Factorization (and Fragmentation)

Muon Collider Benchmarks Workshop

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0. Factorization?
1. Outline of factorization in an unbroken gauge theory
2. A "broken gauge theory": what happens when your beam is all one color?
3. Just a comment on fragmentation

   Questions, corrections, discussion

One view of how concepts underlying factorization perturbative QCD can be thought of at as applying at a very high-energy lepton collider. Quite frankly based on an incomplete consultation of the literature, & non-expert interpretation of what I've seen so far. Discussion (& correction) is very much welcome. What's correct below has surely appeared in the literature.

0. Factorization?

- **The template: At the LHC (and before):**

$$\sigma_{pp\to Q}(Q) \;=\; \sum_{i,j=u,d,s\ldots G} \int dx_1 dx_2\ f_{i/P}(x_1,\mu)\ \hat\sigma_{ij\to Q}(x_1p_1, x_2p_s,\mu)\ f_{j/P}(x_2,\mu)$$

  where $f_{i/P}(x_1,\mu)$ is not perturbatively calculable, but evolves in a calculable way:

$$\frac{d}{d\ln(\mu)} f_{i/P}(x_1,\mu) \;=\; P_{ik}\otimes f_{k/P}(x_1,\mu)$$

  which enables us to extrapolate in energy.

- **This factorization also holds with incoming, color-averaged partonic reactions ($p \to u, d, s \ldots G$), which is how we compute $\hat\sigma$, using dimensional or other regularization. (Not QCD, but has the same short-distance structure. Actually, close to the topic at hand . . .)**

- **What about at a lepton collider? We ought to have:**

$$\sigma_{l\bar l\to Q}(Q) \;=\; \sum_{i,j=u,d,s\ldots G} \int dx_1 dx_2\ f_{i/l}(x_1,\mu)\ \hat\sigma_{ij\to Q}(x_1p_1, x_2p_s,\mu)\ f_{j/\bar l}(x_2,\mu)$$

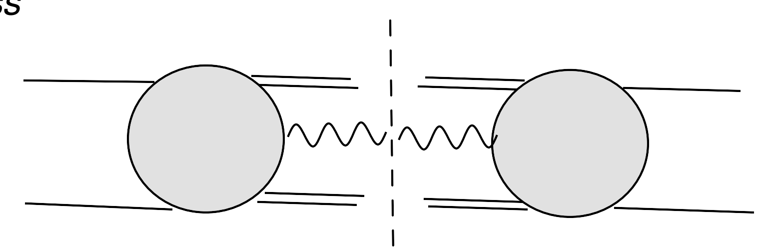
  where now we can compute everything in principle! We know our initial state really well, and EW theory is perturbative in the Standard Model.

- **But it's natural to ask, when we calculate in EW theory, will it really have this form? Let's go down this road, starting with a QCD-like example.**

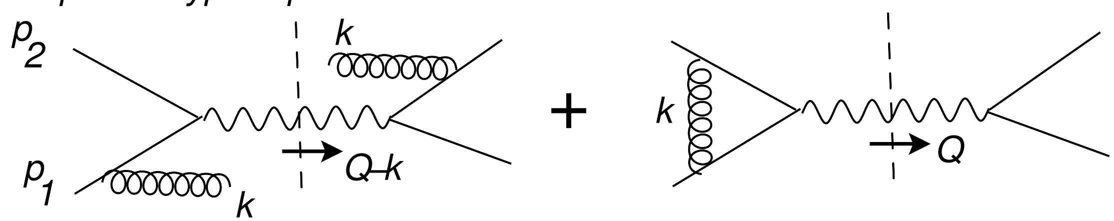
1. Outline of factorization in an unbroken gauge theory

- **Suppose we had an unbroken $SU(2)$ Dirac-gauge theory, with massless "quarks" of two colors. The quarks have EM charge, not part of the gauge group.**
- **How would factorization go? Say for $q\bar{q} \to \gamma^*(Q^2) + X$ ("Drell Yan-like")**
- **Strategy (to be illustrated next slide):**
  - **Locality and analyticity: separate loop momentum and phase space into separate regions that give long-distance behavior in an arbitrary diagram. Call each such region a "part", defined using a mass scale. A single part includes a number of final states. Work on each part and then combine them.**
- **Unitarity: Sum over diagrams that have the same "parts" : cancels "final state" singularities.**
- **Causality and gauge invariance: use gauge invariance to isolate soft-collinear-hard subprocesses.**
- **Causality and gauge invariance: Cancel soft so only collinear-hard is left.**
- **Combine different "parts" to get the whole: factorized cross section**
- **Demand independence of the mass scale used to define the "parts": evolution.**

The process



Simple but typical parts

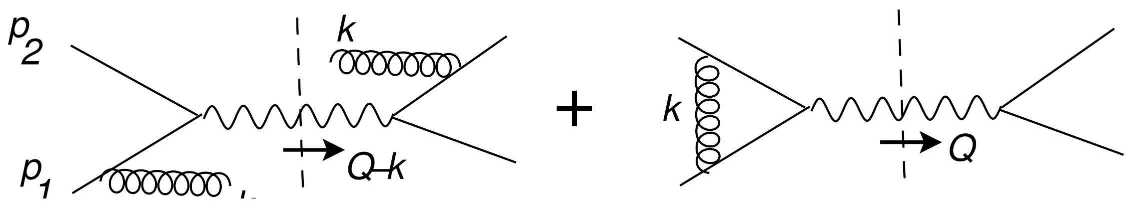


k may be collinear to p_1 or p_2 or soft.

All three parts appear in both diagrams

- **Take** $p_1^\mu = Q\delta_{\mu+}$, $p_2^\mu = Q\delta_{\mu-}$, **so** $p_1^2 = p_2^2 = 0$, **but suppose** $m^2_{\text{vector}} = m^2$.
- **Let's concentrate on** k **parallel to** p_1**:** $k^+ > k_\perp$, $Q \gg k^-$. **Also require** $k_\perp < \mu$ **to "define the part".**

- **The two integrals are almost identical:**



$$\text{Real} \;+\; \text{Virtual} \;=\; Q^2 \int \frac{d^4k}{(2\pi)^4}\,(2\pi)\delta\left(2k^+k^- - k_\perp^2 - m^2\right)\,\theta(Q-k^-)\,\theta(k^+-k_\perp)\,\theta(\mu-k_\perp)$$
$$\times\left(\frac{1}{2k^-(k^+-Q)-k_\perp^2}\right)\left(\frac{1}{-2Qk^+}\right)\left(\frac{1}{2Q(Q-k^+)} - \frac{1}{Q^2}\right)$$
$$\approx \frac{Q^2}{(2\pi)^2}\int_{m^2}^{\mu^2}\frac{dk_\perp^2}{k_\perp^2+m^2}\int_{k_\perp}^{Q}\frac{dk^+}{2k^+}\left(\frac{1}{2Q(Q-k^+)} - \frac{1}{Q^2}\right)$$

- **Or, defining $x = k^+/Q$, this is just like a parton distribution convoluted with a smooth function ($1/x$, in this simpified case).**

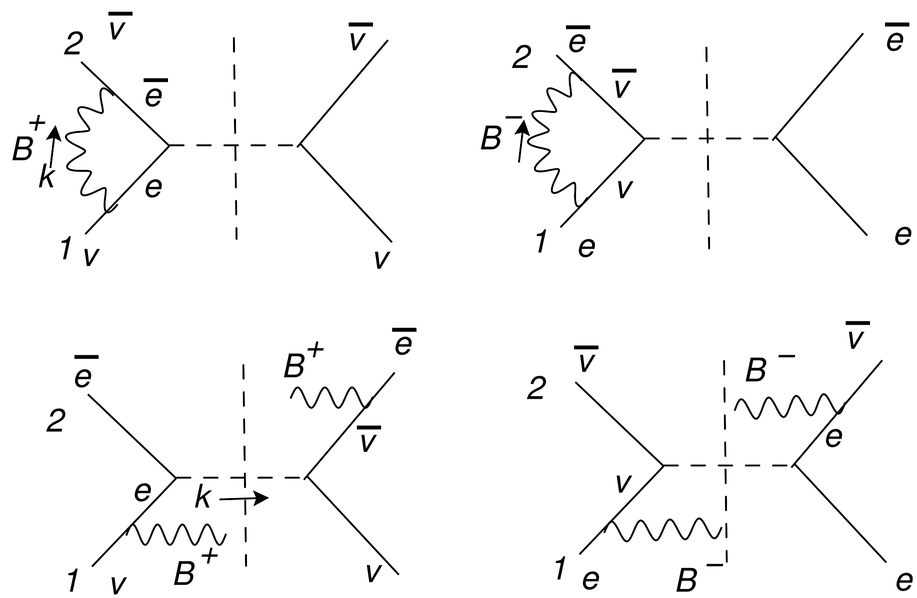
$$\text{Real} \;+\; \text{Virtual} \;\approx\; \ln\frac{\mu^2}{m^2}\int\frac{dx}{[1-x]_+}\,\frac{1}{x}$$

- **Other "parts" follow suit, and can be reassembled into the whole factorized cross section. $d/d\mu \to 0$ on the cross section implies evolution.**

- **Call our SU(2) gauge field B, and our 2-color "quark field", L:**

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}$$

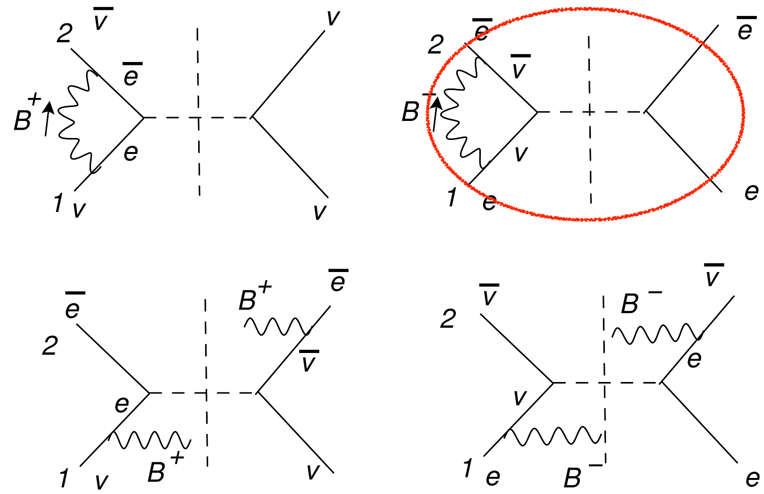
- **In the cross section, we always average over our "colors" ν and e, and get four terms**



- **Cancellations occur between top left, bottom left and top right, bottom right to get "parton distriutions" as above in the k collinear to p_1 region. Nice!**
- **This is how factorization works. Then evolution follows from independence of μ.**

2. Broken gauge theories: what happens when your beam is all one color?

- **But suppose, we could only build a beam of "e-quarks". We'd only have one of the processes.**



- **From this one diagram, we get just the "virtual" integral from the k collinear to p_1 region, which doesn't have the cancellation we found before:**

$$\text{Virtual alone} \approx -\frac{1}{(2\pi)^2}\int_{m^2}^{\mu^2}\frac{dk_\perp^2}{k_\perp^2+m^2}\int_{k_\perp}^{Q}\frac{dk^+}{2k^+}$$

$$\approx -\frac{1}{2(2\pi)^2}\,\ln\frac{\mu^2}{m^2}\,\ln\frac{Q^2}{m^2}$$

- **A typical "Sudakov" double log – we haven't allowed our "e-quark" to radiate, because in the opposing "$\bar{e}$-quark" beam, this radiation can't be absorbed – you need a "$\bar{\nu}$-quark" beam!**

- **What to do? Very schematically ...**

- **Add the $\bar{\nu}$-quark beam, and then subtract it. And while we're at it, add a ν-quark beam, and then subtract that too.**

- **In other words, rewrite the $e\bar{e}$ beam is a fully symmetric cross section plus cross sections cross sections in which there are one or two antisymmetric components.**

- **Like this ...**

$$\sigma_{e\bar{e}} \;=\; \frac{1}{4}\left(\sigma_{(e+\nu)(\bar{e}+\bar{\nu})} \;+\; \sigma_{(e-\nu)(\bar{e}+\bar{\nu})} \;+\; \sigma_{(e+\nu)(\bar{e}-\bar{\nu})} \;+\; \sigma_{(e-\nu)(\bar{e}-\bar{\nu})}\right)$$

- **The first term is "the unbroken theory" and factorizes into parton distributions that evolve according to familiar DGLAP equations.**

- **The other three always involve a difference rather than a sum of real and virtual:**

$$\text{Virtual} \;-\text{Real} \;\approx\; -2\times\frac{1}{2(2\pi)^2}\;\ln\frac{\mu^2}{m^2}\,\ln\frac{Q^2}{m^2}$$

- **Terms like $\sigma_{(e-\nu)(\bar{e}-\bar{\nu})}$ should all be suppressed by exponentials of double logs at large Q, but at finite $Q \not\gg v$ provide the approach to asymptotic behavior.**

- **The factorization procedure outlined at the start can be applied to the "antisymmetric" terms.**

- **I expect an analogy to Q_T-resummation factorization in impact parameter space, but with $1/m_B$ replacing the impact parameter.**

- **I believe these effects are built into more practical treatments: Ciafaloni, Ciafaloni and Comelli (hep-ph/0505047; Bauer, Ferland & Webber (1703.08562), Han, Ma & Xi (2007.14300) . . .**

3. Just a few comments on fragmentation

- **Once organized as above, inclusive sums over final states should result in "standard" jet and fragmentation functions.**

- **In particular, fragmentation functions will have the same "universality" properties if defined as single-particle inclusive**

- **With observed "colors" in the final state, however, ν vs. e, etc., some of the features familiar from parton distributions should recur, as noted in Chien & Li (1801.00395) Baumgarten, Erdogan, Rothstein & Vaidya (1811.04120).**

Questions, corrections or discussion? . . .