eV ダークマター

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Wen Yin (Tohoku University)

Axion domain wall formations and implications

Wen Yin (Tohoku University)

Based on 2012.11576, 2211.06849, 2205.05083, 2306.17146

In collaboration with D. Gonzalez, N. Kitajima, F. Kozai, J. Lee, K. Murai, F. Takahashi,

伝えたいこと

- **ストリング理論由来**のアクションから**宇宙ストリング**を伴わない**ドメインウォール**ができる。
- ドメインウォール問題を解決する宇宙に起こりうる現象の探索は**ストリング理論へのアプローチの可能性**

Plan

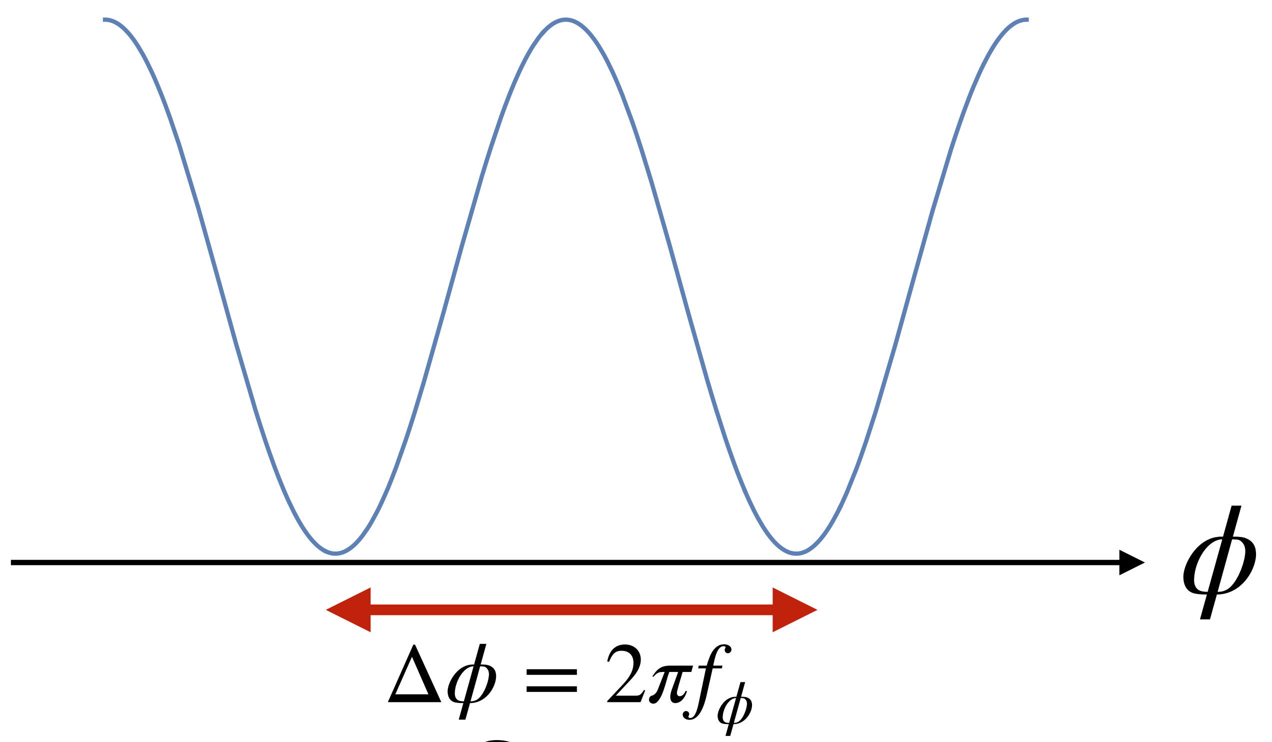
- Introduction
- Axion DW network from inflationary fluctuations is stable —String axion DWs without a string—
- Cosmological implications
- Conclusions

1. Introduction

What is axion, ϕ?

Axion has a periodic field space satisfying $\phi \leftrightarrow \phi + 2\pi f_\phi$,
and an approximate shift symmetry, $\phi \rightarrow \phi + C$.

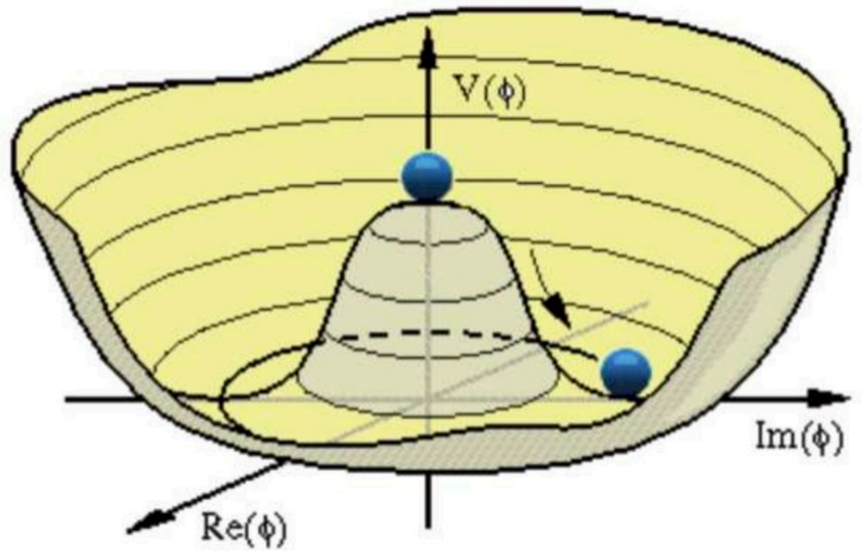
$$V(\phi) = V(\phi + 2\pi f_\phi)$$

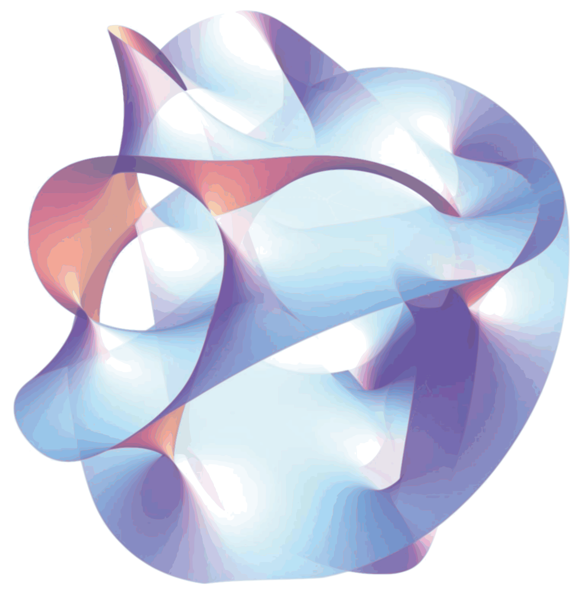


Axion gets periodic potential and small mass from non-perturbative effect.

UV completions:

- U(1)SSB
- String/M theory



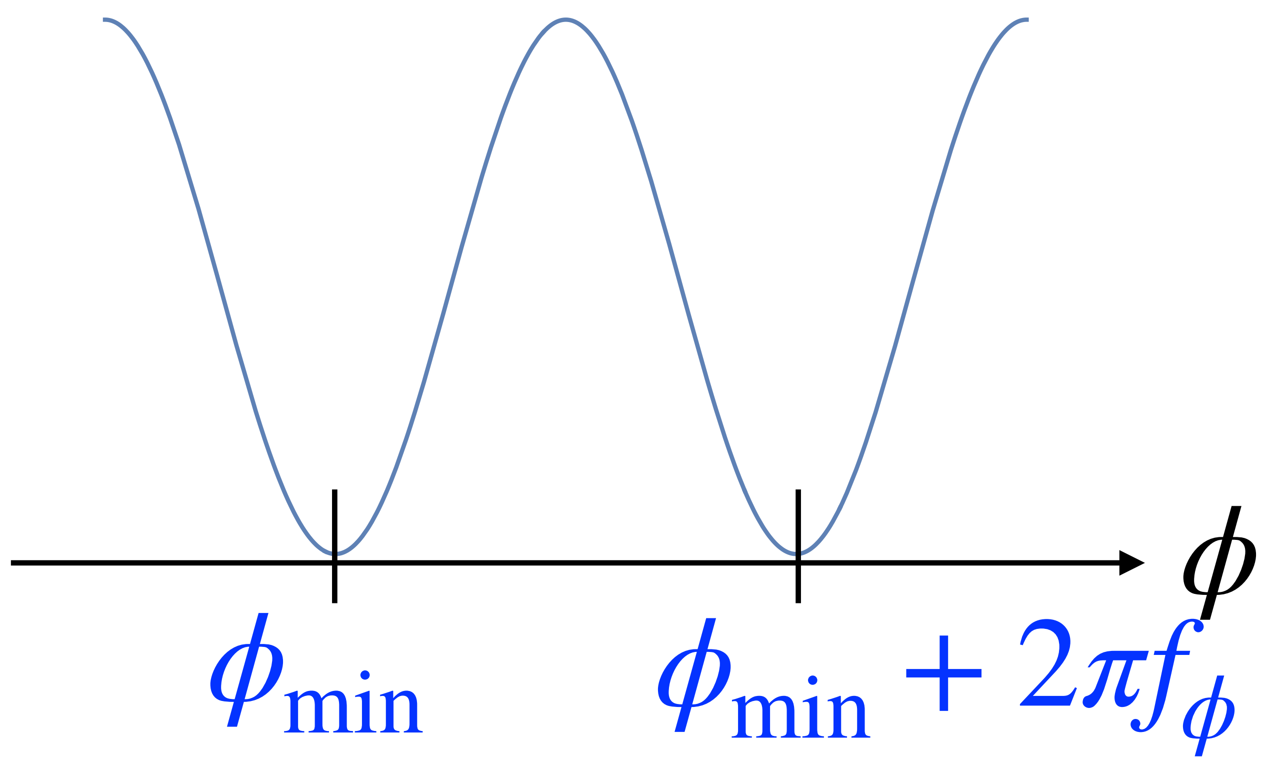


A stable domain wall (DW) configuration must exist in axion theories!

Periodicity predicts degenerate vacua.

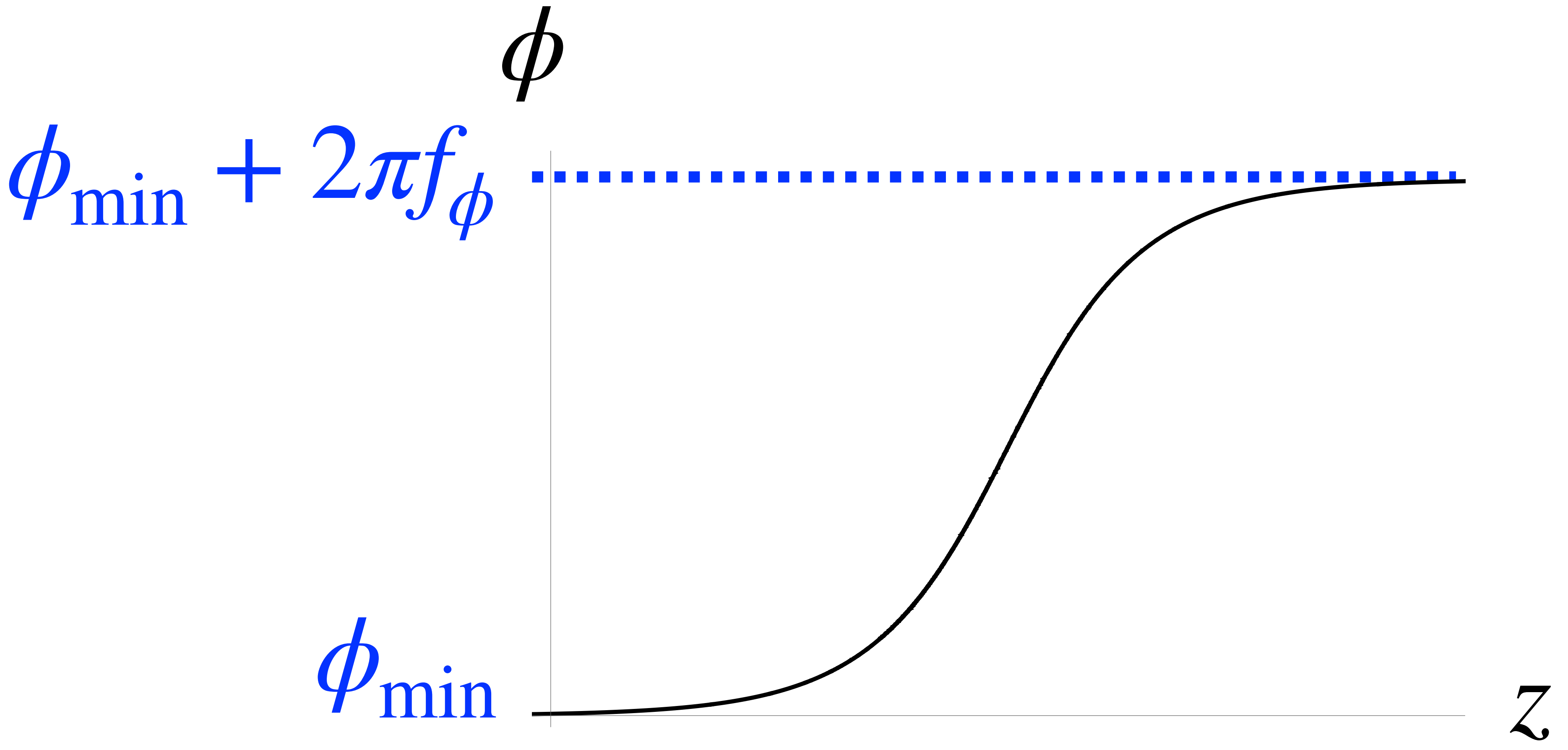
$$V(\phi) = V(\phi + 2\pi f_\phi)$$

$e.g.\ V(\phi) = V_0(1 - \cos(\phi/f_\phi))$



Configuration connecting the vacua gives domain wall.

$$dz = d\phi/\sqrt{2V(\phi)}$$



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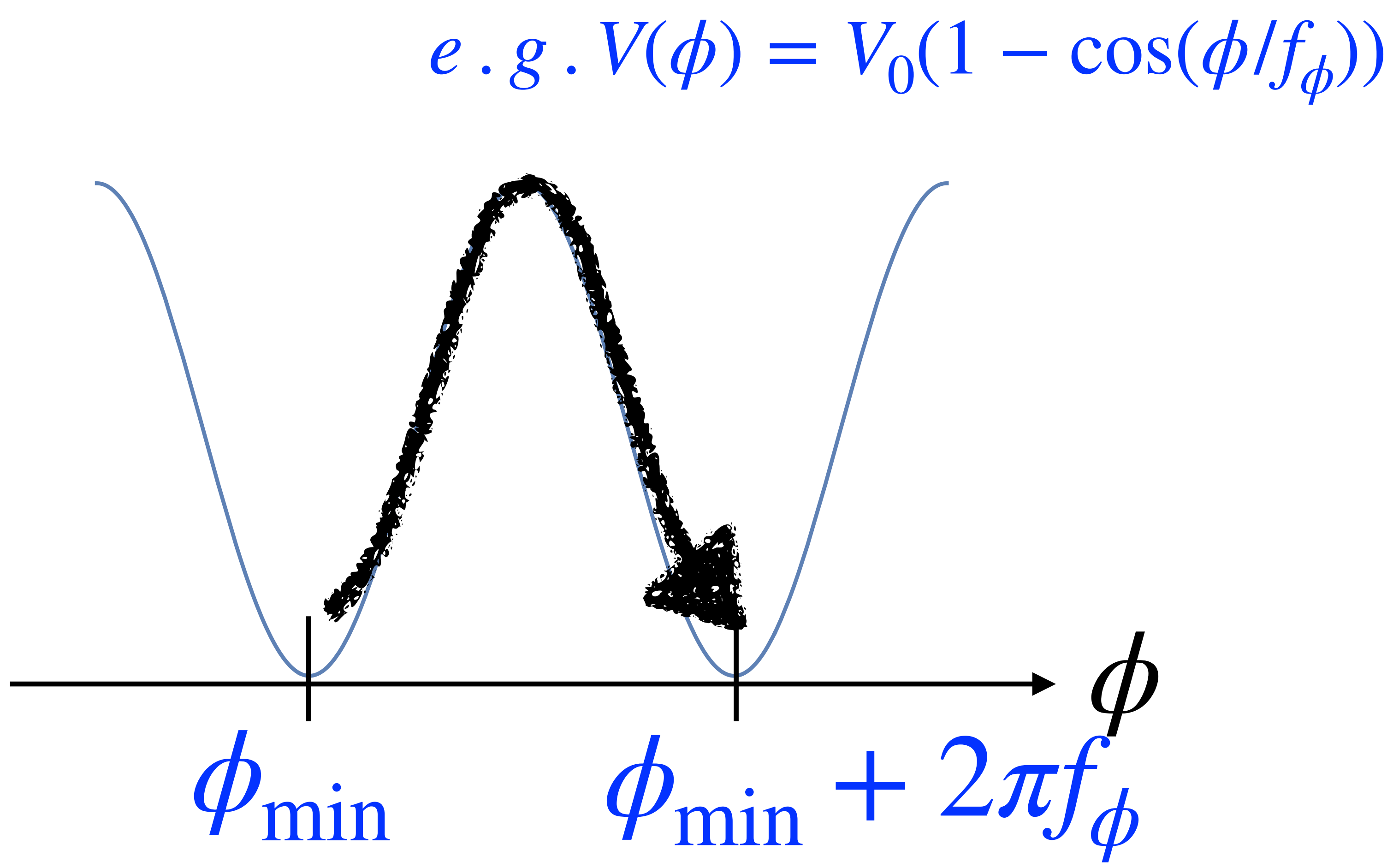

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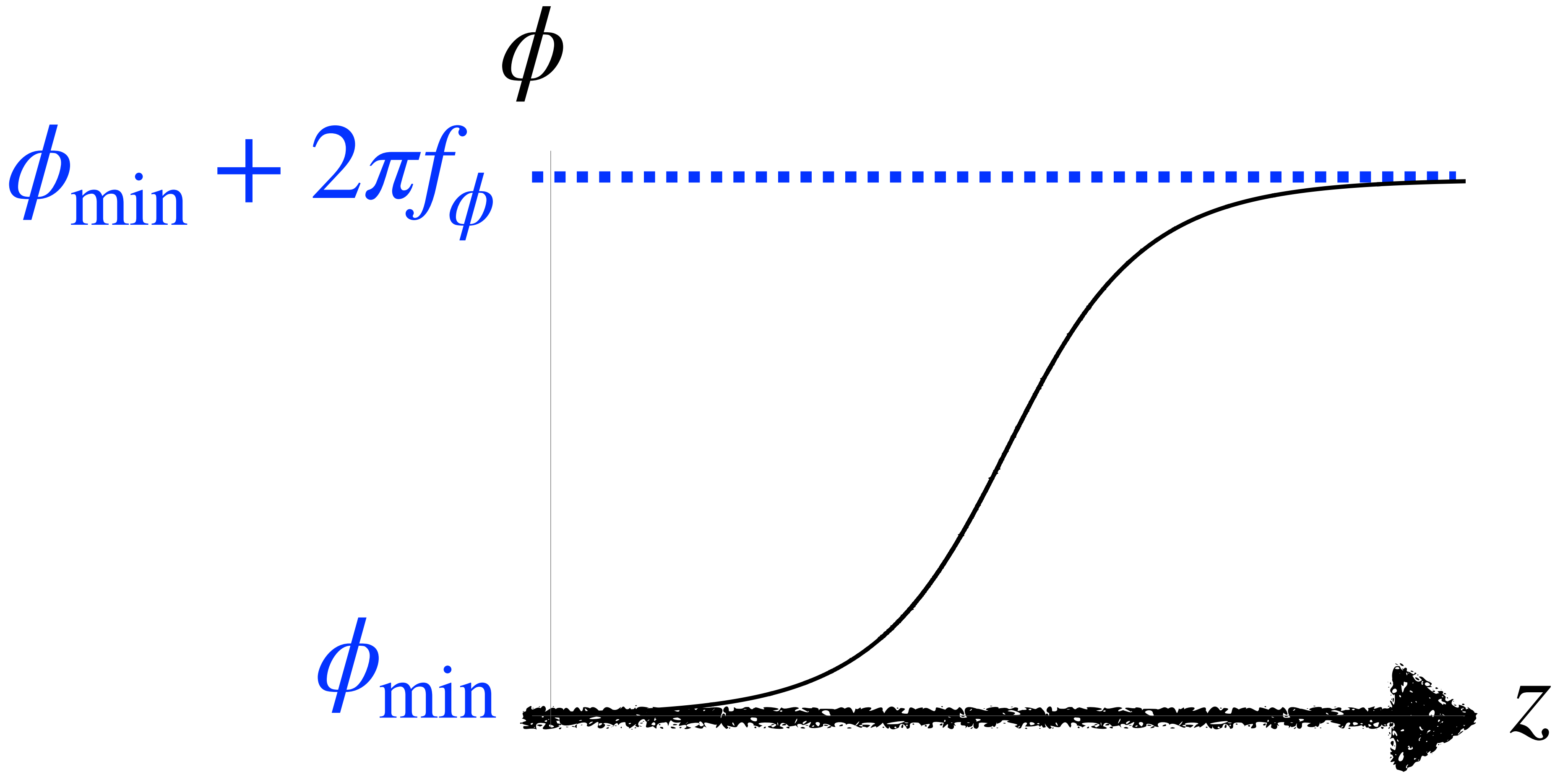

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Number of degenerate vacua in $[0,2\pi f_\phi)$ is DW number, $N_{\rm DW}$.

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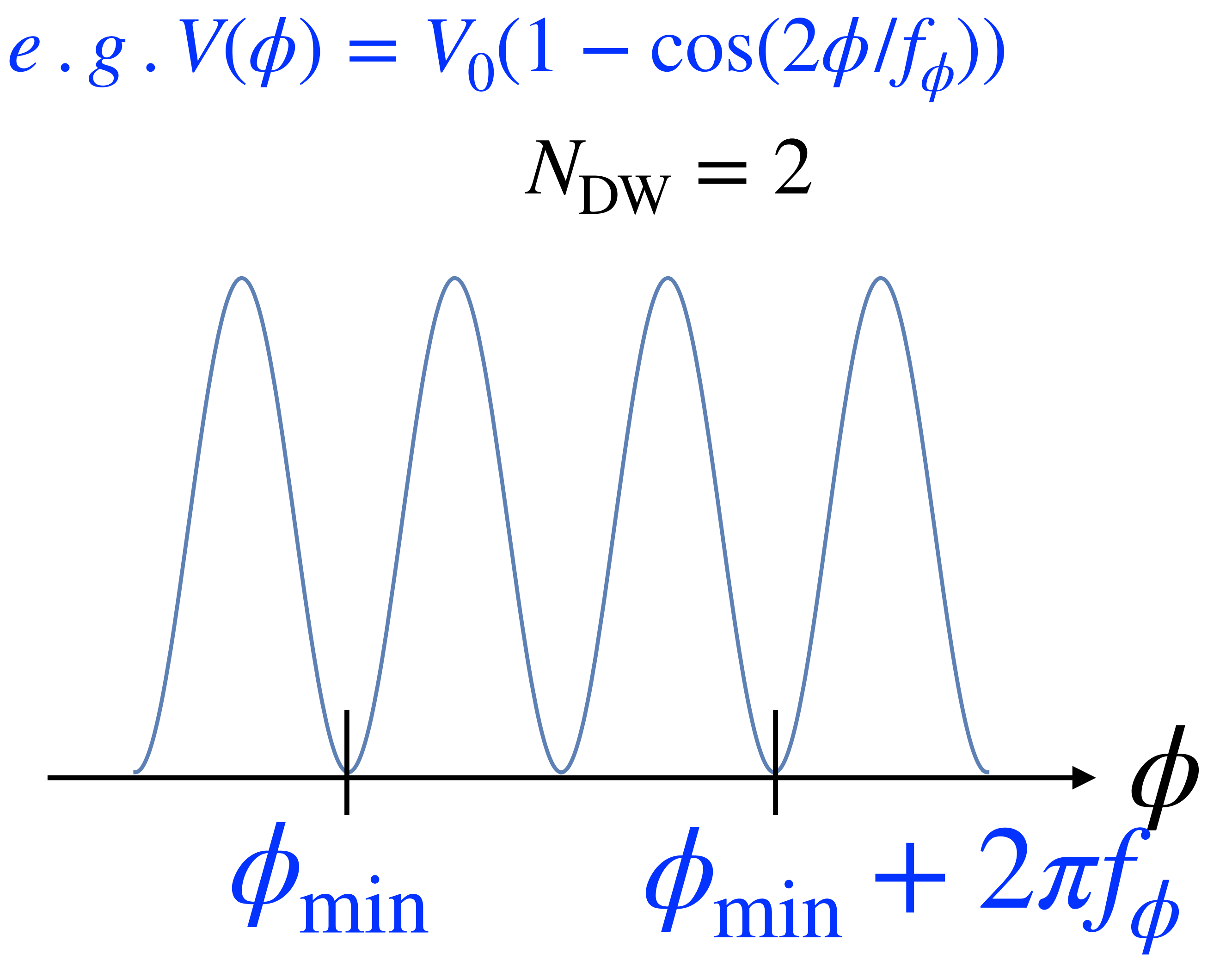

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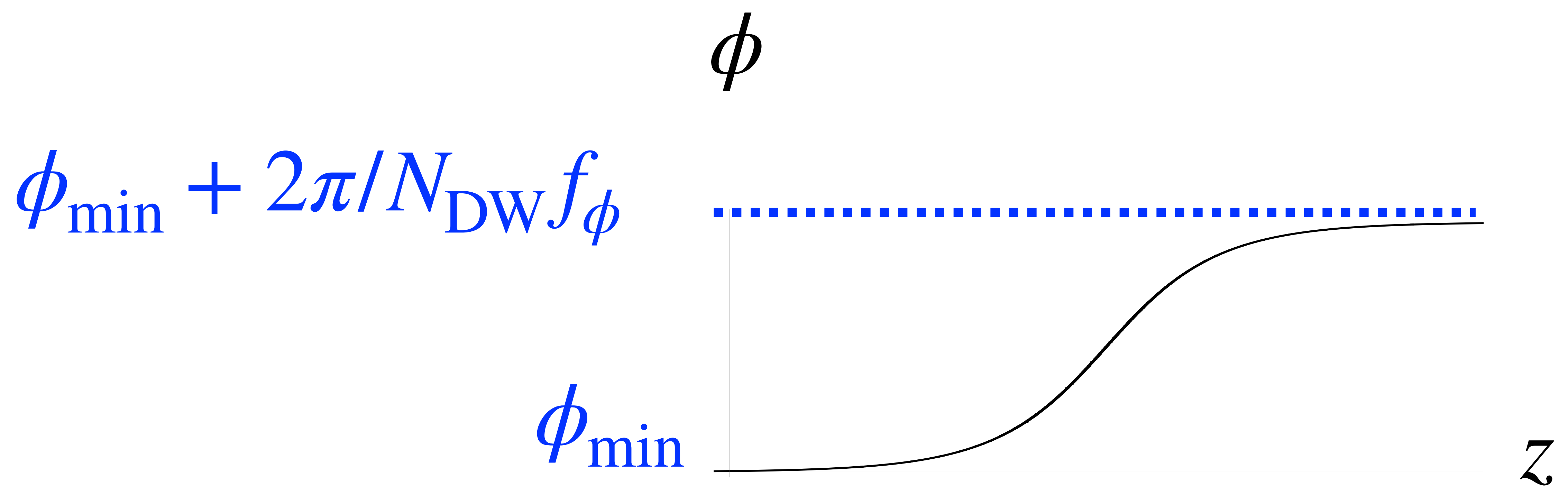

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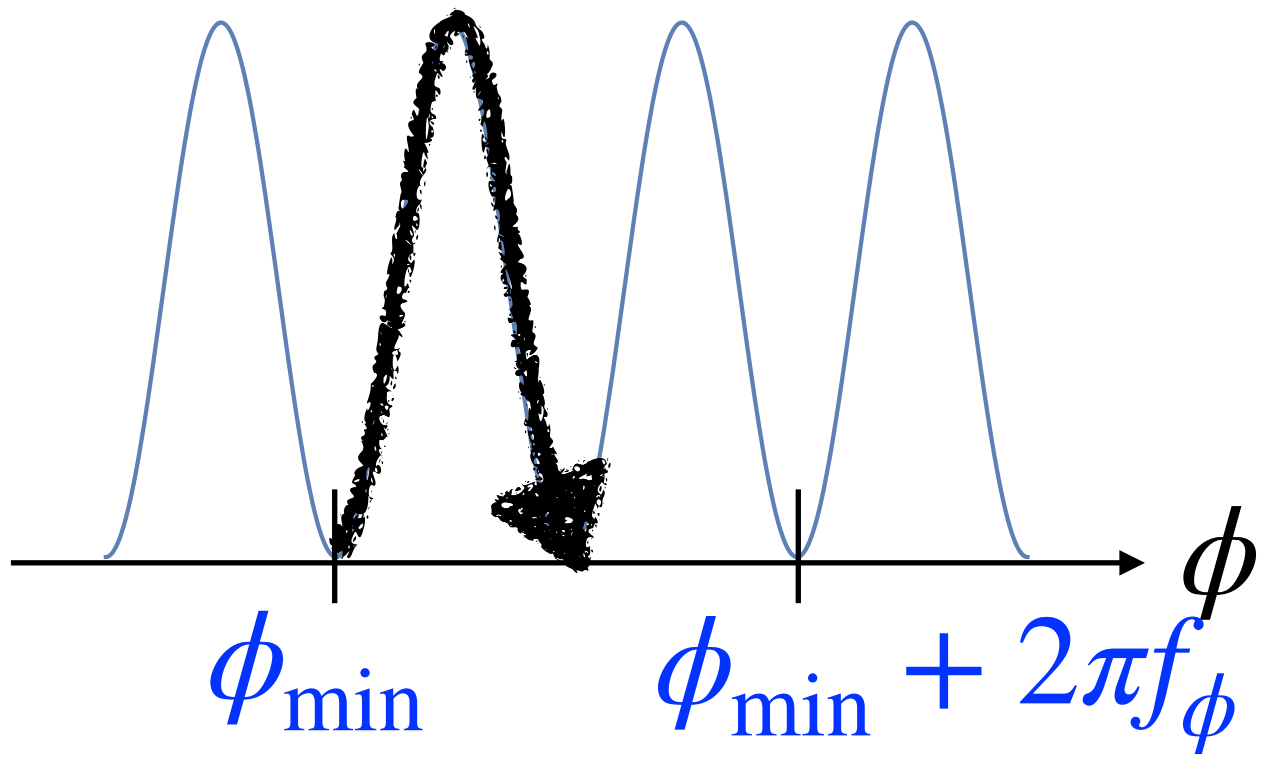

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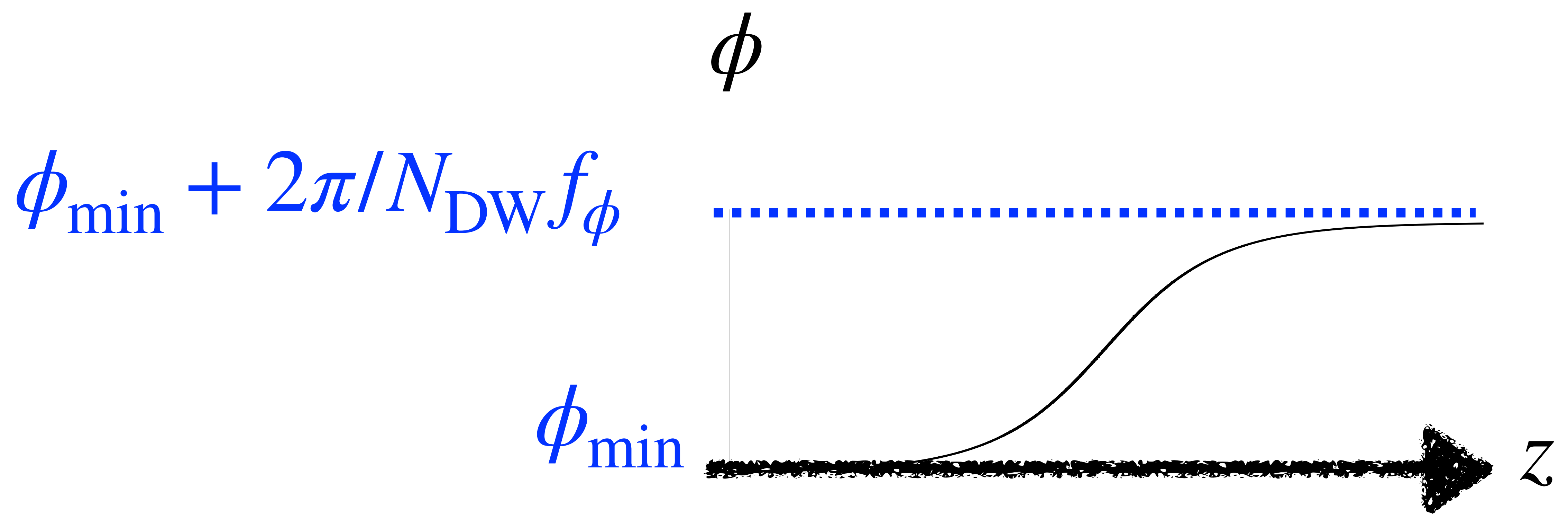
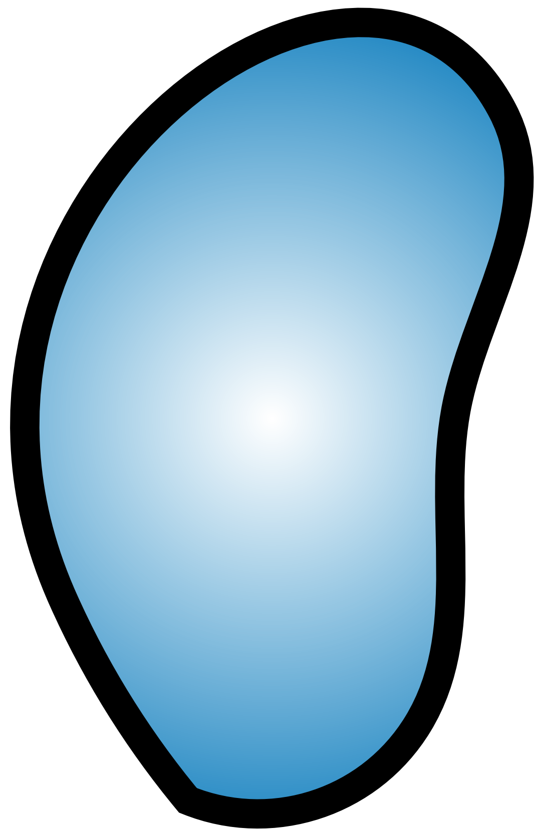


DW network formation in the early Universe

Phase transition of the approximate U(1) Kibble, Zurek

Strings + DWs



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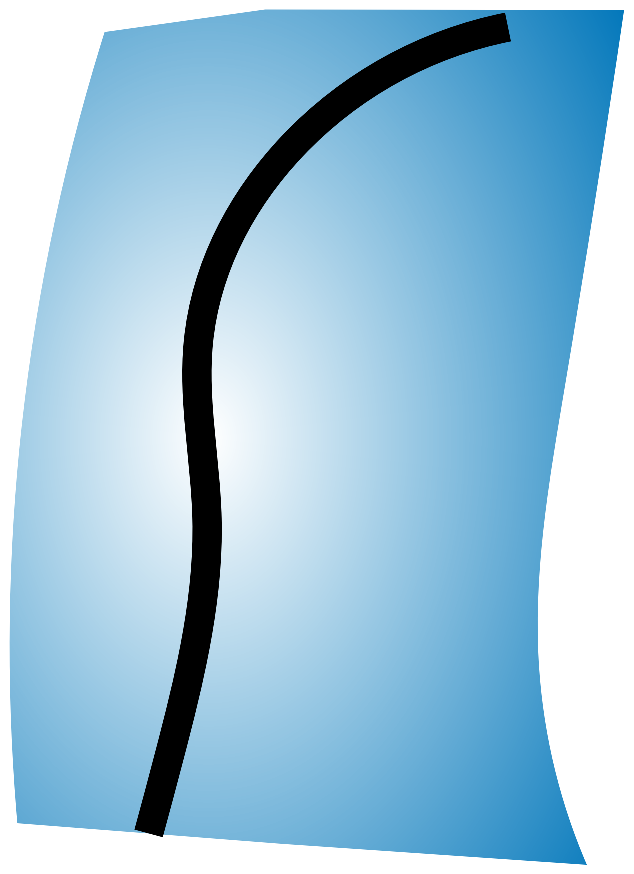


$N_{\rm DW}=2$

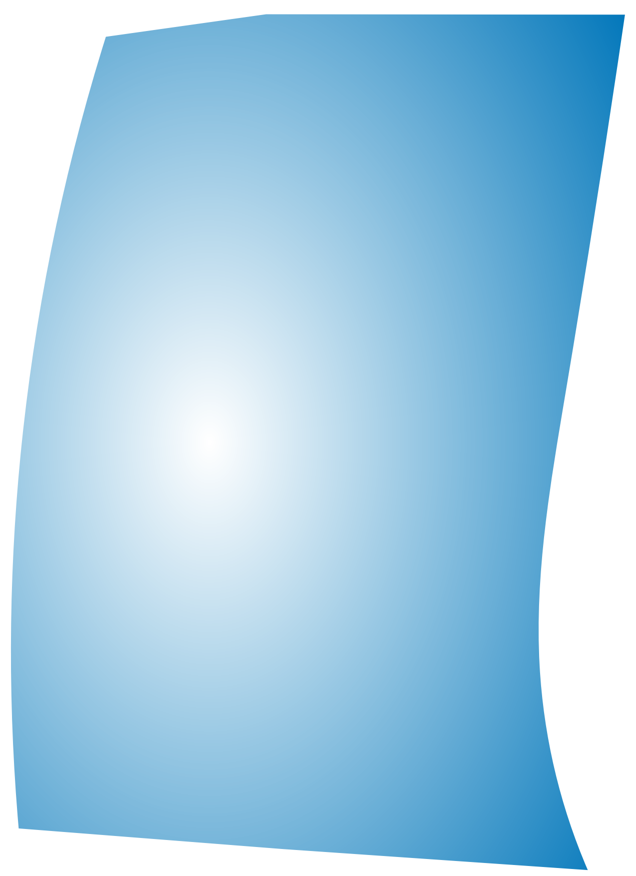
$N_{\rm DW}\geq 2$:stable string-DW network

Applicable to:

- U(1)SSB $V\propto 1-\cos[N_{\rm DW}\phi/f_\phi]$

Inflationary fluctuation in axion EFT.

DWs without a string



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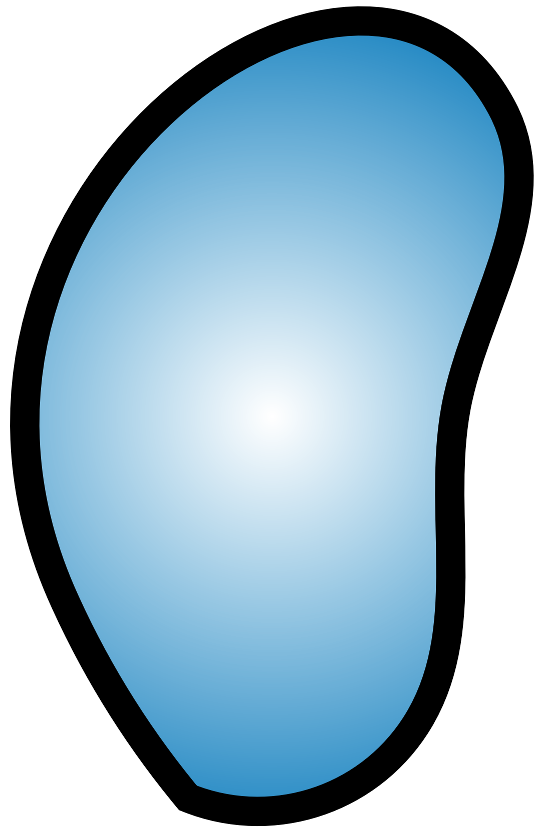

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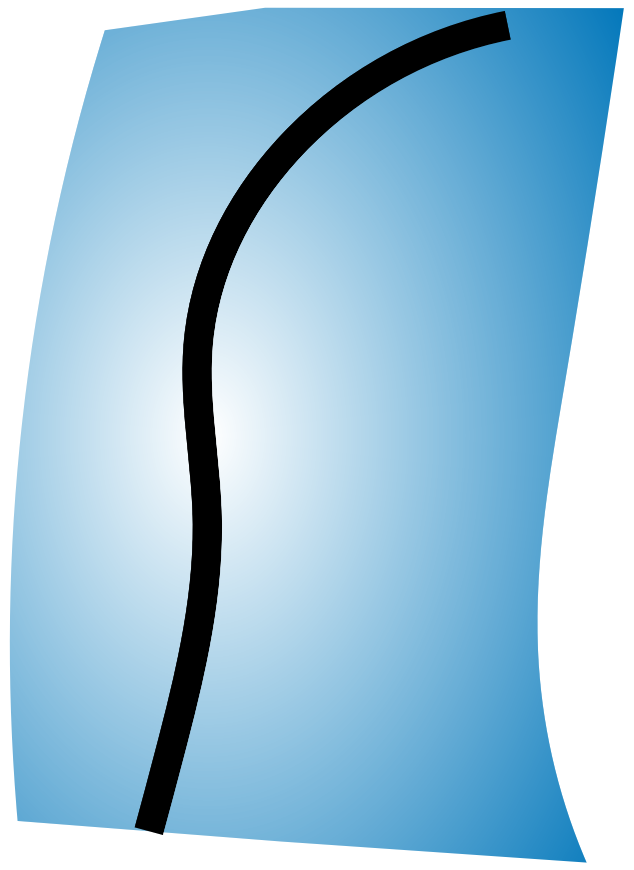

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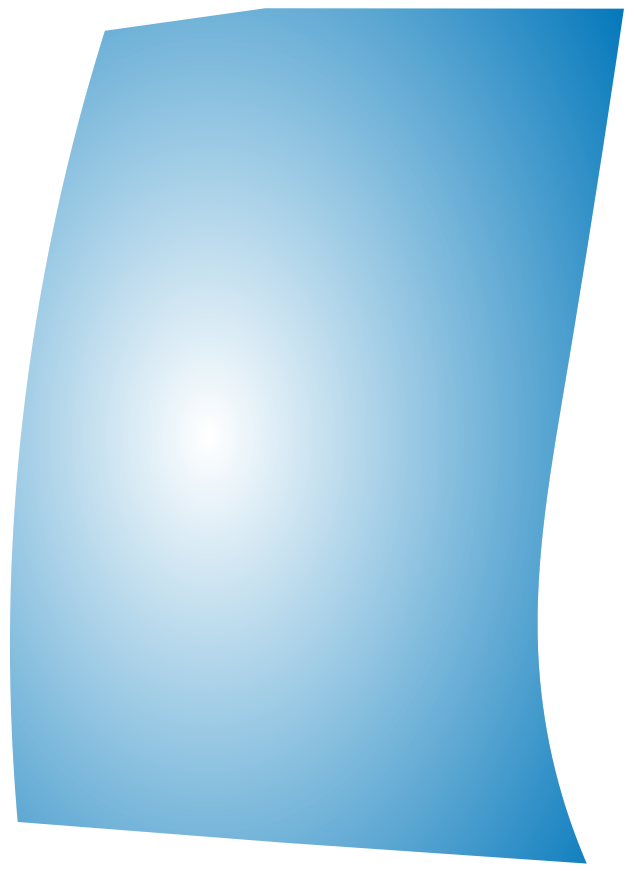

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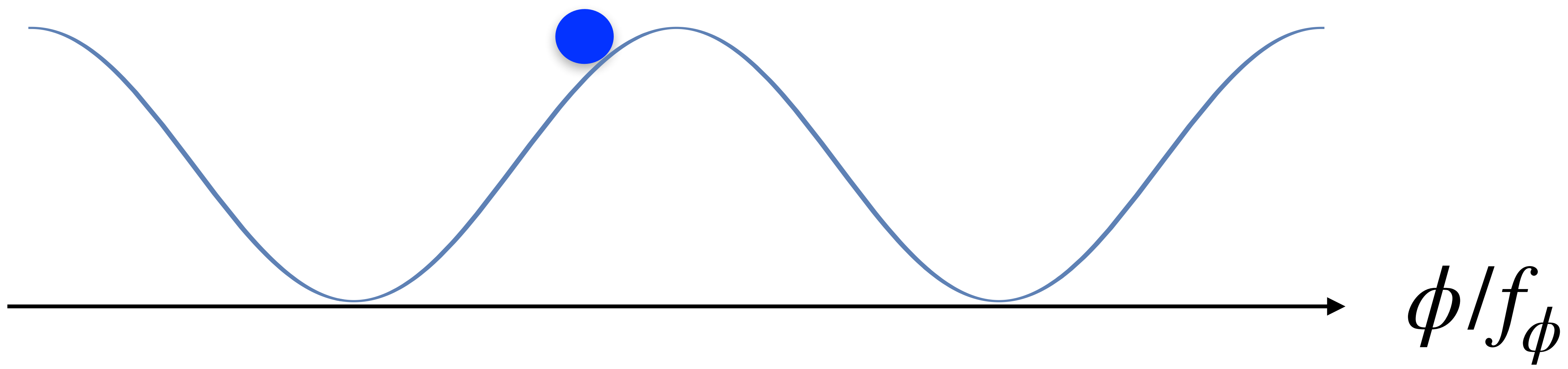
DW from inflationary fluctuations

Let us consider axion EFT (U(1) symmetry never restore.)

- If $m_\phi \ll H_{\rm inf}$, after inflation, axion field values are naturally different at different position.
- In the observable Universe, the values follow a typical distribution around an averaged field value.



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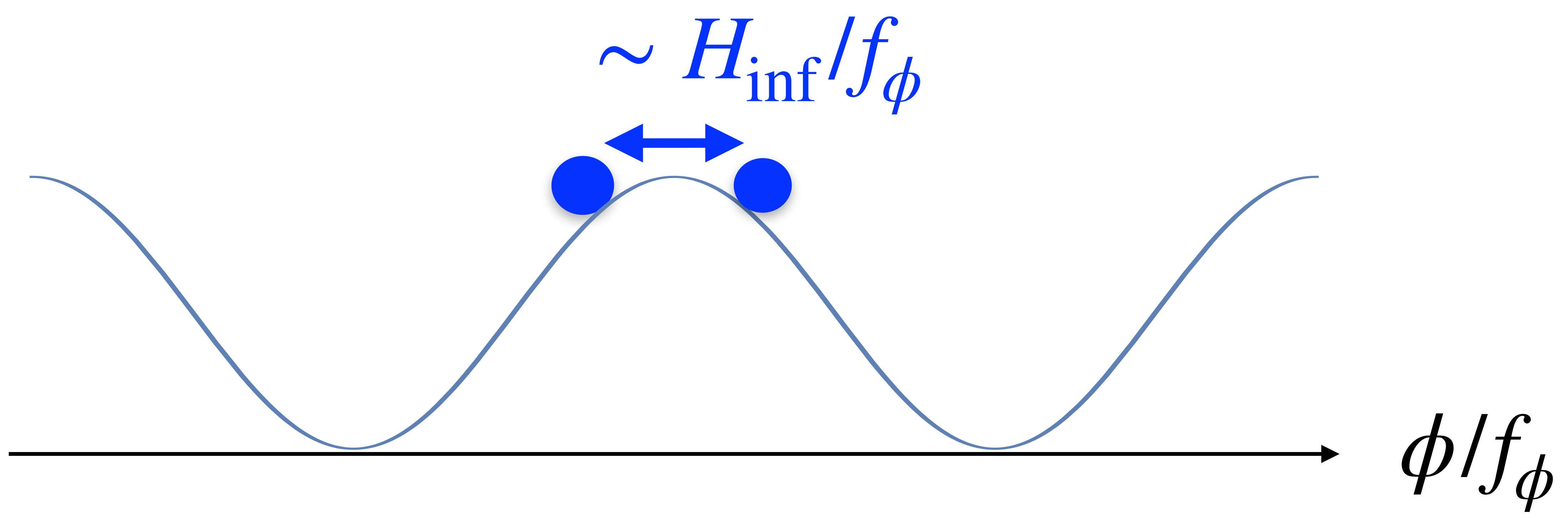

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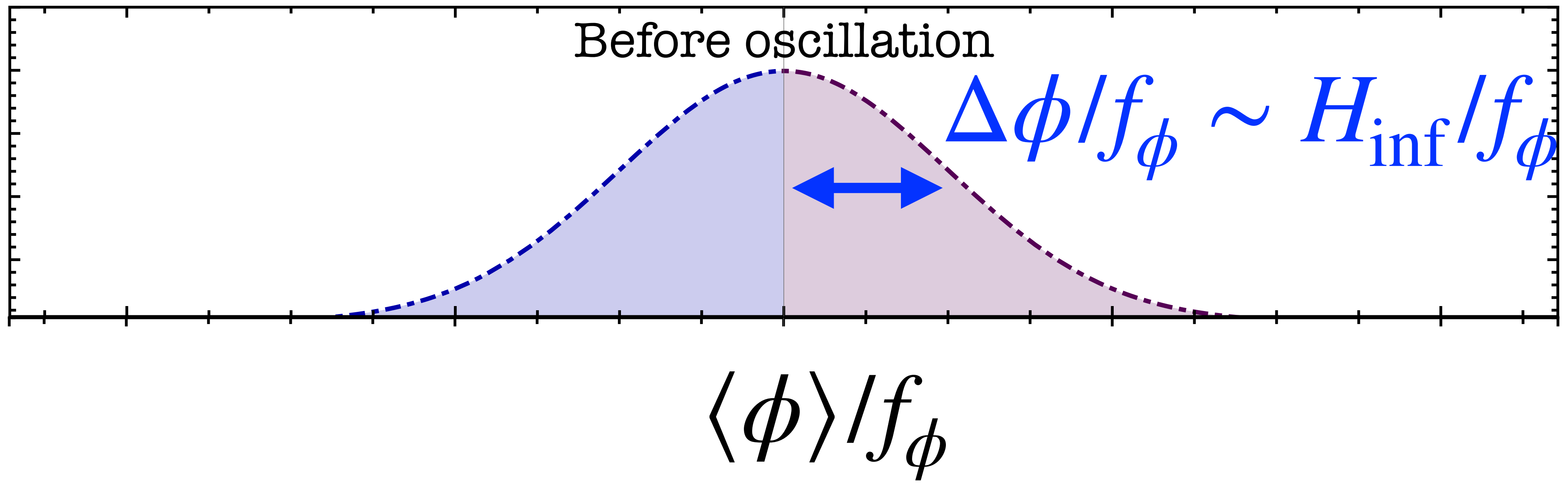

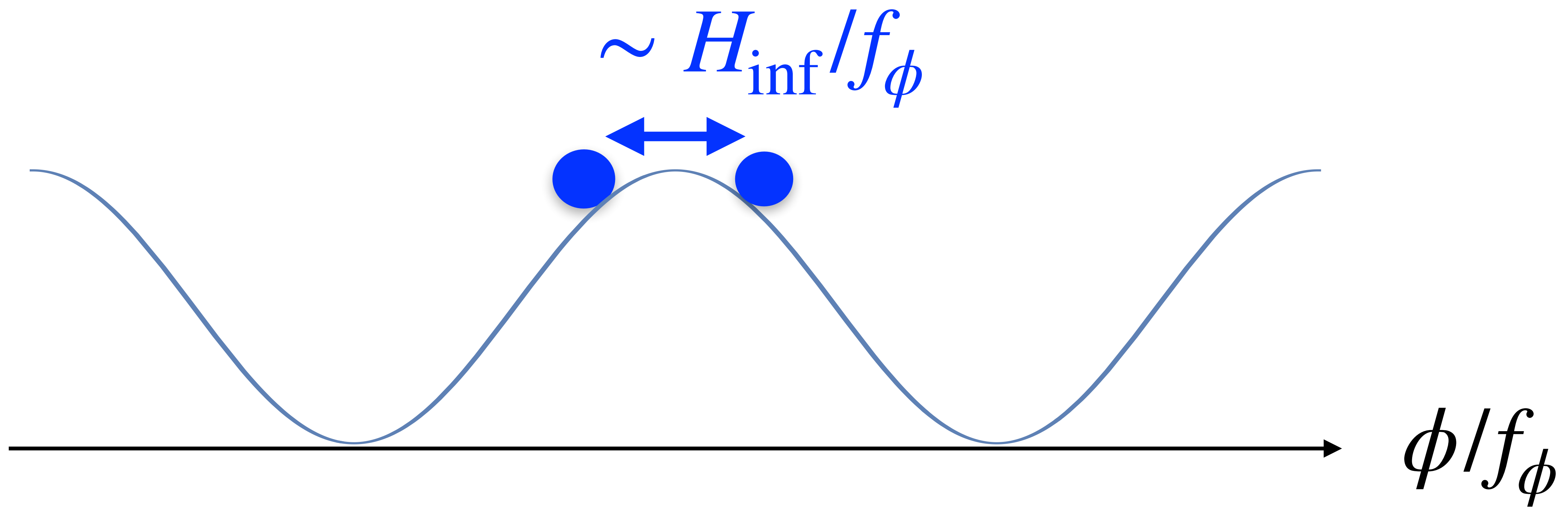

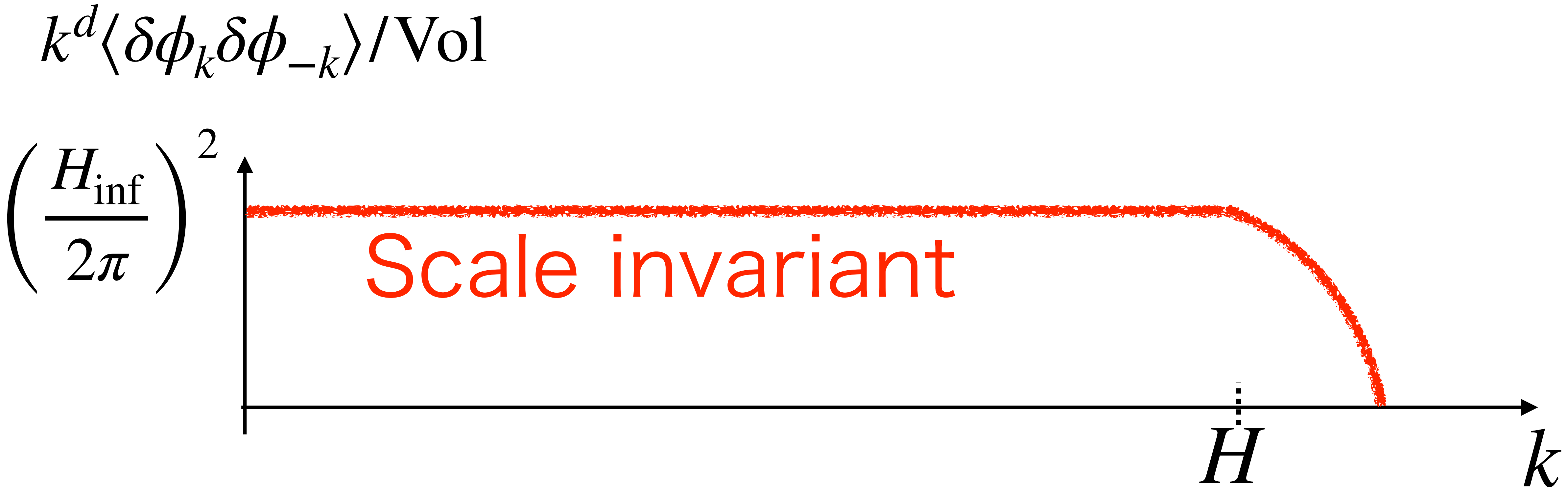

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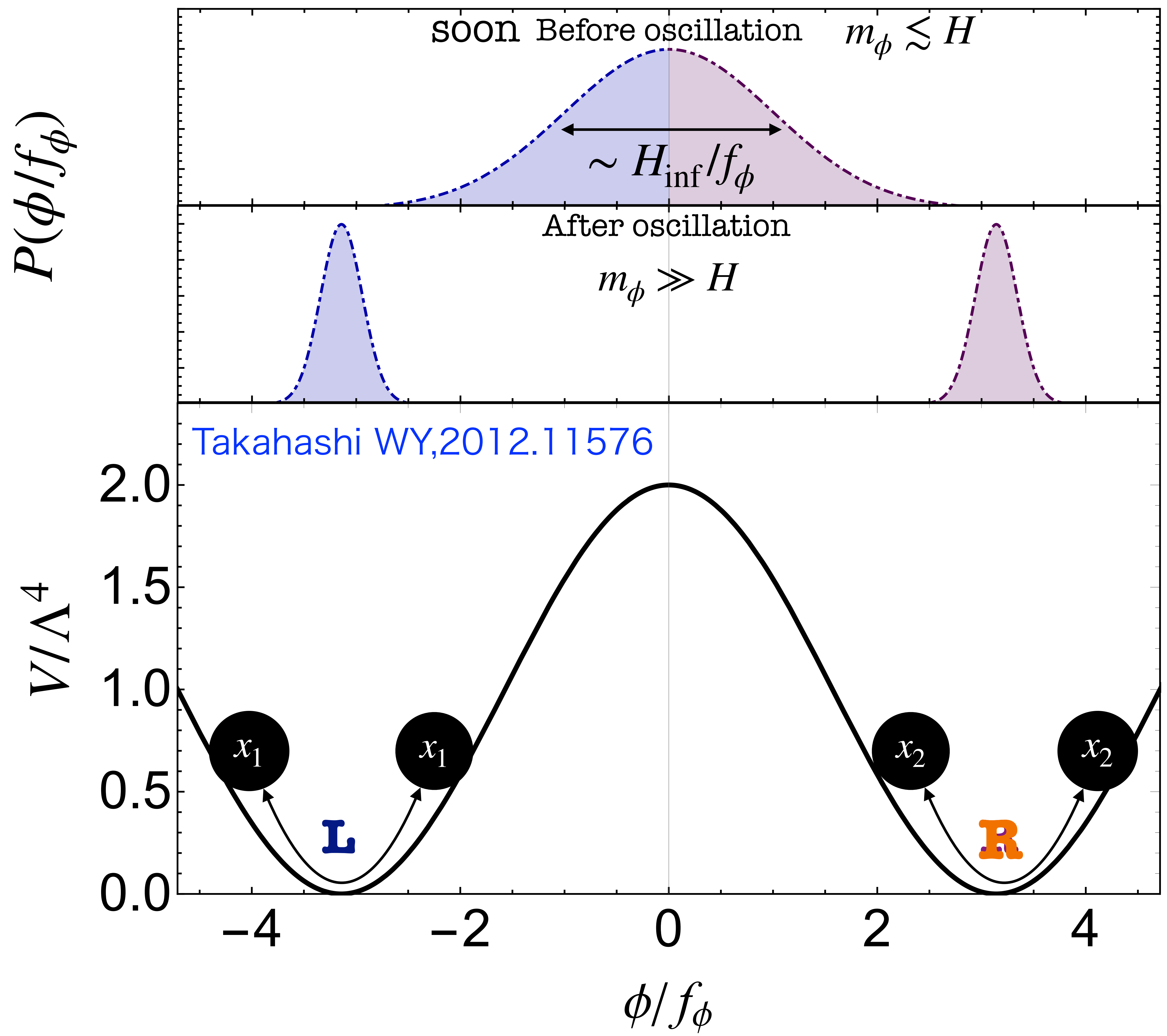
- **When $m_\phi \sim H$, domain walls form!**
- **O(1) DWs in 1 Hubble volume.**
- **No cosmic string!**



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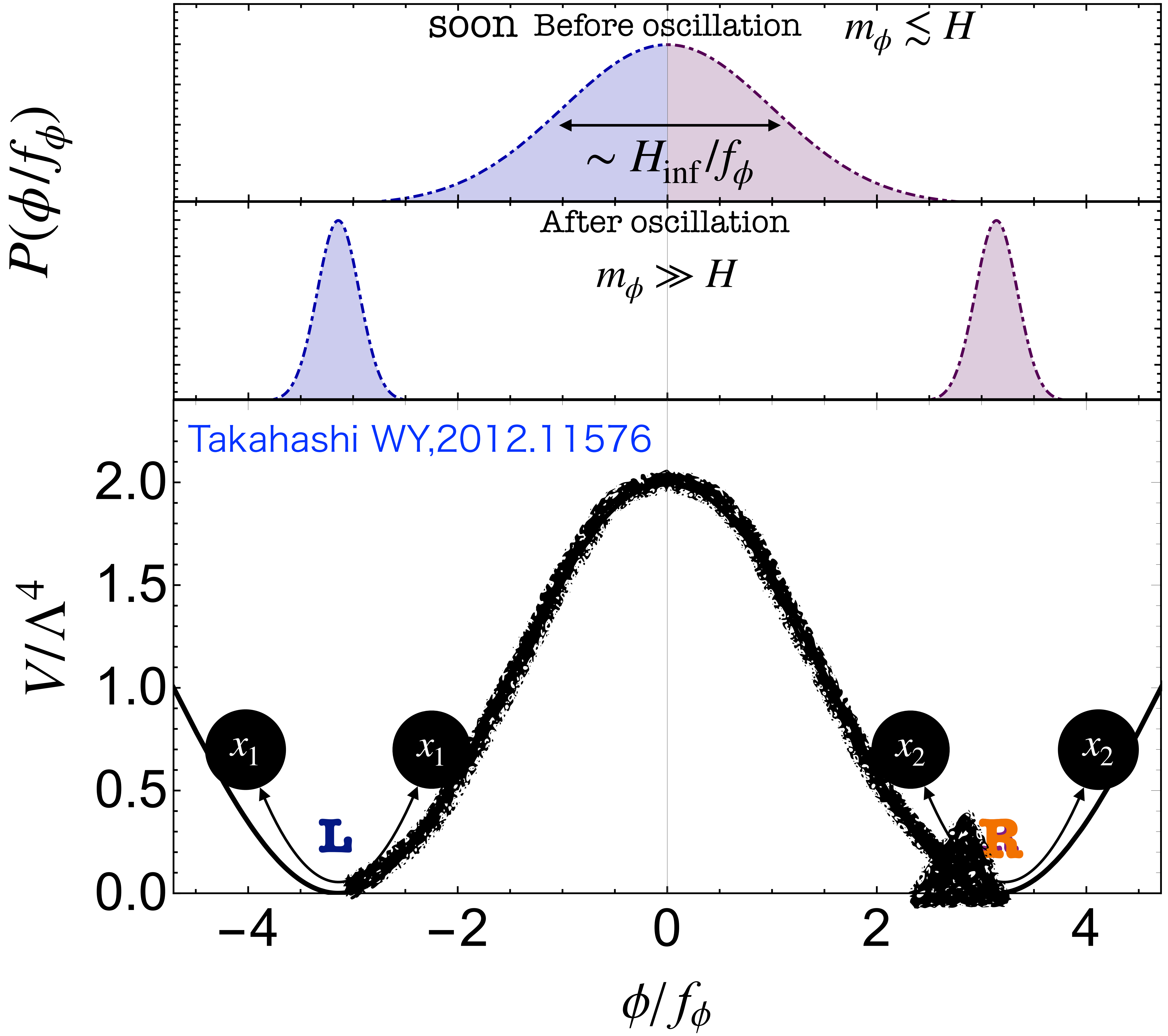
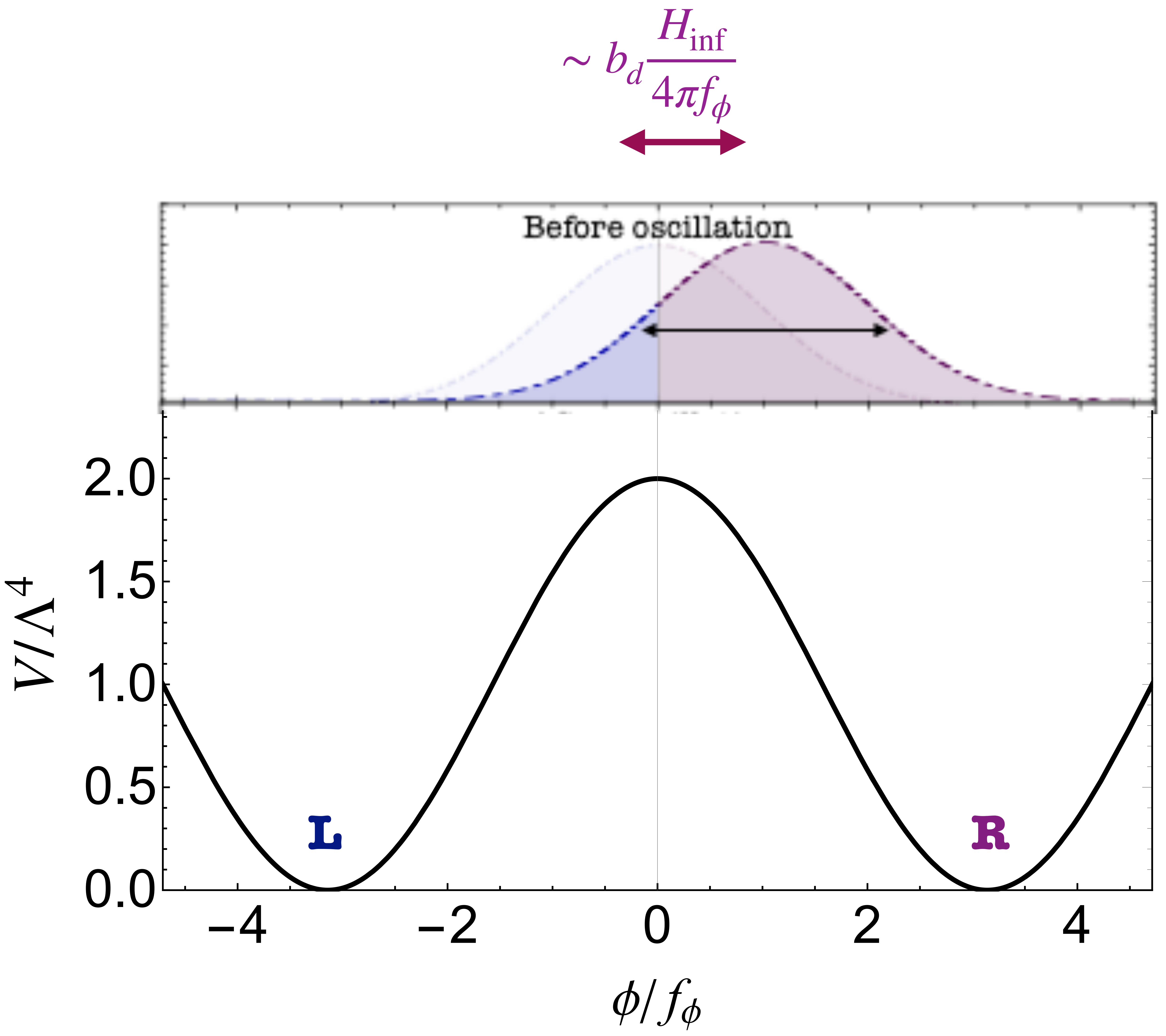


It was considered that stable DW from inflationary fluctuation requires serious fine-tuning.

e.g. Lalak et al, 95, Coulson, et al 96

-Random jump naturally provides $<\phi> \neq 0$.

- $<\phi> \neq 0$, i.e. population bias, the DW network soon decay.



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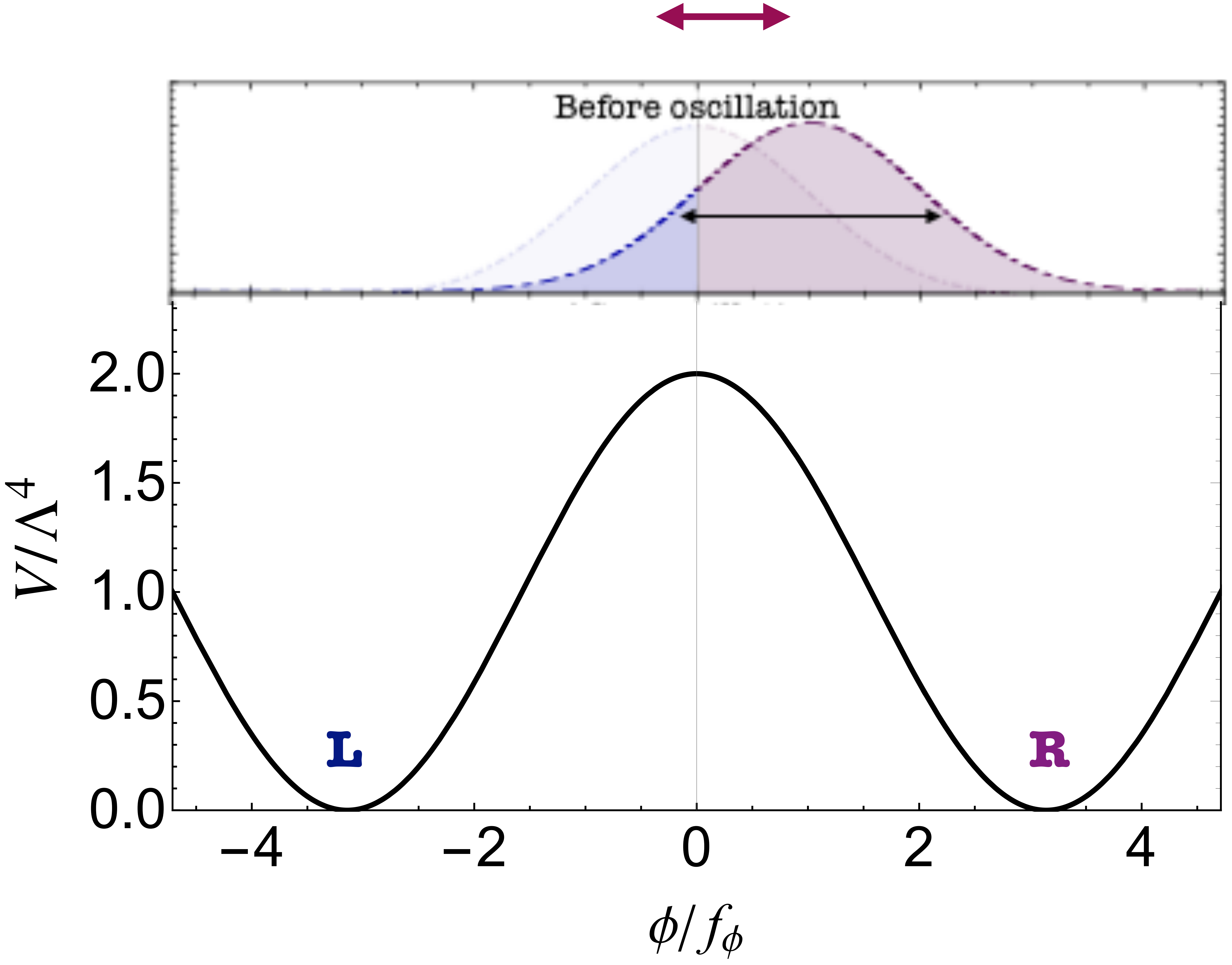
No significant tuning!

e.g. Lalak et al, 95, Coulson, et al 96

$\sim b_d \frac{H_{\rm inf}}{4\pi f_\phi}$



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Not true for inflationary fluctuation!

- 2. Axion DW network from inflationary fluctuations is stable

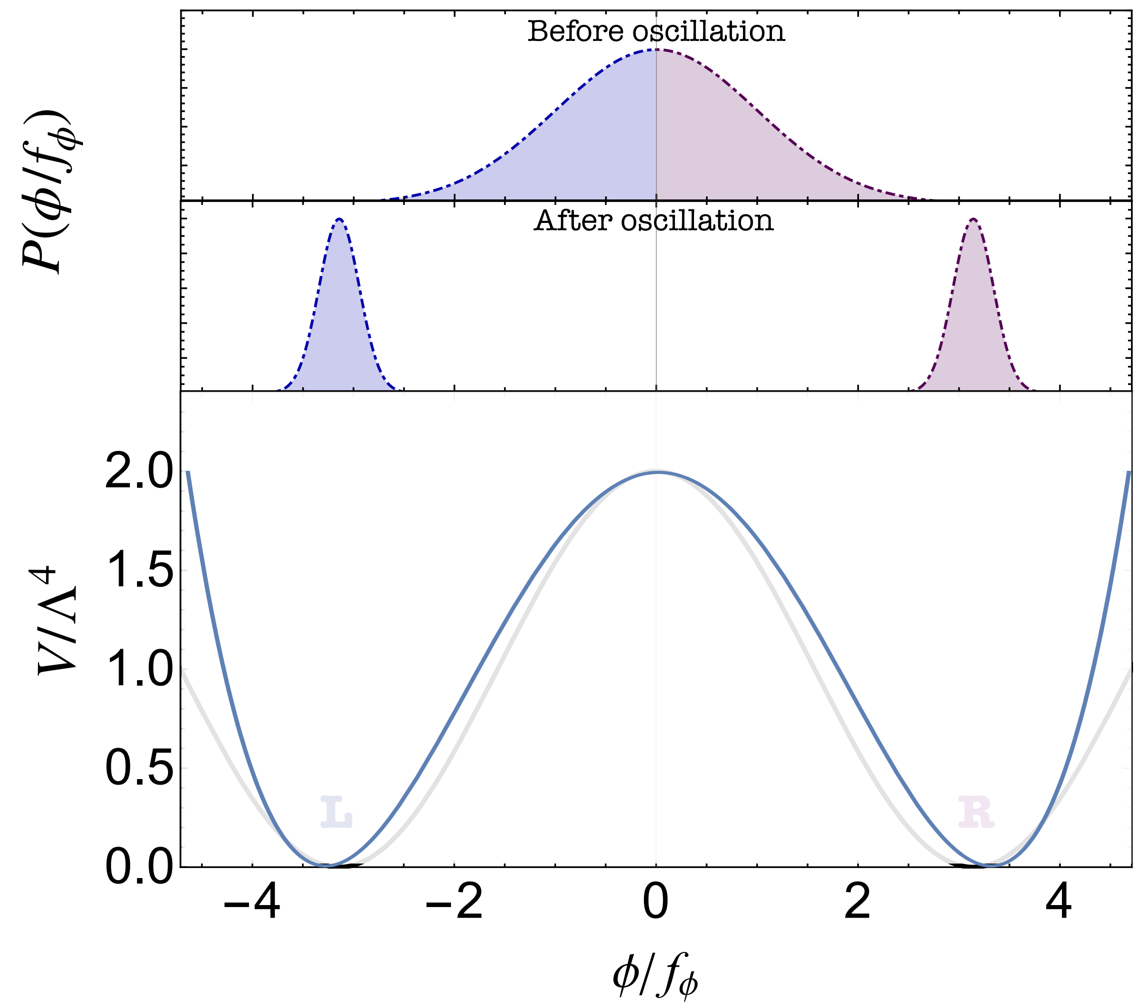
—String axion DWs without a string—

Gonzalez, Kitajima, Takahashi, WY, 2211.06849

Lattice simulation of DW evolutions

We use Z_2symmmetric, ϕ^4 theory to approximate system.

$$V(\phi) = V_0 - \frac{1}{2}m_0^2\phi^2 + \frac{\lambda}{4}\phi^4 .$$



The essential difference between the two formation mechanisms are **initial conditions of $\phi_{k\neq 0}$ modes.**

Thermal fluctuation

with $\mathrm{k} \ll H$

White noise:

$$k^d < \phi_k \phi_{-k} > \propto k^d$$

Inflationary fluctuation

with $\mathrm{k} \ll H$

Gonzalez, Kitajima, Takahashi, WY, 2211.06849

Scale invariant:

$$k^d < \phi_k \phi_{-k} > \propto \left(\frac{H_{\rm inf}}{2\pi}\right)^2$$

DW network with $<\phi> = 0$ ($b_d = 0$).

Both cases have O(1) DW in a Hubble patch, but **the structures are quite different.**

Gonzalez, Kitajima, Takahashi, WY, 2211.06849

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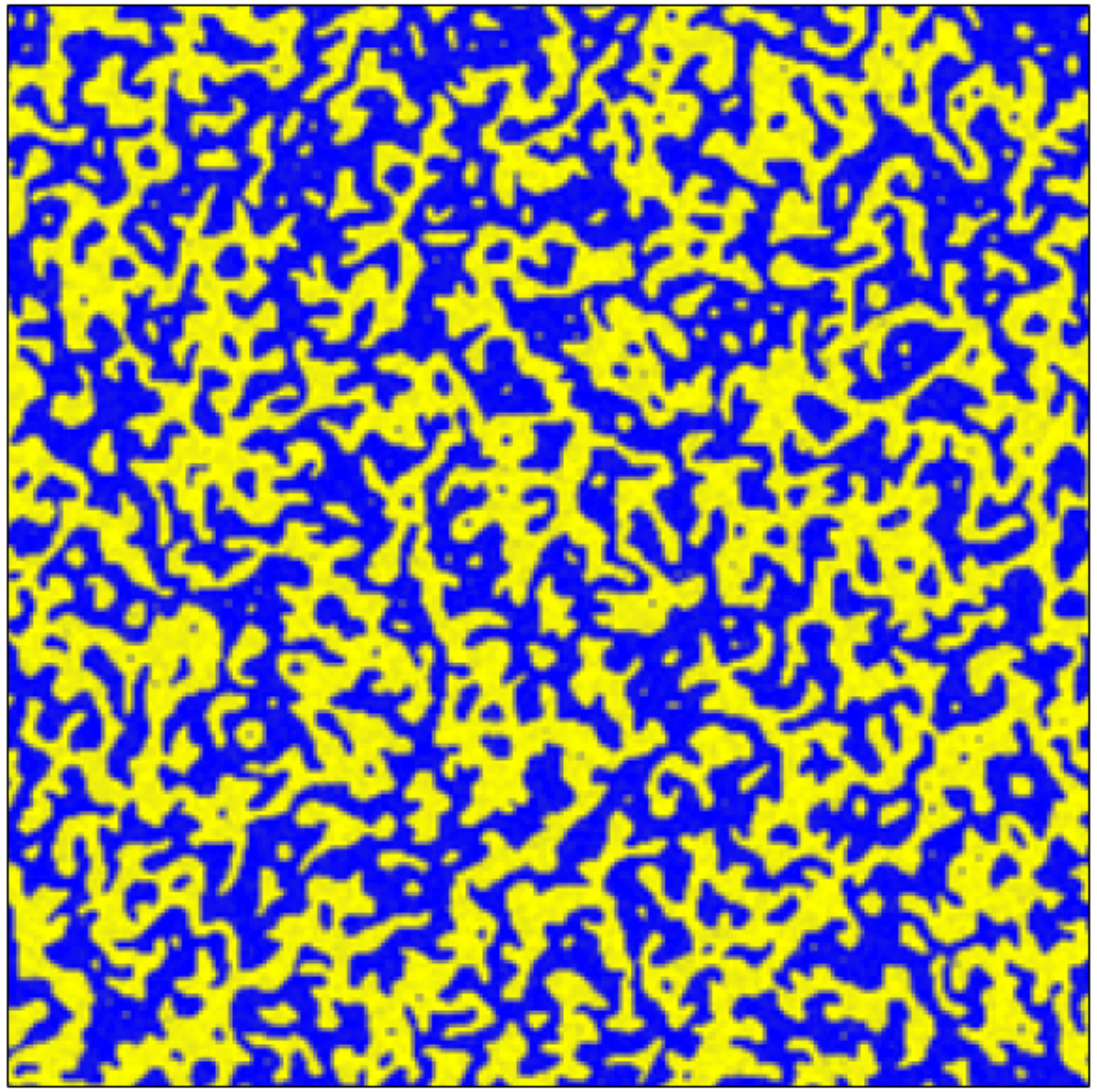

$@\tau = 10/m_0$ H^{-1}

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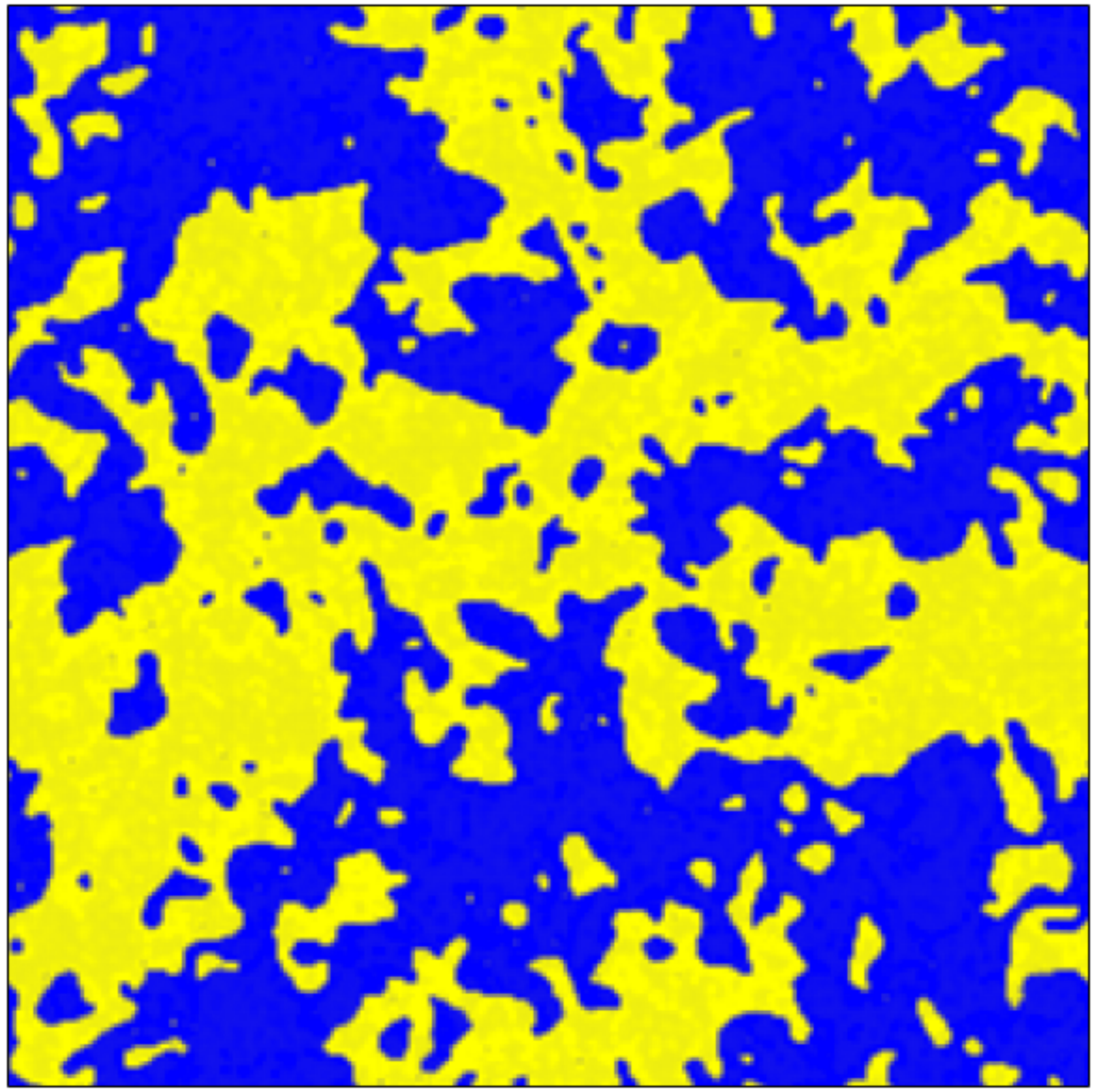

H^{-1}

DW network from inflation is long-lived,

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The key point is the **correlated superhorzion modes** that have been omitted so far.

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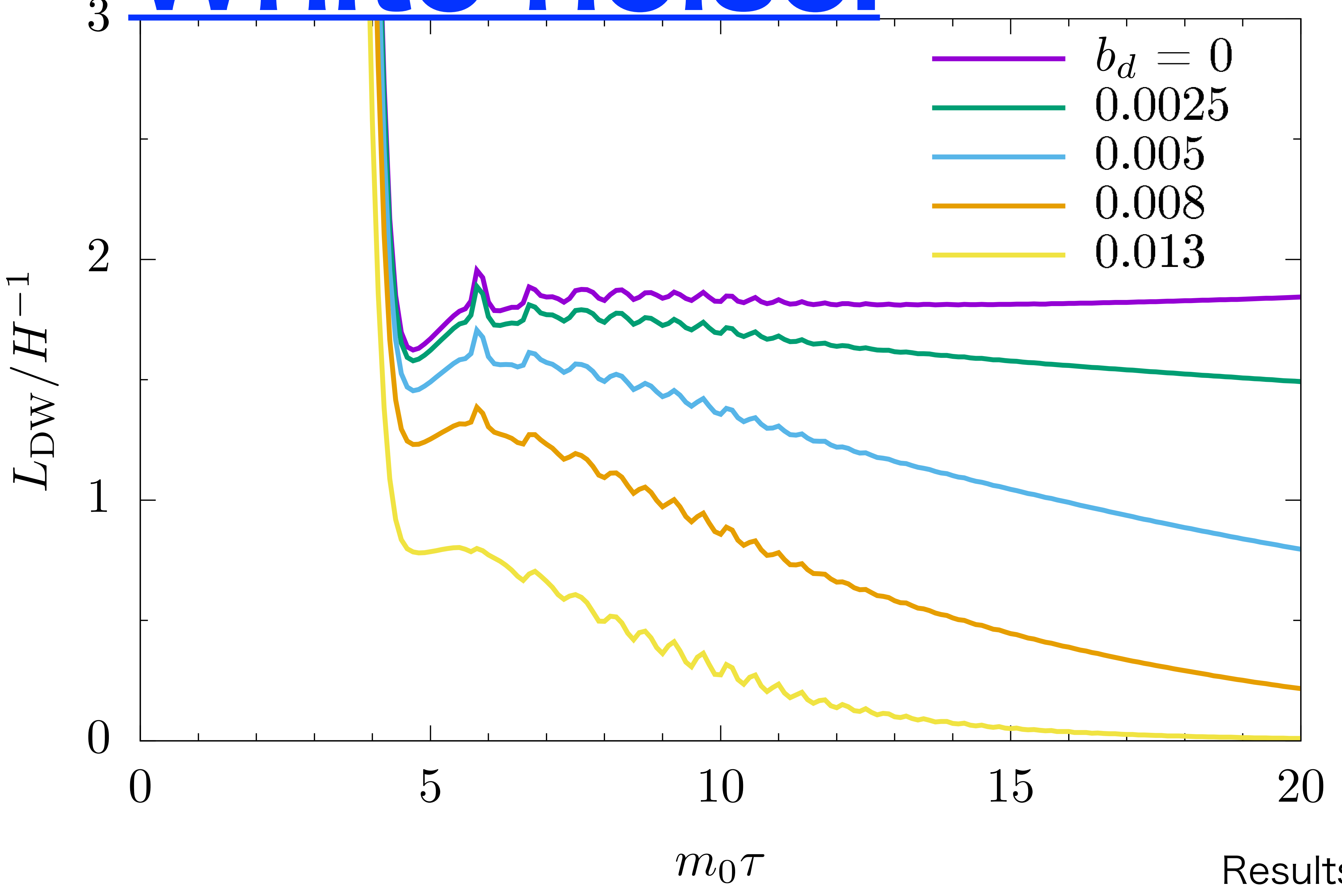

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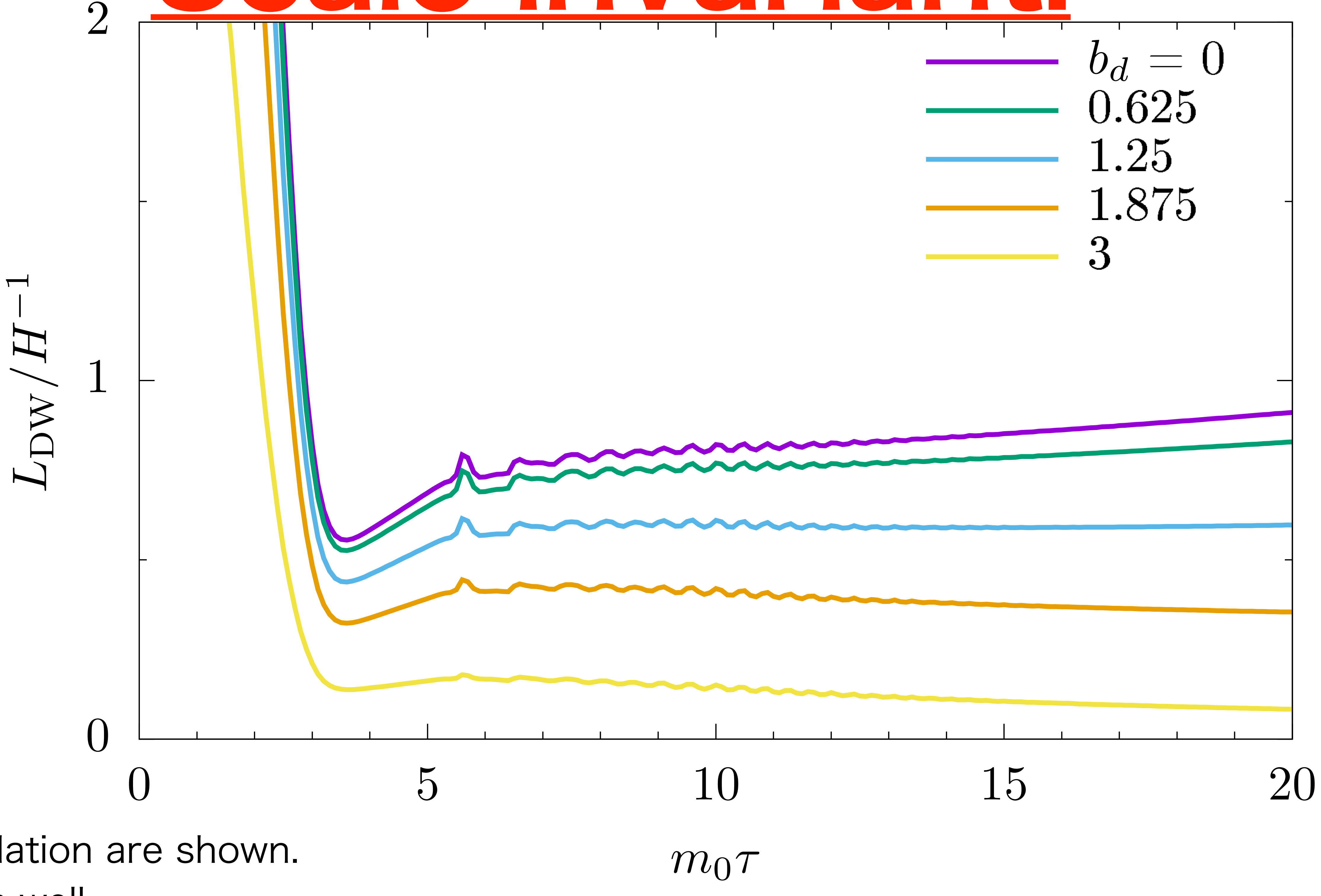

Results from 2D simulation are shown.
3D case is checked as well.

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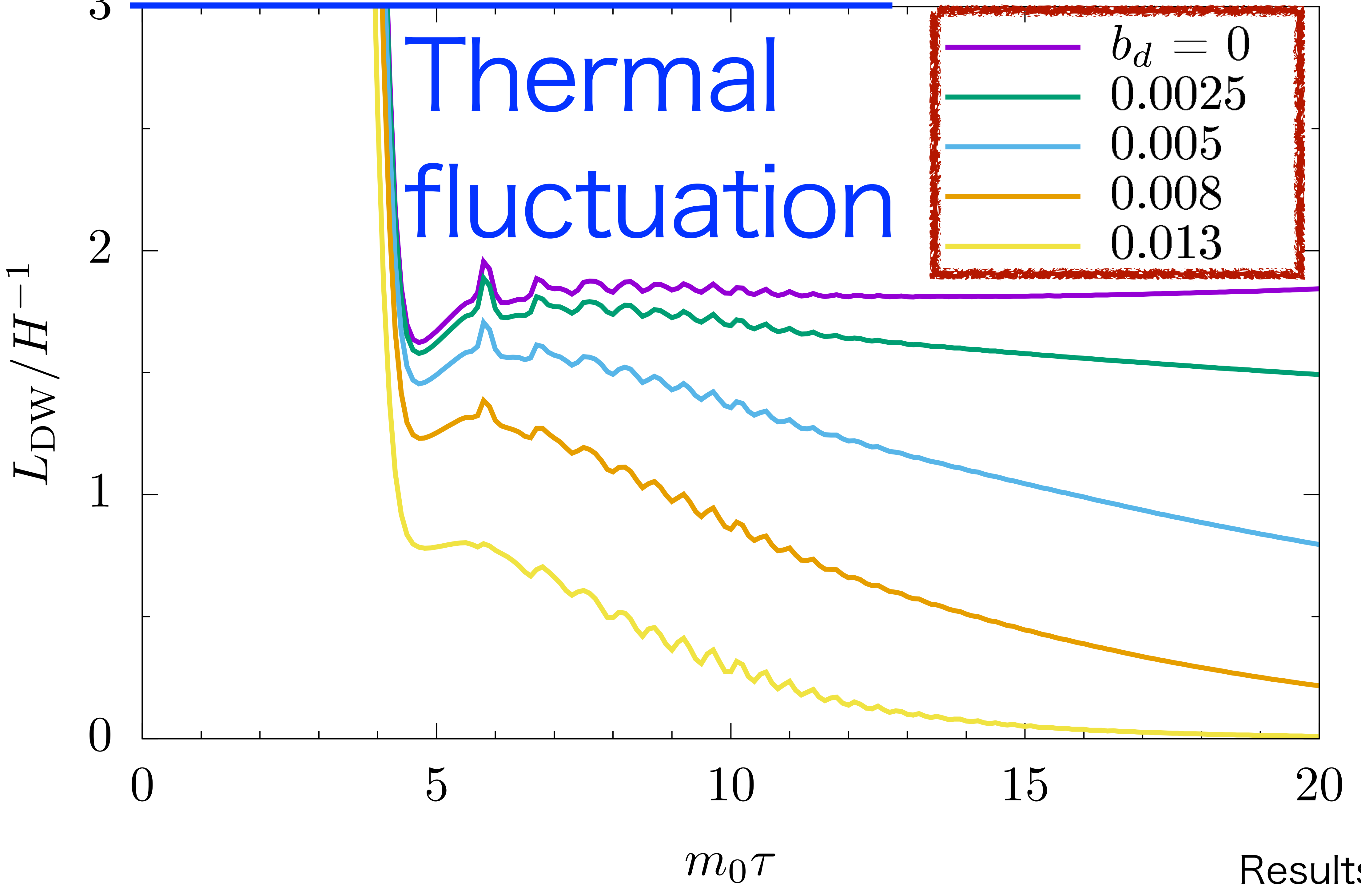

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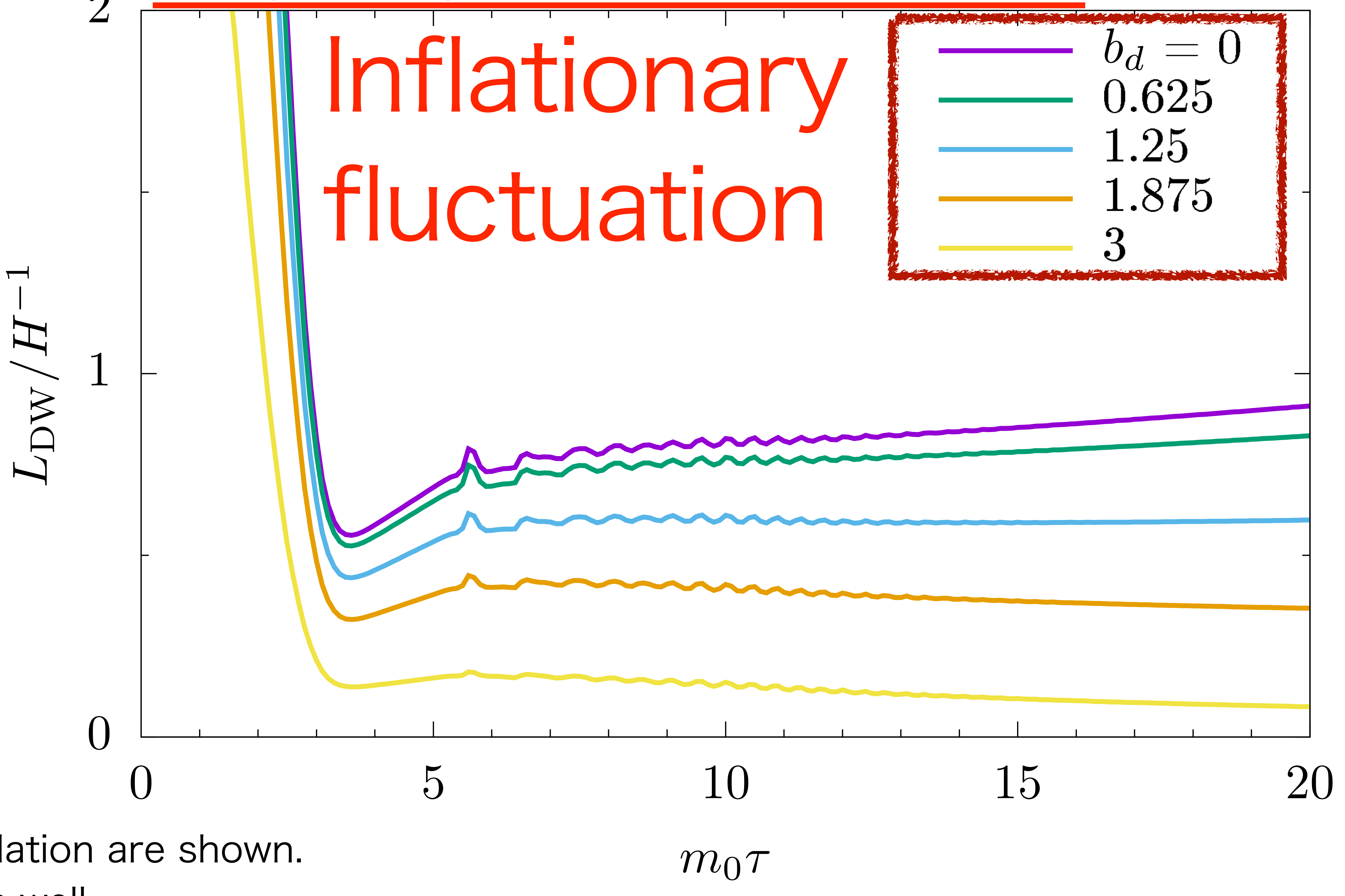

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Scenarios of DWs from inflationary fluctuations

For $b_d \lesssim O(1)$,

1. $f_\phi \sim H_{\rm inf}$

   Reminder : $H_{\rm inf} \lesssim 10^{13}\, GeV$ (tensor-to-scalar ratio)

2. $f_\phi^{\rm inf} \sim H_{\rm inf}$, with time-dependent f_ϕ.

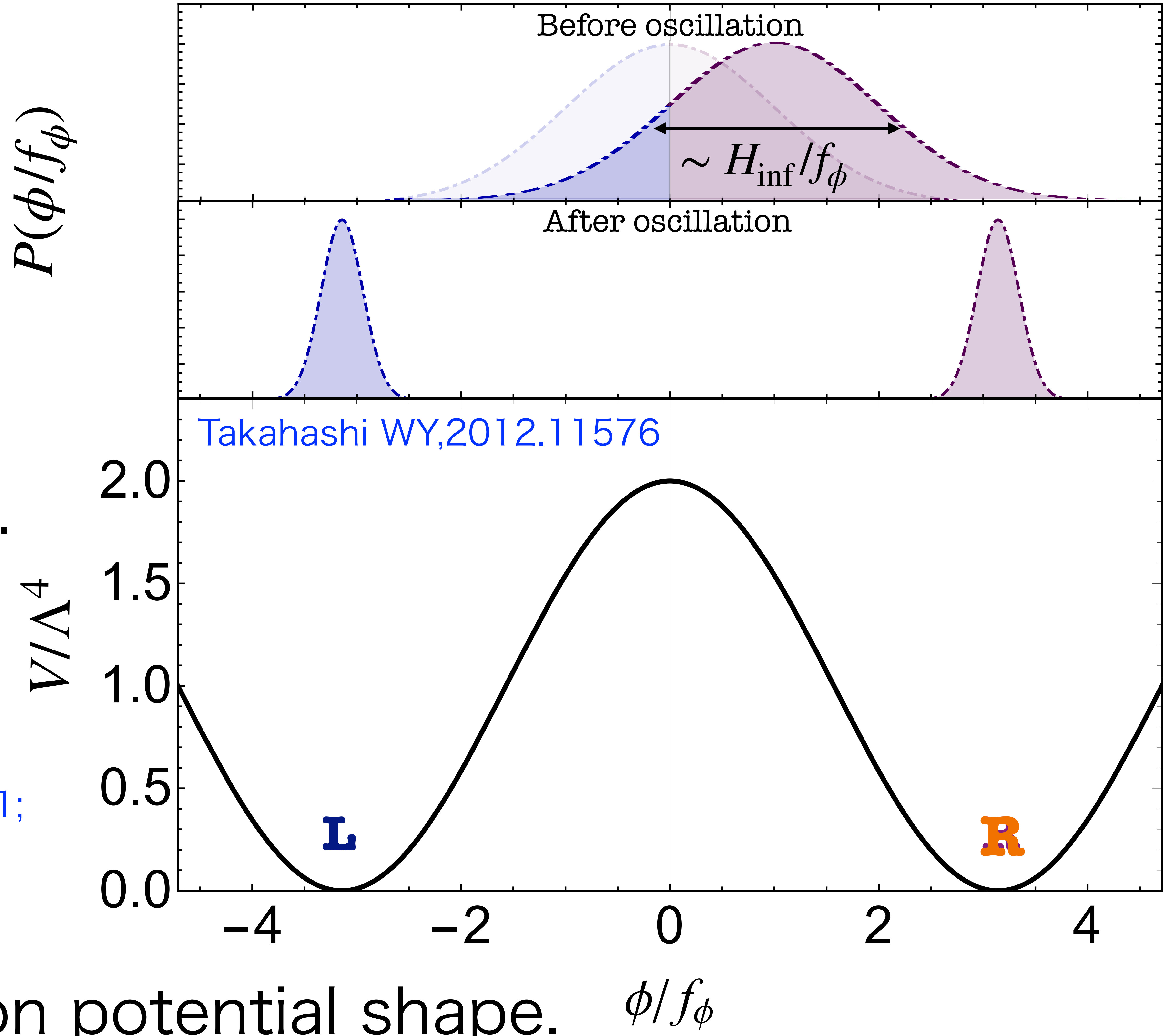
   Takahashi WY,2012.11576

3. $N_{\rm axion} H_{\rm inf} \gtrsim f_\phi$, i.e., many light axions.

   e.g. $N_{\rm axion} \gtrsim 10^2$ for $H_{\rm inf} = 10^{13}\,{\rm GeV}$, $f_\phi = 10^{15}\,{\rm GeV}$

4. Mixing induced shift of ϕ/f_ϕ by π.

Daido, Takahashi, WY, 1702.03284 ; Takahashi, WY, 1908.06071; Nakagawa, Takahashi, WY, 2002.12195 ; Murai, Takahashi, WY, 2305.18677; Narita Takahashi, WY, 2308.12154;



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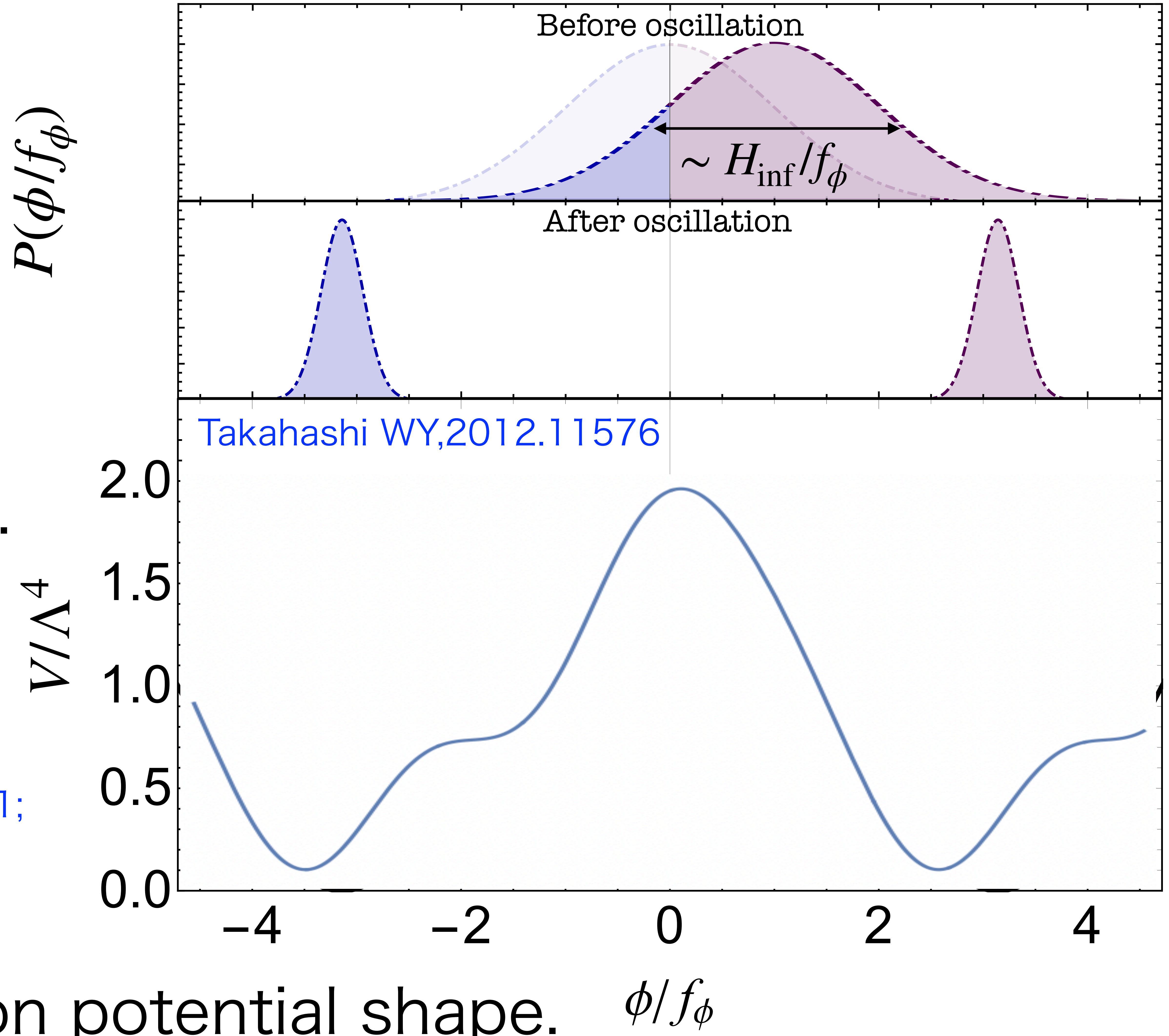

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DWs from string axion have $f_\phi = 10^{15-17}\, GeV$.

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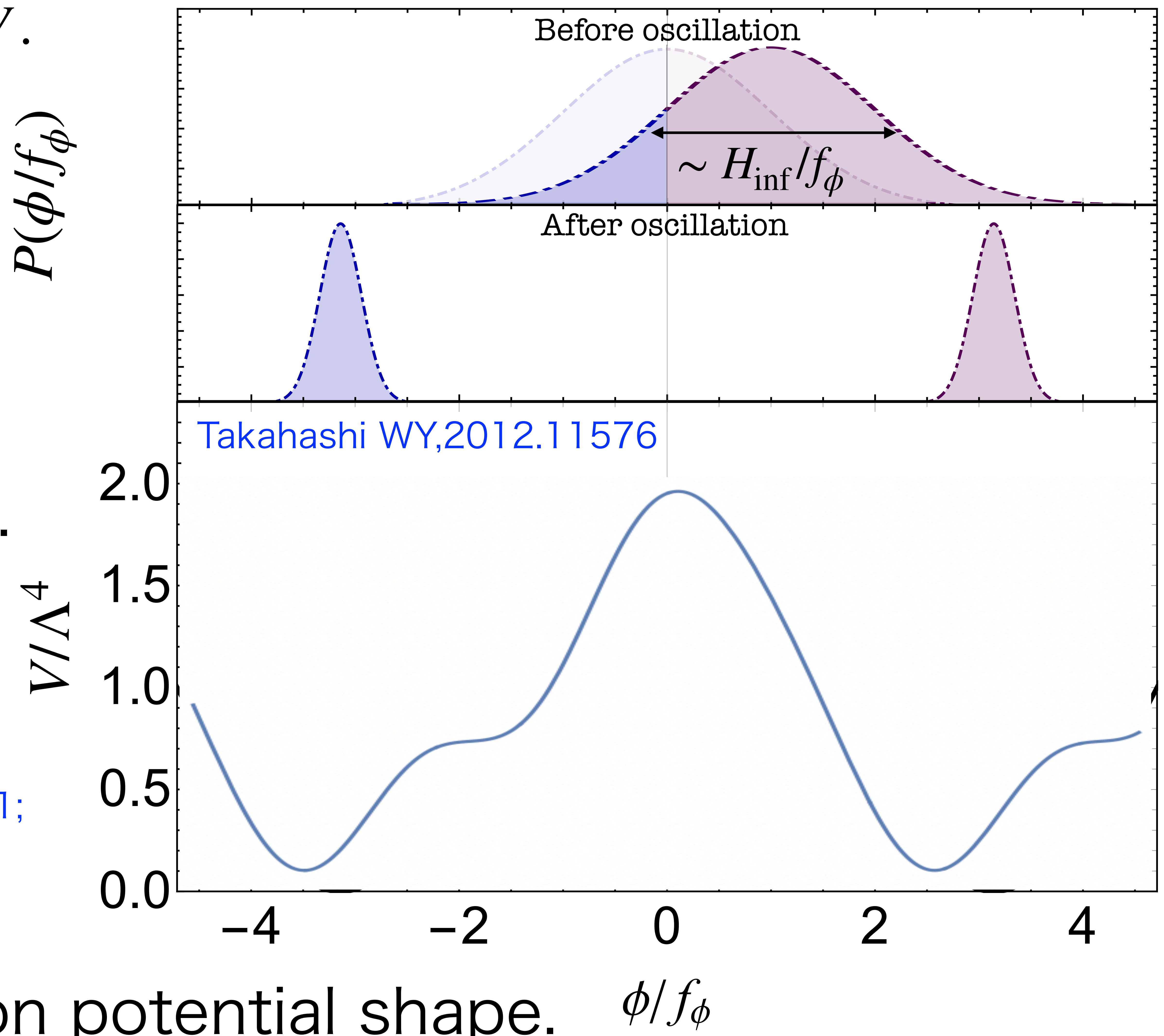

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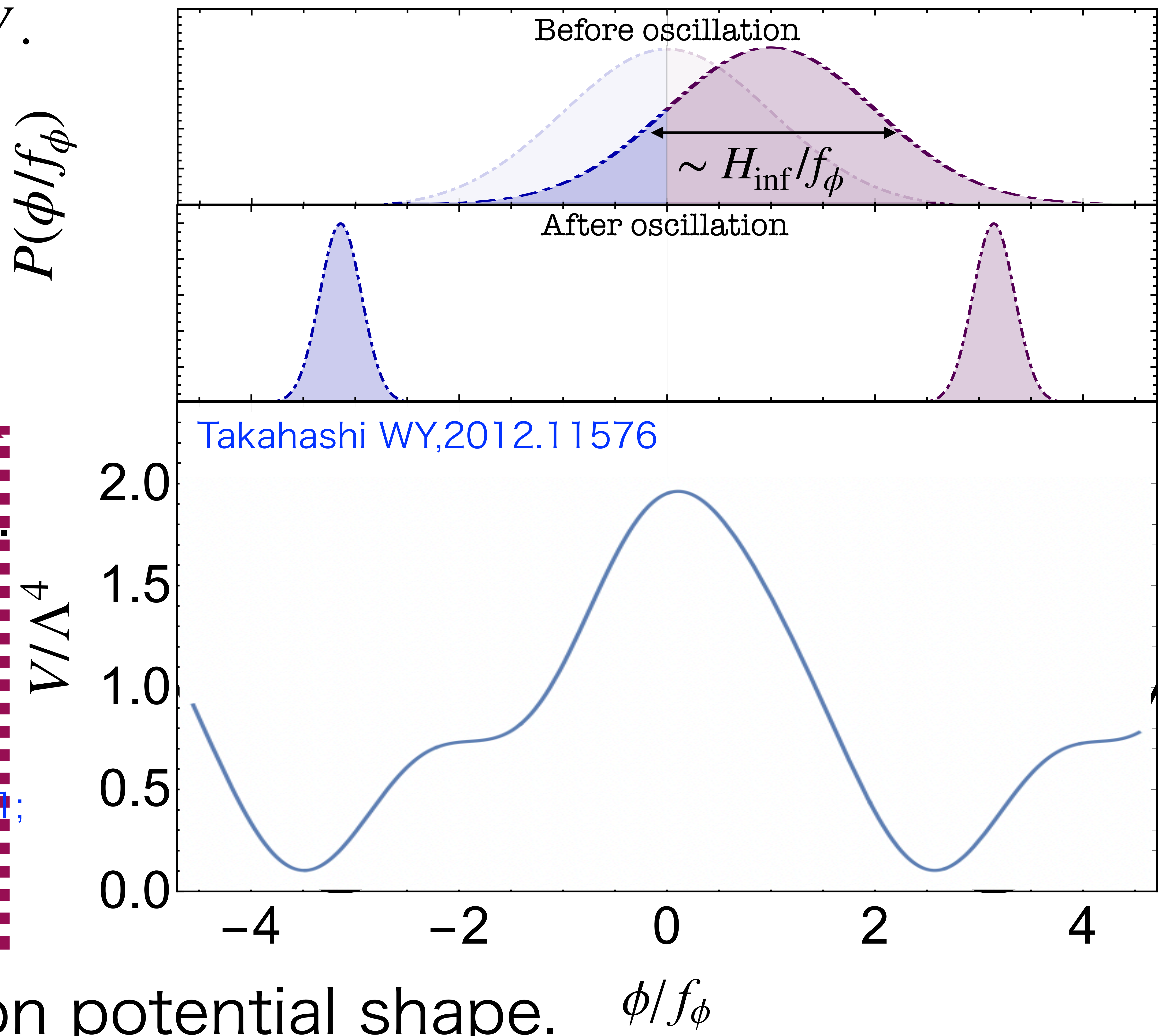

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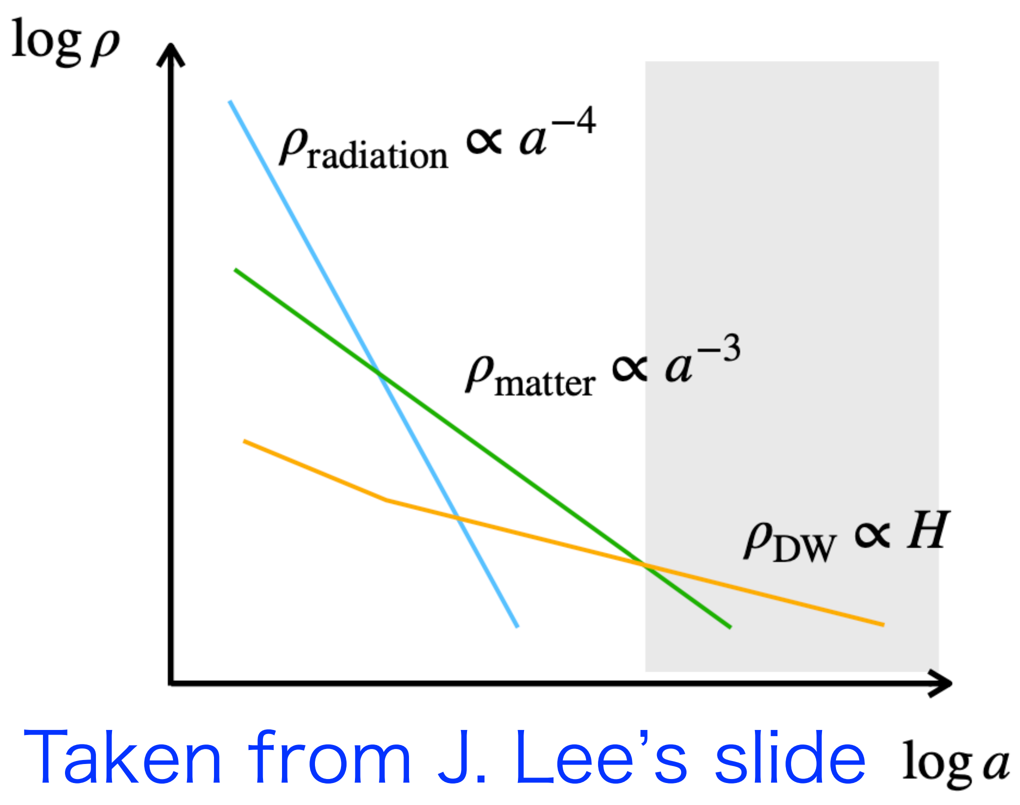
- 3. Cosmological implications

Once the DW network is formed, we must deal with the DW problem.



Taken from J. Lee's slide

DW problem

Zel'dovich, Kobzarev, Okun `74, Kibble `76, ...

DW network decays in early Universe → Gravitational waves

*Potential bias does not work for DW with $N_{\rm DW} = 1$.

Small tension $\sigma < MeV^3$ → Birefringence

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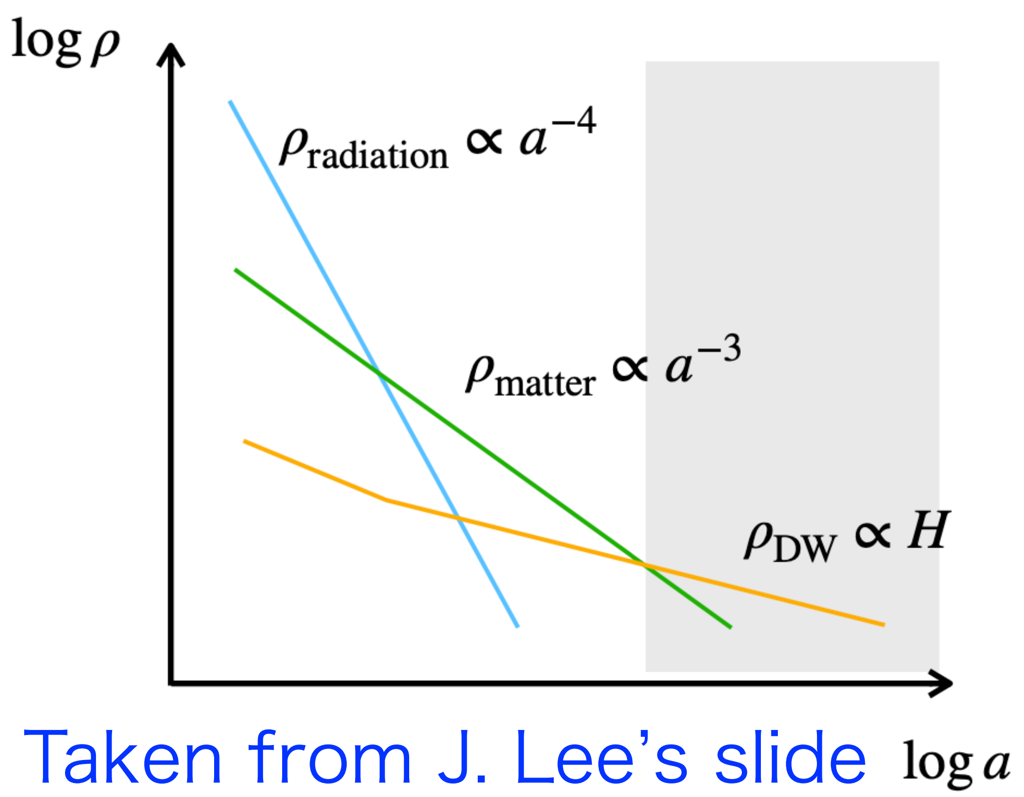

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Gravitational Waves (GWs) from DW collapse

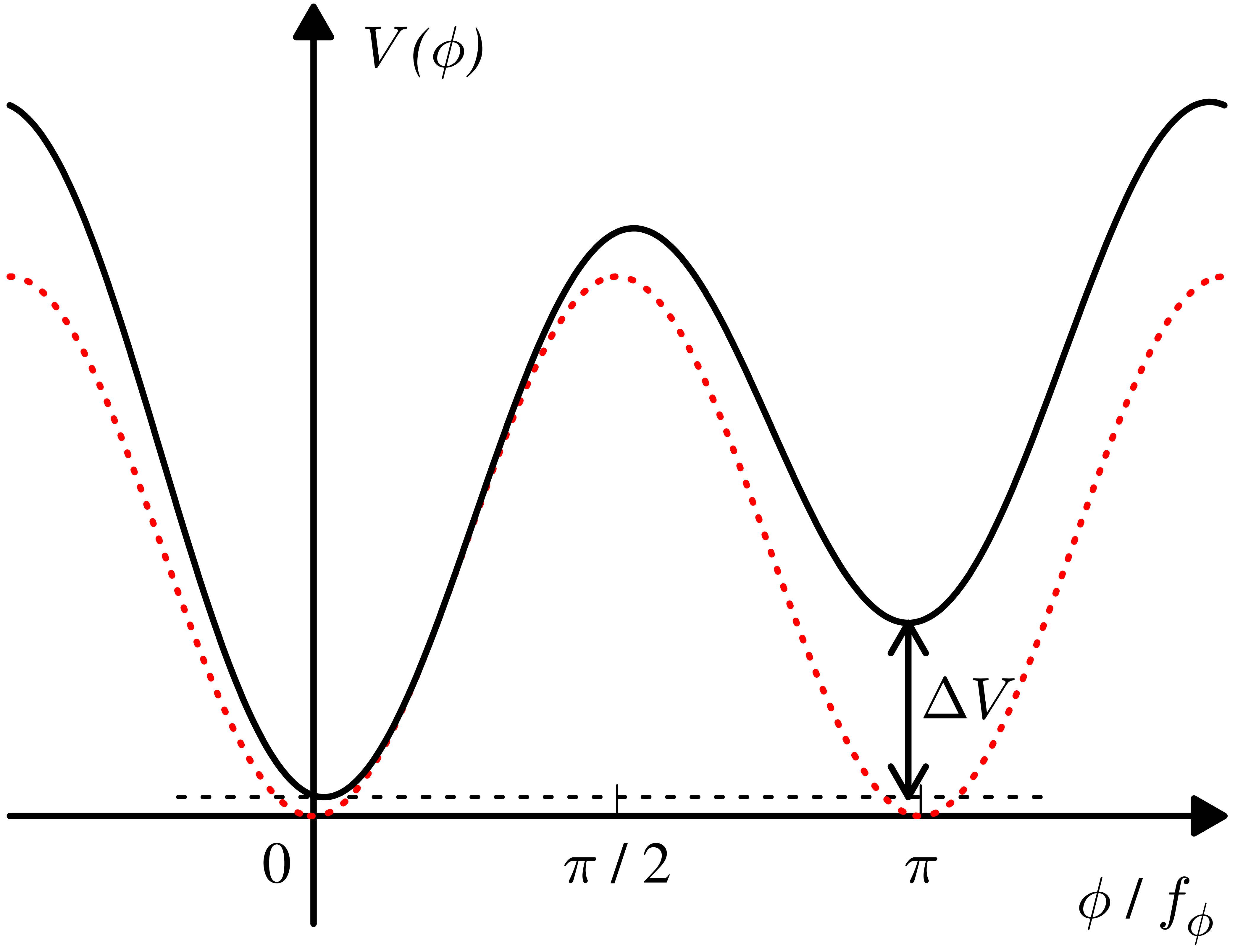
For instance,

$$V = V_0\left(1 - \cos(\frac{2\phi}{f_\phi})\right) + \epsilon(1 - \cos(\frac{\phi}{f_\phi} + \theta))$$

- Potential bias makes DW collapse at $\sigma \times H \sim \Delta V$ at which GWs can be dominantly produced.
- Only GWs from scaling DWs with $\Delta V = 0$ have been numerically studied so far.

e.g. Hiramatsu, Kawasaki, Saikawa, 1002.1555; 1309.5001;

- The numerical lattice simulation is difficult for the full system because **scaling solution vs. potential bias, calculation time vs. resolution.**

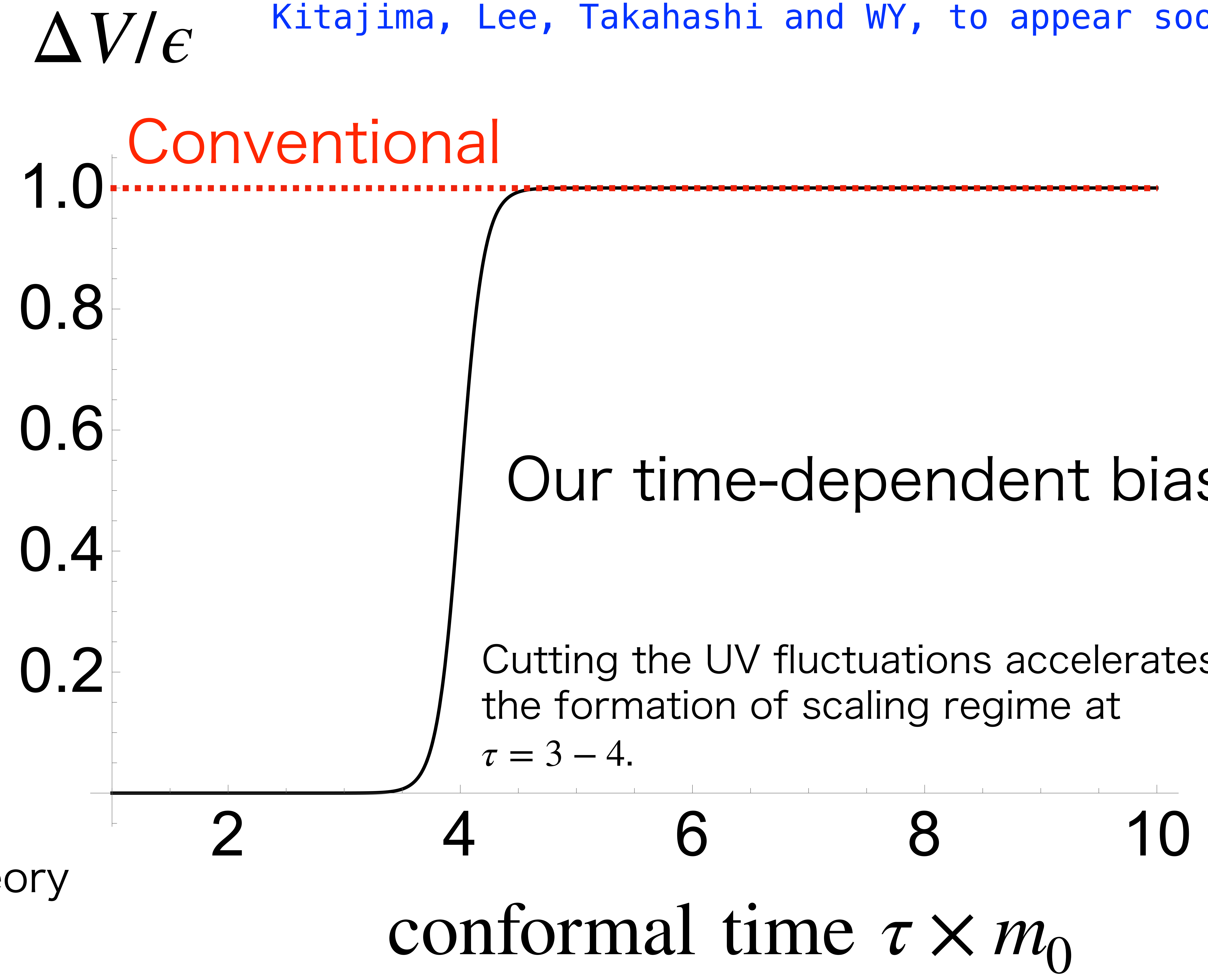


The first lattice simulation of the GW from decaying DW!

The key point of our analysis: time-dependent bias.

Kitajima, Lee, Murai, Takahashi and WY,2306.17146;
Kitajima, Lee, Takahashi and WY, to appear soon;

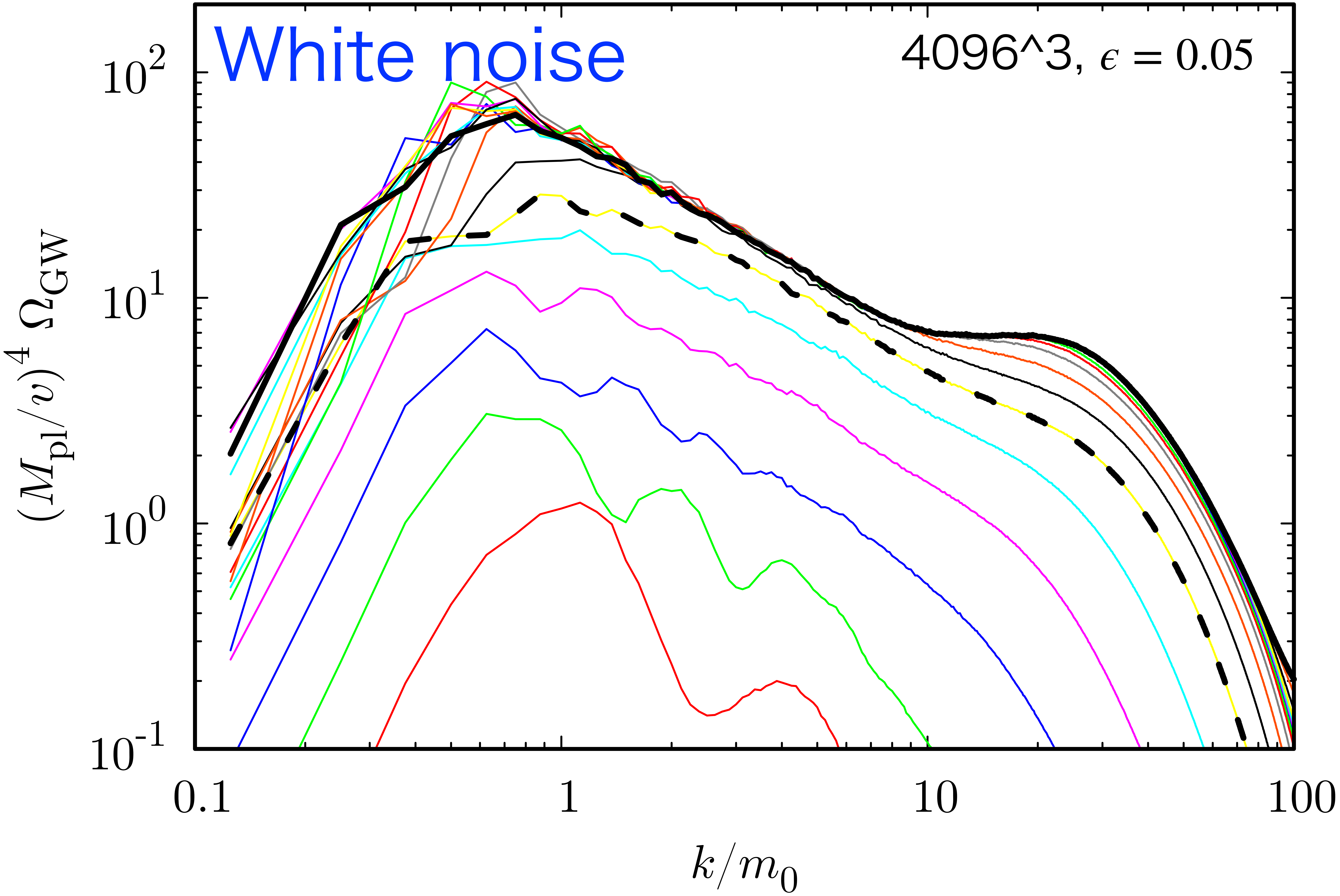
- DW collapse after scaling regime ✔
- DW collapse in a short time ✔
- Good approximation if DW bias has a QCD-axion like potential $\Delta V \propto \chi(T)$.



Reminder: We use Z_2symmmetric, ϕ^4 theory to approximate system.

New dominant contribution to the GW from decaying DW!

Kitajima, Lee, Murai, Takahashi and WY,2306.17146;



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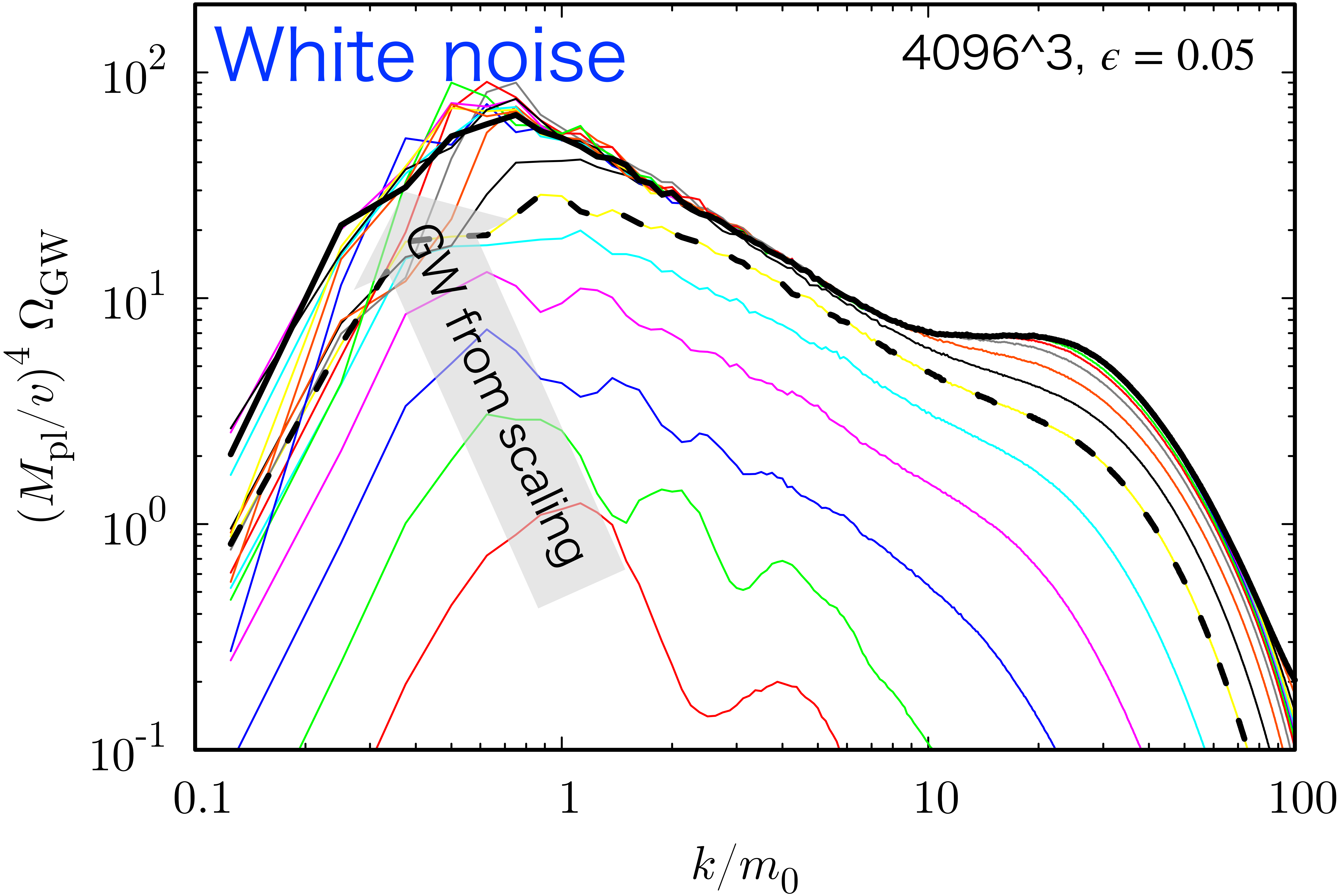

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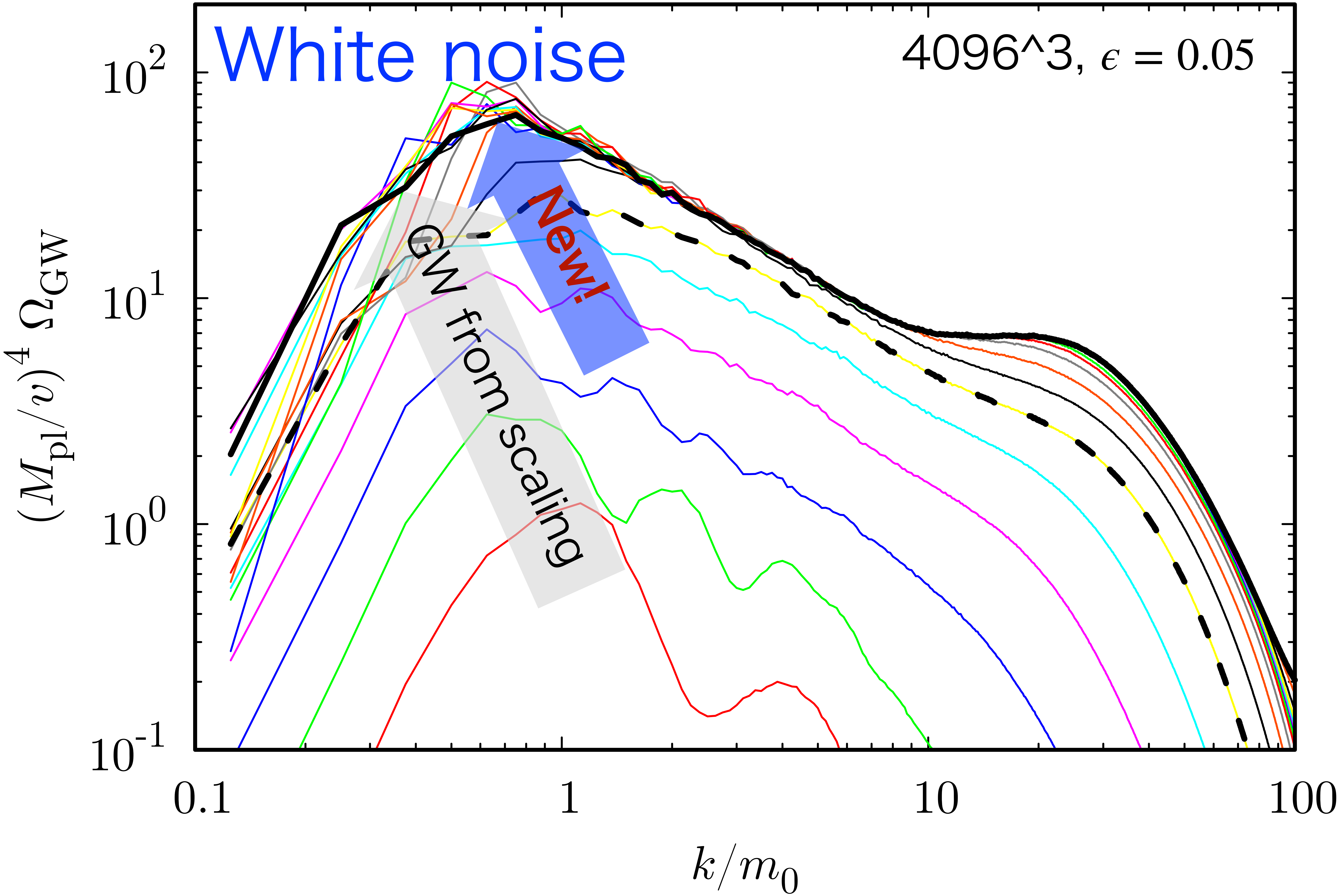


Application: time-dependent bias is natural to explain NANOGrav data!

Kitajima, Lee, Murai, Takahashi and WY,2306.17146;

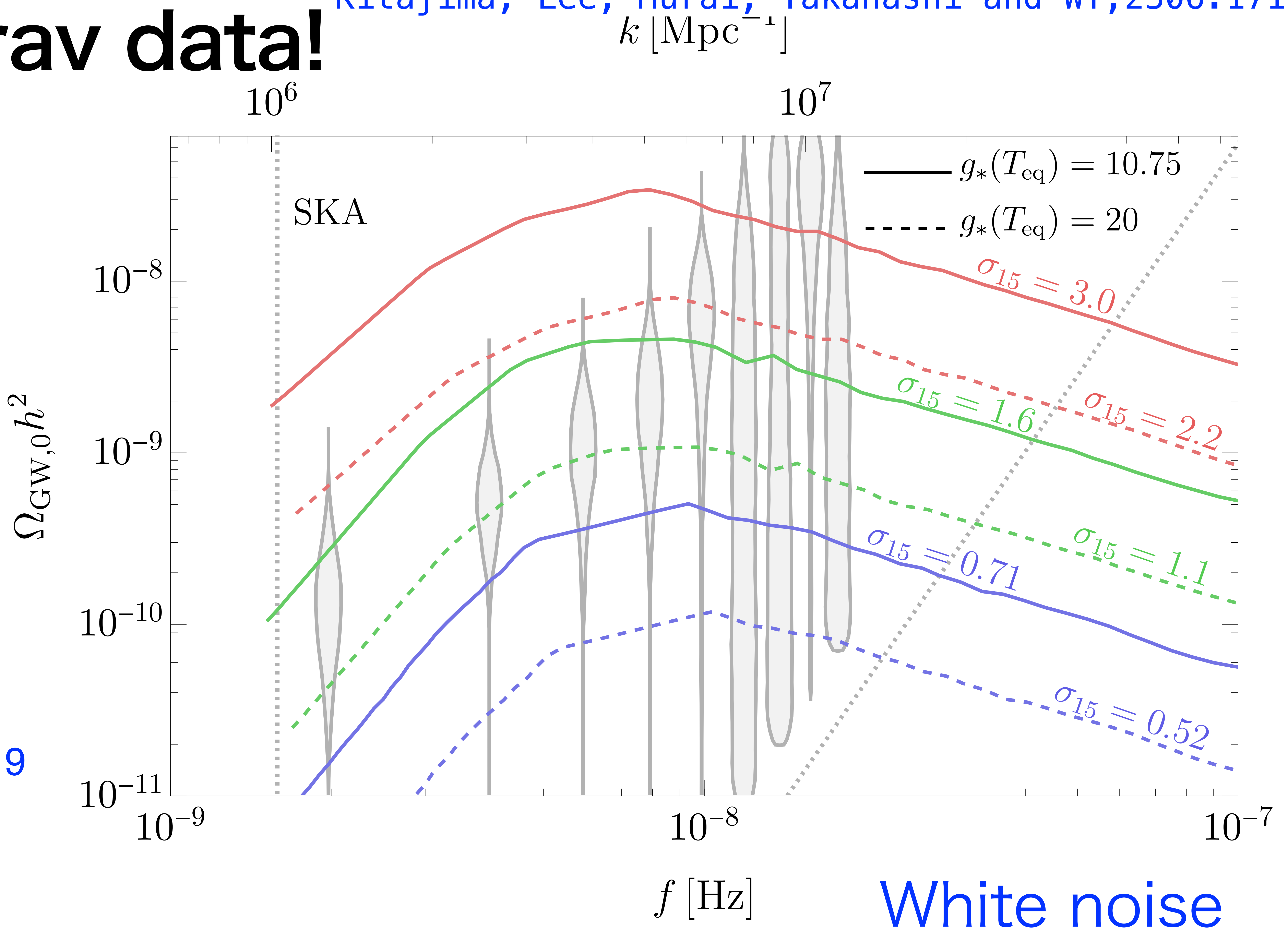
- $\delta\mathscr{L} = \alpha_s \frac{\phi}{8\pi f_\phi} G\tilde{G}$

  $\rightarrow \delta V = \chi(T)\cos(\frac{\phi}{f_\phi} + \theta_{QCD})$

- DW decay induced by QCDPT predicts nHz GW!
- It naturally explains the NANOGrav data!

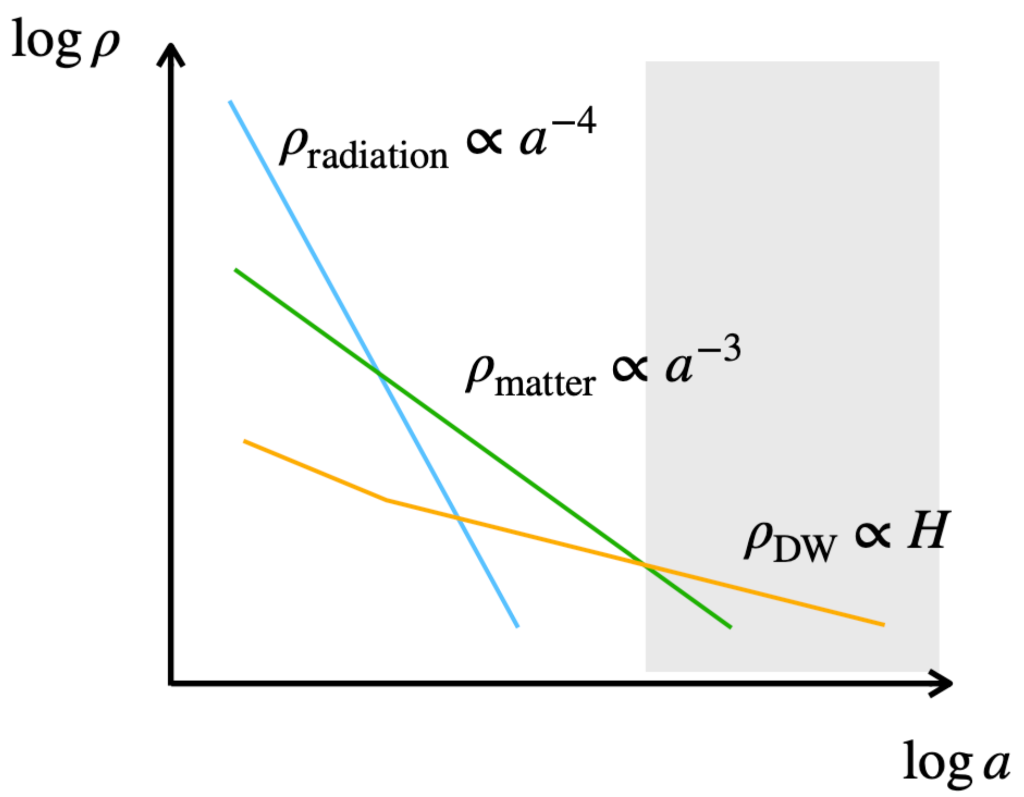
  NANOGrav collaboration, 2306.16219

- It is unlikely the string axion, becasue $f_\phi < 10^8\, GeV$



White noise

Once DW network is formed we have to deal with DW problem.



DW problem

Zel’dovich, Kobzarev, Okun `74, Kibble `76, ...

DW network decays in early Universe. → Gravitational waves

*Does not work for DW with $N_{\text{DW}} = 1$.

Small tension $\sigma < MeV^3$ → Birefringence

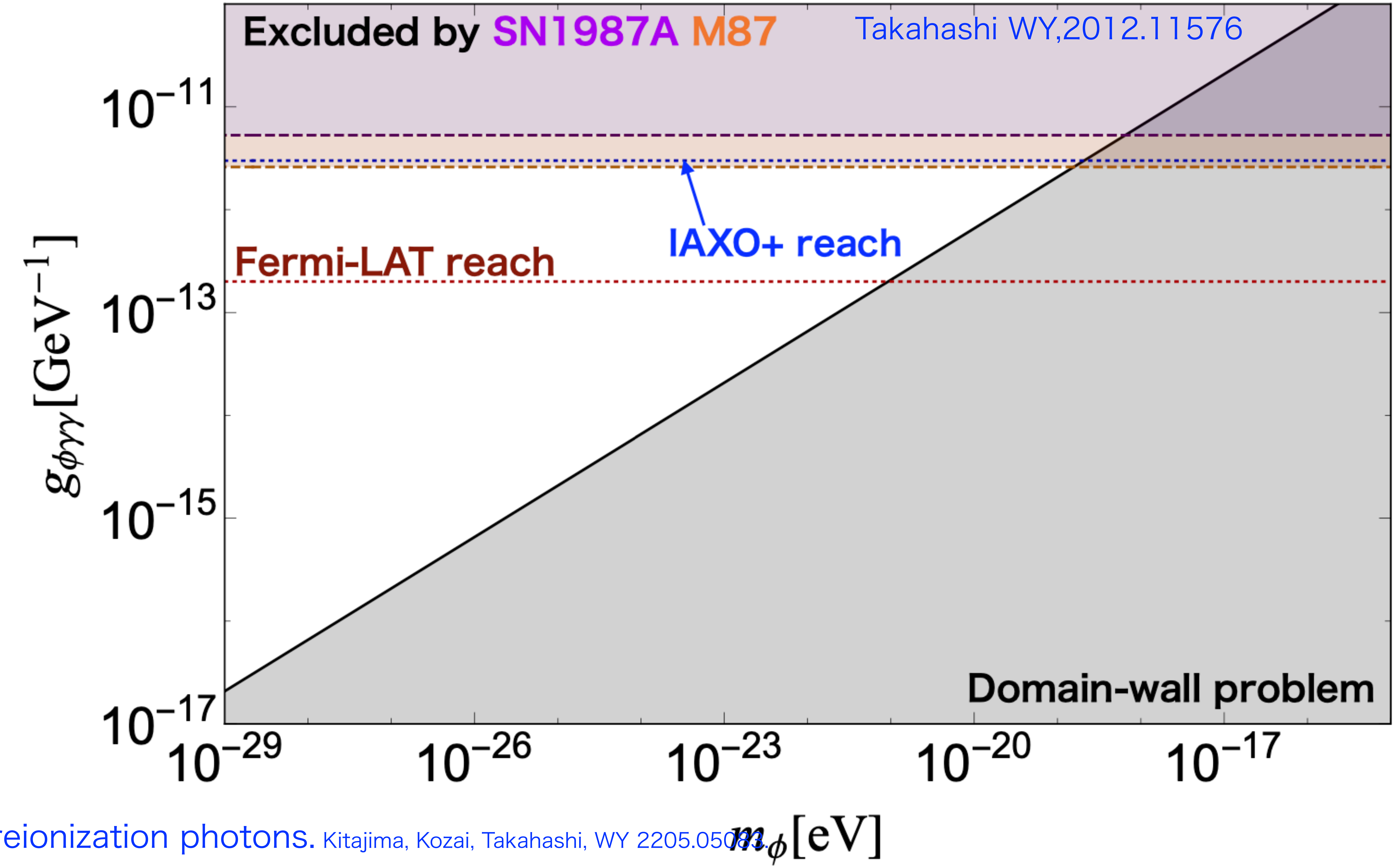
Stable DW with small tension $\sigma < MeV^3$

Such DW can induce observable effects if there is coupling to Standard Model particle.

Let us consider photon coupling

$$\mathscr{L} \supset -\frac{g_{\phi\gamma\gamma}}{4}\phi F\tilde{F}$$

$$\equiv \frac{\alpha\phi}{8\pi f_\phi}F\tilde{F}$$



With $m_\phi = 10^{-33} - 10^{-29}\, eV$,

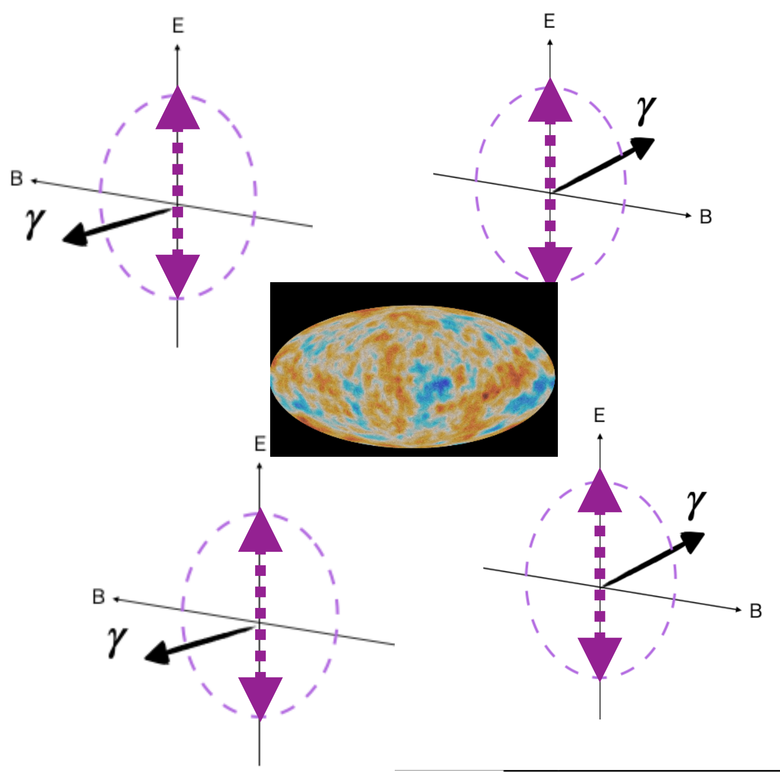
we can have anisotropic CB from reionization photons. Kitajima, Kozai, Takahashi, WY 2205.05083

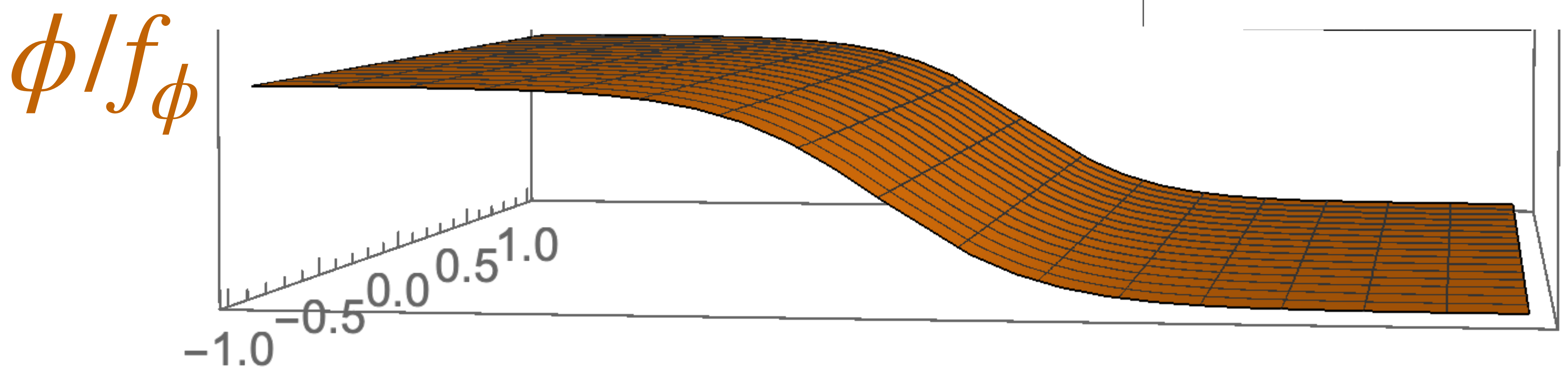
Cosmic birefringence (CB) from axion domain walls

Changes of background axion the field rotate propagating photon polarization.

Carroll, Field, Jackiw,1990; Harari, Sikivie,1992; Carroll,1998;

$$\Phi(\Omega) = 0.42\ \deg \times c_\gamma \left(\frac{\phi_{\rm Earth} - \phi_{\rm LSS}(\Omega)}{2\pi f_\phi} \right),$$





- **Observables:**

-Isotropic CB

$$\beta \equiv \frac{1}{4\pi} \int d\Omega \Phi[\Omega]$$

$$\beta_{\rm obs} = 0.36 \pm 0.11 \deg$$

Minami and Komatsu, 2006.15982, Diego-Palazuelos et al, 2201.07682.

- **-Anisotropic CB**

e.g. $C_\ell^\Phi \leftrightarrow \langle \Phi(\overrightarrow{\Omega})\Phi(\overrightarrow{\Omega}') \rangle$

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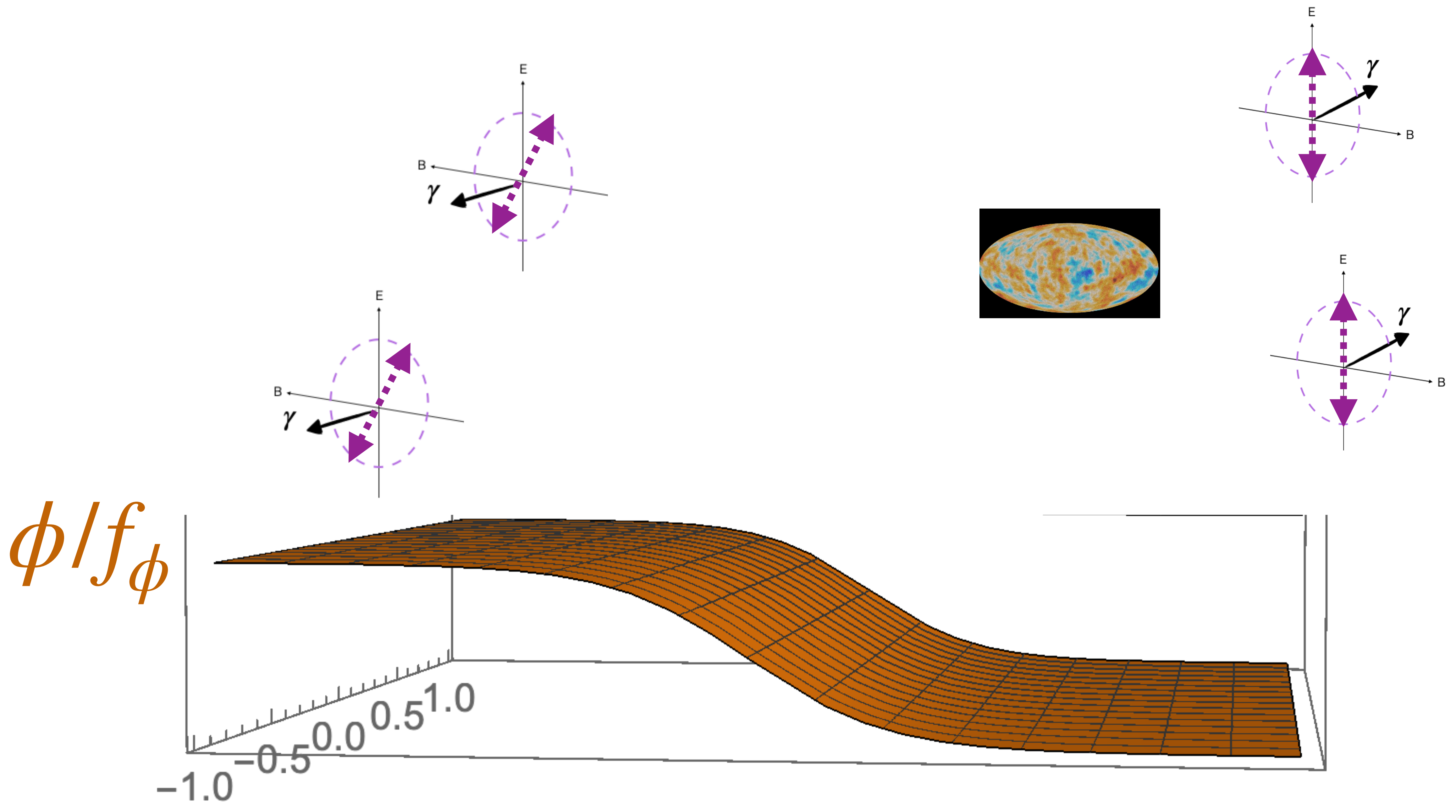

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- **-Anisotropic CB**

e.g. $C_\ell^\Phi \leftrightarrow < \Phi(\overrightarrow{\Omega})\Phi(\overrightarrow{\Omega}') >$

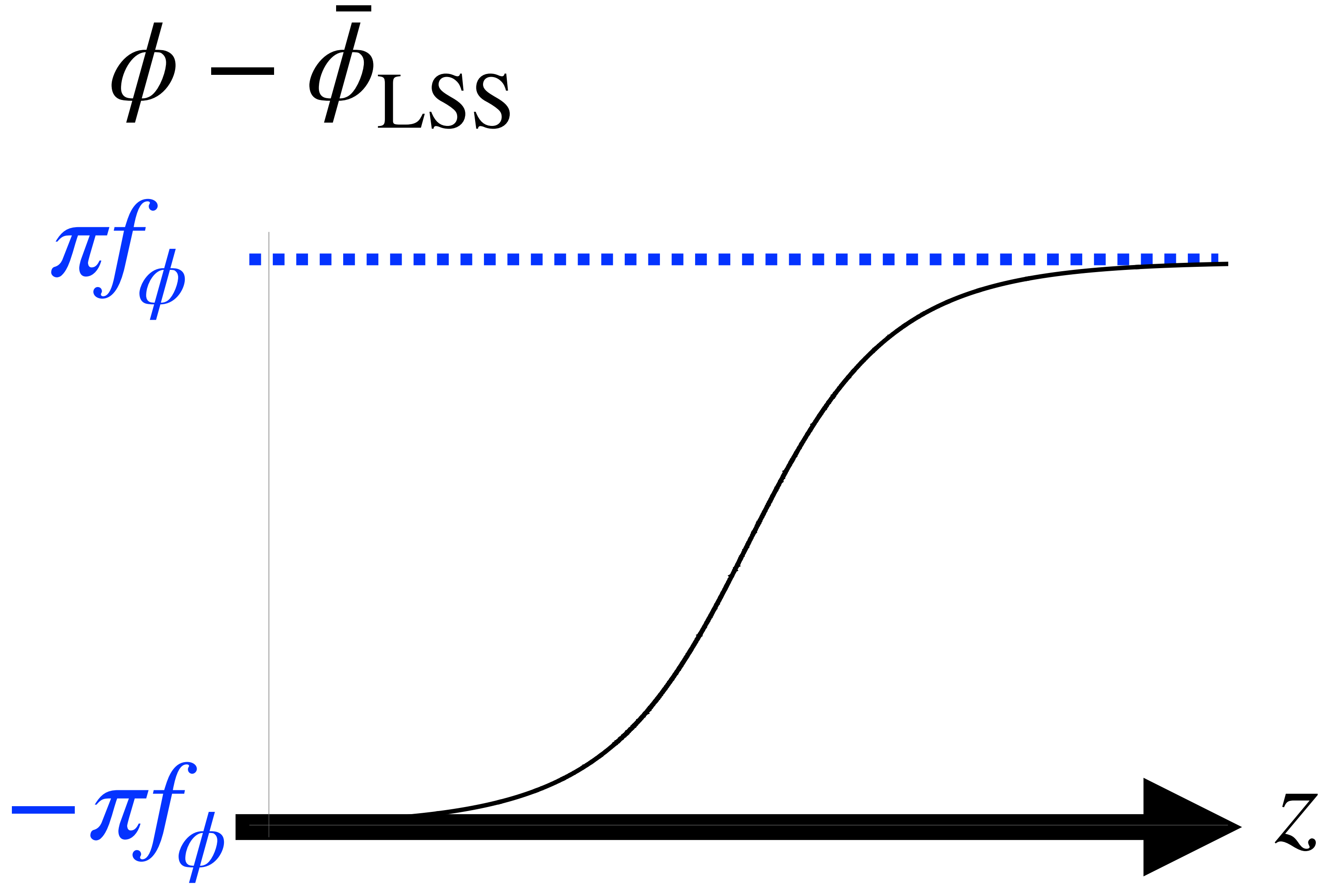
Scaling DW without a string naturally explains isotropic CB!

Takahashi, WY, 2012.11576

$$\beta \simeq 0.21\ \mathrm{deg} c_\gamma \left(\frac{\phi_{\rm earth} - \bar{\phi}_{\rm LSS}}{\pi f_\phi} \right)$$

$$\mathrm{c.f.}\ \beta_{\rm obs} = 0.36 \pm 0.11 \mathrm{deg}$$

Minami and Komatsu, 2006.15982,
Diego-Palazuelos et al, 2201.07682.

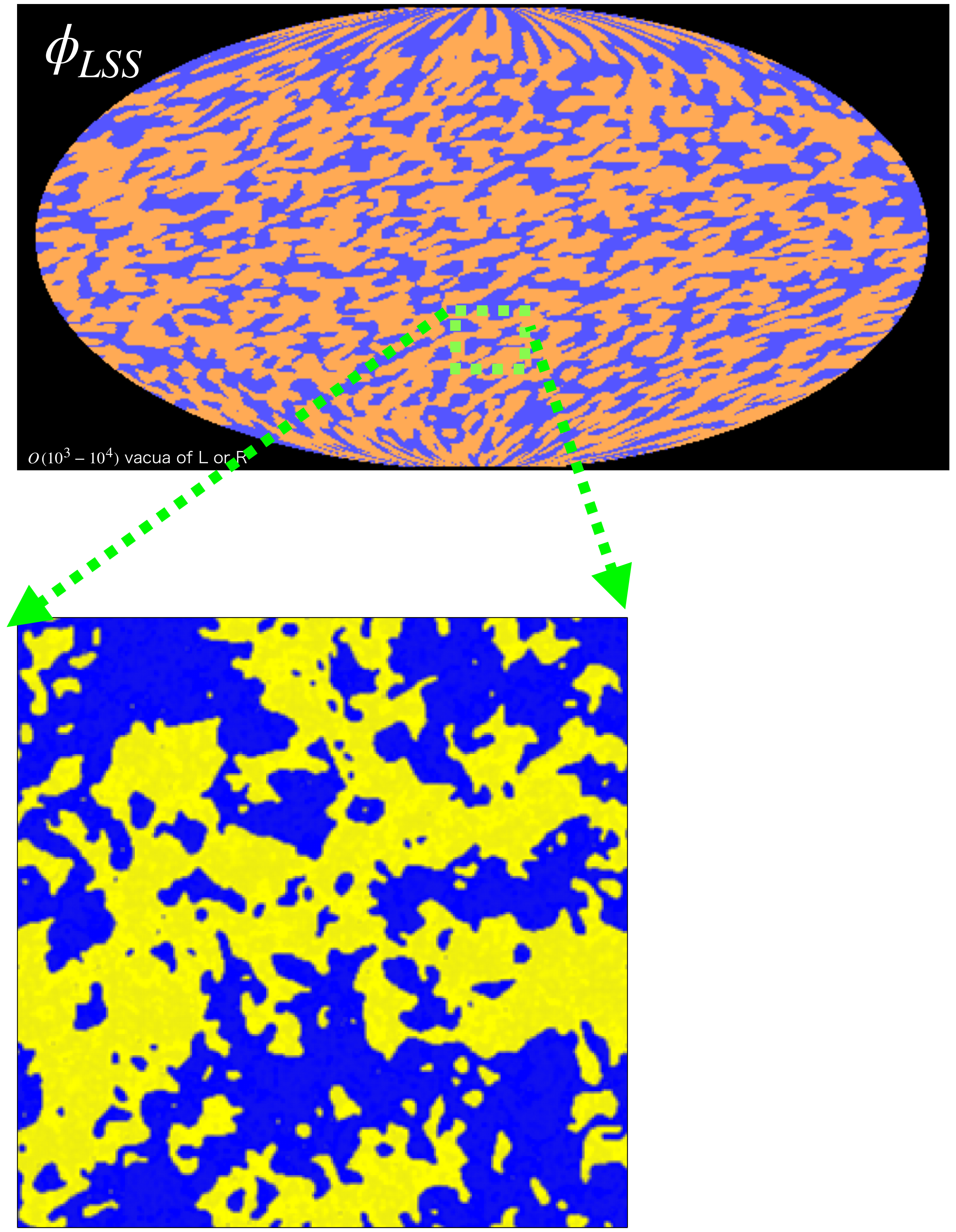


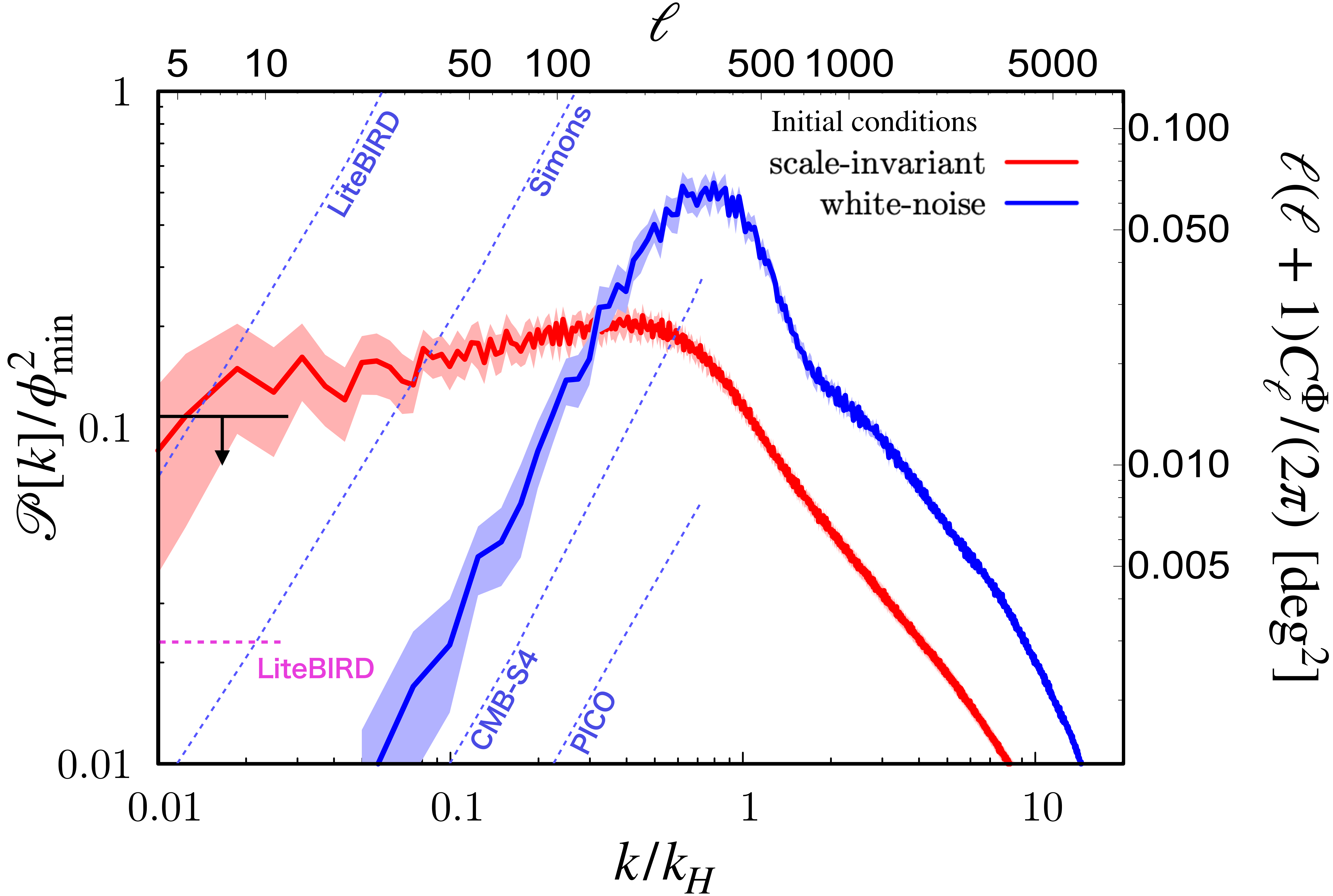
DWs +string from U(1) symmetry is difficult to explain it.

Agrawal, et al,1912.02823, Takahashi, WY, 2012.11576, Jain et al, 2208.08391

Anisotropic CB will be probed soon.

Takahashi, WY, 2012.11576; Takahashi, Kitajima and Kozai, WY, 2205.05083; Gonzalez, Kitajima, Takahashi, WY, 2211.06849





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Conclusions:

Axion models always involve stable domain wall (DW) configurations.

- **Due to inflationary fluctuations, DW formation is natural in string axiverse with many axions.**
- Avoiding the DW problem by potential bias predicts gravitational waves (GWs). From lattice simulation, we found a new dominant contribution to GWs.
- Avoiding the DW problem by a small tension, the stable DW without a string can explain the isotropic cosmic birefringence (CB). This will be tested by future anisotropic CB.
- They both may be the probes of string axiverse because $f_\phi = 10^{15-17}\,\mathrm{GeV}$ can also be probed as well.

Backup

Axion domain wall from phase transition for dark charge conjugation, $C_{\rm dark}$, breaking.

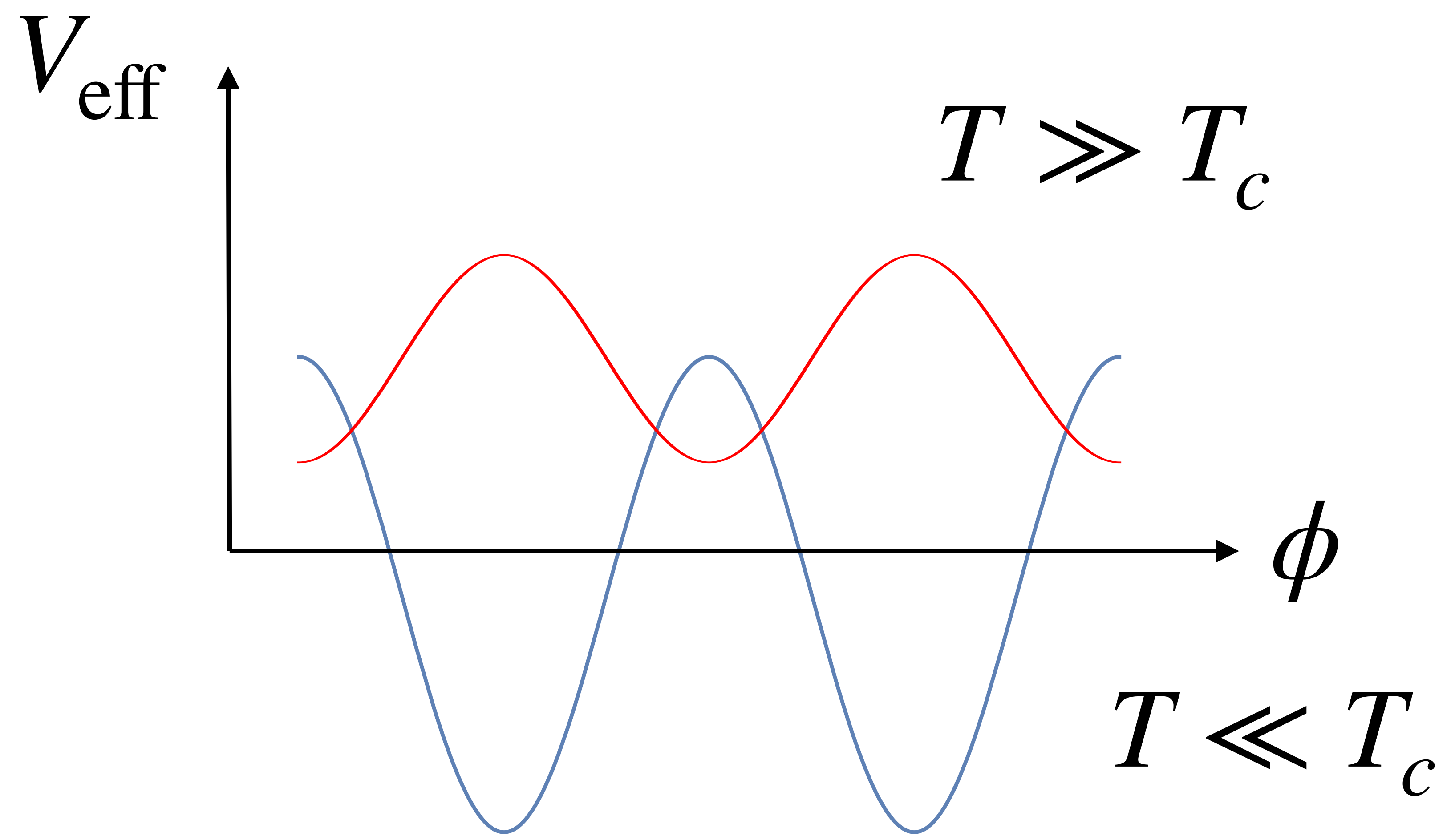
$U(1)_{\rm PQ}$ breaking but $C_{\rm dark}$ conserving interaction in the UV model,

$$\mathcal{L} = -y\Phi\bar{\Psi}_L\Psi_R - M\bar{\Psi}_L\Psi_R + \text{h.c.}$$

y, and M are real.

$$\delta V_{\rm eff} \supset -\frac{T^2}{24}M_{\rm eff}^2 \supset -\frac{T^2}{24}\sqrt{2}yf_aM\cos(a/f_a).$$

Gonzalez, Kitajima, Takahashi, WY, 2211.06849



This results in white noise DW without string if $U(1)_{\rm PQ}$ never restores.

Axion induced birefringence

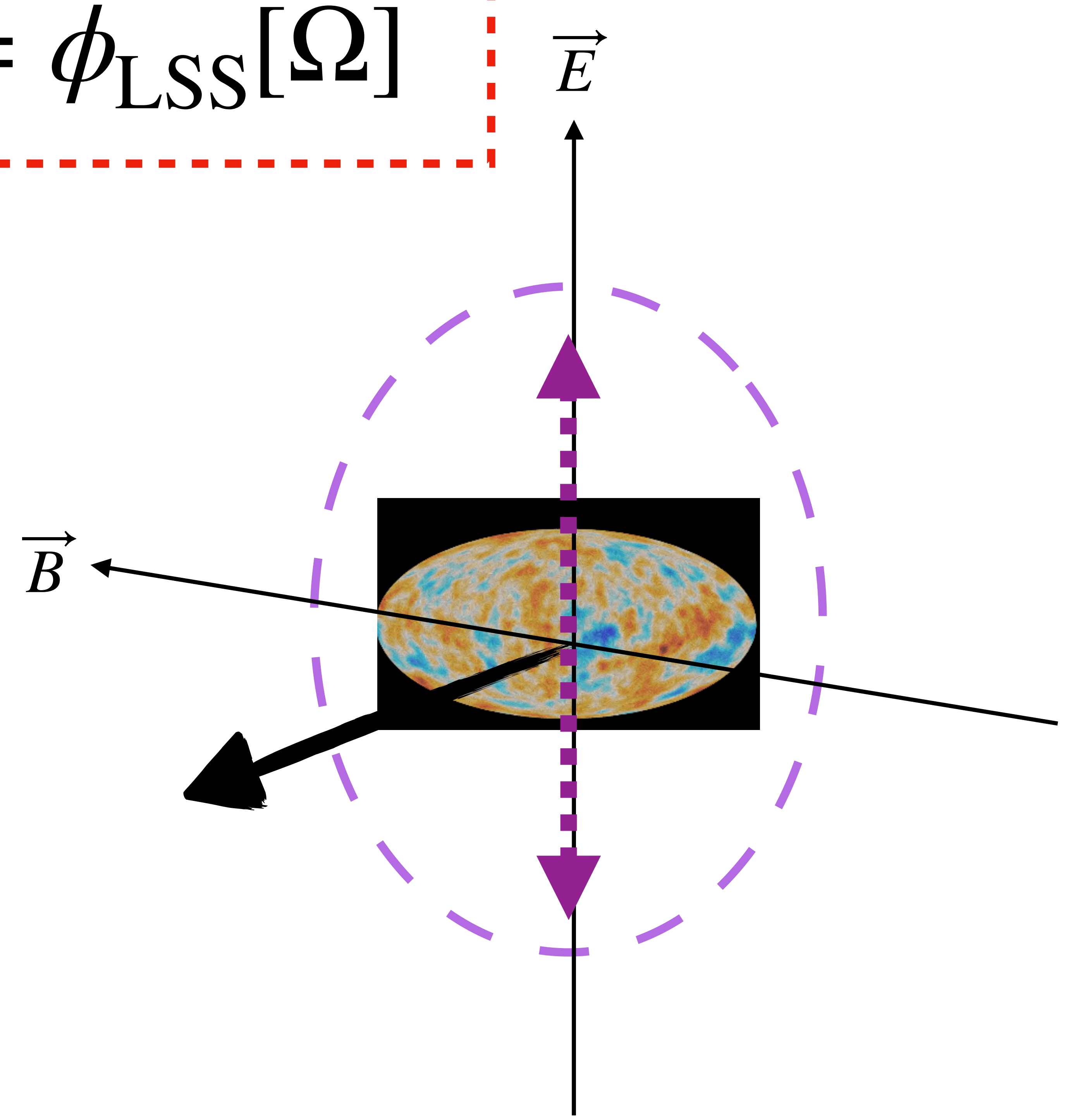
Carroll, Field, Jackiw, 1990; Harari, Sikivie, 1992; Carroll, 1998;

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - c_\gamma \frac{\alpha}{4\pi}\frac{\phi}{f_\phi}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$= \frac{1}{2}(\vec{E}^2 - \vec{B}^2) - \frac{1}{4}g_{\phi\gamma\gamma}\phi(\vec{E}\cdot\vec{B})$$

$$\approx \frac{1}{2}\left[\underbrace{(\vec{E} + \frac{1}{2}g_{\phi\gamma\gamma}\phi\vec{B})}_{''D''}{}^2 - \underbrace{(\vec{B} - \frac{1}{2}g_{\phi\gamma\gamma}\phi\vec{E})}_{''H''}{}^2\right]$$

$$\phi[\vec{x}_\gamma[t_{\rm rec}], t_{\rm rec}] = \phi_{\rm LSS}[\Omega]$$



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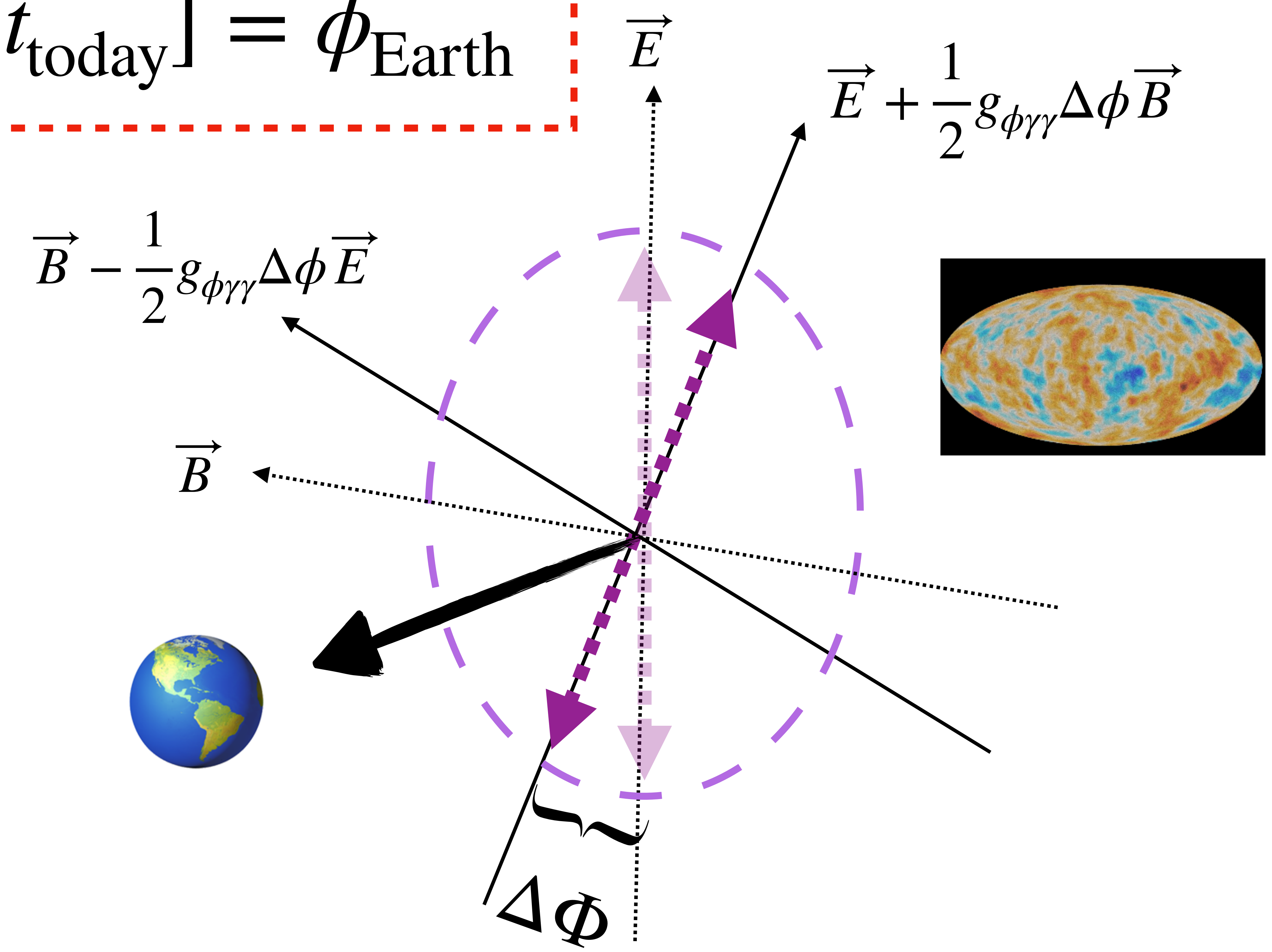
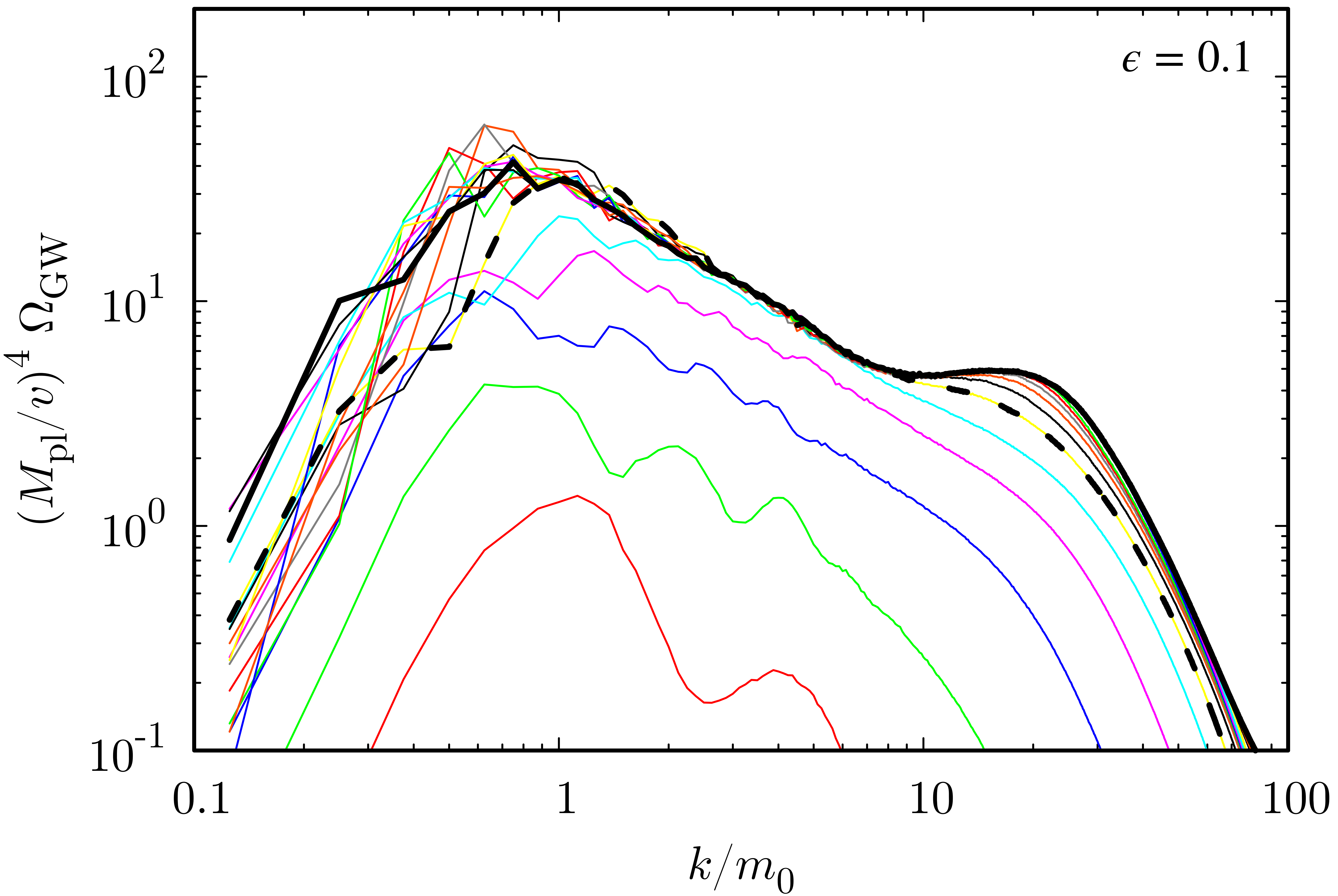
$$\phi[\vec{x}_\gamma[t_{\rm today}], t_{\rm today}] = \phi_{\rm Earth}$$

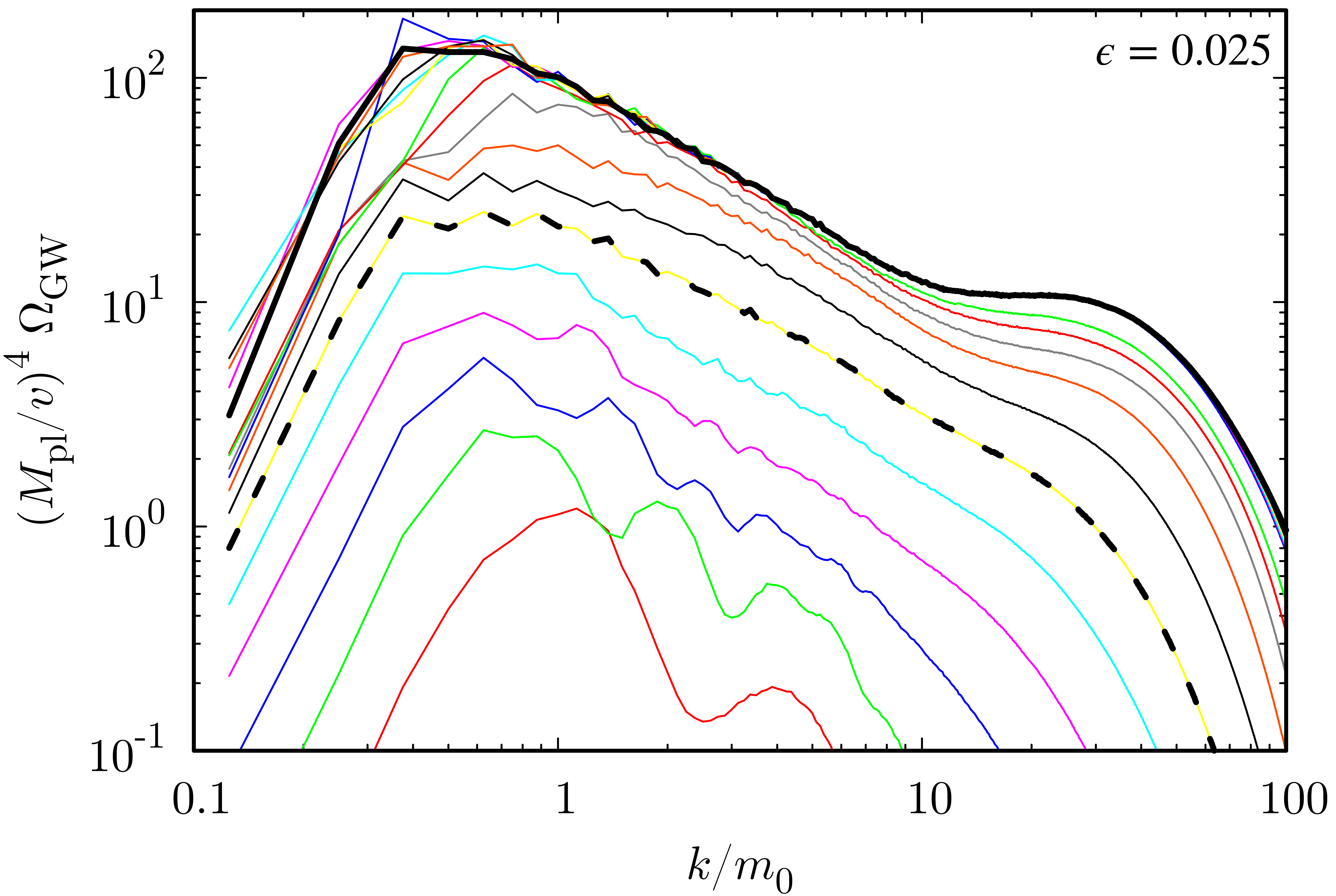


(if ALP background changes adiabatically.)

$$\Delta\Phi(\vec{\Omega}) = \frac{1}{2}g_{\phi\gamma\gamma}\int_{\rm LSS}^{\rm today} d\phi = 0.42\ {\rm deg} \times c_\gamma \left(\frac{\phi_{\rm Earth} - \phi_{\rm LSS}(\vec{\Omega})}{2\pi f_\phi}\right),$$



$\epsilon = 0.1$
10^2
10^1
10^0
10^{-1}
$(M_{\rm pl}/v)^4\,\Omega_{\rm GW}$
0.1
1
10
100
k/m_0



$\epsilon = 0.025$
10^2
10^1
10^0
10^{-1}
$(M_{\rm pl}/v)^4\,\Omega_{\rm GW}$
0.1
1
10
100
k/m_0