Theoretical Prediction for Double Higgs Production via Photon Fusion at Muon Colliders

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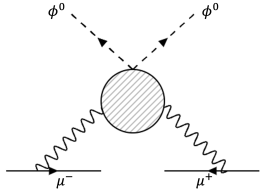
Introduction

1. Double Higgs production is important for testing the Higgs self-coupling which is responsible for giving mass to elementary particles and the shape of the Higgs potential.
2. Tri-linear Higgs coupling is challenging to measure directly, as it requires the production of two or more Higgs bosons simultaneously.
3. To measure the tri-linear Higgs coupling at the LHC requires high luminosity.
4. Muon colliders can reach higher center of mass energies than proton colliders ($\sqrt{s} \sim \mathcal{O}(10\,TeV)$) , which can increase the production rate of triple Higgs events.

(E. Asakawa, CO 2008)



Higgs Triplet Model

1. In the Higgs triplet Model, alongside the Standard Model weak doublet, represented as $\Phi \sim (1, 2, 1/2)$, there is an additional Higgs triplet denoted as $\Delta \sim (1, 3, 2)$ which transforms under the $SU(2)_L$ gauge group.
2. The motivation for the Type II Seesaw Model stems from the observation that two doublets can be decomposed into a triplet and a singlet representation $(2 \otimes 2 = 3 \oplus 1)$.

$$\Delta = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ \delta^0 & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}$$

Under the gauge transformation $U(x)$, H and Δ transform as $H \rightarrow U(x)H$ and $\Delta \rightarrow U(x)\Delta U^\dagger(x)$.

Higgs Triplet model

1. Kinetic term

$$\mathcal{L}_k = \mathrm{Tr}[(D_\mu\Delta)^\dagger(D^\mu\Delta)],$$

$$D_\mu\Delta = \partial_\mu\Delta + i\frac{g}{2}[\sigma^a W_\mu^a, \Delta] + i\frac{g^{'}}{2}Y_\Delta B_\mu\Delta.$$

2. The general scalar potential term , $V(\Phi, \Delta)$,

$$\begin{aligned} V(\Phi,\Delta) = &-\mu_\Phi^2\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2 + \mu_\Delta^2\mathrm{Tr}(\Delta^\dagger\Delta) \\ &+ [\mu(\Phi^T i\sigma^2\Delta^\dagger\Phi) + \mathrm{h.c}] + \lambda_1(\Phi^\dagger\Phi)Tr(\Delta^\dagger\Delta) + \lambda_2(Tr\Delta^\dagger\Delta)^2 \\ &+ \lambda_3 Tr(\Delta^\dagger\Delta)^2 + \lambda_4\Phi^\dagger\Delta\Delta\Phi \end{aligned}$$

3. There are seven physical Higgs states in mass basis and those can be categorized such as the charged Higgs Bosons $(H^\pm, H^{\pm\pm})$ and the neutral Higgs Bosons (h^0, H^0) and a pseudo scalar (A^0)

(A. Arhrib, CO 2011)

Constraints-ρ parameter

1. We can establish two distinct parameter spaces as follows:

$$\mathcal{P}_1 = \{\mu, \lambda, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \tan\beta, \cos\alpha\},$$

$$\mathcal{P}_2 = \{M_{h^0} = 125\mathrm{GeV}, M_{H^0}, M_{A^0}, M_{H^\pm}, M_{H^{\pm\pm}}, v_\Delta, v_\Phi, \cos\alpha\}.$$

   μ - LFV parameter.

   α, β denote the rotation angles respectively in the CP-even and CP-odd sectors.

2. The ρ parameter

$$\rho = \frac{1 + \frac{1}{2}\tan^2\beta}{1 + \tan^2\beta}.$$

   Where $\tan\beta = 2v_\Delta / v_\Phi$. The experimental value of the rho parameter, $\rho^{exp} = 1.0008^{+0.0017}_{-0.0007}$, being close to unity, leads to the bound $\tan\beta \lesssim 0.0633$.

Constraints-Vacuum stability

In the Higgs Triplet model, vacuum stability is required to ensure that the electroweak symmetry breaking minimum of the Higgs potential is stable and that the vacuum dies decay into a lower-energy state.

$$\lambda \geq 0; \lambda_2 + \lambda_3 \geq 0; \lambda_2 + \frac{\lambda_3}{2} \geq 0$$

$$\lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0; \lambda_1 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} \geq 0$$

$$\lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0; \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} \geq 0$$

Perturbativity dictates that these dimensionless parameters should be less than 4π.

Constraints-Tachyonic modes

To prevent the occurrence of tachyonic Higgs states, we establish the following constraints. These relationships ensure that the mass-squared values of the Higgs states remain positive: For the neutral pseudoscalar field (A^0),

$$\mu > 0 = \mu_0,$$

for the singly charged field ($H^{\pm}$),

$$\mu > \frac{\lambda_4 M_W s_W}{4\sqrt{2\pi\alpha}} \frac{\tan\beta}{\sqrt{1+\frac{\tan^2\beta}{2}}} = \mu_1,$$

for the doubly charged field ($H^{\pm\pm}$)

$$\mu > \frac{M_W s_W}{4\sqrt{2\pi\alpha}} \frac{(2\lambda_4 \tan\beta + \lambda_3 \tan^3\beta)}{\sqrt{1+\frac{\tan^2\beta}{2}}} = \mu_2.$$

Constraints-Tachyonic modes

To avoid the tacyonic modes of Heavy Higgs state, $\mu \in [\mu_-, \mu_+]$.

$$\mu_\pm = \frac{M_W S_W}{8\tan\beta\sqrt{\pi\alpha}(1+\frac{\tan^2\beta}{2})}\Big[\lambda + 2(\lambda_1+\lambda_4)\tan^2\beta$$
$$\pm\sqrt{2\lambda^2 + 8\lambda\tan^2\beta(\lambda_1+\lambda_4+\lambda_2\tan^2\beta+\lambda_3\tan^2\beta)}\Big]$$

The lower bound of μ

$$\mu_L = max\{\mu_0, \mu_1, \mu_2, \mu_-\}$$

Therefore, the boundaries of the lepton number violating parameter are,

$$\mu \in [\mu_L, \mu_+]$$

The cross section for $\gamma\gamma \to \phi\phi$ and the computation method

1. The process of the Higgs pair production in photon collision is denoted by

$$A_\mu(k_1) + A_\nu(k_2) \to \phi(k_3) + \phi(k_4),$$

where $\phi \in \{h^0, A^0\}$ The tensor amplitude and the invariant amplitude are expressed as follows:

$$\mathcal{M}_{\mu\nu} = (\mathcal{M}_{\mu\nu}^{box} + \mathcal{M}_{\mu\nu}^{triangle} + \mathcal{M}_{\mu\nu}^{bubble} + \mathcal{M}_{\mu\nu}^{quartic})$$

$$\mathcal{M} = \mathcal{M}_{\mu\nu}\epsilon^\mu(k_1, \lambda_1)\epsilon^\nu(k_2, \lambda_2).$$

$$\hat{\sigma}(\hat{s}, \gamma\gamma \to \phi\phi) = \frac{1}{32\pi\hat{s}^2}\int_{\hat{t^-}}^{\hat{t^+}} dt \sum_{spins} |\mathcal{M}|^2$$

Where $\hat{t^\pm} = (M_\phi^2 - \hat{s}/2) \pm \sqrt{(\hat{s}/2 - M_\phi^2)^2 - M_\phi^4}.$

Total cross sections $\mu^+\mu^- \to \gamma\gamma \to \phi\phi$

Since the Higgs pair production via photon-photon collisions is a subprocess of $\mu^+\mu^-$ collisions at the muon collider, the total cross section of this process can be conveniently obtained by utilizing the expression

$$\sigma(s, \mu^+\mu^- \to \gamma\gamma \to \phi\phi) = \int_{\frac{2M_\phi^2}{\sqrt{s}}}^{1} d\tau \frac{d\mathcal{L}_{\gamma\gamma}}{d\tau} \hat{\sigma}(\hat{s} = \tau s, \gamma\gamma \to \phi\phi),$$

along with the photon luminosity

$$\frac{d\mathcal{L}_{\gamma\gamma}}{d\tau} = \int_{\tau}^{1} \frac{dx}{x} f_{\gamma/\mu}(x) f_{\gamma/\mu}(\frac{\tau}{x}),$$

The computational processes were facilitated by the utilization of the FeynArts and FormCalc Mathematica packages. To replace the use of CompAZ for the effective photon approximation, we implemented our own subroutine.

Equivalent Photon Approximation

1. The case of collinear photon emission from an electron at leading order can be described using the equivalent photon approximation (EPA) or Weizsacker-Williams Approximation

(M Chiesa, CO 2021)

$$f_{\gamma,l}(x) \approx \frac{\alpha}{2\pi} P_{\gamma,l}(x) \ln \frac{E^2}{m_l^2}$$

2. Where the splitting function are, $P_{\gamma,l} = (1 + (1 - x^2)/x$ for $l \to \gamma$ Initially, we examine the partonic cross sections involving $\gamma\gamma$ fusions leading to the production of Higgs pairs in the Higgs Triplet Model.

Iterative Solutions for QED

By solving iteratively the DGLAP equations, the approximate solutions for the PDF (for our case),

(F Garosi, CO 2023)

$$f_\gamma(x,t) = \frac{\alpha}{2\pi} t P_{vf}^{f} + \frac{1}{2}\left(\frac{\alpha t}{2\pi}\right)^2 [(P_\gamma^v + P_f^v) P_{vf}^f + I_{vfff}].$$

where,

$$t = \alpha log\left(\frac{E^2}{M_\mu}\right), P_f^v = 3/2, P_f^v = -40/9, P_{vf}^f = \frac{1+(1-x)^2}{x}$$

$$I_{vfff} = \left(\frac{3}{2} + 2log(1-x)\right) P_{vf}^f + \frac{(1-x)(2x-3)}{x} + (2-x)log(x)$$

Numerical Results

In our analysis, we explore various hierarchies between $M_{H^\pm}$ and $M_{H^{\pm\pm}}$, primarily determined by the sign of λ_4 at $\tan\beta << 1$ values.

$$M^2_{H^\pm} - M^2_{H^{\pm\pm}} = \frac{M_W^2 S_W^2}{4\alpha\pi}\lambda_4 + \frac{\mu M_W S_W}{\sqrt{2\pi\alpha}}\tan\beta + \mathcal{O}(\tan^2\beta)$$

$\lambda_4 > 0$	$M_{H^{\pm\pm}} < M_{H^\pm} < M_H \approx M_{A^0}$
$\lambda_4 = 0$	$M_{H^{\pm\pm}} = M_{H^\pm} = M_H \approx M_{A^0}$
$\lambda_4 < 0$	$M_{H^\pm} < M_{H^{\pm\pm}} < M_H \approx M_{A^0}$

Numerical Results

We have used the following input values: $\tan\beta = 0.001$, $\lambda = 0.51$, $\lambda_1 = 10$, $\lambda_2 = 1$, $\lambda_3 = -1$, and $\mu = [0.2, 1.5]$. The table below displays the mass ranges corresponding to these input values.

	$M_{H^\pm}$ GeV	$M_{H^{\pm\pm}}$ GeV	M_{H^0} GeV	M_{A^0} GeV
$\lambda_4 = 1.82$	201-672	114-651	261-692	261-692
$\lambda_4 = 0$	261-692	261-692	261-692	261-692
$\lambda_4 = -1.82$	309-712	351-731	261-692	261-692

According to the ATLAS collaboration, searches for di-W bosons are excluded from the mass region where $M_{H^{\pm\pm}}$ lies between 200 GeV and 220 GeV for $Br(H^{\pm\pm} \to W^\pm W^\pm)$.

(P Siddharth, CO 2023)

$\sigma(\mu^+\mu^- \to \gamma\gamma \to h^0 h^0)$

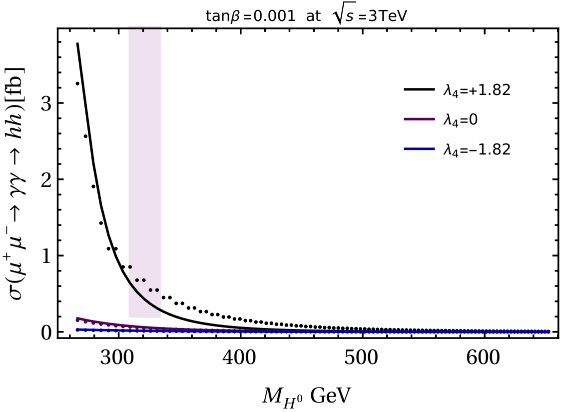


Figure: The total cross section $\sigma(\mu^+\mu^- \to \gamma\gamma \to h^0 h^0)$ as a function of M_{H^0}.

The pink region in the plot: In this scenario, $H^{\pm\pm}$ meets the exclusion limits from 200 GeV to 220 GeV. The corresponding H^0 mass values range from 315 GeV to 335 GeV.

$\sigma(\mu^+\mu^- \to \gamma\gamma \to A^0A^0)$

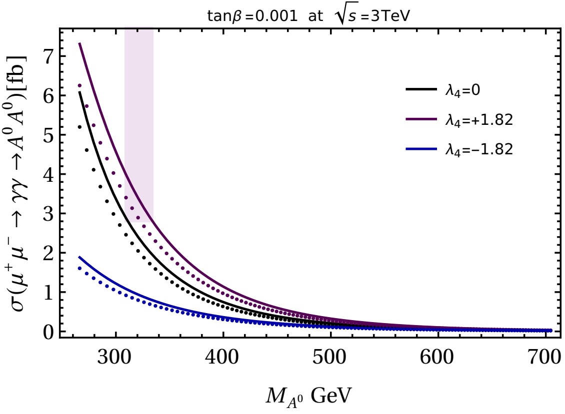


Figure: The total cross section $\sigma(\mu^+\mu^- \to \gamma\gamma \to A^0A^0)$ as a function of M_{A^0}.

The pink region in the plot: In this scenario, $H^{\pm\pm}$ meets the exclusion limits from 200 GeV to 220 GeV. The corresponding A^0 mass values range from 315 GeV to 335 GeV.

Decoupling Limits

1. Trilinear SM-like Higgs Coupling at $\alpha \approx 0$

$$\lambda_{hhh}^{HTM} = -\frac{3}{2}\frac{M_W S_W}{\sqrt{\pi\alpha}}\lambda + \frac{3}{8}\lambda\frac{M_W S_W}{\sqrt{\pi\alpha}}\tan^2\beta + \mathcal{O}(\tan^4\beta)$$

If we define,

$$\lambda^{'} = -\frac{3\pi\alpha}{M_W^2 S_W^2}\left(\frac{M_h^2}{-\frac{3}{2} + \frac{3}{8}\tan^2\beta + \mathcal{O}(\tan^4\beta)}\right)$$

we can observe

$$\lambda_{hhh}^{HTM}\Big|_{\lambda=\lambda^{'}} \rightarrow \lambda_{hhh}^{SM}$$

Numerically, when $\lambda \approx 0.51$, $\left|\frac{\Delta\lambda_{hhh}}{\lambda_{hhh}^{SM}}\right| \times 100 \approx 0.6\%$

Decoupling Limits

1. The $H^0h^0h^0$ coupling at $\alpha \approx 0$ is,

$$\lambda_{Hhh}^{HTM} = \sqrt{2}\mu - \frac{(\lambda_1 + \lambda_4)M_W S_W}{2\sqrt{\alpha\pi}} \tan\beta + \frac{(\lambda_1 + \lambda_4)M_W S_W}{8\sqrt{\alpha\pi}} \tan^3\beta + \mathcal{O}(\tan^5\beta).$$

For $\tan\beta << 1$, if we define

$$\mu^* = \frac{(\lambda_1 + \lambda_4)}{\sqrt{8\pi\alpha}} \tan\beta,$$

the $\lambda_{Hhh}^{HTM}\Big|_{\mu=\mu^*} \to 0$.

λ_4	μ^*	$M_{H^0}(\mathrm{GeV})$	λ_{Hhh}^{HTM}
$+1.82$	1.047	597.98	10^{-6}
0	0.869	544.78	10^{-7}
-1.82	0.725	497.60	10^{-7}

Summary and Outlook

1. The cross sections of $\mu^+\mu^- \to \gamma\gamma \to \phi\phi$ can be enhanced for $M_{H^\pm} > M_{H^{\pm\pm}}$. The lowest cross sections can be observed for $M_{H^{\pm\pm}} > M_{H^\pm}$.
2. The percentage difference in cross sections for h^0h^0 and A^0A^0 production processes using the two PDFs can be summarized as follows:
   1. At $\lambda_4 = 0$ and $\lambda_4 = -1.82$, the difference in cross sections is approximately 4%.
   2. At $\lambda_4 = 1.82$, the difference in cross sections for h^0h^0 production increases significantly to 30%.
3. The $R = \frac{d\mathcal{L}_{\gamma\gamma}^{LO+2^{nd}}}{d\tau} / \frac{d\mathcal{L}_{\gamma\gamma}^{LO}}{d\tau}$ value in the range $0 \leq R \leq 30$ at 3 TeV is one numerical factor influencing the 30% difference observed at $\lambda_4 = 1.82$. Additionally, this emphasizes the necessity of including third-order correction terms for the Effective Photon Approximation (EPA).

Thank you!

Appendix A

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi \\ \delta \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta' & \sin\beta' \\ -\sin\beta' & \cos\beta' \end{pmatrix} \begin{pmatrix} \phi^\pm \\ \delta^\pm \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi \\ \eta \end{pmatrix}$$

Appendix B

$$v_{\phi}^2 = \frac{1}{1 + \frac{1}{2}\tan^2\beta}\frac{S_W^2 M_W^2}{\pi\alpha}$$

$$v_{\Delta}^2 = \frac{\tan^2\beta}{1 + \frac{1}{2}\tan^2\beta}\frac{S_W^2 M_W^2}{4\pi\alpha}$$

$$\mu = \frac{\sqrt{2}v_{\Phi}}{v_{\Phi}^2 + 4v_{\Delta}^2}M_A^2$$

$$\lambda = -\frac{2}{v_{\Phi}^2}(c_{\alpha}^2 M_h^2 + s_{\alpha}^2 M_H^2)$$

$$\lambda_1 = -\frac{2}{v_\Phi^2 + 4v_\Delta^2} M_A^2 + \frac{2}{v_\Phi^2 + 2v_\Delta^2} M_{H^\pm}^2 + \frac{\sin 2\alpha}{2 v_\Delta v_\Phi} (M_h^2 - M_H^2)$$

$$\lambda_2 = \frac{1}{2v_\Delta^2} \left(c_\alpha^2 M_h^2 + s_\alpha^2 M_H^2 + \frac{v_\Delta^2}{v_\Phi^2 + 4v_\Delta^2} M_A^2 - \frac{4v_\Delta^2}{v_\Phi^2 + 2v_\Delta^2} M_A^2 + 2M_{H^{\pm\pm}}^2 \right)$$

$$\lambda_3 = \frac{1}{v_\Delta^2} \left(-\frac{v_\Phi^2}{v_\Phi^2 + 4v_\Delta^2} M_A^2 + \frac{2v_\Phi^2}{v_\Phi^2 + 2v_\Delta^2} M_{H^\pm}^2 - M_{H^{\pm\pm}}^2 \right)$$

$$\lambda_4 = \frac{4}{v_\Phi^2 + 4v_\Delta^2} M_A^2 - \frac{4}{v_\Phi^2 + 2v_\Delta^2} M_{H^\pm}^2$$