Did we hear the sound of the Universe boiling? Analysis using the full fluid velocity profiles and NANOGrav 15-year data

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THE SPECTRUM OF GRAVITATIONAL WAVES







North American Nanohertz Observatory for Gravitational Waves (NANOGrav)

Detecting gravitational waves via regular observations of an ensemble of millisecond pulsars using the Green Bank Telescope, Arecibo Observatory, and the Very Large Array —>

In collaboration with international partners in the Parkes Pulsar Timing Array (PPTA) in Australia, the European Pulsar Timing Array (EPTA), and the Indian Pulsar Timing Array as part of the International Pulsar Timing Array (IPTA).

BLACK HOLE BINARY







First Order Phase Transitions

Starting from the unbroken phase, as the Universe cools, the potential energy undergoes a transformation, leading to the emergence of a new minimum that is distinct from the original one. This newly formed minimum is separated from the original by a potential energy barrier.

The phase transition proceeds through a process known as bubble nucleation. In this process, tiny regions within the Universe spontaneously transition from the old phase to the new one, forming bubbles of the new phase within the surrounding unbroken phase. These bubbles of the new phase then grow and expand, eventually dominating the Universe's state as it continues to cool.

$$S_{3}(\vec{\phi},T) = 4\pi \int drr^{2} \left[\frac{1}{2} \left(\frac{d\vec{\phi}(t)}{dr} \right)^{2} + V_{\text{eff}}(\vec{\phi},T) \right] \longrightarrow$$
$$\alpha(T_{n}) = \frac{1}{\rho_{\text{rad}}} \left(\Delta V_{\text{eff}} - \frac{1}{4} \frac{d\Delta V_{\text{eff}}}{dT} \right) \bigg|_{T_{n}} \qquad \beta = H_{n}T_{n} \frac{d(S_{3}/T)}{dT}$$



Bubble wall velocity:
$$v_w$$

$$H_n^2 = \frac{8\pi G}{2} \rho_{\rm rad}(T_n)$$

B

Strength of the FOPT: α_n

Bubble nucleation rate: β



$$\frac{S_3(T_n)}{T_n} \approx 140$$



First Order Phase Transitions and Gravitational Waves

First Order Phase Transition (FOPT) can give three sources for GW:

- Bubble Collision
- Sound Waves
- Turbulence

FOPT gravitational waves from sound waves:

- Stirred Plasma as Source for GW
- Long-lasting after phase transition
- Much larger than bubble collisions



arXiv:2006.08802



Gravitational Waves from FOPT: Sound Shell Model

During the Universe's cooling process, a new energy minimum forms, separated from the original by a potential energy barrier. Bubble nucleation happens, with small regions transitioning to the new phase, forming bubbles of the new phase within the old phase. These bubbles grow due to the lower energy of the new phase, converting more of the old phase. Gravitational waves result from the motion of the expanding bubble walls, generating spacetime ripples that carry away energy and momentum, creating a burst of gravitational waves.

Constructing the velocity field involves superimposing self-similar velocity and enthalpy density profiles, originating from randomly nucleated, expanding bubbles. These profiles, based on $\xi = R/T - R$ (distance from the bubble's center) and T (time since nucleation), dynamically evolve. When these bubbles collide, the scalar field driving the self-similar profile vanishes, turning it into the initial condition for unfettered sound wave propagation.

Computing the velocity field a convolution of the single-bubble velocity field with the distribution of bubble lifetimes —> Getting the velocity power spectrum —> Calculating the GW power spectrum using the velocity power spectrum!

arXiv:1909.10040





Gravitational Waves from NANOGrav: Power Law

Power Law Approximation for GW signal of NANOGrav:

$$\Omega = \frac{8\pi^4 f^5 \Phi(f)}{H_0^2 \Delta f} \qquad \Phi = \frac{A^2}{12\pi^2 T_{\text{obs}}} \left(\frac{f}{\text{yr}^{-1}}\right)^{-\gamma} \text{yr}^3$$

 $\Delta f = 1/T_{obs}$ and $H_0 = h \times 100 \text{ km s}^{-1} \text{Mpc}^{-1}$

At first approximation NANOGrav and EPTA giving the best fit for a power law spectrum:

$$\gamma = \begin{cases} 3.2 \\ 3.1 \\ - \\ - \\ 10^{-1} \end{cases}$$
$$A = \begin{cases} 6.4 \\ - \\ 10^{-1} \end{cases}$$

 ± 0.6 NANOGrav EPTA $^{+4.2}_{-2.7} \times 10^{-15}$ NANOGrav -14.13 ± 0.12 EPTA



Gravitational Waves from FOPT: Full Sound Shell Model

Relic density of GW from FOPT:

$$\Omega_{\rm GW}h^2 = 8.5 \times 10^{-6} \left(\frac{100}{g_*(T_n)}\right)^{1/3} \Gamma^2 \bar{U}_f^4 \left[\frac{H_n}{\beta(v_w)}\right] v_w \times \Upsilon \times S(f)$$

 $\Upsilon = 1 -$

Single broken power-law spectral form:

 $S(f) = \left(\frac{f}{f}\right)$

$$f_p = 8.9 \times 10^{-6} \frac{1}{v_w} \left(\frac{\beta}{H_n}\right) \left(\frac{z_p}{10}\right) \left(\frac{T_n}{100 \text{GeV}}\right) \left(\frac{g_*(T_n)}{100}\right)^{1/6} \text{Hz}$$

$$\frac{1}{\sqrt{1+2\tau_{\rm sh}H_s}}$$

$$\left(\frac{f}{\frac{c}{p}}\right)^3 \left(\frac{7}{4+3(f/f_p)^2}\right)^{7/2}$$

Nucleation temperature: T_n Strength of the FOPT: α_n Bubble nucleation rate: β Bubble wall velocity: v_w Root mean square fluid velocity: \bar{U}_f Adiabatic index: $\Gamma \sim 4/3$ Suppression factor from the finite lifetime (τ_{sh}) of the sound waves: Υ





Fitting FOPT to NANOGrav Data



Frequentist approach: $\chi_{\text{fit}}^2 = \sum_{k=1}^{N} \frac{\log_1 k}{k}$

Scanning over the parameter space of v_w , β/H_n , α , T_n from numerical estimation of GW using full velocity profile:



$$\frac{10 \Omega_{\rm th} h^2 - \log_{10} \Omega_{\rm exp} h^2}{2\bar{\sigma}_i^2}$$

We also did the Bayesian approach analysis to find the maximum likelihood!

Best Fit - NANOGrav

Numerical		Analytical	
Full Sound shell		Broken power law fit	
Parameter	Best fit value	Parameter	Best fit value
$lpha_n$	0.85	$lpha_n$	0.89
eta/H_*	42	eta/H_*	5.17
T_n	$133 { m ~MeV}$	T_n	$142 \mathrm{MeV}$
v_w	0.09	v_w	0.67
$\chi_{ m fit}$	1.4	$\chi_{ m fit}$	1.59

A possible (dark) phase transition at low temperatures!

OP.