Probing Exotic Phases Via Stochastic Gravitational Wave Spectra

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Gravitational waves as probes of the early universe

- CMB can directly probe the universe back to recombination $T \sim$ eV.
- Observations of the abundance of light elements allows us to probe the universe up to temperatures $T \sim$MeV.
- Gravitational waves produced in the pre-BBN era are the most suitable probes for accessing higher temperatures.
- This is because gravitational waves only interact via gravity and this makes the universe transparent to gravitational waves.

Causality-limited GWs and their evolution

- Causality-limited GWs : $\lambda \gg 1/\beta$.

$$h''(\mathbf{k},\tau) + 2\mathcal{H}(\tau)h'(\mathbf{k},\tau) + k^2 h(\mathbf{k},\tau) = J(\mathbf{k},\tau)$$

- Sub-horizon modes ($k \gg \mathcal{H}_*$), we find (arXiv:2010.03568)

$$h(k,\tau) \approx \frac{a(\tau_*)J_*}{a(\tau)k}\sin(k(\tau-\tau_*))$$

- When modes that were super-horizon at the time of production enter the horizon

$$h(k,\tau) \approx \frac{\Gamma(n-\frac{1}{2})J_*\tau_*}{2\sqrt{\pi}}\left(\frac{2}{k\tau}\right)^n \cos\left(k\tau - \frac{n\pi}{2}\right)$$

where $n = 2/(1+3w)$

Power spectrum and $\Omega_{\mathsf{GW}}(k)$ vs k

- $$\Omega_{\mathsf{GW}}(k) \equiv \frac{d\Omega_{\mathsf{GW}}}{d\log k}$$
- Sub-horizon modes: P_h is proportional to k^{-2}.

$$\implies \Omega_{\mathsf{GW}}(k) \propto k^5 P_h \propto k^3$$

- Super-horizon modes: P_h is proportional to k^{-2n}.

$$\implies \Omega_{\mathsf{GW}}(k) \propto k^{3-2\left(\frac{1-3w}{1+3w}\right)}$$

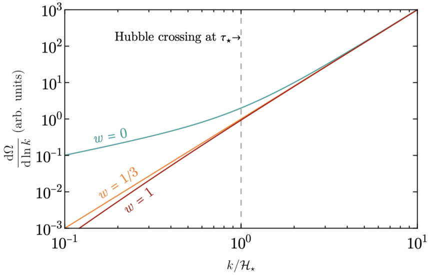

Figure 1: Scaling of $\Omega_{\mathsf{GW}}(k)$ versus $k/\mathcal{H}_*$ for different equations of state w. (arXiv:2010.03568)

Weak-Confined Standard Model (WCSM)

- In the WCSM the $SU(2)_L$ component of the electroweak force is strongly coupled.
- The relevant kinetic term in the Lagrangian is

$$\mathcal{L} = -\frac{1}{2}\Big(\frac{1}{g^2} - \frac{\hat{\varphi}}{M}\Big)\mathrm{Tr}(W^{\mu\nu}W_{\mu\nu})$$

$$\frac{1}{g_{\text{eff}}^2} = \frac{1}{g^2} - \frac{\langle\hat{\varphi}\rangle}{M}$$

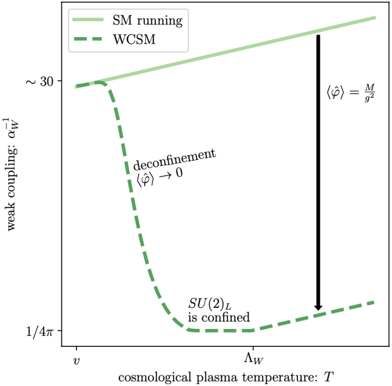

Figure 2: The WCSM phase (arXiv:1906.05157).

Lots of pions in the WCSM!

- The WCSM, just like the Standard Model, has
  - $\mathsf{U}(1)_Y \times \mathsf{SU}(2)_L \times \mathsf{SU}(3)_c$ gauge bosons
  - 12 $\mathsf{SU}(2)_L$-doublet fermions
  - 1 $\mathsf{SU}(2)_L$-doublet scalar
  - 21 $\mathsf{SU}(2)_L$-singlet fermions
- 12 $\mathsf{SU}(2)_\mathsf{L}$-doublet fermions $\longrightarrow$ composite states $\longrightarrow$ scalar fields Σ_{ij}
- The Σ_{ij} are assumed to acquire a non-zero vacuum expectation value.
- Symmetry breaking : SU(12) $\longrightarrow$ Sp(12)
- 65 broken generators $\Longrightarrow$ 65 Goldstone bosons (pions)

Interactions and masses

- The non-isospin gauge group gets spontaneously broken as $\mathsf{U}(1)_Y \times \mathsf{SU}(3)_c \to \mathsf{U}(1)_Q \times \mathsf{SU}(2)_c$.
- Out of 9 gauge bosons, 5 become massive and 5 pions are eaten.
- Out of the remaining 60 pions, 58 acquire mass through gauge and Yukawa interactions.
- The left-handed fermions become mostly composite, while the right-handed states are mostly elementary.

Change in w with T

The number density of these states get exponentially suppressed as the universe cools. As a result, w of the fluid pervading the universe changes with temperature.

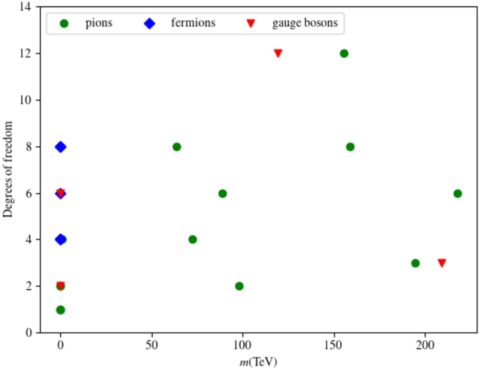

Figure 3: Mass of the various degrees of freedom in the WCSM spectrum.

δw in the WCSM phase

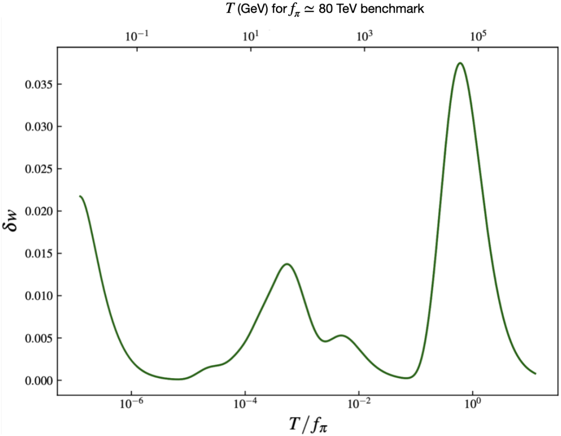

Figure 4: Typical behaviour of δw $(= w - 1/3)$ with respect to temperature during the WCSM phase.

Model and approximations

- We consider gravitational waves emanating from first-order phase transitions in the early universe for our calculations.
- The latent heat of the phase transition percolates into the cosmic fluid leading to sound waves with power spectrum

$$h^2\Omega_{\mathsf{sw}}(f) = 2.65 \times 10^{-6}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3} v_w S_{\mathsf{sw}}(f)$$

$$S_{\mathsf{sw}}(f) = (f/f_{\mathsf{sw}})^3\left(\frac{7}{4+3(f/f_{\mathsf{sw}})^2}\right)^{7/2}$$

The evolution of h

- With $\delta w \neq 0$, the conformal Hubble rate

$$\mathcal{H} \approx \frac{1}{\tau + \frac{3}{2}\int_{\tau_*}^{\tau} d\tau' \delta w(\tau')}$$

- Plugging this in the Einstein equation allows us to solve for h numerically.

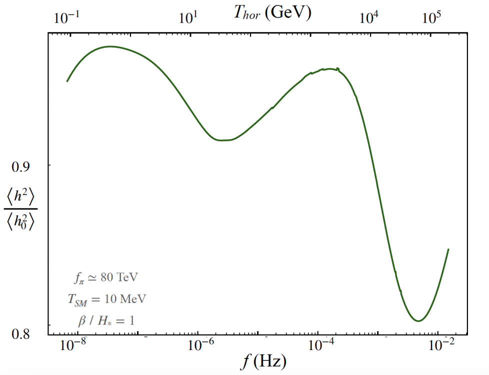

Figure 5: Ratio of $\langle h^2 \rangle$ with respect to $\langle h_0^2 \rangle$ for different values of f.

Sensitivity to signal

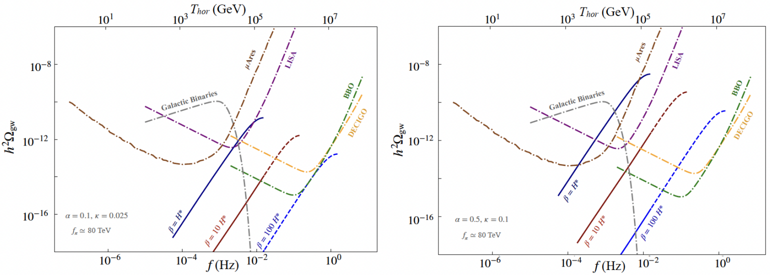

Figure 6: Sensitivity of the signal along with a background signal of galactic binaries compared to the sensitivity reach for future detectors. The fit for the galactic binaries background was obtained from arXiv:2106.05984.

Concluding remarks

- Gravitational waves from first-order phase transitions occurring in the pre-BBN era can tell us about the early universe with $T \gg$ MeV.

- Changes in the number of relativistic degrees of freedom affect the primordial spectrum of causality-limited gravitational waves, which can possibly be detected by future experiments. This can be used to ascertain if the universe underwent a phase of $SU(2)_L$ confinement.

- A similar analysis has been carried out in arXiv:1812.07577.

Einstein equation

- General solution:

$$h(\mathbf{k},\tau) = \int d\tau' \frac{e^{-\mathcal{H}(\tau-\tau')}}{\sqrt{k^2-\mathcal{H}^2}} \sin\left((\tau-\tau')\sqrt{k^2-\mathcal{H}^2}\right) J(\mathbf{k},\tau')$$

  assuming $h(k,\tau)=0$ for $\tau<\tau_*$.
- Initial conditions assumed in our work:

$$h(k,\tau_*)=0 \qquad\qquad h'(k,\tau_*)=J_*$$