



Precision physics at a muon collider

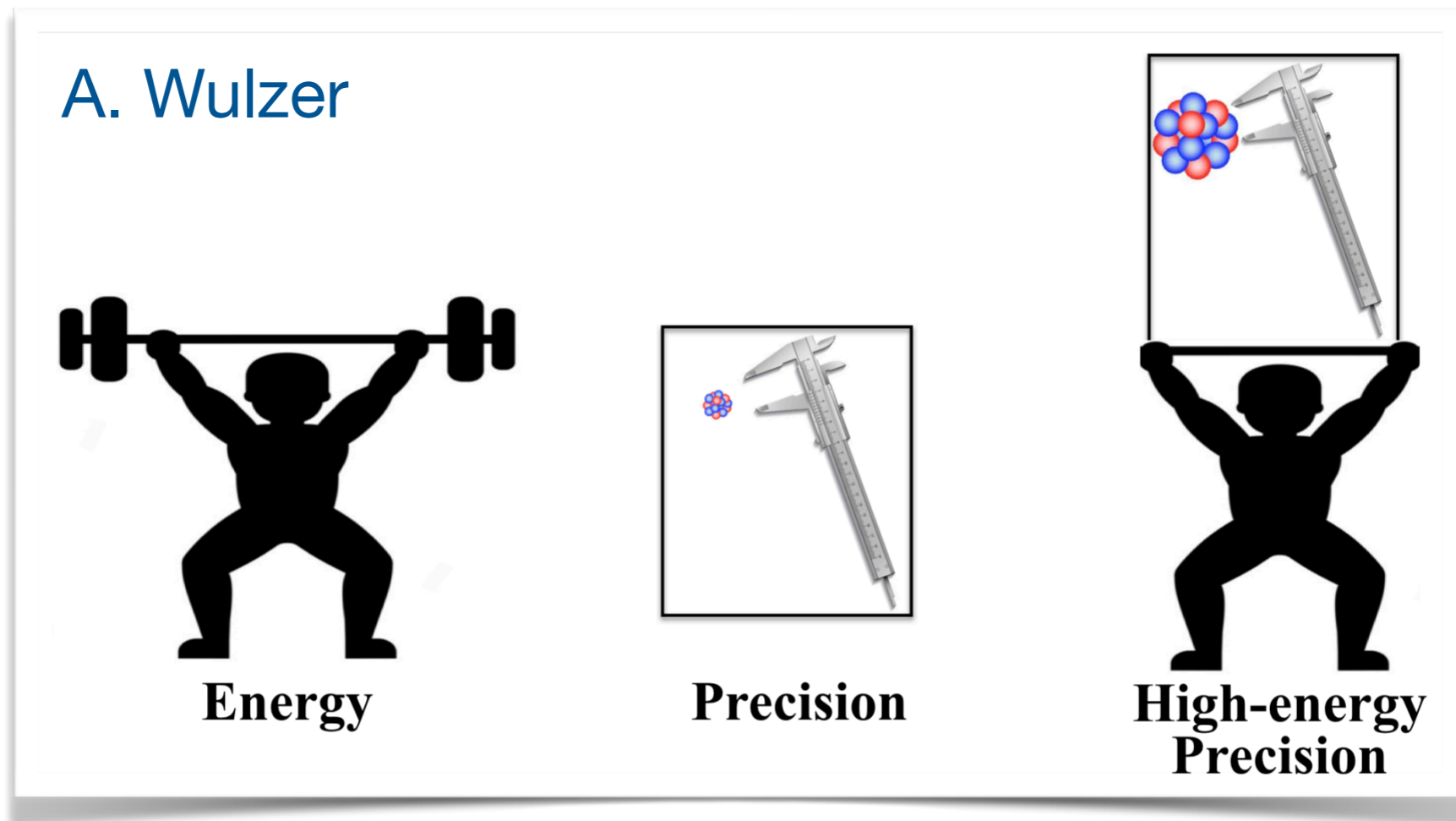
Flavor, Higgs, $g-2$, etc...

Dario Buttazzo

Energy AND Precision

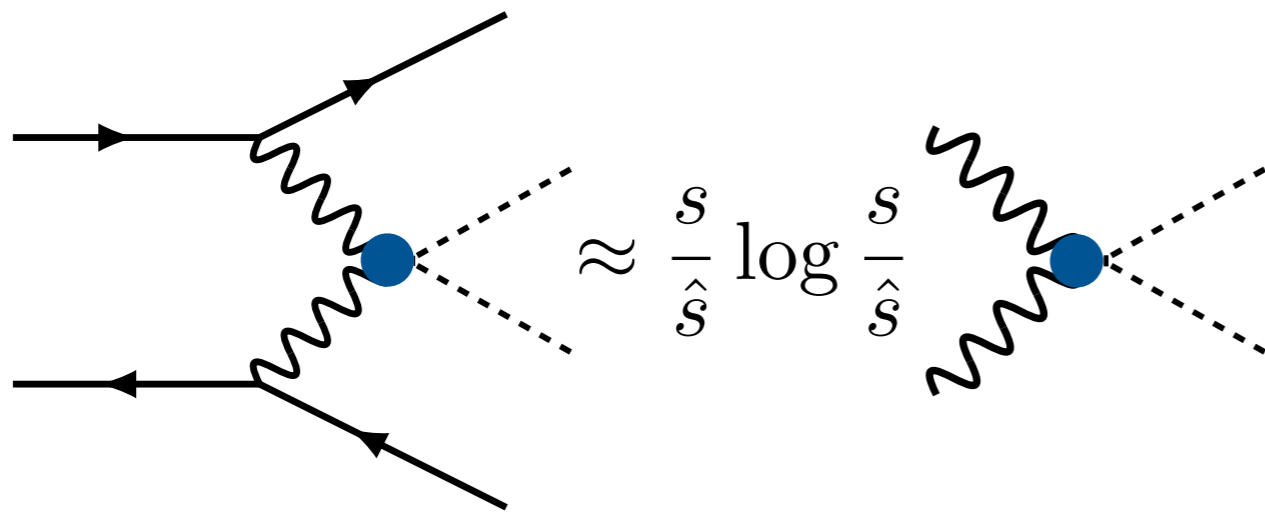
A muon collider is an amazing discovery machine:
production of EW particles up to several TeV of mass...

... but also a tool for precision measurements



Two paths to precision

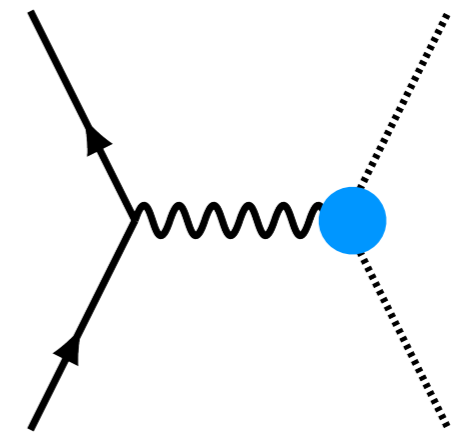
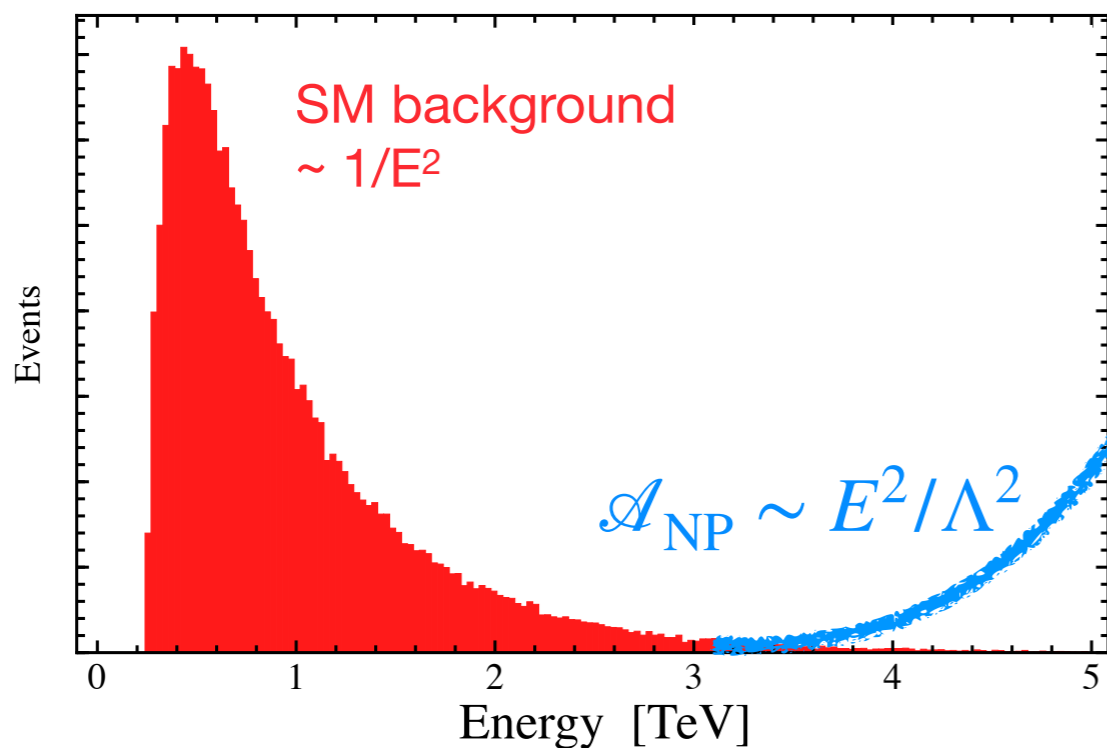
- High rate: more events = better precision



For “soft” SM final state $\hat{s} \sim m_{EW}^2$
cross-section is enhanced

$$\sigma_{SM} \sim \log(s/m_{EW}^2) / m_{EW}^2$$

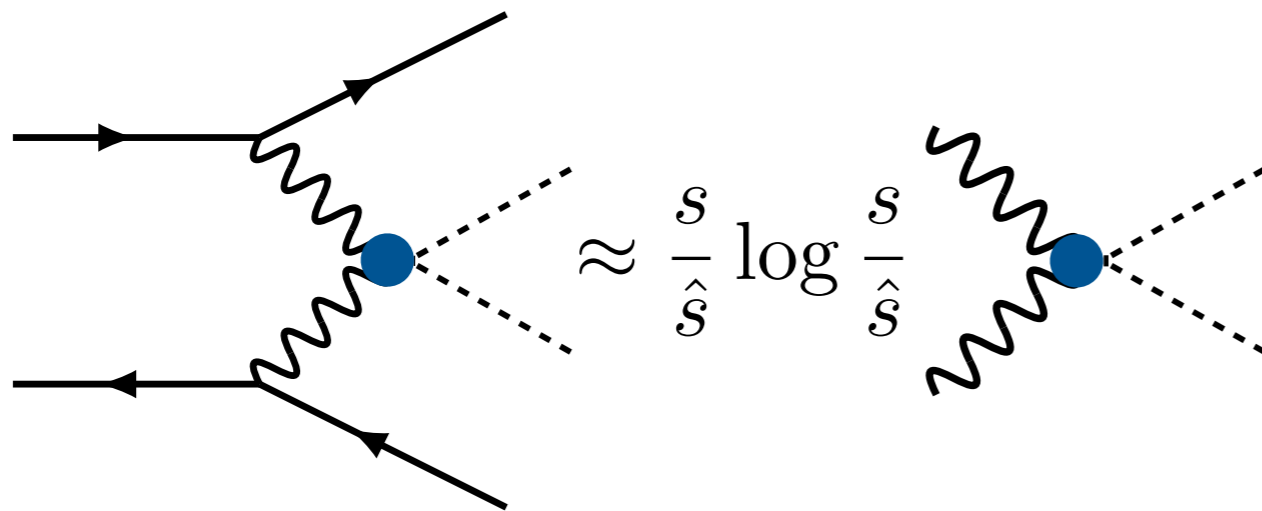
- High energy: new physics effects grow



Hard scattering $\sigma_{SM} \sim 1/s$

High rate probes

- High rate: more events = better precision



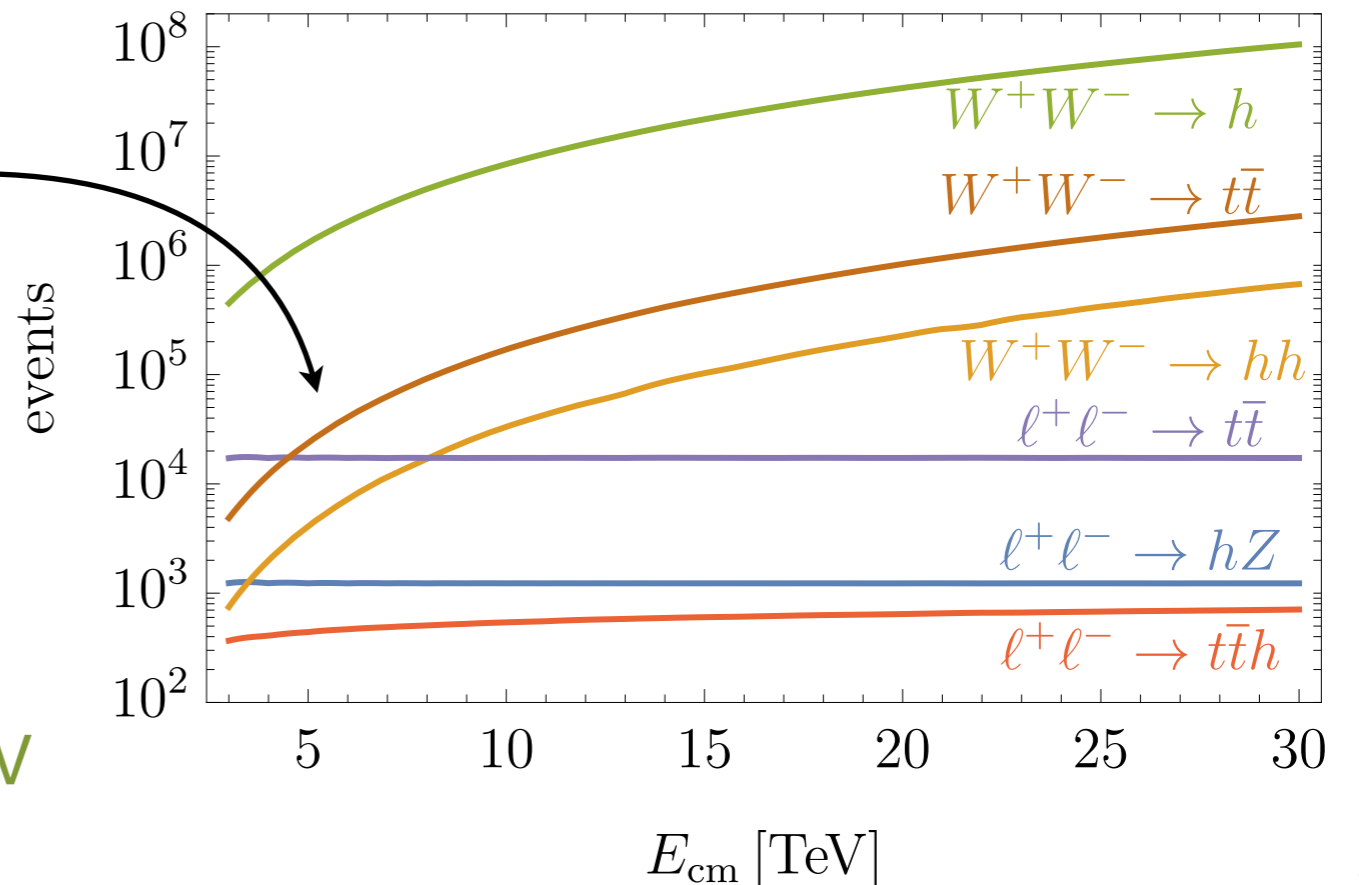
A High Energy Lepton Collider is a “vector boson collider”

For “soft” SM final state $\hat{s} \sim m_{EW}^2$ cross-section is enhanced

Dawson 1985

Above few TeV the VBF cross-section dominates over the hard $2 \rightarrow 2$

- Huge single Higgs rate in vector-boson-fusion: 10^7 - 10^8 Higgs bosons at 10-30 TeV



High rate probes: Higgs physics

A 10+ TeV muon collider is a perfect Higgs factory!



◆ Signal-only estimate: $\sim 10^7$ Higgses at 10 TeV + efficiencies, BR

➔ rough estimate: 10^{-3} for dominant decay channels @ 10 TeV

(as a comparison: 1.7×10^7 Z bosons @ LEP)

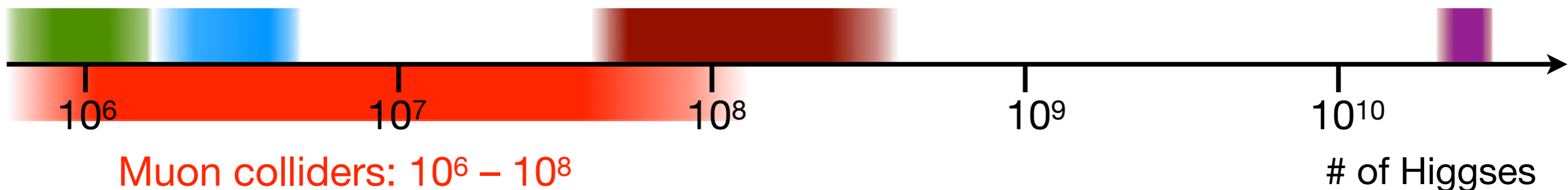
Low energy
 e^+e^- factories
(FCC-ee, CEPC,
ILC, CLIC380)

TeV-scale
 e^+e^- factories
(CLIC, ILC1000)

LHC: few $\times 10^7$

HL-LHC: few $\times 10^8$

FCC-hh:
few $\times 10^{10}$



High rate probes: Higgs physics

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“Muon smasher guide” 2103.14043

Forslund, Meade 2203.09425

κ -0 fit	HL-LHC	ILC			CLIC			CEPC	FCC-ee		FCC-ee/ eh/hh	$\mu^+\mu^-$ 10000
		250	500	1000	380	1500	3000		240	365		
κ_W [%]	1.7	1.8	0.29	0.24	0.86	0.16	0.11	1.3	1.3	0.43	0.14	0.06
κ_Z [%]	1.5	0.29	0.23	0.22	0.5	0.26	0.23	0.14	0.20	0.17	0.12	0.23
κ_g [%]	2.3	2.3	0.97	0.66	2.5	1.3	0.9	1.5	1.7	1.0	0.49	0.15
κ_γ [%]	1.9	6.7	3.4	1.9	98*	5.0	2.2	3.7	4.7	3.9	0.29	0.64
$\kappa_{Z\gamma}$ [%]	10.	99*	86*	85*	120*	15	6.9	8.2	81*	75*	0.69	1.0
κ_c [%]	—	2.5	1.3	0.9	4.3	1.8	1.4	2.2	1.8	1.3	0.95	0.89
κ_t [%]	3.3	—	6.9	1.6	—	—	2.7	—	—	—	1.0	6.0
κ_b [%]	3.6	1.8	0.58	0.48	1.9	0.46	0.37	1.2	1.3	0.67	0.43	0.16
κ_μ [%]	4.6	15	9.4	6.2	320*	13	5.8	8.9	10	8.9	0.41	2.0
κ_τ [%]	1.9	1.9	0.70	0.57	3.0	1.3	0.88	1.3	1.4	0.73	0.44	0.31

dominant channels
~ other Higgs
factories

rare modes
much better
(~ hadron collider)

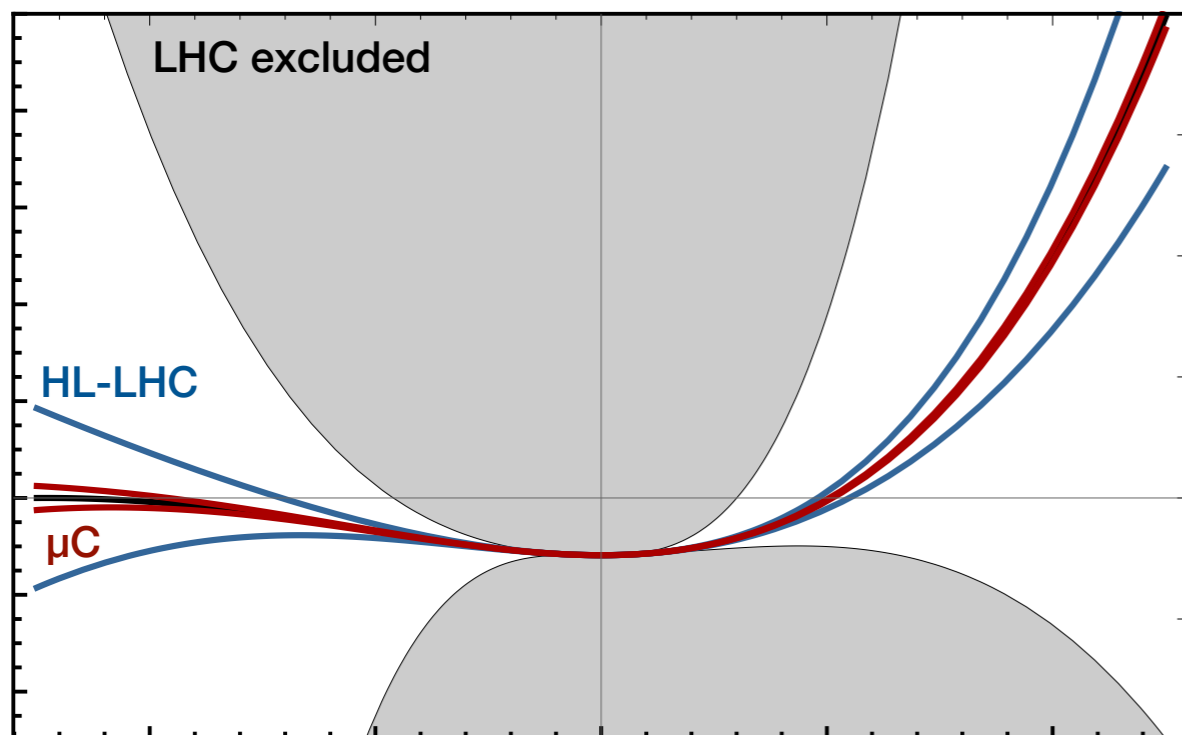
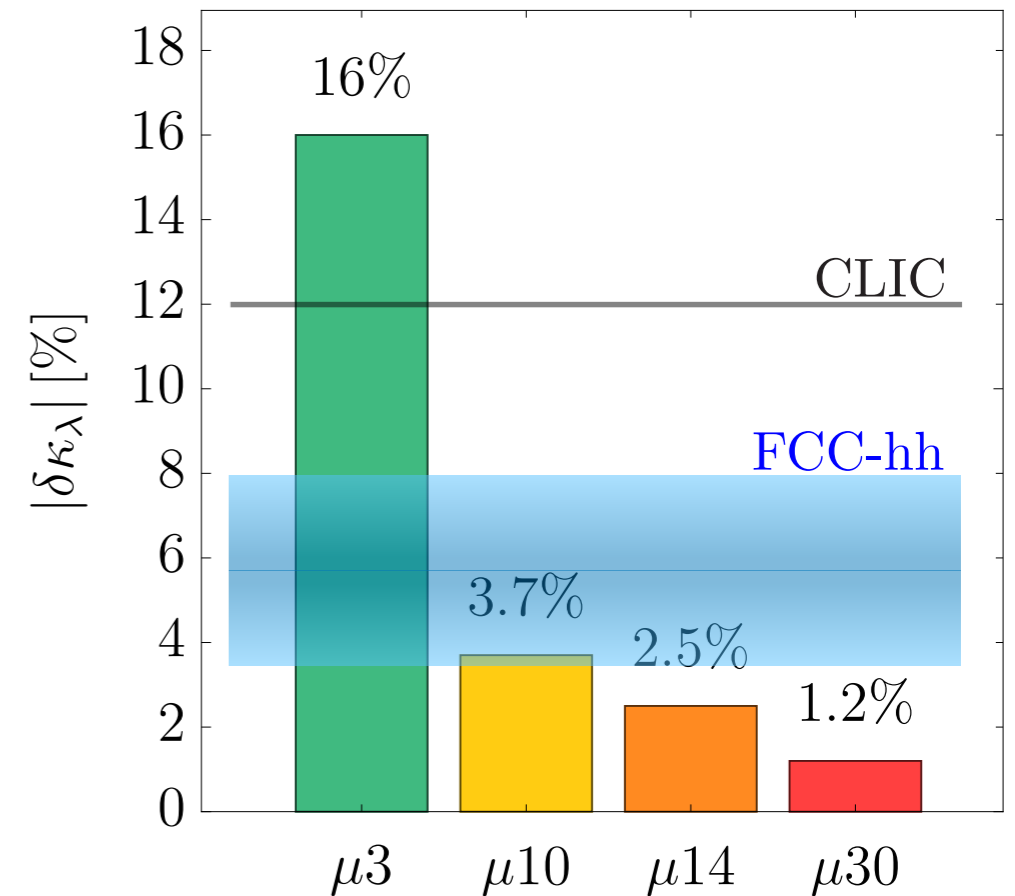
Double Higgs production

- Large double Higgs VBF rate: Higgs trilinear coupling from $hh \rightarrow 4b$

E [TeV]	\mathcal{L} [ab ⁻¹]	N_{rec}	$\delta\kappa_3$
3	5	170	~ 10%
10	10	620	~ 4%
14	20	1340	~ 2.5%
30	90	6'300	~ 1.2%

B, Franceschini, Wulzer 2012.11555,

Han et al. 2008.12204, Costantini et al. 2005.10289



credits: Craig, Petrossian-Byrne

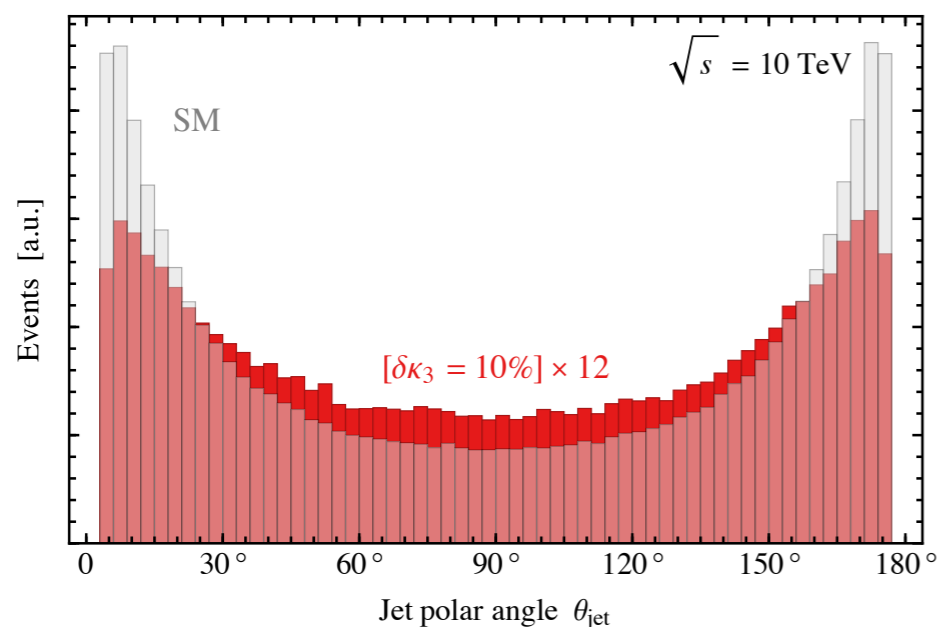
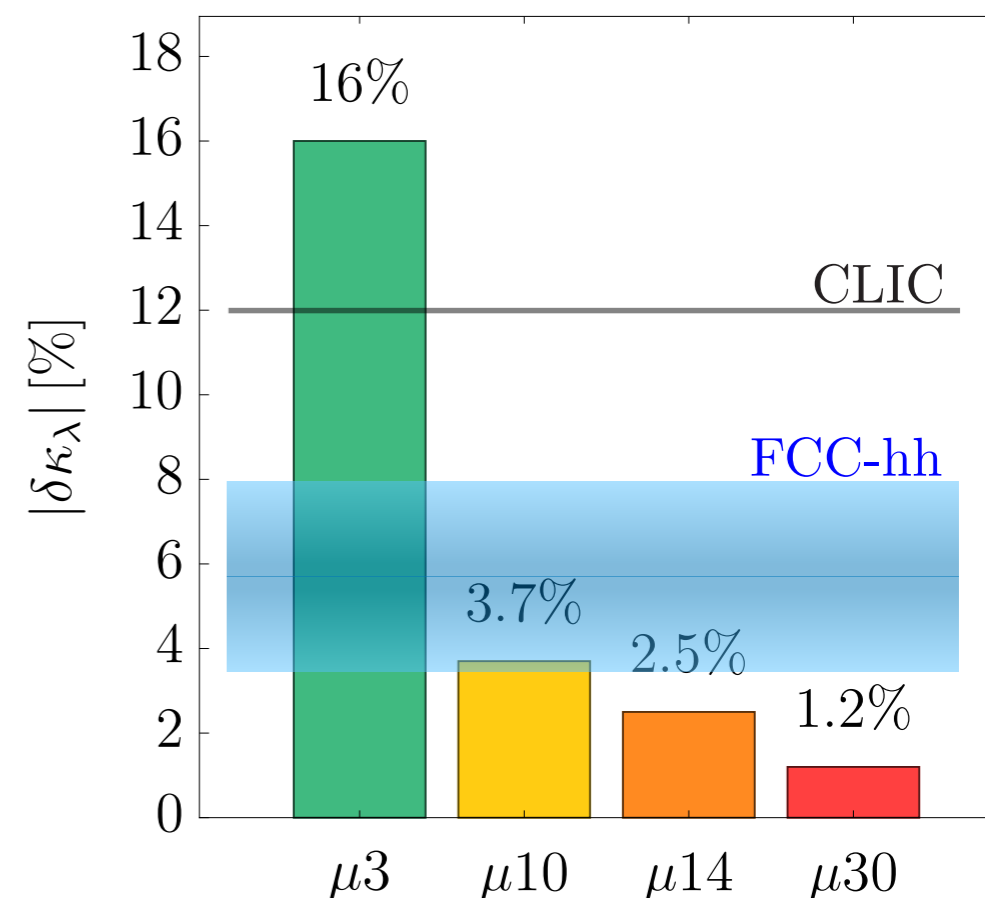
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- Weak dependence on angular acceptance (signal is in the central region)
- Some dependence on detector resolution (to remove backgrounds)

B, Franceschini, Wulzer 2012.11555

see also CLIC study 1901.05897

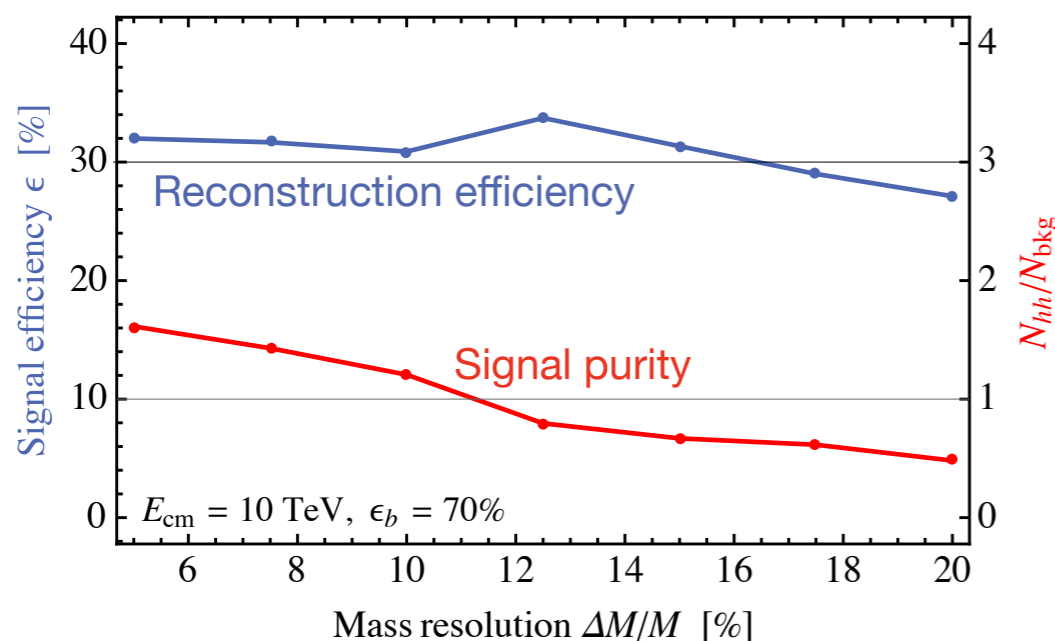
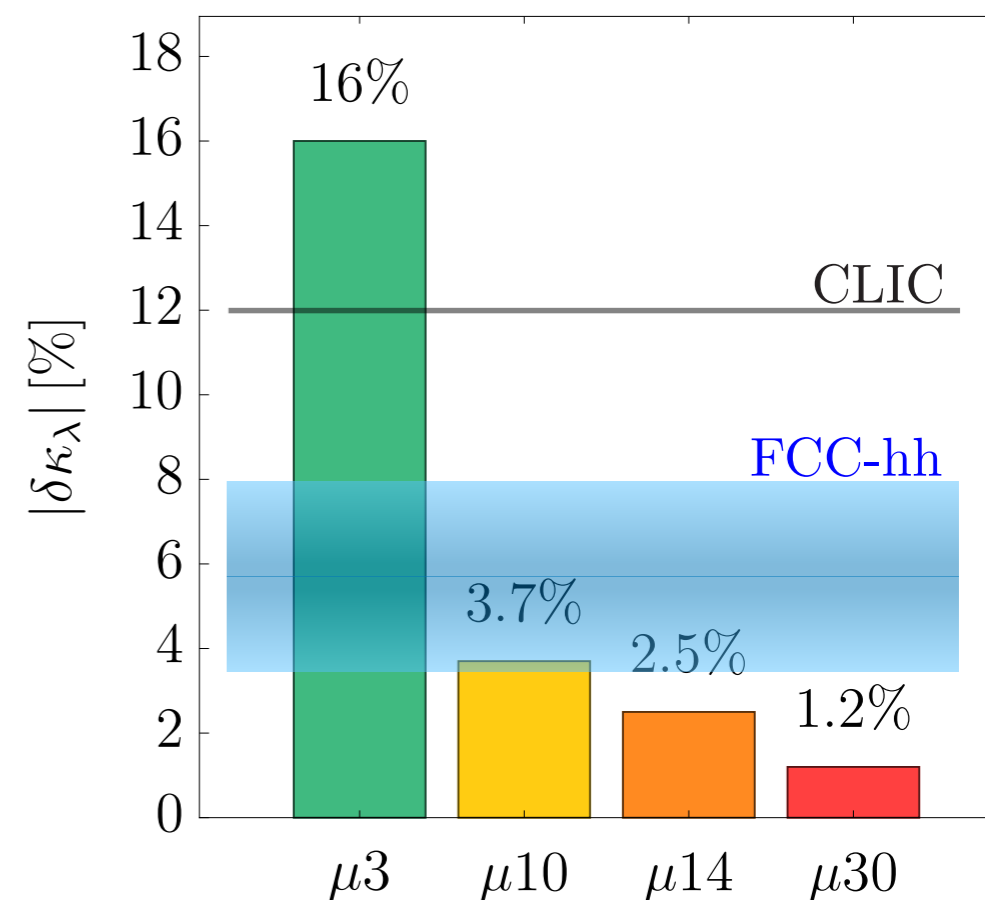
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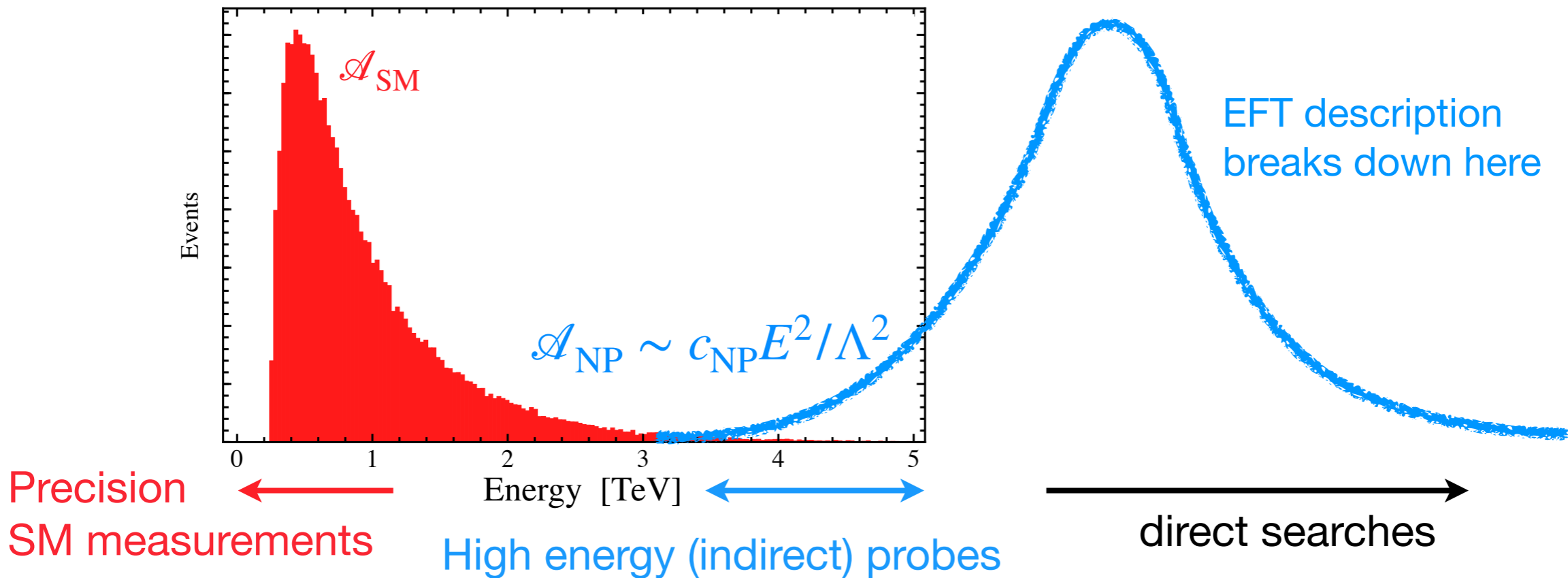
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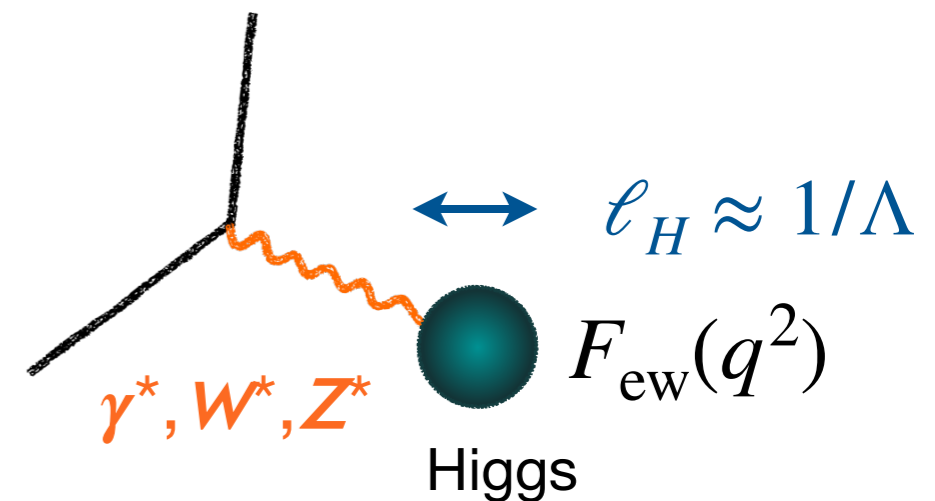
High-energy probes

- NP effects are more important at high energies

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \sum C_i \mathcal{O}_i$$



$$\frac{\Delta\sigma(E)}{\sigma_{\text{SM}}(E)} \propto \frac{E^2}{\Lambda_{\text{BSM}}^2} \approx \begin{cases} 10^{-2}, & E \sim 10 \text{ TeV} \\ 10^{-6}, & E \sim 100 \text{ GeV} \end{cases}$$



- EW high-energy probes: μC can test scales of 100 TeV or more!

Example: Double Higgs at high mass

◆ Di-Higgs production is affected by two operators:

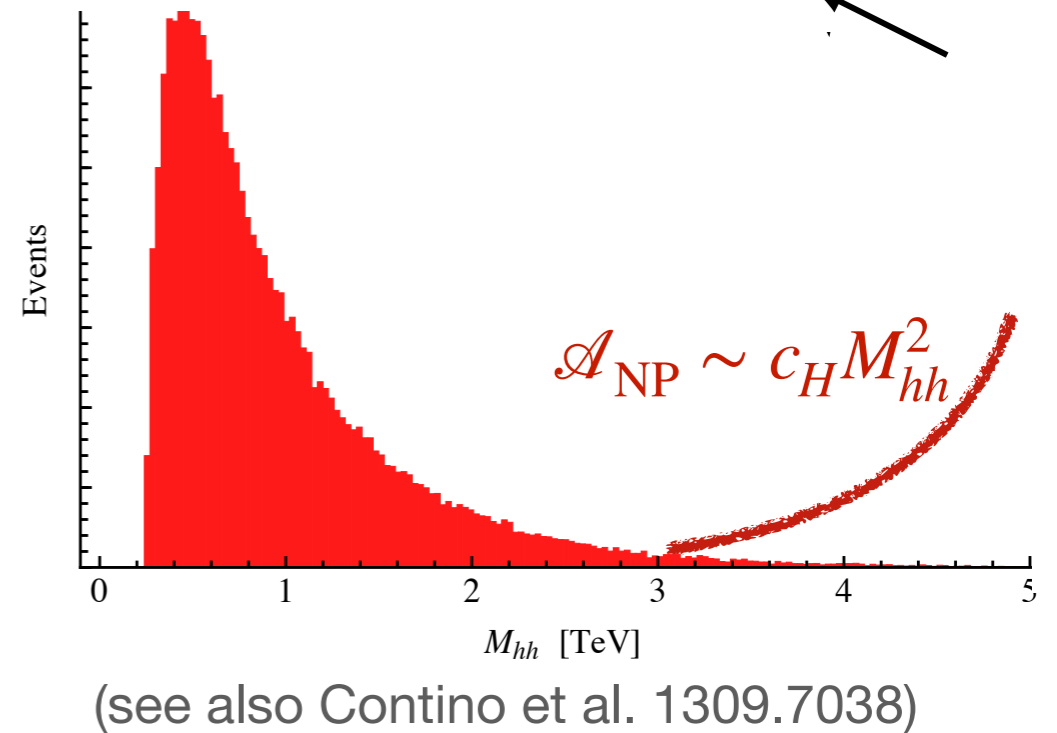
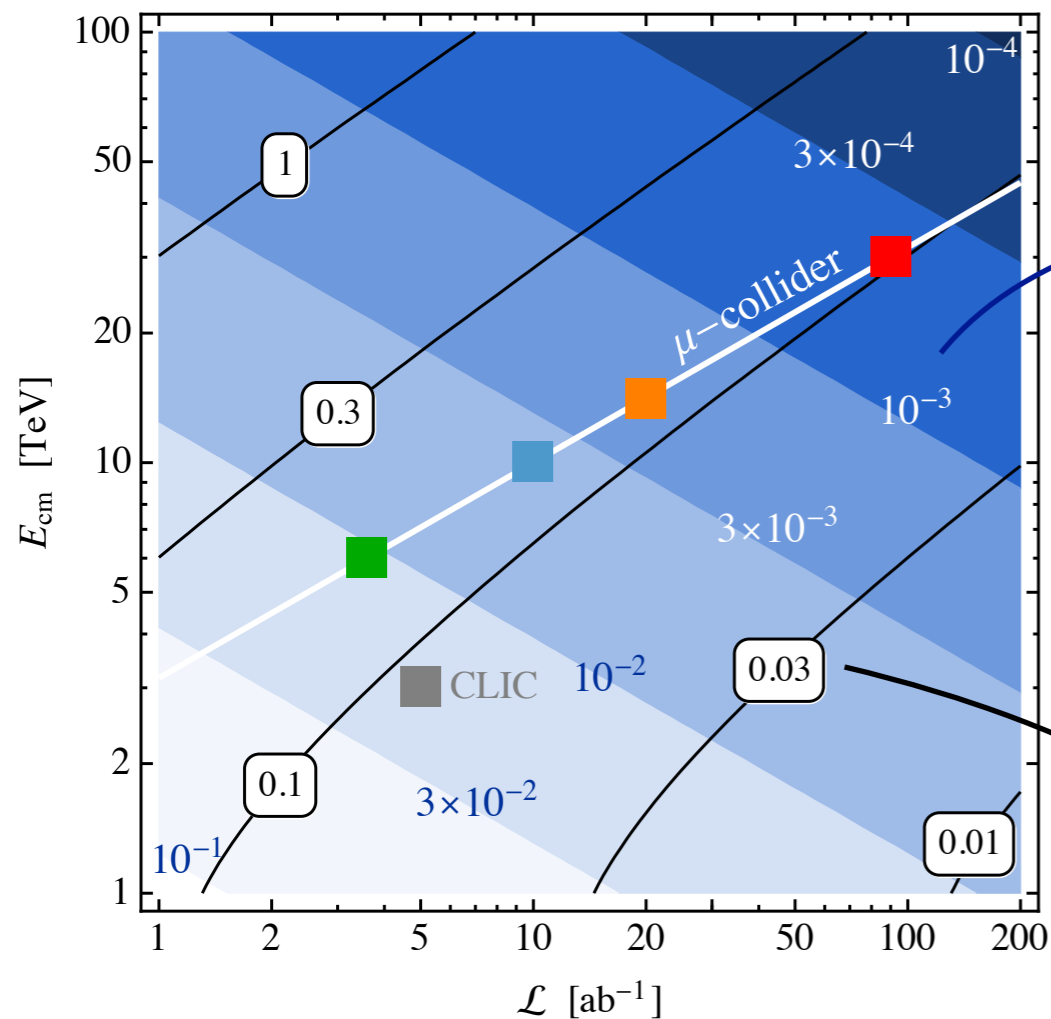
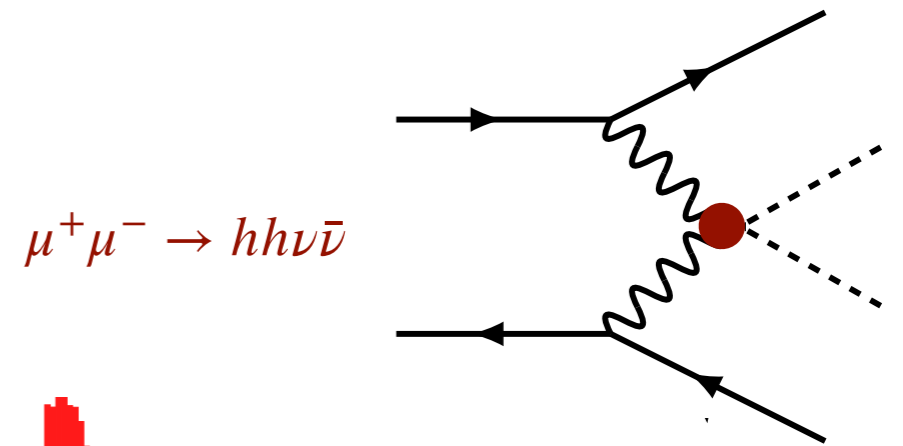
$$\mathcal{O}_6 = -\lambda |H|^6$$

◆ NP contribution from \mathcal{O}_H grows as E^2 :

$$\mathcal{O}_H = \frac{1}{2} (\partial_\mu |H|^2)^2$$

high mass tail gives a *direct* measurement of C_H

High-energy $WW \rightarrow hh$ more sensitive than Higgs pole physics at energies $\gtrsim 10$ TeV



S/B low-precision measurement

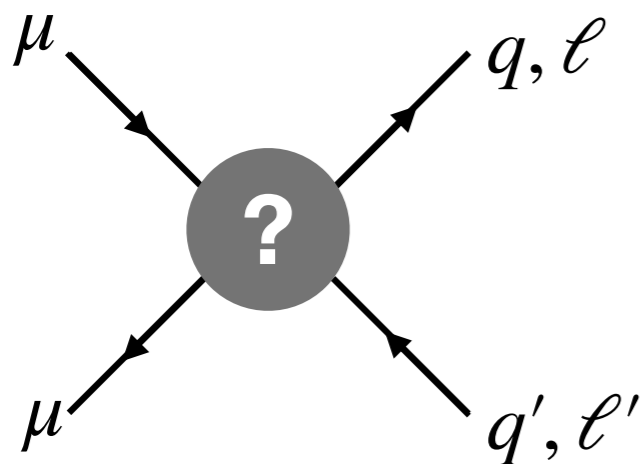
Flavour: muons vs. electrons

- ◆ New Physics (especially if related to the Higgs sector) could distinguish the different families of fermions.
- ◆ EW interactions are flavour-universal: an accidental property of the gauge lagrangian, *not* a fundamental symmetry of nature!
 - ▶ Example: Yukawa couplings, the only non-gauge interactions in the SM, violate flavour universality maximally!

$$m_u \sim \begin{pmatrix} \cdot & \cdot & \bullet \end{pmatrix}$$

$$m_d \sim \begin{pmatrix} \cdot & \cdot & \bullet \end{pmatrix}$$

$$m_\ell \sim \begin{pmatrix} \cdot & \cdot & \bullet \end{pmatrix}$$



A muon collider collides 2nd generation particles:

- ◆ can have access to flavor processes than cannot be efficiently probed elsewhere
- ◆ could test flavour structure

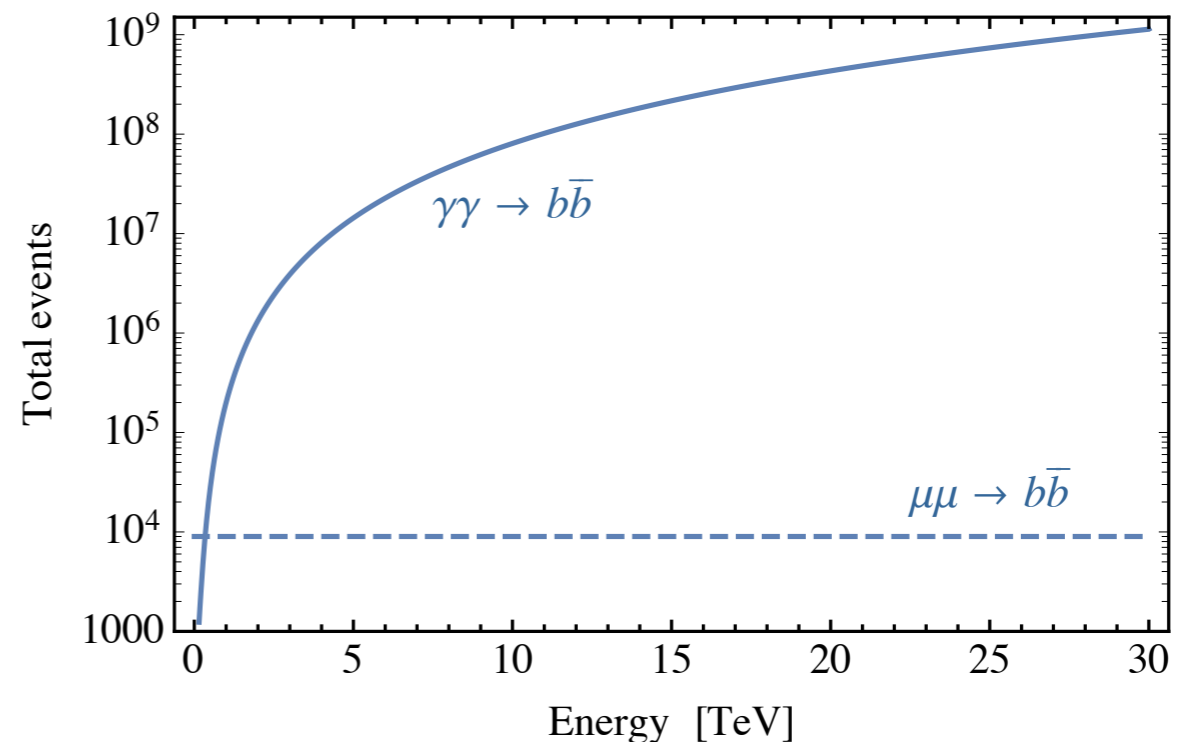
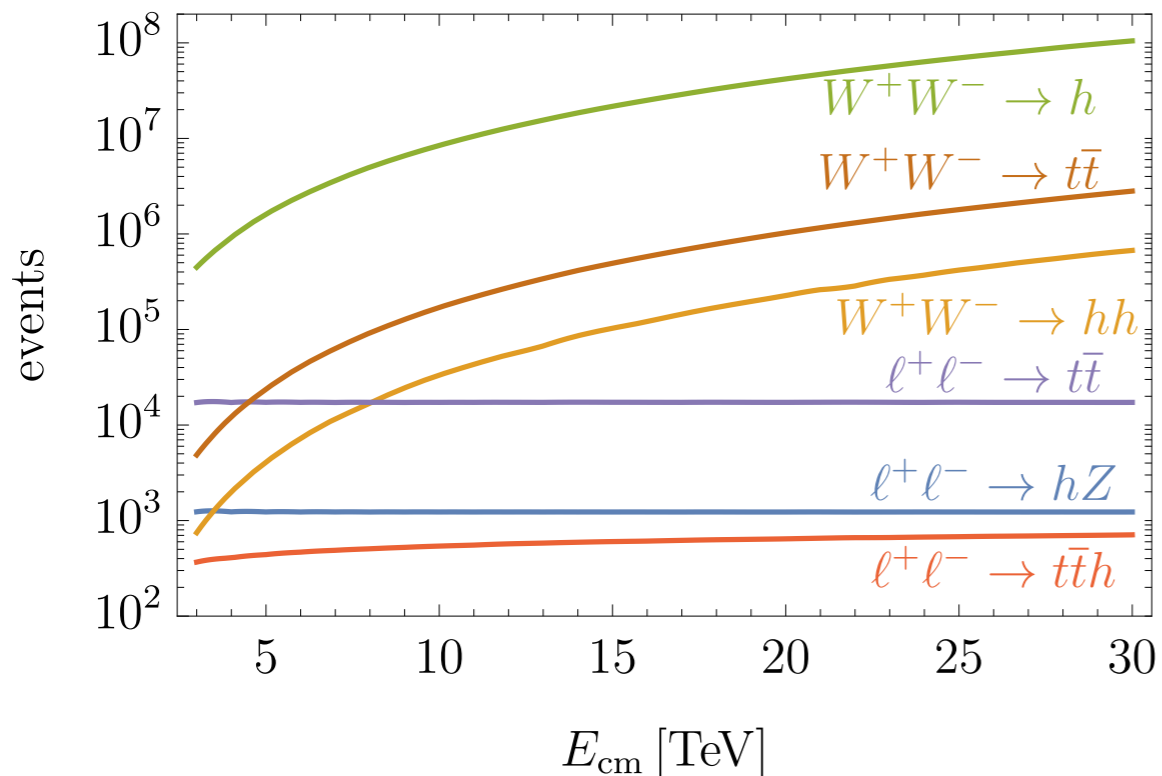
Flavor and precision

- ♦ Flavor processes: rare decays & tiny effects

$$\text{BR}(B_s \rightarrow \mu\mu) \sim 10^{-9}, \quad \text{BR}(\tau \rightarrow 3\mu) \lesssim 10^{-8}, \quad \Delta a_\mu \approx 10^{-9}$$

➔ need billions of events, usually probed by means of high-intensity experiments

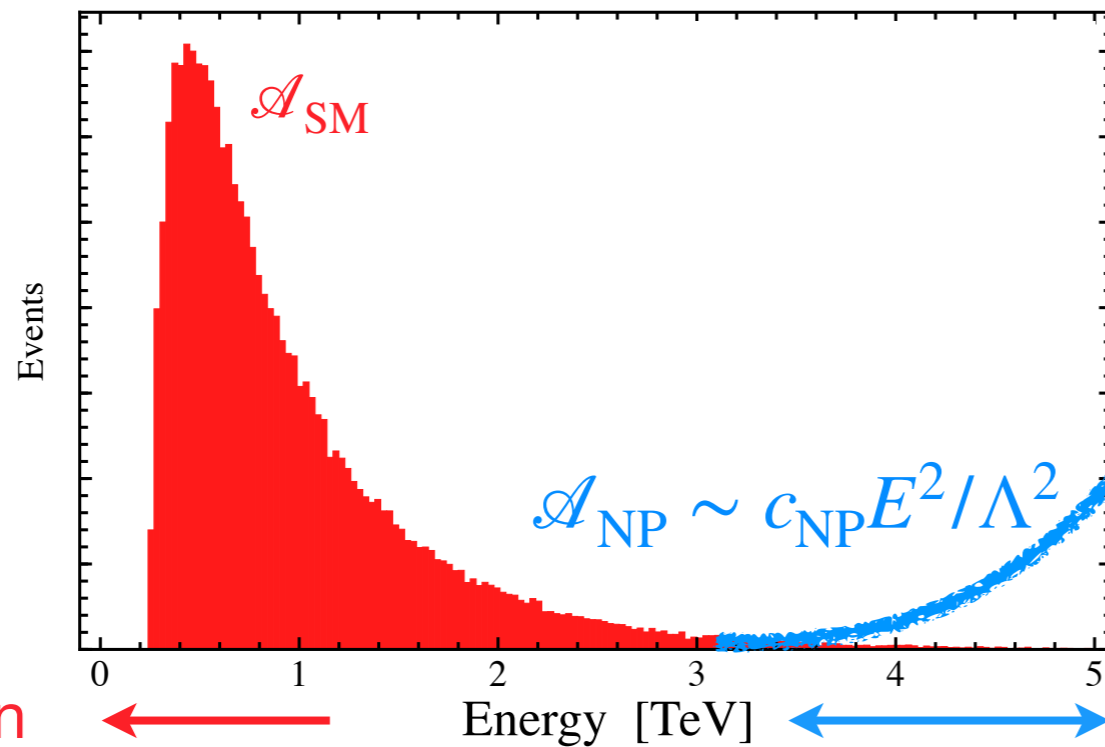
- ♦ Muon-collider: very large number of (clean) EW particles, but overall event rate not comparable to flavor factories



High-energy probes

- NP effects are more important at high energies

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \sum C_i \mathcal{O}_i$$



EFT description
breaks down here

Precision
SM measurements

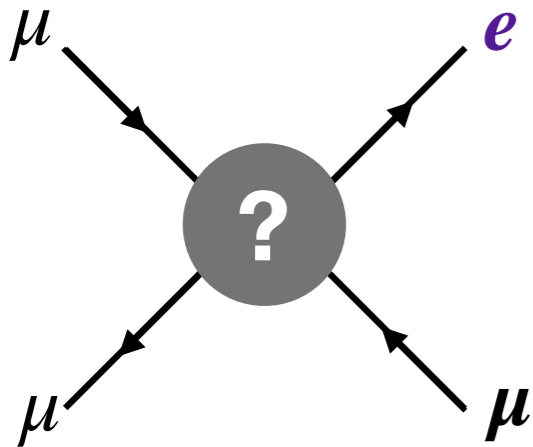
High energy (indirect) probes

direct searches

$$\frac{\Delta\sigma(E)}{\sigma_{\text{SM}}(E)} \propto \frac{E^2}{\Lambda_{\text{BSM}}^2} \approx \begin{cases} 10^{-2}, & E \sim 10 \text{ TeV} \\ 10^{-6}, & E \sim 100 \text{ GeV} \\ 10^{-10}, & E \sim 1 \text{ GeV} \end{cases}$$

- very powerful at a μ -collider with 10's of TeV
- taken to the extreme for flavor processes: gain can be as large as $(E/m_\mu)^2$

Lepton flavor violation



Four-fermion interactions: muon current coupled to flavor-violating bilinear

$$\frac{c_{e3\mu}}{\Lambda^2} (\bar{e}_{L,R} \Gamma \mu_{L,R}) (\bar{\mu}_{L,R} \Gamma \mu_{L,R})$$

- ◆ Charged lepton flavor violating processes: $\tau \rightarrow \ell \gamma$, $\mu \rightarrow e \gamma$, $\mu \rightarrow 3e$
some of the strongest bounds on (generic) BSM interactions

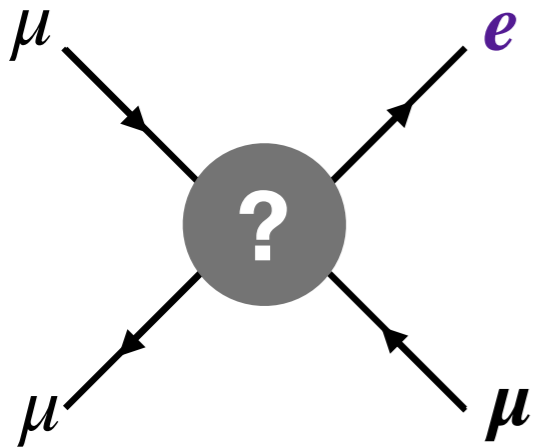
$$\text{BR}(\mu \rightarrow e \gamma) \lesssim 4 \times 10^{-13}$$

$$\text{BR}(\mu \rightarrow 3e) \lesssim 10^{-12}$$

$$\Lambda \gtrsim 10^6 \text{ TeV} \times \frac{\alpha}{4\pi}$$

not in reach of any collider experiment 😞

Lepton flavor violation



Four-fermion interactions: muon current coupled to flavor-violating bilinear

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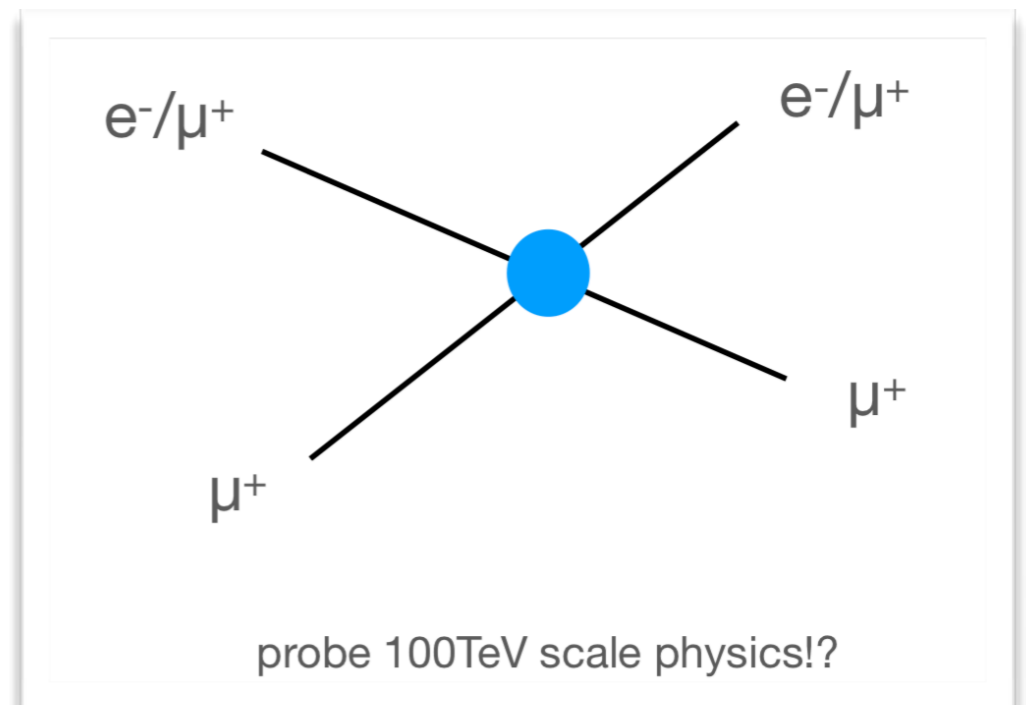
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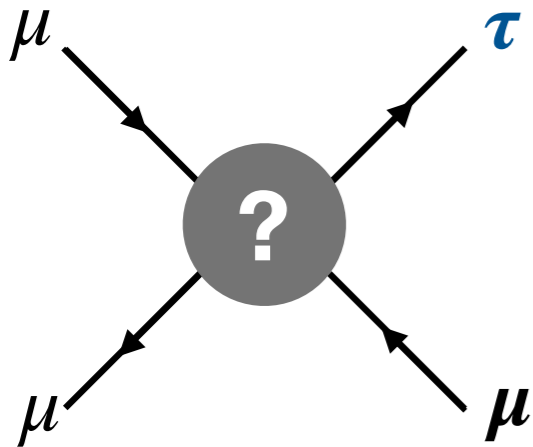
$$\Lambda \gtrsim 10^6 \text{ TeV} \times \frac{\alpha}{4\pi} \times \text{small}$$

can probe larger set of operators than $\mu \rightarrow e \gamma$, $\mu \rightarrow 3e$? 🤔



👉 talk by Ryuichiro

Lepton flavor violation

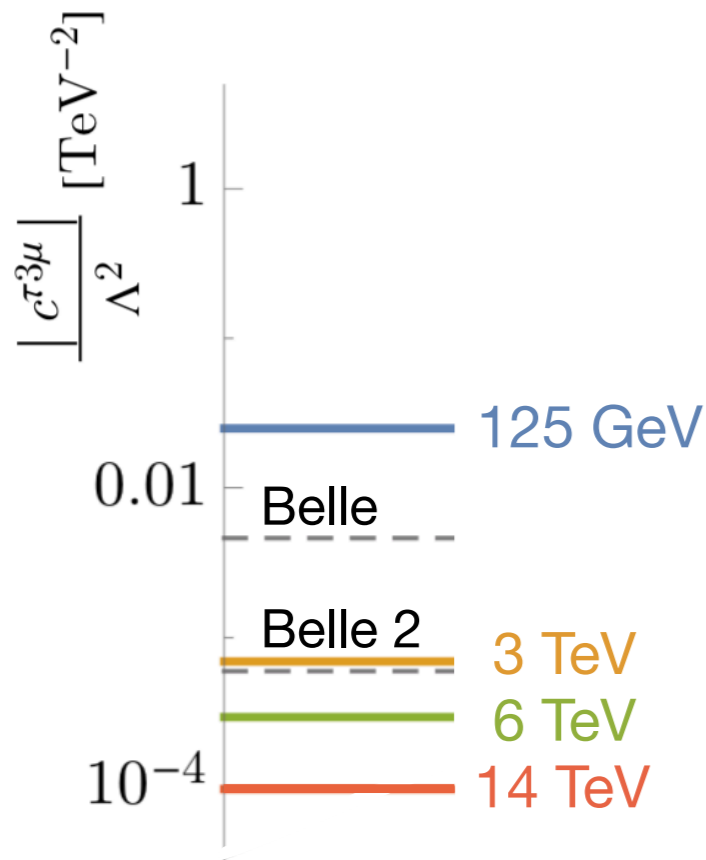


Four-fermion interactions: muon current coupled to flavor-violating bilinear

$$\frac{c_{\tau 3\mu}}{\Lambda^2} (\bar{\tau}_{L,R} \gamma^\rho \mu_{L,R}) (\bar{\mu}_{L,R} \gamma_\rho \mu_{L,R})$$

☞ talk by Sam yesterday!

- Charged lepton flavor violating processes: $\tau \rightarrow \ell \gamma$, $\mu \rightarrow e \gamma$, $\mu \rightarrow 3e$
some of the strongest bounds on (generic) BSM interactions



- 3rd generation less severely constrained:
 $\tau \rightarrow 3\mu$ constrains NP scale $\Lambda > 15$ TeV [Belle]

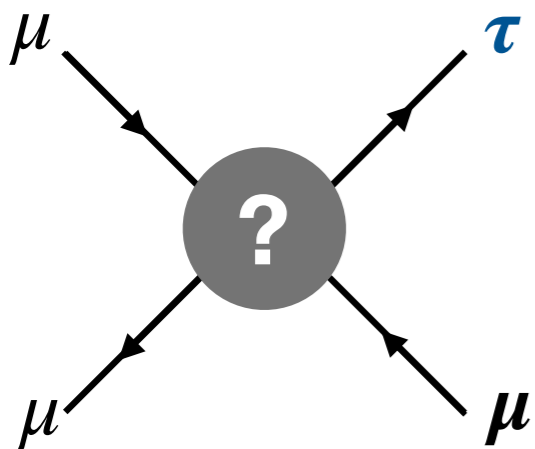
$$\text{BR}(\tau \rightarrow 3\mu) \sim \frac{m_W^4}{\Lambda^4} \quad \sigma(\mu\bar{\mu} \rightarrow \tau\bar{\mu}) \sim \frac{E^2}{\Lambda^4}$$

already at 3 TeV the same sensitivity as Belle II,

$\Lambda > 40$ TeV

“Muon smasher guide” 2103.14043
Homiller, Lu, Reece 2203.08825

Lepton flavor violation

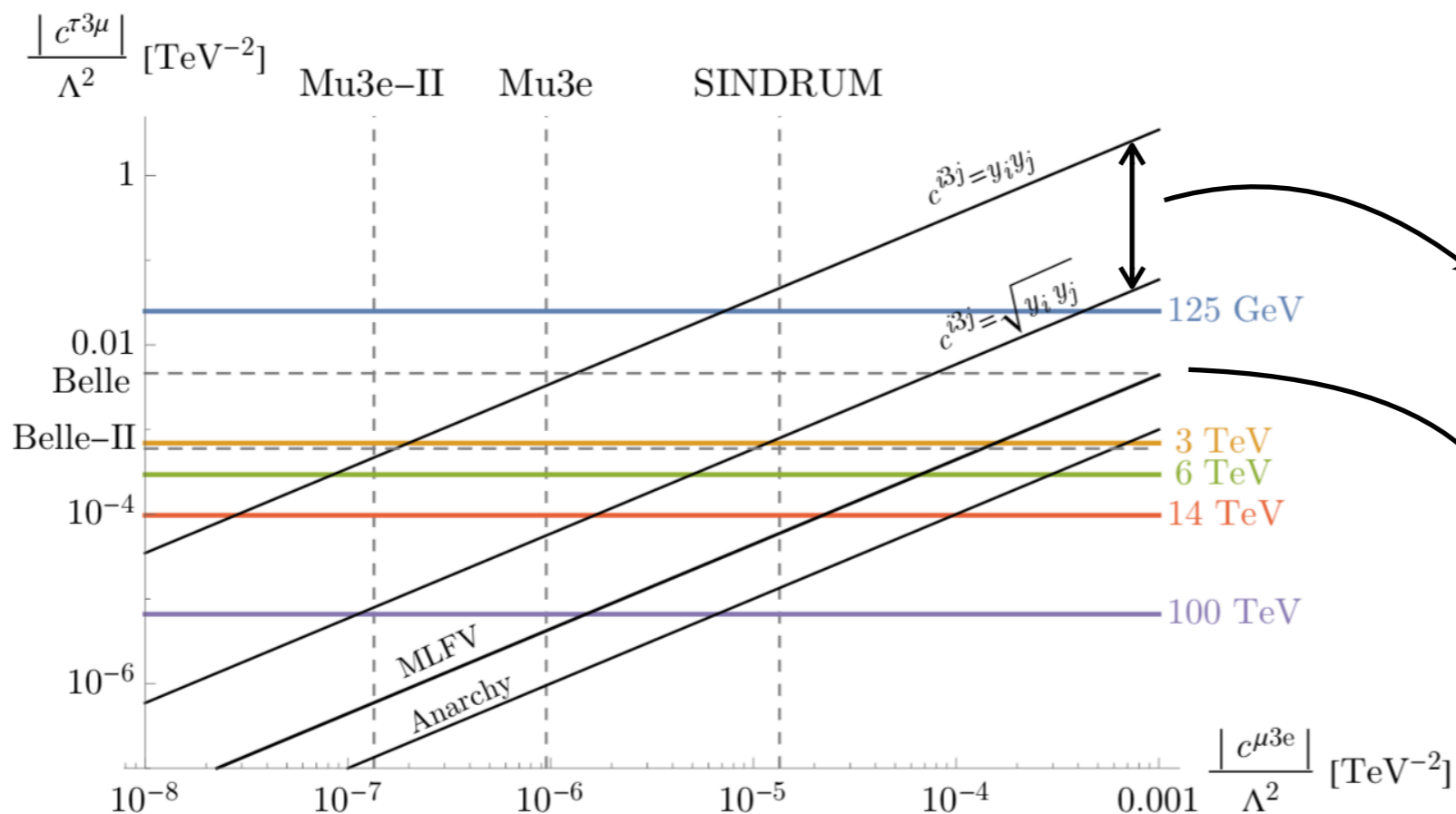


Four-fermion interactions: muon current coupled to flavor-violating bilinear

$$\frac{c_{\tau 3\mu}}{\Lambda^2} (\bar{\tau}_{L,R} \gamma^\rho \mu_{L,R}) (\bar{\mu}_{L,R} \gamma_\rho \mu_{L,R})$$

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- Charged lepton flavor violating processes: $\tau \rightarrow \ell \gamma$, $\mu \rightarrow e \gamma$, $\mu \rightarrow 3e$
- some of the strongest bounds on (generic) BSM interactions



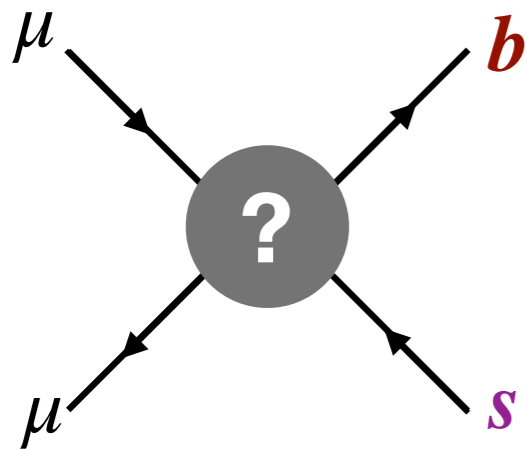
Correlation with $\mu \rightarrow 3e$:
assumes flavor structure

~ quark sector:
competes with Mu3e

~ neutrino sector

“Muon smasher guide” 2103.14043
Homiller, Lu, Reece 2203.08825

Quark flavor violation



Four-fermion interactions: muon current coupled to flavor-violating bilinear

$$\frac{c_{bs}}{\Lambda^2} (\bar{b}_{L,R} \gamma^\rho s_{L,R}) (\bar{\mu}_{L,R} \gamma_\rho \mu_{L,R})$$

- Contributes to (semi-)leptonic rare B decays $b \rightarrow s \mu \mu$: branching ratios & angular observables of various hadronic processes

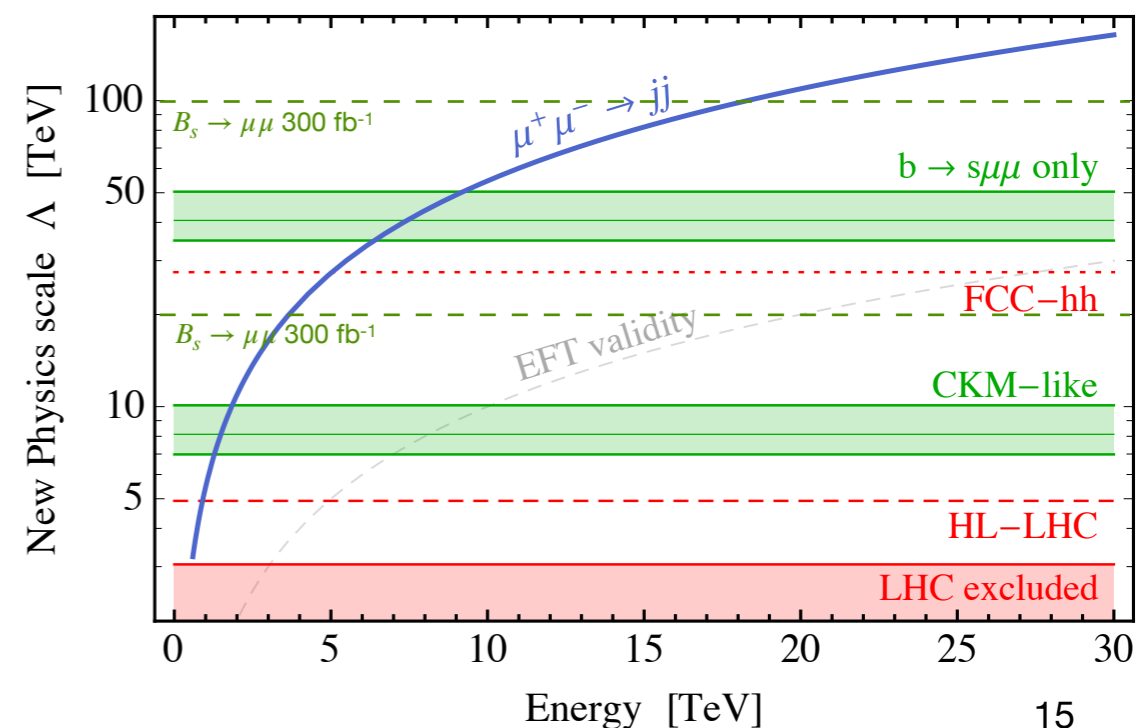
$$B_s \rightarrow \mu\mu, \quad B \rightarrow K^{(*)} \mu\mu, \quad B_s \rightarrow \phi \mu\mu, \quad \Lambda_b \rightarrow \Lambda \mu\mu$$

- Theory & systematic uncertainties: rare decays cannot improve indefinitely

$$\text{BR}(B \rightarrow K \mu\mu) \sim \frac{m_W^4}{\Lambda^4} V_{ts}, \quad \sigma(\mu\bar{\mu} \rightarrow jj) \sim \frac{E^2}{\Lambda^4}$$

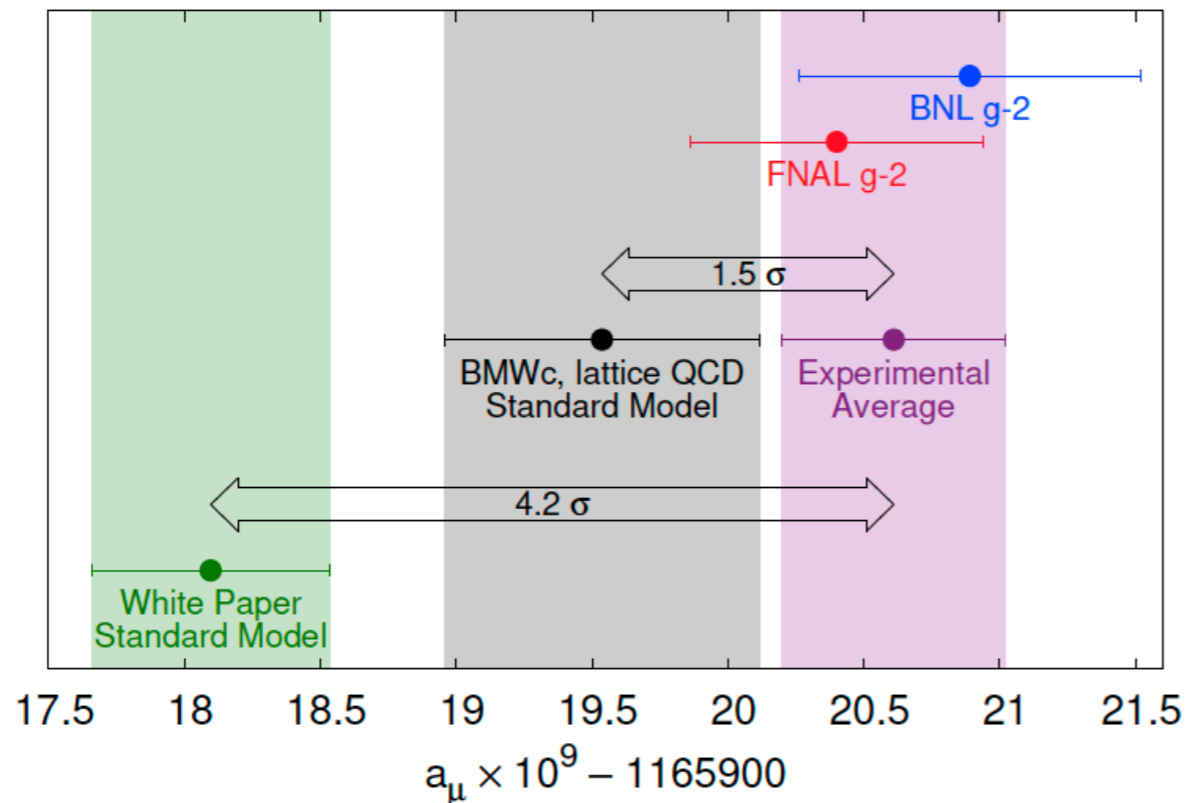
Azatov, Garosi, Greljo, Marzocca,
Salko, Trifinopoulos 2205.13552

see also Altmannshofer et al. 2306.15017



The muon g-2

- ◆ Example: muon g-2. Can it be tested at high energies at a muon collider?



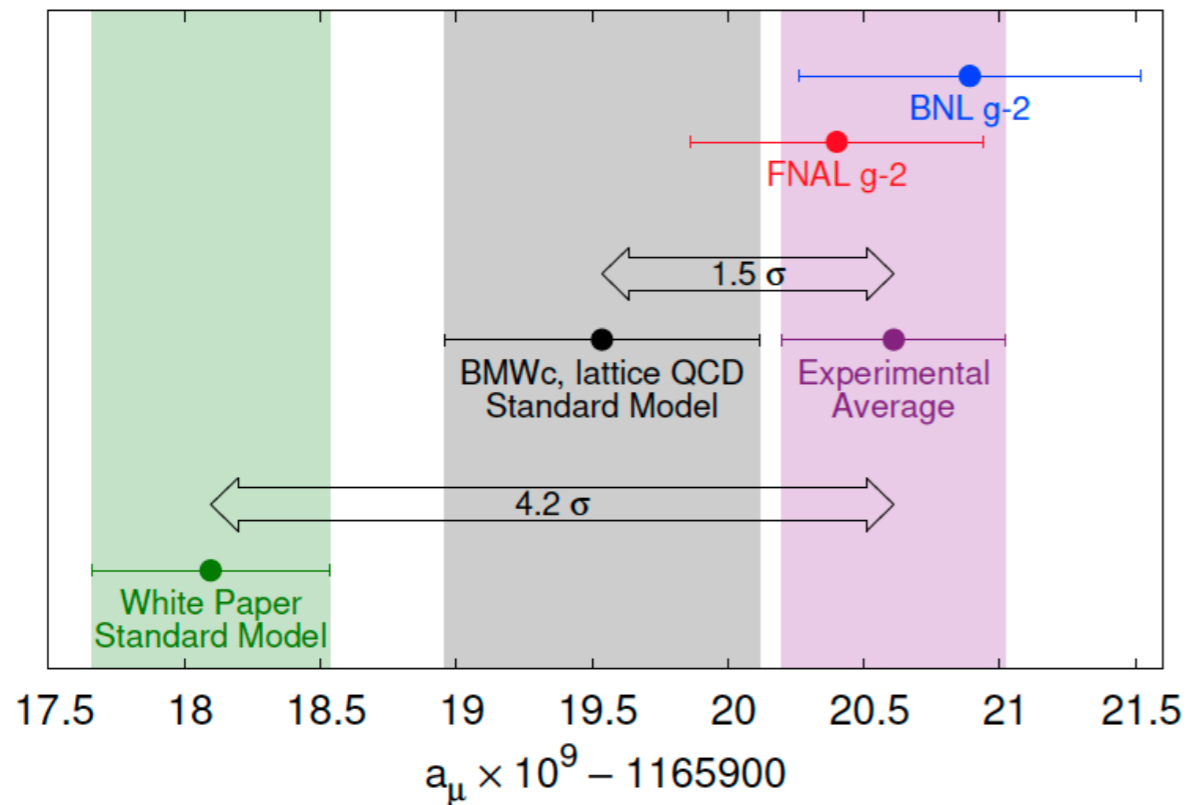
$$\Delta a_\mu = 251(59) \times 10^{-11}$$

Theoretical/systematic errors need to be controlled at the level of $\Delta a_\mu \approx 10^{-9}$

➔ Independent test of Δa_μ is desirable (ideally with different sys. & th. errors)

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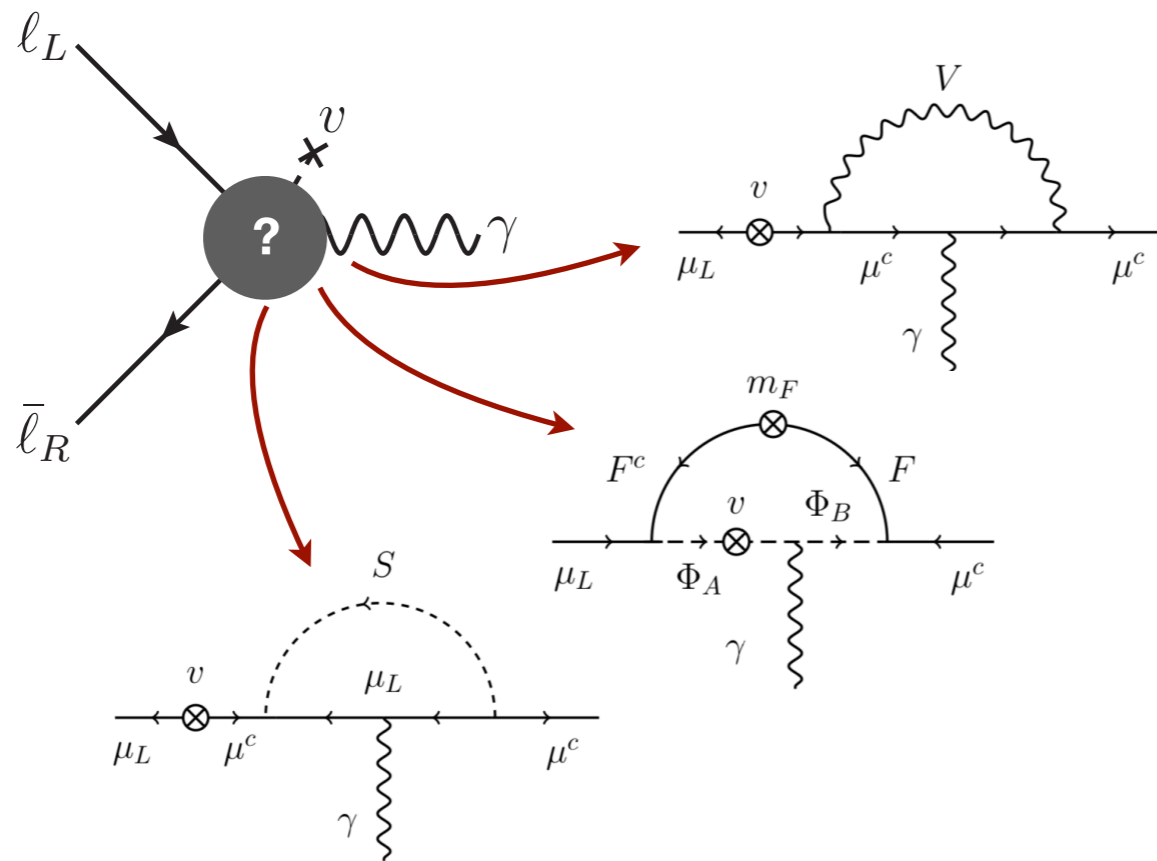
$$\Delta a_\mu = ???$$

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Muon g-2 @ muon collider

- ◆ Example: muon g-2. Can it be tested at high energies at a muon collider?
- ◆ If new physics is light enough (i.e. weakly coupled), a Muon Collider can directly produce the new particles ➡ direct searches: model-dependent



Capdevilla et al. 2006.16277
2101.10334

classify New Physics that
can enter the loop
(under reasonable assumptions)

weakly coupled models w/ MFV
⇒ new states below ~ 20 TeV

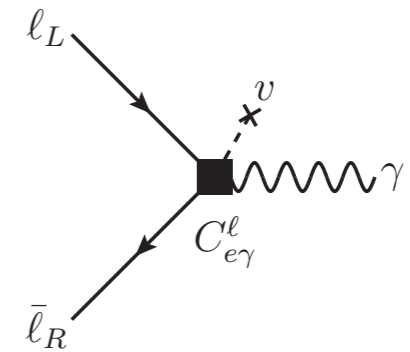
Muon g-2 @ muon collider

- ◆ Example: muon g-2. Can it be tested at high energies at a muon collider?
- ◆ If new physics is heavy: EFT!

One dim. 6 operator contributes at tree-level: $\mathcal{L}_{g-2} = \frac{C_{e\gamma}}{\Lambda^2} H (\bar{\ell}_L \sigma_{\mu\nu} e_R) e F^{\mu\nu} + \text{h.c.}$

At low energy

$$\Delta a_\mu = \frac{4m_\mu v}{\Lambda^2} C_{e\gamma} \approx 3 \times 10^{-9} \times \left(\frac{140 \text{ TeV}}{\Lambda} \right)^2 C_{e\gamma}$$



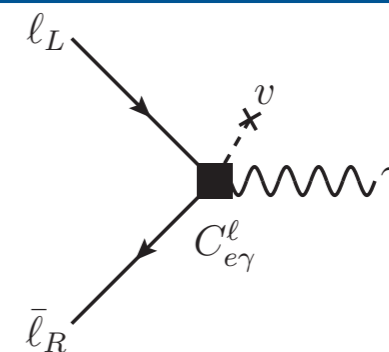
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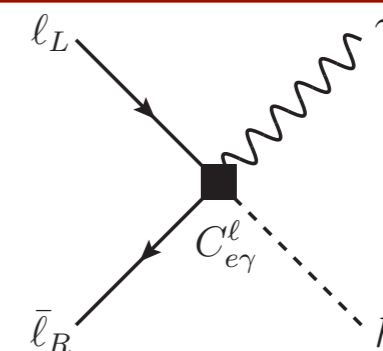


Dipole operator generates both Δa_μ and $\mu\mu \rightarrow h\gamma$

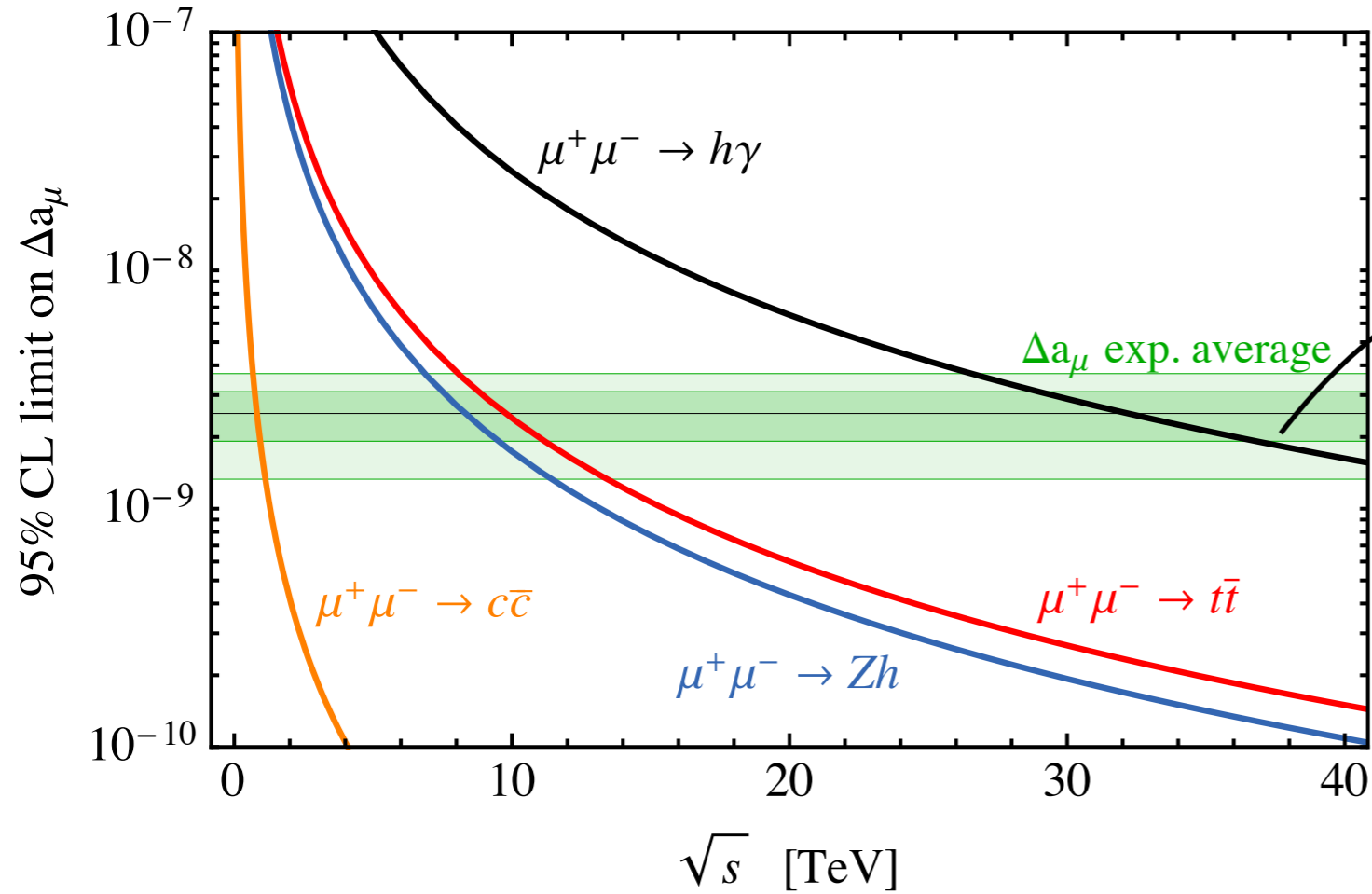
At high energy

$$\sigma_{\mu^+\mu^- \rightarrow h\gamma} = \frac{s}{48\pi} \frac{|C_{e\gamma}|^2}{\Lambda^4} \approx 0.7 \text{ ab} \left(\frac{\sqrt{s}}{30 \text{ TeV}} \right)^2 \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2$$

$$N_{h\gamma} = \sigma \cdot \mathcal{L} \approx \left(\frac{\sqrt{s}}{10 \text{ TeV}} \right)^4 \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \quad \text{need } E > 10 \text{ TeV}$$



Muon g-2 @ muon collider



Exp. value of Δa_μ can be tested at 95% CL at a 30 TeV collider!
(with reasonable assumptions on detector performance)

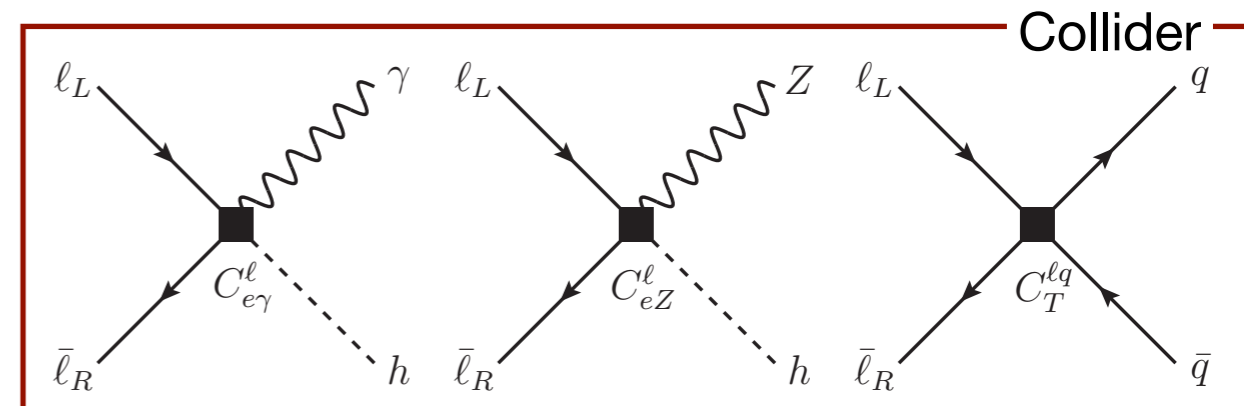
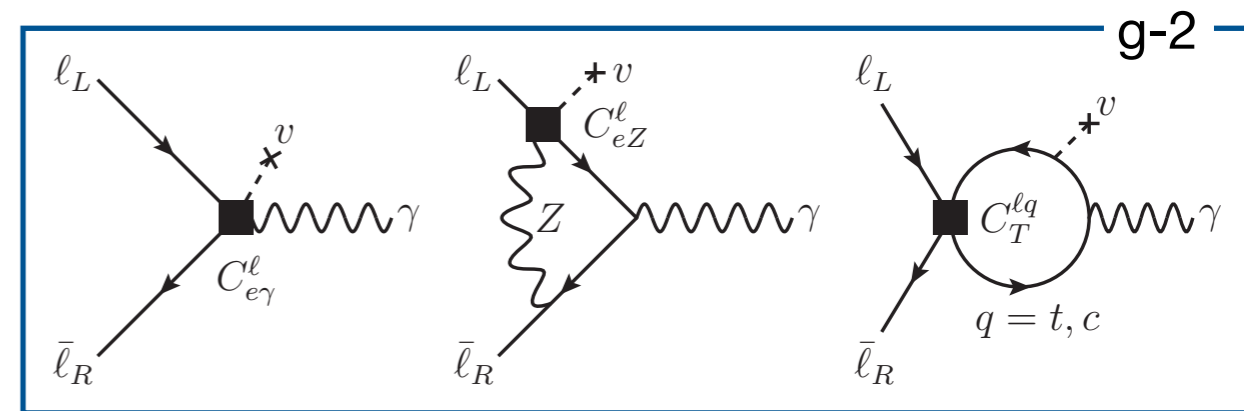
This result is completely model-independent!

B, Paradisi 2012.02769

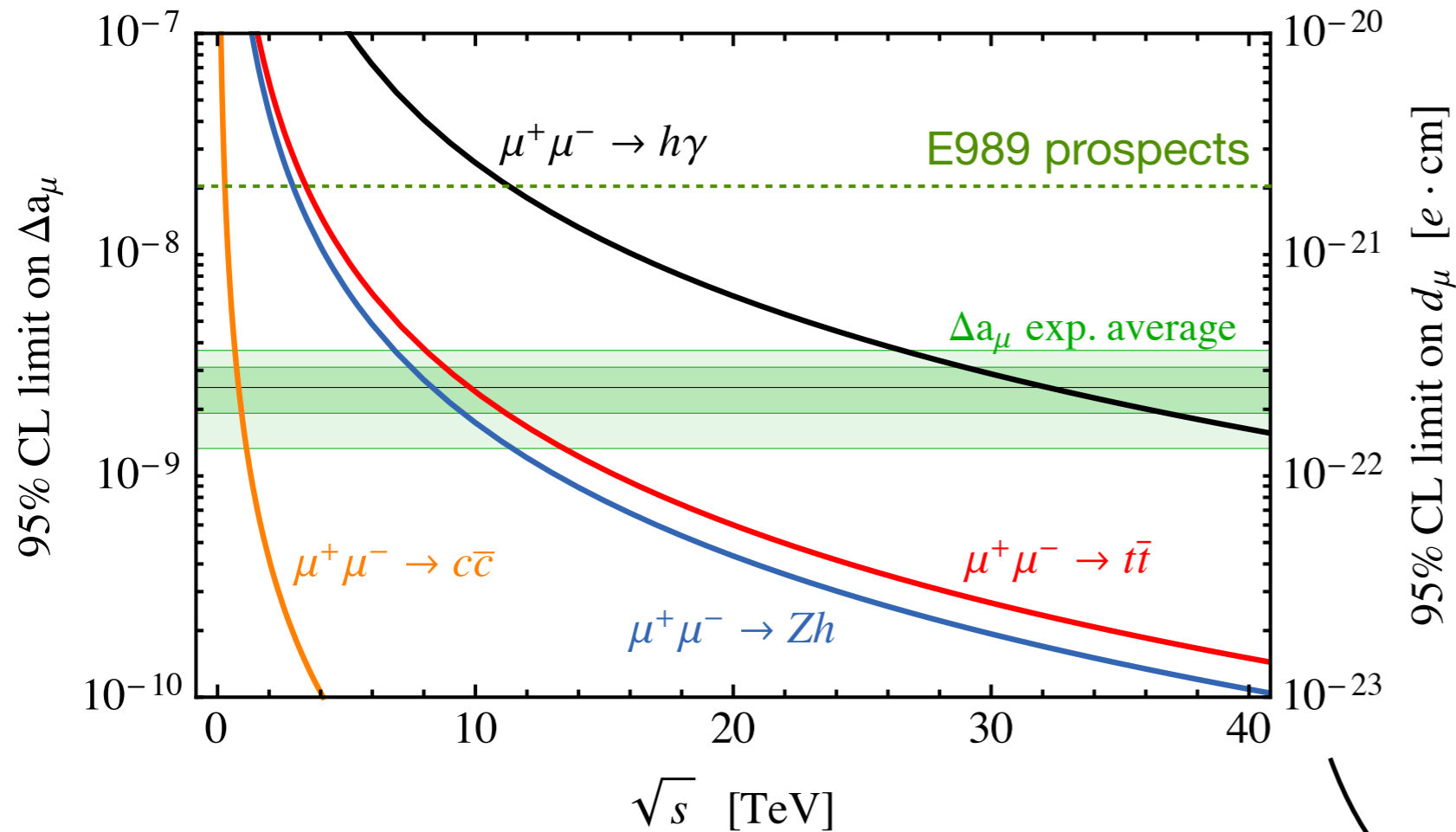
- ◆ Other operators enter g-2 at 1 loop:

$$\Delta a_\mu \approx \left(\frac{250 \text{ TeV}}{\Lambda^2} \right)^2 \left(C_{e\gamma} - \frac{C_{Tt}}{5} - \frac{C_{Tc}}{1000} - \frac{C_{eZ}}{20} \right)$$

- ◆ Full set of operators with $\Lambda \gtrsim 100 \text{ TeV}$ can be probed at a high-energy muon collider



Muon EDM @ muon collider



Exp. value of Δa_μ can be tested at 95% CL at a 30 TeV collider!

This result is completely model-independent!

B, Paradisi 2012.02769

Muon EDM for free!

$$\Delta a_\mu = \frac{4\nu m_\mu \text{Re}(C_{e\gamma})}{\Lambda^2}$$

$$d_\mu = \frac{2\nu \text{Im}(C_{e\gamma})}{\Lambda^2} = \frac{\Delta a_\mu}{2m_\mu} \tan \phi_\mu e$$

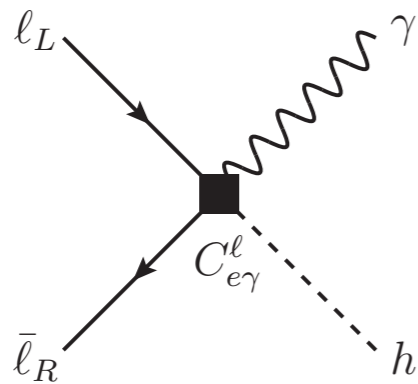
Collider constrains $|C_{e\gamma}|^2$

$$\Rightarrow d_\mu \lesssim 10^{-22} e \cdot \text{cm}$$

3 o.o.m. stronger than present bound!

Lepton $g-2$ from rare Higgs decays

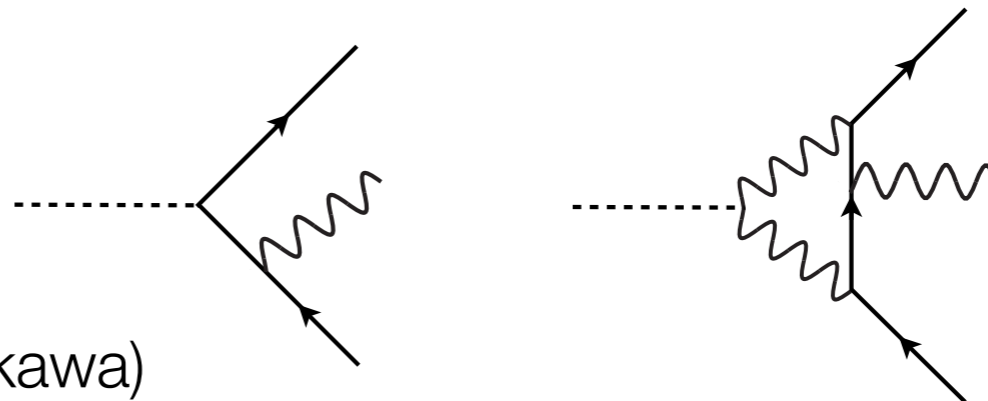
- ◆ Dipole operator contributes also to $h \rightarrow \ell\ell\gamma$ decays



$$\Gamma_{h \rightarrow \ell^+ \ell^- \gamma}^{(\text{NP})} = \frac{\alpha |C_{e\gamma}|^2 m_h^5}{192\pi^2 \Lambda^4}$$

$$\Gamma_{h \rightarrow \ell^+ \ell^- \gamma}^{(\text{int})} = \frac{\alpha m_\ell \text{Re}(C_{e\gamma}) m_h^3}{16\pi^2 v \Lambda^2}$$

$$\Gamma_{h \rightarrow \ell^+ \ell^- \gamma}^{(\text{SM})} = \Gamma_{\text{tree}}^{(\text{SM})} + \Gamma_{\text{loop}}^{(\text{SM})}$$



Han, Wang
1704.00790

(tree-level is suppressed by lepton Yukawa)

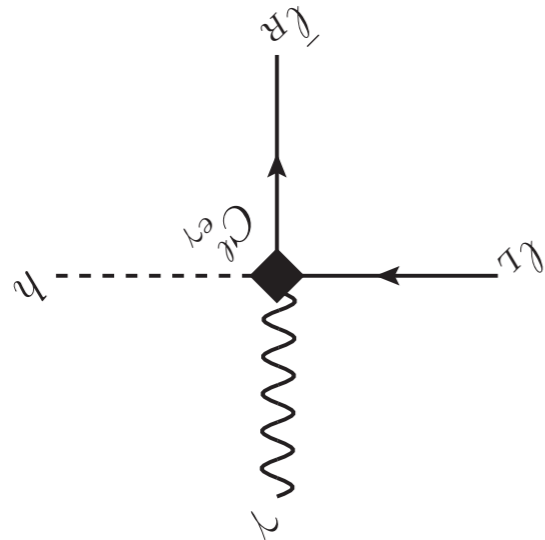
- ▶ Muon $g-2$: rate is too small 😞

$$\text{BR}_{h \rightarrow \mu^+ \mu^- \gamma}^{(\text{SM})} \approx 10^{-4} \quad (\text{mainly one-loop})$$

$$\text{BR}_{h \rightarrow \mu^+ \mu^- \gamma}^{(\text{NP})} \approx 5 \times 10^{-10} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right)$$

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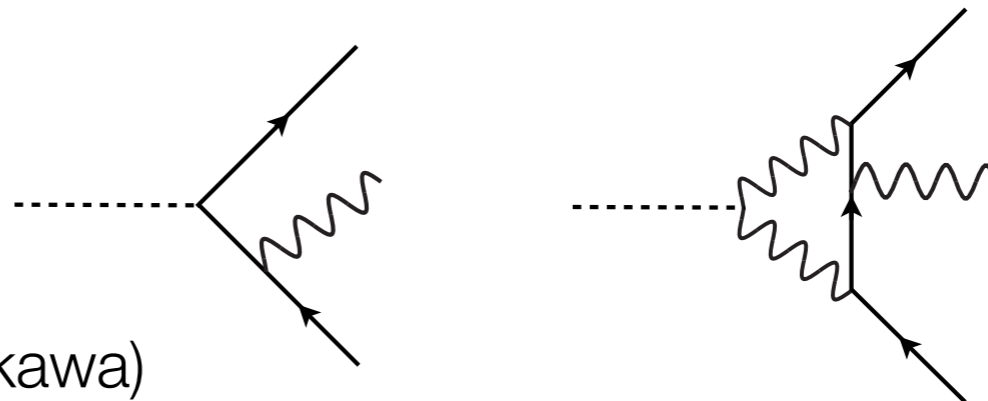
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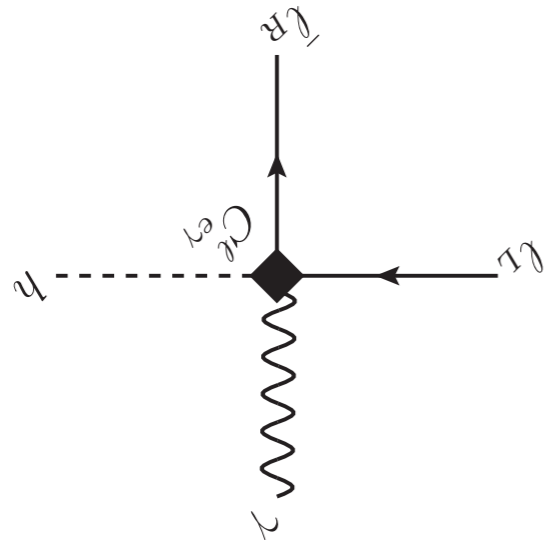
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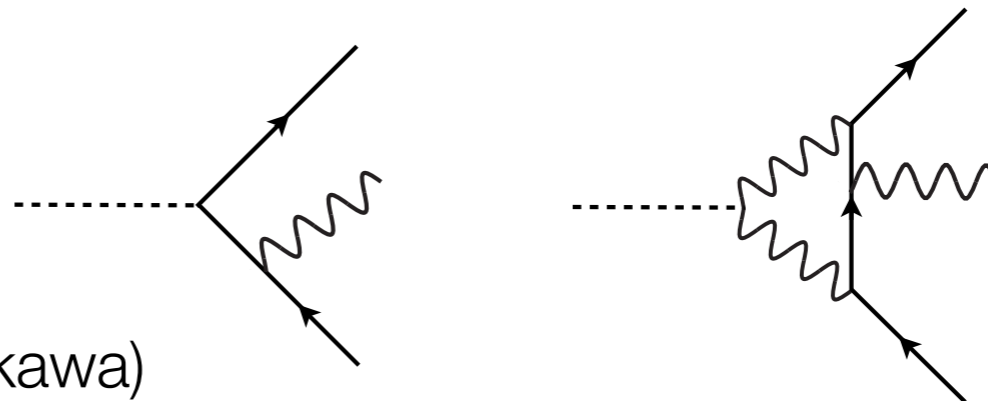
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👉 What about the tau?

Lepton g-2 from rare Higgs decays

- ◆ Tau magnetic dipole moment: enhanced due to the larger mass

$$\Delta a_\tau = \frac{4v m_\tau}{\Lambda^2} C_{e\gamma}^\tau \approx \Delta a_\mu \frac{m_\tau^2}{m_\mu^2} \approx 10^{-6}$$

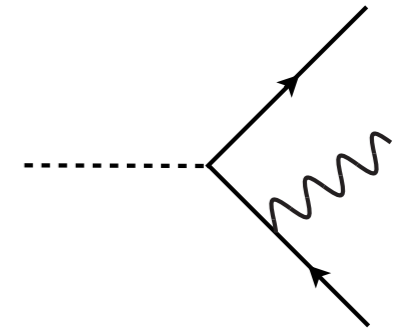
if $C_{e\gamma}^\ell$ scales as y_ℓ

Present bound: $\Delta a_\tau \lesssim 10^{-2}$
 from LEP $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$
 hep-ex/0406010

Can be improved to few 10^{-3}
 at HL-LHC 1908.05180

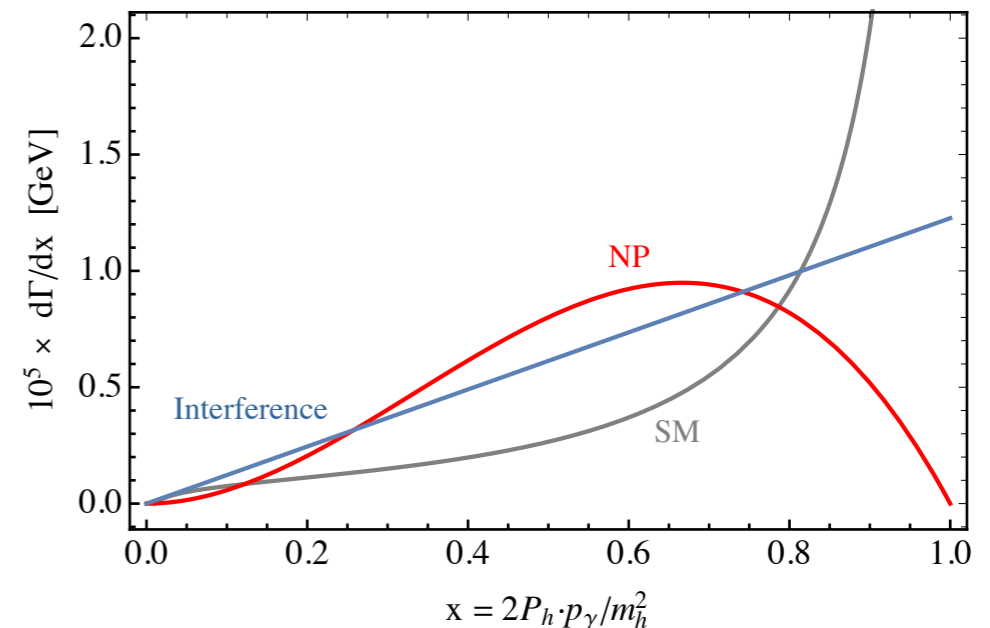
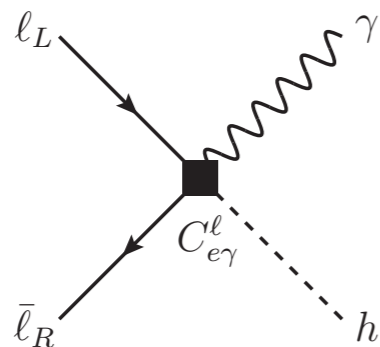
- ◆ Contribution to $h \rightarrow \tau\tau\gamma$ decays:

$$\text{BR}_{h \rightarrow \tau^+\tau^-\gamma}^{(\text{SM})} \approx 5 \times 10^{-4} \quad (\text{with cut on soft collinear photon})$$



could be measured at few % level by Higgs factory

$$\text{BR}_{h \rightarrow \tau^+\tau^-\gamma}^{(\text{NP})} \approx 0.2 \times \Delta a_\tau$$



Lepton $g-2$ from rare Higgs decays

$$\text{BR}_{h \rightarrow \tau^+ \tau^- \gamma}^{(\text{SM})} \approx 5 \times 10^{-4}$$

$$\text{BR}_{h \rightarrow \tau^+ \tau^- \gamma}^{(\text{NP})} \approx 0.2 \times \Delta a_\tau$$

♦ **MuC:** 10^7 Higgs bosons @ 10 TeV \Rightarrow 5k $H \rightarrow \tau\tau\gamma$ events, 2% precision on SM,

$$\Delta a_\tau \lesssim 3 \times 10^{-5} \quad (\text{signal only})$$

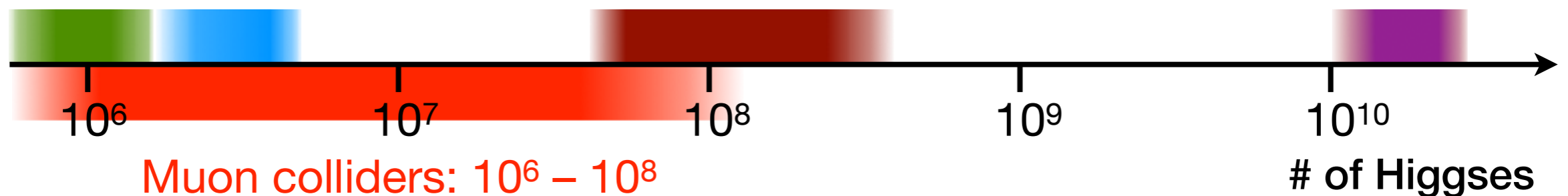
3 o.o.m. improvement of current limit!

Low energy
e⁺e⁻ factories
(FCC-ee, CEPC,
ILC, CLIC380)

TeV-scale
e⁺e⁻ factories
(CLIC, ILC1000)

LHC: few $\times 10^7$
HL-LHC: few $\times 10^8$

FCC-hh:
few $\times 10^{10}$



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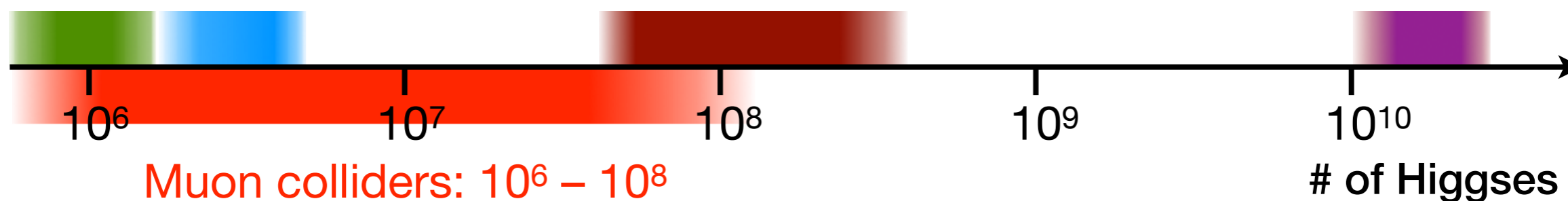
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LHC: few $\times 10^7$
HL-LHC: few $\times 10^8$

FCC-hh:
few $\times 10^{10}$



- ♦ **e^+e^- factory:** $\sim 400 H \rightarrow \tau\tau\gamma$ events \Rightarrow 5% precision on SM, $\Delta a_\tau \lesssim \text{few} \times 10^{-4}$

- ♦ **LHC:** large number of Higgs bosons, but large backgrounds

Rescaling $H \rightarrow \tau\tau$ searches ~ 350 reconstructed $H \rightarrow \tau\tau\gamma$ events at HL-LHC,
but 10x more background \Rightarrow 20% precision on SM, $\Delta a_\tau \lesssim 5 \times 10^{-4}$

Lepton $g-2$ from rare Higgs decays

$$\text{BR}_{h \rightarrow \tau^+ \tau^- \gamma}^{(\text{SM})} \approx 5 \times 10^{-4}$$

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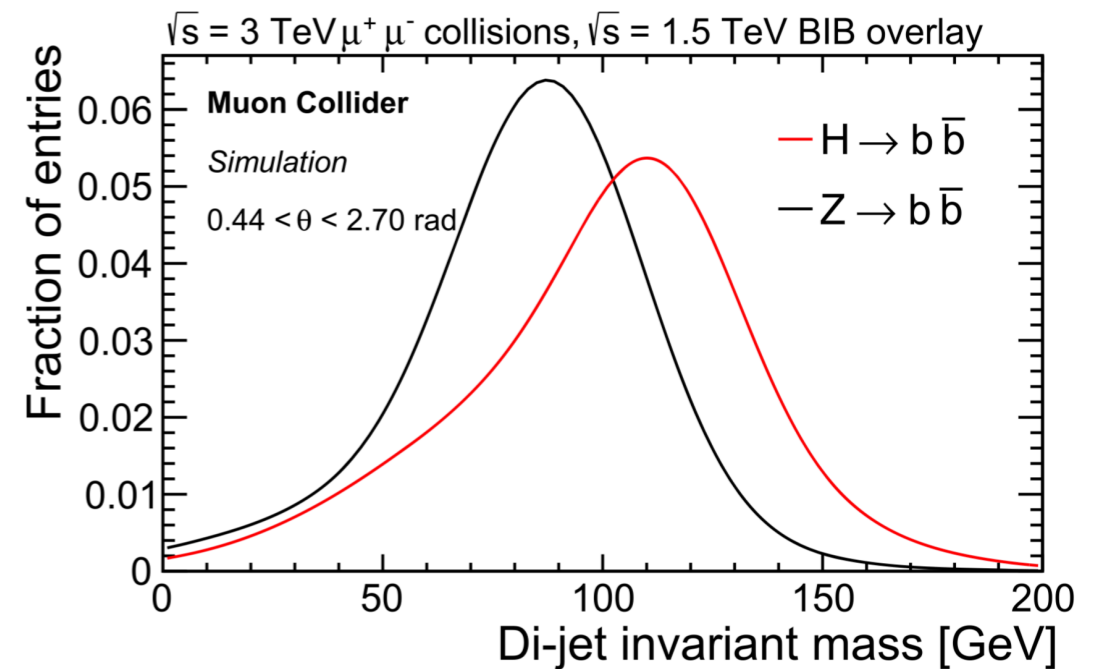
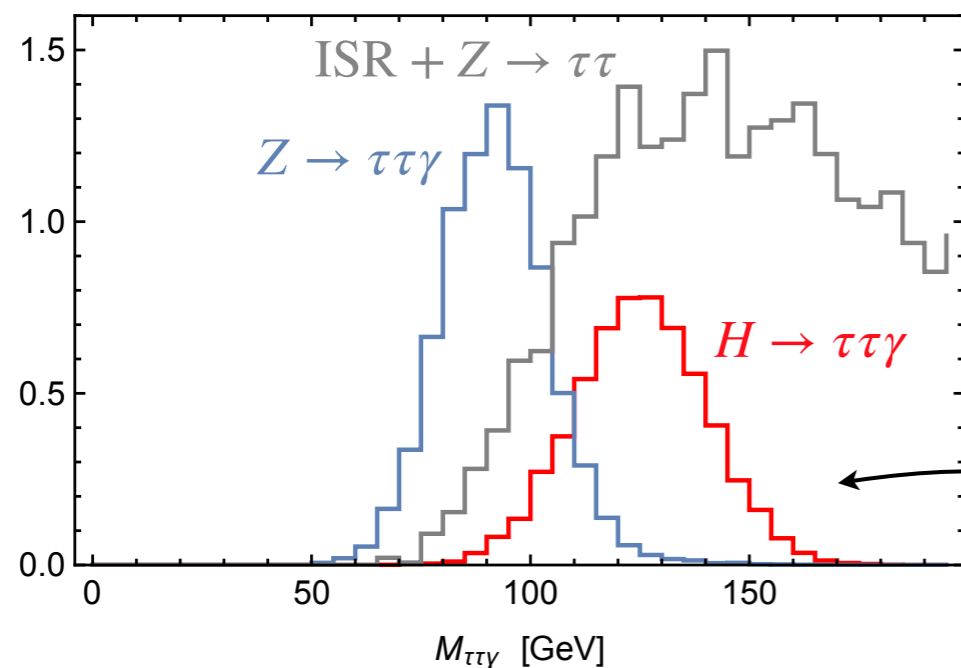
◆ **MuC:** 10^7 Higgs bosons @ 10 TeV \Rightarrow 5k $H \rightarrow \tau\tau\gamma$ events, 2% precision on SM,

$$\Delta a_\tau \lesssim 3 \times 10^{-5} \quad (\text{signal only})$$

3 o.o.m. improvement of current limit!

◆ Caveat: need to be able to reconstruct Higgs mass in di-tau channel w/ reasonable precision

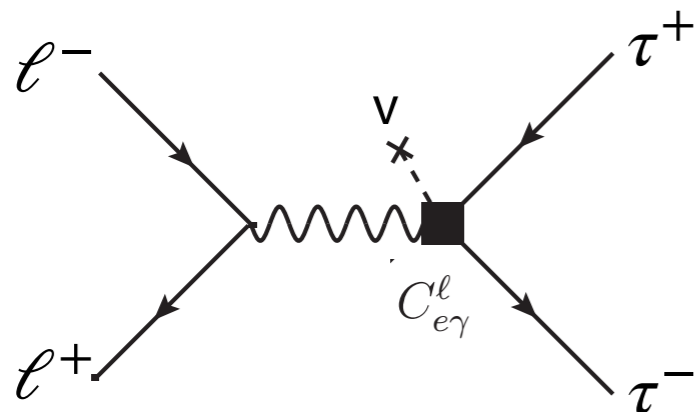
◆ Reducible background: $\gamma + Z$



Tau g-2 from high-energy probes

Further possibilities to measure Δa_τ precisely from high-energy probes

◆ Pair production



$$\sigma_{\text{SM}} \sim \frac{4\pi\alpha^2}{3s}$$

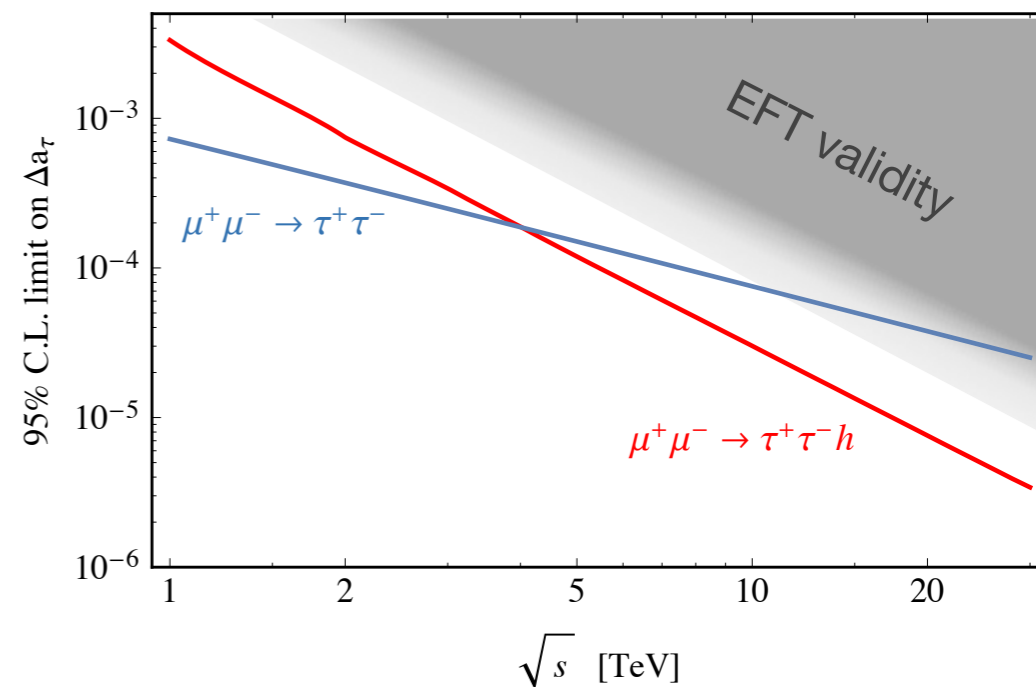
$$\sigma_{\text{NP}} = \frac{4\pi\alpha^2}{3} \frac{|C_{e\gamma}^\ell|^2 v^2}{\Lambda^4} \sim \frac{\pi\alpha^2 \Delta a_\ell^2}{6m_\ell^2}$$

Limit on g-2: $\Delta a_\ell \lesssim \frac{\text{const.}}{\sqrt{\mathcal{L}}} \sim E^{-1}$

▶ equivalently, $\Lambda \sim \sqrt{E}$

EFT description breaks down above few TeV!

Could probe $\Delta a_\tau \sim \text{few } 10^{-5}$

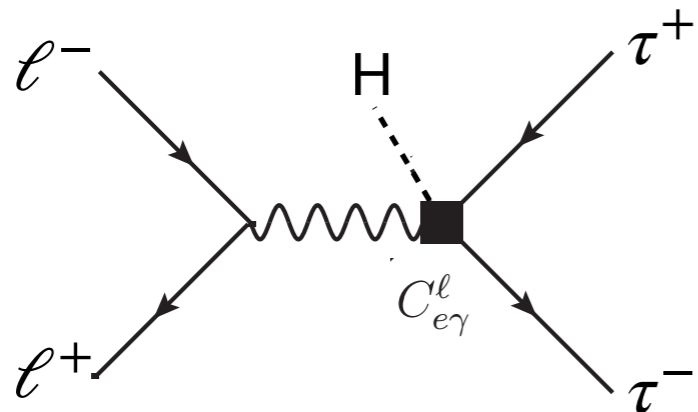


w/ Levati, Maltoni, Paradisi, Wang

Tau g-2 from high-energy probes

Further possibilities to measure Δa_τ precisely from high-energy probes

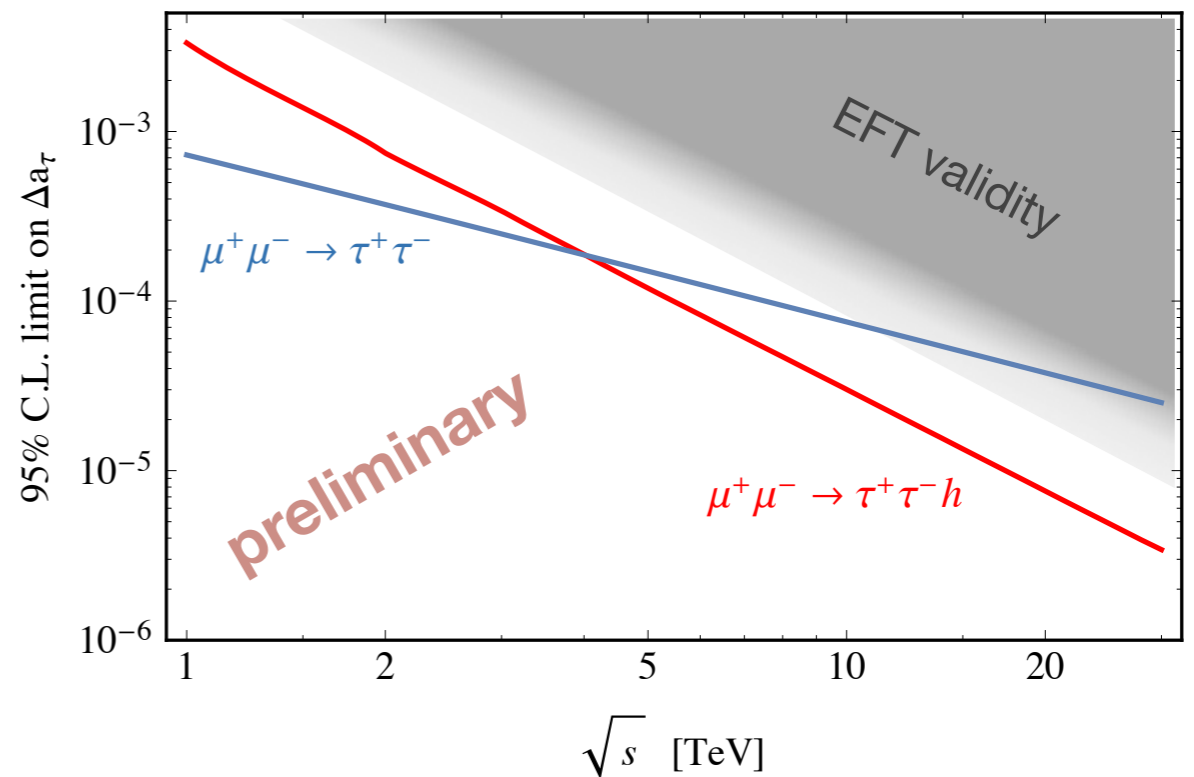
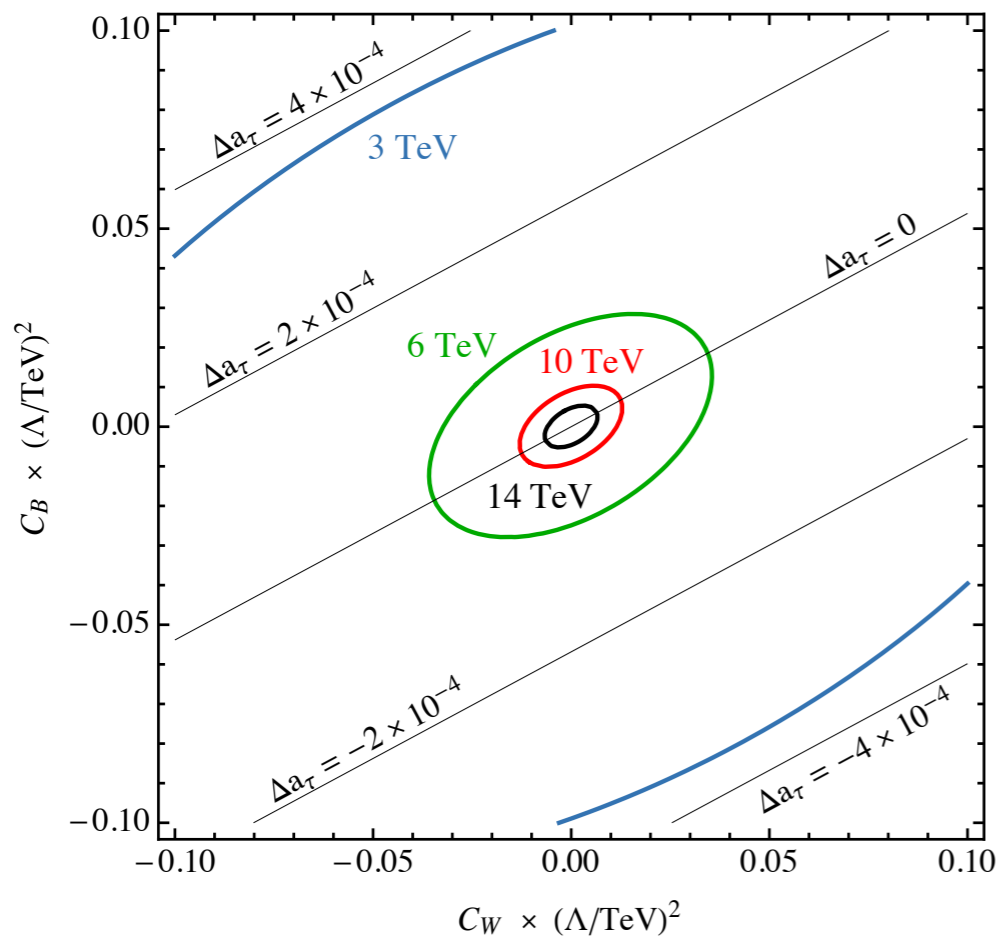
◆ $H\tau\tau$ associated production



DB, Levati, Paradisi, Maltoni, Wang to appear...

- ▶ Main background from $\mu\mu \rightarrow Z\gamma$ (where Z is mistaken for H)

Could probe $\Delta a_\tau \sim 10^{-5}$ @ 10 TeV

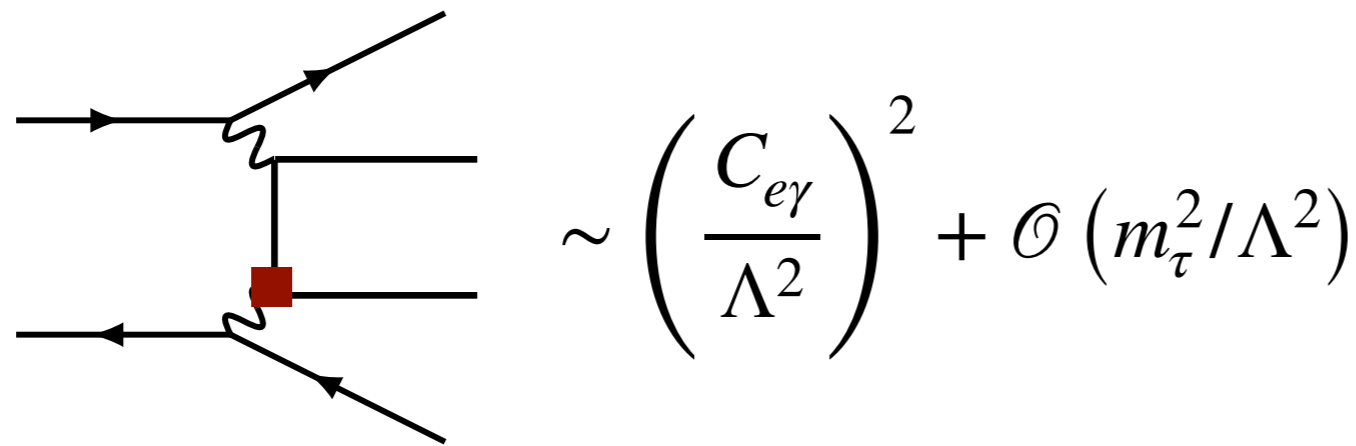


also a bound on tau EDM!

Tau g-2 from high-intensity probes

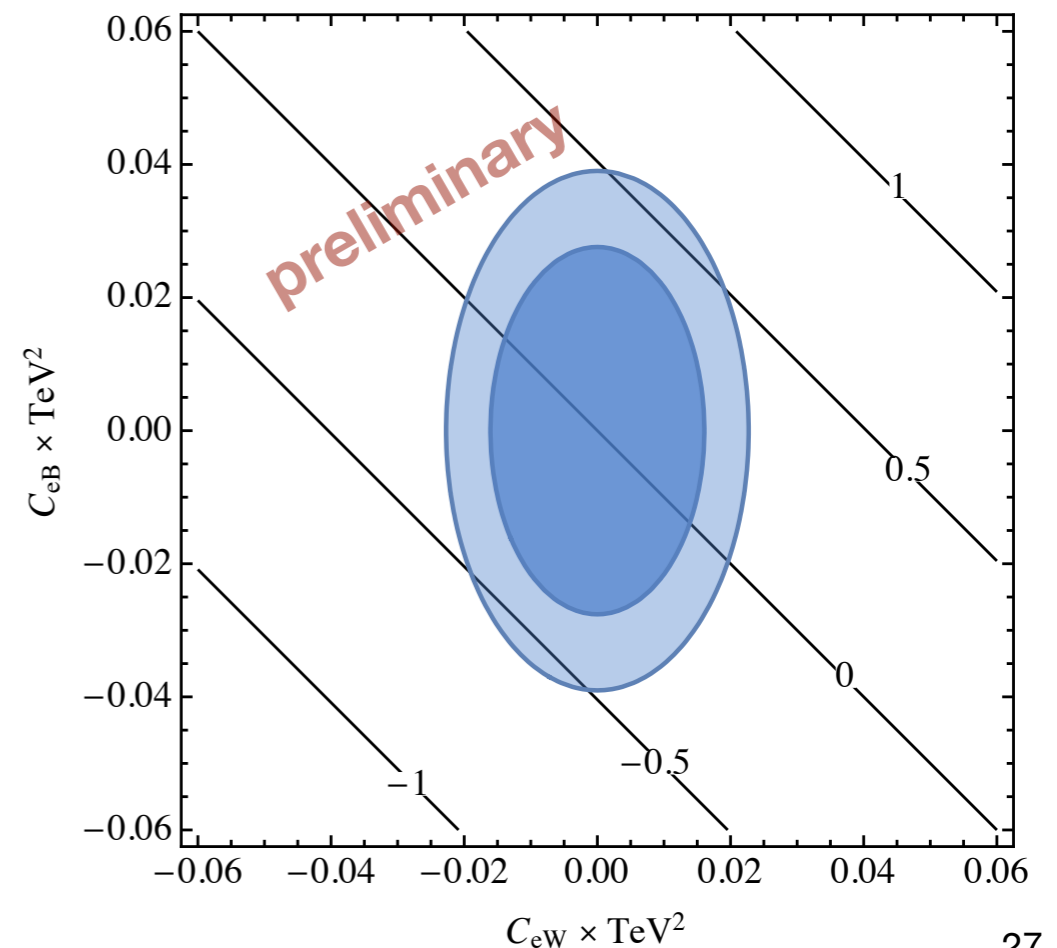
A high-energy lepton collider has a huge VBF rate!

- ◆ Δa_τ from vector boson scatterings $\ell^+\ell^- \rightarrow \ell^+\ell^-\tau^+\tau^-, \nu\bar{\nu}\tau^+\tau^-$
(same as LEP bound)



work in progress with Levati,
Paradisi, Maltoni, Wang

$\Delta a_\tau \times 10^4$ from $\ell^+\ell^- \rightarrow \tau^+\tau^-\nu\bar{\nu}$



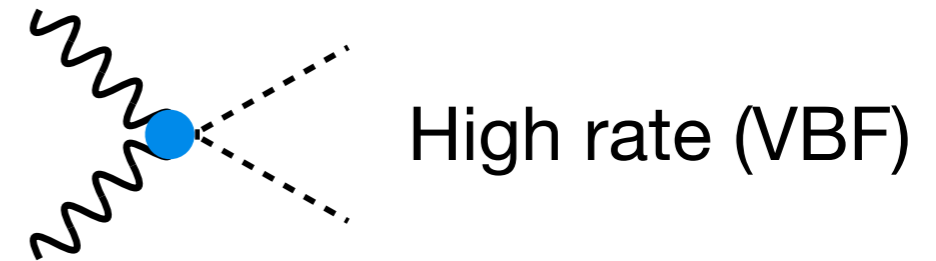
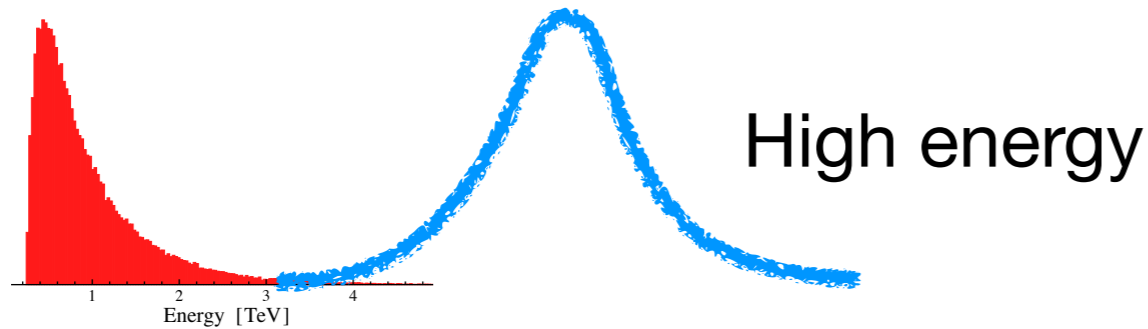
- ▶ Caveat: VBF is a “soft” process,
EFT mainly affects high-mass region

Still, could probe $\Delta a_\tau \sim \text{few } 10^{-5}$

charged and neutral channel can
separately constrain C_{eB} and C_{eW}

Summary

- High energy muon collider offers two paths to precision measurements



- New ways to test flavor @ μ Collider:

gain from high energy probes

access to second generation processes

- Bounds on lepton- and quark-flavor violating interactions can surpass low-energy precision measurements!
- Can exploit rare Higgs decays + Higgs production @ high energy to access 3rd generation physics (tau)
- Can improve lepton $g-2$ and EDM bound by orders of magnitude





Backup

Radiation & high-energy neutrinos

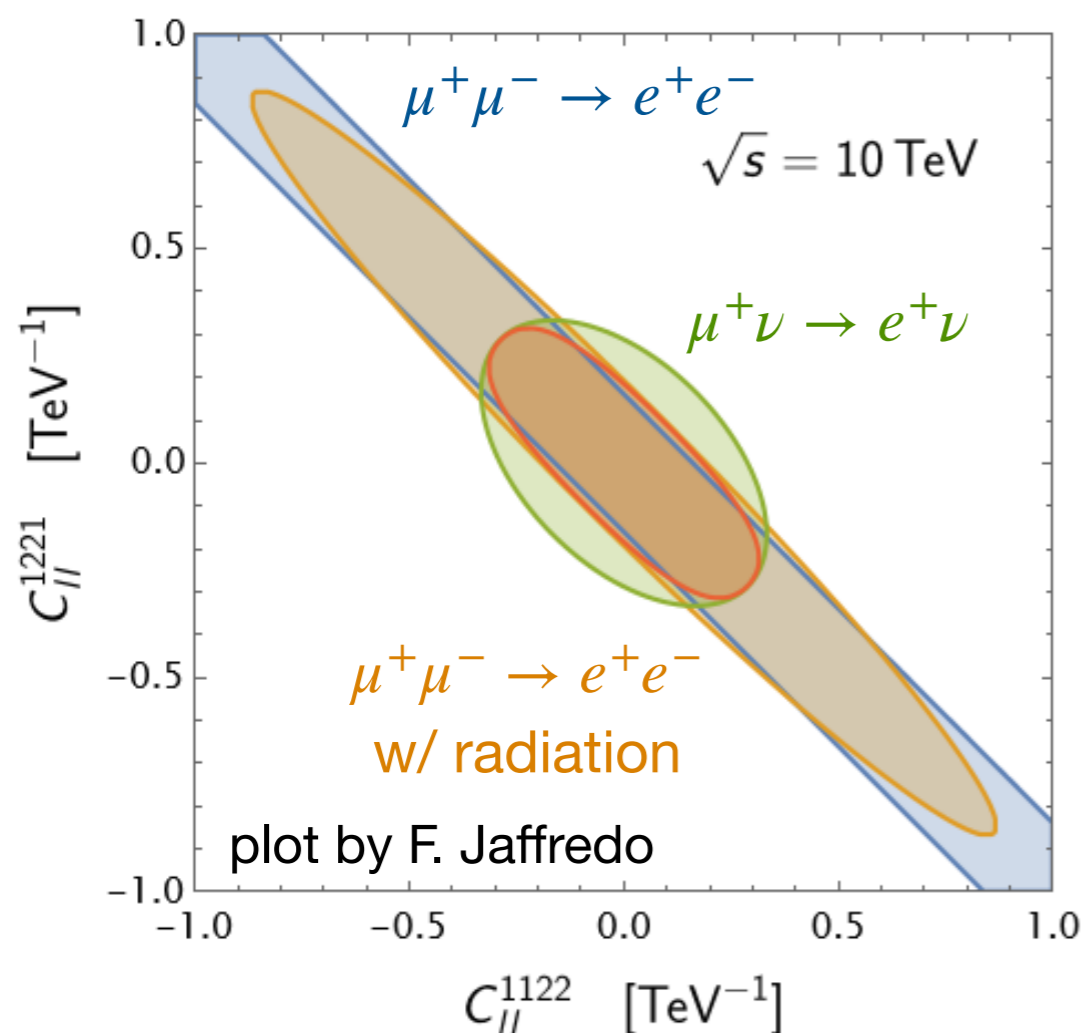
- ◆ Radiation effects are important.

☞ talks by David, Andrea, Davide

- ◆ Hard neutrinos from $\mu^\pm \rightarrow W^\pm \nu$ have a large PDF

⇒ can access neutrino initial state at high energy!

see also Chen, Glioti, Rattazzi,
Ricci, Wulzer 2202.10509



- ◆ Directly probe neutrino interactions at high energy: usual E^2 gain

- ◆ can put strong constraints on BSM neutrino physics

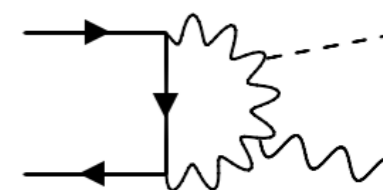
☞ previous talk by Zahra

- ◆ complementary to $\mu^+ \mu^-$ scattering, can remove flat directions and probe operators with different flavor structure

Muon g-2 @ muon collider

- SM irreducible background is small: $\sigma_{\mu^+\mu^-\rightarrow h\gamma}^{(SM)} \approx 10^{-2} \text{ ab} \left(\frac{30 \text{ TeV}}{\sqrt{s}} \right)^2$

tree-level is suppressed by muon mass; loop contribution dominant



- Main background from $\mu\mu \rightarrow Z\gamma$ (where Z is mistaken for H)
(large due to transverse Z polarizations)

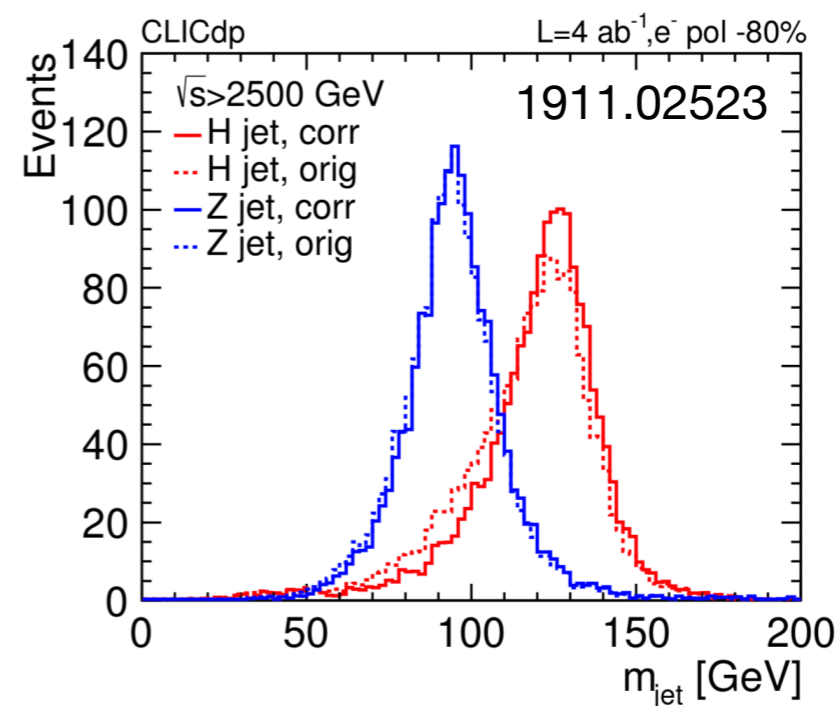
Search in $h \rightarrow b\bar{b}$ channel:

$$\epsilon_b \approx 80\% \quad |\cos \theta_{\text{cut}}| < 0.6 \quad \text{BR}_{h \rightarrow b\bar{b}} = 58\%$$

At 30 TeV, 90 ab^{-1} , for $\Delta a_\mu = 3 \times 10^{-9}$:

$$N_S = 22, \quad N_B = 886 \times p_{Z \rightarrow h}$$

Δa_μ can be tested at 95% CL at a 30 TeV collider if $Z \rightarrow h$ mistag probability < 10-15%



Beyond tree-level

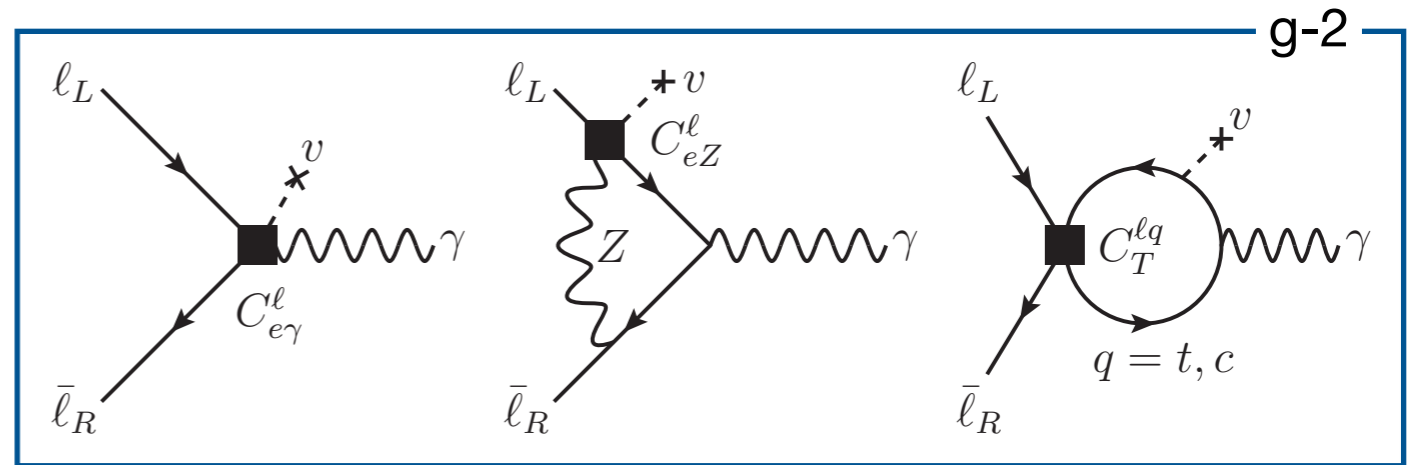
- Other operators contribute to g-2 at one loop:

$$\mathcal{L} = \frac{C_{eB}}{\Lambda^2} (\bar{\ell}_L \sigma^{\mu\nu} e_R) H B_{\mu\nu} + \frac{C_{eW}}{\Lambda^2} (\bar{\ell}_L \sigma^{\mu\nu} e_R) \tau^I H W_{\mu\nu}^I + \frac{C_{qT}}{\Lambda^2} (\bar{\ell}_L \sigma^{\mu\nu} e_R) \epsilon (\bar{q}_L \sigma_{\mu\nu} u_R)$$

(+ other effects suppressed by y_μ)

$$|H|^2 W_{\mu\nu}^a W^{\mu\nu,a} \quad |H|^2 B_{\mu\nu} B^{\mu\nu}$$

$$(H^\dagger \sigma^a H) W_{\mu\nu}^a B^{\mu\nu}$$



Including 1-loop running:

$$\Delta a_\mu \simeq \frac{4m_\mu v}{e\Lambda^2} \left(C_{e\gamma}(m_\mu) - \frac{3\alpha}{2\pi} \frac{c_W^2 - s_W^2}{s_W c_W} C_{eZ} \log \frac{\Lambda}{m_Z} \right) - \sum_{q=c,t} \frac{4m_\mu m_q}{\pi^2} \frac{C_{Tq}}{\Lambda^2} \log \frac{\Lambda}{m_q}$$

$$\simeq \left(\frac{250 \text{ TeV}}{\Lambda^2} \right)^2 (C_{e\gamma} - 0.2 C_{Tt} - 0.001 C_{Tc} - 0.05 C_{eZ})$$

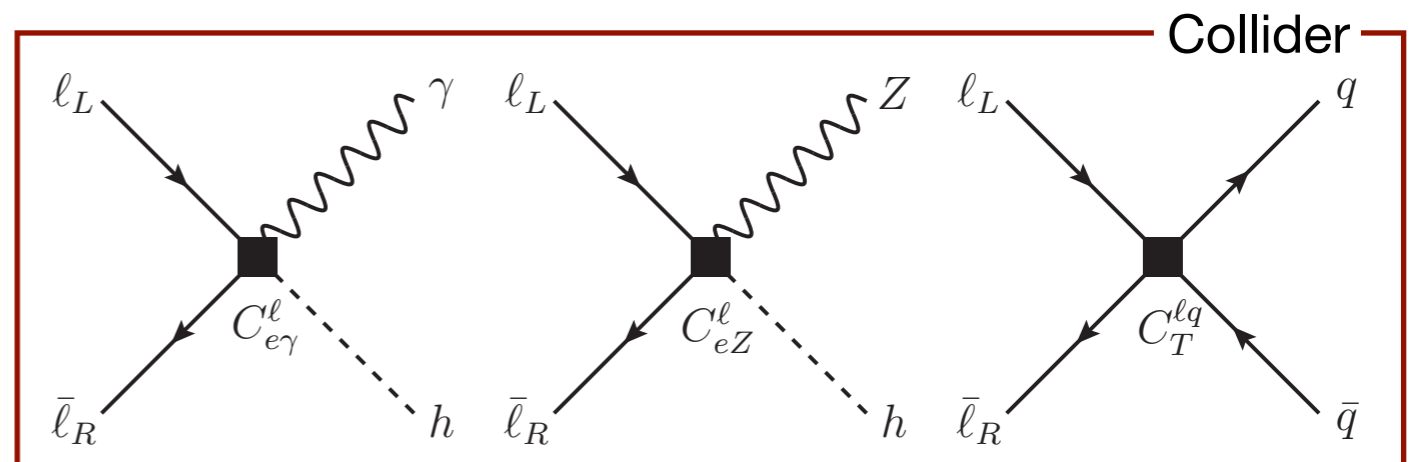
B, Paradisi 2012.02769

Full set of operators
can be probed
at high energy

$$\mu^+ \mu^- \rightarrow h\gamma$$

$$\mu^+ \mu^- \rightarrow hZ$$

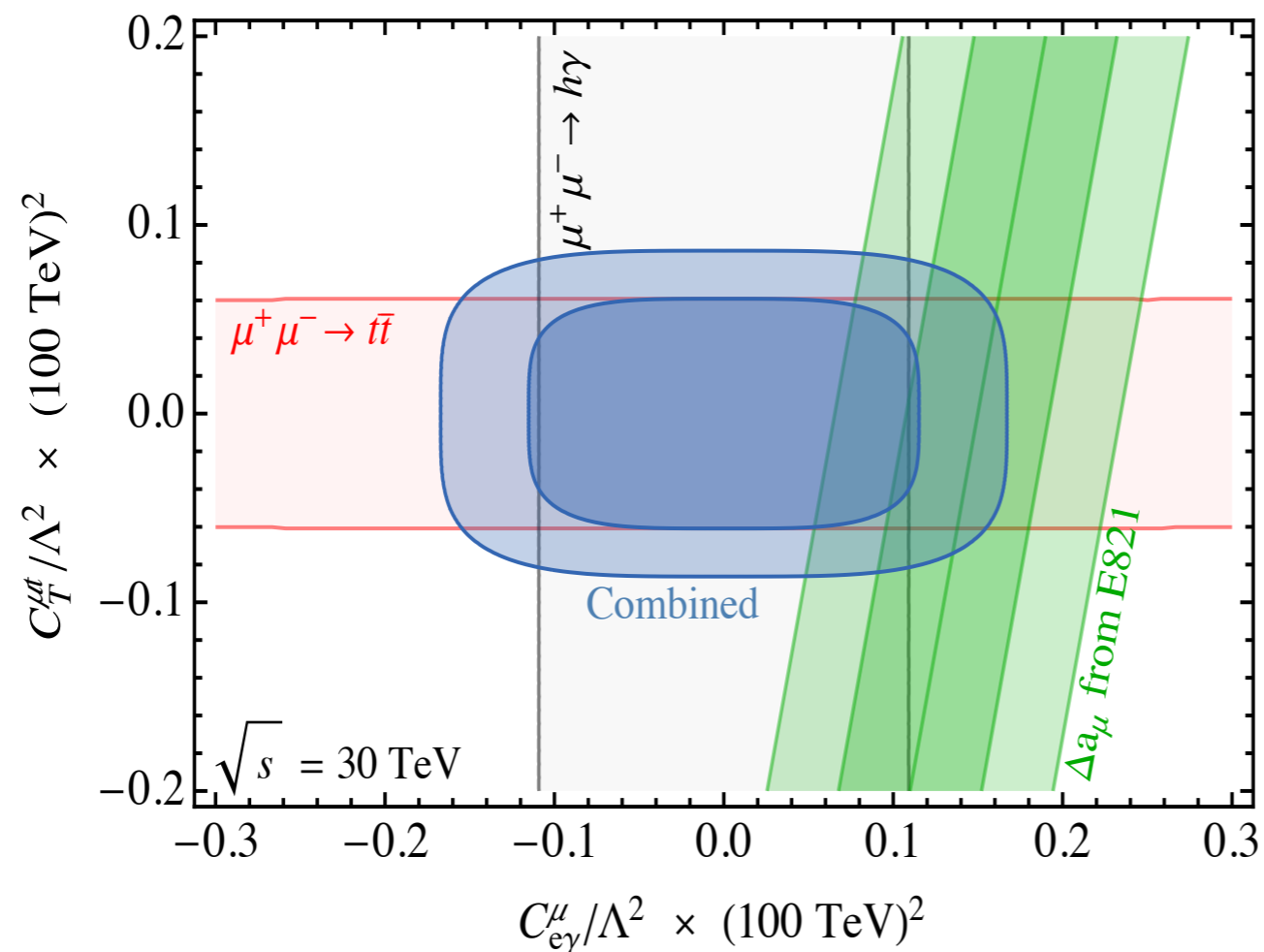
$$\mu^+ \mu^- \rightarrow q\bar{q}$$



Muon g-2 @ muon collider

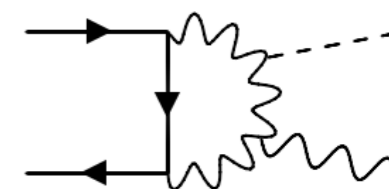
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- Full set of operators with $\Lambda \gtrsim 100$ TeV can be probed at a high energy muon collider



Muon g-2 @ muon collider

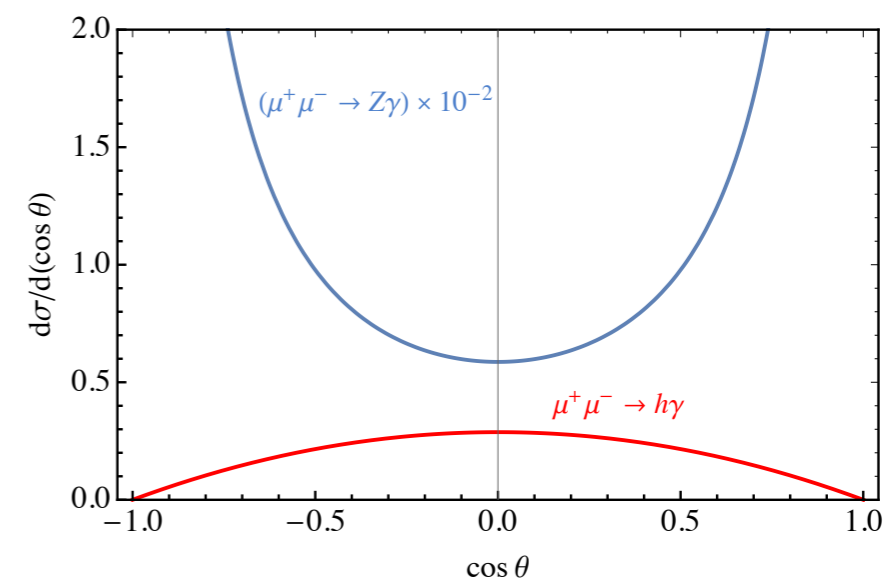
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- Main background from $\mu\mu \rightarrow Z\gamma$ (where Z is mistaken for H)
(large due to transverse Z polarizations)

$$\frac{d\sigma_{\mu\mu\rightarrow h\gamma}}{d\cos\theta} = \frac{|C_{e\gamma}^\mu(\Lambda)|^2}{\Lambda^4} \frac{s}{64\pi} (1 - \cos^2\theta)$$

$$\frac{d\sigma_{\mu\mu\rightarrow Z\gamma}}{d\cos\theta} = \frac{\pi\alpha^2}{4s} \frac{1 + \cos^2\theta}{\sin^2\theta} \frac{1 - 4s_W^2 + 8s_W^4}{s_W^2 c_W^2}$$



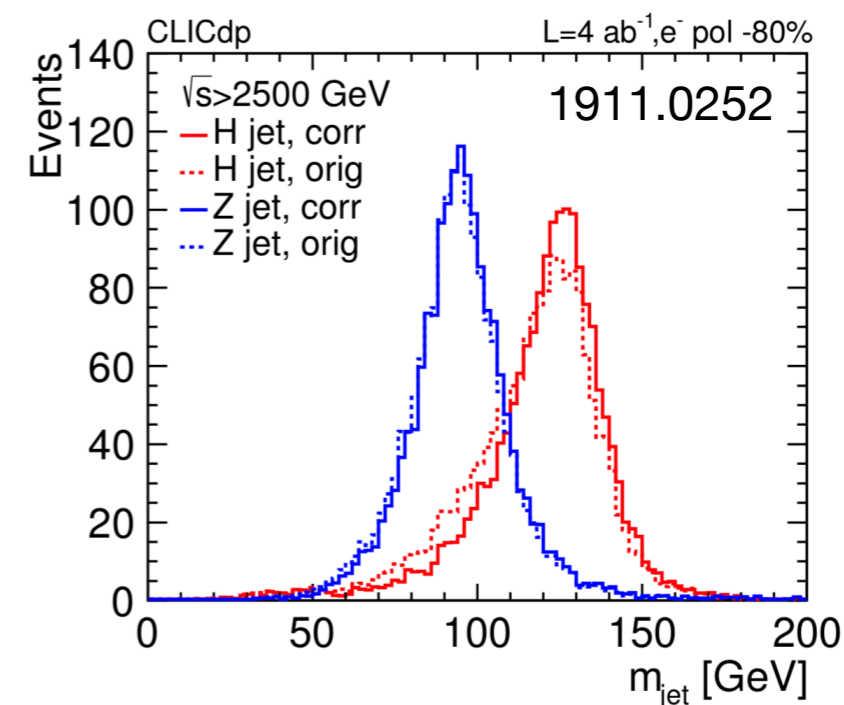
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H $\tau\tau$ signal

New physics:
$$\frac{d\sigma_{NP}}{dx_1 dx_2} = \frac{1}{192\pi^3} \frac{Q^2}{\Lambda^4} \left[(Q_\mu e C_\gamma)^2 + \left(\frac{g C_Z}{4c_w} \right)^2 \right] (2x_1 x_2 - x_1 - x_2 + 1)$$

Interference:

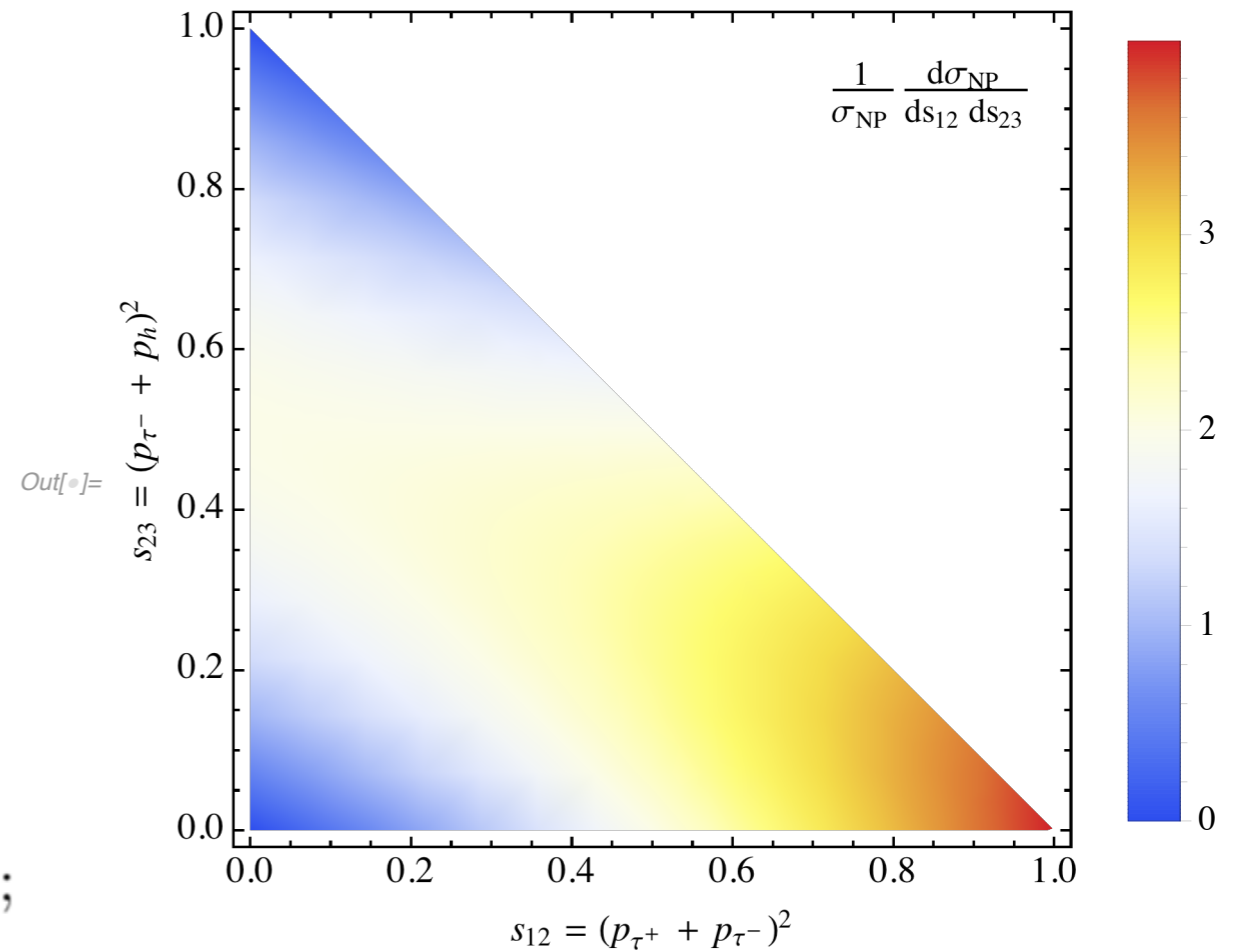
$$\frac{d\sigma_{N\gamma}}{dx_1 dx_2} = -\frac{\sqrt{2}}{96\pi^3} \frac{m_\tau}{v} \frac{e^3}{\Lambda^2} Q_\mu C_\gamma;$$

Standard Model:

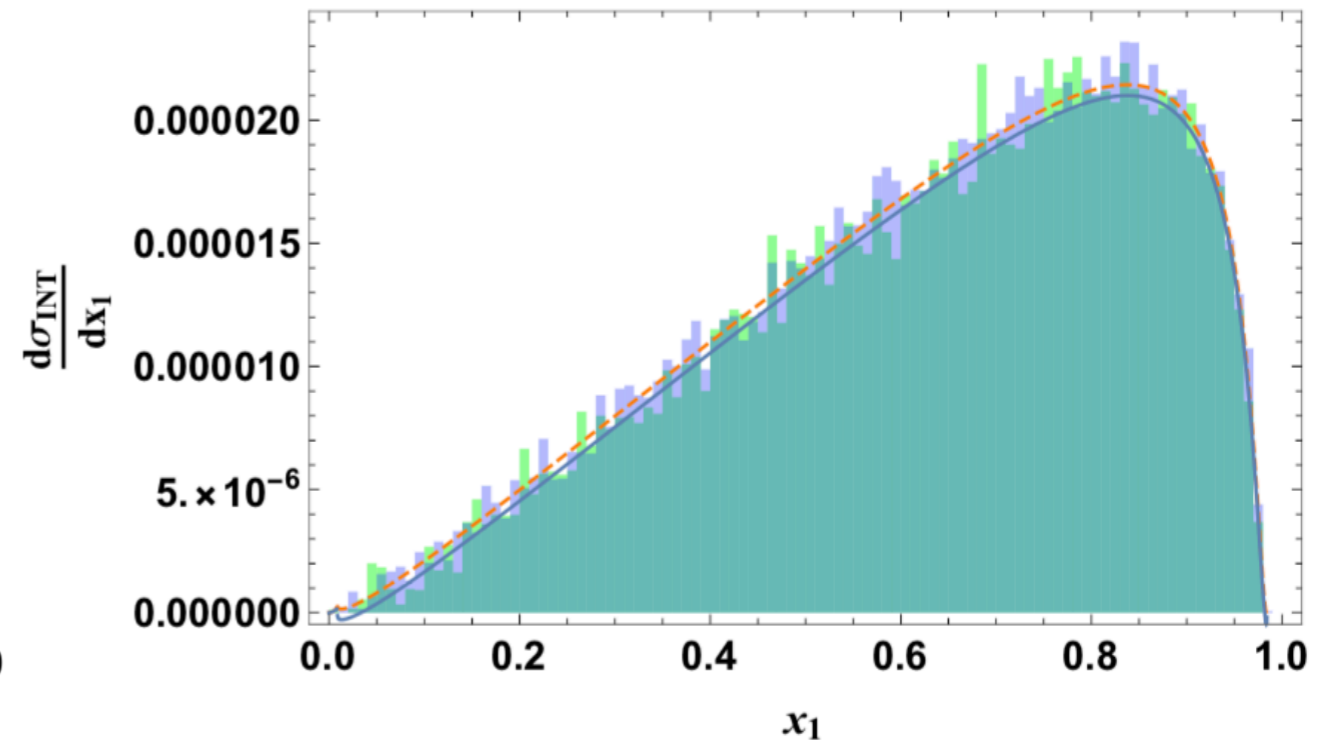
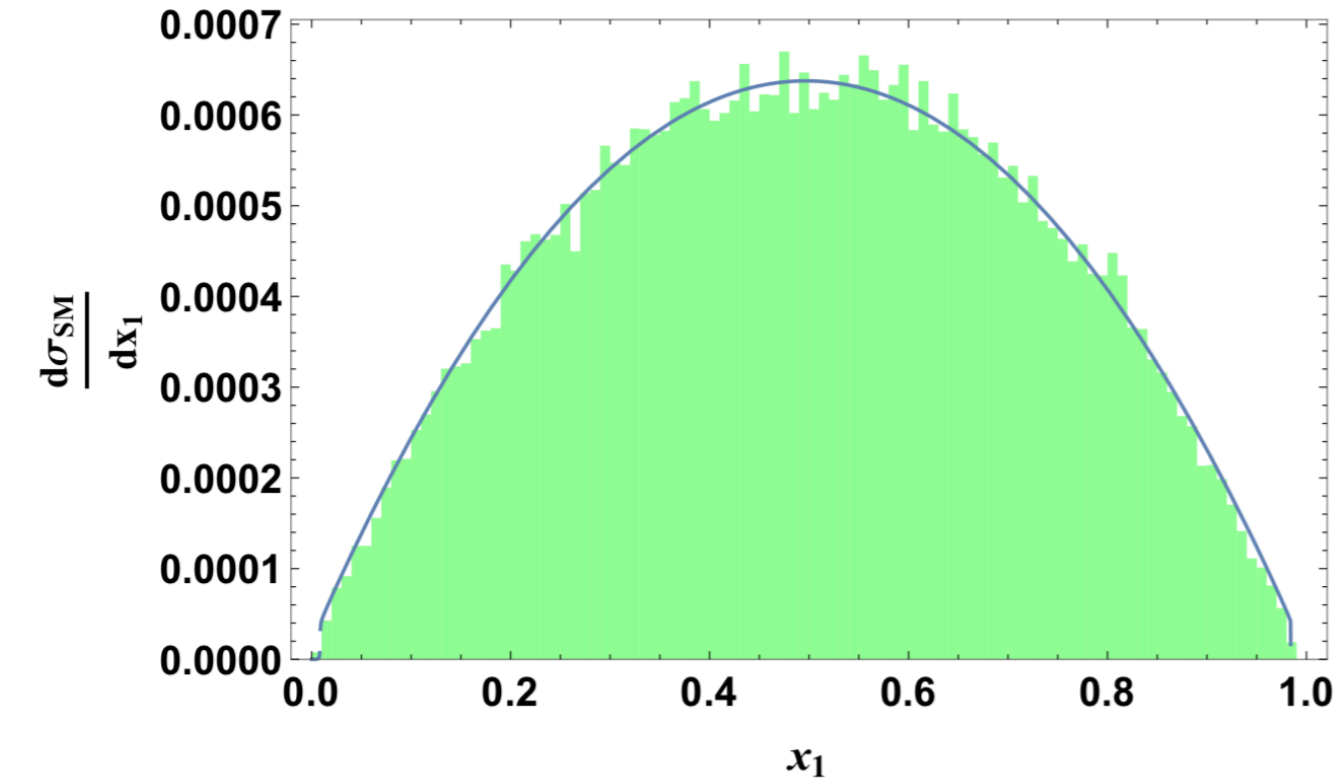
$$\frac{d\sigma_\gamma}{dx_1 dx_2} = \frac{1}{192\pi^3} \frac{m_\tau^2 e^4}{v^2} \frac{1}{Q^2} \left[\frac{1-x_1}{1-x_2} + 2 + \frac{1-x_2}{1-x_1} \right];$$

$$\frac{d\sigma_2}{dx_1 dx_2} = \frac{1}{3 \cdot 2^{14} \pi^3} \frac{m_\tau^2 g^4}{v^2 c_w^4} \frac{1}{Q^2} \left[\frac{1-x_1}{1-x_2} - 2 + \frac{1-x_2}{1-x_1} \right];$$

$$\frac{d\sigma_3}{dx_1 dx_2} = \frac{1}{6 \cdot 2^{16} \pi^3} \frac{g^8}{c_w^8} \frac{v^2}{Q^4} \left[\frac{x_1 x_2 + x_1 + x_2 - 1}{(x_1 + x_2 - 1)^2} \right];$$



H $\tau\tau$ signal



(c)

