QUANTUM ANNEALING SPEEDUP

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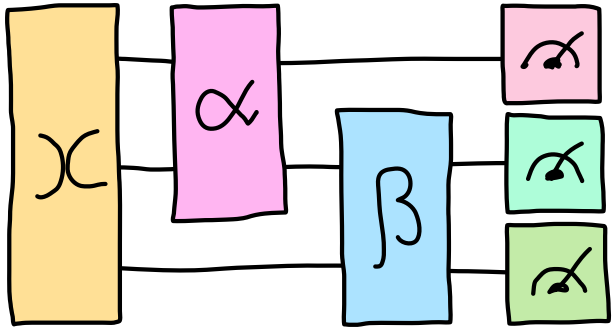
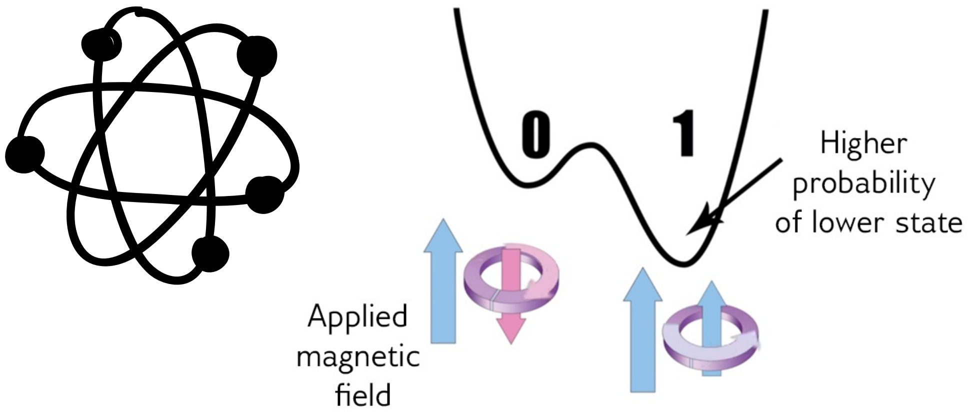
Quantum Computation

The circuit model is not
the only paradigm for quantum computation.



GATE BASED



ADIABATIC COMPUTING and ANNEALING

evolve a system towards low-energy states

Quantum Annealing

we love to describe systems with **Hamiltonians** $\hat{\mathcal{H}}$

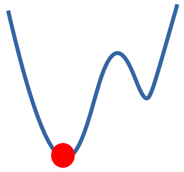
Seems a good idea to diagonalize it, ya?

$$\hat{\mathcal{H}} \left|\psi_i\right\rangle = E_i \left|\psi_i\right\rangle$$

E_i is the energy of (eigen)state $\left|\psi_i\right\rangle$

The eigenvector associated to the lowest energy eigenvalue is called the **ground state**.

$$\left|GS\right\rangle$$



An Hamiltonian is an operator that describes the quantum system in terms of energy. Thus, **the eigenvalues are the energy of our system**.

KEY IDEA OF **QA** and **QAC**

- encode the target problem as $\mathcal{H}_p$
- each state is a solution to the problem
- the GS is the **optimal solution**

Example problems: last mile resupply problem, layout planning, route planning, molecular unfolding, machine learning, lattice gauge, ...

Further reading
https://docs.dwavesys.com/docs/latest/c_gs_2.html
https://www.dwavesys.com/learn/featured-applications/

How to do it?

The system evolves in time, thus $\hat{\mathcal{H}}(t)$:

$$t = 0 \qquad\qquad t = \tau$$

$$\mathcal{H}_i \xrightarrow{\text{time}} \mathcal{H}_p$$

We can prepare a state here... … and measure the final state

Quantum Annealing (QA): the initial state is metaheuristic, the transverse field allows to explore the hypersurface of solutions

Adiabatic Quantum Computation (AQC): the initial state is ground-state and easy to prepare, the system evolves slowly

Life is not easy for any of us.

Adiabatic theorem(s)

Such theorems bound the **minimum total evolution time**, such as:

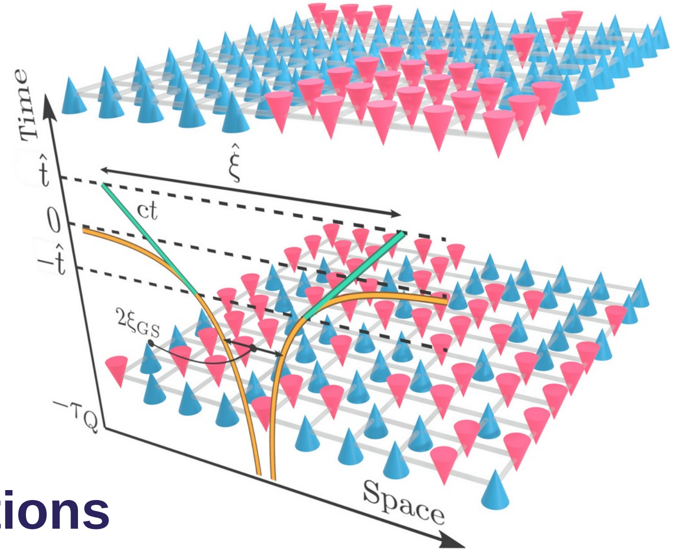
$$\tau \geq \frac{1}{\delta}\left(\int_0^s \left[\frac{||\partial_s^2 H(\sigma)||}{\Delta^2(s)} + 7\frac{||\partial_s H(\sigma)||^2}{\Delta^3(s)}\right] d\sigma + B\right)$$

to guarantee that

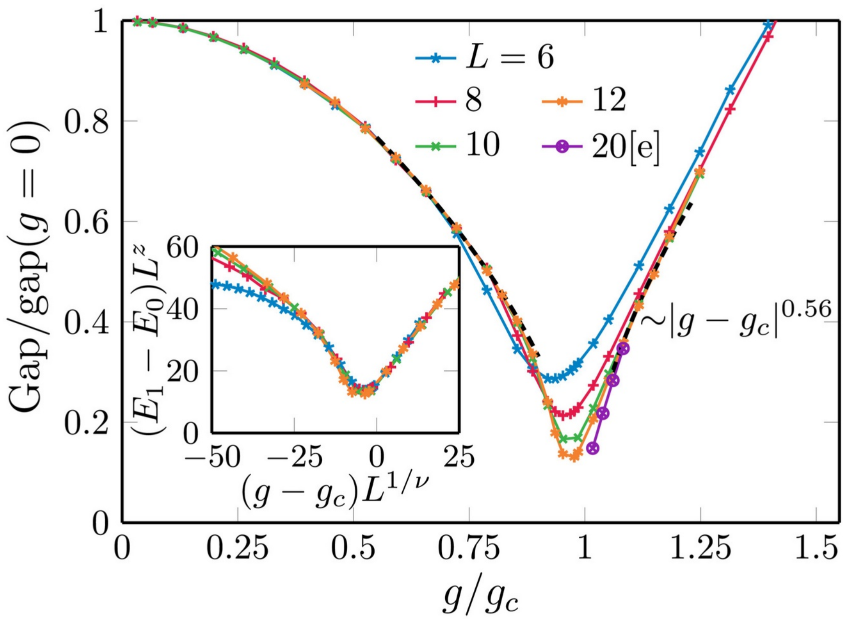
$$|\langle\psi(s)|\psi^T(s)\rangle| \geq 1-\delta \qquad \forall s \in [0,1]$$

where $|\psi(s)\rangle$ instantaneous GS of $\mathcal{H}$(s)

$\Delta(s)$ gap between GS and first excited eigenstate



Phase transitions



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Possible solutions

QUANTUM OPTIMAL CONTROL

Use numerically optimized paths to avoid gap closures.

SHORTCUTS TO ADIABATICITY (STA)

Remove diabatic transitions instantaneously.

COUNTERDIABATIC DRIVING (CD)

or transitionless driving

Suppress losses that arise due to fast deformations of the system via analytical compensation.

$$\mathcal{H}_{CD} = \mathcal{H}_0(\lambda) + \dot{\lambda}\hat{\mathcal{A}} \quad \text{st} \quad \hat{\mathcal{A}} = i\hbar \sum_{m \neq l} \frac{\langle \epsilon_m(\lambda) | \partial_\lambda \mathcal{H}_0(\lambda) | \epsilon_l(\lambda) \rangle}{\epsilon_l(\lambda) - \epsilon_m(\lambda)} |\epsilon_m(\lambda)\rangle \langle \epsilon_l(\lambda)|$$

APPROXIMATE COUNTERDIABATIC DRIVING

$$\mathcal{A} \simeq \alpha \sum_i \sigma_i^y + \gamma \sum_i (\sigma_i^x \sigma_{i+1}^y + \sigma_i^y \sigma_{i+1}^x) + \xi \sum_i (\sigma_i^y \sigma_{i+1}^z + \sigma_i^z \sigma_{i+1}^y)$$

⚠ $$\min_{\mathcal{A}} \mathrm{Tr}[G_t(\mathcal{A})^2]$$

$$G_t = -\partial_t \mathcal{H}_0 + \frac{i}{\hbar}[\mathcal{A}, \mathcal{H}_0]$$

Target solution

Counterdiabatic Optimized Local Driving

Ieva Čepaitė, Anatoli Polkovnikov, Andrew J. Daley, Callum W. Duncan

$$\mathcal{H}_{\mathrm{COLD}}(t,\beta) = H_0(t) + \underbrace{\boldsymbol{\alpha}(t,\beta)\mathcal{O}_{\mathrm{LCD}}}_{\text{CD}} + \underbrace{f(t,\beta)\mathcal{O}_{\mathrm{opt}}}_{\text{QOC}}$$

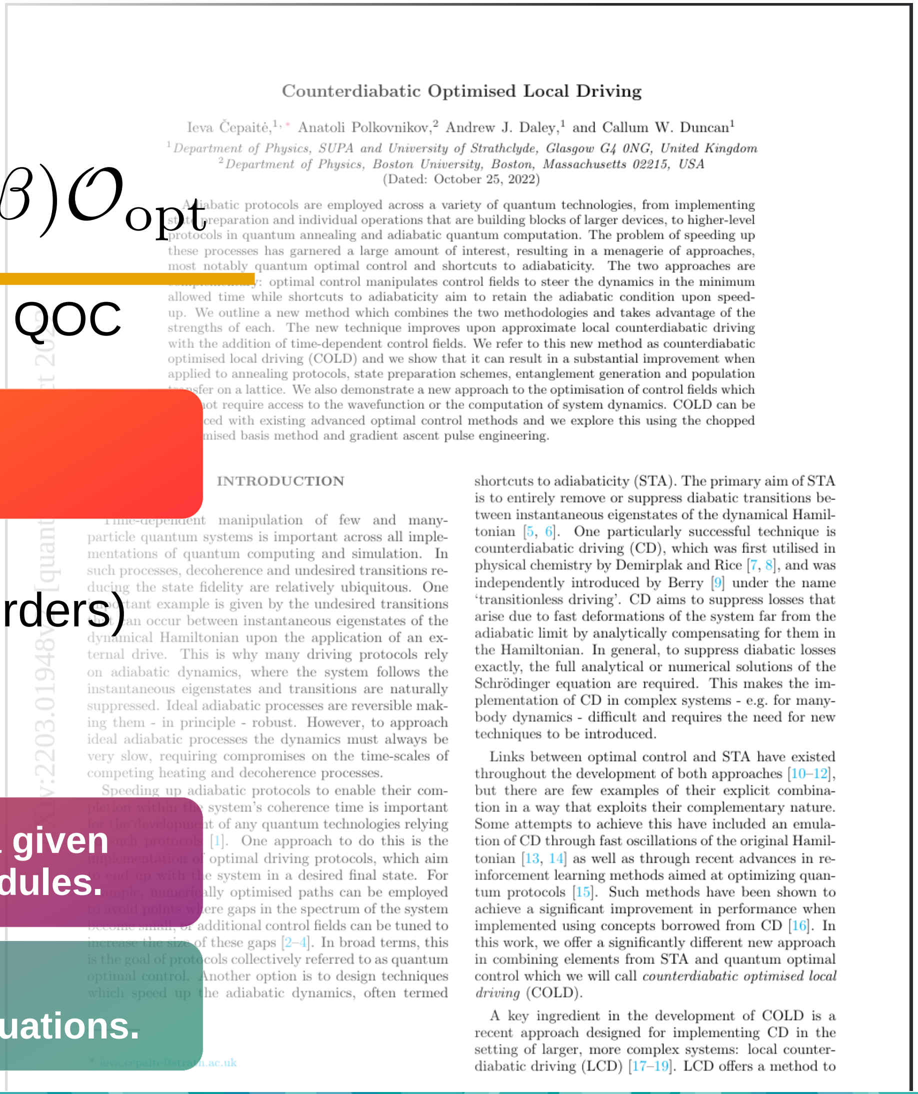
My contribution — **A library to manage the simulation of arbitrary spin systems annealing in the COLD formulation.**

$\mathcal{O}_{\mathrm{LCD}}$: can be local (first order) or non-local (higher orders)

$\mathcal{O}_{\mathrm{opt}}$: additional degrees of freedom

My contribution — **The Gauge potential is computed symbolically for a given configuration, allowing fast prototyping of QA schedules.**

My contribution — **The QOC optimization is carried out with bayesian sampling methods, to reduce the loss function evaluations.**



Counterdiabatic Optimised Local Driving

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(Dated: October 25, 2022)

...iabatic protocols are employed across a variety of quantum technologies, from implementing ...preparation and individual operations that are building blocks of larger devices, to higher-level protocols in quantum annealing and adiabatic quantum computation. The problem of speeding up these processes has garnered a large amount of interest, resulting in a menagerie of approaches, most notably quantum optimal control and shortcuts to adiabaticity. The two approaches are ...: optimal control manipulates control fields to steer the dynamics in the minimum allowed time while shortcuts to adiabaticity aim to retain the adiabatic condition upon speed-up. We outline a new method which combines the two methodologies and takes advantage of the strengths of each. The new technique improves upon approximate local counterdiabatic driving with the addition of time-dependent control fields. We refer to this new method as counterdiabatic optimised local driving (COLD) and we show that it can result in a substantial improvement when applied to annealing protocols, state preparation schemes, entanglement generation and population ...sfer on a lattice. We also demonstrate a new approach to the optimisation of control fields which ...ot require access to the wavefunction or the computation of system dynamics. COLD can be ...ced with existing advanced optimal control methods and we explore this using the chopped ...mised basis method and gradient ascent pulse engineering.

INTRODUCTION

... manipulation of few and many-particle quantum systems is important across all implementations of quantum computing and simulation. In such processes, decoherence and undesired transitions reducing the state fidelity are relatively ubiquitous. One ...tant example is given by the undesired transitions ...an occur between instantaneous eigenstates of the dynamical Hamiltonian upon the application of an external drive. This is why many driving protocols rely on adiabatic dynamics, where the system follows the instantaneous eigenstates and transitions are naturally suppressed. Ideal adiabatic processes are reversible making them - in principle - robust. However, to approach ideal adiabatic processes the dynamics must always be very slow, requiring compromises on the time-scales of competing heating and decoherence processes.

Speeding up adiabatic protocols to enable their com-... system's coherence time is important ...t of any quantum technologies relying ... [1]. One approach to do this is the ... optimal driving protocols, which aim ...e system in a desired final state. For ...ally optimised paths can be employed ...ere gaps in the spectrum of the system ... additional control fields can be tuned to ... of these gaps [2–4]. In broad terms, this ...cols collectively referred to as quantum optimal control. Another option is to design techniques ...he adiabatic dynamics, often termed shortcuts to adiabaticity (STA). The primary aim of STA is to entirely remove or suppress diabatic transitions between instantaneous eigenstates of the dynamical Hamiltonian [5, 6]. One particularly successful technique is counterdiabatic driving (CD), which was first utilised in physical chemistry by Demirplak and Rice [7, 8], and was independently introduced by Berry [9] under the name 'transitionless driving'. CD aims to suppress losses that arise due to fast deformations of the system far from the adiabatic limit by analytically compensating for them in the Hamiltonian. In general, to suppress diabatic losses exactly, the full analytical or numerical solutions of the Schrödinger equation are required. This makes the implementation of CD in complex systems - e.g. for many-body dynamics - difficult and requires the need for new techniques to be introduced.

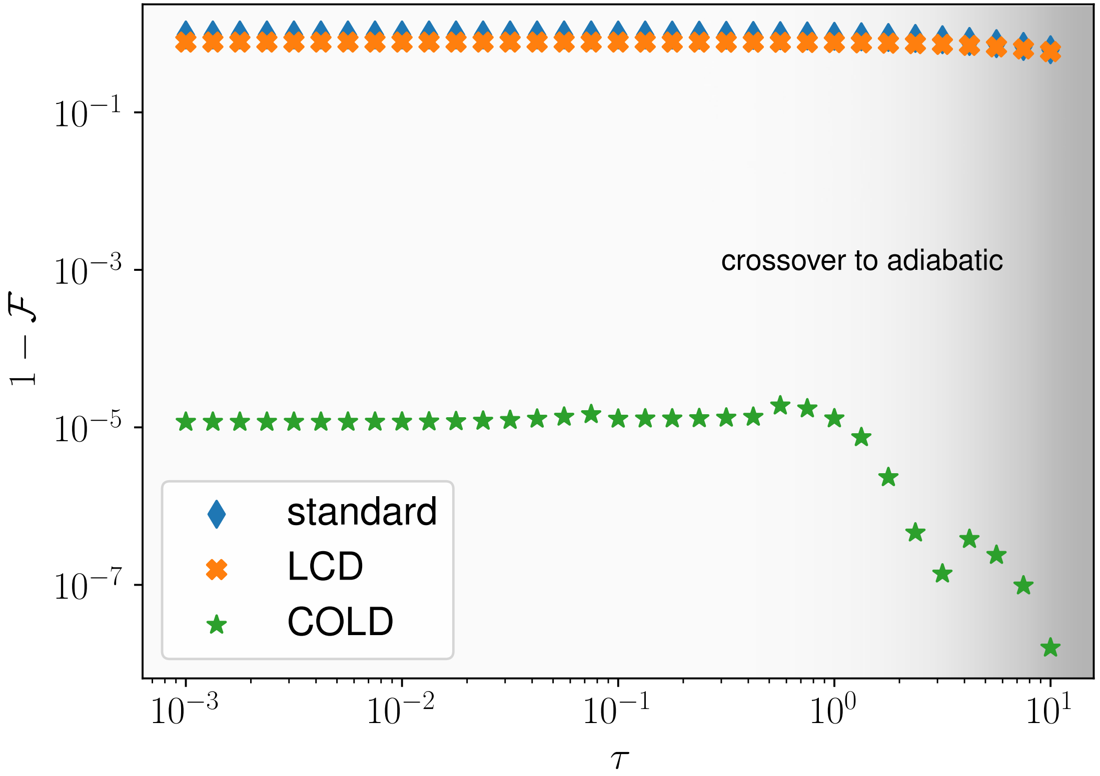
Links between optimal control and STA have existed throughout the development of both approaches [10–12], but there are few examples of their explicit combination in a way that exploits their complementary nature. Some attempts to achieve this have included an emulation of CD through fast oscillations of the original Hamiltonian [13, 14] as well as through recent advances in reinforcement learning methods aimed at optimizing quantum protocols [15]. Such methods have been shown to achieve a significant improvement in performance when implemented using concepts borrowed from CD [16]. In this work, we offer a significantly different new approach in combining elements from STA and quantum optimal control which we will call *counterdiabatic optimised local driving* (COLD).

A key ingredient in the development of COLD is a recent approach designed for implementing CD in the setting of larger, more complex systems: local counterdiabatic driving (LCD) [17–19]. LCD offers a method to

Results

ISING MODEL

$$\mathcal{H} = -J(t)\sum_i \sigma_i^z \sigma_{i+1}^z + X(t)\sum_i \sigma_i^x + Z(t)\sum_i \sigma_i^z$$



$$-\sum_i \sigma_i^x \longrightarrow \mathcal{H}_{\text{Ising}}$$

1D lattice: 5 spins, nearest neighbor interactions

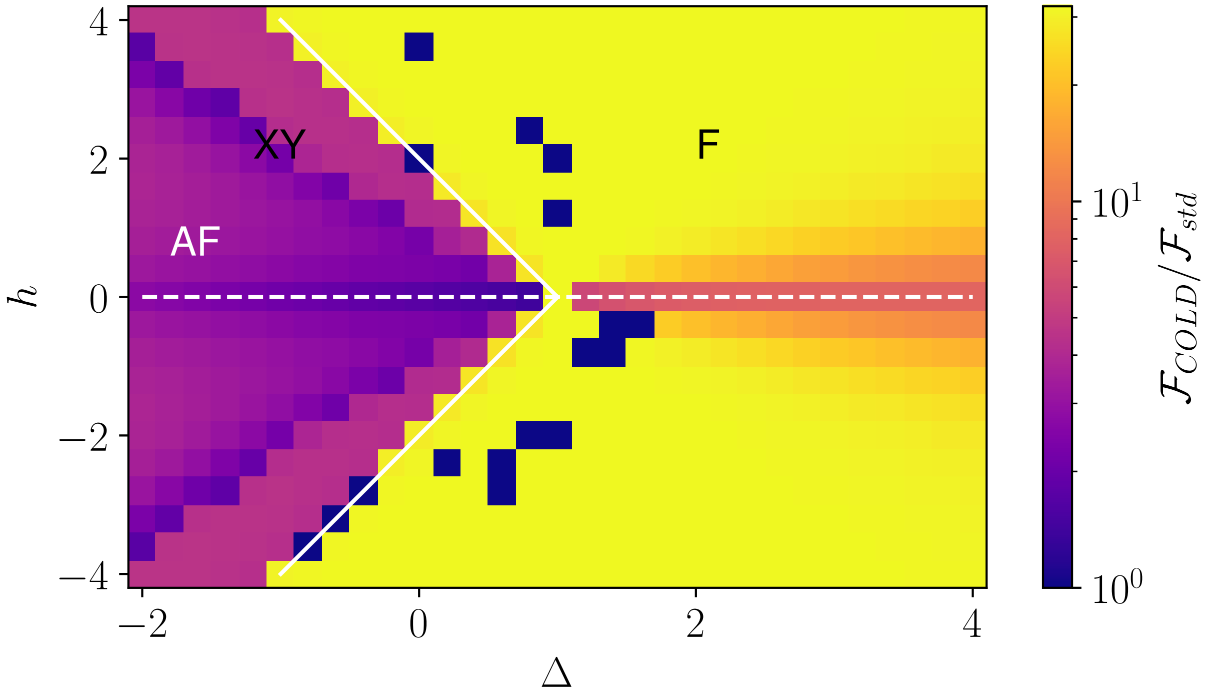
Simulated annealing: > 2000 steps, min timestep constant in magnitude

Results

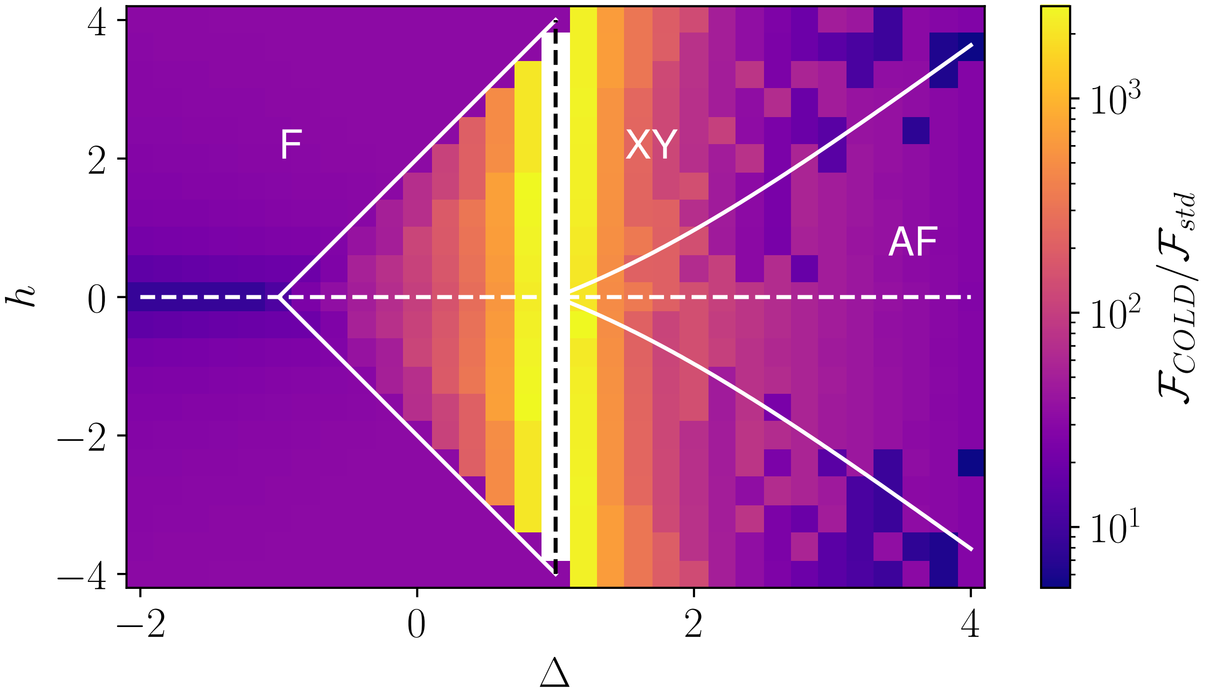
XXZ MODEL

$$\mathcal{H} = J(t)\left(\sum_i \sigma_i^x \sigma_{i+1}^x + \sum_i \sigma_i^y \sigma_{i+1}^y\right) + \Delta(t)\sum_i \sigma_i^z \sigma_{i+1}^z + h(t)\sum_i \sigma_i^z$$

$$-\sum_i \sigma_i^x \longrightarrow \mathcal{H}(J=1, -h, -\Delta)$$



$$-\sum_i \sigma_i^x \longrightarrow -\mathcal{H}(J=1)$$

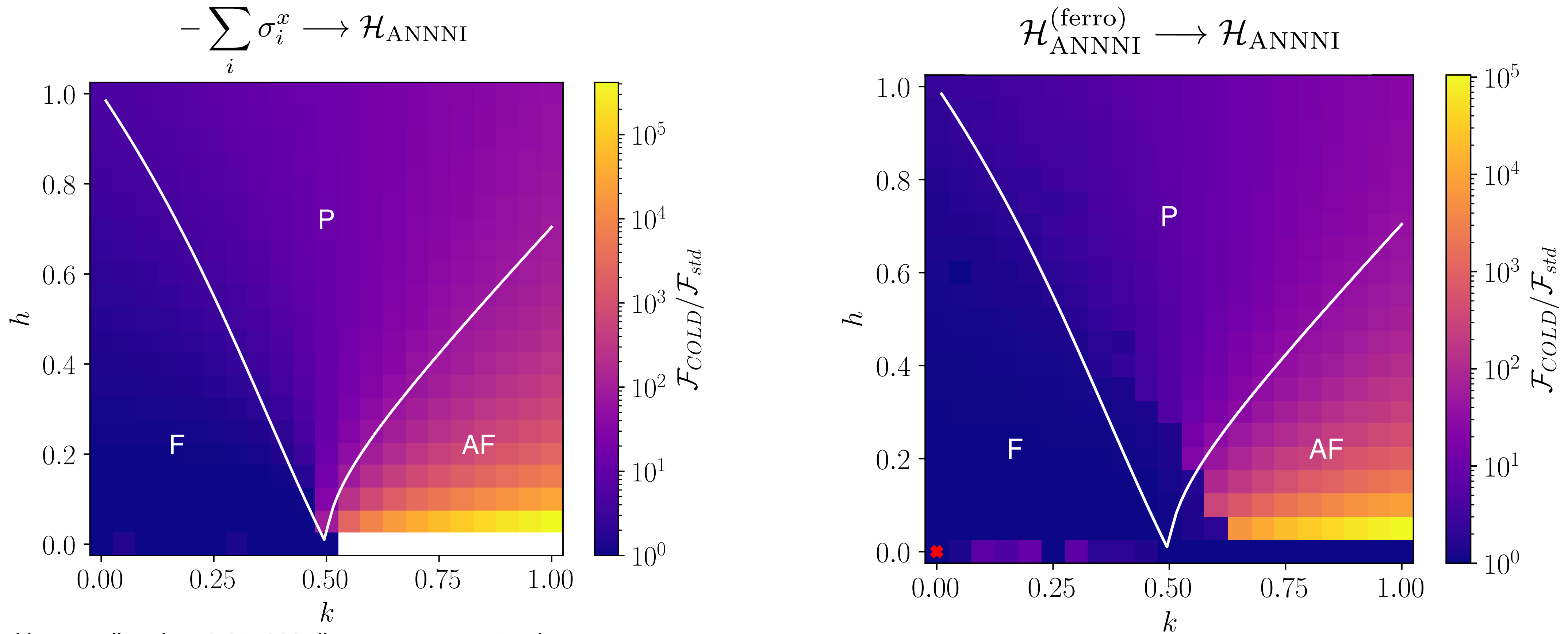


ⓘ Simulation with annealing time 0.01, 200 discrete steps on 5 spin systems.

Results

ANNNI MODEL

$$\mathcal{H} = -J(t)\sum_i \sigma_i^z\sigma_{i+1}^z + k(t)\sum_i \sigma_i^z\sigma_{i+2}^z + h(t)\sum_i \sigma_i^z$$



ⓘ Simulation with annealing time 0.01, 200 discrete steps on 5 spin systems.

Goal: prepare a good initial state

for QPE algorithms

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I would like to say

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Bibliography

- [Cepaite2022] **Counterdiabatic Optimized Local Driving** (PRXQuantum.4.010312)
- [Hartmann2022] **Polynomial scaling enhancement in the ground-state preparation of Ising spin models via counterdiabatic driving**
- [Prielinger2021] **Two-parameter counter-diabatic driving in quantum annealing**
- [Schmitt2022] **Quantum phase transition dynamics in the two-dimensional transverse-field Ising model** (doi.org/10.1126/sciadv.abl6850)
- **D-Wave Systems** (https://www.dwavesys.com/)

Attributions

Slide 2 – Pennylane illustration of quantum circuit
Slide 3 – Einstein illustration, credits to Albert Vectors by Vecteezy