The Standard Model Precision Parameters at 200 GeV

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INTRODUCTION

- Summarize our quantitative knowledge of SM in terms of the Lagrangian parameters
- $\overline{\text{MS}}$ renormalization scheme and Landau gauge
- `SMDR` (Standard Model in Dimensional Regularization)
- Interpolation formulas

WHAT IS SMDR?

- A public computer code library which does state-of-the-art calculations relating the $\overline{\text{MS}}$ inputs to on-shell observables.
- All known multi-loop contributions to the physical masses of the Higgs boson, the W and Z bosons, and the top quark, the fine structure constant and weak mixing angle, and the renormalization group equations and threshold matching relations for the gauge couplings, fermion masses, and Yukawa couplings.

SMDR can be downloaded from: [https://www.niu.edu/spmartin/SMDR/]

Ref: arXiv:1907.02500

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SMDR (Standard Model in Dimensional Regularization) v1.2

On-shell quantities read from "ReferenceModel.dat":

Mt = 172.500;
Mh = 125.250;
MZ = 91.18760;
alpha_S_5_MZ = 0.117900;
GFermi = 1.16637870 10^-5;
alpha = 1/137.03599908;
Delta_hadronic^(5) alpha(MZ) = 0.027660;
mb(mb) = 4.18000;       mc(mc) = 1.27000;
ms(2 GeV) = 0.09300;    mu(2 GeV) = 0.002160;   md(2 GeV) = 0.004670;
Mtau = 1.776860;        Mmuon = 0.10565837;     Melectron = 0.000510998946;
After fit by iteration:

MW = 80.332091;  (* pole mass *)
MW = 80.352476;  (* PDG convention *)

MSbar parameters:
Q = 200.000000;
gauge  couplings: g3 = 1.152514;      g = 0.646832;      gp = 0.358852;
Yukawa couplings: yt = 0.923778;     yb = 0.015335;    ytau = 0.01000655;
                  yc = 0.0033618;    ys = 0.00028886;   ymu = 0.000589088;
                  yu = 0.0000066738; yd = 0.000014505;   ye = 0.00000279634;
Higgs self-coupling lambda = 0.123533;
Higgs VEV = 246.216725;
Higgs mass^2 parameter m2 = -8672.606033 = -(93.126828)^2;
Tree-level vacuum energy Lambda = 123847259.871422 = (105.492504)^4;
Delta_hadronic^(5) alpha(MZ) = 0.027660

Total time: 113.95 seconds
```

LOOP CALCULATION OF M_h

The Higgs squared pole mass M_h can be obtained from

$$M_h^2 - i\Gamma_h M_h = 2\lambda v^2 + \frac{1}{16\pi^2}\Delta^{(1)}_{M_h^2} + \dots$$

$$\begin{aligned}\Delta^{(1)}_{M_h^2} &= 3y_t^2(4t-s)B(t,t) - 18\lambda^2 v^2 B(h,h)\\ &+\frac{1}{2}(g^2+g'^2)[(s-3Z-\frac{s^2}{4Z})B(Z,Z) - \frac{s}{2Z}A(Z) + 2Z]\\ &+g^2[(s-3W-\frac{s^2}{4W})B(W,W) - \frac{s}{2W}A(W) + 2W]\end{aligned}$$

SMDR does similar calculations at 2 loops and beyond.

WHY NOT JUST USE SMDR?

- Long Computational Times
- For instance, it took 6 hours (on the NICADD cluster) to produce 100 values to make a plot of λ vs M_t (shown later)
- Not having access to a machine that runs SMDR
- Learning Curve for SMDR
- Interpolation formulas give the "error budget" for the parameters. Know exactly how each input parameter affects each output parameter, as a number.

INTERPOLATION FORMULAS

- We evaluate the $\overline{\mathrm{MS}}$ Lagrangian parameters at 200 GeV
- 200 GeV is chosen since it is safely above M_t
- Can be scaled using renormalization group equations
- The parameters which will be our outputs include:

Higgs sector: λ, m^2,

gauge couplings: g_3, g, g',

quark Yukawa couplings: $y_t, y_b, y_c, y_s, y_u, y_d$,

lepton Yukawa couplings: y_τ, y_μ, y_e .

INTERPOLATION FORMULAS

The quantities that are in the most direct correspondence to the Lagrangian parameters and which will be our inputs include:

fine-structure constant: $\alpha = 1/137.035999084\ldots$ and $\Delta\alpha^{(5)}_{\text{had}}(M_Z)$

Fermi decay constant: G_F

5-quark QCD coupling: $\Delta\alpha^{(5)}_{\text{s}}(M_Z)$

heavy particle physical masses: M_t, M_h, M_z, M_W

running light quark masses:

$$m_b(m_b), m_c(m_c), m_s(2\text{GeV}), m_u(2\text{GeV}), m_d(2\text{GeV})$$

lepton pole masses: M_τ, M_μ, M_e

Values taken from RPP (Review of Particle Physics)

INTERPOLATION FORMULAS

We define a set of benchmark on-shell inputs:

$$\alpha_0 = 1/137.035999084, \qquad \Delta\alpha^{(5)}_{\text{had}}(M_Z)_0 = 0.027660,$$

$$G_{F0} = 1.1663787 \times 10^{-5}, \qquad \alpha^{(5)}_{S0}(M_Z) = 0.1179,$$

$$M_{t0} = 172.5\ \text{GeV}, \qquad M_{h0} = 125.25\ \text{GeV}, \qquad M_{Z0} = 91.1876\ \text{GeV},$$

$$m_b(m_b)_0 = 4.18\ \text{GeV}, \qquad m_c(m_c)_0 = 1.27\ \text{GeV}, \qquad m_s(2\ \text{GeV})_0 = 93\ \text{MeV},$$

$$m_u(2\ \text{GeV})_0 = 2.16\ \text{MeV}, \qquad m_d(2\ \text{GeV})_0 = 4.67\ \text{MeV},$$

$$M_{\tau 0} = 1.77686\ \text{GeV}, \qquad M_{\mu 0} = 0.1056583745\ \text{GeV},$$

$$M_{e0} = 0.5109989461\ \text{MeV}$$

Values taken from RPP (Review of Particle Physics)

INTERPOLATION FORMULAS

We next define the following dimensionless quantities:

$$\begin{aligned}
\delta_Z &= (M_Z - M_{Z0})/(0.001\ \mathrm{GeV}), \\
\delta_t &= (M_t - M_{t0})/(1\ \mathrm{GeV}), \\
\delta_h &= (M_h - M_{h0})/(0.1\ \mathrm{GeV}), \\
\delta_S &= 1000\left[\alpha_S^{(5)}(M_Z) - \alpha_{S0}^{(5)}(M_Z)\right], \\
\delta_a &= 10^4\left[\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z) - \Delta\alpha_{\mathrm{had},0}^{(5)}(M_Z)\right].
\end{aligned}$$

Most contribution.

INTERPOLATION FORMULAS

For our data set, we formed a grid of $N = 5^5 = 3125$ inputs by taking all possible combinations of 5 different values of the 5 on-shell experimental quantities, M_Z, M_t, M_h, $\alpha_S^{(5)}(M_Z)$ and $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ when it came to finding interpolation formulas for M_W, λ, m^2, g_3, g, g' and y_t.

The 5 different values were chosen by varying the values in δ_k by increments of 2.5 times the experimental uncertainty σ both in the positive and negative directions.

INTERPOLATION FORMULAS

When it came to the creating a data set for the other parameters, namely the Yukawa couplings of light quarks and leptons, we formed a grid $3^5 \times 5 = 1215$ inputs.

We took all possible combinations of 3 different values of the 5 on-shell experimental quantities and 5 different values of the respective masses.

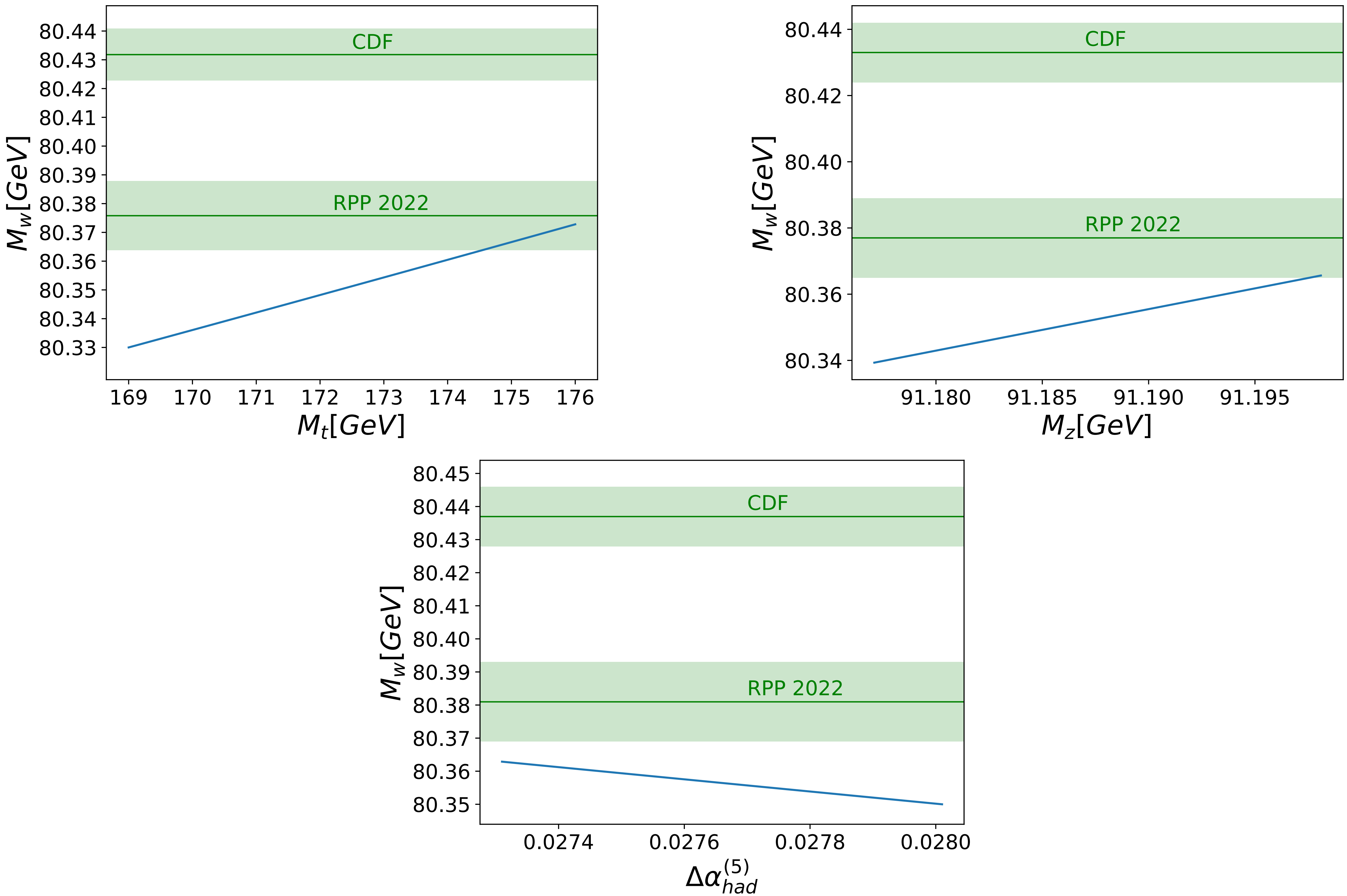
INTERPOLATION FORMULA FOR THE W-BOSON MASS

$$M_W = M_{W0}\left(1 + c^t_{M_W}\delta_t + c^Z_{M_W}\delta_Z + c^a_{M_W}\delta_a + c^S_{M_W}\delta_S + c^h_{M_W}\delta_h + c^{tt}_{M_W}\delta_t^2\right)$$

$$c^t_{M_W} = 7.61\times10^{-5},\qquad c^Z_{M_W} = 1.56\times10^{-5},\qquad c^a_{M_W} = -\,2.29\times10^{-5},$$
$$c^S_{M_W} = -\,8.8\times10^{-6},\qquad c^h_{M_W} = -\,5.9\times10^{-7},\qquad c^{tt}_{M_W} = 1.3\times10^{-7}\,.$$

Reproduces the results of SMDR to better than 0.1 MeV, which is much smaller than the current theoretical and experimental uncertainties

INTERPOLATION FORMULA FOR THE W-BOSON MASS



INTERPOLATION FORMULA FOR λ

$$\begin{aligned}\lambda \;=\; & \lambda_0\left[1 + c_\lambda^h\delta_h + c_\lambda^t\delta_t + c_\lambda^Z\delta_Z + c_\lambda^S\delta_S + c_\lambda^a\delta_a + c_\lambda^{tt}\delta_t^2 + c_\lambda^{tS}\delta_t\delta_S + c_\lambda^{hh}\delta_h^2\right.\\ & \left.+c_\lambda^{ht}\delta_h\delta_t + c_\lambda^{SS}\delta_S^2 + c_\lambda^{hS}\delta_h\delta_S + c_\lambda^{ttt}\,\delta_t^3 + c_\lambda^{ttS}\,\delta_t^2\delta_S + c_\lambda^b\,\Delta_b + c_\lambda^{G_F}\,\Delta_{G_F}\right]\end{aligned}$$

$$\Delta_f = \frac{m_f}{m_{f0}} - 1$$

Where, f = b, c, s, u, d, τ, μ, e

$$\Delta_{G_F} = \frac{G_F}{G_{F0}} - 1$$

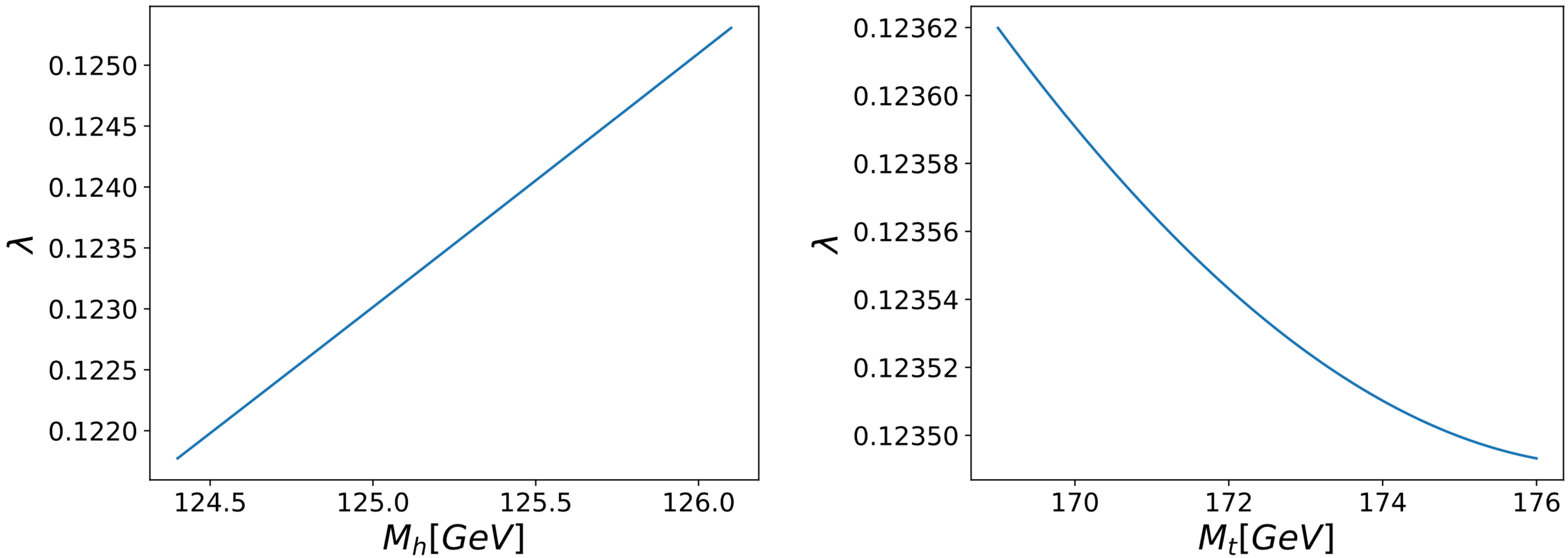
INTERPOLATION FORMULA FOR λ

$$\lambda \;=\; \lambda_0\Big[1 + c_\lambda^h\delta_h + c_\lambda^t\delta_t + c_\lambda^Z\delta_Z + c_\lambda^S\delta_S + c_\lambda^a\delta_a + c_\lambda^{tt}\delta_t^2 + c_\lambda^{tS}\delta_t\delta_S + c_\lambda^{hh}\delta_h^2$$
$$+c_\lambda^{ht}\delta_h\delta_t + c_\lambda^{SS}\delta_S^2 + c_\lambda^{hS}\delta_h\delta_S + c_\lambda^{ttt}\,\delta_t^3 + c_\lambda^{ttS}\,\delta_t^2\delta_S + c_\lambda^b\,\Delta_b + c_\lambda^{G_F}\,\Delta_{G_F}\Big]$$

$$c_\lambda^h = 1.6823\times10^{-3}, \qquad c_\lambda^t = -1.488\times10^{-4}, \qquad c_\lambda^Z = -3.5\times10^{-7},$$
$$c_\lambda^S = -2.2\times10^{-7}, \qquad c_\lambda^a = 3.4\times10^{-7}, \qquad c_\lambda^{tt} = 1.528\times10^{-5},$$
$$c_\lambda^{tS} = -4.02\times10^{-6}, \qquad c_\lambda^{hh} = 7.0\times10^{-7}, \qquad c_\lambda^{ht} = -6.1\times10^{-7},$$
$$c_\lambda^{SS} = 3.0\times10^{-7}, \qquad c_\lambda^{hS} = 6.4\times10^{-8}, \qquad c_\lambda^{ttt} = 1.9\times10^{-7},$$
$$c_\lambda^{ttS} = -7.6\times10^{-8}, \qquad c_\lambda^b = 4.5\times10^{-5}, \qquad c_\lambda^{G_F} = 0.95.$$

Agrees to better than 10^{-6} fractional precision in λ

INTERPOLATION FORMULA FOR λ



(a) λ vs M_h

(b) λ vs M_t

The time taken by SMDR to produce 100 values for the λ vs M_t plot = 6.3 hours

INTERPOLATION FORMULA FOR g_3

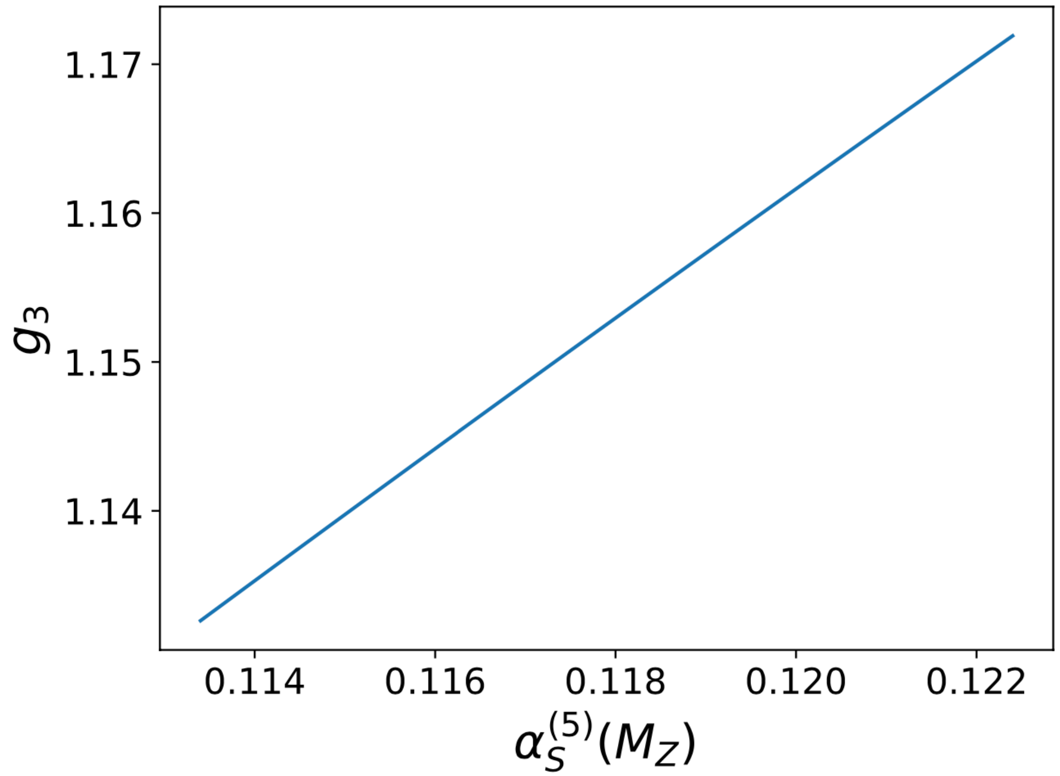
$$g_3 = g_{30}\left(1 + c_{g_3}^{S}\delta_S + c_{g_3}^{t}\delta_t + c_{g_3}^{SS}\delta_S^2 + c_{g_3}^{h}\delta_h + c_{g_3}^{Z}\delta_Z + c_{g_3}^{a}\delta_a\right)$$

$$c_{g_3}^{S} = 3.7875\times 10^{-3}, \qquad c_{g_3}^{t} = -3.98\times 10^{-5}, \qquad c_{g_3}^{SS} = -1.07\times 10^{-5},$$

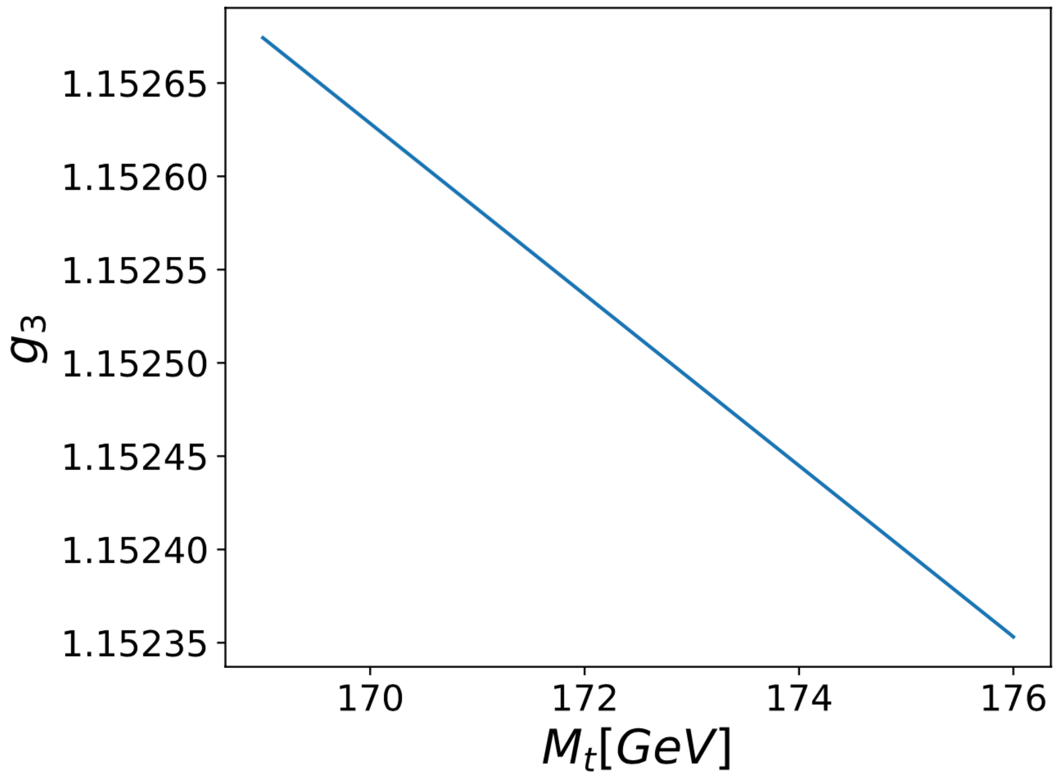
$$c_{g_3}^{h} = 2.5\times 10^{-8}, \qquad c_{g_3}^{Z} = 2.7\times 10^{-9}, \qquad c_{g_3}^{a} = -2.0\times 10^{-9}.$$

Agrees to better than 10^{-5} fractional precision

INTERPOLATION FORMULA FOR g_3



(a) g_3 vs $\alpha_S^{(5)}(M_Z)$



(b) g_3 vs M_t

CONCLUSION

- Help satisfy basic curiosity about the fundamental parameters of the Standard Model
- Help in matching to various candidate ultra-violet completions of the Standard Model,
- The results also can be viewed as providing the parametric error budget for the defining couplings of the Standard Model Lagrangian.

THANK YOU!