Decoherence of Wave Packets in Neutrino Oscillation

Kin-ya Oda (Tokyo Woman's Christian University)

+**Haruhi Mitani** (TWCU→Fujitsu): PLB (2023)

Also

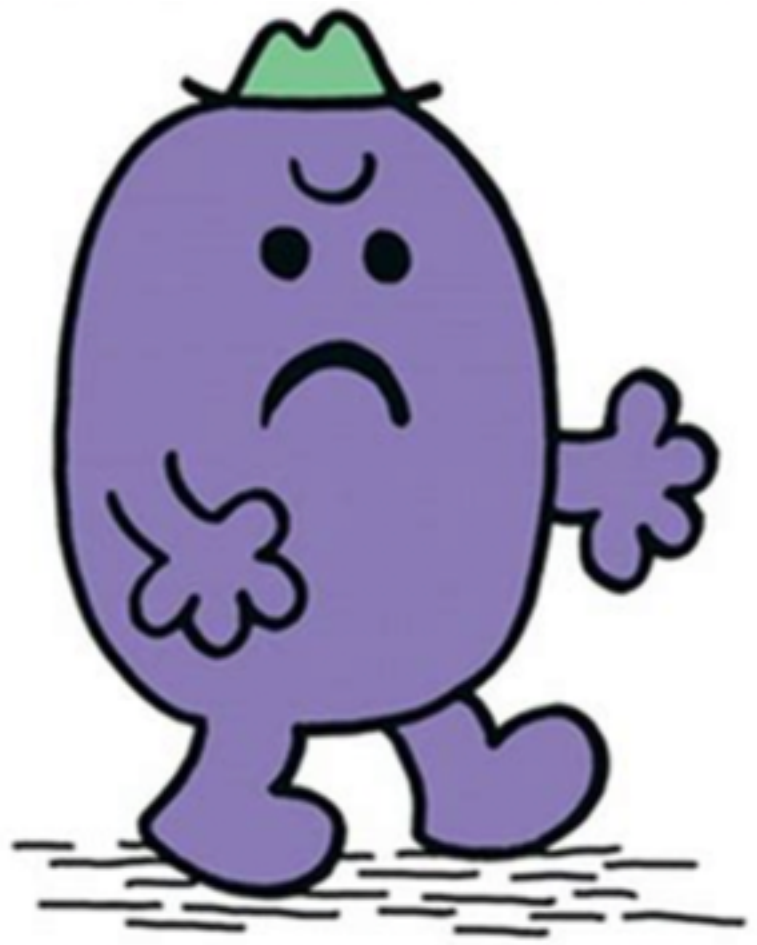
+**Juntaro Wada** (Tokyo): EPJC (2021); arXiv:2307.05932

+**Kenzo Ishikawa** (Hokkaido): PTEP (2018);

+ KI, **Kenji Nishiwaki** (Shiv Nadar): PTEP (2020); arXiv:2102.12032 to appear in PRD (finally!)

A grumble about the one finally appearing on PRD…

Better have scientist editor



Referee A: All remarks of previous reports have been fully addressed and the revised manuscript has been tremendously improved by streamlining the discussion. In this way the manuscript is accessible to the larger community. As the overlooked effect has a vast impact on various branches of physics such as neutrino physics, astrophysics and biophysics I can fully recommend this article for publication in Physical Review Letters.

Referee C: In my understanding, this result sheds new light on several problems of quantum field theory and the related topics. In particular, the interpretation in terms of Lefschetz thimble decomposition is what the physicists should have done in old days. The paper is well written and organized so as to be readable for the readers from other fields. I recommend the editor to publish the paper in this journal.

Referees B says, *I find it hard to conclude whether this result has the relevance and depth that would make me say it ⟪"should be" as opposed to "could be"⟫ published in PRL,* and finishes the letter by the following sentence: *But of course I leave to the Editor the final decision based also on the other referees' observations.* Referee D did not recommend, the whole report being as follows:

Referee D: I would agree with the assessment of Referee B. This is a very old subject and its not clear to me that the intricacies the authors are concerned with are of sufficient interest to a wide enough audience. Nor do I think will it have any overwhelming impact in a sub-field that would overcome the aforementioned shortcoming.

[Here and hereafter, all pictures from web unless otherwise stated]

Introduction

What is spacetime in quantum mechanics?

- We all learn in QFT kindergarten:
  - **Failure** of **relativistic quantum mechanics**
- You still remember why/how?

Plan

1. Position space is mass-dependent

2. What is flavor/interaction eigenstate?

3. Decoherence in neutrino oscillation

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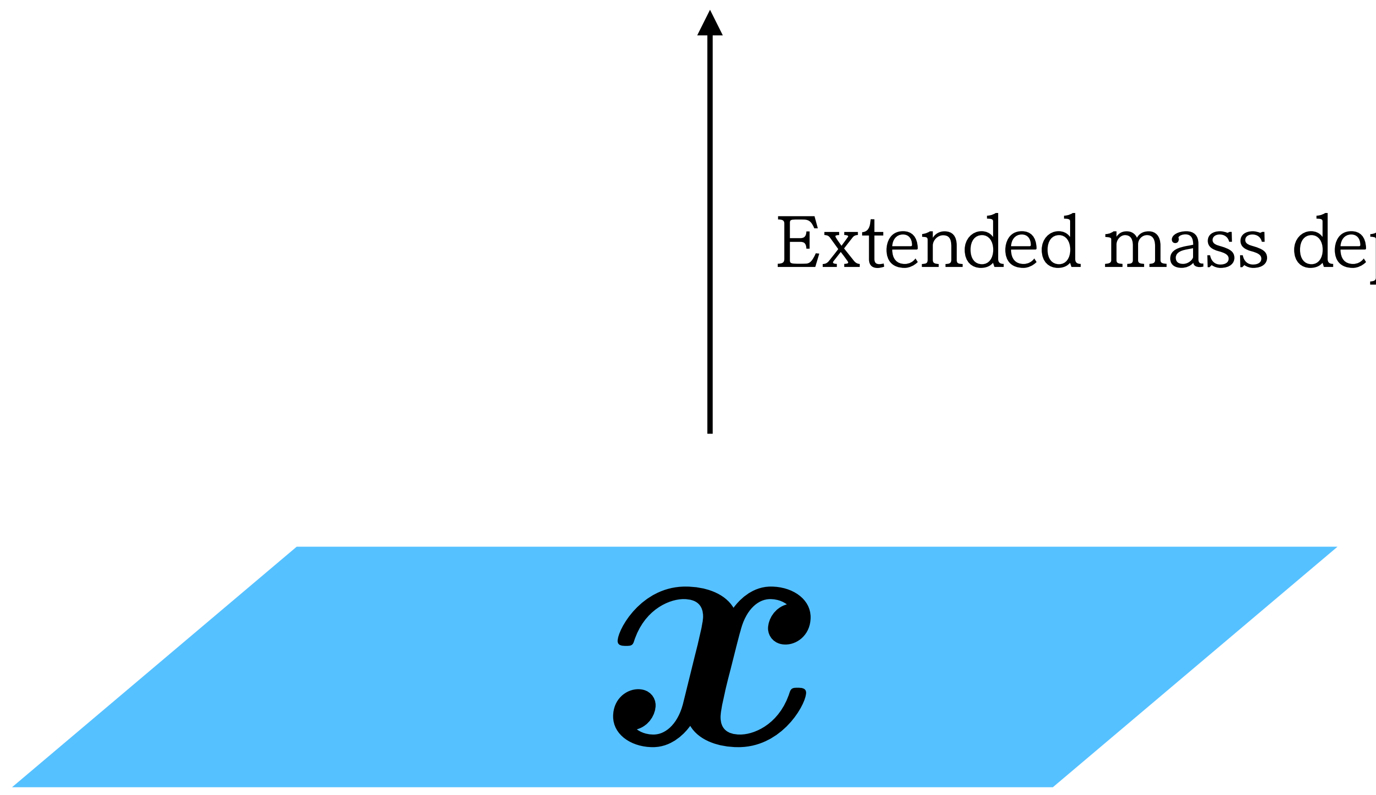
Spacetime is mass-dependent in QM

- Wave function in QM: $\psi(x) = \langle x \,|\, \psi \rangle$

$$x = (t, \boldsymbol{x})$$



- Position eigenbasis extended from **a space-like hypersurface**, say, at $t = 0$:

$$\langle x | = \langle \boldsymbol{x} | \, e^{-i\hat{E}t}$$

- Extension is mass dependent:

$$\langle x | = e^{-i\sqrt{m^2 - \nabla^2}\,t} \, \langle \boldsymbol{x} |$$

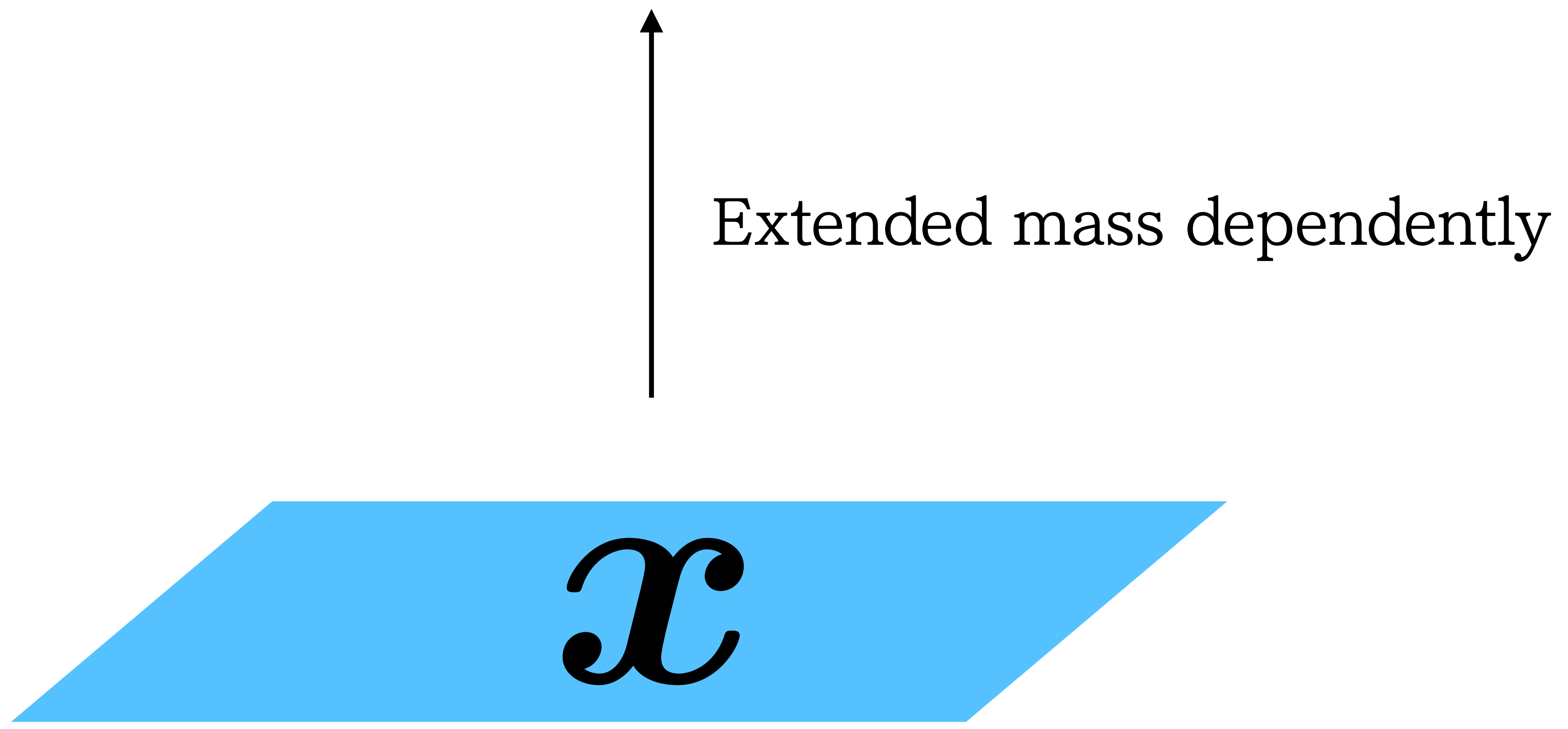
Position operator non-Hermitian

- Covariant **position operator** becomes **non-Hermitian** in one-particle subspace

$$\hat{\chi} = \hat{\boldsymbol{x}} - i\frac{\hat{\boldsymbol{p}}}{2E_{\hat{\boldsymbol{p}}}^2}$$

[Oda, Wada (2021)]:

$$x = (t, \boldsymbol{x})$$



So we need QFT for
full Lorentz covariance

Plane-wave basis

- Free 1-particle "state" $|\mathbf{p}\rangle$:

$$|\boldsymbol{p}\rangle = \hat{a}^{\dagger}(\boldsymbol{p})\,|0\rangle$$

- 1-particle position basis $|\mathbf{x}\rangle$ defined by

$$\langle \boldsymbol{x} | \, \boldsymbol{p} \rangle \propto e^{i\boldsymbol{p}\cdot\boldsymbol{x}}$$

- "States" $|\mathbf{p}\rangle$, $|\mathbf{x}\rangle$ are NOT an element of Hilbert space: $\langle \mathbf{p}|\mathbf{p}\rangle = \delta(0) = \infty$

4D notation

- Throughout this talk, interchangeably,
  - spacetime coordinates:
    - $x = (x^0, \mathbf{x}) = (t, \mathbf{x})$,
  - spacetime center of wave packet:
    - $X = (X^0, \mathbf{X}) = (T, \mathbf{X})$.

Backup

- Interaction-picture eigenbasis:

$$\langle x| = \langle \boldsymbol{x}|\, e^{-iH_{\mathrm{free}}t}$$

$$\langle x|\boldsymbol{p}\rangle = \langle \boldsymbol{x}|\, e^{-iH_{\mathrm{free}}t}\,|\boldsymbol{p}\rangle \propto \left. e^{ip\cdot x}\right|_{p^0=E(\boldsymbol{p})}$$

- Scalar field expanded as

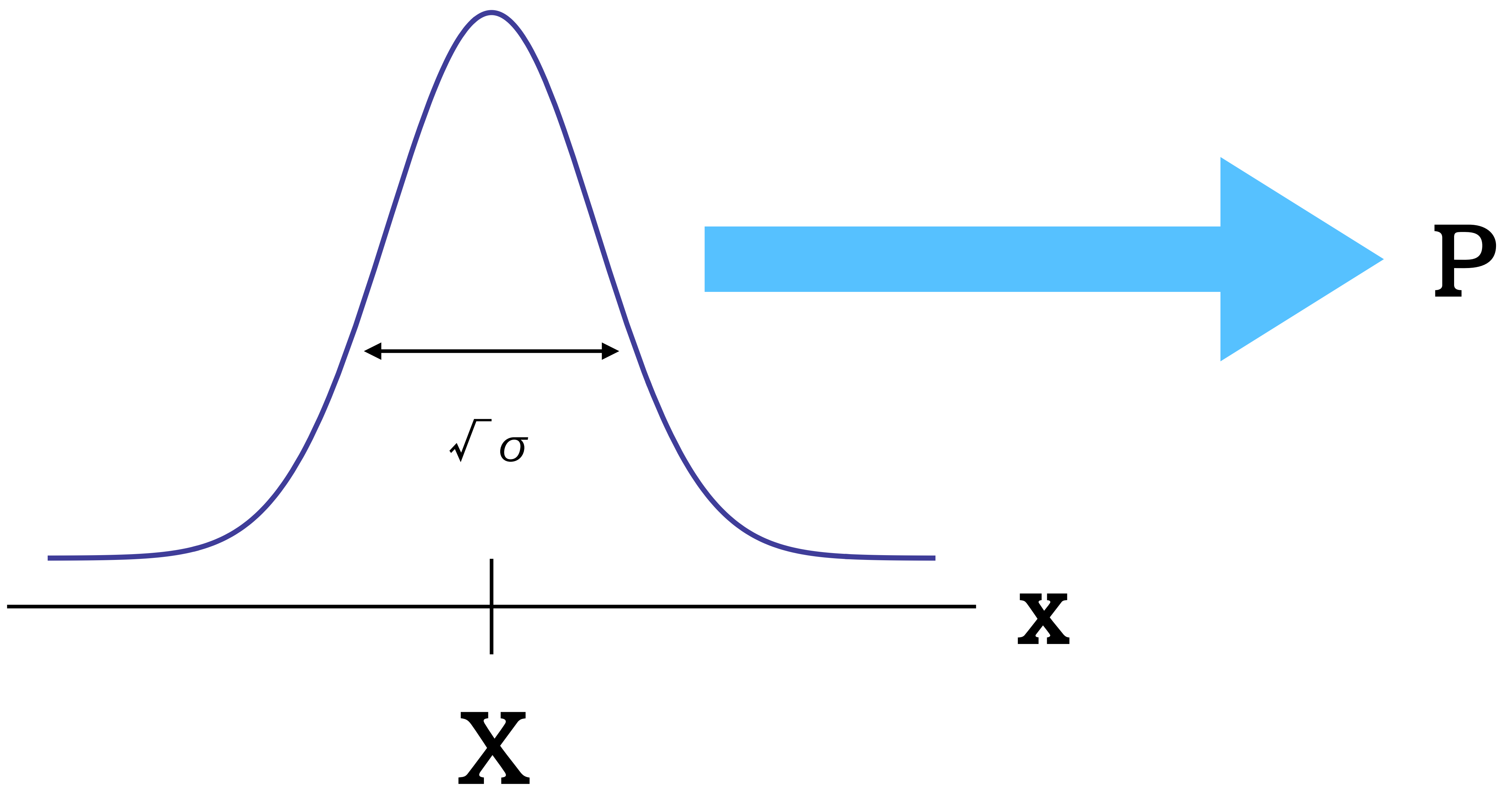
$$\hat{\phi}(x) = \int \frac{\mathrm{d}^3\boldsymbol{p}}{\sqrt{2E(\boldsymbol{p})}\,(2\pi)^{3/2}} \Big[\, \langle x|\boldsymbol{p}\rangle\, \hat{a}(\boldsymbol{p}) + \mathrm{h.c.} \Big]$$

How about wave-packet basis?

Gaussian basis

- Gaussian basis state $|\sigma, \mathbf{X}, \mathbf{P}\rangle$ defined (roughly) by

$$\langle \boldsymbol{x} | \sigma, \boldsymbol{X}, \boldsymbol{P} \rangle \propto e^{i\boldsymbol{P}\cdot(\boldsymbol{x}-\boldsymbol{X})} e^{-\frac{(\boldsymbol{x}-\boldsymbol{X})^2}{2\sigma}}$$



Characteristics

- Non-orthogonal:

$$\langle \sigma, \boldsymbol{X}, \boldsymbol{P} | \sigma', \boldsymbol{X}', \boldsymbol{P}' \rangle \propto e^{-\frac{1}{4\sigma_A}(\boldsymbol{X}-\boldsymbol{X}')^2} e^{-\frac{\sigma_I}{4}(\boldsymbol{P}-\boldsymbol{P}')^2} \times e^{\frac{i}{2\sigma_I}(\sigma \boldsymbol{P}+\sigma' \boldsymbol{P}')\cdot(\boldsymbol{X}-\boldsymbol{X}')}$$

Details irrelevant

- Form a(n over-)complete basis:

$$\int \frac{d^3\boldsymbol{X}\, d^3\boldsymbol{P}}{(2\pi)^3} |\sigma, \boldsymbol{X}, \boldsymbol{P}\rangle \langle \sigma, \boldsymbol{X}, \boldsymbol{P}| = \hat{1}$$

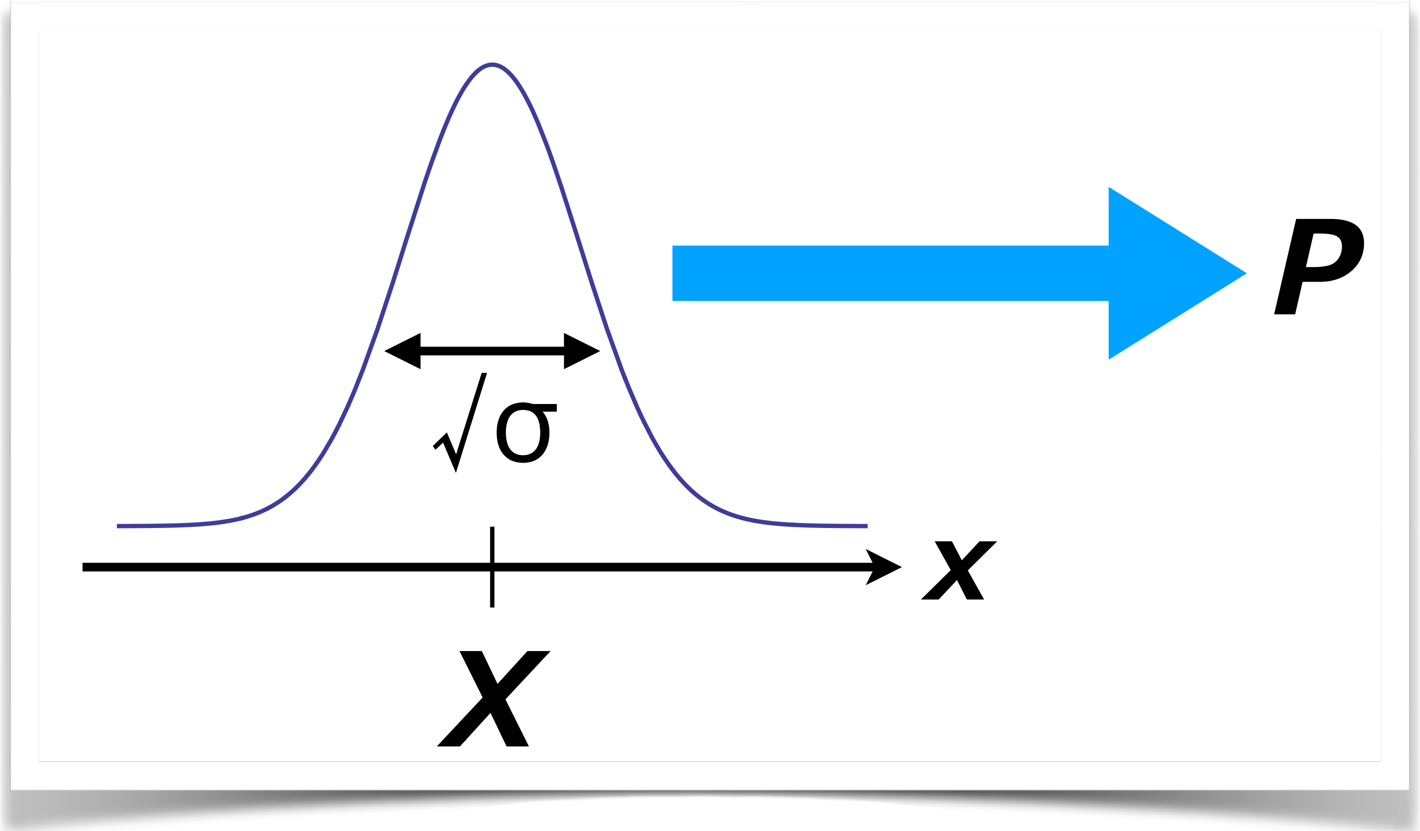
- **Normalizable!** Namely, itself an element of Hilbert space: $\langle \sigma, \mathbf{X}, \mathbf{P} | \sigma, \mathbf{X}, \mathbf{P} \rangle = 1$.

Gaussian expansion

- Expansion by free Gaussian wave functions:

$$\hat{\phi}(x) = \int \frac{\mathrm{d}^3\boldsymbol{X}\,\mathrm{d}^3\boldsymbol{P}}{(2\pi)^3}\left[f_{\sigma,X,\boldsymbol{P}}(x)\,\hat{A}(\sigma,X,\boldsymbol{P}) + \mathrm{h.c.}\right]$$

- In plane-wave limit $\sigma \to \infty$,
    - $f_{\sigma,X,\mathbf{P}}(x) \to$ Gaussian wave packet,
        - centered around $\mathbf{X}+(t-T)\mathbf{V}$.



Lorentz in(/co)variant generalization?

- Lorentz invariant scalar wave packet [KO, Wada, EPJC (2021)]
  - Forms complete set
  - Uncertainty relation modified
- Lorentz covariant spinor wave packet [KO, Wada, arXiv:2307.05932]
  - Irreducibility under Lorentz-transformation maintained
  - Forms complete set: Fully covariant completeness relation

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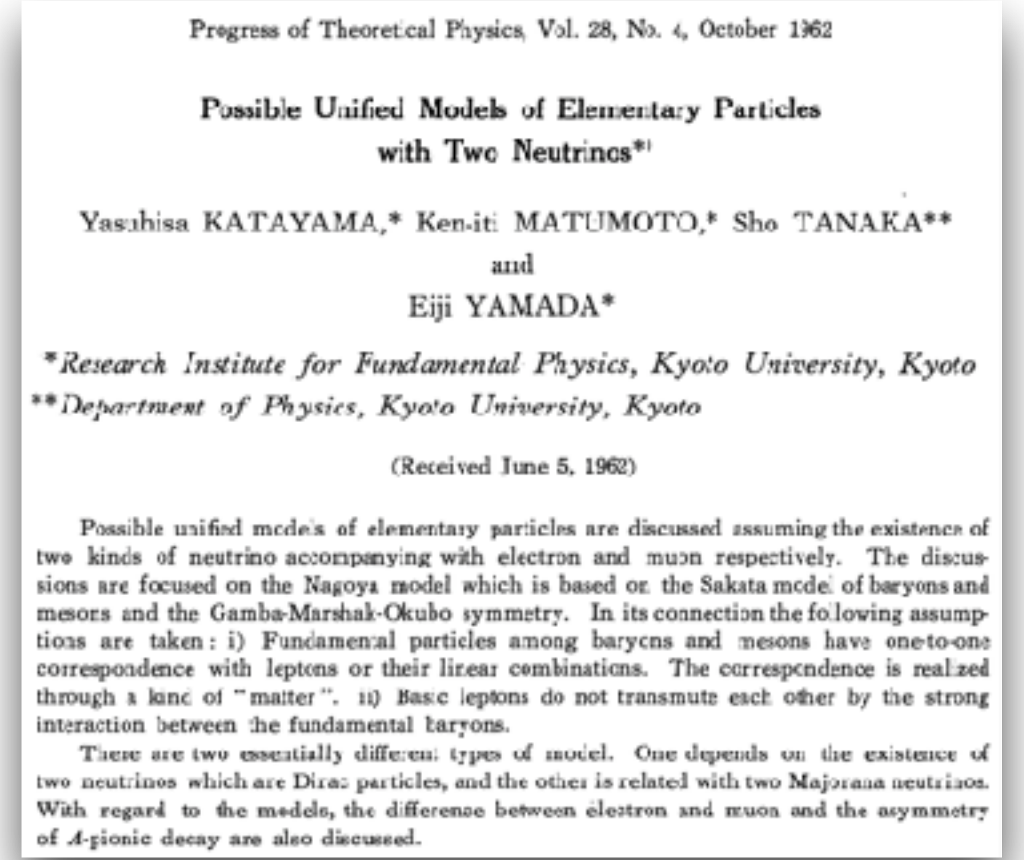
P(KMTY)MNS Matrix



Progress of Theoretical Physics, Vol. 28, No. 4, October 1962

Possible Unified Models of Elementary Particles with Two Neutrinos[*)]

Yasuhisa KATAYAMA,[*] Ken-iti MATUMOTO,[*] Sho TANAKA[**] and Eiji YAMADA[*]

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(Received June 5, 1962)

Possible unified models of elementary particles are discussed assuming the existence of two kinds of neutrino accompanying with electron and muon respectively. The discussions are focused on the Nagoya model which is based on the Sakata model of baryons and mesons and the Gamba-Marshak-Okubo symmetry. In its connection the following assumptions are taken: i) Fundamental particles among baryons and mesons have one-to-one correspondence with leptons or their linear combinations. The correspondence is realized through a kind of "matter". ii) Basic leptons do not transmute each other by the strong interaction between the fundamental baryons.

There are two essentially different types of model. One depends on the existence of two neutrinos which are Dirac particles, and the other is related with two Majorana neutrinos. With regard to the models, the difference between electron and muon and the asymmetry of Λ-pionic decay are also discussed.

- In QFT, what are mixed are **not states** but **fields**
  - Not really true:

$$|\nu_\alpha\rangle = \sum_\alpha U^*_{\alpha I} |\nu_I\rangle$$

  - True:

$$\widehat{\nu}_\alpha(x) = \sum_I U_{\alpha I} \widehat{\nu}_I(x)$$

More precise mixing

$$\widehat{\nu}_\alpha(x) = \sum_I U_{\alpha I} \widehat{\nu}_I(x)$$

$$\widehat{\nu}_I(x) = \sum_s \int \frac{\mathrm{d}^d \boldsymbol{p}}{(2\pi)^{\frac{d}{2}}} \left[u_I(\boldsymbol{p}, s)\, e^{-iE_I(p)x^0 + i\boldsymbol{p}\cdot\boldsymbol{x}}\, \widehat{a}_I(\boldsymbol{p}, s) + v_I(\boldsymbol{p}, s)\, e^{iE_I(p)x^0 - i\boldsymbol{p}\cdot\boldsymbol{x}}\, \widehat{a}_I^{\mathrm{c}\dagger}(\boldsymbol{p}, s) \right]$$

$$\langle 0 | \, \widehat{\nu}_\alpha(x) \, | \boldsymbol{p}, s; I \rangle = \sum_I U_{\alpha I} \frac{u_I(\boldsymbol{p}, s)\, e^{-iE_I(p)x^0 + i\boldsymbol{p}\cdot\boldsymbol{x}}}{(2\pi)^{\frac{d}{2}}},$$

$$\langle \boldsymbol{p}, s; I | \, \widehat{\nu}_\alpha(x) \, | 0 \rangle = \sum_I U_{\alpha I} \frac{v_I(\boldsymbol{p}, s)\, e^{iE_I(p)x^0 - i\boldsymbol{p}\cdot\boldsymbol{x}}}{(2\pi)^{\frac{d}{2}}}.$$

Mass-dependence issue

- This is partly covered in [KO, Wada, arXiv:2307.05932]
- More on neutrino mixing will be presented [Nishiwaki, KO, Wada].
- Stay tuned!

Ultra-relativistic limit

- At least in the ultra-relativistic limit, mass-dependence in energy, etc. will drop out

- In [Mitani, KO, 2023], we work in this limit:

$$|X, \boldsymbol{P}; \sigma, \nu_\alpha\rangle = \sum_I U^*_{\alpha I} |X, \boldsymbol{P}; \sigma, I\rangle$$

(Pity that easy-going papers are accepted far faster than more fundamental ones⋯)

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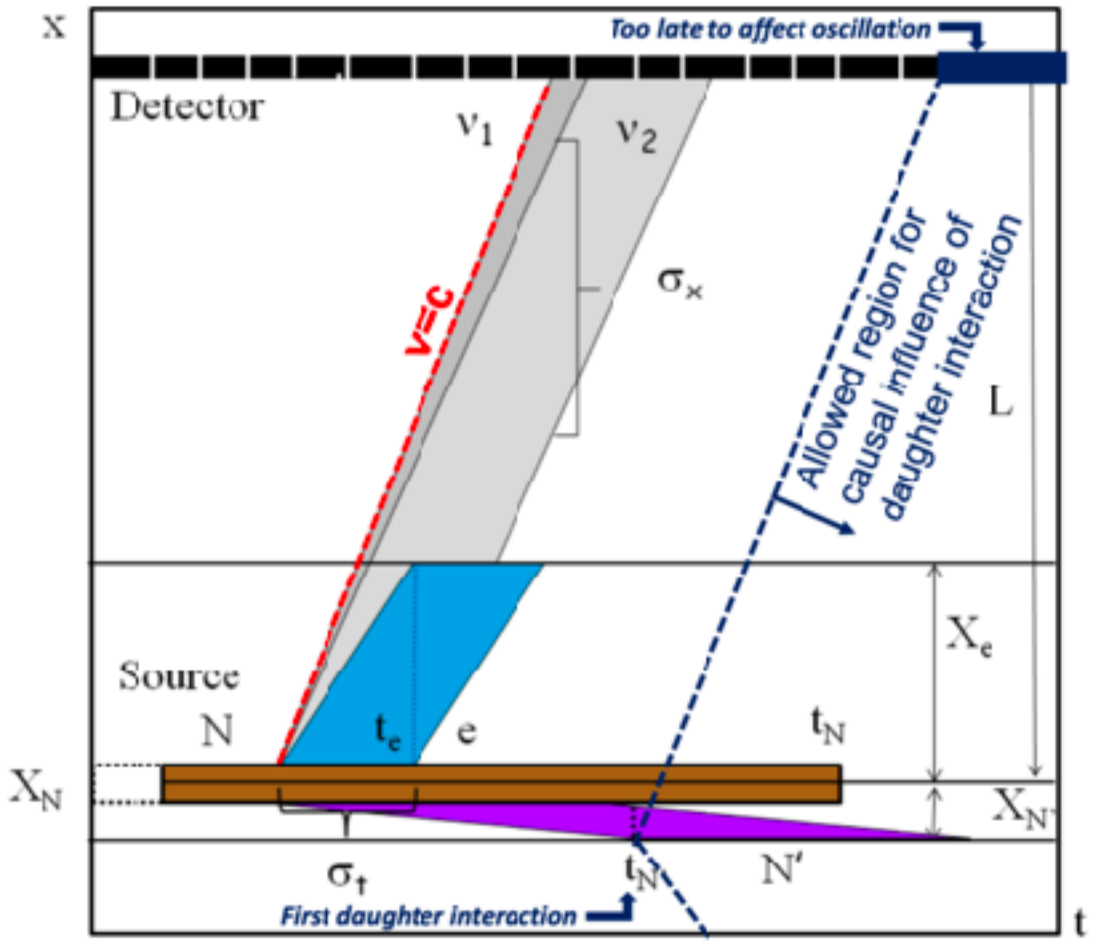
Decoherence

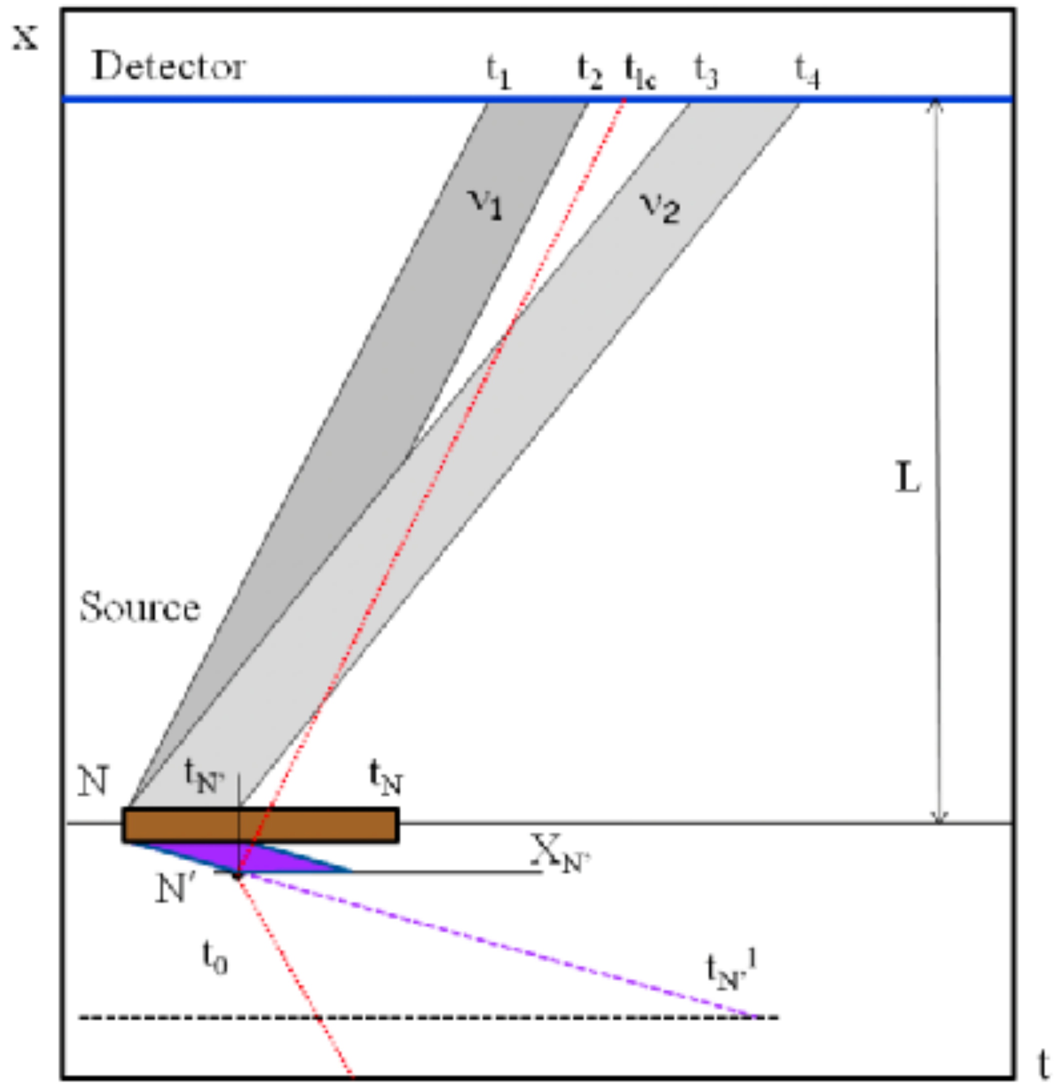
- And oscillation frozen

Neutrino decoherence under hot debate

- Akhmedov, Smirnov (2022): "Damping of neutrino oscillations, decoherence and the lengths of neutrino wave packets"
- Jones (2022): "Comment on "Damping of neutrino oscillations, decoherence and the lengths of neutrino wave packets""
- Akhmedov, Smirnov (2022): "Reply to "Comment on "Damping of neutrino oscillations, decoherence and the lengths of neutrino wave packets"""



Jones (2022)



Akhmedov, Smirnov (2022)

詳しくは三谷さんの修論を

What we do

- So far, only neutrino wave function: $\langle x \,|\, X, \boldsymbol{P}, \sigma \rangle$
- We, for the first time, treated both production and detection:

$$\left\langle X', \boldsymbol{P}', \sigma' \,\middle|\, X, \boldsymbol{P}, \sigma \right\rangle$$

What we obtained: general result

$$P(\alpha \to \beta) = \left(\frac{\sigma_{\rm red}}{\sigma_{\rm sum}}\right)^{\frac{d}{2}} e^{-\sigma_{\rm red}(\boldsymbol{P}_{\rm D}-\boldsymbol{P}_{\rm S})^2} \sum_{I,J} U_{\alpha I} U^*_{\beta I} U^*_{\alpha J} U_{\beta J} e^{i\left(\bar{E}_I - \bar{E}_J\right)T}$$
$$\times \frac{\exp\left[-\frac{\boldsymbol{L}_\perp^2}{\sigma_{\rm sum}} - \frac{\left(L_\parallel - \frac{\bar{v}_I + \bar{v}_J}{2}T\right)^2}{\sigma_{\rm sum}} - \frac{(\bar{v}_I - \bar{v}_J)^2}{4\sigma_{\rm sum}}T^2\right]}{C_I C_J^*},$$

$$\sigma_{\rm sum} := \sigma_{\rm S} + \sigma_{\rm D}, \qquad \sigma_{\rm red} := \frac{\sigma_{\rm S}\sigma_{\rm D}}{\sigma_{\rm S} + \sigma_{\rm D}},$$

$$C_I := \left(1 - i\frac{T}{\sigma_{\rm sum}\bar{E}_I}\right)^{\frac{d-1}{2}} \sqrt{1 - i\frac{\left(1-\bar{v}_I^2\right)T}{\sigma_{\rm sum}\bar{E}_I}}$$

- Larger (smaller) spatial width counts for positions (momenta)
- $L_{/\!/} \sim T$
- Exponential suppression for $T > L_{{\rm coh},IJ} \sim \sqrt{\sigma_{\rm sum}} / |v_I - v_J|$

Two-flavor example in ultra-relativistic limit

- Decoherence (trivially) disappears at the leading ultra-relativistic limit:

$$P(\alpha \to \alpha) \approx \left(\frac{\sigma_\text{red}}{\sigma_\text{sum}}\right)^{\frac{d}{2}} \frac{e^{-\sigma_\text{red}(\boldsymbol{P}_\text{D}-\boldsymbol{P}_\text{S})^2 - \frac{\boldsymbol{L}_\perp^2}{\sigma_\text{sum}} - \frac{\left(L_\parallel - T\right)^2}{\sigma_\text{sum}}}}{\left(1+\frac{T^2}{\sigma_\text{sum}^2 \bar{P}^2}\right)^{\frac{d-1}{2}}}$$

$$\times \left(\cos^4\theta\, e^{-\frac{m_1^2\left(L_\parallel - T\right)T}{\sigma_\text{sum}\bar{P}^2}} + \sin^4\theta\, e^{-\frac{m_2^2\left(L_\parallel - T\right)T}{\sigma_\text{sum}\bar{P}^2}}\right.$$

$$\left. + 2\cos\left[\left(\bar{E}_1 - \bar{E}_2\right)T\right]\cos^2\theta\sin^2\theta\, e^{-\frac{\left(m_1^2+m_2^2\right)\left(L_\parallel - T\right)T}{2\sigma_\text{sum}\bar{P}^2}}\right).$$

Summary

1. Position space is mass-dependent
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Discussion

- Decoherence (trivially) **disappears** at leading order in **ultra-relativistic limit** ($v_I \to v_J$):
  - $L_{coh,IJ} \sim \sqrt{\sigma_{sum}} / |v_I - v_J| \to \infty$
- What if we take into account **mass dependence** in **energy** and **Dirac wave function**?
  - How is **interaction eigenstate** defined? [Nishiwaki, KO, Wada]
  - Need **Lorentz-covariant packet** [KO, Wada 2023]?
- Where to test **experimentally**? (DUNE, etc. appear challenging)


Thank you!