宇宙ひもによるダークフォトンDM生成と重力波生成

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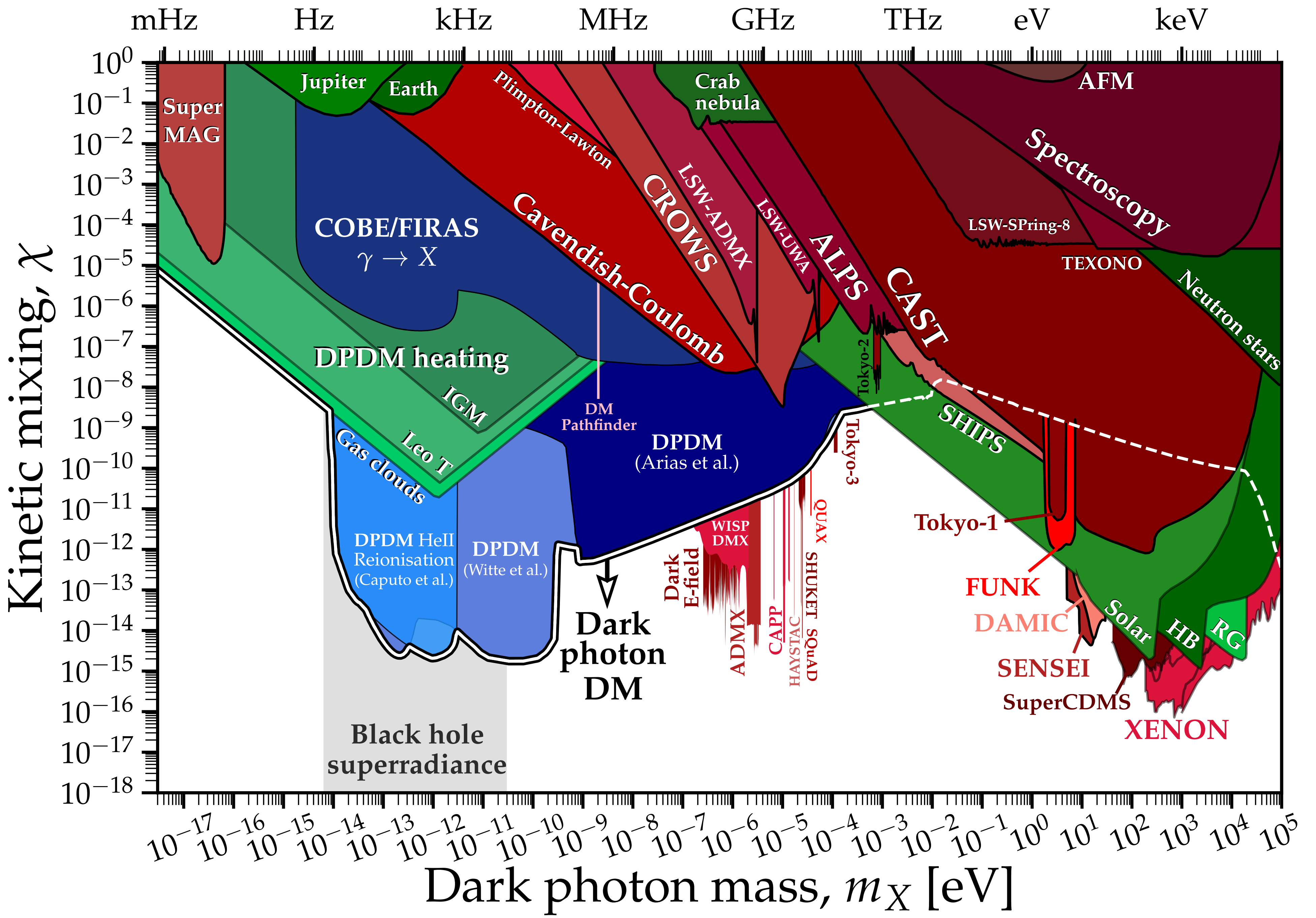
NK, Kazunori Nakayama (Tohoku U.), 2212.13573, 2306.17390

阿蘇ワークショップ 2023 11/12-15

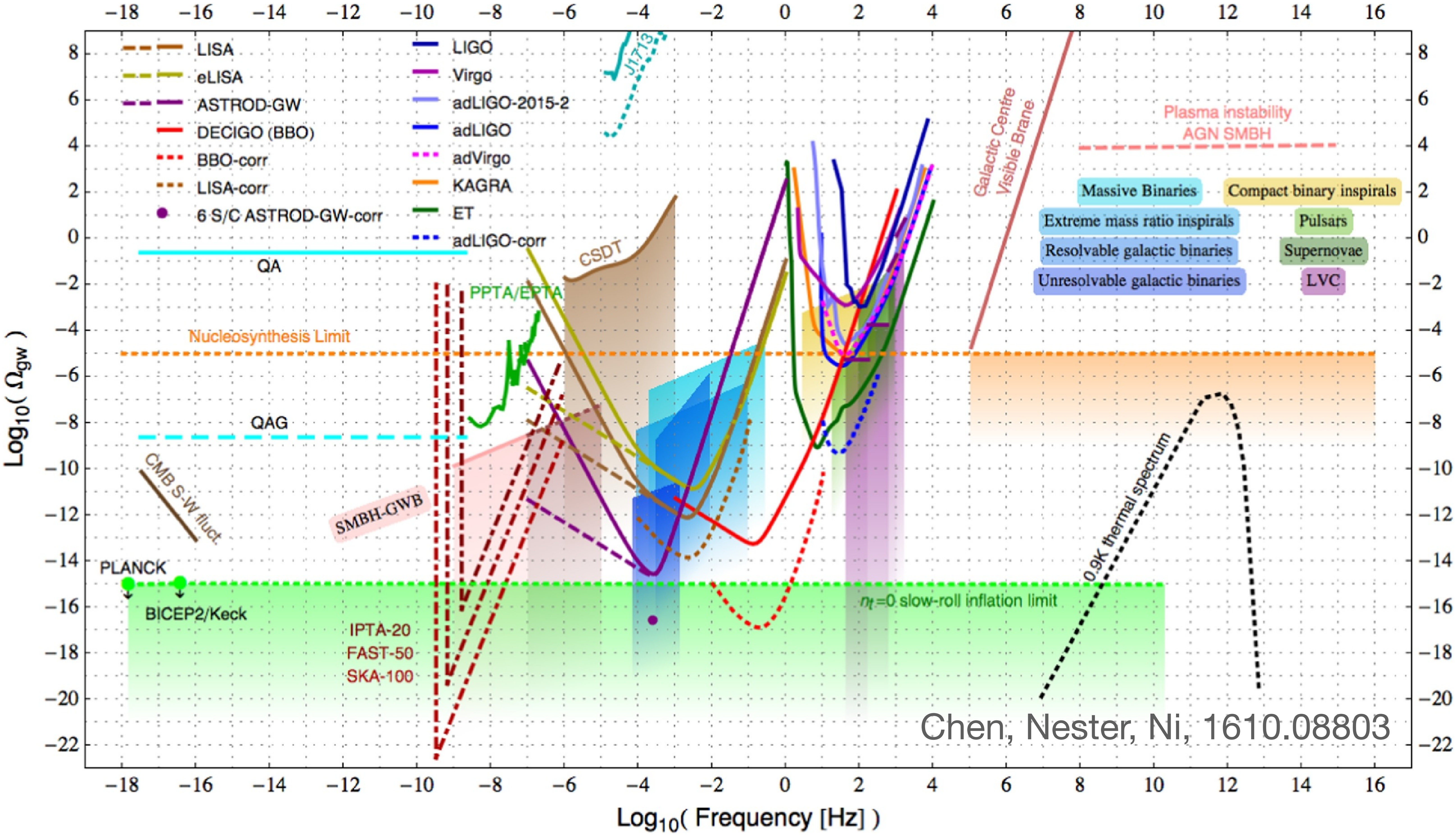
Dark photon search

$$\mathcal{L} \ni \frac{1}{2}\chi F^{\mu\nu} X_{\mu\nu}$$



Caputo et al 2105.04565

Gravitational wave proves

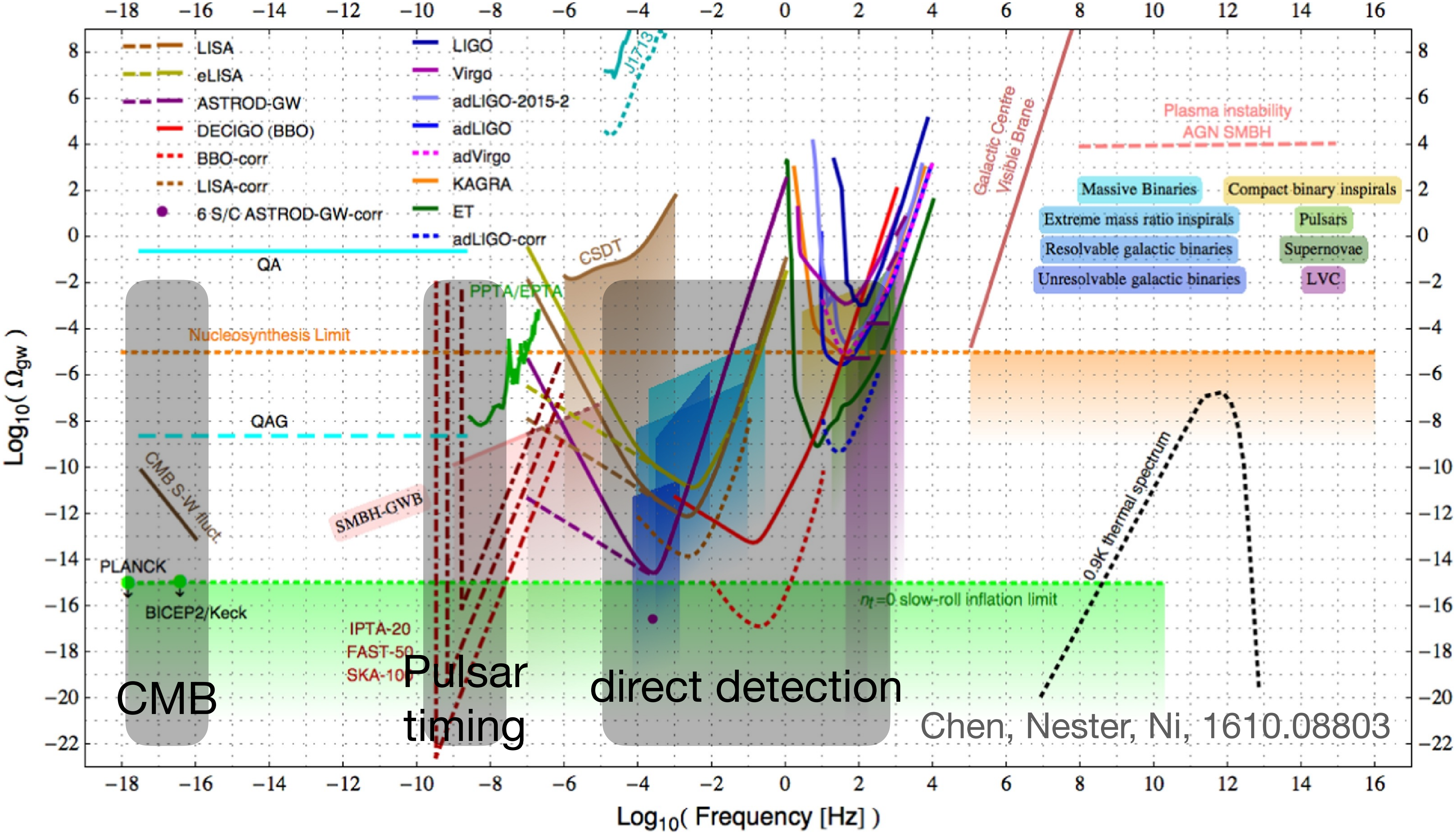


Chen, Nester, Ni, 1610.08803

⟷ more than 20 digits observable range

⟶ BSM search

Gravitational wave proves



CMB

Pulsar timing

direct detection

Chen, Nester, Ni, 1610.08803

more than 20 digits observable range

→ BSM search

Dark photon DM production

- Gravitational particle production during inflation / reheating

Graham, Mardon, Rajendran (2016) / Ema, Nakayama, Tang (2019)
Sato, Takahashi, Yamada (2022)

$$\Omega_{\gamma'} \simeq \Omega_{\rm DM}\sqrt{\frac{m_{\gamma'}}{6\,\mu{\rm eV}}}\left(\frac{H_{\rm inf}}{10^{14}\,{\rm GeV}}\right)^2$$ —> lower limit on dark photon mass

- Resonant production from scalar field

Axion : Agrawal, NK, Reece, Sekiguchi, Takahashi (2020)
Co, Pierce, Zhang, Zhao (2019), Bastro-Gil, Santiago, Ubaldi, Vega-Morales (2019)
NK, Takahashi (2023)

Higgs : Harigaya, Narayan (2019), Nakayama Yin (2021)

Spectator : Nakai, Namba, Obata (2022)

- Misalignment production Nakayama (2019), Nakayama (2020), NK, Nakayama (2023)

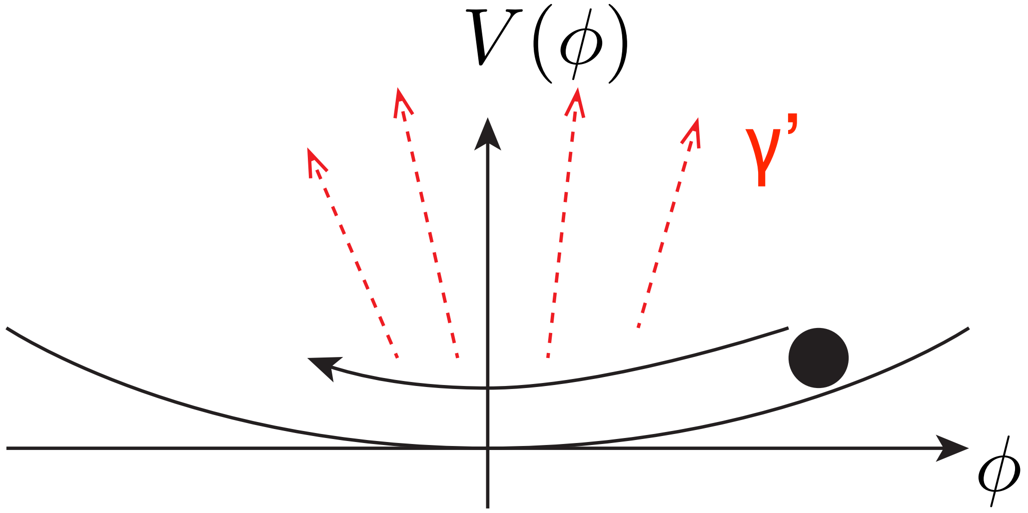
- Production from cosmic strings Long, Wang (2019), NK, Nakayama (2022)

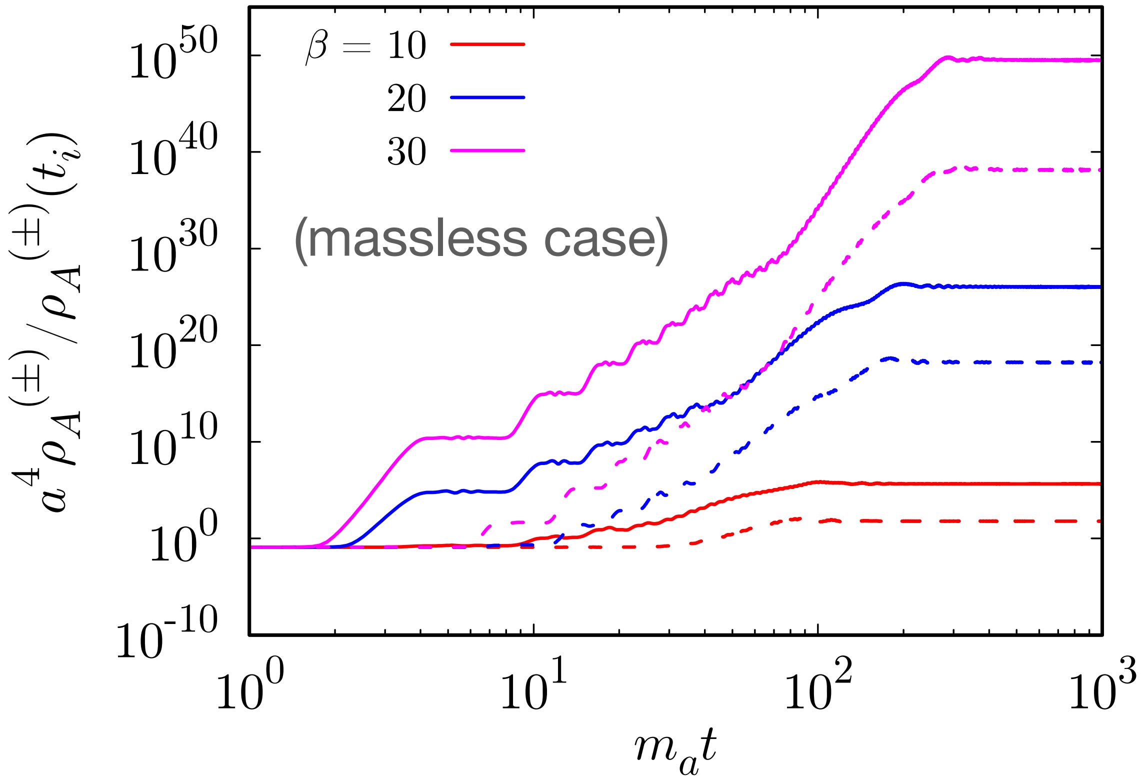
Resonant dark photon production from axion

Agrawal, NK, Reece, Sekiguchi, Takahashi, 1810.07188
Co, Pierce, Zhang, Zhao, 1810.07196
Bastero-Gil, Santiago, Ubaldi, Vega-Morales, 1810.07208

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_{\gamma'}^2 A_\mu A^\mu - \frac{\beta}{4f_a}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

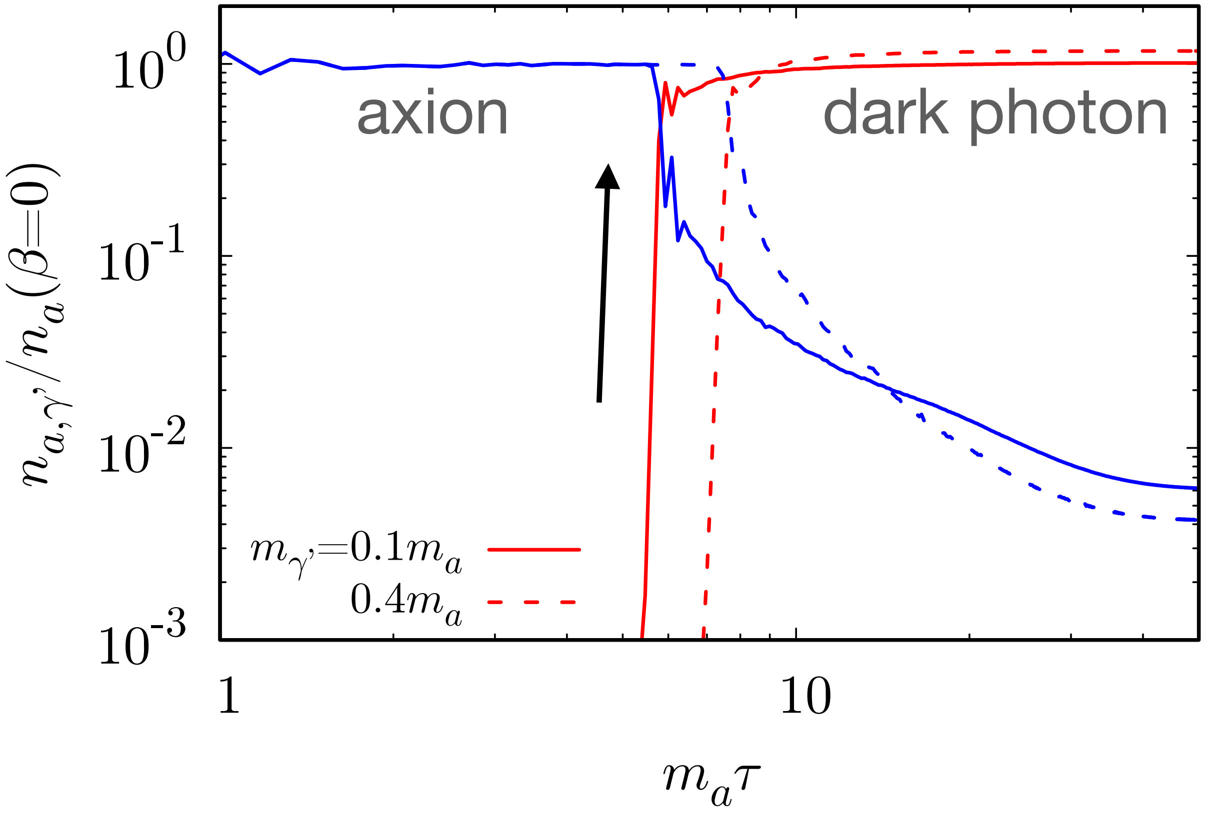
$$\longrightarrow \ddot{\mathbf{A}}_{\mathbf{k},\pm} + H\dot{\mathbf{A}}_{\mathbf{k},\pm} + \left(m_{\gamma'}^2 + \frac{k^2}{a^2} \mp \frac{k}{a}\frac{\beta\dot{\phi}}{f_a}\right)\mathbf{A}_{\mathbf{k},\pm} = 0$$



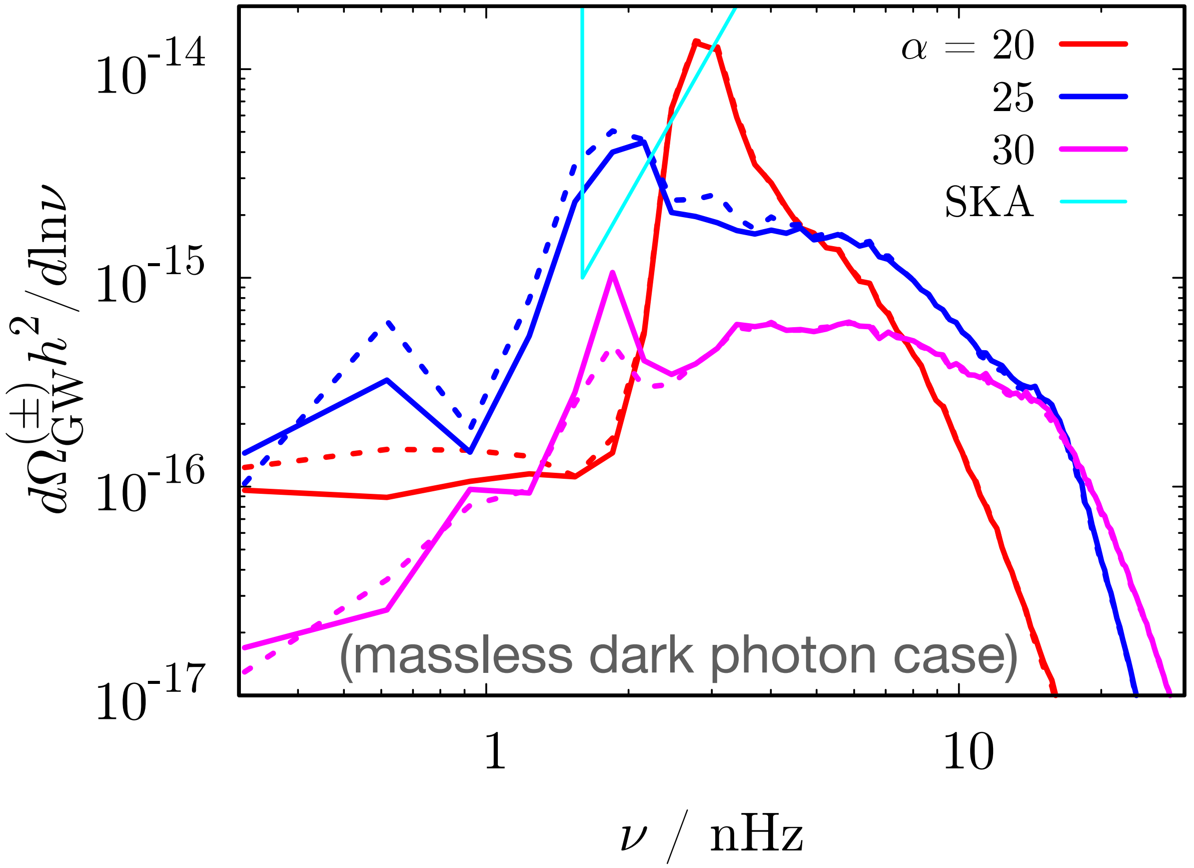


—> magnetogenesis Fujita+(2015), Kamada+(2019), Patel+(2020), …

non-linear evolution



gravitational wave



- Axion abundance is suppressed & dark photon is dominant

Agrawal, NK, Reece, Sekiguchi, Takahashi, 1810.07188
(see also NK, T. Sekiguchi, F. Takahashi, 1711.06590)

- Produced dark photons can stabilize the dark Higgs $V(\Phi) \ni |\boldsymbol{A}|^2|\Phi|^2$

—> secondary inflation, early dark energy

NK, Nakagawa, Takahashi, 2111.06696 Nakagawa, Takahashi, Yin, 2209.01107

- GW emission with circular polarization NK, Soda, Urakawa, 2010.10990

see also Machado+ (2019), Salehian+ (2020), Ratzinger+ (2020), Namba+ (2020)

Coherent vector DM production

Nakayama (2019), Nakayama (2020), NK, Nakayama (2023)

$$\mathcal{L} = -\frac{f^2(\phi)}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m_A^2 A_\mu A^\mu - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

$f^2 \propto a^\alpha, \ \bar{A}_i = f A_i / a, \ R_A = \frac{\rho_A}{\rho_\phi}$

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V \left(1 + \frac{\alpha R_A}{2\epsilon_V}\right) = 0 \qquad \epsilon_V = \frac{M_P^2}{2}\left(\frac{\partial_\phi V}{V}\right)^2$$

(slow-roll parameter)

$$\ddot{\bar{A}}_i + 3H\dot{\bar{A}}_i + \left(\frac{m_A^2}{f^2} - \frac{(\alpha+4)(\alpha-2)}{4} H^2 + \frac{2-\alpha}{2}\dot{H}\right)\bar{A}_i = 0$$

Statistical anisotropy $\mathcal{P}_\zeta(\boldsymbol{k}) = \mathcal{P}_\zeta^{(\mathrm{iso})}(k)(1 + g_k \sin^2\theta_k), \quad \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{A}} = \cos\theta_k$

$\& \quad g_k \propto R_A$

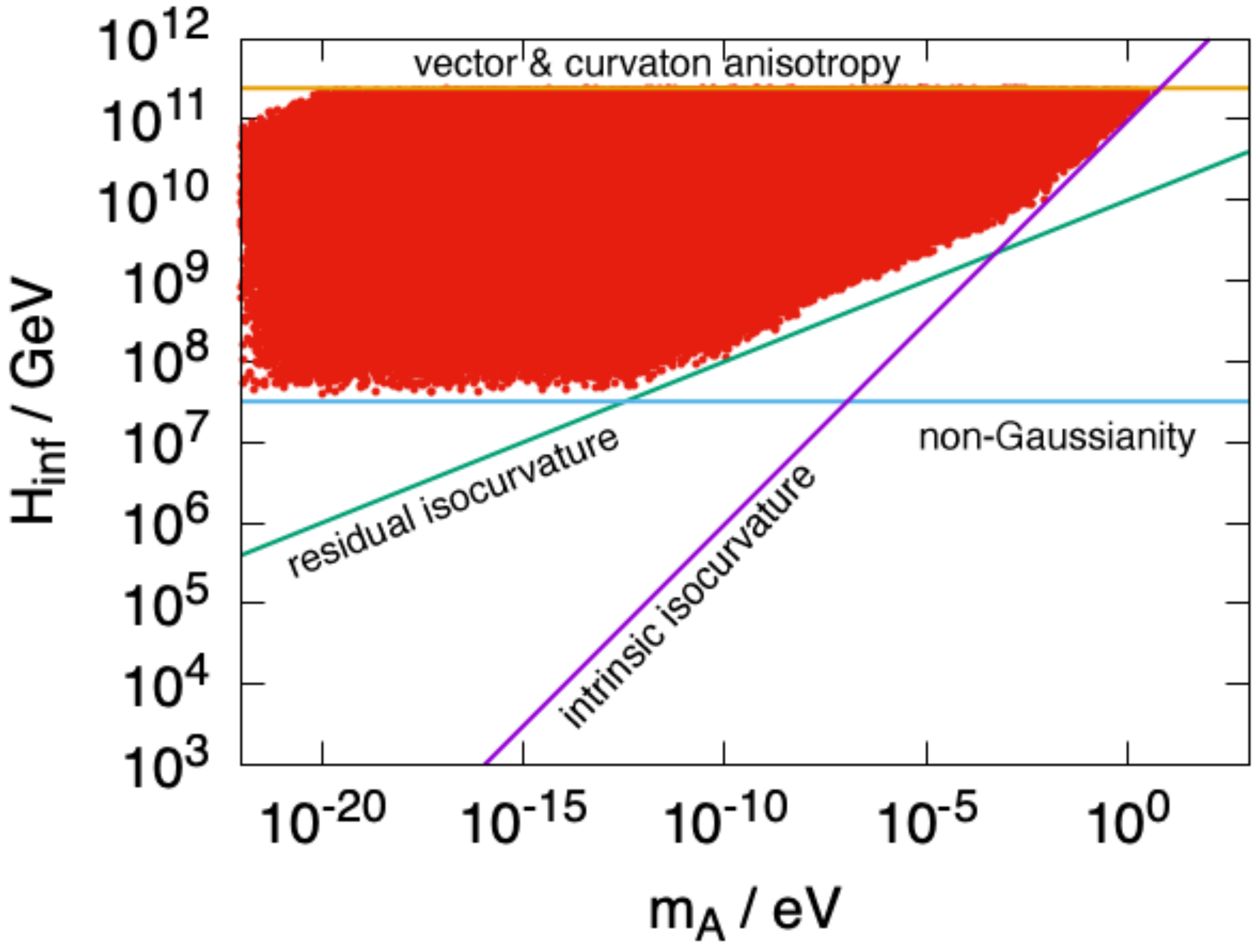
DM isocurvature perturbation $S = \frac{\delta\rho_A}{\bar{\rho}_A} \sim \frac{H_{\mathrm{inf}}}{\pi \bar{A}_i} \propto R_A^{-1}$

CMB observation —> $g_k \lesssim 0.01, \quad S \lesssim 0.1\zeta$

“Viable” coherent vector DM scenario

NK, Nakayama, 2303.04287

curvaton scenario : introduction of an additional scalar field responsible for the curvature perturbation



Abelian-Higgs model

$$\mathcal{L} = (\mathcal{D}_\mu \Phi)^* \mathcal{D}^\mu \Phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\Phi), \;\; V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2$$

$$(\mathcal{D}_\mu = \partial_\mu - ieA_\mu, \;\; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$$

Higgs mechanism may give the dark photon mass

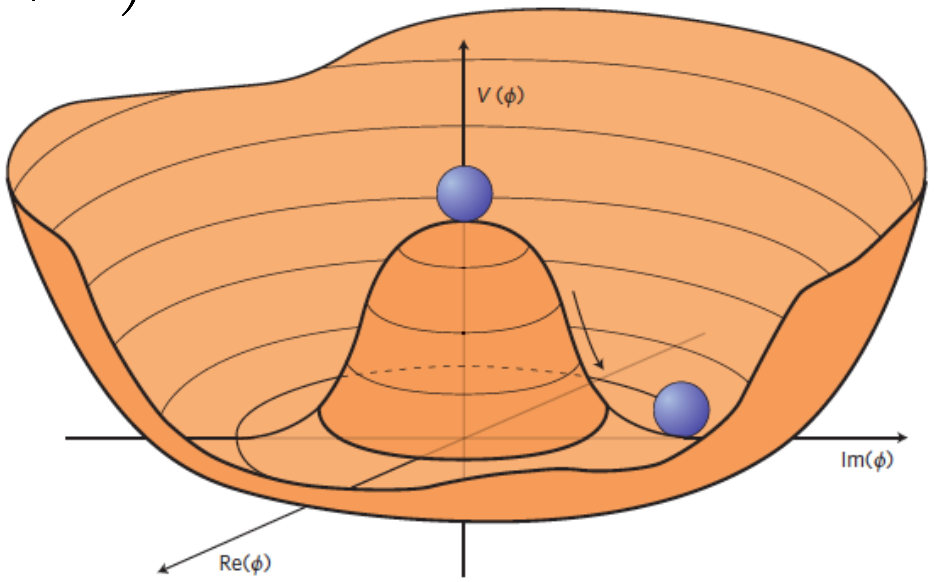
→ formation of cosmic string

(if symmetry breaking occurs after inflation)

Assumption : $m_A \ll m_\Phi \quad (e \ll \sqrt{\lambda})$

(Type II string)

—> dark photon production



Ellis, Gaillard, Nanopoulos 1504.07217

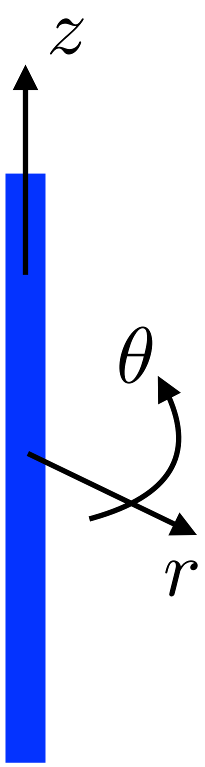
Abelian-Higgs string (local string)

$$\mathcal{L} = (\mathcal{D}_\mu\Phi)^*\mathcal{D}^\mu\Phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\Phi),\ \ V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2$$

$$(\mathcal{D}_\mu = \partial_\mu - ieA_\mu,\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$$

static solution: $\Phi(r,\theta) = ve^{in\theta}f(r),\ \ \boldsymbol{A}(r,\theta) = \frac{n\alpha(r)}{er}\hat{\boldsymbol{e}}_\theta$

with boundary condition $f(0) = \alpha(0) = 0,\ f(\infty) = \alpha(\infty) = 1$



energy of local string :

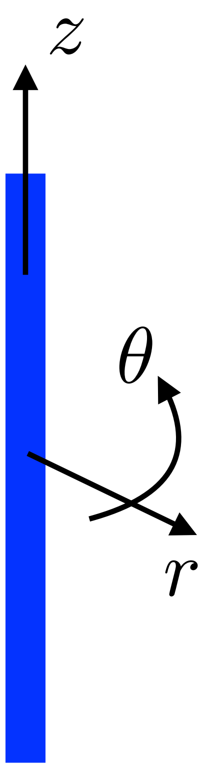
$$\begin{aligned}\mathcal{E} &= \int d^3x\left(|\mathcal{D}_i\Phi|^2 + V(\Phi) + \frac{1}{2}|\nabla\times\boldsymbol{A}|^2\right)\\ &= 2\pi L\int rdr\left(f'^2 + \frac{n^2(\alpha-1)^2}{r^2}f^2 + V(f) + \frac{n^2\alpha'^2}{2e^2r^2}\right)\\ &\equiv \mu L\ \ \text{(string tension × string length)}\end{aligned}$$

Global (axion) string

$$\boxed{\mathcal{L} = (\partial_\mu \Phi)^* \partial^\mu \Phi - V(\Phi),\ \ V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2}$$

static configuration: $\Phi(r,\theta) = v e^{in\theta} f(r) \ \ \text{with} \ \ f(0) = 0 \ \ \& \ \ f(\infty) = 1$

energy of global string :



$$\mathcal{E} = \int d^3x \left(|\partial_i \Phi|^2 + V(\Phi)\right) = 2\pi L \int r dr \left(f'^2 + \frac{n^2 f^2}{r^2} + V(f) \right)$$

log-divergence

$$\approx 2\pi L v^2 \int_\delta^\Lambda dr \frac{1}{r} = 2\pi L v^2 \log\left(\frac{\Lambda}{\delta}\right)$$

$$\xrightarrow[\Lambda \to t]{} \qquad \mu = 2\pi v^2 \log\left(\frac{t}{\delta}\right)$$

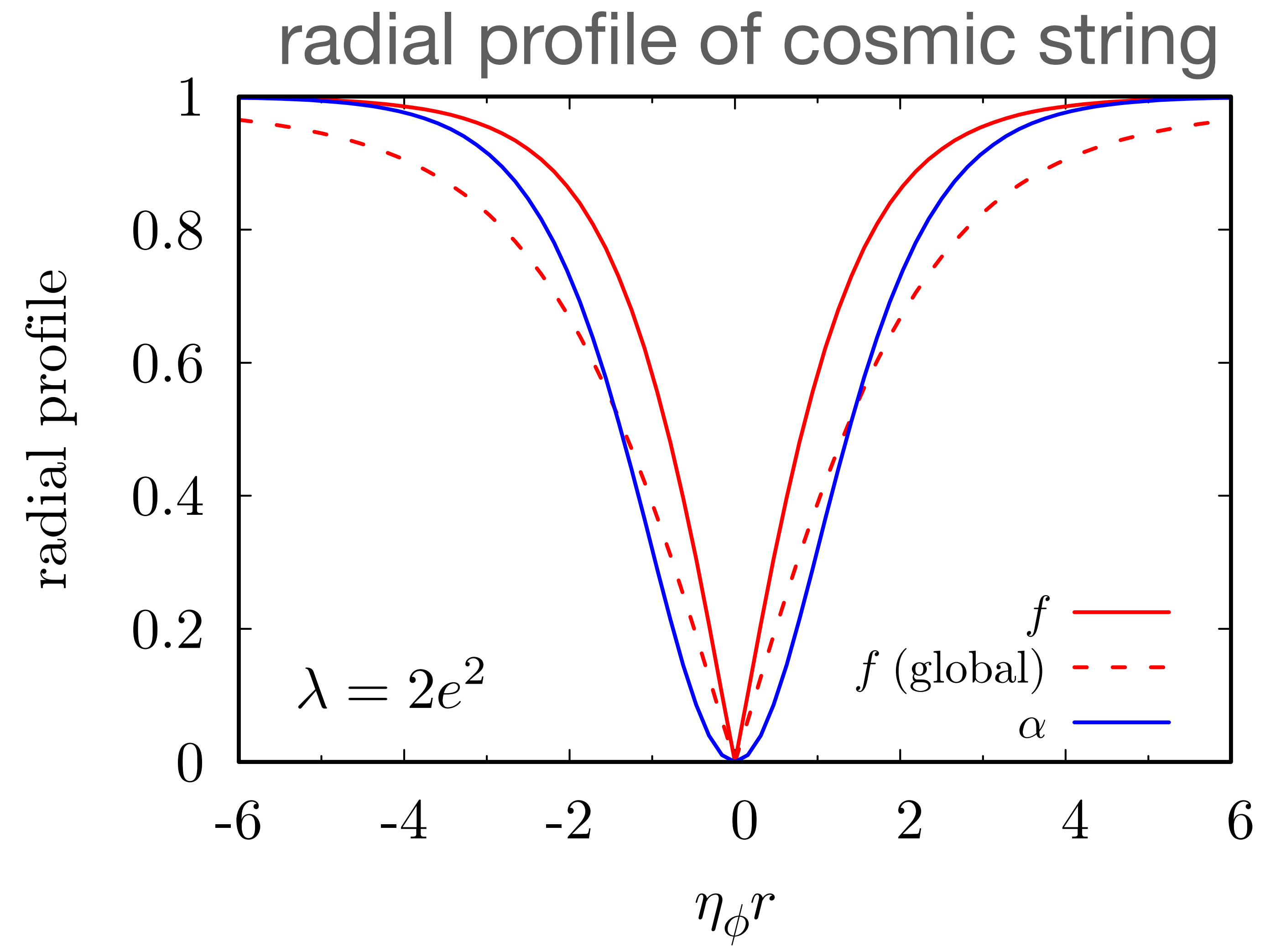
(log-time-dependent)

(cutoff scale ~ horizon scale)

energy minimization
(local string)

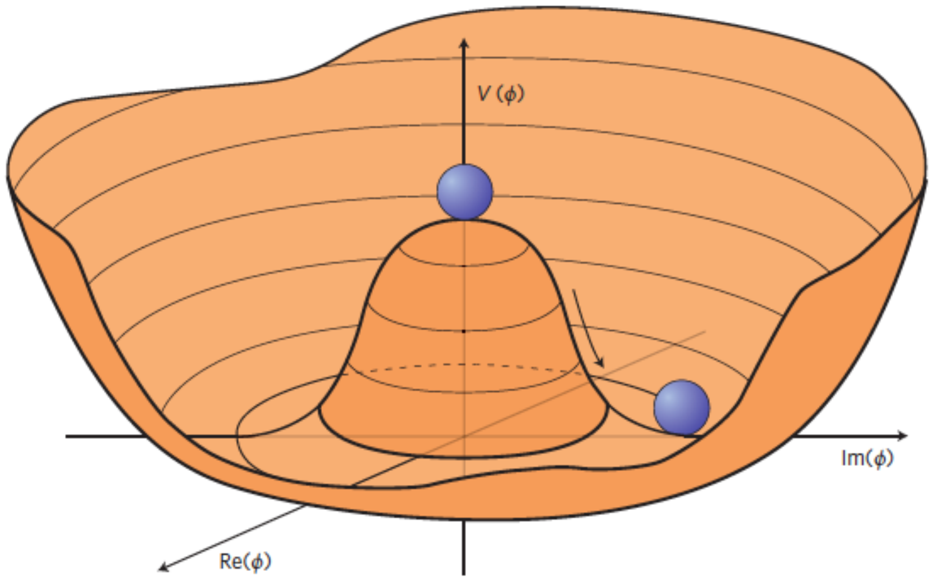
$$f'' + \frac{f'}{r} - \frac{n^2(\alpha-1)^2 f}{r^2} - \frac{1}{2}\frac{\partial V}{\partial f} = 0$$

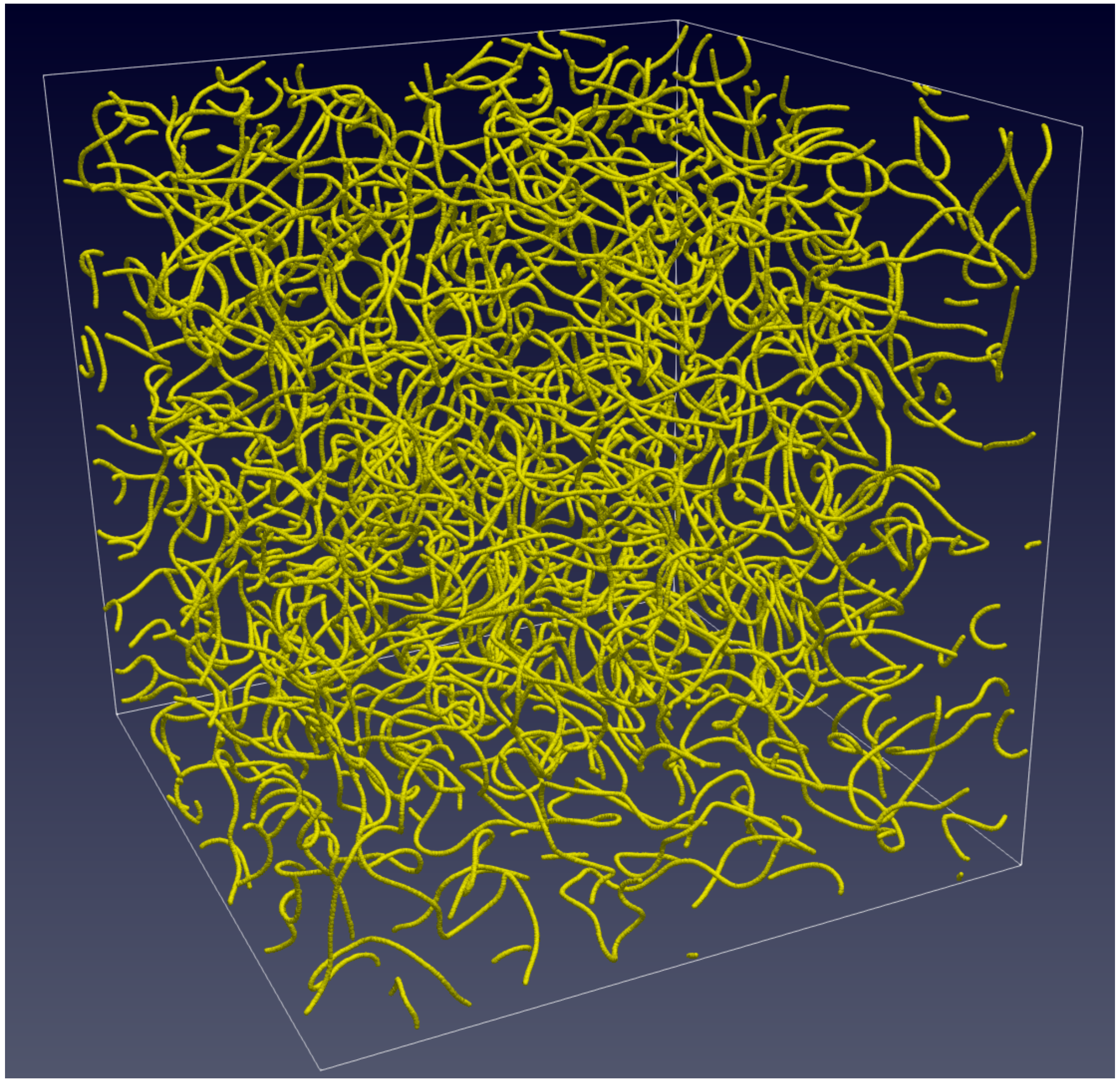
$$\alpha'' - \frac{\alpha'}{r} - 2e^2 f^2(\alpha - 1) = 0$$


radial profile of cosmic string

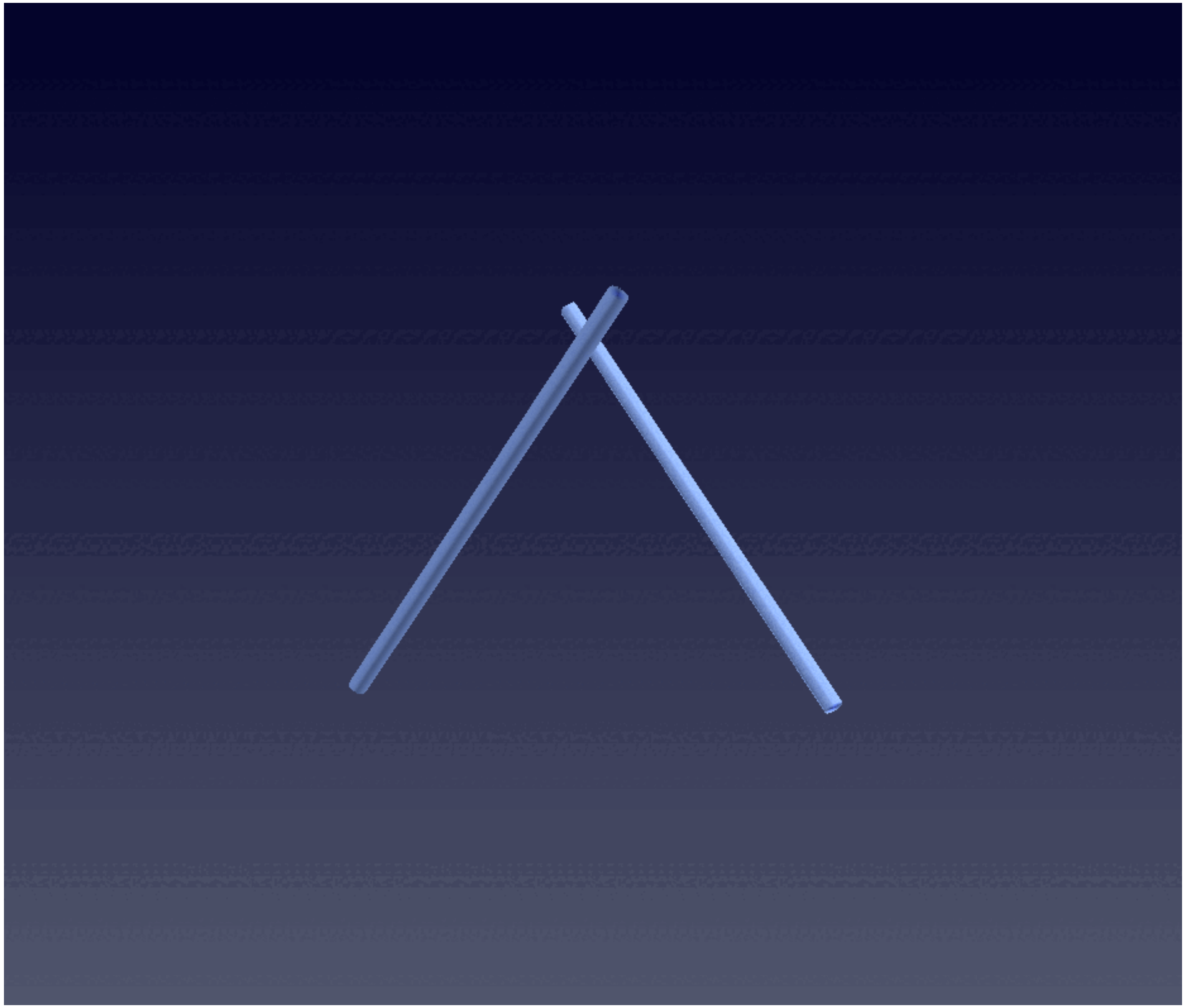
local string is more localized

Cosmic string “network”



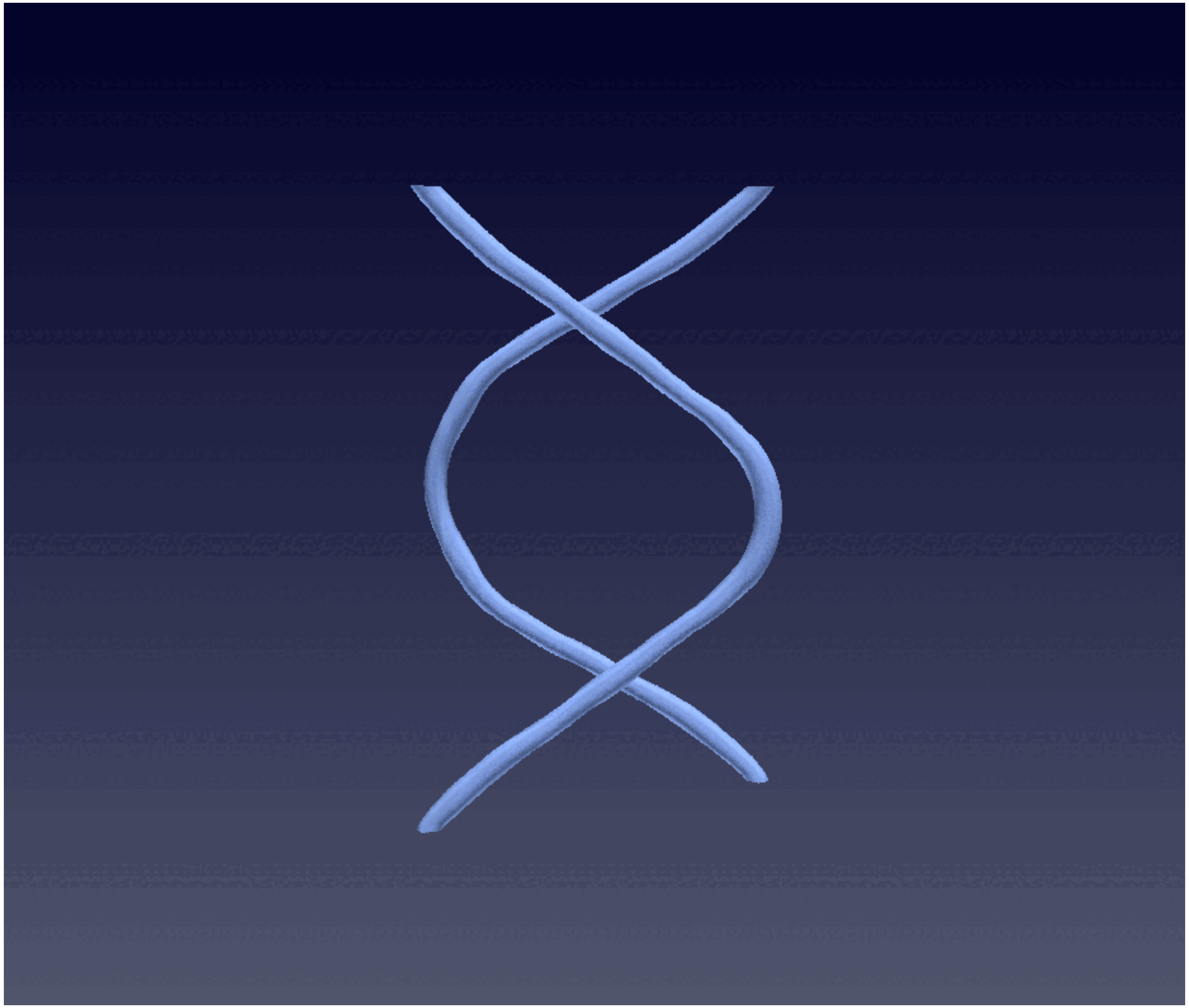


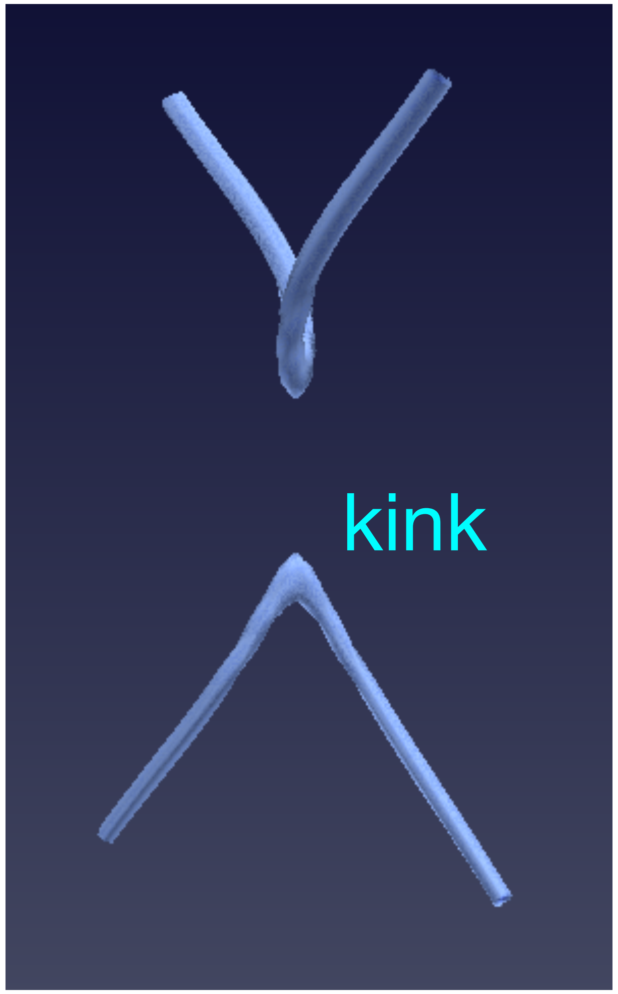
collision of straight strings



pair annihilation of string and anti-string (opposite winding number)

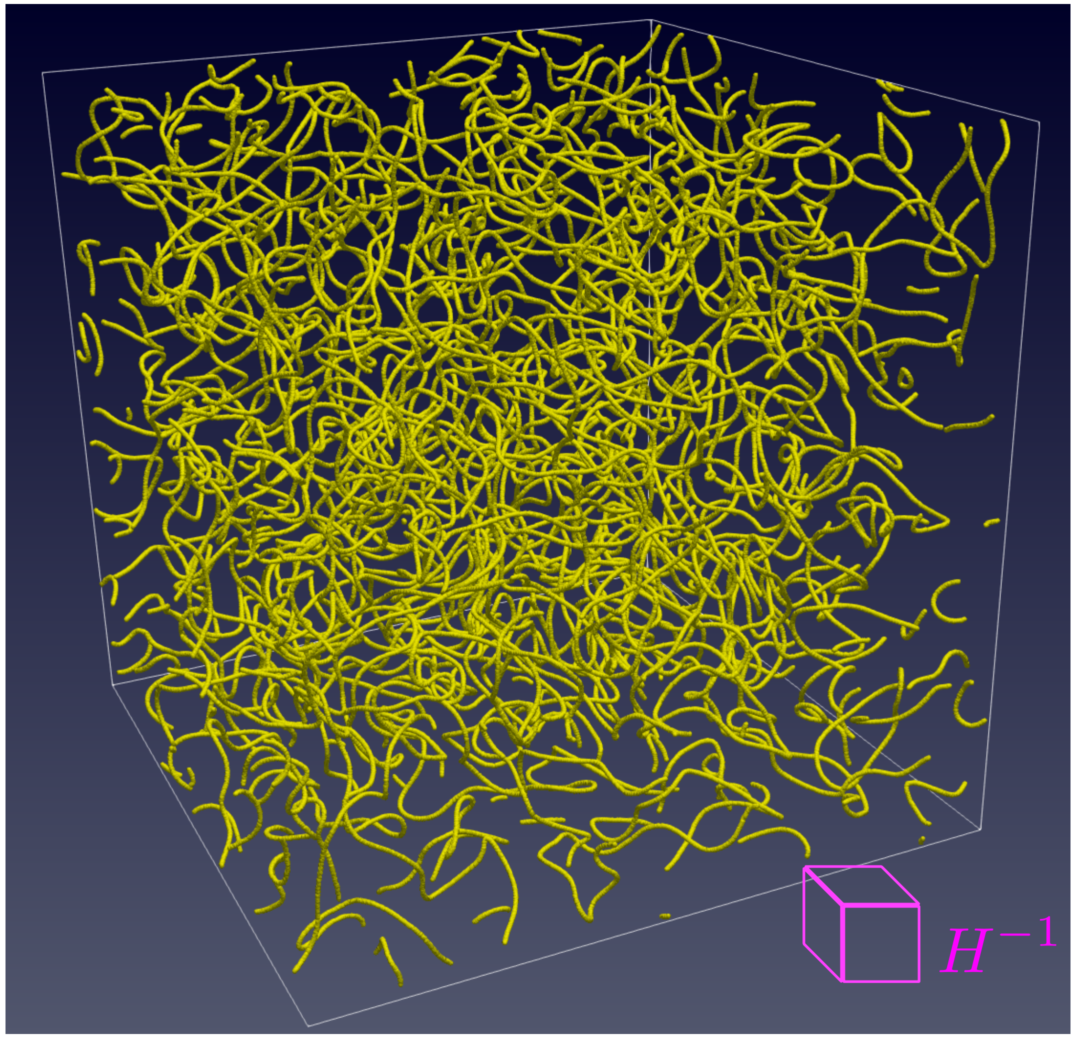
collision of curved strings

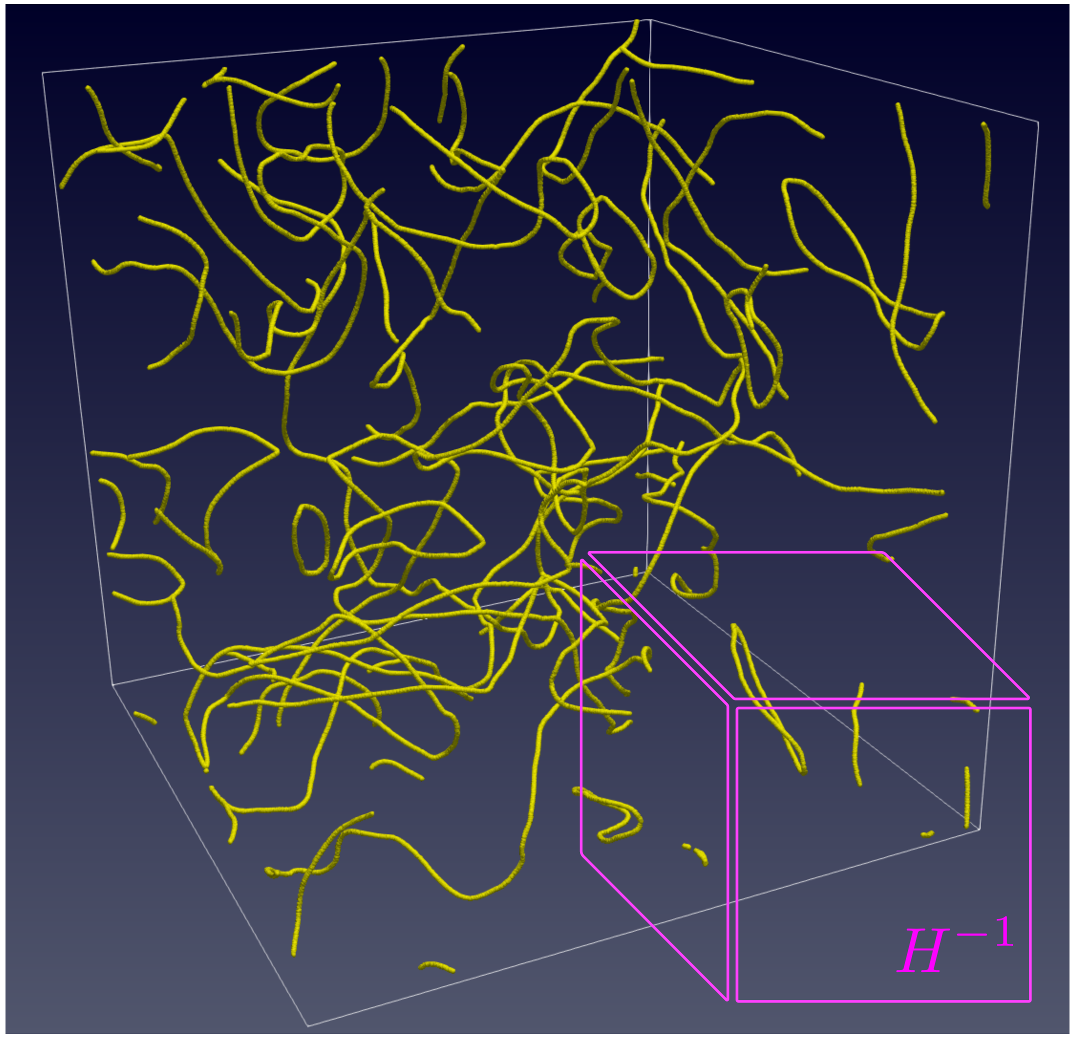




—> source of gravitational waves

Cosmic string network





Scaling law : O(1) strings per Hubble patch

Scenario

Long, Wang 1901.03312
NK, Nakayama 2212.13573

- "Light" dark photons can be produced by cosmic strings

small gauge coupling i.e. $m_A \ll m_\Phi \quad (e \ll \sqrt{\lambda})$ (Type II string)
e = 0 limit corresponds to the massless NG boson emission (global string case)

- Dark photon production becomes inefficient for $\ell_{\rm loop} \gtrsim m_A^{-1}$

i.e. loop oscillation frequency becomes smaller than the mass $\rightarrow \quad H \lesssim m_A$

Dark photon abundance is fixed

cf. axion DM are produced through domain wall decay

- After that, string evolves like "local" string

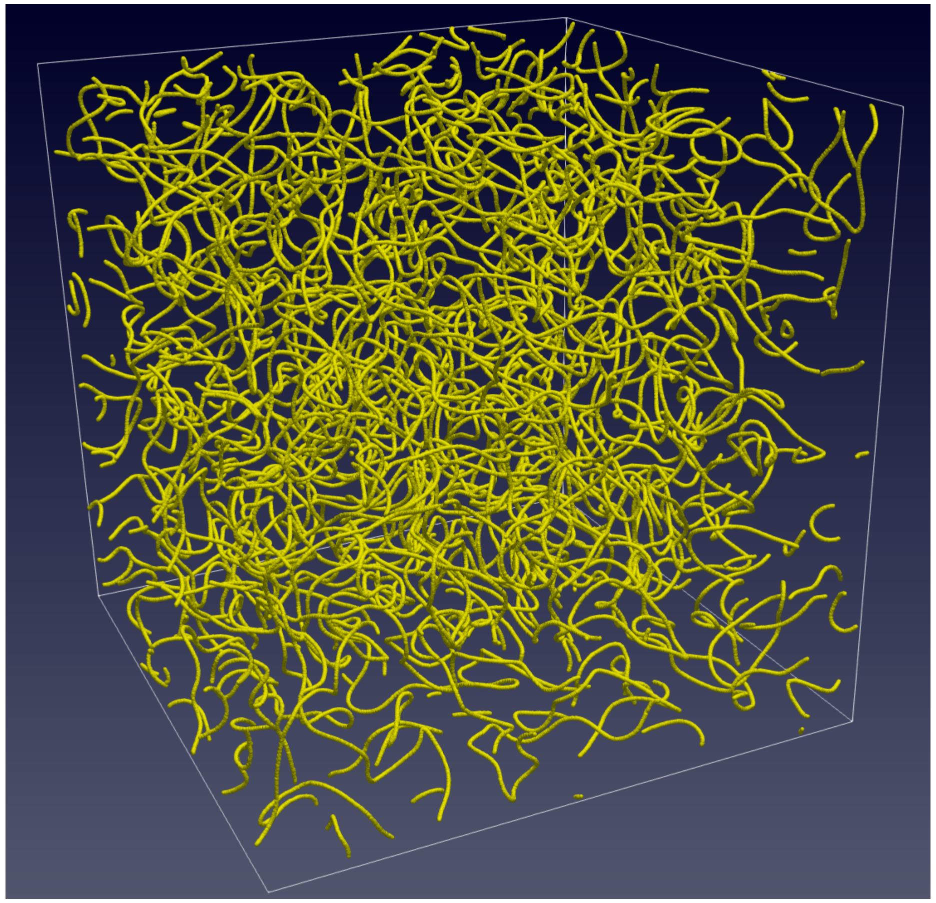
network loses the energy only through the GW emission (Nambu-Goto limit)

Field theoretic (Abelian-Higgs) simulation

$$\Phi'' + 2\mathcal{H}\Phi' - D_i D_i \Phi + a^2 \frac{\partial V}{\partial \Phi^*} = 0,$$
$$E_i' + \partial_j F_{ij} - 2ea^2\,\mathrm{Im}(\Phi^* D_i \Phi) = 0,$$
$$\partial_i E_i - 2ea^2\,\mathrm{Im}(\Phi^* \Phi') = 0$$

Moore+ (2001)
Bevis+(2007)
Dufaux(2010)
Hiramatsu+(2013)
Correia+(2020)
and more

* Lattice-gauge formulation, Gauss's law is satisfied



- AOBA (SX-Aurora TUBASA) in Tohoku U.

4,032 Vector Engine

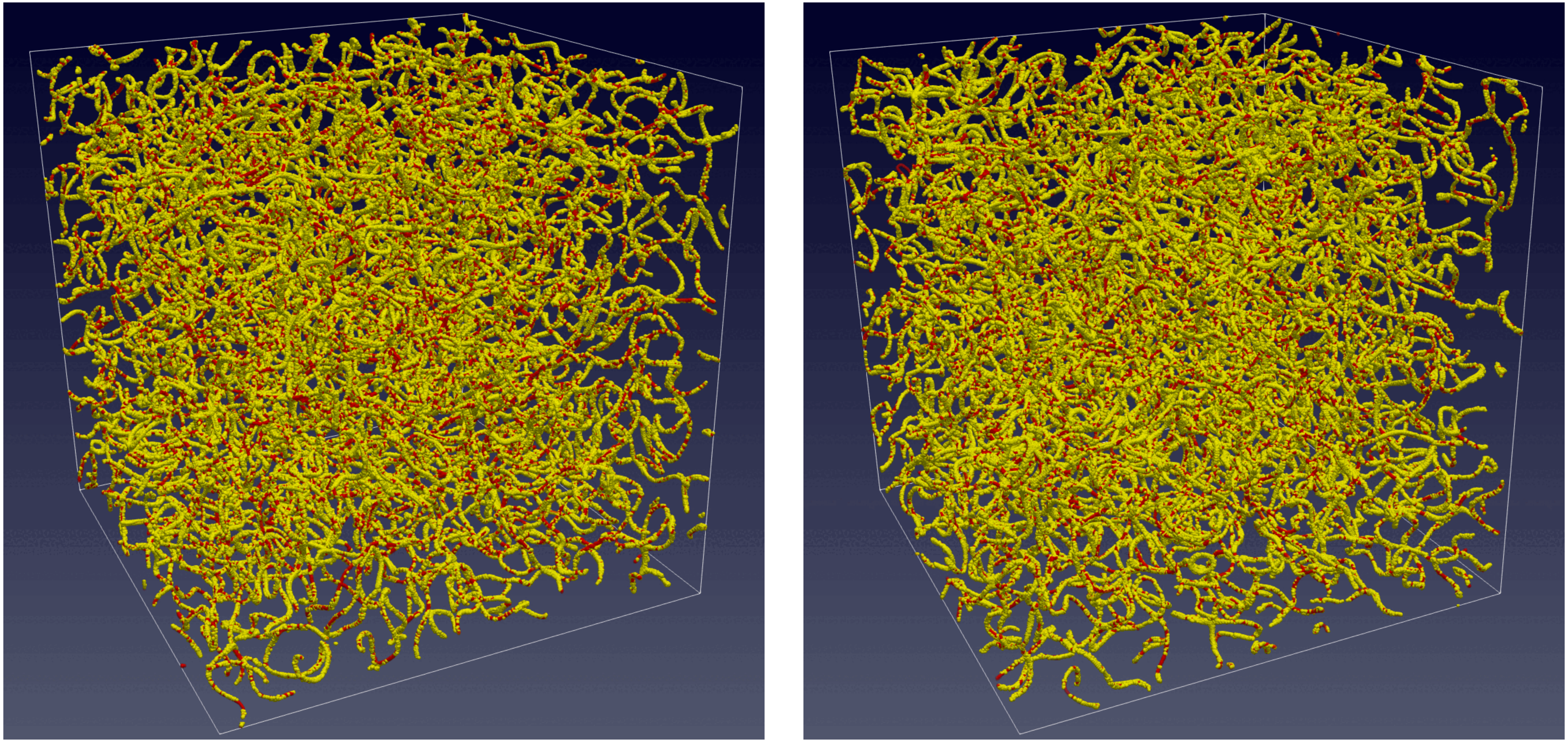
16 cores, 256 vector length, 96GB / 1VE

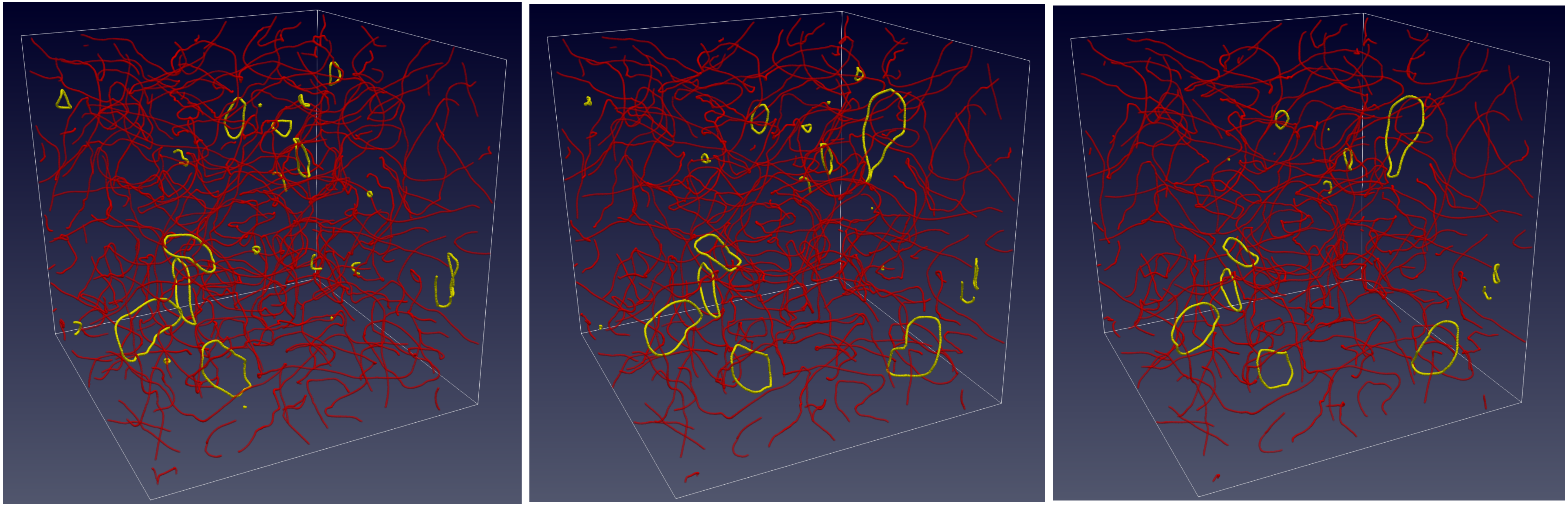
- FUGAKU supercomputer (Riken) (trial use)

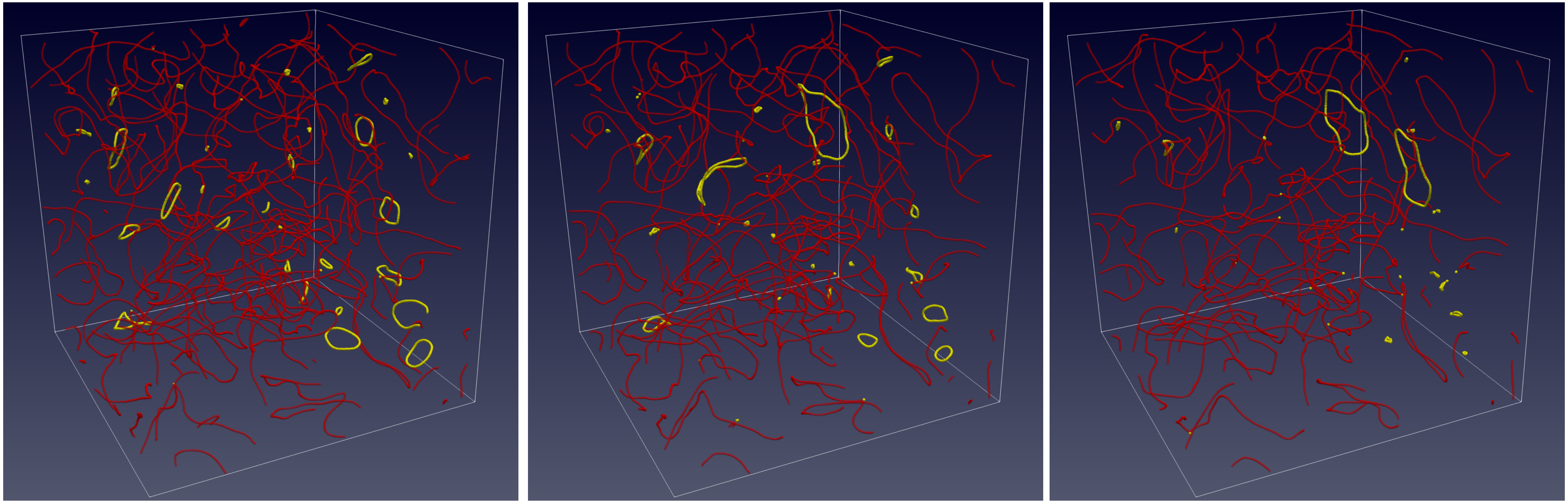
158.976 nodes

48 cores, 32GiB / 1node

Loop production & decay







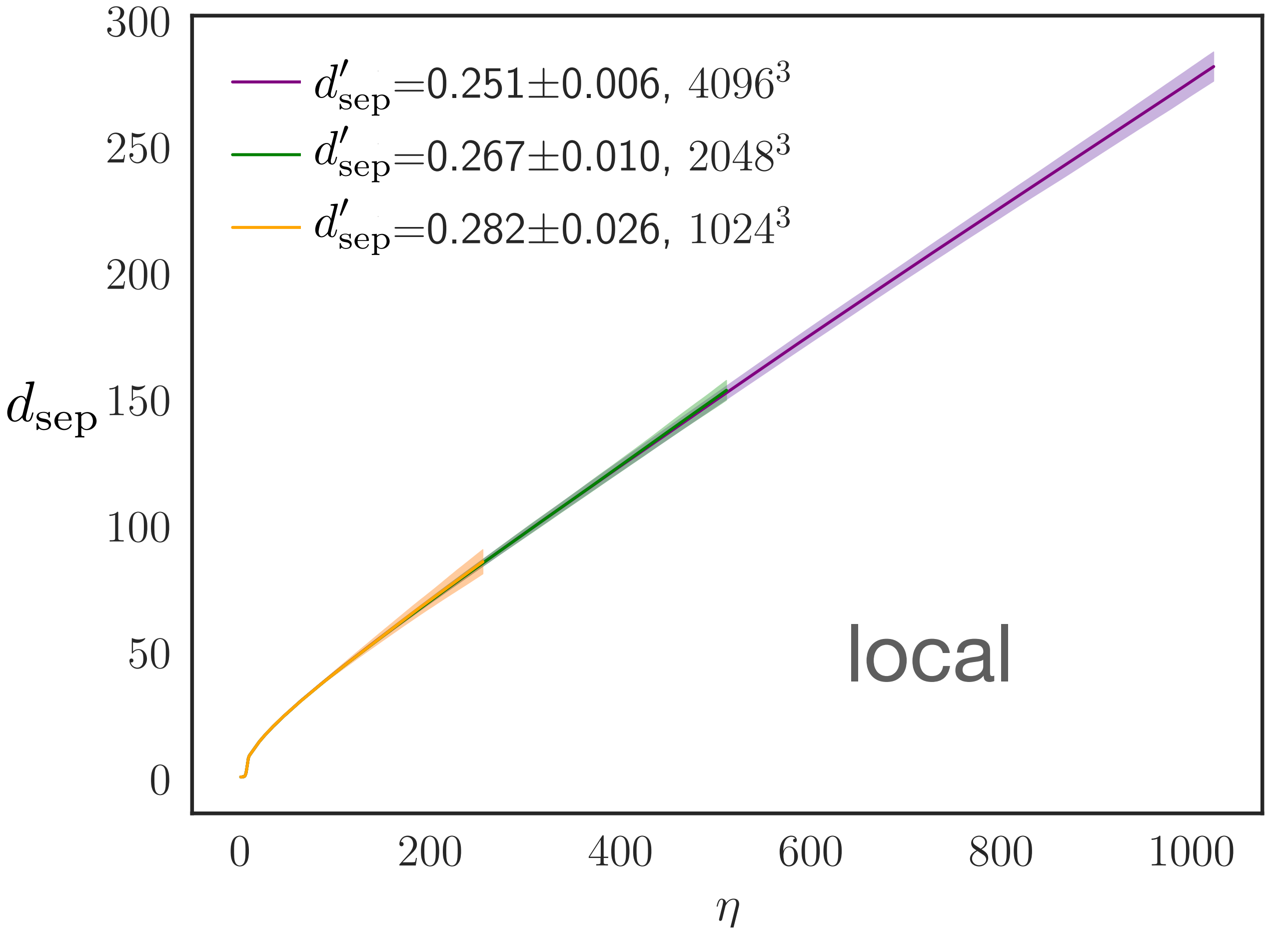
Scaling law of local / global string

Mean separation of neighboring strings: $d_{\rm sep} = \sqrt{\dfrac{V^{(c)}}{\ell^{(c)}_{\rm str}}}$

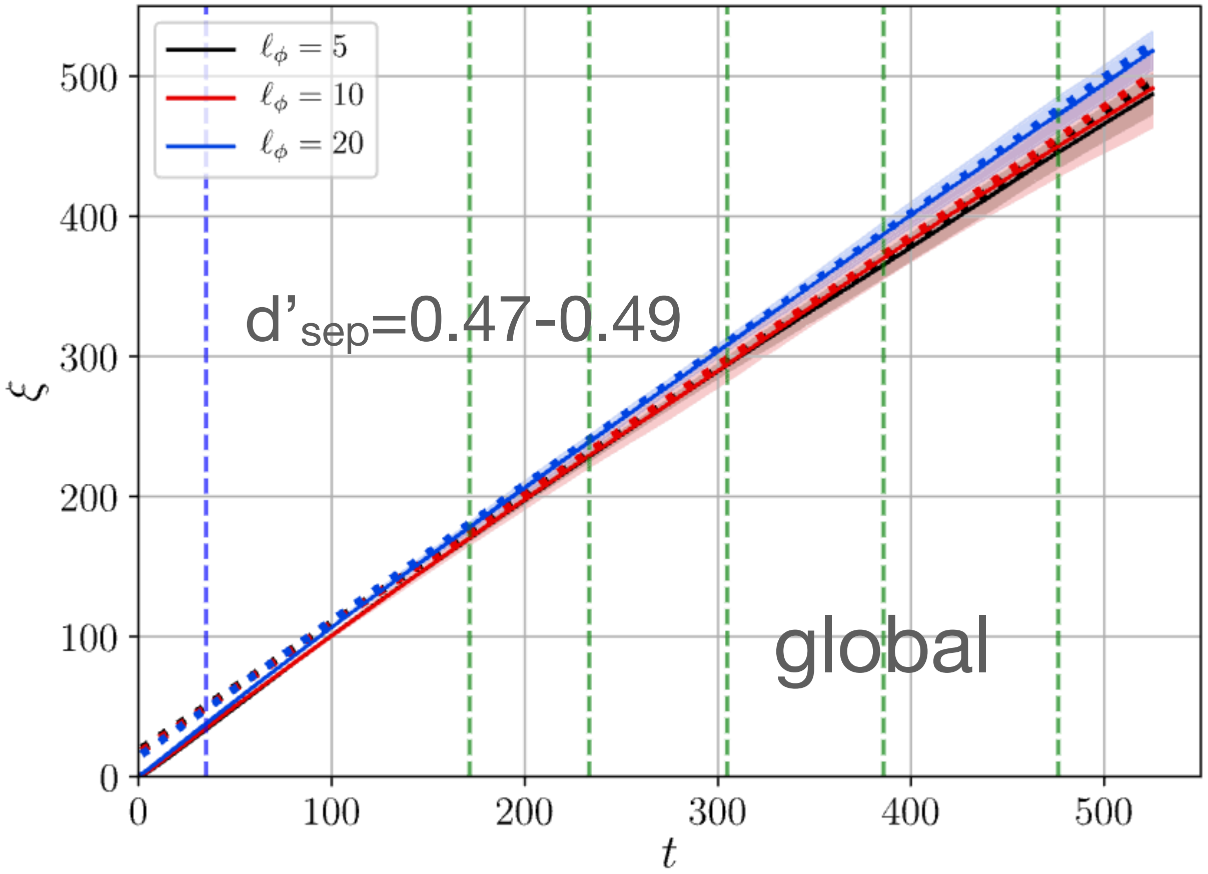
(comoving / physical length)

Scaling law —> $d_{\rm sep} \propto \tau, \quad \text{i.e.} \quad d'_{\rm sep} = \text{const}$

(conformal / physical time)



Correia+ arXiv:2005.14454



Hindmarsh+ arXiv:1908.03522

Global string & scaling violation?

Number of strings per Hubble volume: $\xi = \dfrac{\ell_{\rm str} t^2}{V}$

$\xi = \text{const}$ (scaling law) vs $\xi = \alpha \log(t) + \beta$ (scaling violation)

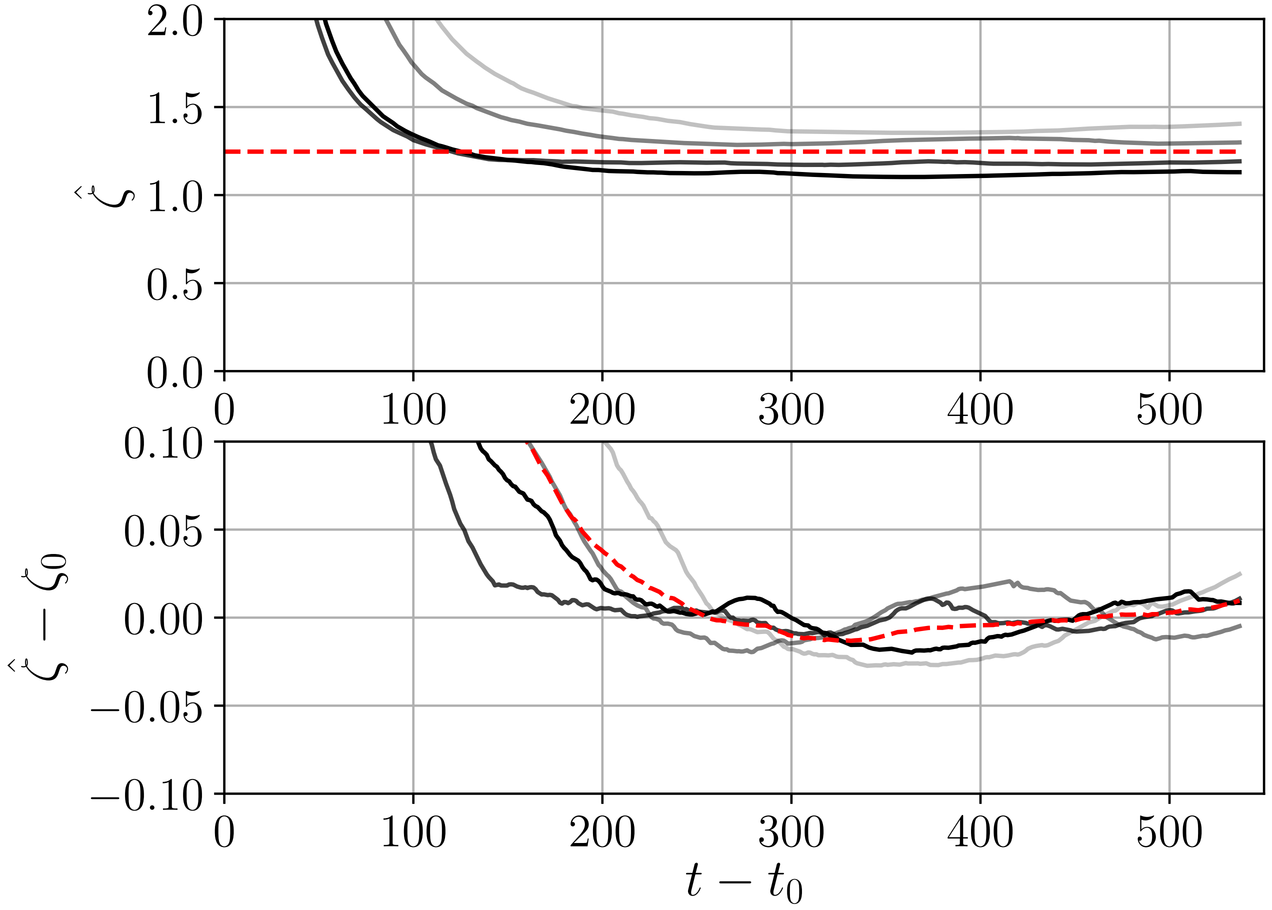
Hindmarsh et al, 1908.03522

Hindmarsh et al, 2102.07723

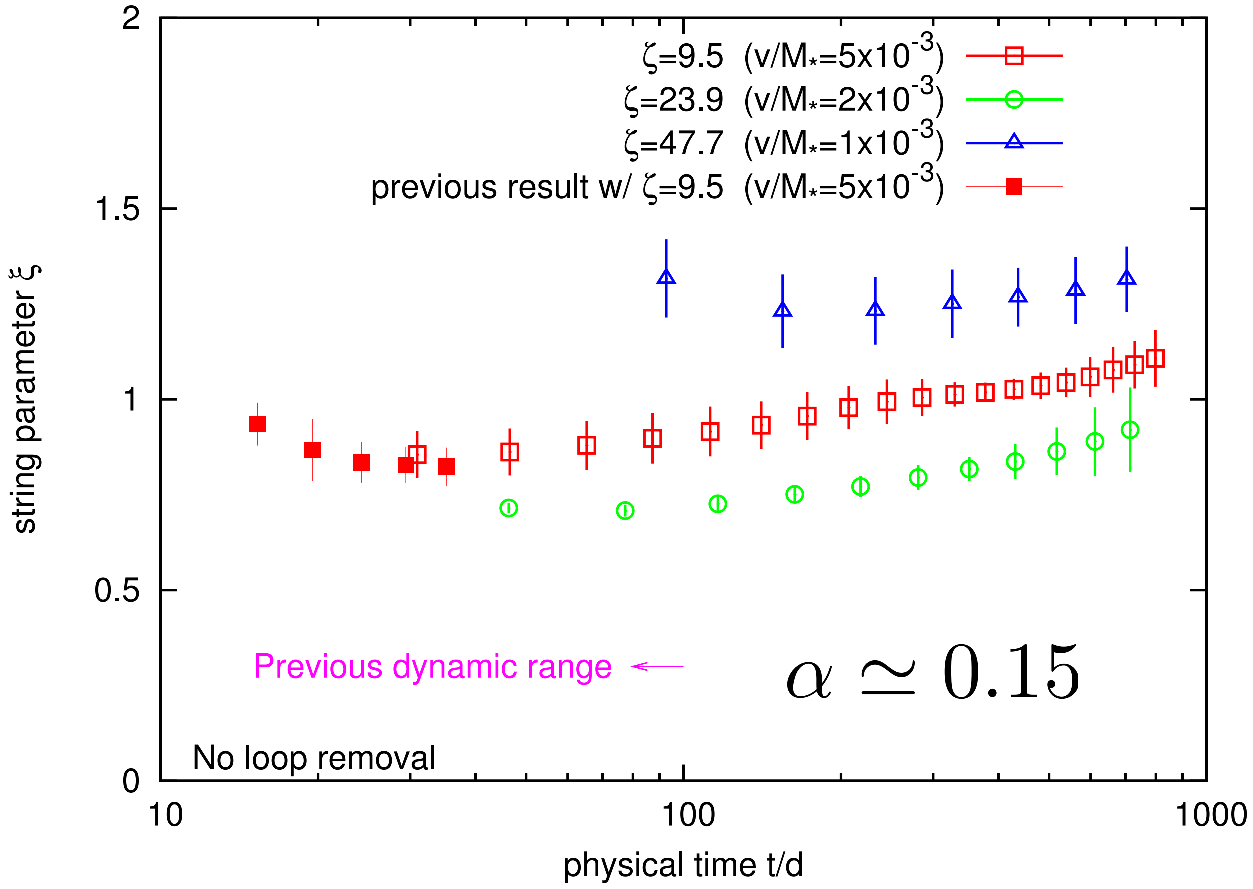
Gorghetto et al, 1806.04677

Kawasaki et al, 1806.05566

Buschmann et al, 2108.05368



Hindmarsh+ 1908.03522



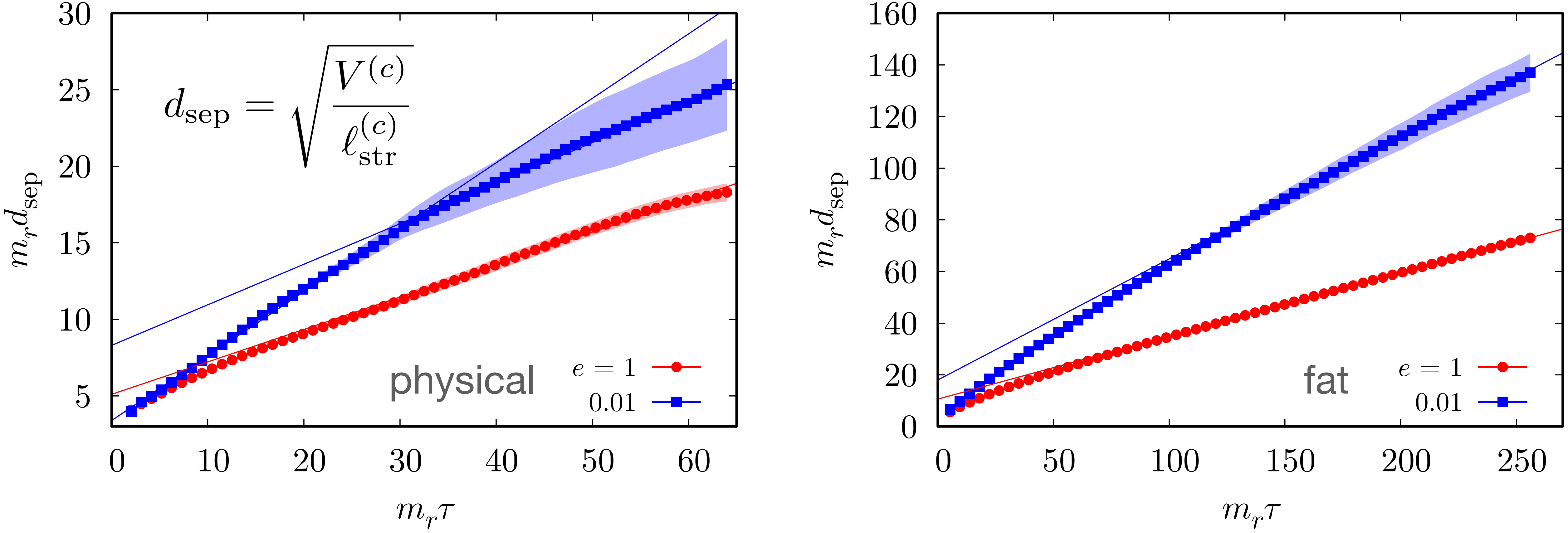
$\alpha \simeq 0.15$

Kawasaki+ 1806.05566

Mean separation

NK, Nakayama 2212.13573

global str —> local str

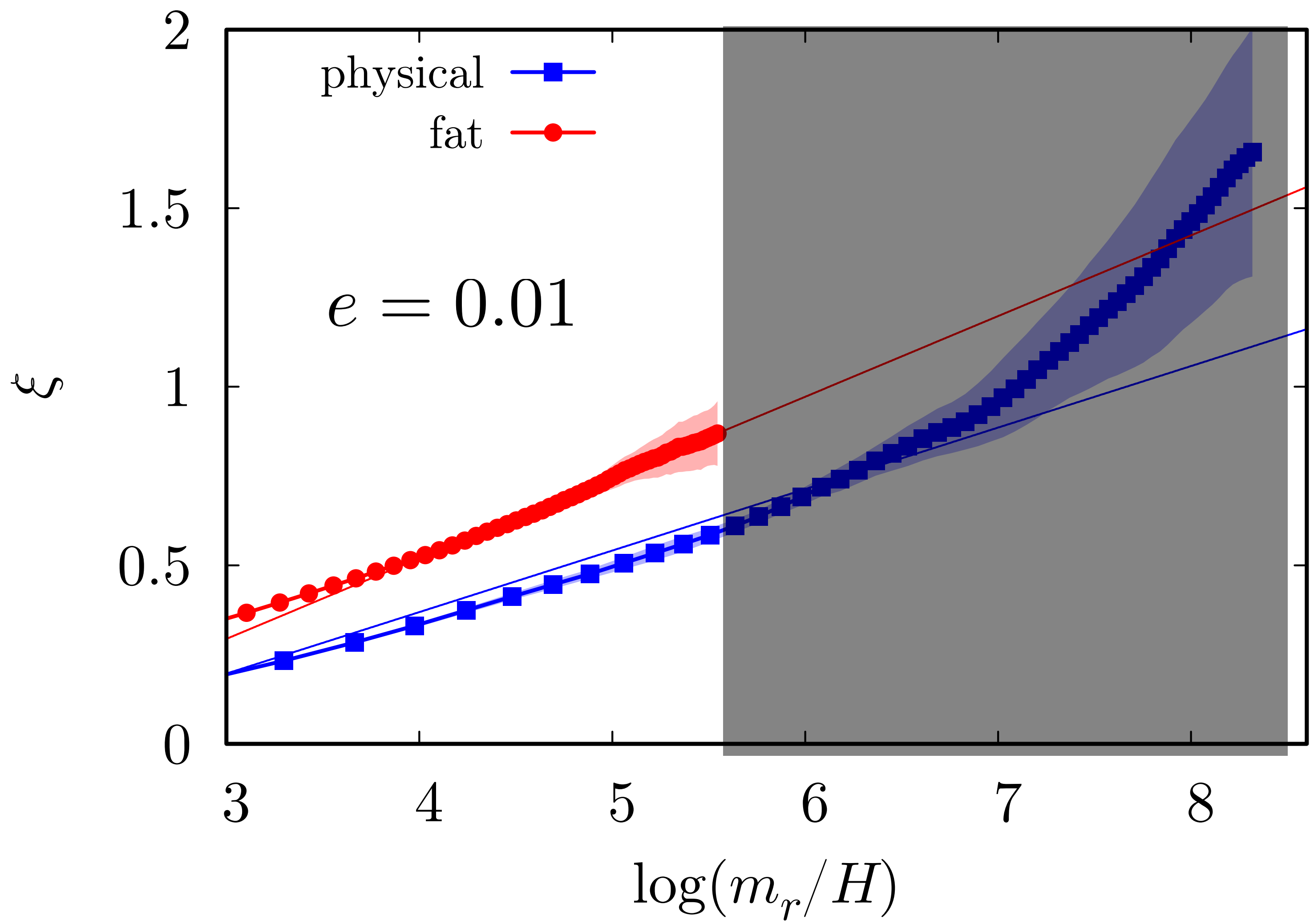


	grid	$m_r L$	$m_r \Delta x$	e	$m_r \tau$	a	b
physical string	4096^3	64	1/64	1	33.6 - 64.0	0.21 ± 0.0043	5.1 ± 0.21
physical string	4096^3	64	1/64	0.01	2.05 - 32.5	0.42 ± 0.0038	3.4 ± 0.073
physical string	4096^3	64	1/64	0.01	33.6 - 64.0	0.27 ± 0.019	8.4 ± 0.96
fat string	1024^3	512	1/2	1	133 - 256	0.24 ± 0.0013	11 ± 0.25
fat string	1024^3	512	1/2	0.01	133 - 256	0.47 ± 0.0081	18 ± 1.6

Table 1: Simulation setup and linear fitting parameters of the mean string separation in terms of the conformal time, defined by $m_r d_{\rm sep} = a m_r \tau + b$.

String density

$$\xi = \frac{\ell_{\rm str} t^2}{V}$$



	grid	$m_r L$	$m_r \Delta x$	e	$m_r \tau$	α	β
physical string	4096^3	64	$1/64$	0.01	2.05 - 32.5	0.17 ± 0.0034	-0.32 ± 0.019
fat string	1024^3	512	$1/2$	0.01	133 - 256	0.23 ± 0.018	-0.38 ± 0.096

Table 1: Log fitting parameters of the string length parameter, defined by $\xi = \alpha \log(m_r/H) + \beta$.

NK, Nakayama 2212.13573

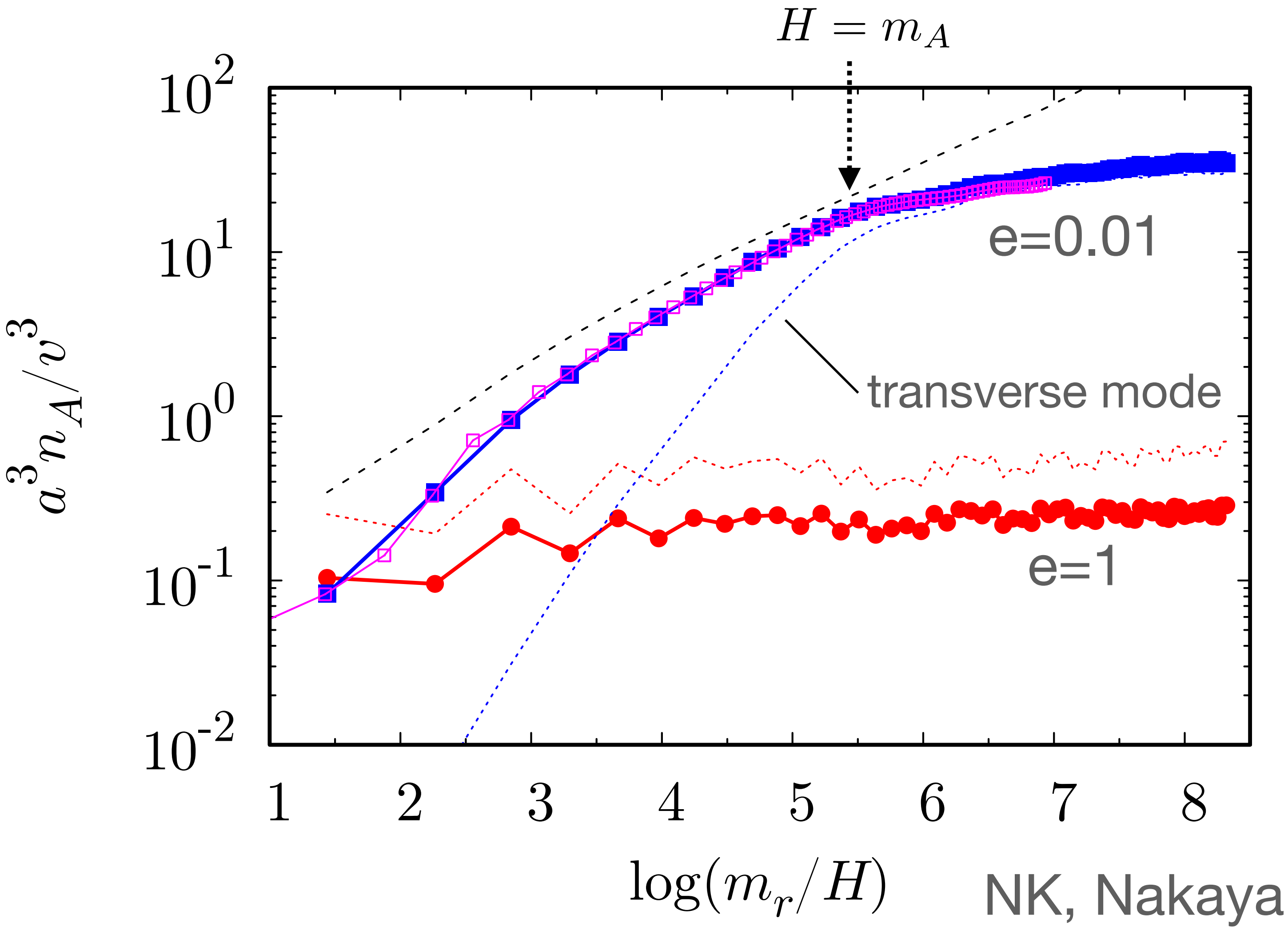
Emission of (longitudinal) vector boson

$$\rho_A^{(\mathrm{L})} = \frac{|\Phi|^2}{v^2}\left[\frac{2}{a^2}\left(\frac{\mathrm{Im}(\Phi^*\Phi')}{|\Phi|}\right)^2 + \frac{1}{a^4}\left(E_i^{(\mathrm{L})}\right)^2\right].$$

$$n_A = \int dk \frac{dn_A}{dk} = \int dk \frac{1}{E_A(k)}\frac{d\rho_A}{dk} \qquad n_A^{(\mathrm{L})}(t) \simeq \frac{8\xi\mu H}{\bar{E}_A/H}$$

(analytic estimation)
Long, Wang 1901.03312

$H = m_A$

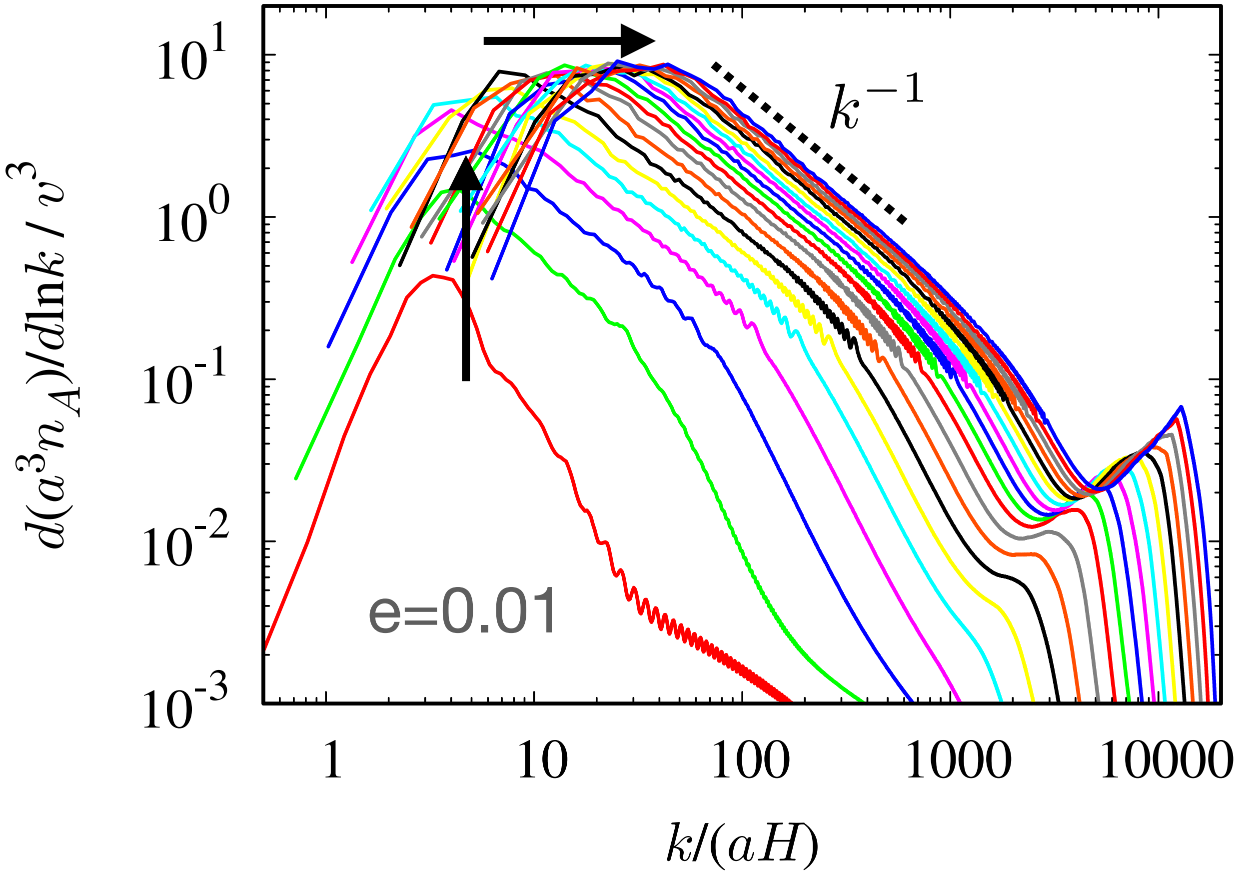


NK, Nakayama 2212.13573

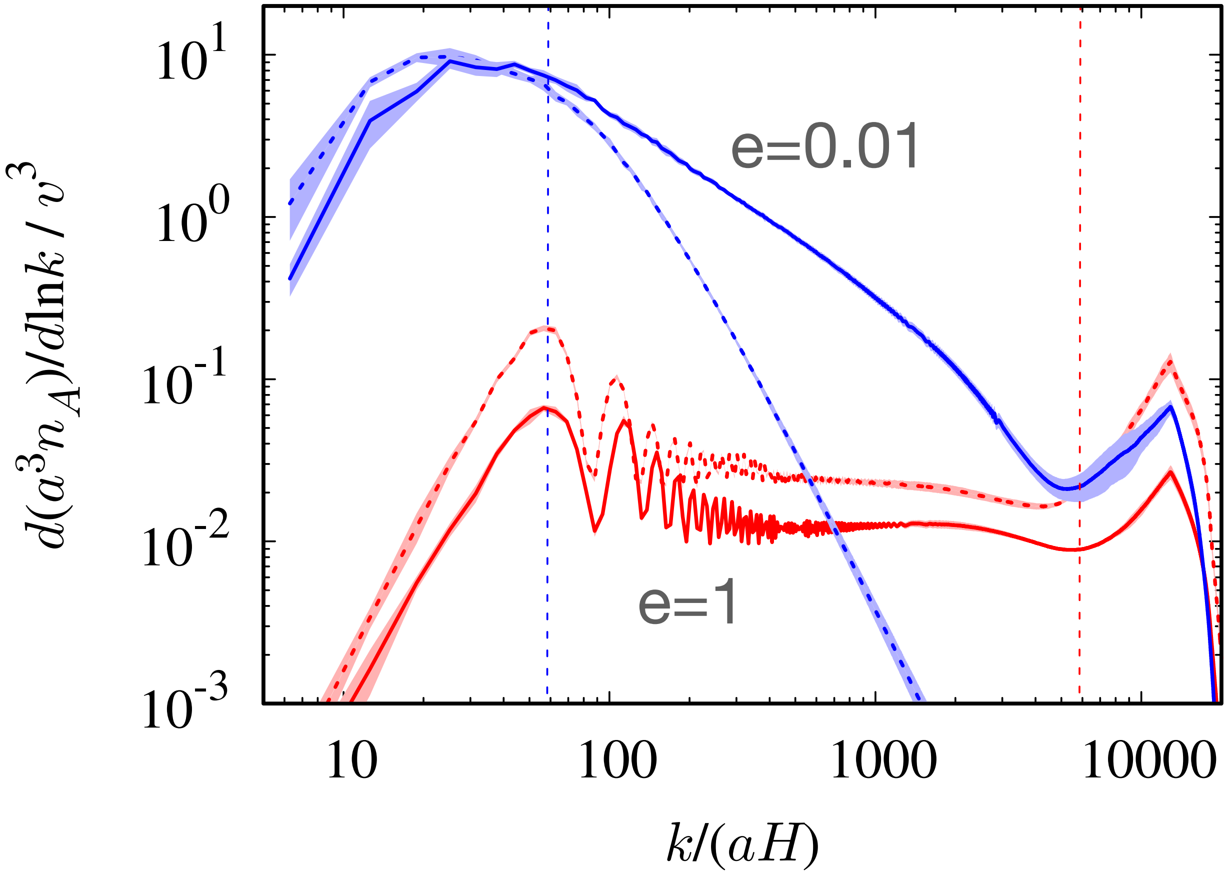
Spectrum of emitted dark photon

NK, Nakayama 2212.13573

time evolution (from bottom to top)



final value



peak wavenumber: $k/a \sim 10H$ ↔ typical loop size: $\ell \sim 0.1H^{-1}$

Relic abundance

$$\Omega_A h^2 = \frac{m_A (n_{A,0}/s_0) h^2}{\rho_{\rm cr,0}/s_0} \simeq 0.091 \left(\frac{\xi}{12}\right) \left(\frac{m_A}{10^{-13}\,\mathrm{eV}}\right)^{1/2} \left(\frac{v}{10^{14}\,\mathrm{GeV}}\right)^2$$

$\xi = \mathrm{const}$ (scaling law)

Hindmarsh et al, 1908.03522
Hindmarsh et al, 2102.07723

$$\xi = 0.15 \log\left(\frac{m_r}{m_A}\right) \simeq 12 + 0.15 \log\left[\left(\frac{m_r}{10^{14}\,\mathrm{GeV}}\right)\left(\frac{10^{-13}\,\mathrm{eV}}{m_A}\right)\right]$$

(scaling violation)

Gorghetto et al, 1806.04677
Kawasaki et al, 1806.05566
Buschmann et al, 2108.05368

GW emission from cosmic strings

Credit: Daniel Dominguez/CERN

Quadrupole formula for GW emission: $\dot{E}_{\rm GW} \sim G(\dddot{D})^2$

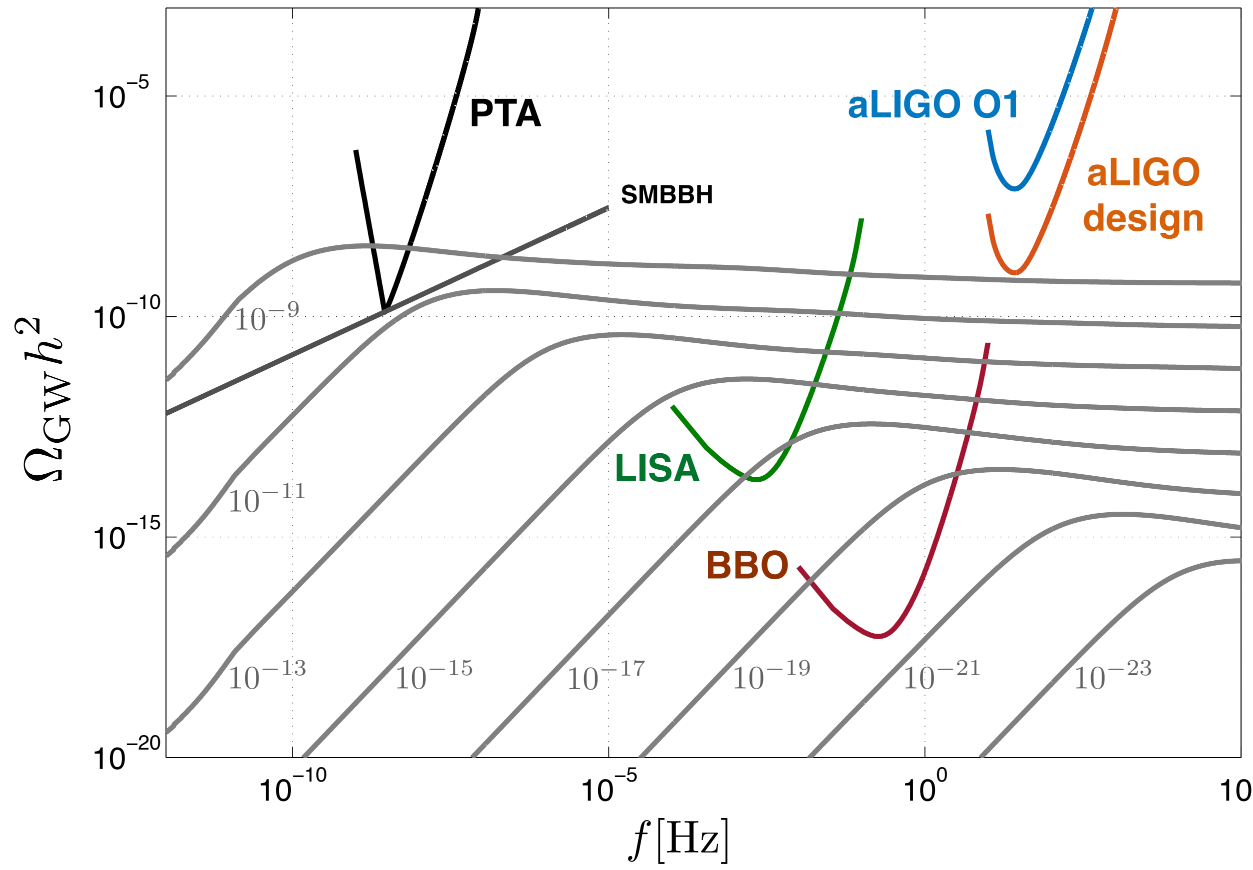
quadrupole moment: $D \sim ML^2 \sim \mu L^3, \quad \dddot{D} \sim \omega^3 D \sim L^{-3} D$

L : typical loop size ~ (typical oscillation frequency)$^{-1}$

GW emission rate: $\dot{E}_{\rm GW} \sim G\mu^2 \equiv \Gamma_{\rm GW} G\mu^2$

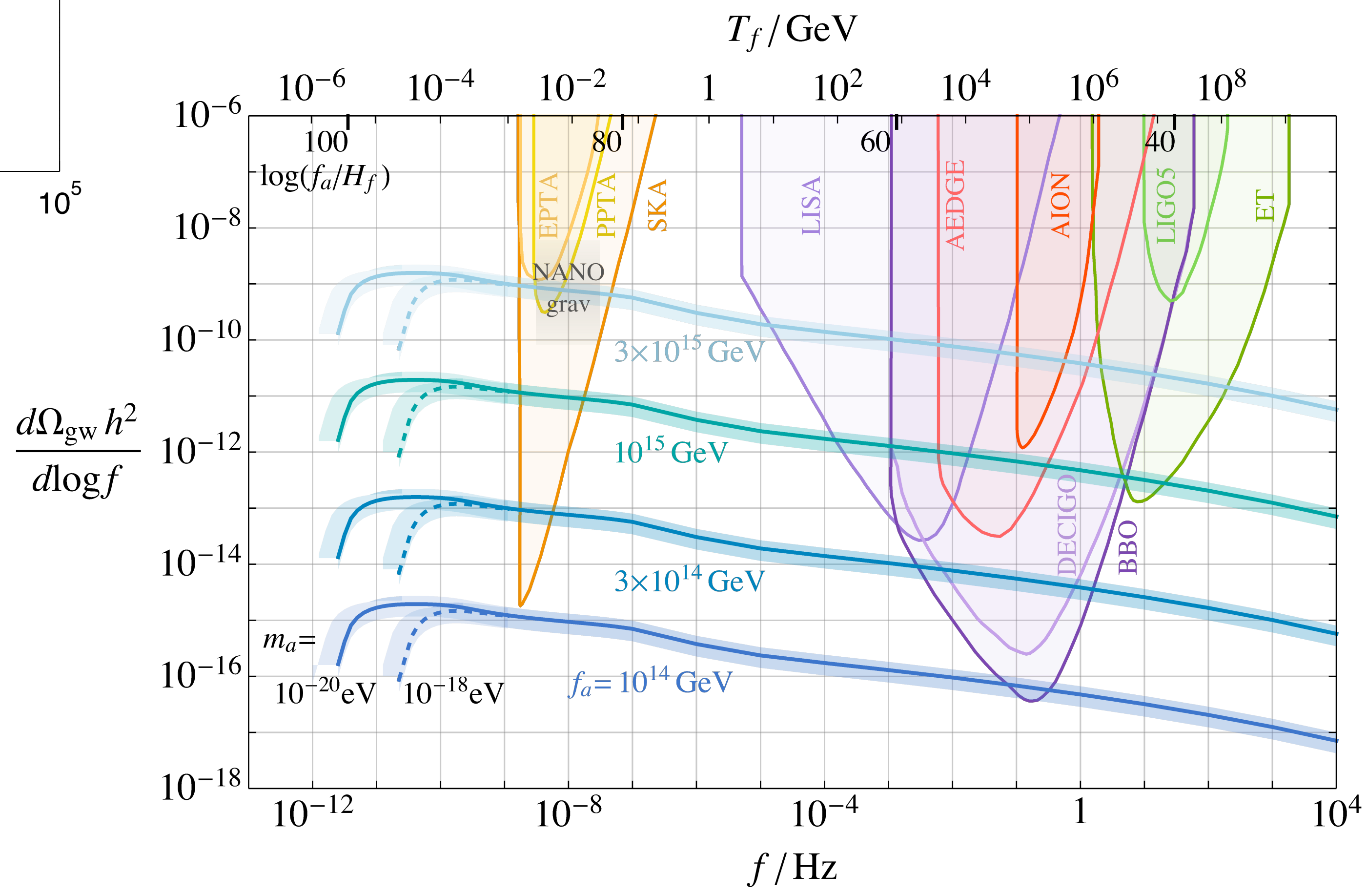
$G\mu \sim (v/M_P)^2 \sim 10^{-7}(v/10^{15}{\rm GeV})^2$

GW spectrum from local/global strings



Blanco-Pillado+ 1709.02434

Gorghetto et al, 2101.11007



—> Edward’s talk

energy loss of loops = GW emission + vector boson emission

$$\frac{dE_\ell}{dt} = -\Gamma_{\rm GW} G\mu^2 - \Gamma_{\rm vec} v^2 \theta(1 - m_A \ell)$$

$$\Gamma_{\rm GW} \sim 50, \quad \Gamma_{\rm vec} \sim 65$$

Loops shorter than m_A^{-1} can emit vector bosons

(i.e. loop oscillation frequency should be larger than m_A)

—> short lived & GW emission is suppressed

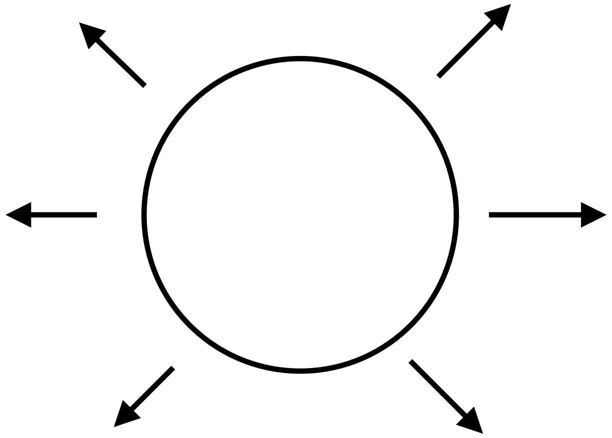
loop lifetime:

$$\tau(\ell) \sim \frac{E_\ell}{\dot{E}_\ell} \sim \begin{cases} \dfrac{\ell}{\Gamma_{\rm GW} G\mu} & \text{for} \quad m_A \ell > 1 \\ \dfrac{\pi \log(m_r/H)\ell}{\Gamma_{\rm vec}} & \text{for} \quad m_A \ell < 1 \end{cases}$$

(GW emission) $\tau \gtrsim 10^6 \ell \gg t$ — long-lived

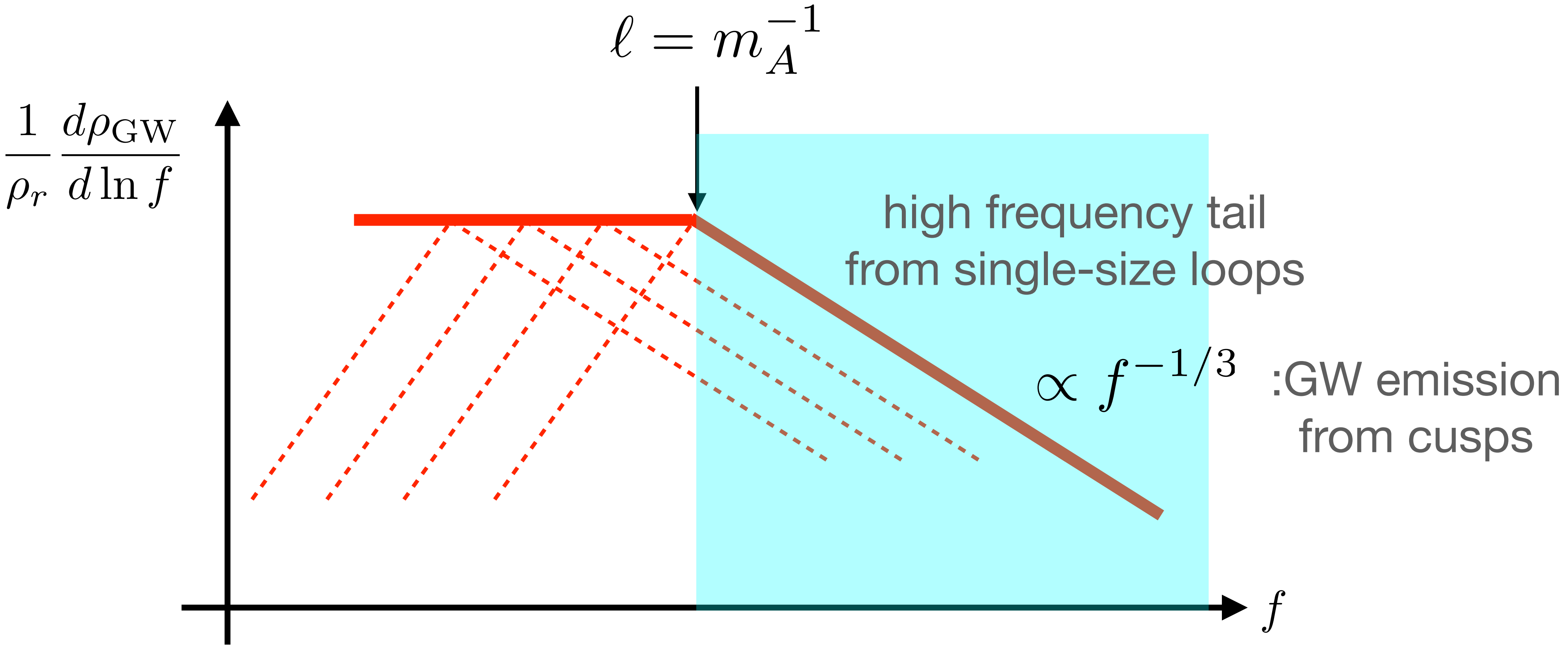
(vector boson emission) $\tau \sim \ell < t$ — short-lived

Power of GW emission by single loop: $\dfrac{dP_{\rm GW}(\ell)}{d\ln f} = G\mu^2 f\ell S(f\ell)$



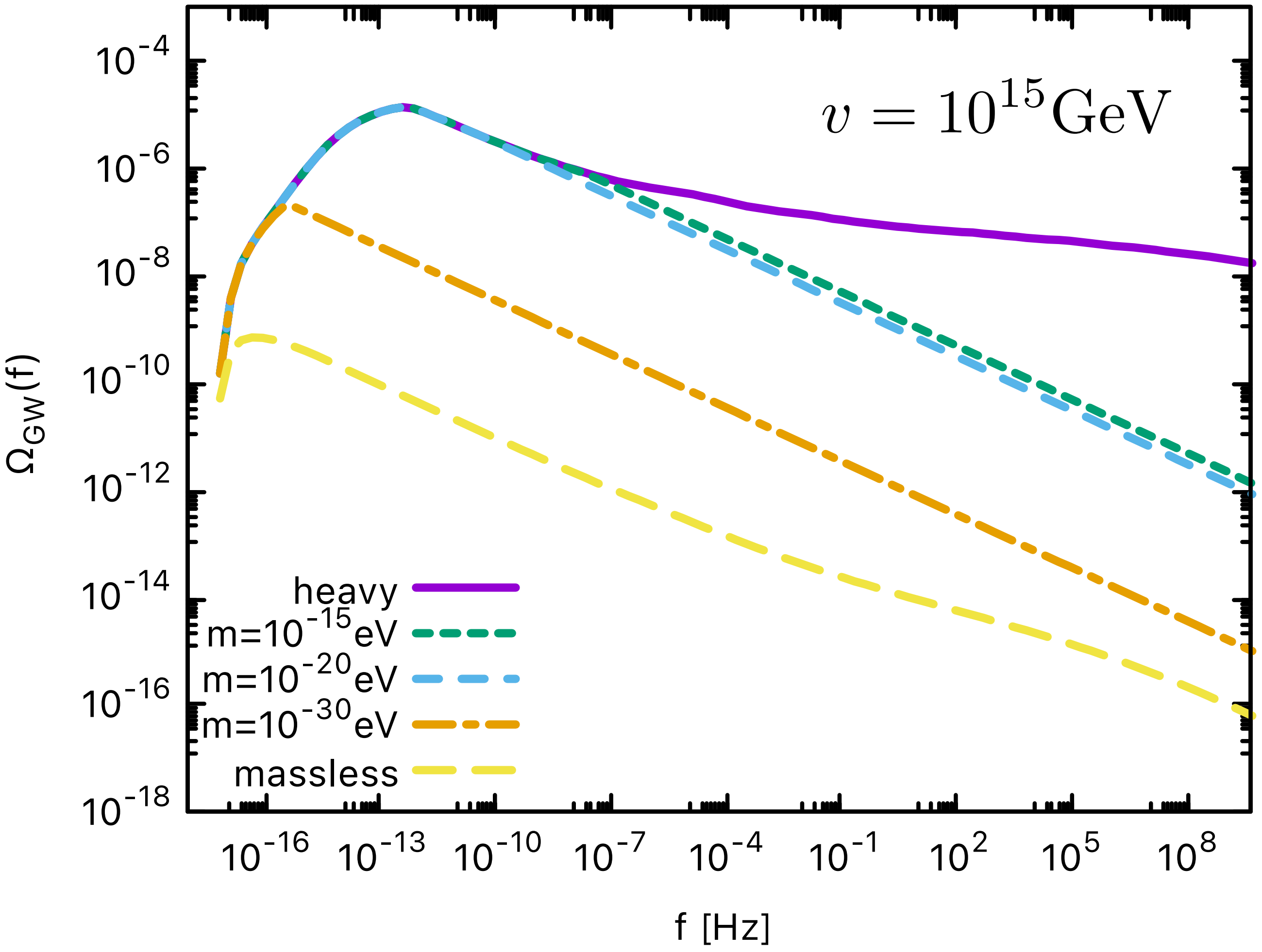
$$S(x) = (q-1)2^{q-1}\Gamma_{\rm GW}\frac{\theta(x-2)}{x^q}$$

$$q = 4/3 \quad (\text{cusp}), \quad q = 5/3 \quad (\text{kink})$$

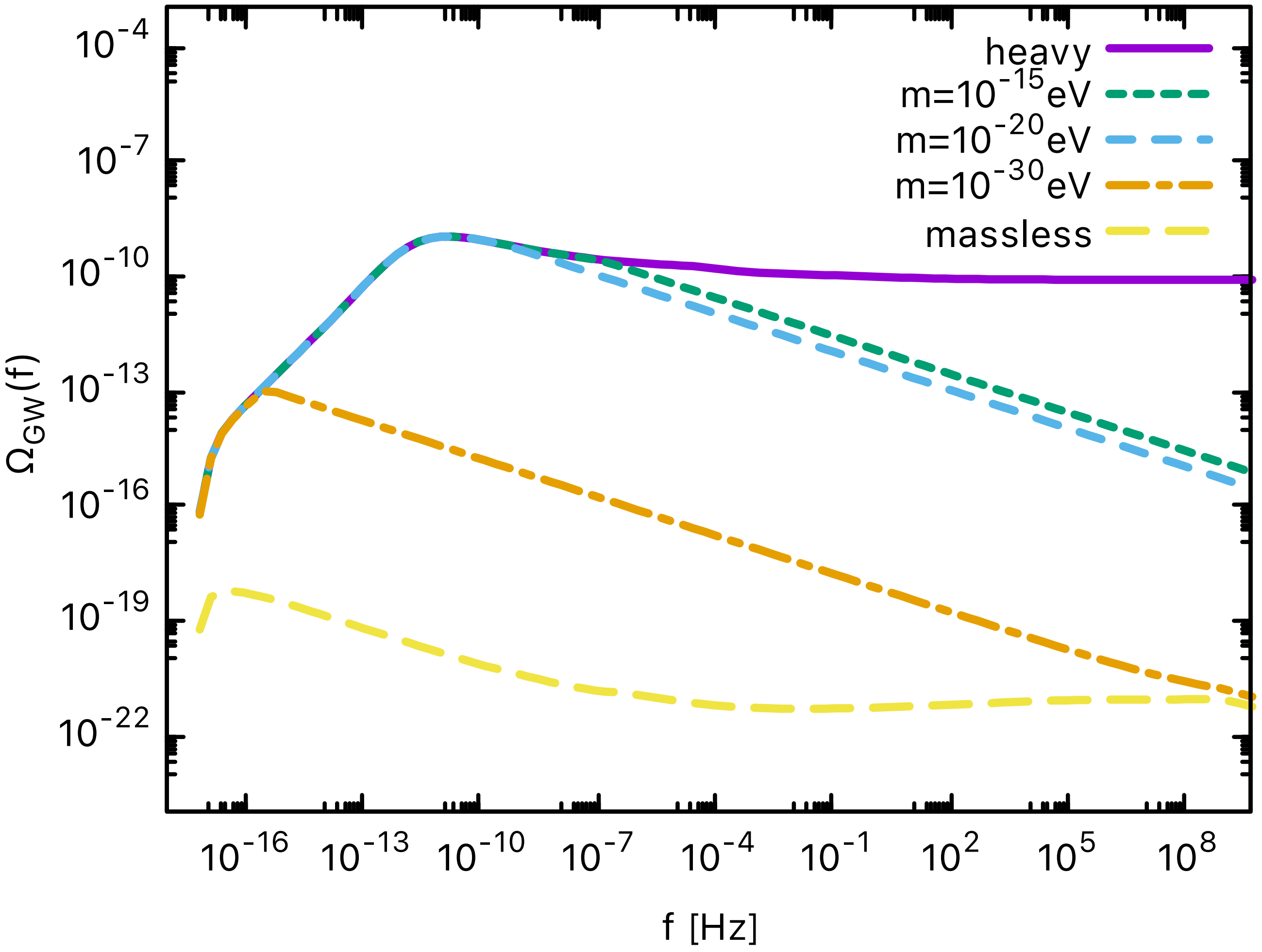


GW spectrum : our scenario

NK, Nakayama 2212.13573



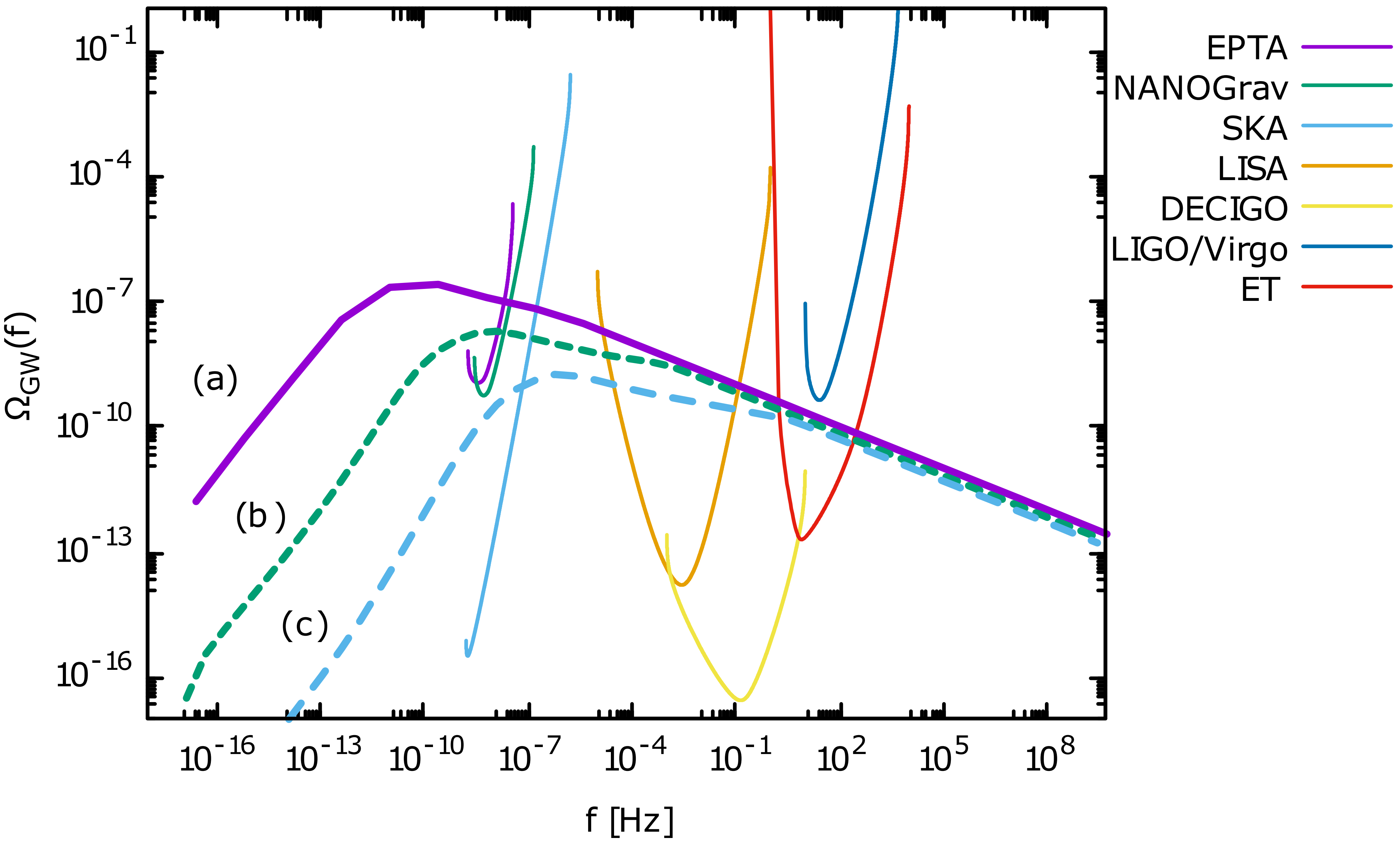
log-enhanced (scaling violation)



no-scaling violation

GW spectrum : our scenario

NK, Nakayama 2212.13573

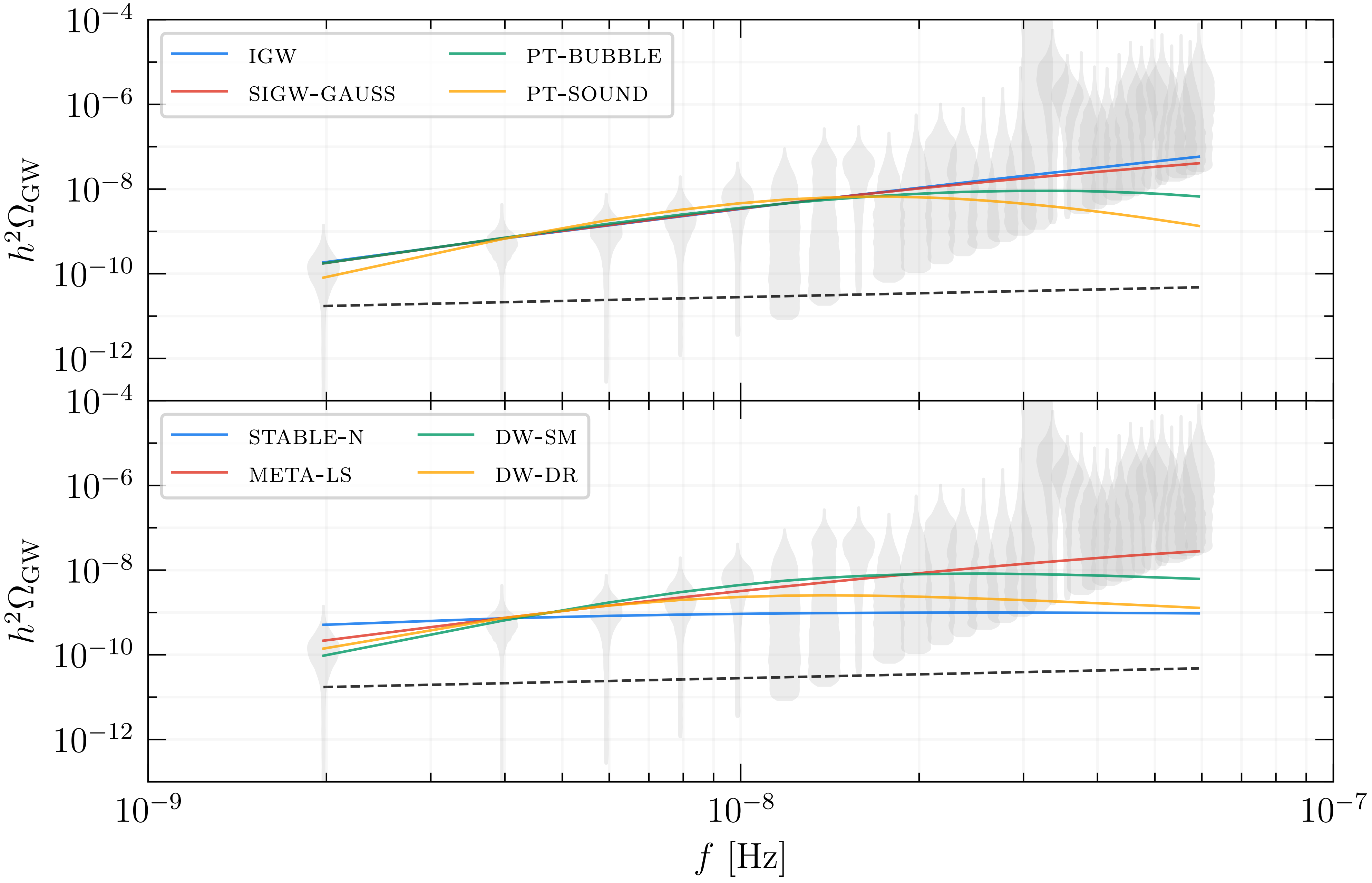


$(a)\ v = 10^{15}\,\mathrm{GeV},\ m_A = 10^{-14}\,\mathrm{eV}$

$(b)\ v = 10^{13}\,\mathrm{GeV},\ m_A = 10^{-10}\,\mathrm{eV}$

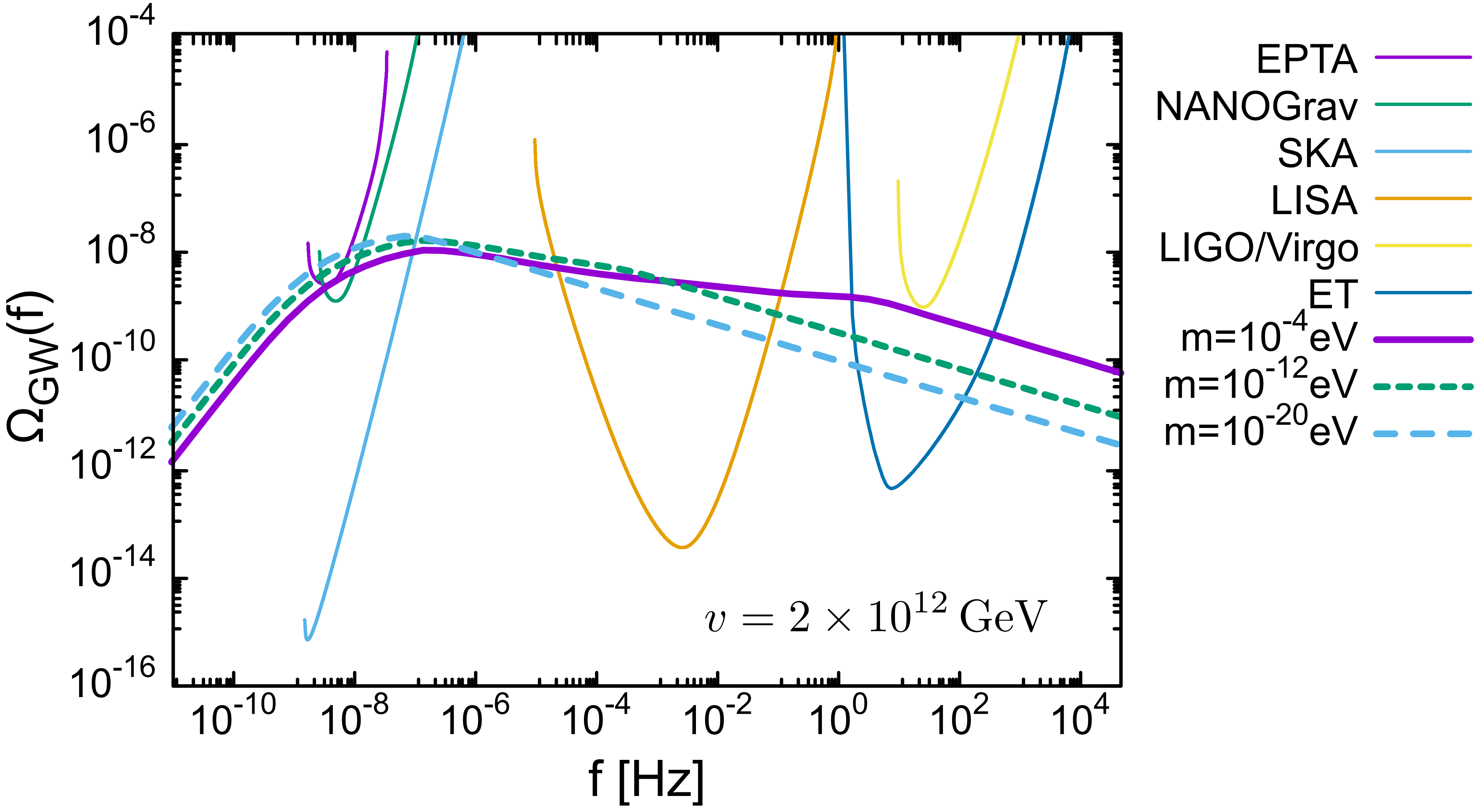
$(c)\ v = 10^{12}\,\mathrm{GeV},\ m_A = 10^{-5}\,\mathrm{eV}$

NANOGrav 15 yr data 2306.16219



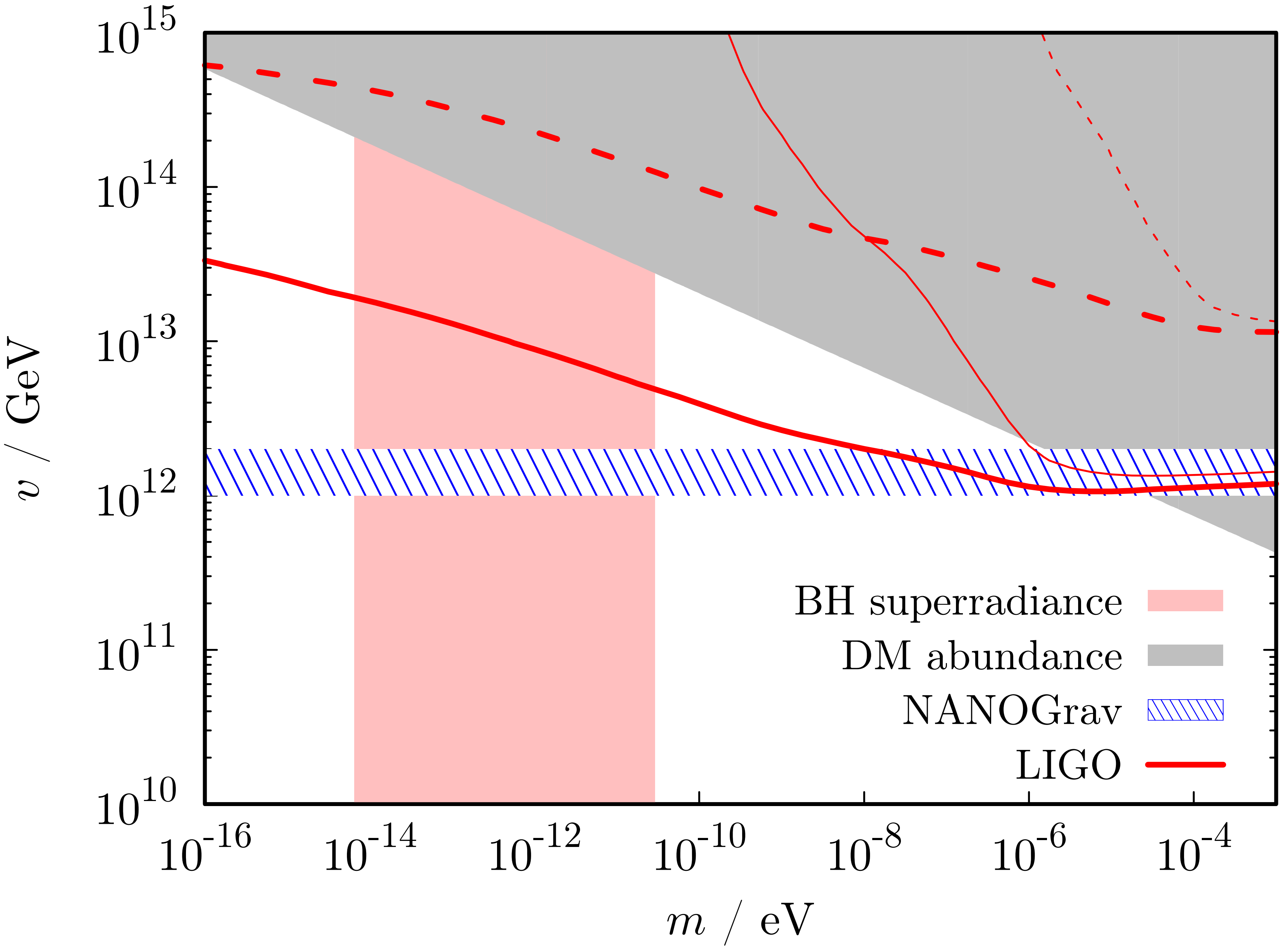
GW spectrum : implication for NANOGrav 15 yr data

NK, Nakayama 2306.17390



Viable parameter space

NK, Nakayama 2306.17390



Sweet spot is m~1-10μeV

Discussion

We need more precise study …

- Scaling violation ?
- Time-dependence of the tension
- Loop formation and dark photon production rate

  especially near the transition : H ~ m_A (global —> local)

- Initial loop size distribution (monochromatic or extended?)
- Spectral function of GW from individual loop (cusp- or kink-like?)

  (because it is crucial for high frequency region)

- Loop lifetime (deviation from Nambu-Goto string)

  discussed in Hindmarsh et al (2017), Matsunami et al (2019)

Summary

- Dark photon can be produced from the network of cosmic strings (via loop collapse)

  & production stops when H < m_A

  (i.e. the dark photon emission is kinematically suppressed)

  —> Relic abundance is fixed at that time
  Observed abundance is reproduced for e.g.

$$v \sim 10^{12}\text{-}10^{14}\text{GeV},\ m_A \sim 10^{-14}\text{-}10^{-5}\text{eV}$$

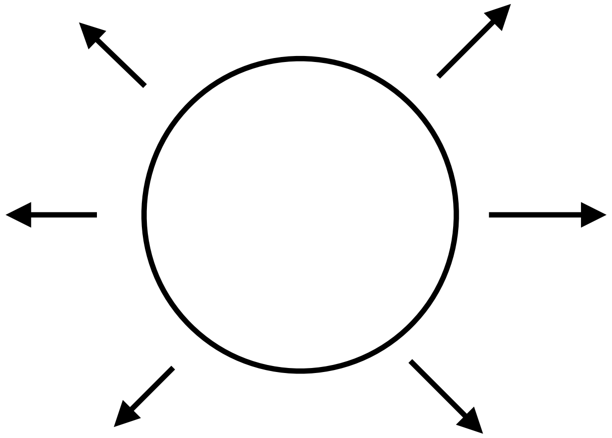
- Gravitational waves are emitted as a signal of this scenario

  Spectrum is different from both local and global one
  It can be tested by combining pulsar timing and direct detection

- NANOGrav data can be explained & tested by future aLIGO

GW emission

Power of GW emission by single loop: $$\frac{dP_{\rm GW}(\ell)}{d\ln f} = G\mu^2 f\ell S(f\ell)$$



$$S(x) = (q-1)2^{q-1}\Gamma_{\rm GW}\frac{\theta(x-2)}{x^q}$$

$$q = 4/3 \quad ({\rm cusp}), \quad q = 5/3 \quad ({\rm kink})$$

$$\frac{d\rho_{\rm GW}(t_0)}{df_0} = \int dt \int d\ell G\mu^2(t) S\left(\frac{\ell f_0 a_0}{a(t)}\right)\frac{dn_\ell(t)}{d\ln\ell}\left(\frac{a(t)}{a_0}\right)^3$$

loop number density

$$\longrightarrow \quad \Omega_{\rm GW}(f_0) = \frac{1}{\rho_{\rm crit}}\frac{d\rho_{\rm GW}(t_0)}{d\ln f_0}$$

$$\frac{dn_\ell(t)}{d\ln\ell} = \int^t d\ln t_i \left[\frac{dn_\ell(t_i)}{d\ln\ell_i}\right]_{\rm prod} \left(\frac{a(t_i)}{a(t)}\right)^3 \theta(t-t_i)$$

$$\left[\frac{dn_\ell(t_i)}{d\ln\ell_i}\right]_{\rm prod} = \frac{\tau(\ell)}{t+\tau(\ell)}\frac{C_{\rm loop}\xi}{\alpha_0 t^3}\ell\delta(l-\alpha_0 t)$$

with $\rho_{\rm str}(t) = \xi(t)\dfrac{\mu(t)}{t^2}$ & $\rho_{\rm loop}(t) = C_{\rm loop}\rho_{\rm str}(t)$

$$\longrightarrow \qquad \frac{dn_\ell(t)}{d\ln\ell} \simeq \underset{\text{loop lifetime}}{\underline{\frac{\tau(\ell)}{\tau(\ell)+t_i}}}\frac{C_{\rm loop}\xi(t_i)}{\alpha_0 t_i^3}\left(\frac{a(t_i)}{a(t)}\right)^3 \theta\left(t-\frac{\ell_i}{\alpha_0}\right)$$

$$\frac{d\rho_{\rm GW}(t_0)}{df_0} = \int dt \int d\ell G\mu^2(t) S\left(\frac{\ell f_0 a_0}{a(t)}\right)\underline{\frac{dn_\ell(t)}{d\ln\ell}}\left(\frac{a(t)}{a_0}\right)^3$$