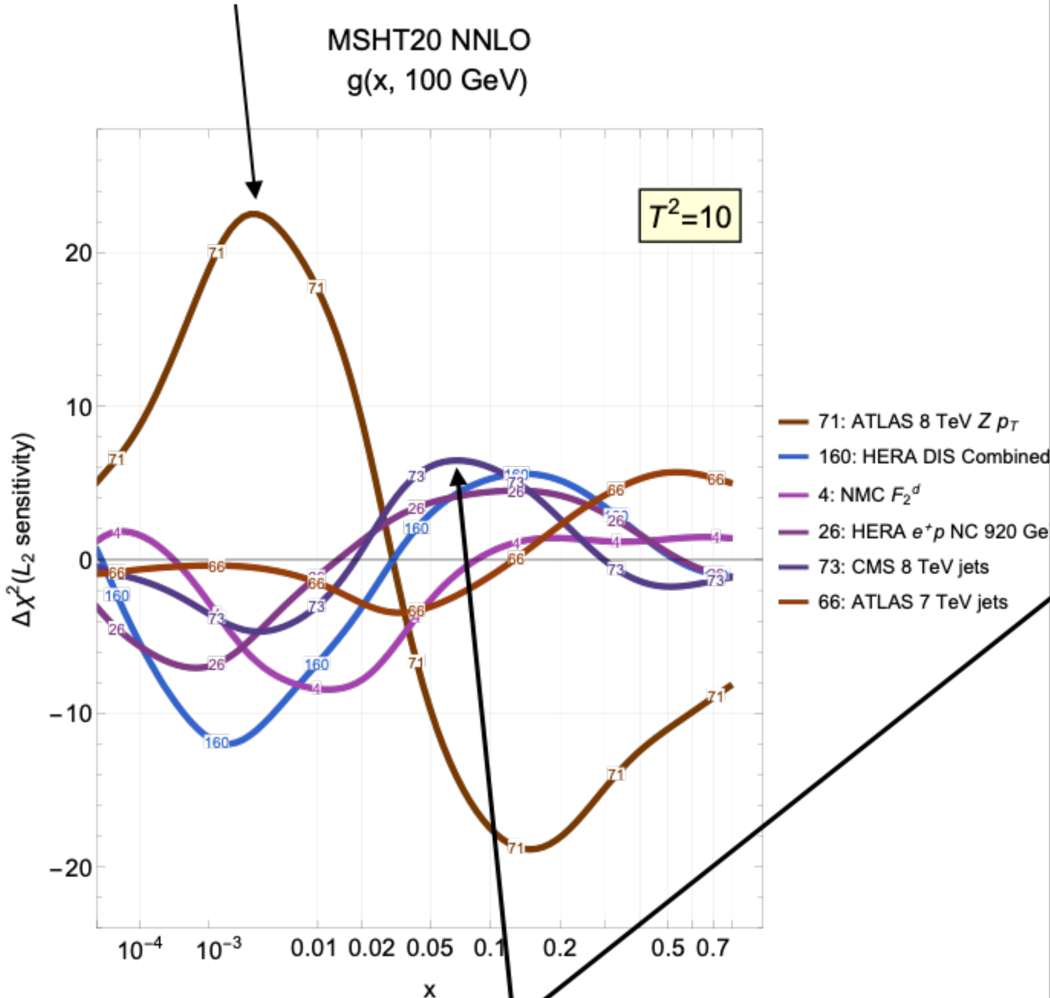


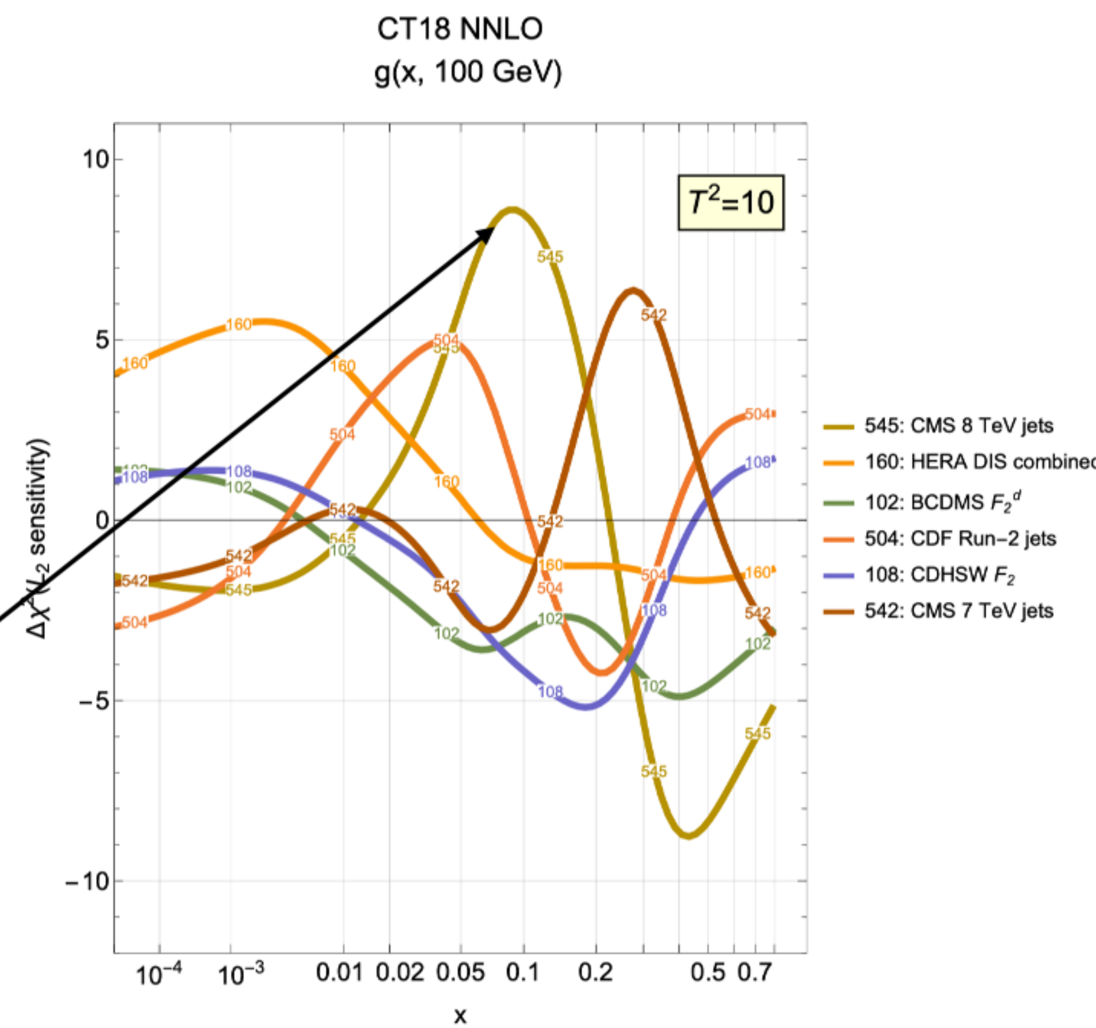
Pattern of pull w/o lattice input changed by the preference for a lower R_s at largish x from the lattice. That positive pull is compensated by a negative one due to net strangeness.

For more lattice-related sensitivity: PDFSense [*Phys.Rev.D* 98, 094030 (2018)]

Comparison among groups with Hessian methodology

Also Z pT data pulls against other experiments at NNLO.
see talk by L. Harland-Lang



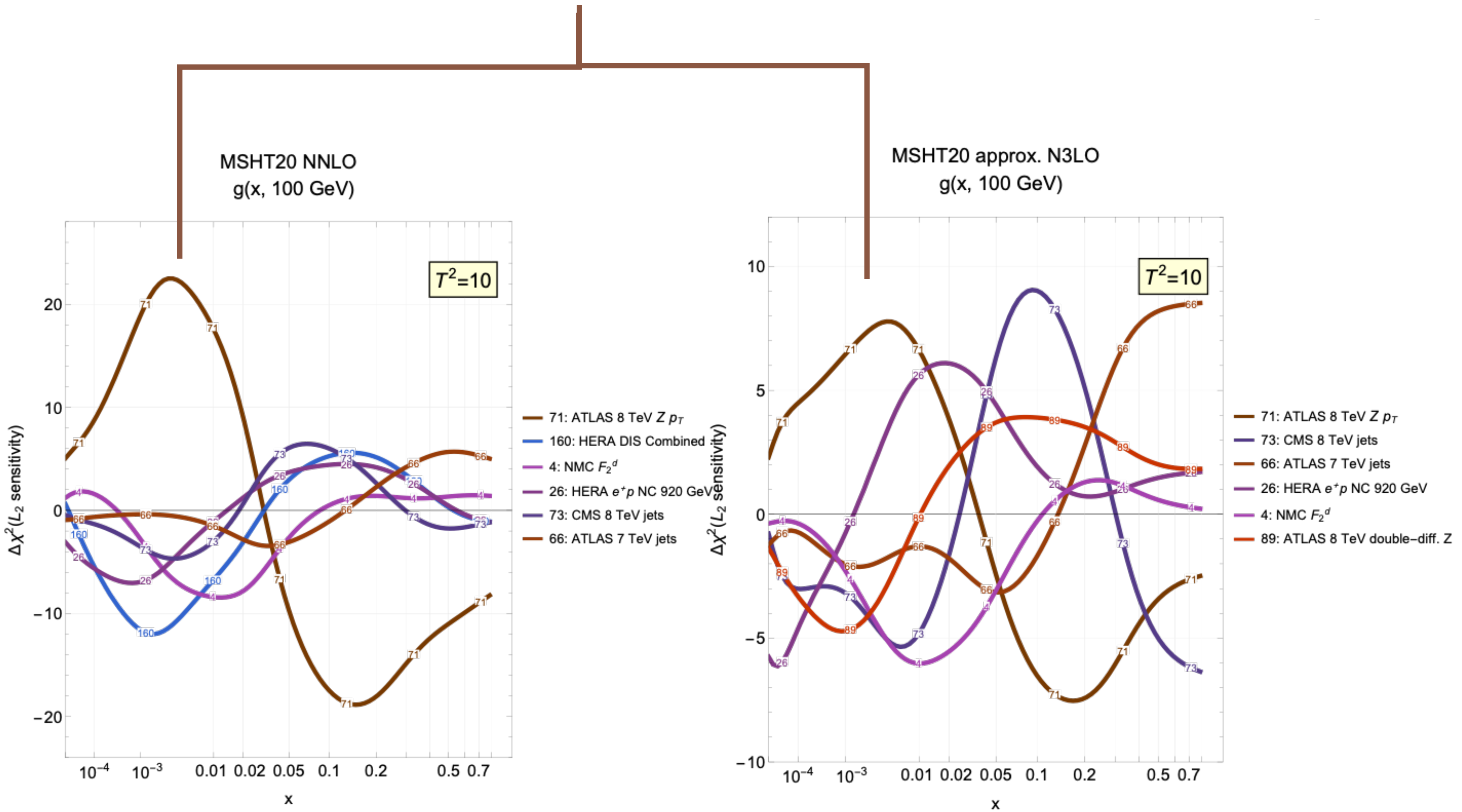


CMS 8 TeV jet data play a similar role as in CT18

ATLAS Z p_T not one of 6 most important experiments (more restrictive kinematic region)

MSHT gluon at NNLO and aN3LO

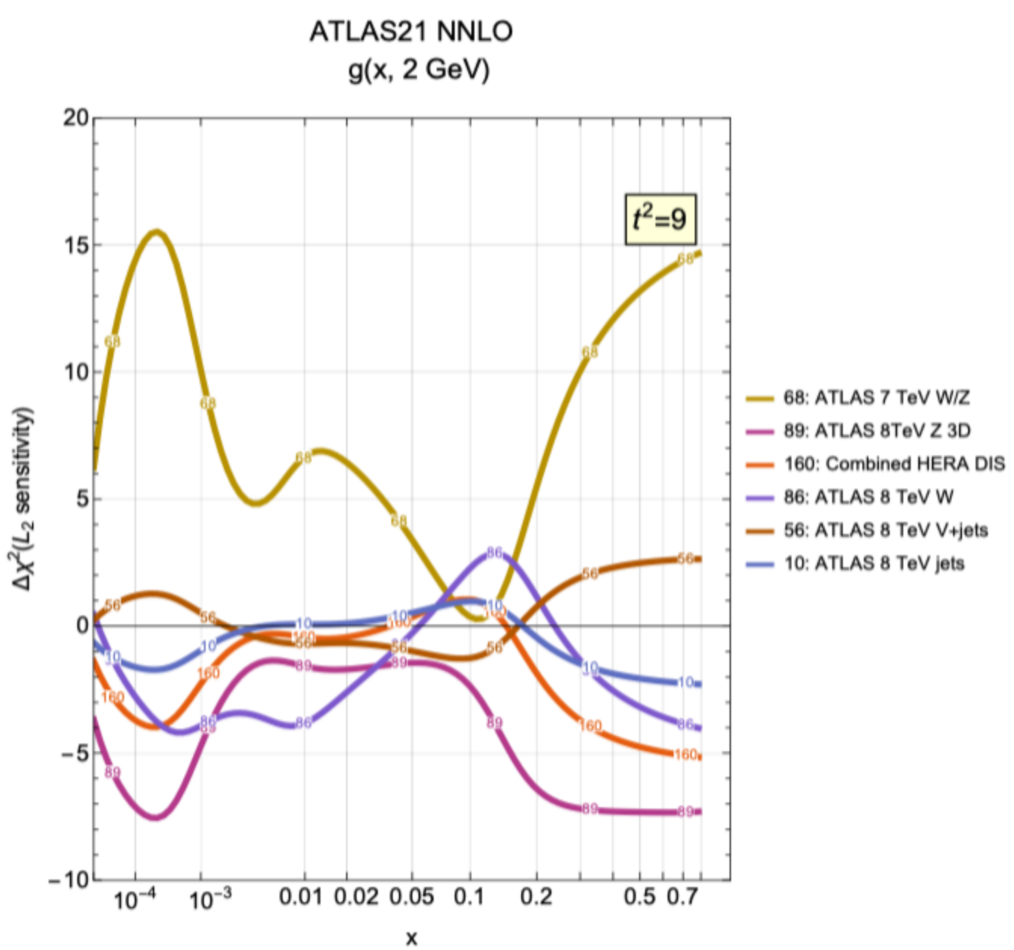
The rôle of ATLAS 8TeV ZpT is reduced when going to aN3LO — sensitivity pattern drastically changed.

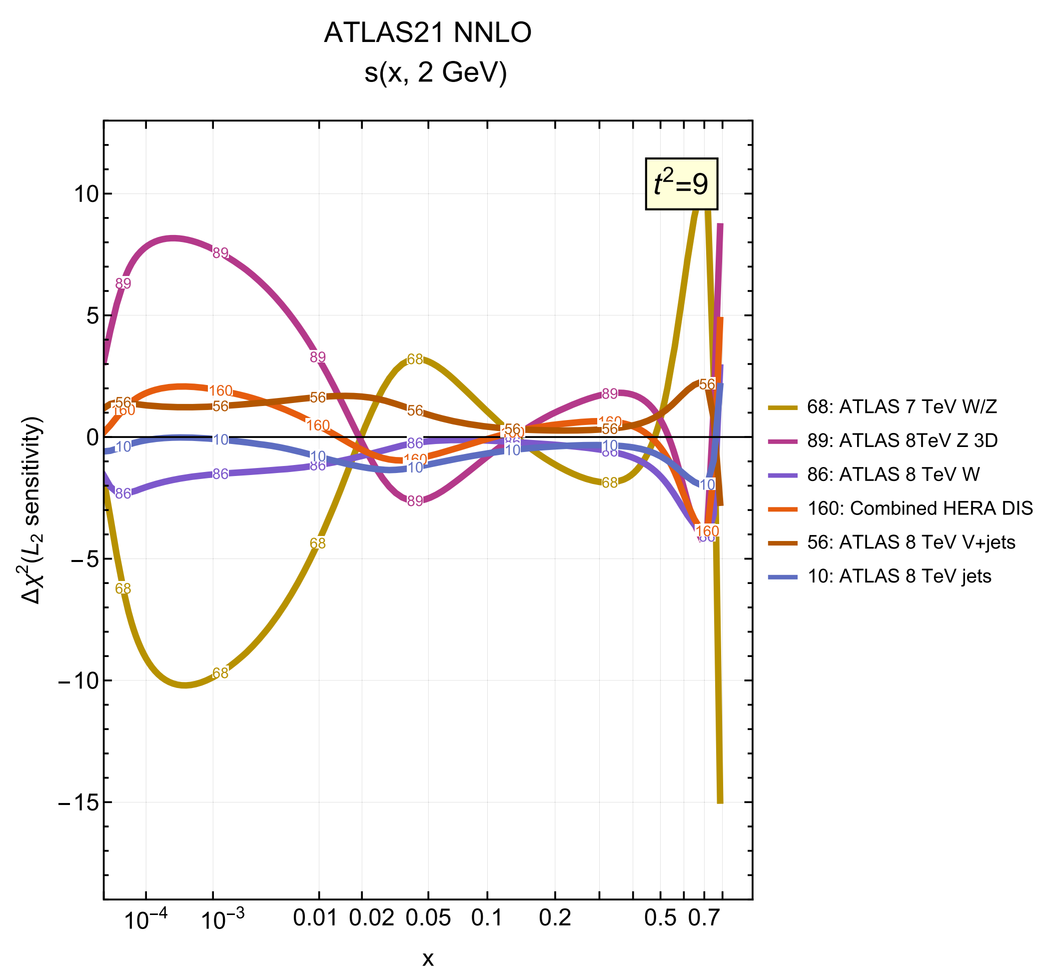


ATLASpdf21

ATLAS PDF fits are based on a more limited set of data, with HERA inclusive as the backbone

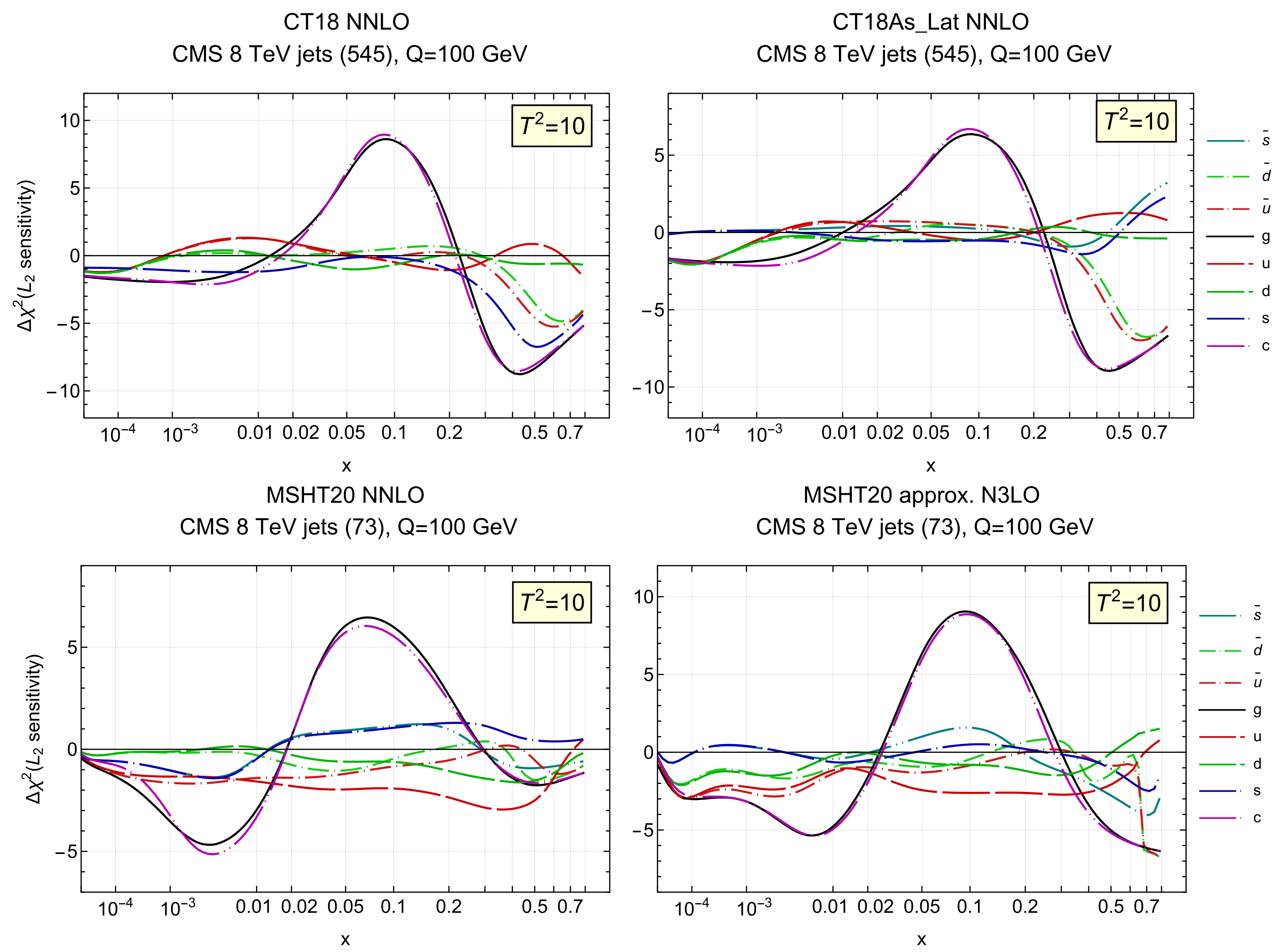
Full information on correlated systematic sources of uncertainty used (not available to to other PDF fits)





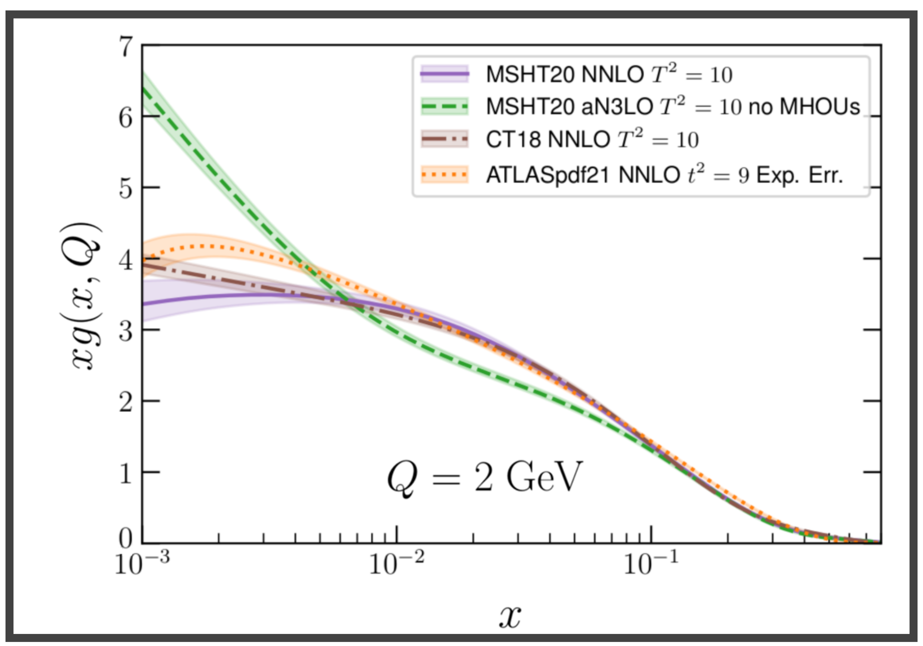
L_2 sensitivities per experiment — global comparisons

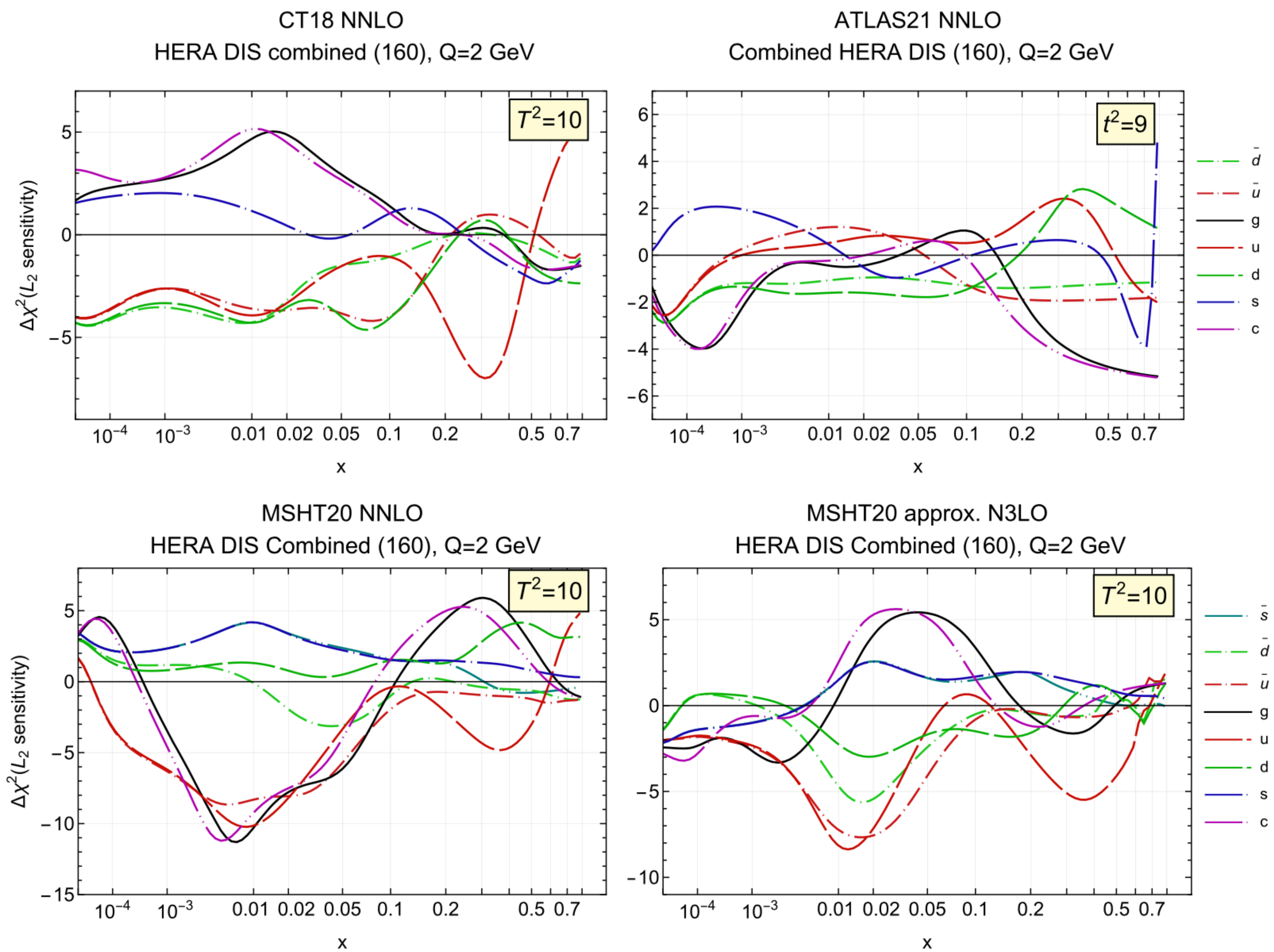
Some data sets lead to similar patterns among all fits and groups!



L_2 sensitivities per experiment — global comparisons

… but most don't: L_2 patterns may help understand the differences in the PDF sets.

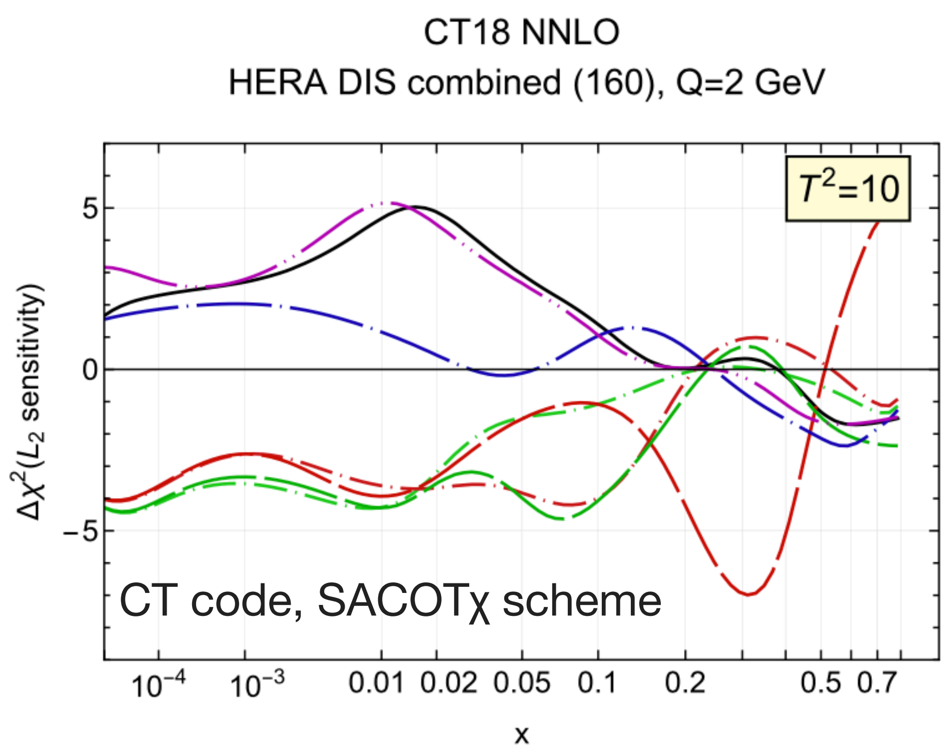


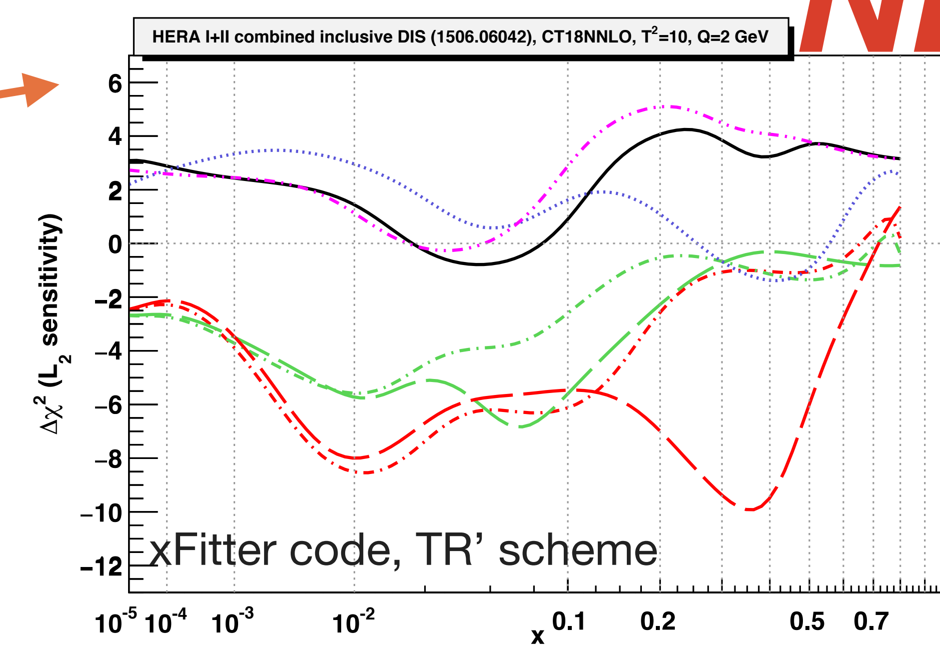


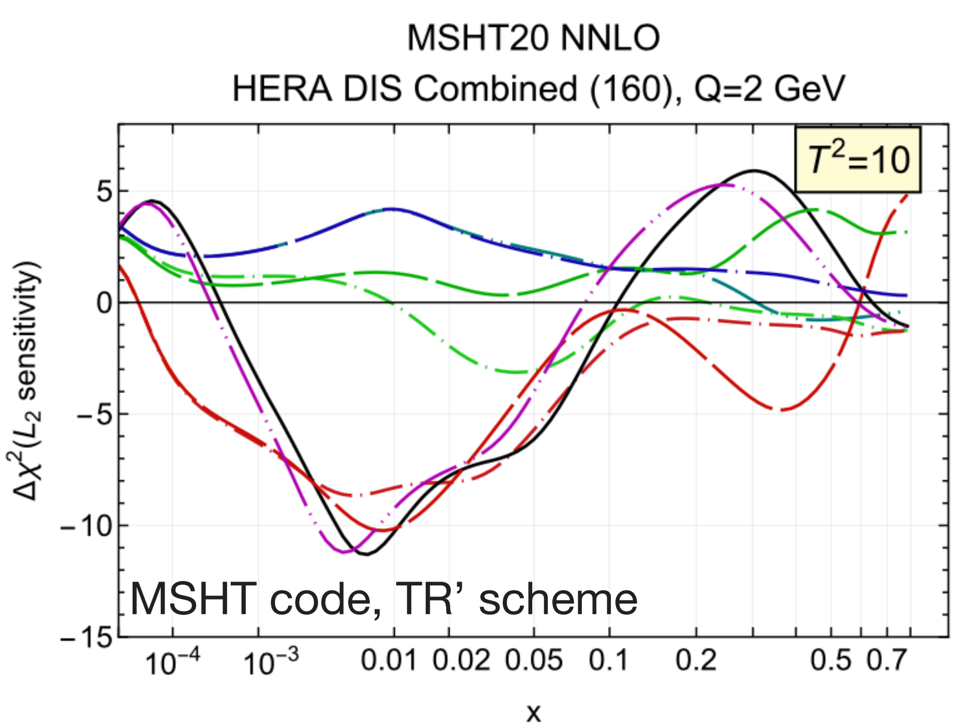
L_2 sensitivities evaluated in xFitter

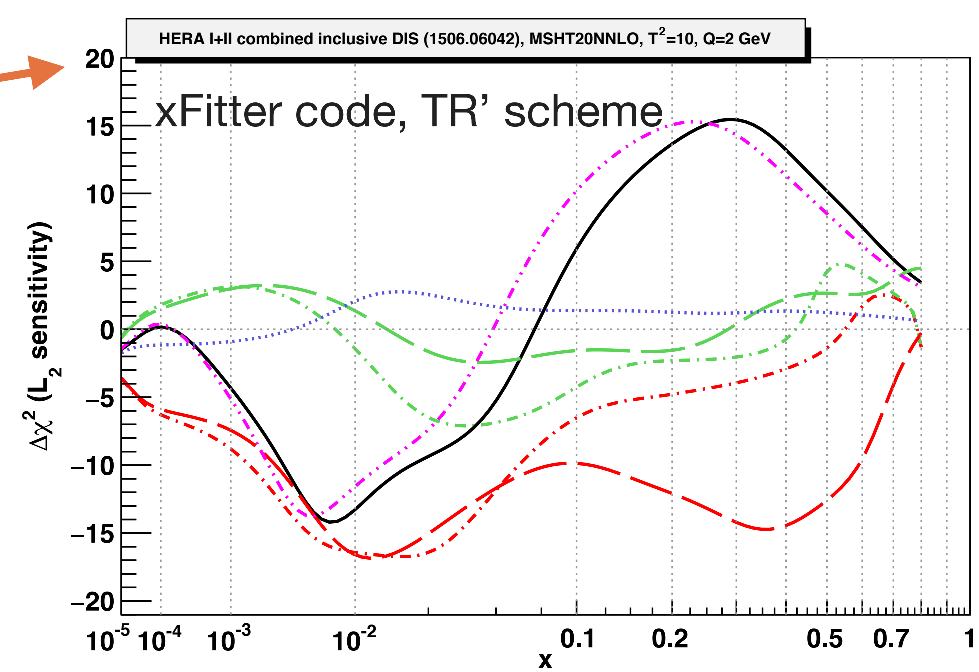
from Lucas Kotz (SMU)

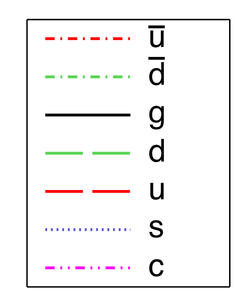
NEW









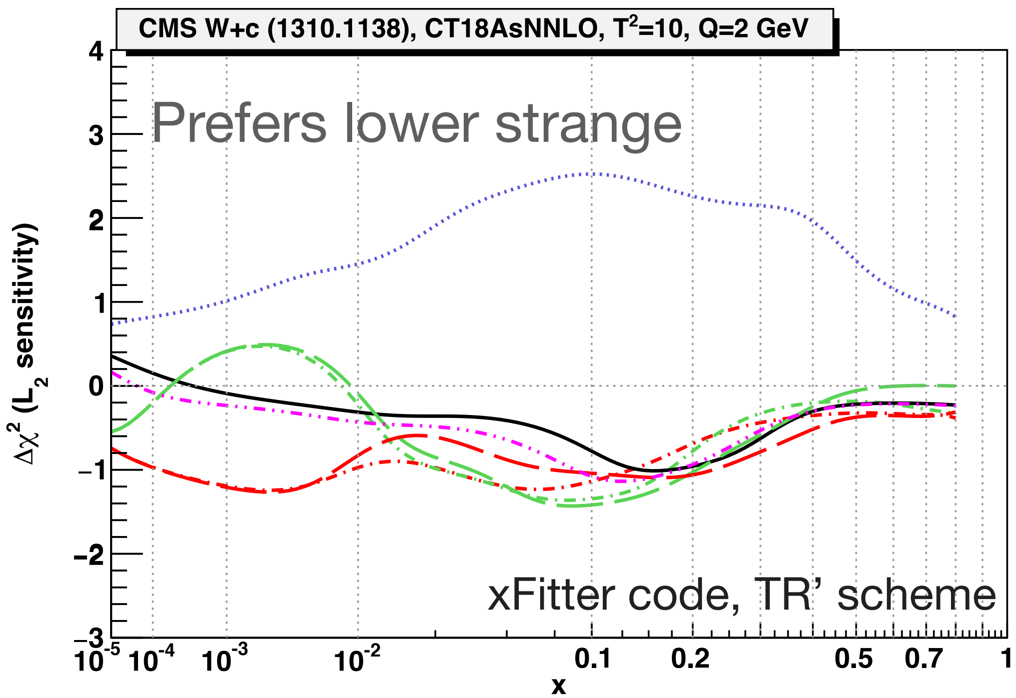


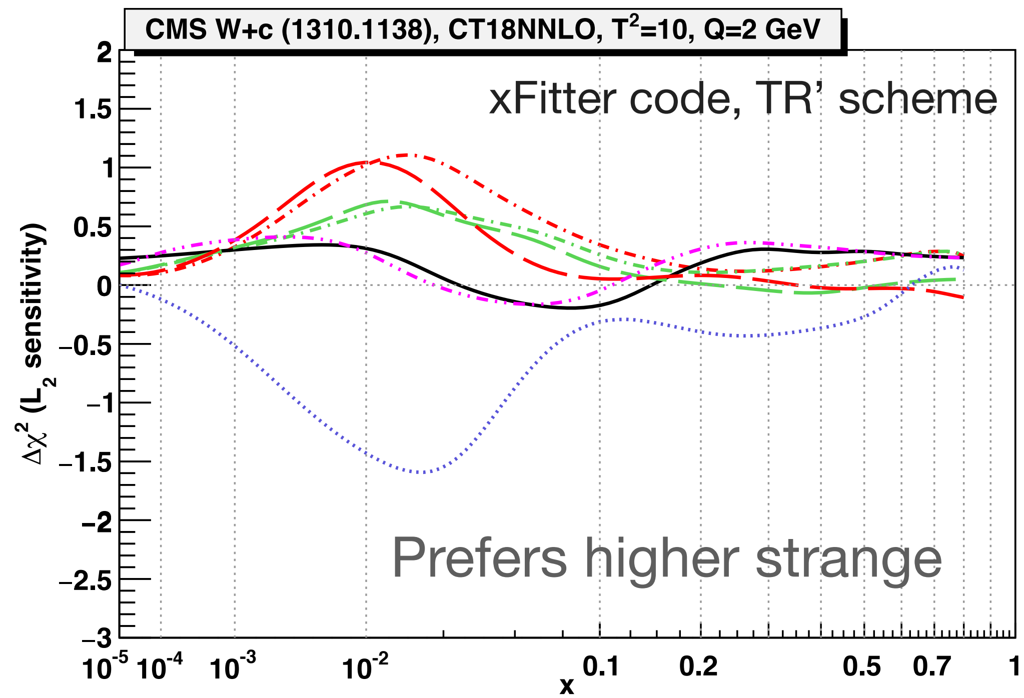
L_2 sensitivities evaluated in xFitter

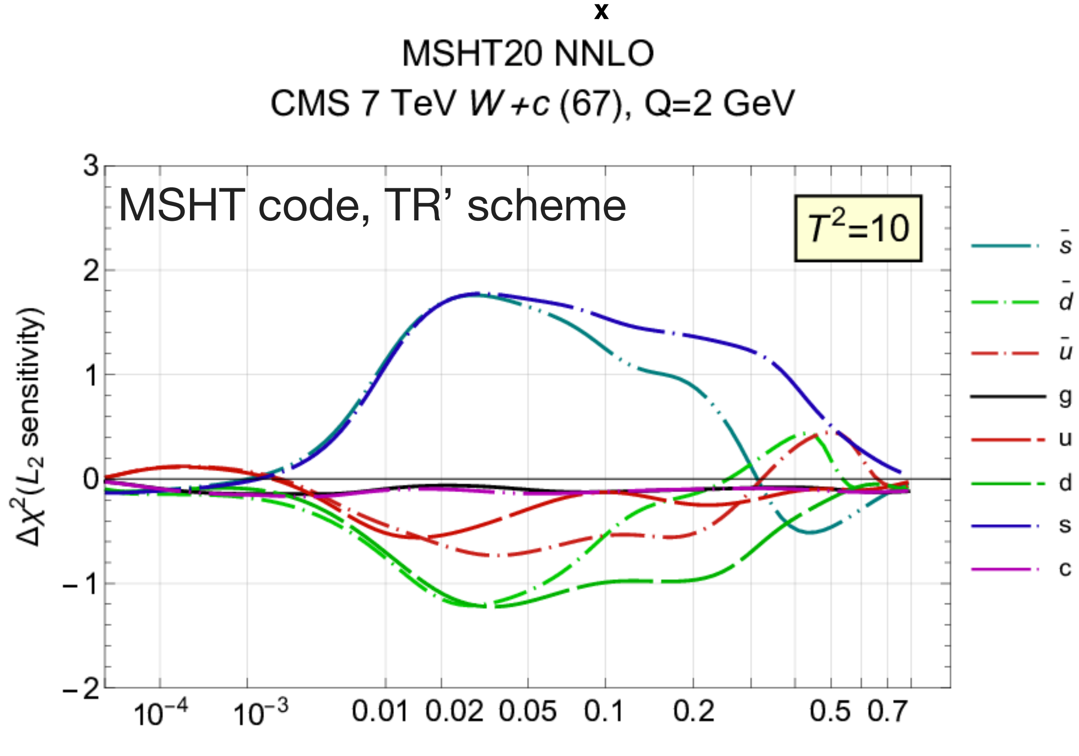
from Lucas Kotz (SMU)

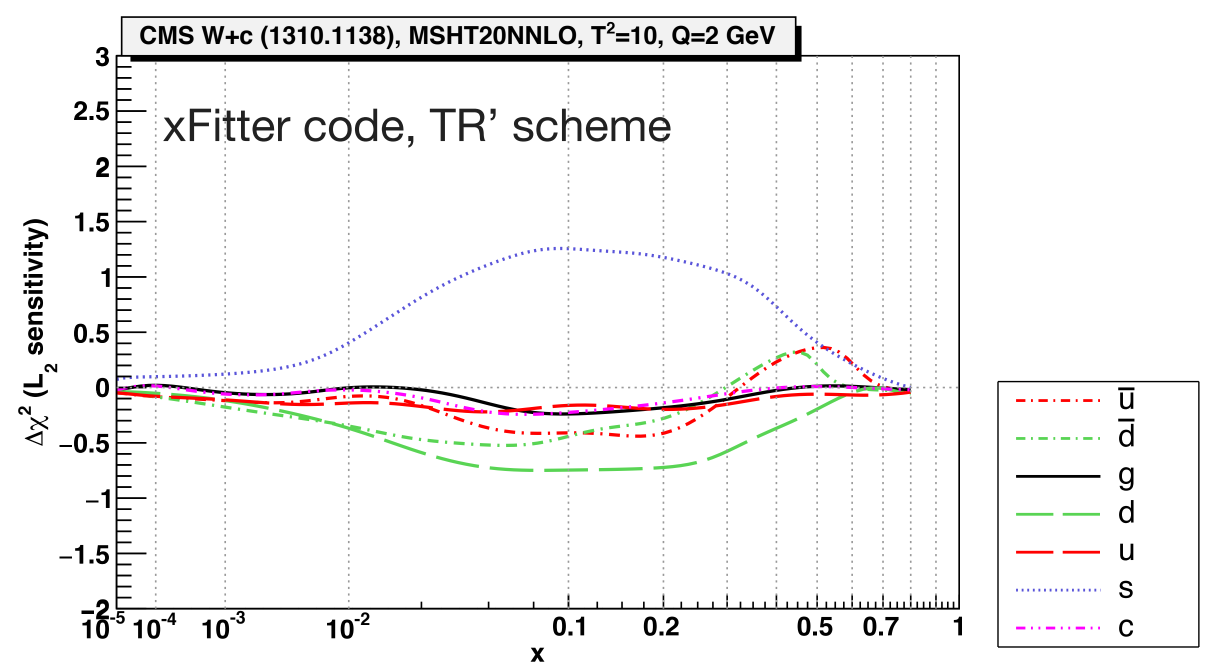
Sensitivity for a set that is not included in CT analysis but is in MSHT

NEW









Conclusions

The goal to achieve precision and accuracy in PDF determination is ultimately related to the shape of the likelihood in the multidimensional space of acceptable solutions.

Approaches to determine the shape of the likelihood vary from Hessian to MC, methodological choices,…

see talk by P. Nadolsky

The L_2 sensitivity performs a likelihood-ratio test *after* the fit:

$$P(D|a) \propto \exp\left(-\frac{1}{2\,T^2}(\chi^2(D,a) - \chi_0^2)\right)$$

The technique is fast and accessible to many users — requirement: Hessian PDF grids and χ^2 values.

More plots available at the following links:

https://www.physics.smu.edu/nadolsky/work/pdf4lhc21/L2sens/index2.html
https://www.physics.smu.edu/nadolsky/work/pdf4lhc21/L2sens/index3.html

BACKUP SLIDES

Likelihoods in PDF global analyses

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Approaches to determine the shape of the likelihood vary from Hessian to MC, methodological choices,…

see talk by P. Nadolsky

Likelihood probability

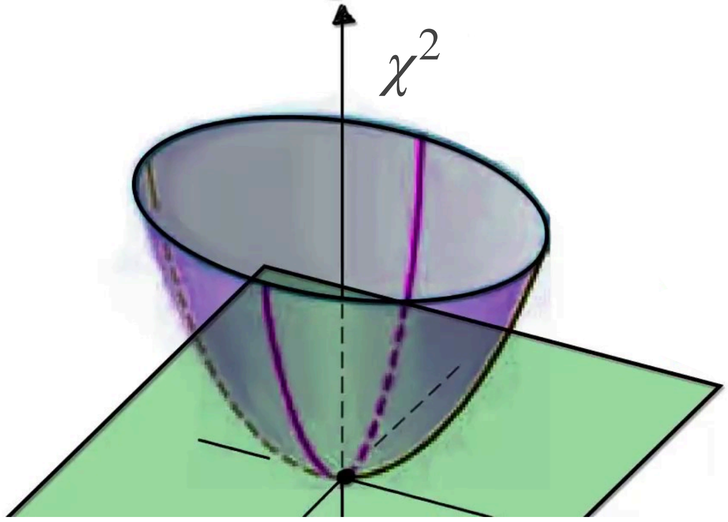
$$P(D|a) \propto \exp\left(-\frac{1}{2}\chi^2(D,a)\right)$$

Posterior probability

$$P(a|D) \propto P(D|a)\,P(a)$$

$$\Leftrightarrow \exp(-\chi^2_{\rm aug}/2) \propto \exp(-\chi^2/2)\,\exp(-\chi^2_{\rm prior}/2)$$

$$\Rightarrow \chi^2_{\rm aug} = \chi^2 + \chi^2_{\rm prior}$$



[Lepage et al., NPB Proc.Suppl.106(2002) 12-20]

Likelihoods ratios — outside-the-fit test of PDFs

On which basis are PDFs accepted or rejected?

Likelihood ratios:
two replicas can be ordered according to their relative likelihood or relative prior.

$$\frac{P(T_2|D)}{P(T_1|D)} = \frac{P(D|T_2)}{P(D|T_1)} \times \frac{P(T_2)}{P(T_1)}$$

$\equiv r_{\text{posterior}}$ $\equiv r_{\text{likelihood}}$ $\equiv r_{\text{prior}}$

aleatory epistemic + aleatory probabilities

Prior: replica can be discarded based on $P(T_2) < P(T_1)$ even for $r_{likelihood} \sim 1$

Likelihood: replica can be accepted based on $r_{likelihood} = \dfrac{P(D\,|\,T_2)}{P(D\,|\,T_1)} \sim 1$ when $P(T_2) \sim P(T_1)$

Likelihoods in PDF global analyses

Likelihood probability with tolerance prescription

$$P(D|a) \propto \exp\left(-\frac{1}{2\,T^2}(\chi^2(D,a) - \chi_0^2)\right)$$

Through the likelihood-ratio test, PDFs with a low, but not the lowest, χ^2 can be acceptable with some probability determined by the tolerance prescription

Tolerance historically emerged from tension among experimental data.

Data from two measurements can both be very precise, but the result of adding both to the PDF fit can be an increase in the PDF uncertainty if the data are in tension with each other.

Beyond the $\Delta\chi^2 = 1$ criterion —CT (Tier 2 penalty) and MSHT (dynamic tolerance)

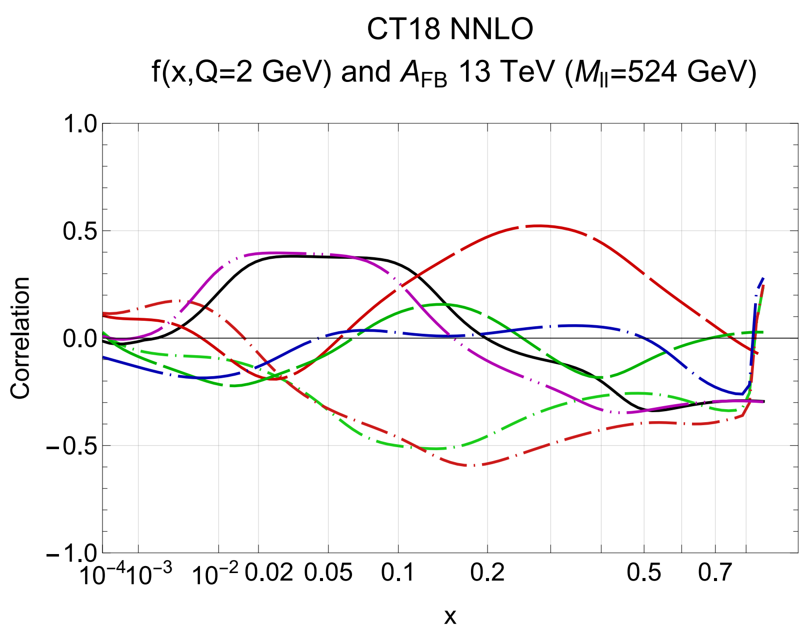
For data from an experimental measurement to influence the PDF fit in a particular region of x and Q^2, two conditions usually must be met:

1. the parton-level dynamics underlying the measurement must substantially depend on a particular PDF
2. the measurement must have sufficient resolving power to nontrivially contribute to the likelihood function

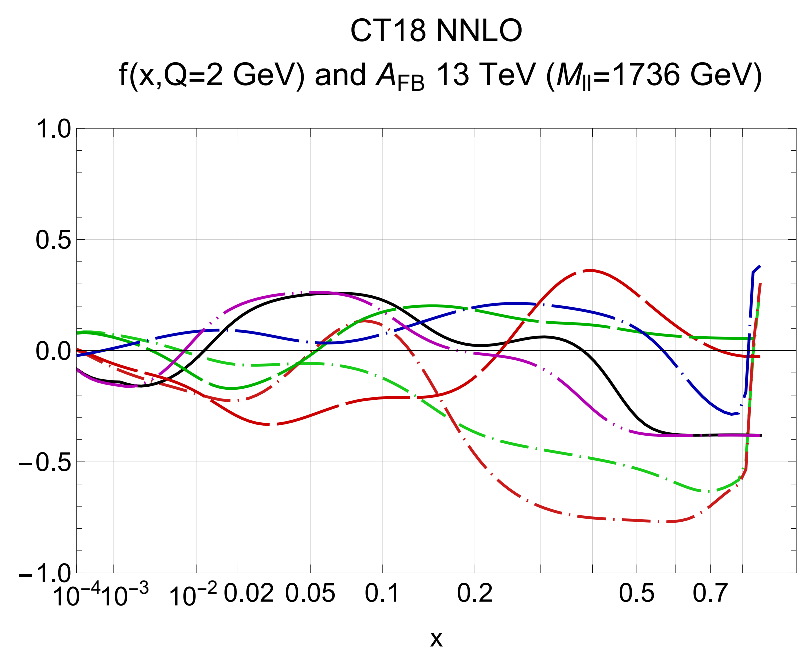
Forward-backward asymmetry

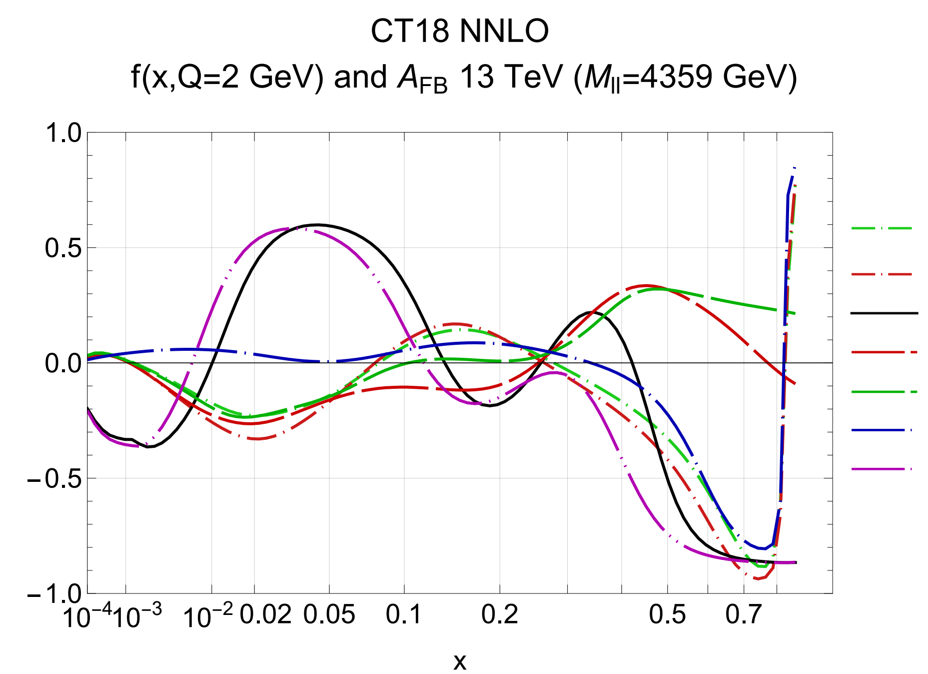
NEW:
based on predictions from Fu, Hou, Yuan, et al.

Drell-Yan backward-forward dilepton production is sensitive to light sea and gluon for increasing M_{ll}.



Strong anti-correlation of AFB with $\bar{u}$ and $\bar{d}$ (and gluon) at large x for increasing M_{ll}.





PRELIMINARY

Growing correlation of AFB with gluon at $x < 0.2$.

see also [Ball et al, EPJC 2022 82]

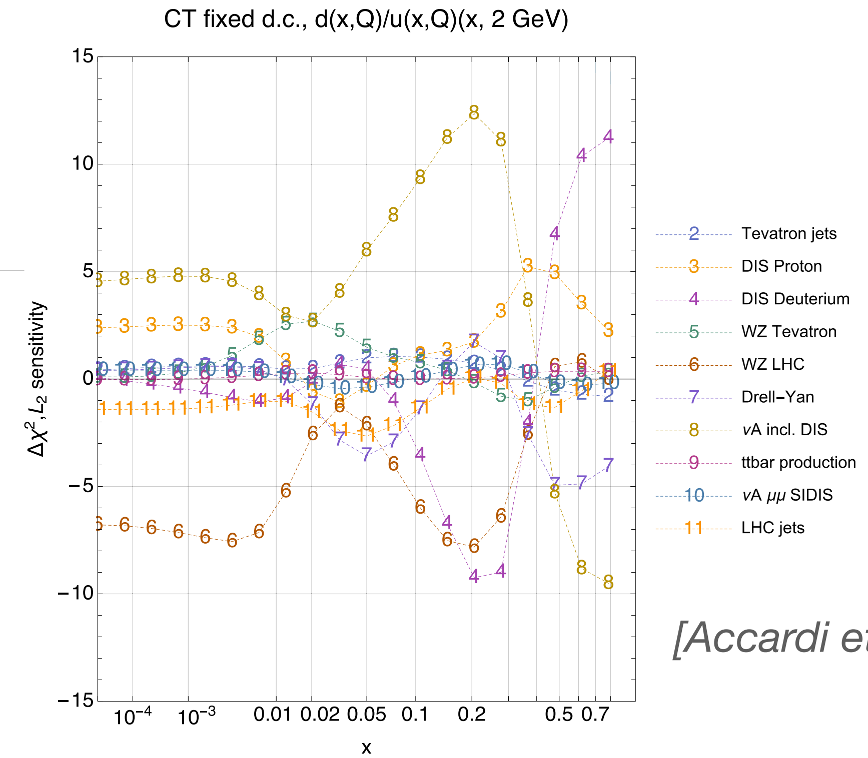
Toward lower Q^2

Mid-Q^2 analyses encounter additional radiative contributions:

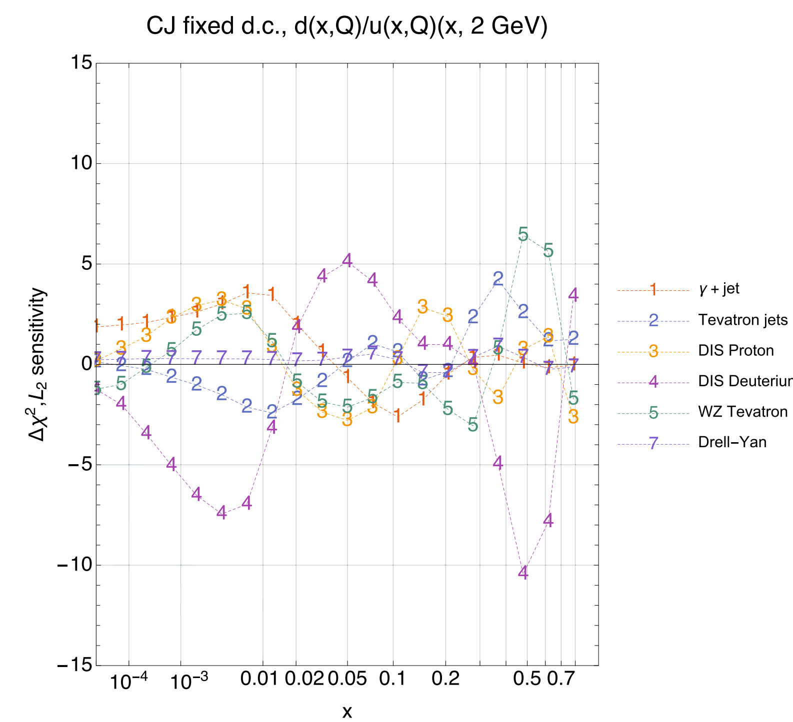
- target mass corrections
- higher-twist corrections — $\mathcal{O}(M^2/Q^2)$
- nuclear corrections

Large-x PDFs determined from high Q^2 offer a possibility to systematically test the leading-power PDFs toward lower Q^2.

CT has studied the impact of various corrections, by analyzing CT vs. CJ (highlight on deuteron corrections), or examining the quark counting rules at mid-Q^2.



[Accardi et al, EPJC81]



Pulls affected by cuts, *e.g.,* on deuteron data sets

L_2 sensitivity shows the correlation between a given PDF configuration and objective function.
Pulls on χ^2 when $f(x) \to f(x) + \Delta f(x)$.

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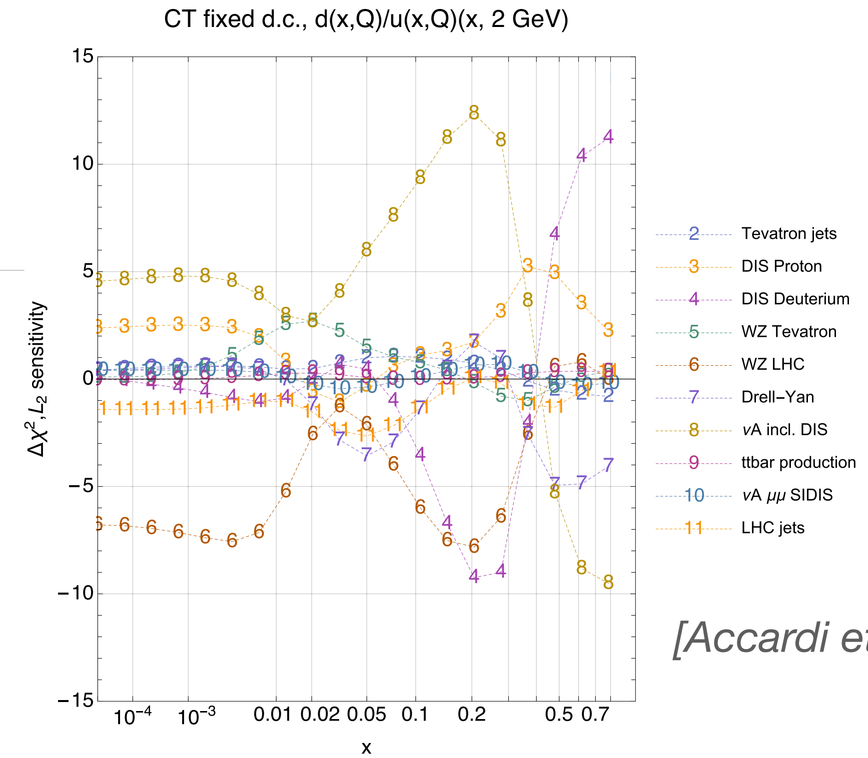

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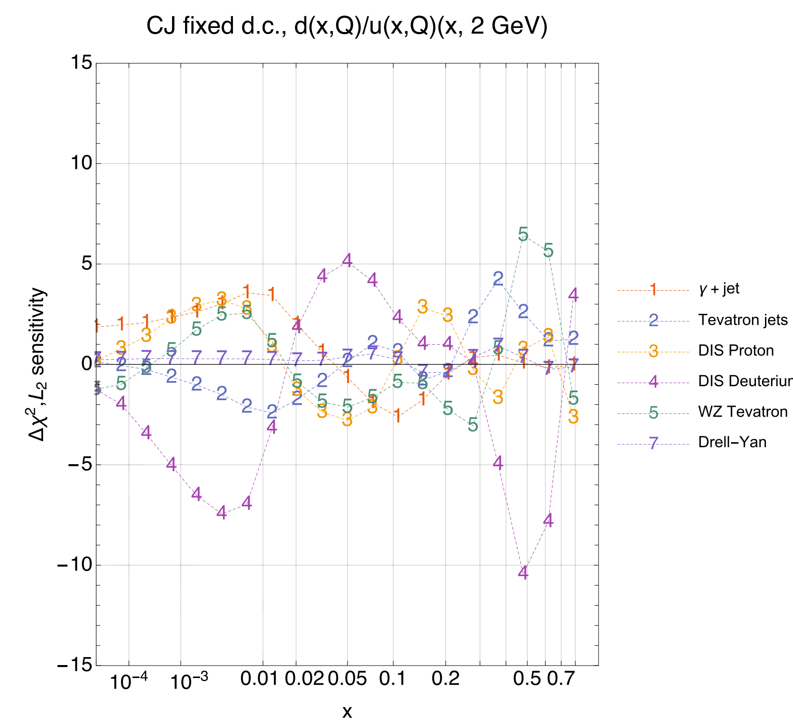

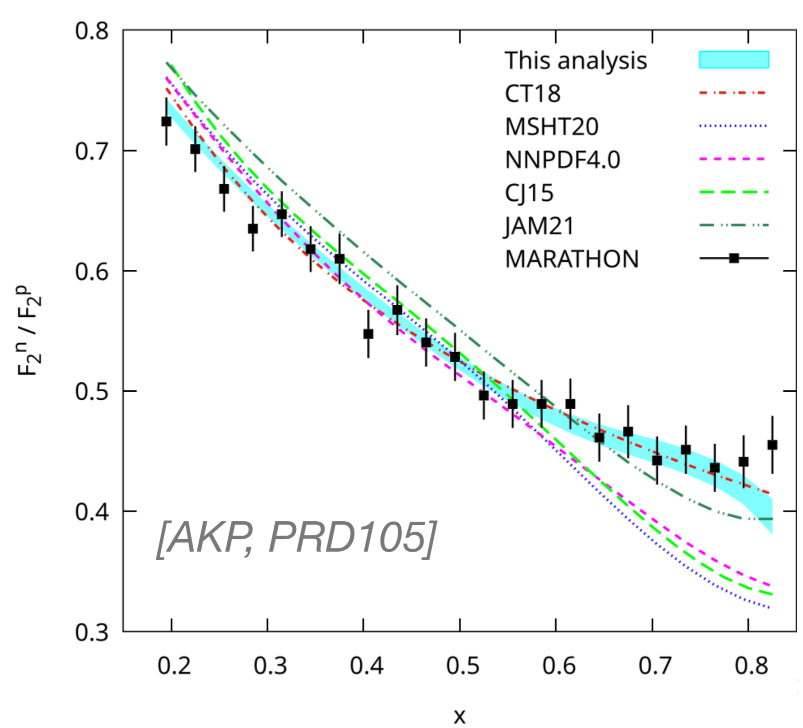

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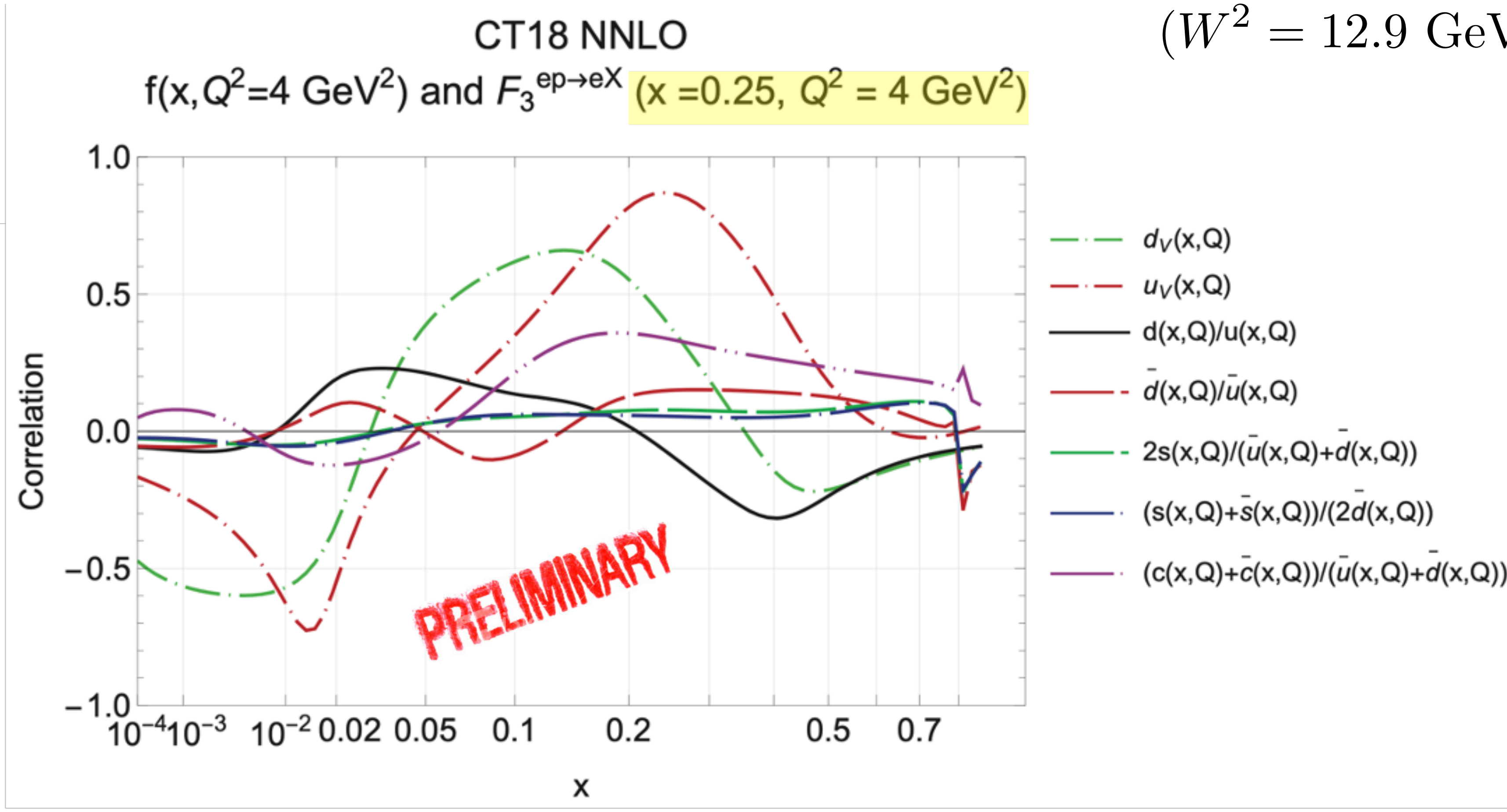
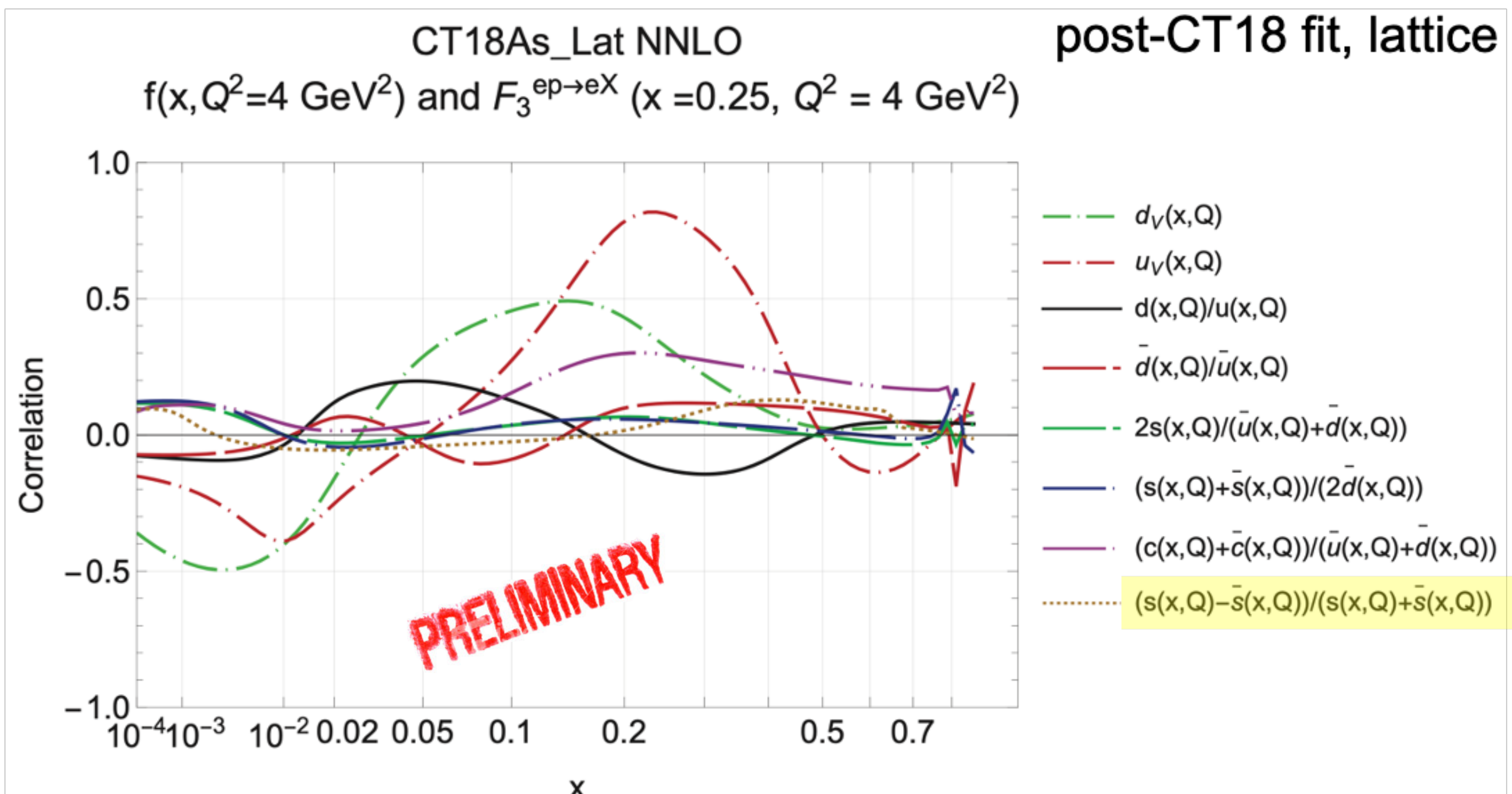
$(W^2 = 12.9 \text{ GeV}^2)$



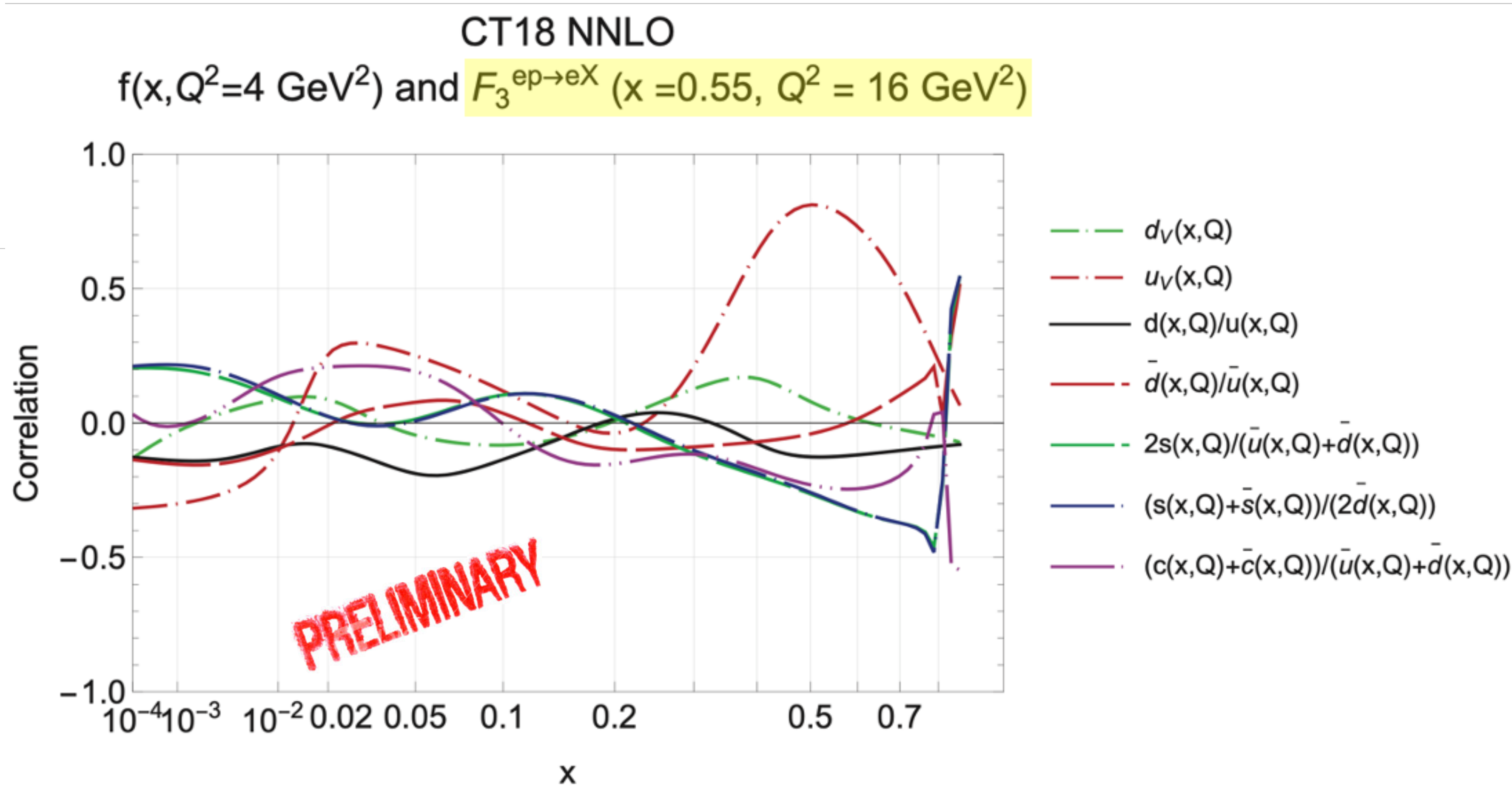
realistic APV PDF impact studies will require careful understanding of systematics

→ as a proxy, consider PDF correlations with $F_3^{\gamma Z} \sim q - \bar{q}$

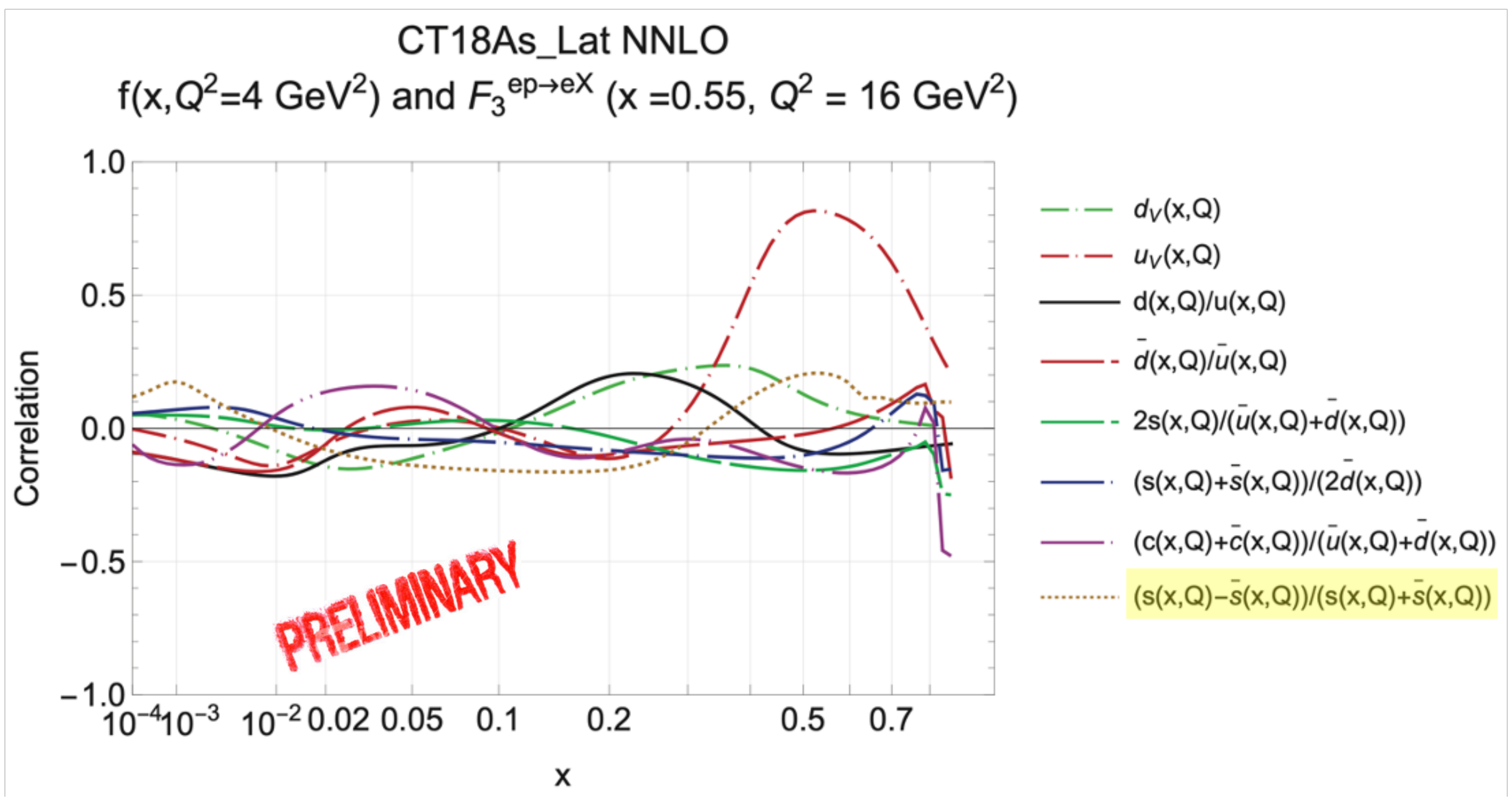
post-CT18 fit, lattice constraints to $s \neq \bar{s}$



NEW:
CT18 NNLO correlations with $F_3^{\gamma Z}$



→ PDF correlations have sharp dependence on sampled x, Q^2



→ PDF correlations suggest strong potential sensitivity to high-*x* valence-like combinations

NEW:
CT18 NNLO correlations with $F_3^{\gamma Z}$

Tolerance hypersphere in the PDF space

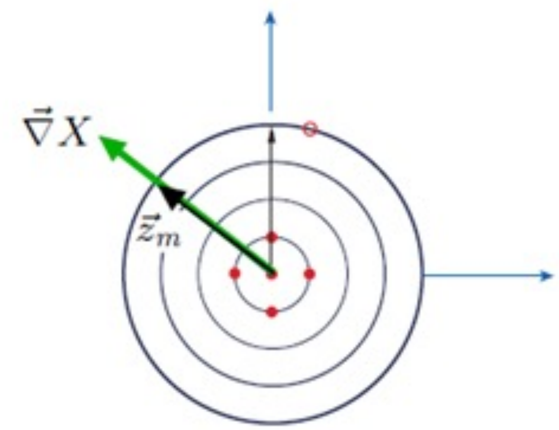
2-dim (i,j) rendition of N-dim (26) PDF parameter space

Hessian method: Pumplin et al., 2001

A symmetric PDF error for a physical observable X is given by

$$\Delta X = \vec{\nabla} X \cdot \vec{z}_m = \left|\vec{\nabla} X\right|$$

$$= \frac{1}{2}\sqrt{\sum_{i=1}^{N}\left(X_i^{(+)} - X_i^{(-)}\right)^2}$$



(b)
Orthonormal eigenvector basis

Correlation cosine for observables X and Y:

hep-ph/0110378

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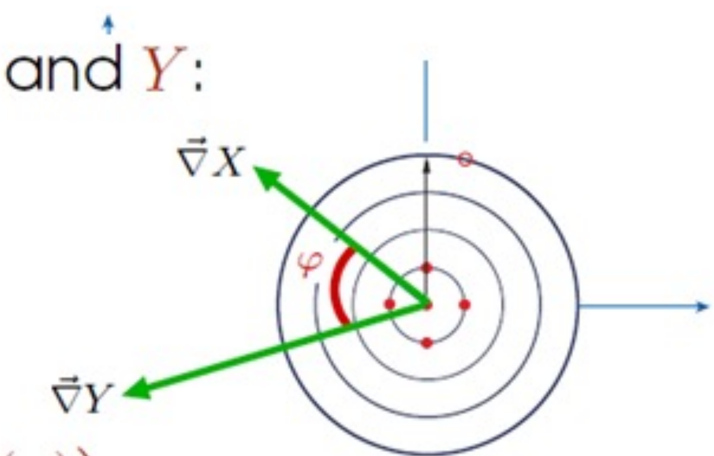

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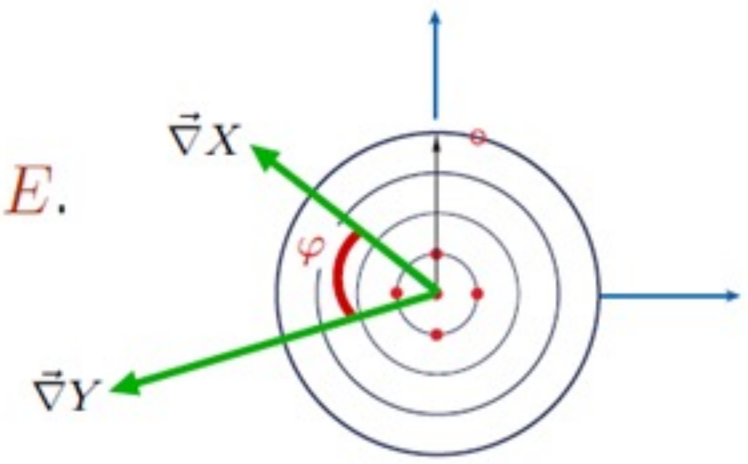
Orthonormal eigenvector basis

L_2 sensitivity, definition

$S_{f,L_2}(E)$ for experiment E is the estimated $\Delta\chi^2_E$ for this experiment when a PDF $f_a(x_i, Q_i)$ increases by the +68% c.l. Hessian PDF uncertainty

Take $X = f_a(x_i, Q_i)$ or $\sigma(f)$; $Y = \chi^2_E$ for experiment E.



$$S_{f,L_2} \equiv \Delta Y(\vec{z}_{m,X}) = \vec{\nabla}Y \cdot \vec{z}_{m,X} = \vec{\nabla}Y \cdot \frac{\vec{\nabla X}}{|\vec{\nabla X}|} = \Delta Y \cos\varphi.$$

A fast version of the Lagrange Multiplier scan of χ^2_E along the direction of $f_a(x_i, Q_i)$!