

g-2, LFV, EDMと フレーバー対称性

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Based on Morimitsu Tanimoto(Niigata U.), KY [2310.16325](#) [hep-ph]

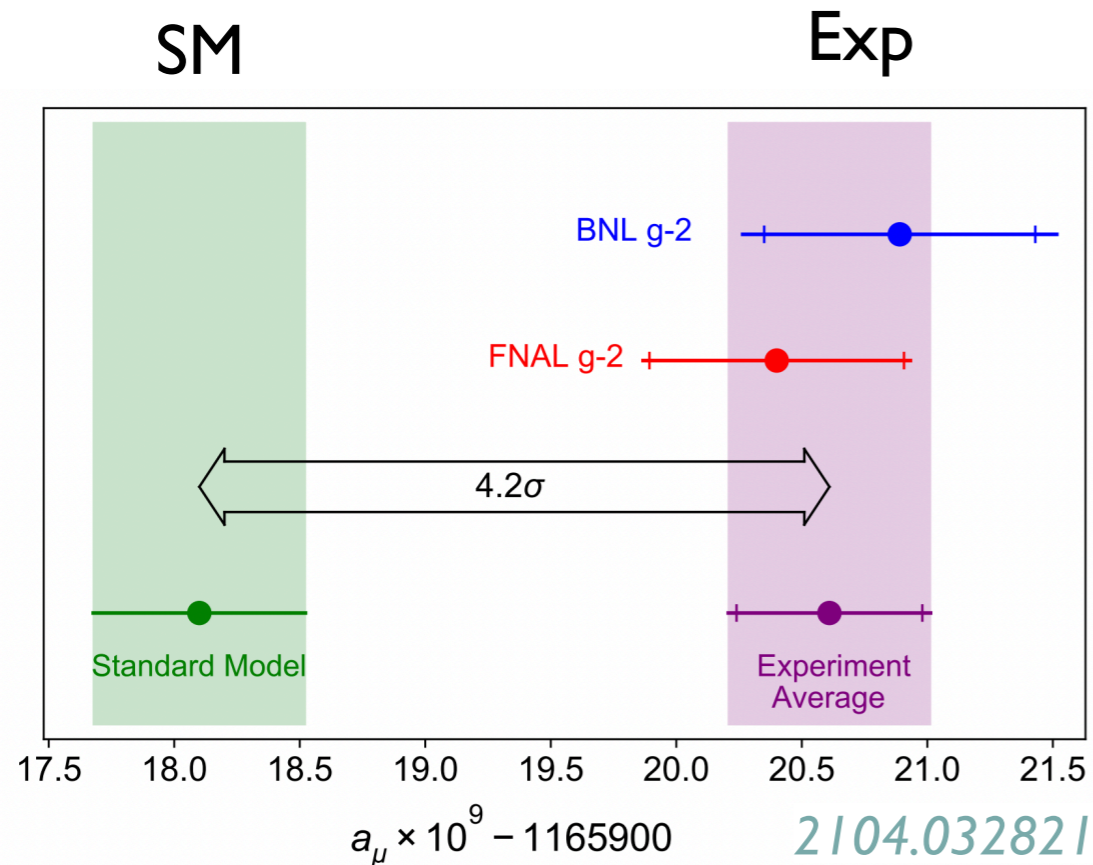
話すこと

Muon $(g - 2)_\mu$ anomaly が新物理の寄与によるものであったとき

- 新物理のレプトンフレーバー構造に与える示唆は何か？
- フレーバー対称性との整合性は？
- 他の物理量への予言は？

Muon $(g - 2)_\mu$ anomaly

anomalous magnetic dipole moment of the muon $a_\mu \equiv \frac{(g - 2)_\mu}{2}$



$$\Delta a_\mu = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}$$

Muon $(g - 2)_\mu$ anomaly ?

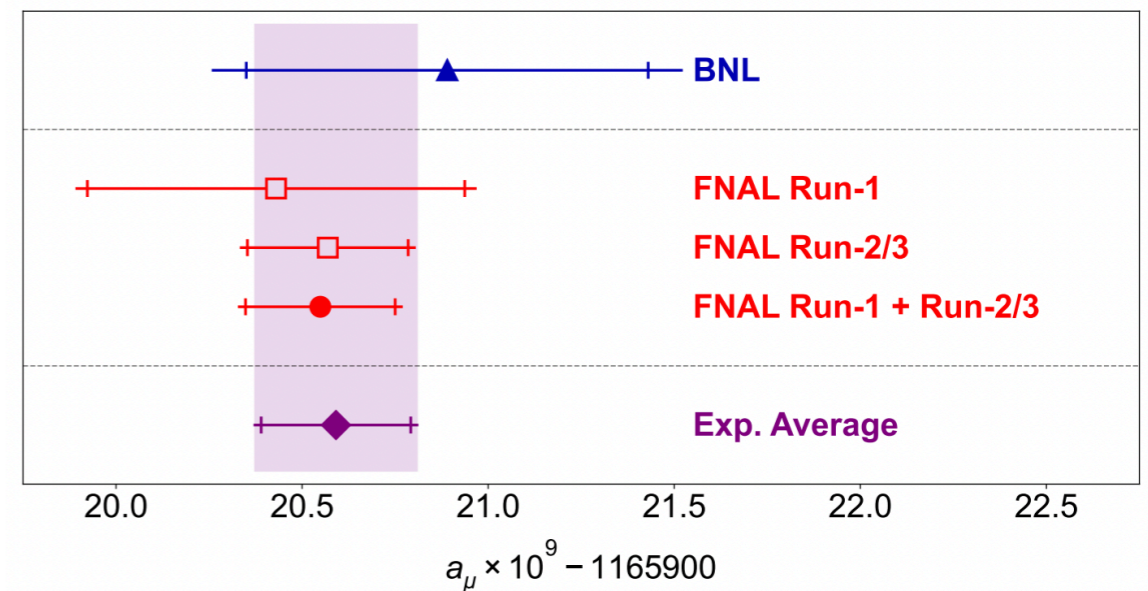
Lattice QCD result

on Hadron Vacuum Polarization

→ smaller discrepancy

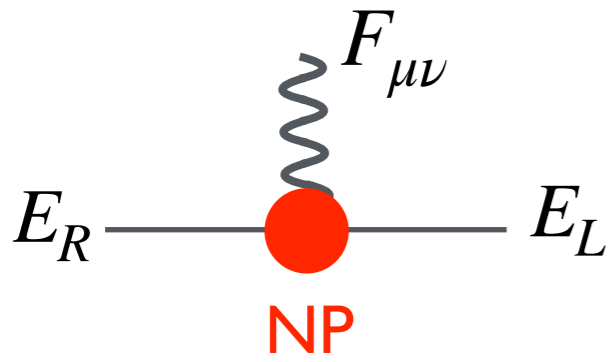
New FNAL result

2308.06230



新物理のフレーバー構造 from $(g - 2)_\ell$, LFV and EDM

Dipole operator



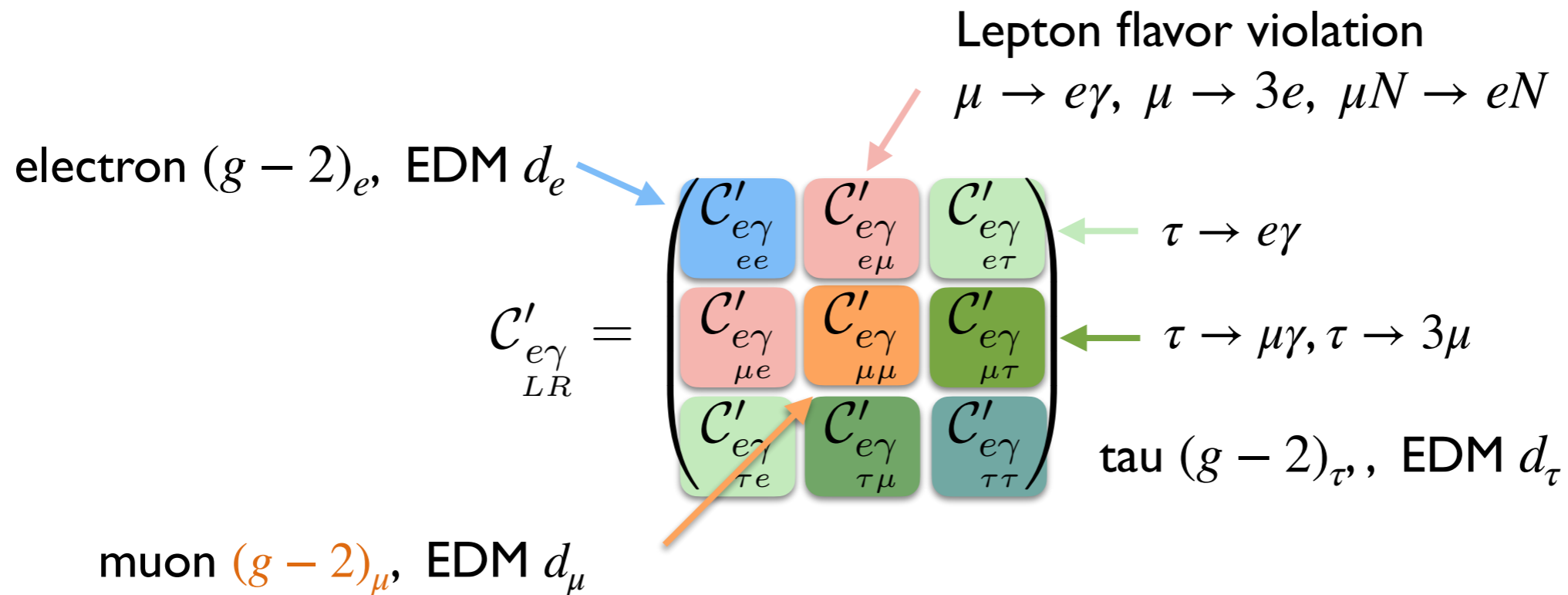
$$\mathcal{O}_{LR}^{e\gamma} = \frac{v}{\sqrt{2}} \bar{E}_L \sigma^{\mu\nu} E_R F_{\mu\nu}$$

$$\mathcal{L}_{\text{dipole}} = \frac{1}{\Lambda^2} \left(c'_{LR}{}^{e\gamma} \mathcal{O}_{LR}^{e\gamma} + c'_{RL}{}^{e\gamma} \mathcal{O}_{RL}^{e\gamma} \right)$$

$$c'_{LR}{}^{e\gamma} = \begin{pmatrix} c'_{ee}{}^{e\gamma} & c'_{e\mu}{}^{e\gamma} & c'_{e\tau}{}^{e\gamma} \\ c'_{\mu e}{}^{e\gamma} & c'_{\mu\mu}{}^{e\gamma} & c'_{\mu\tau}{}^{e\gamma} \\ c'_{\tau e}{}^{e\gamma} & c'_{\tau\mu}{}^{e\gamma} & c'_{\tau\tau}{}^{e\gamma} \end{pmatrix} \begin{matrix} e \\ \mu \\ \tau \end{matrix}$$

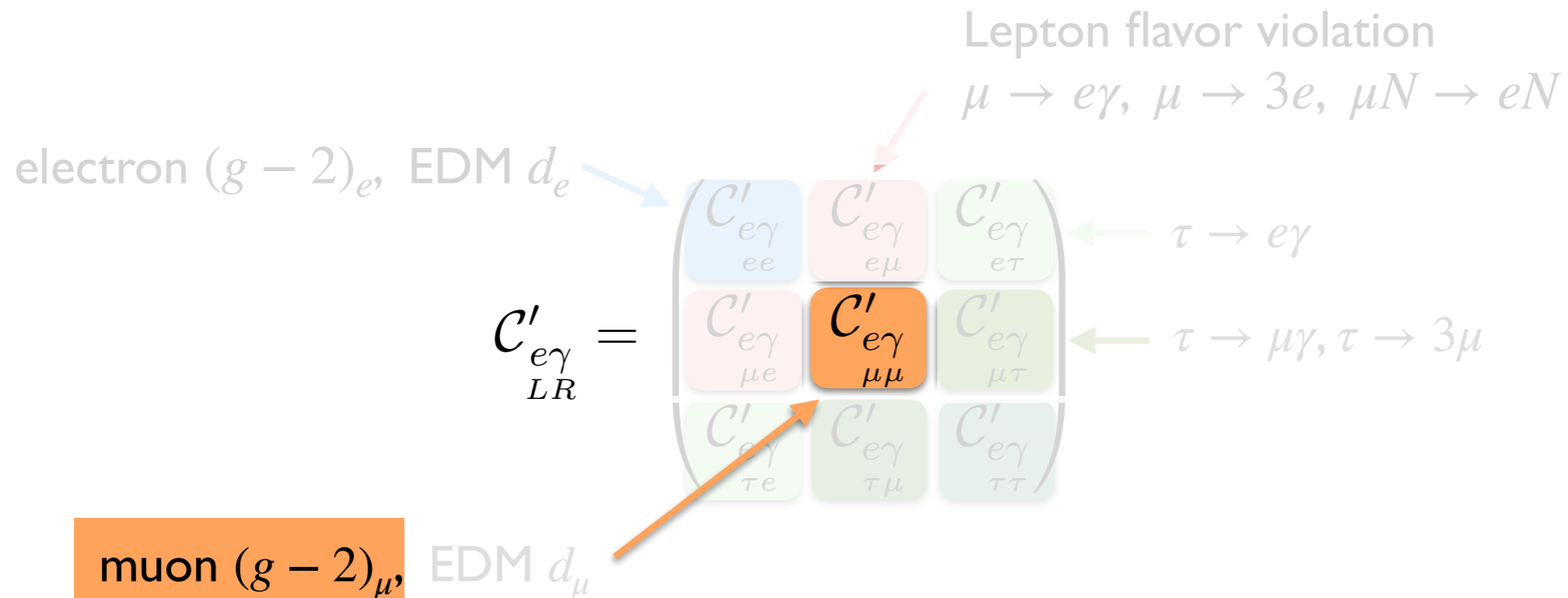
新物理のフレーバー構造 from $(g - 2)_\ell$, LFV and EDM

Muon $(g - 2)_\mu$ 以外にも様々な現象が引き起こされる



新物理のフレーバー構造 from $(g - 2)_\ell$, LFV and EDM

muon $(g - 2)_\mu$



$$\Delta a_\mu = \frac{4m_\mu}{e} \frac{v}{\sqrt{2}} \frac{1}{\Lambda^2} \text{Re} [C'_{e\gamma}_{\mu\mu}]$$

$$\Delta a_\mu = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}$$

FNAL, BNL

$$\longrightarrow \frac{1}{\Lambda^2} \text{Re} [C'_{e\gamma}_{\mu\mu}] \approx 1.0 \times 10^{-5} \text{ TeV}^{-2}$$

新物理のフレーバー構造 from $(g - 2)_\ell$, LFV and EDM

muon $(g - 2)_\mu$ & $\mu \rightarrow e\gamma$

electron $(g - 2)_e$, EDM d_e

Lepton flavor violation

$\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$, $\mu N \rightarrow eN$

$$C'_{e\gamma LR} = \begin{pmatrix} C'_{e\gamma ee} & C'_{e\gamma e\mu} \\ C'_{e\gamma \mu e} & C'_{e\gamma \mu\mu} \\ C'_{e\gamma \tau e} & C'_{e\gamma \tau\mu} \end{pmatrix}$$

$$\mathcal{B}(l_r \rightarrow l_s \gamma) = \frac{m_{l_r}^3 v^2}{8\pi\Gamma_{l_r}} \frac{1}{\Lambda^4} \left(|C'_{e\gamma rs}|^2 + |C'_{e\gamma sr}|^2 \right)$$

$$\mathcal{B}(\mu^+ \rightarrow e^+ \gamma) < 4.2 \times 10^{-13} \text{ (90\% C.L.)}$$

MEG

$$\longrightarrow \frac{1}{\Lambda^2} |C'_{e\gamma e\mu(\mu e)}| < 2.1 \times 10^{-10} \text{ TeV}^{-2}$$

muon $(g - 2)_\mu$, EDM d_μ

$$\Delta a_\mu = \frac{4m_\mu}{e} \frac{v}{\sqrt{2}} \frac{1}{\Lambda^2} \text{Re} [C'_{e\gamma \mu\mu}]$$

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新物理のフレーバー構造 from $(g-2)_\ell$, LFV and EDM

muon $(g-2)_\mu$ & $\mu \rightarrow e\gamma \rightarrow$ 特徴的なフレーバー構造 (対角成分 \gg 非対角成分)

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FNAL, BNL

$$\frac{1}{\Lambda^2} \text{Re} [C'_{e\gamma \mu\mu}] \approx 1.0 \times 10^{-5} \text{ TeV}^{-2}$$

Strong flavor alignment

$$\left| \frac{C'_{e\gamma e\mu(\mu e)}}{C'_{e\gamma \mu\mu}} \right| < 2.1 \times 10^{-5}$$

$$C'_{e\gamma LR} = \begin{pmatrix} C'_{e\gamma ee} & C'_{e\gamma e\mu} & C'_{e\gamma e\tau} \\ C'_{e\gamma \mu e} & C'_{e\gamma \mu\mu} & C'_{e\gamma \mu\tau} \\ C'_{e\gamma \tau e} & C'_{e\gamma \tau\mu} & C'_{e\gamma \tau\tau} \end{pmatrix}$$

造 from $(g - 2)_\ell$, LFV and EDM

$$\Delta a_e = \frac{4m_e}{e} \frac{v}{\sqrt{2}} \frac{1}{\Lambda^2} \text{Re} [C'_{e\gamma}]_{ee}$$

$$\Delta a_e^{Cs} = a_e^{\text{Exp}} - a_e^{\text{SM,Cs}} = (-8.8 \pm 3.6) \times 10^{-13}$$

$$\Delta a_e^{Rb} = a_e^{\text{Exp}} - a_e^{\text{SM,Rb}} = (4.8 \pm 3.0) \times 10^{-13}$$

electron $(g - 2)_e$

11 vs 22

$$C'_{LR} = \begin{pmatrix} C'_{e\gamma}_{ee} & C'_{e\gamma}_{e\mu} & C'_{e\gamma}_{e\tau} \\ C'_{e\gamma}_{\mu e} & C'_{e\gamma}_{\mu\mu} & C'_{e\gamma}_{\mu\tau} \\ C'_{e\gamma}_{\tau e} & C'_{e\gamma}_{\tau\mu} & C'_{e\gamma}_{\tau\tau} \end{pmatrix}$$

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$\tau \rightarrow e\gamma$

$\tau \rightarrow \mu\gamma, \tau \rightarrow 3\mu$

muon $(g - 2)_\mu$

$$\Delta a_\mu = \frac{4m_\mu}{e} \frac{v}{\sqrt{2}} \frac{1}{\Lambda^2} \text{Re} [C'_{e\gamma}]_{\mu\mu}$$

$$\Delta a_\mu = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}$$

FNAL, BNL

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新物理のフレーバー構造 from $(g - 2)_\ell$, LFV and EDM

$$[C'_{e\gamma}]_{\mu\mu} \begin{cases} \text{Re} \rightarrow (g - 2)_\mu \\ \text{Im} \rightarrow \text{EDM } d_\mu \end{cases}$$

Lepton flavor violation
 $\mu \rightarrow e\gamma, \mu \rightarrow 3e, \mu N \rightarrow eN$

electron $(g - 2)_e$, EDM d_e

$$C'_{e\gamma LR} = \begin{pmatrix} C'_{e\gamma ee} & C'_{e\gamma e\mu} & C'_{e\gamma e\tau} \\ C'_{e\gamma \mu e} & C'_{e\gamma \mu\mu} & C'_{e\gamma \mu\tau} \\ C'_{e\gamma \tau e} & C'_{e\gamma \tau\mu} & C'_{e\gamma \tau\tau} \end{pmatrix}$$

$\leftarrow \tau \rightarrow e\gamma$
 $\leftarrow \tau \rightarrow \mu\gamma, \tau \rightarrow 3\mu$

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$$\Delta a_\mu = \frac{4m_\mu}{e} \frac{v}{\sqrt{2}} \frac{1}{\Lambda^2} \text{Re} [C'_{e\gamma}]_{\mu\mu}$$

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$$\longrightarrow \frac{1}{\Lambda^2} \text{Re} [C'_{e\gamma}]_{\mu\mu} \approx 1.0 \times 10^{-5} \text{ TeV}^{-2}$$

EDM d_μ

$$d_\mu = -\sqrt{2} \frac{v}{\Lambda^2} \text{Im} [C'_{e\gamma}]_{\mu\mu}$$

$$|d_\mu/e| < 1.8 \times 10^{-19} \text{ cm} \quad \text{BNL}$$

$$\longrightarrow \frac{1}{\Lambda^2} \text{Im} [C'_{e\gamma}]_{\mu\mu} < 2.7 \times 10^{-2} \text{ TeV}^{-2}$$

$$\Delta a_e = \frac{4m_e}{e} \frac{v}{\sqrt{2}} \frac{1}{\Lambda^2} \text{Re} [C'_{e\gamma}]_{ee}$$

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electron $(g - 2)_e$

$$d_e = -\sqrt{2} \frac{v}{\Lambda^2} \text{Im} [C'_{e\gamma}]_{ee}$$

$$|d_e/e| \lesssim 1.1 \times 10^{-29} \text{ cm} = 5.6 \times 10^{-13} \text{ TeV}^{-1}$$

$$\frac{1}{\Lambda^2} \text{Im} [C'_{e\gamma}]_{ee} < 1.6 \times 10^{-12} \text{ TeV}^{-2}$$

EDM d_e

$$C'_{e\gamma LR} = \begin{pmatrix} C'_{e\gamma ee} & C'_{e\gamma e\mu} & C'_{e\gamma e\tau} \\ C'_{e\gamma \mu e} & C'_{e\gamma \mu\mu} & C'_{e\gamma \mu\tau} \\ C'_{e\gamma \tau e} & C'_{e\gamma \tau\mu} & C'_{e\gamma \tau\tau} \end{pmatrix}$$

← $\tau \rightarrow e\gamma$
← $\tau \rightarrow \mu\gamma, \tau \rightarrow 3\mu$

muon $(g - 2)_\mu$

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FNAL, BNL

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新物理のフレーバー構造 from $(g - 2)_\ell$, LFV and EDM

$$\mathcal{B}(l_r \rightarrow l_s \gamma) = \frac{m_{l_r}^3 v^2}{8\pi\Gamma_{l_r}} \frac{1}{\Lambda^4} \left(|C'_{e\gamma}_{rs}|^2 + |C'_{e\gamma}_{sr}|^2 \right)$$

$$\mathcal{B}(\tau^\pm \rightarrow \mu^\pm \gamma) < 4.4 \times 10^{-8} \text{ (90\% CL)}$$

BaBar

$$\frac{1}{\Lambda^2} |C'_{e\gamma}_{\mu\tau(\tau\mu)}| < 2.7 \times 10^{-6} \text{ TeV}^{-2}$$

electron $(g - 2)_e$, EDM d_e

$$C'_{e\gamma}_{LR} =$$

$C'_{e\gamma}_{ee}$	$C'_{e\gamma}_{e\mu}$	$C'_{e\gamma}_{e\tau}$
$C'_{e\gamma}_{\mu e}$	$C'_{e\gamma}_{\mu\mu}$	$C'_{e\gamma}_{\mu\tau}$
$C'_{e\gamma}_{\tau e}$	$C'_{e\gamma}_{\tau\mu}$	$C'_{e\gamma}_{\tau\tau}$

$\tau \rightarrow \mu\gamma, \tau \rightarrow 3\mu$

muon $(g - 2)_\mu$, EDM d_μ

$$\Delta a_\mu = \frac{4m_\mu}{e} \frac{v}{\sqrt{2}} \frac{1}{\Lambda^2} \text{Re} [C'_{e\gamma}_{\mu\mu}]$$

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FNAL, BNL

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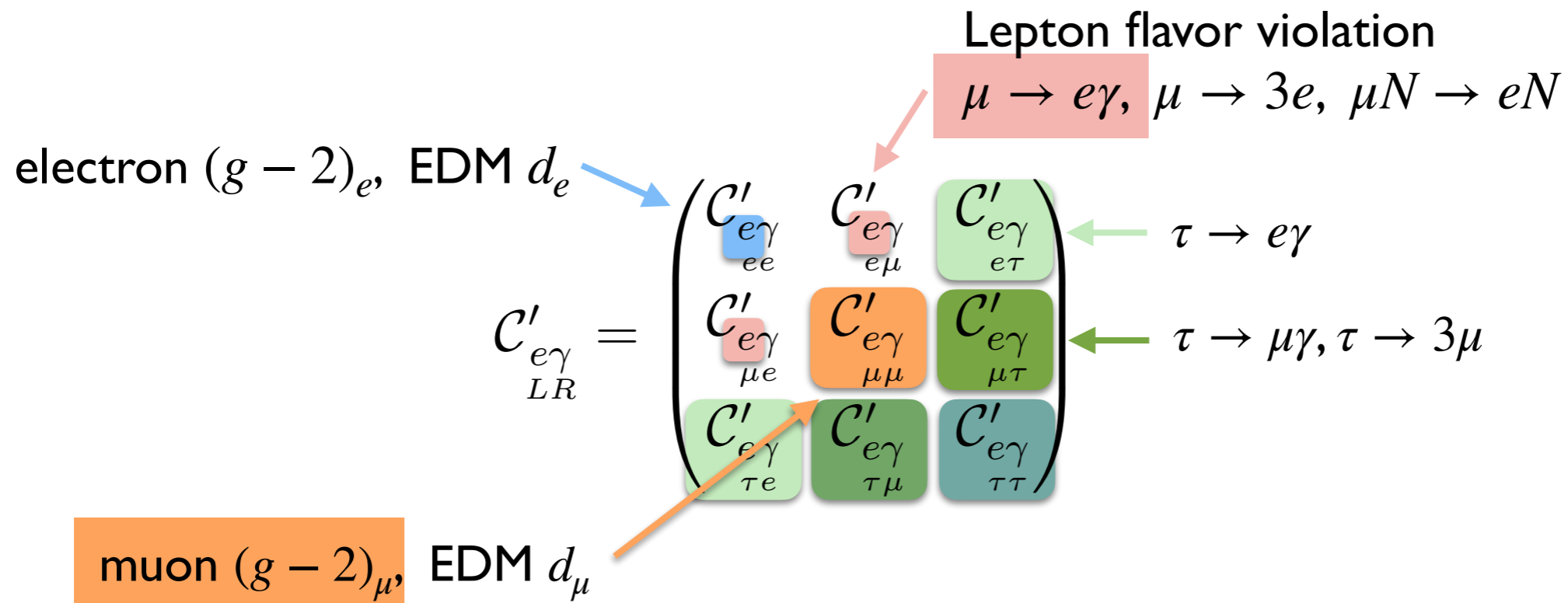
$$\left| \frac{C'_{e\gamma}_{\mu\tau(\tau\mu)}}{C'_{e\gamma}_{\tau\tau}} \right| < 1.6 \times 10^{-2} \times \left| \frac{y_\tau C'_{e\gamma}_{\mu\mu}}{y_\mu C'_{e\gamma}_{\tau\tau}} \right|$$

natural expectation

$$|C'_{e\gamma}_{\tau\tau}|/y_\tau \sim |C'_{e\gamma}_{\mu\mu}|/y_\mu$$

新物理のフレーバー構造 from $(g - 2)_\ell$, LFV and EDM

Dipole operator によって様々な現象が引き起こされる



muon $(g - 2)_\mu$ & $\mu \rightarrow e\gamma \rightarrow$ 特徴的なフレーバー構造 (対角成分 \gg 非対角成分)

このようなフレーバー構造はどのようにして出すことができるか？

\rightarrow フレーバー対称性

フレーバー対称性

SM flavor problem

$$M_{u,d,e} \sim \begin{pmatrix} \cdot & & \\ \cdot & \cdot & \\ & \cdot & \cdot \end{pmatrix} \quad V_{CKM} \sim \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

階層的構造 なぜ？

NP flavor problem

現在、フレーバー物理量において有意なずれは見つかっていない

→ 新物理に対して厳しい制限

新物理は特殊なフレーバー構造を持っている？

その背後には**フレーバー対称性**があるかもしれない

imposing **flavor symmetry** $\left\{ \begin{array}{l} \text{U(2) flavor symmetry} \\ \text{A4 modular flavor symmetry} \end{array} \right.$ Isidori, Pages and Wilsch[2111.13724]
Tanimoto, KY [2310.16325](#)
Kobayashi, Otsuka, Tanimoto, KY [2204.12325]

NP (realize muon $(g-2)_\mu$ anomaly
satisfy constraint from LFV



EDM d_e, d_μ electron $(g-2)_e$

$U(2)^5$ flavor symmetry

Barbieri, Isidori, Jones-Perez,
Lodone, Straub [1105.2296]

- $U(2)^5$ フレーバー対称性は、3世代目湯川結合がなぜ大きいのか自然な説明を与える

1st & 2nd 世代のみに作用する

3rd 世代目は対称性によって許される

$$\psi = (\psi_1, \psi_2, \psi_3)$$

SU(2) doublet singlet

- SM Yukawa が良い近似で保っている対称性

exact symmetry for $m_u, m_d, m_c, m_s = 0$ & $V_{CKM} = 1$

⇒ small breaking terms を導入するのみでOK

The SM flavor puzzle

Striking hierarchy

Mass : 3rd > 2nd > 1st

Almost diagonal CKM matrix

$$M_{u,d} \sim \begin{pmatrix} \cdot & & \\ & \cdot & \\ & & \bullet \end{pmatrix}$$

$$V_{CKM} \sim \begin{pmatrix} \bullet & \cdot & \cdot \\ \cdot & \bullet & \cdot \\ \cdot & \cdot & \bullet \end{pmatrix}$$

$U(2)^5$ flavor symmetry

Barbieri, Isidori, Jones-Perez,
Lodone, Straub [1105.2296]

Under $U(2)^3 = U(2)^q \times U(2)^u \times U(2)^d$ symmetry

$$\psi = (\psi_1, \psi_2, \psi_3)$$

SU(2) doublet singlet

$$\mathcal{L}_{\text{Yuk}} = (\bar{Q}^{(2)} \quad \bar{q}^3) Y_u \begin{pmatrix} u^{(2)} \\ t \end{pmatrix} + (\bar{Q}^{(2)} \quad \bar{q}^3) Y_d \begin{pmatrix} d^{(2)} \\ b \end{pmatrix}$$

$$\begin{aligned} Q^{(2)} = (Q^1, Q^2) &\sim (2, 1, 1) & Q^3 &\sim (1, 1, 1) \\ u^{(2)} = (u^1, u^2) &\sim (1, 2, 1) & t &\sim (1, 1, 1) \\ d^{(2)} = (d^1, d^2) &\sim (1, 1, 2) & b &\sim (1, 1, 1) \end{aligned}$$

Unbroken symmetry

$$Y_u = y_t \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{matrix} U(2)_q \\ \\ U(2)_u \end{matrix}$$

After U(2) breaking

$$\begin{pmatrix} \Delta_u & & V_q \\ \hline 0 & 0 & 1 \end{pmatrix}$$

U(2) breaking (Spurion)

$$V_q \sim (2, 1, 1),$$

$$\Delta_u \sim (2, \bar{2}, 1),$$

$$\Delta_d \sim (2, 1, \bar{2})$$

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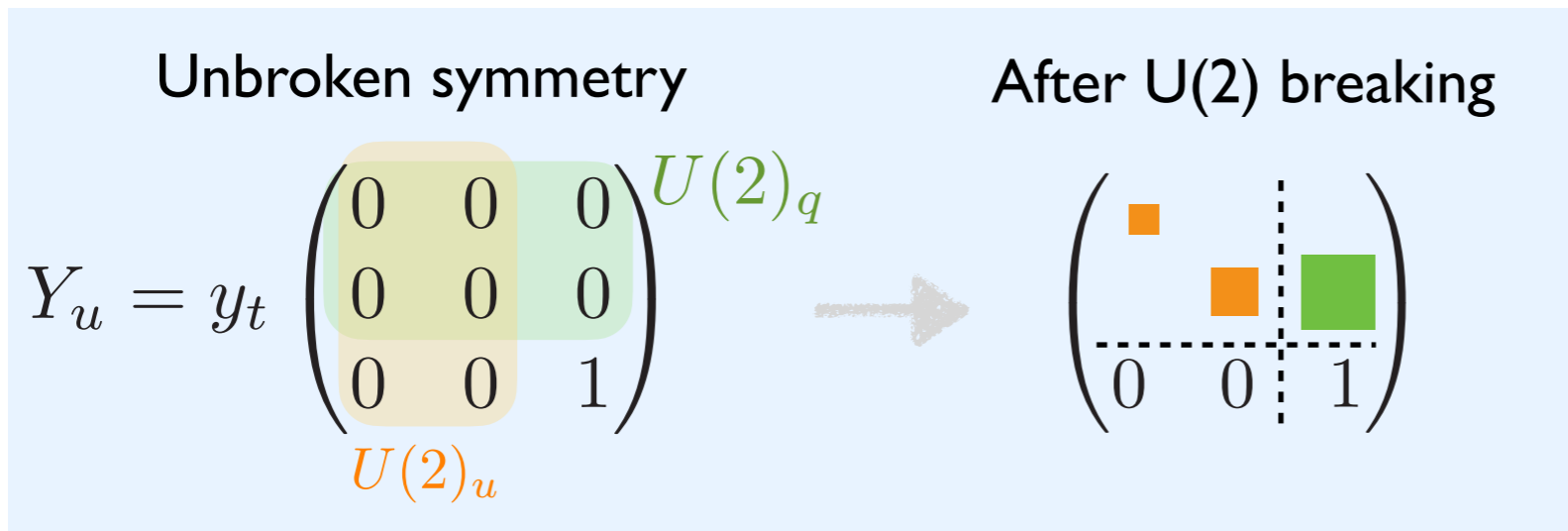
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U(2) breaking (Spurion)

$$\begin{aligned} \blacksquare &: |V_q| \sim |V_{ts}| \sim \mathcal{O}(10^{-1}) \\ \blacksquare &: |\Delta_u| \sim \begin{pmatrix} y_u/y_t & \\ & y_c/y_t \end{pmatrix} \\ &\sim \begin{pmatrix} \mathcal{O}(10^{-3}) & \\ & \mathcal{O}(10^{-2}) \end{pmatrix} \end{aligned}$$

spurion order : $1 \gg \blacksquare \gg \blacksquare \gg \blacksquare > 0$

$$\mathcal{O}(10^{-1}) \quad \mathcal{O}(10^{-2}) \quad \mathcal{O}(10^{-3})$$

NP LR flavor structure in U(2)

新物理のレプトンフレーバー構造もU(2)フレーバー対称性でコントロールされていると考える → 新物理の高次元オペレーターのフレーバー構造をU(2) breaking spurionで記述する

$$U(2)_{L_L} \otimes U(2)_{E_R} \text{ breaking (Spurion)} \quad V_\ell \sim (2,1), \quad \Delta_e \sim (2,\bar{2})$$

LR flavor structure at order $\mathcal{O}(V_\ell^2 \Delta_e)$ $X_{\alpha\beta}^n (\bar{\ell}_\alpha \Gamma e_\beta) \eta^n$ ($n = Y, e\gamma$)

$$X^n = \left(\begin{array}{c|c} (2,\bar{2}) & (2,1) \\ \hline (1,2) & (1,1) \end{array} \right)_{LR}$$

NP LR flavor structure in U(2)

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LR flavor structure at order $\mathcal{O}(V_\ell^2 \Delta_e)$ $X_{\alpha\beta}^n (\bar{\ell}_\alpha \Gamma e_\beta) \eta^n$ ($n = Y, e\gamma$)

$$X^n = \left(\begin{array}{c|c} \frac{C_\Delta^n (\Delta_e)_{\alpha\beta} + C_{VVV}^n (V_\ell)_\alpha (V_\ell^\dagger)_\gamma (\Delta_e)_{\gamma\beta}}{C_{V\Delta}^n (V_\ell^\dagger)_\alpha (\Delta_e)_{\alpha\beta}} & \frac{C_V^n (V_\ell)_\alpha + C_{VVV}^n (V_\ell) (V_\ell^\dagger) (V_\ell)_\alpha}{C^n + C_{VV}^n (V_\ell^\dagger V_\ell)} \end{array} \right)_{LR}$$

$C, C_V, C_\Delta, C_{VV}, C_{V\Delta}, C_{VV\Delta} : \mathcal{O}(1)$ NP coefficients

NP LR flavor structure in U(2)

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$$X^n = \left(\begin{array}{c|c} C_\Delta^n (\Delta_e)_{\alpha\beta} + C_{VVV}^n (V_\ell)_\alpha (V_\ell^\dagger)_\gamma (\Delta_e)_{\gamma\beta} & C_V^n (V_\ell)_\alpha + C_{VVV}^n (V_\ell) (V_\ell^\dagger) (V_\ell)_\alpha \\ \hline C_{V\Delta}^n (V_\ell^\dagger)_\alpha (\Delta_e)_{\alpha\beta} & C^n + C_{VV}^n (V_\ell^\dagger V_\ell) \end{array} \right)_{LR}$$

parametrization

$C, C_V, C_\Delta, C_{VV}, C_{V\Delta}, C_{VV\Delta} : \mathcal{O}(1)$ NP coefficients

$$V_\ell = \begin{pmatrix} 0 \\ \epsilon_\ell \end{pmatrix}, \quad \Delta_e = O_e^T \begin{pmatrix} \delta'_e & 0 \\ 0 & \delta_e \end{pmatrix}, \quad O_e = \begin{pmatrix} c_e & s_e \\ -s_e & c_e \end{pmatrix}$$

$$= \left(\begin{array}{ccc} C_\Delta^{e\gamma} c_e \delta'_e & -C_\Delta^{e\gamma} s_e \delta_e & 0 \\ s_e \delta'_e (C_\Delta^{e\gamma} + C_{VV\Delta}^{e\gamma} \epsilon_\ell^2) & c_e \delta_e (C_\Delta^{e\gamma} + C_{VV\Delta}^{e\gamma} \epsilon_\ell^2) & (C_V^{e\gamma} \epsilon_\ell + C_{VVV}^{e\gamma} \epsilon_\ell^3) \\ C_{V\Delta}^{e\gamma} (s_e \epsilon_\ell \delta'_e) & C_{V\Delta}^{e\gamma} (c_e \epsilon_\ell \delta_e) & C^{e\gamma} + C_{VV}^{e\gamma} \epsilon_\ell^2 \end{array} \right)_{LR}$$

parameters : $\frac{\delta'_\ell}{\delta_\ell} \simeq \frac{y_e}{y_\mu}$ ϵ_ℓ and s_e are not constrained, but presume from quark sector
 $s_e = 0.01 - 0.1, \quad \epsilon_\ell = 0.01 - 0.1$

$(g - 2)_\mu$ & $\mu \rightarrow e\gamma$ in $U(2)$

$$\mathcal{C}_{\mu\mu}^{e\gamma} \simeq |C_\Delta^{e\gamma}| \delta_e \left[\frac{1}{c_e} + c_e \epsilon_\ell^2 \left(\left| \frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} \right| \cos(\arg C_{VV\Delta}^{e\gamma}) - \left| \frac{C_{VV\Delta}^y}{C_\Delta^y} \right| \frac{s_e^2}{c_e^2} \cos(\arg C_{VV\Delta}^y) \right) \right]$$

$$\mathcal{C}_{e\mu}^{e\gamma} = [U_L^\dagger X^{e\gamma} U_R]_{12} \simeq |C_\Delta^{e\gamma}| s_e \delta_e \epsilon_\ell^2 \left(\frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} - \frac{C_{VV\Delta}^y}{C_\Delta^y} \right)$$

$$C'_{e\gamma LR} = \begin{pmatrix} C'_{e\gamma ee} & C'_{e\gamma e\mu} & C'_{e\gamma e\tau} \\ C'_{e\gamma \mu e} & C'_{e\gamma \mu\mu} & C'_{e\gamma \mu\tau} \\ C'_{e\gamma \tau e} & C'_{e\gamma \tau\mu} & C'_{e\gamma \tau\tau} \end{pmatrix}$$

muon $(g - 2)_\mu$ and $\mu \rightarrow e\gamma$

strong flavor alignment

$$\begin{pmatrix} C'_{e\gamma e\mu} \\ C'_{e\gamma \mu\mu} \end{pmatrix} \simeq \left| \frac{s_\theta}{c_\theta} \epsilon_\ell^2 \left(\frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} - \frac{C_{VV\Delta}^y}{C_\Delta^y} \right) \right| < 2.1 \times 10^{-5}$$

$s_e = \mathcal{O}(\sqrt{m_e/m_\mu}) \sim 10^{-1}$ $\epsilon_\ell \sim 10^{-1}$
 $\rightarrow 10^{-3}$ suppression by $U(2)$ spurions

from $\mu \rightarrow e\gamma$

$$\frac{1}{\Lambda^2} |C'_{e\gamma e\mu(\mu e)}| < 2.1 \times 10^{-10} \text{ TeV}^{-2}$$

from muon $(g - 2)_\mu$

$$\frac{1}{\Lambda^2} \text{Re} [C'_{e\gamma \mu\mu}] \approx 1.0 \times 10^{-5} \text{ TeV}^{-2}$$

$$\left| \frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} - \frac{C_{VV\Delta}^y}{C_\Delta^y} \right| \lesssim 10^{-2} \quad \text{tight alignment condition}$$

Isidori, Pages and Wilsch
[2111.13724]

$(g - 2)_\mu$ & $\mu \rightarrow e\gamma$ in U(2)

$$\mathcal{C}_{\mu\mu}^{e\gamma} \simeq |C_\Delta^{e\gamma}| \delta_e \left[\frac{1}{c_e} + c_e \epsilon_\ell^2 \left(\left| \frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} \right| \cos(\arg C_{VV\Delta}^{e\gamma}) - \left| \frac{C_{VV\Delta}^y}{C_\Delta^y} \right| \frac{s_e^2}{c_e^2} \cos(\arg C_{VV\Delta}^y) \right) \right]$$

$$\mathcal{C}_{e\mu}^{e\gamma} = [U_L^\dagger X^{e\gamma} U_R]_{12} \simeq |C_\Delta^{e\gamma}| s_e \delta_e \epsilon_\ell^2 \left(\frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} - \frac{C_{VV\Delta}^y}{C_\Delta^y} \right) - s_e \delta_e \epsilon_\ell^2 C_{3rd}$$

$$C'_{e\gamma LR} = \begin{pmatrix} C'_{e\gamma ee} & C'_{e\gamma e\mu} & C'_{e\gamma e\tau} \\ C'_{e\gamma \mu e} & C'_{e\gamma \mu\mu} & C'_{e\gamma \mu\tau} \\ C'_{e\gamma \tau e} & C'_{e\gamma \tau\mu} & C'_{e\gamma \tau\tau} \end{pmatrix}$$

3世代目を含んだ対角化で出てくる効果

muon $(g - 2)_\mu$ and $\mu \rightarrow e\gamma$

$$C_{3rd} = \frac{C_\Delta^{y*} C_V^y + C_{V\Delta}^{y*} C^y}{|C^y|^2} \left(\frac{C^{e\gamma}}{C^y} C_V^y - C_V^{e\gamma} \right) + \frac{C_V^y}{C^y} \left(\frac{C_V^{e\gamma}}{C^y} C_{V\Delta}^y - C_{V\Delta}^{e\gamma} \right)$$

strong flavor alignment

$$\begin{pmatrix} C'_{e\gamma e\mu} \\ C'_{e\gamma \mu\mu} \end{pmatrix} \approx \left| \frac{s_\theta}{c_\theta} \epsilon_\ell^2 \left(\frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} - \frac{C_{VV\Delta}^y}{C_\Delta^y} \right) \right| < 2.1 \times 10^{-5}$$

$s_e = \mathcal{O}(\sqrt{m_e/m_\mu}) \sim 10^{-1}$ $\epsilon_\ell \sim 10^{-1}$

$\rightarrow 10^{-3}$ suppression by U(2) spurions

from $\mu \rightarrow e\gamma$

$$\frac{1}{\Lambda^2} |C'_{e\gamma e\mu(\mu e)}| < 2.1 \times 10^{-10} \text{ TeV}^{-2}$$

from muon $(g - 2)_\mu$

$$\frac{1}{\Lambda^2} \text{Re} [C'_{e\gamma \mu\mu}] \approx 1.0 \times 10^{-5} \text{ TeV}^{-2}$$

$$\left| \frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} - \frac{C_{VV\Delta}^y}{C_\Delta^y} \right| \lesssim 10^{-2}$$

C_{3rd} の効果で、たとえ $\frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} = \frac{C_{VV\Delta}^y}{C_\Delta^y}$ であったとしても $C'_{e\gamma e\mu(\mu e)}$ は suppress されない

$(g - 2)_\mu$ & EDM d_e in U(2)

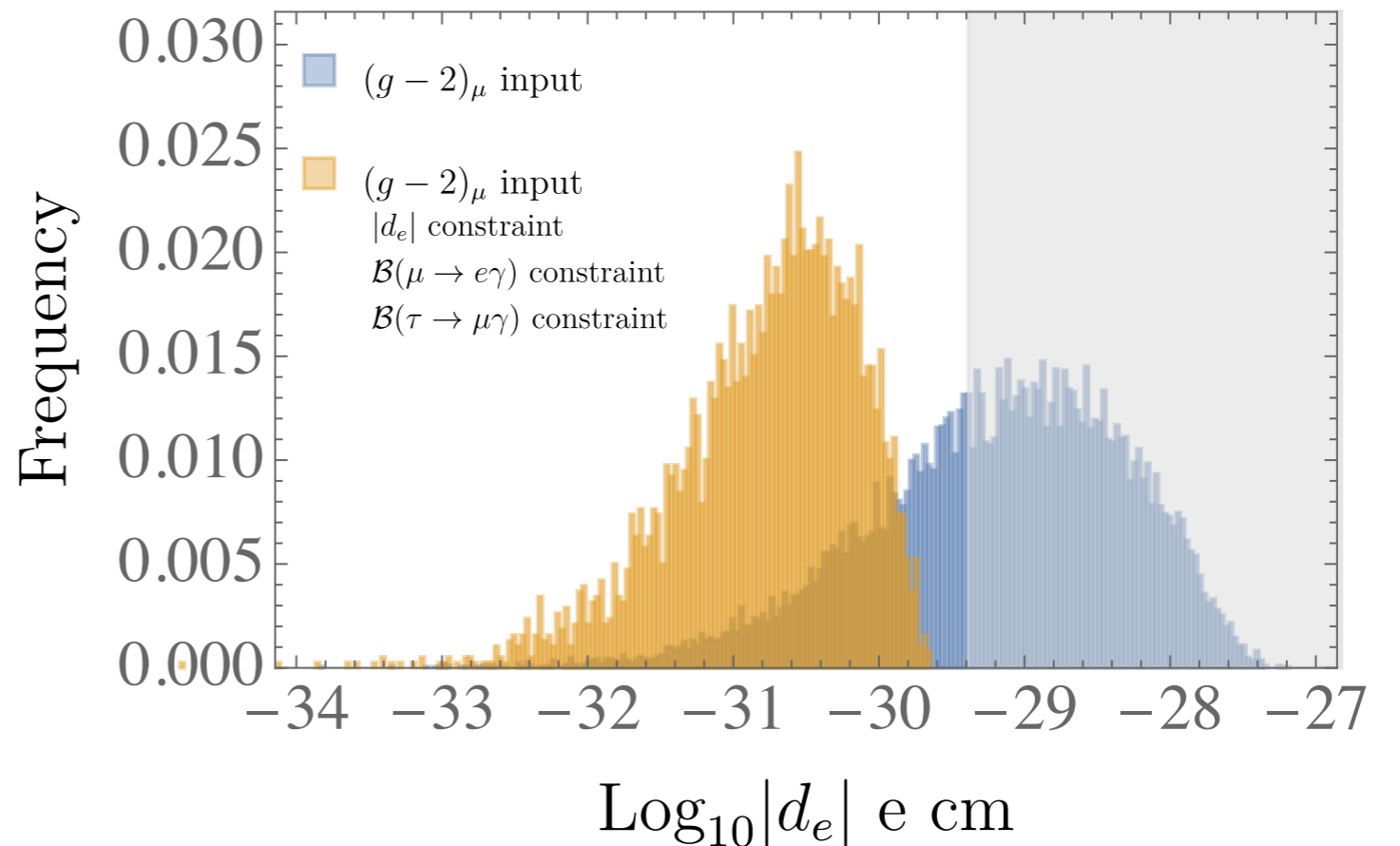
$$C'_{e\gamma LR} = \begin{pmatrix} C'_{e\gamma ee} & C'_{e\gamma e\mu} & C'_{e\gamma e\tau} \\ C'_{e\gamma \mu e} & C'_{e\gamma \mu\mu} & C'_{e\gamma \mu\tau} \\ C'_{e\gamma \tau e} & C'_{e\gamma \tau\mu} & C'_{e\gamma \tau\tau} \end{pmatrix}$$

muon $(g - 2)_\mu$ and EDM d_e

$$\left| \frac{\text{Im } C'_{e\gamma ee}}{\text{Re } C'_{e\gamma \mu\mu}} \right| \simeq s_e^2 \frac{\delta'^2}{\delta} \epsilon_\ell^2 \text{Im} \left(\frac{C_{VV\Delta}^{e\gamma}}{C_\Delta^{e\gamma}} - \frac{C_{VV\Delta}^y}{C_\Delta^y} + \frac{C_{3rd}}{|C_\Delta^{e\gamma}|} \right) < 1.8 \times 10^{-8}$$

$\sim 4 \times 10^{-9}$

constraint from $\mu \rightarrow e\gamma$ more tight
 ∇
 constraint from EDM d_e in U(2)



$(g - 2)_\mu$ & $(g - 2)_e$ in $U(2)$

$$C'_{e\gamma LR} = \begin{pmatrix} C'_{e\gamma ee} & C'_{e\gamma e\mu} & C'_{e\gamma e\tau} \\ C'_{e\gamma \mu e} & C'_{e\gamma \mu\mu} & C'_{e\gamma \mu\tau} \\ C'_{e\gamma \tau e} & C'_{e\gamma \tau\mu} & C'_{e\gamma \tau\tau} \end{pmatrix}$$

muon $(g - 2)_\mu$ and electron $(g - 2)_e$

U(2) relation $\frac{\text{Re } C'_{e\gamma ee}}{\text{Re } C'_{e\gamma \mu\mu}} \approx \frac{\square}{\square} = \frac{m_e}{m_\mu} \simeq 5 \times 10^{-3}$

$$\Delta a_e = \Delta a_\mu \frac{m_e}{m_\mu} \frac{\text{Re } C'_{e\gamma ee}}{\text{Re } C'_{e\gamma \mu\mu}} \approx \Delta a_\mu \times \left(\frac{m_e}{m_\mu} \right)^2 \sim 5.7 \times 10^{-14}$$

naive scaling vs.

$$\Delta a_e^{Cs} = a_e^{\text{Exp}} - a_e^{\text{SM,Cs}} = (-8.8 \pm 3.6) \times 10^{-13}$$

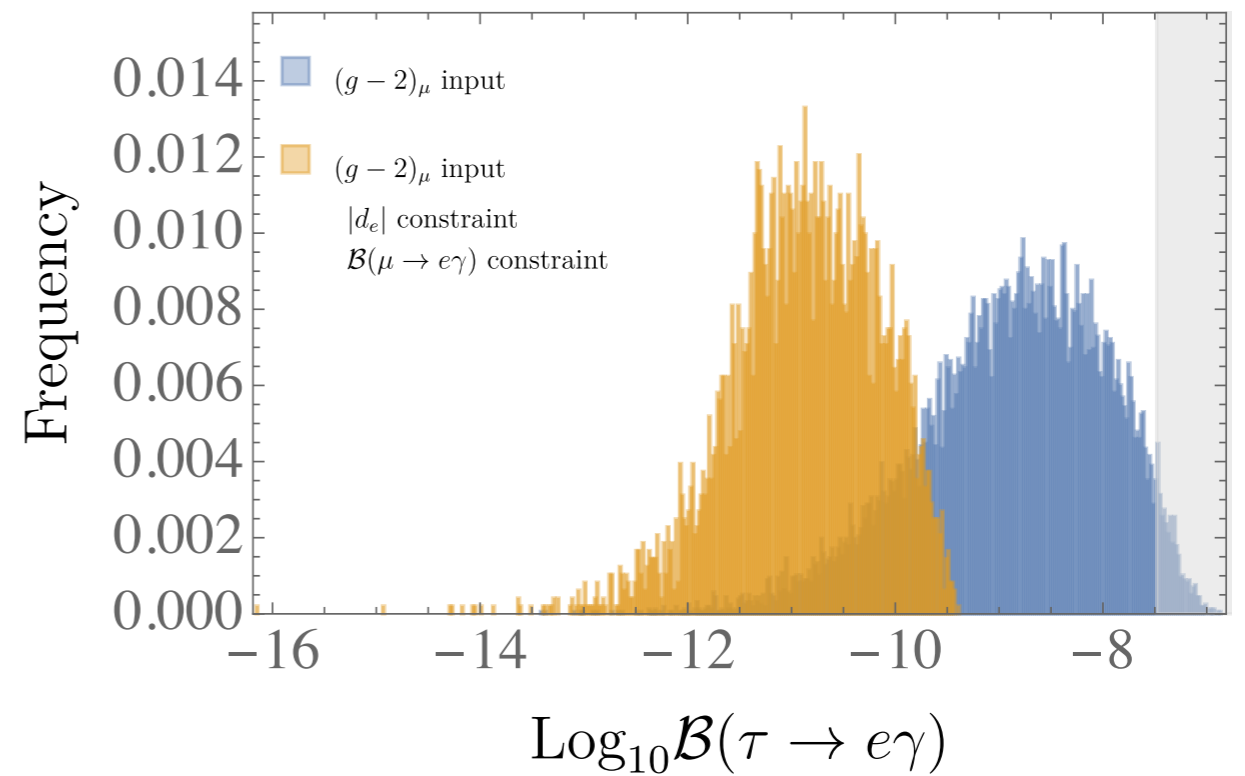
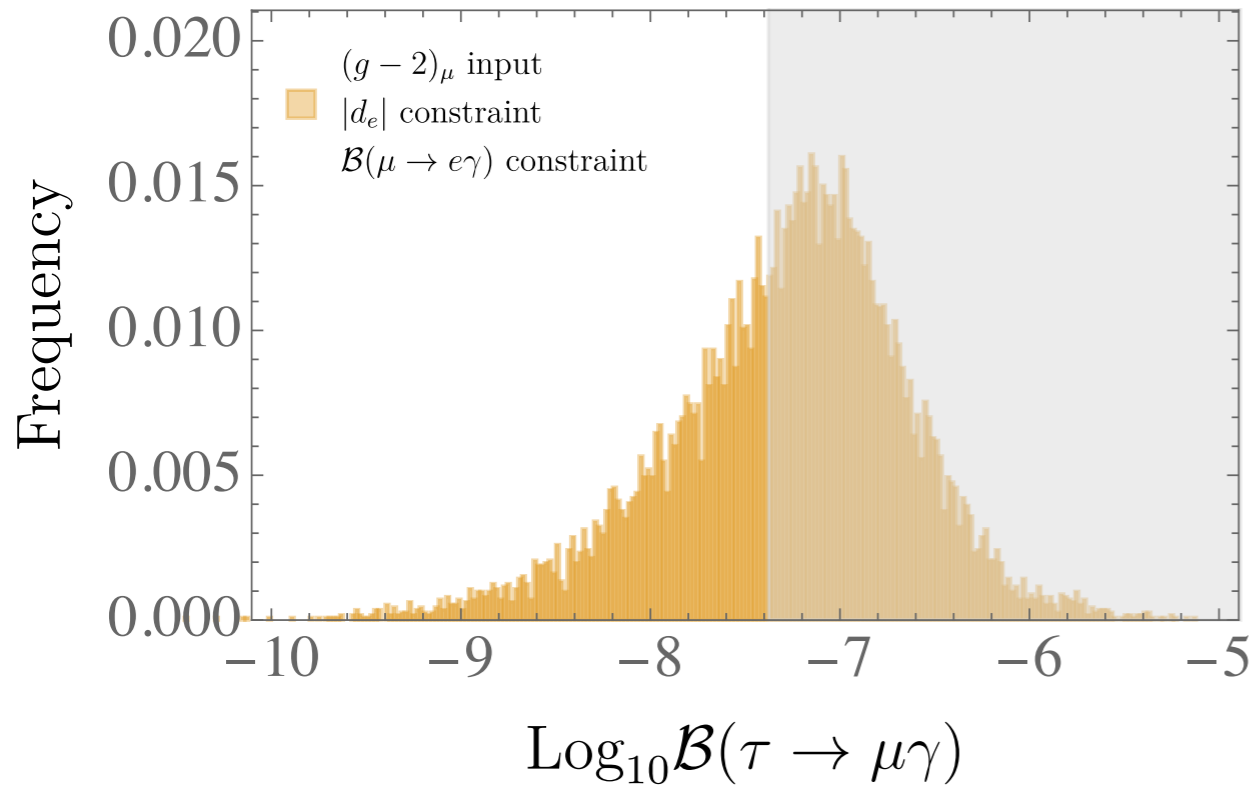
$$\Delta a_e^{Rb} = a_e^{\text{Exp}} - a_e^{\text{SM,Rb}} = (4.8 \pm 3.0) \times 10^{-13} .$$

Predicted value is small of one order compared with the present observed one at present
 Wait for the precise observation of the fine structure constant to test the framework

$(g - 2)_\mu$ & $\tau \rightarrow \mu\gamma, \tau \rightarrow e\gamma$ in $U(2)$

$$C'_{e\gamma LR} = \begin{pmatrix} C'_{e\gamma ee} & C'_{e\gamma e\mu} & C'_{e\gamma e\tau} \\ C'_{e\gamma \mu e} & C'_{e\gamma \mu\mu} & C'_{e\gamma \mu\tau} \\ C'_{e\gamma \tau e} & C'_{e\gamma \tau\mu} & C'_{e\gamma \tau\tau} \end{pmatrix}$$

muon $(g - 2)_\mu$ and $\tau \rightarrow \mu\gamma, \tau \rightarrow e\gamma$



→ Belle II

Summary

- Muon $(g - 2)_\mu$ & $\mu \rightarrow e\gamma \rightarrow$ 新物理のフレーバー構造

Tight bound on flavor alignment

$$C'_{e\gamma LR} = \begin{pmatrix} C'_{e\gamma ee} & C'_{e\gamma e\mu} & C'_{e\gamma e\tau} \\ C'_{e\gamma \mu e} & C'_{e\gamma \mu\mu} & C'_{e\gamma \mu\tau} \\ C'_{e\gamma \tau e} & C'_{e\gamma \tau\mu} & C'_{e\gamma \tau\tau} \end{pmatrix}$$

- そのフレーバー構造の背後にはフレーバー対称性があるかも

U(2) flavor symmetry

input $(g - 2)_\mu$ anomaly

	$\mu \rightarrow e\gamma$	EDM d_e	EDM d_μ	$(g - 2)_e$
<i>U(2)</i>	input	$ d_e/e \lesssim 10^{-30}\text{cm}$	$ d_\mu/e \lesssim 10^{-26}\text{cm}$	$\Delta a_\mu \times \left(\frac{m_e}{m_\mu}\right)^2$ naive scaling
<i>Modular</i>	$BR(\mu \rightarrow e\gamma) \lesssim 10^{-16}$	input	$ d_\mu/e \lesssim 10^{-25}\text{cm}$	

- もしmuon $(g - 2)_\mu$ が新物理のシグナルであるならば、フレーバー対称性に依って、物理量間の相関が異なる形で出てくる