SM PDFs: why and how

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LePDF

Francesco Garosi, D.M., Sokratis Trifinopoulos
JHEP 09 (2023) 107 [**2303.16964**]

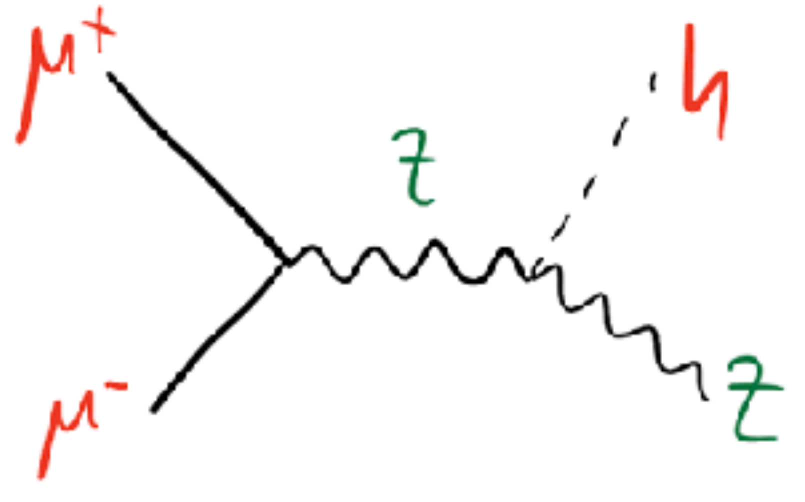
Source + Downloads available at
https://github.com/DavidMarzocca/LePDF

Based on several previous works, most notably:
P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0001142, hep-ph/0505047],
Bauer, Webber [1703.08562, 1808.08831],
Chen, Han, Tweedie [1611.00788], Han, Ma, Xie [2007.14300, **2103.09844**]
Azatov, Garosi, Greljo, DM, Salko, Trifinopoulos [2205.13552]

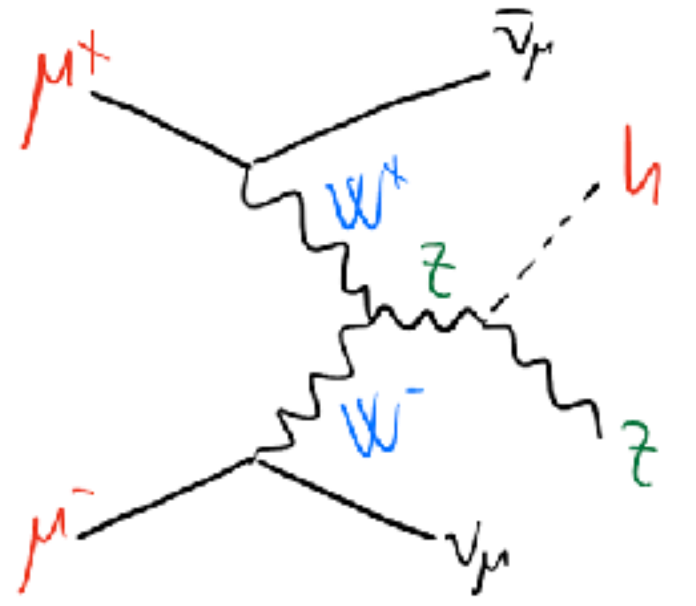
Muon Collider Physics Benchmark Workshop 2023 - 17/11/2023

MuC is a Vector Boson Collider

At muon colliders above ~1 - 5 TeV,
the VBF process dominates over annihilation:
mostly collinear emission.



$$\sigma_{ann} \propto \frac{\alpha^2}{s}$$



$$\frac{d\sigma_{VBF}}{dm_h^2} \propto \frac{\alpha^2}{m_{hZ}^2}\,\alpha^2 \log^2\frac{m_{hZ}^2}{m_W^2}\,\log\frac{s}{m_{hZ}^2}$$

[Costantini et al. 2005.10289, …]

If $p_T(W),\ m_W \ll E_{hard}$ the emission of **EW collinear radiation** (photon, *W*, *Z*, etc..)
off a muon can be factorised from the hard scattering. [Cuomo, Vecchi, Wulzer 1911.12366, …]

This can be described in terms of
generalised Parton Distribution Functions, like for proton colliders:

$$\sigma(\mu\bar{\mu} \to C + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{ij} f_i(x_1,Q)\, f_j(x_2,Q)\, \hat{\sigma}(ij \to C)(\hat{s})$$

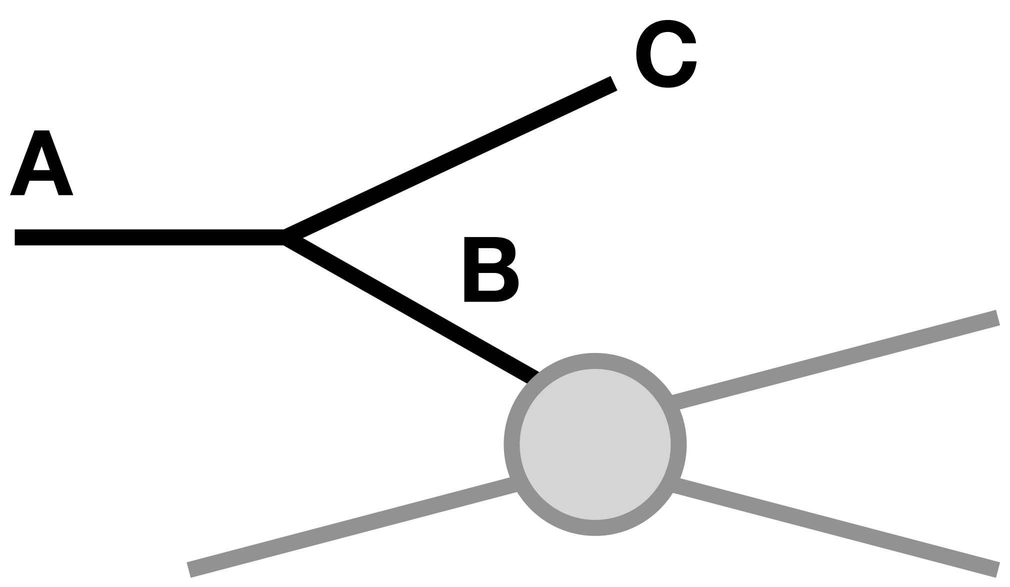
PDFs of a muon

Unlike for protons, since the muon is elementary this can be done **from first principles**.

The **boundary condition** is set by $f_\mu(x, m_\mu) = \delta(1\text{-}x) + O(\alpha)$, $\quad f_{i\neq\mu}(x, m_\mu) = 0 + O(\alpha)$

NLO corrections in Frixione [1909.03886]

The **SM DGLAP equations** describe the evolution of the PDFs (QED+QCD below m_W, full SM above)



$$Q^2 \frac{df_B(x,Q^2)}{dQ^2} = P_B^v\, f_B(x,Q^2) + \sum_{A,C} \frac{\alpha_{ABC}}{2\pi} \widetilde{P}_{BA}^C \otimes f_A + \frac{v^2}{16\pi^2 Q^2} \sum_{A,C} \widetilde{U}_{BA}^C \otimes f_A$$

Virtual corrections

Real emission

ultra-collinear terms (EWSB)

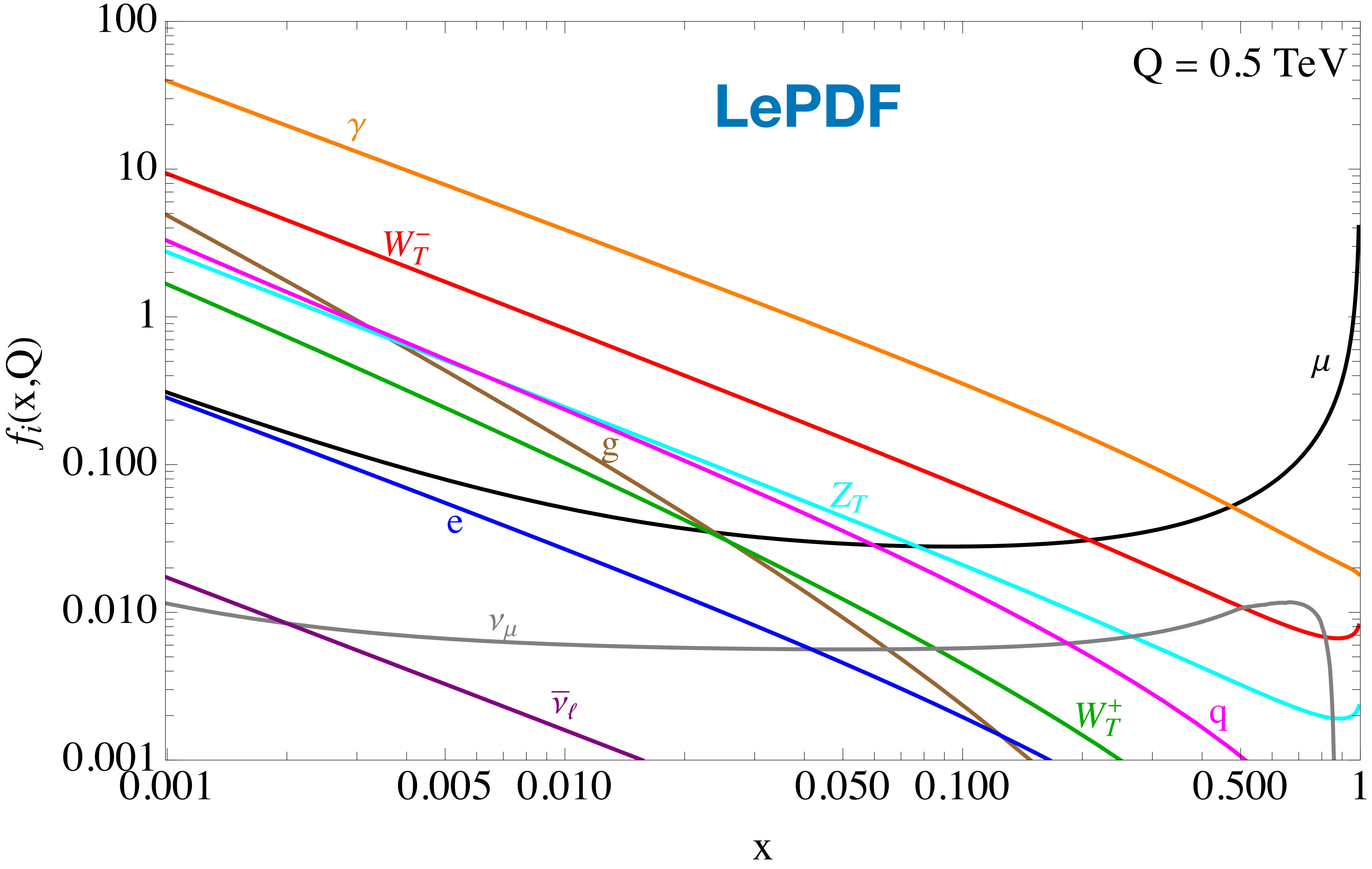
Chen, Han, Tweedie [1611.00788]

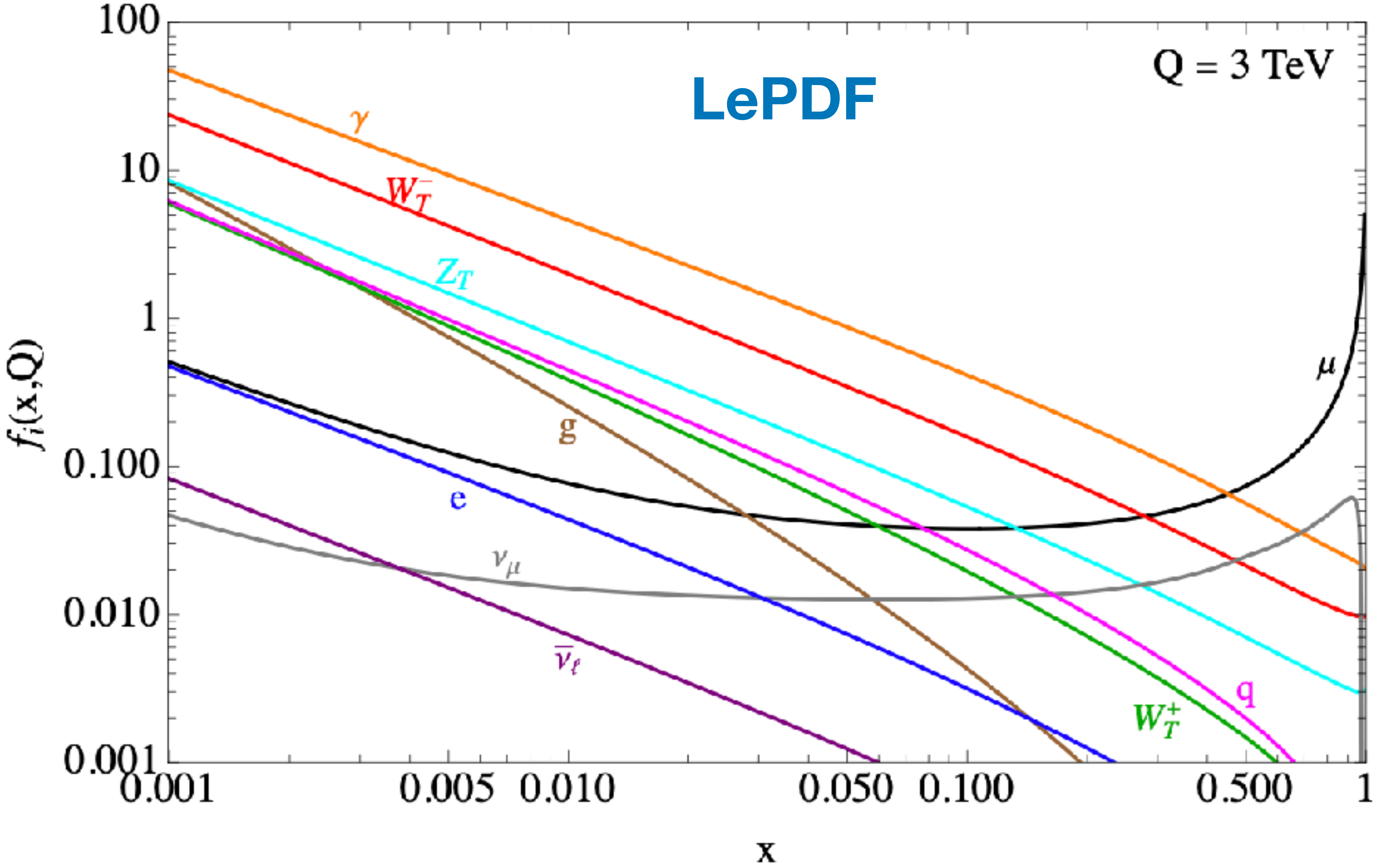
Sudakov double-logs (for initial-state radiation)

In case of collinear W emission they can be implemented (and resummed)
at the **Leading Log** level by putting an **explicit IR cutoff** $z_{max} = 1 - Q_{EW}/Q$

M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0111109]
Bauer, Ferland, Webber [1703.08562]

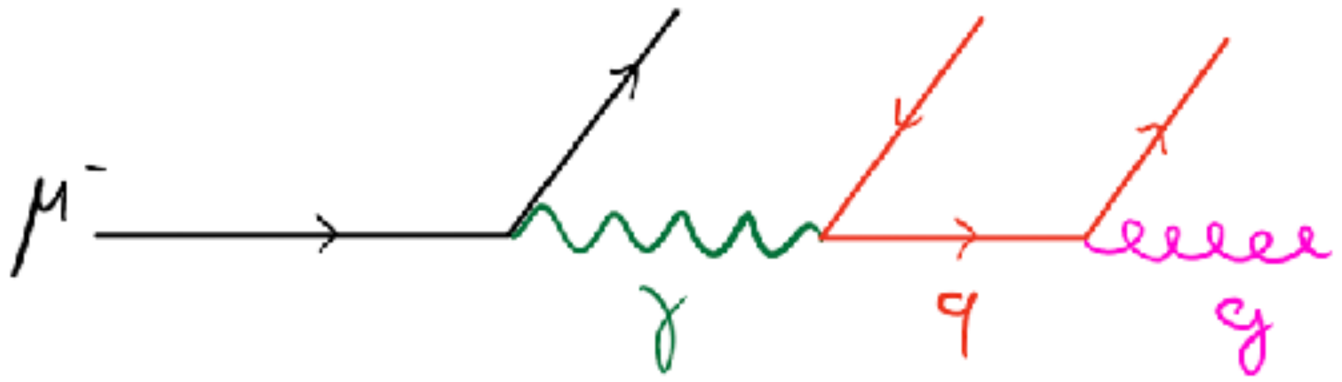
PDFs of a muon





- **Large EW boson PDFs**, above EW scale and small x
- Non negligible **gluon** and **quark** content.

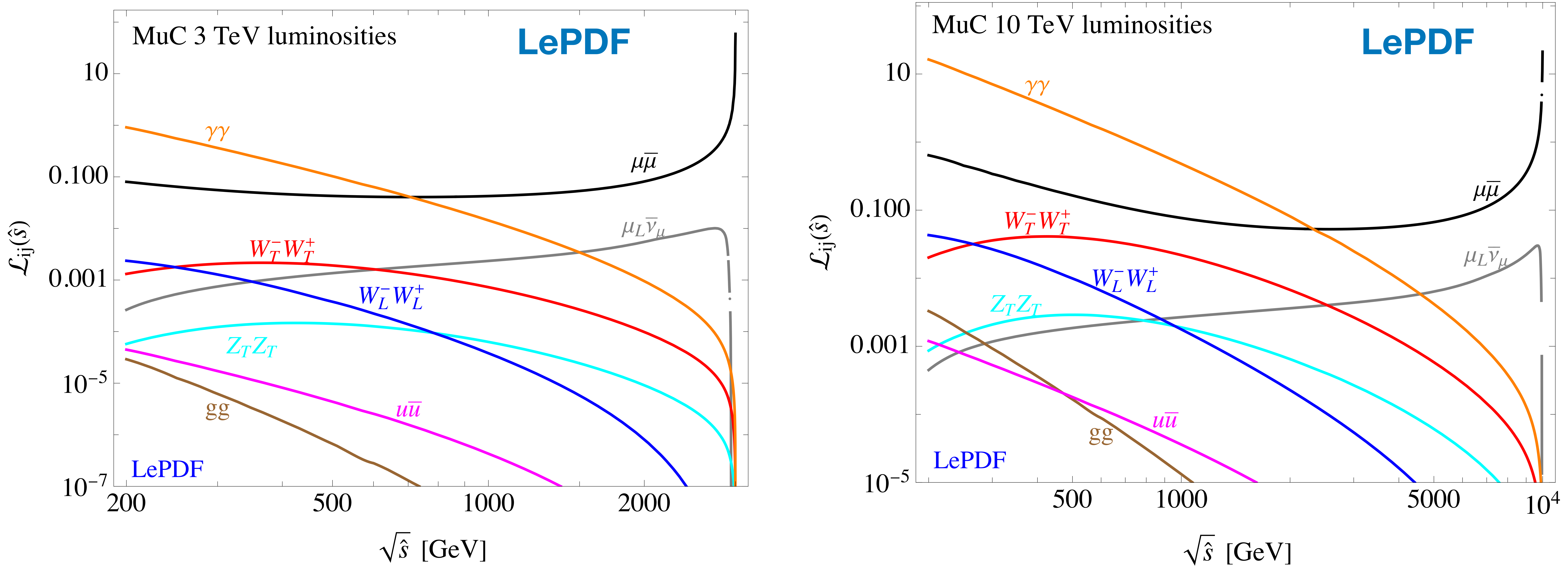
Han, Ma, Xie [2007.14300, 2103.09844]



PDFs of a muon

Some examples of **parton luminosities** for muon colliders.

$$\mathcal{L}_{ij}(\hat{s}) = \int_{\hat{s}/s_0}^{1} dx \frac{1}{x} f_i^{(\mu)}\left(x, \frac{\sqrt{\hat{s}}}{2}\right) f_j^{(\bar{\mu})}\left(\frac{\hat{s}}{x s_0}, \frac{\sqrt{\hat{s}}}{2}\right)$$



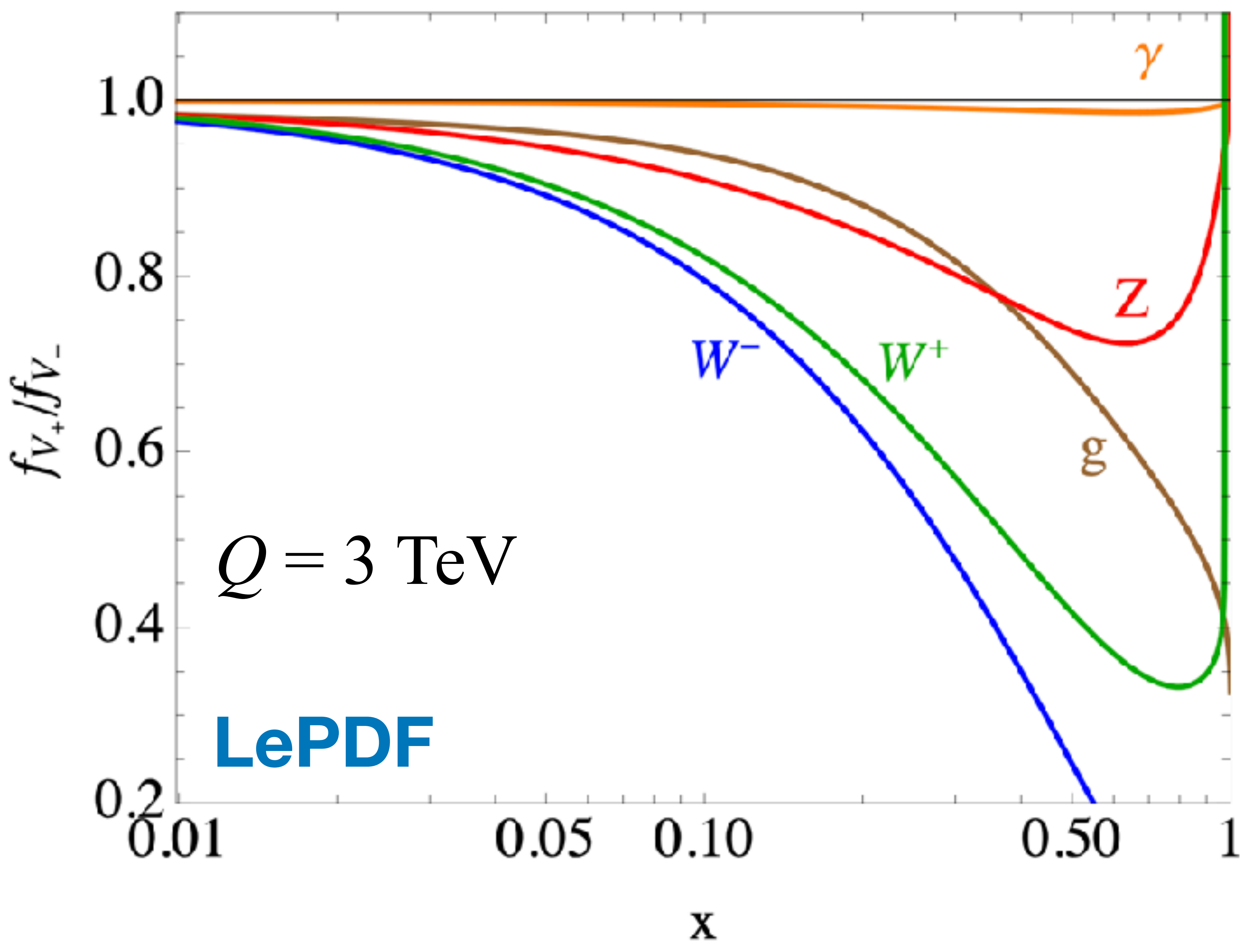
Some comments:

- The **very large $\gamma\gamma$ lumi** could dominate over Z and Z/γ contributions.
- **gluon and quark luminosities are small**: suppressed impact of QCD-induced backgrounds.

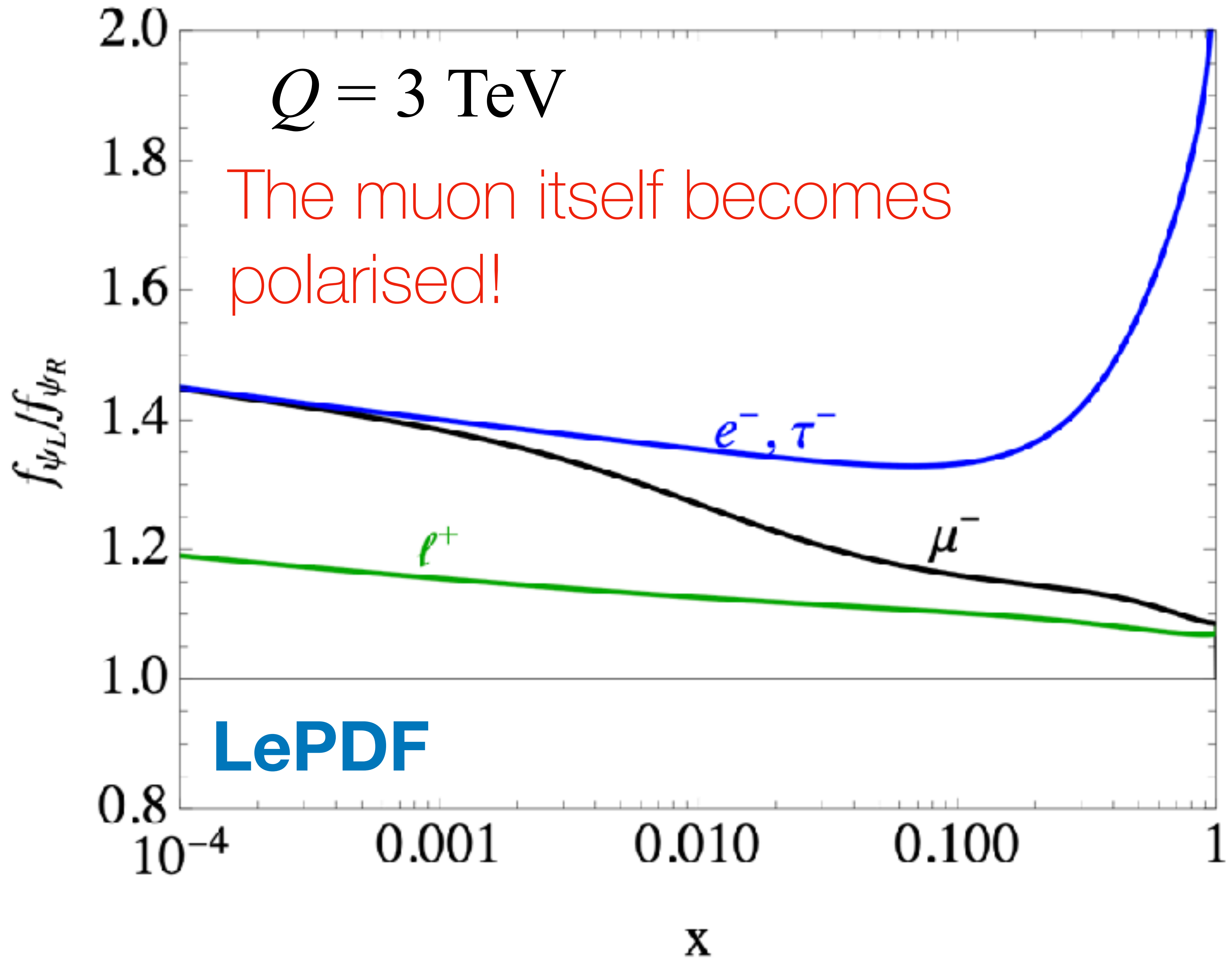
Polarisation

Since EW interactions are chiral, PDFs become polarised. Bauer, Webber [1808.08831]

Vectors polarisation: V_+ / V_-



Fermions polarisation: ψ_L / ψ_R

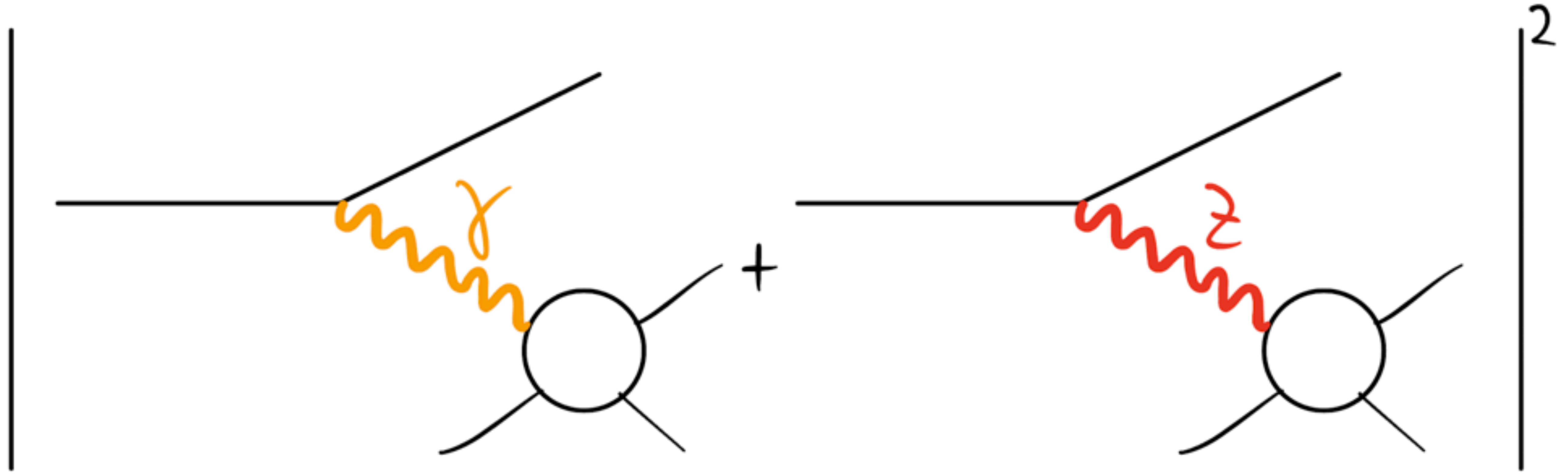


O(1) polarisation effects! (except for photon PDF)

E.g. in case of W^- PDF, coupled to μ_L, the PDF for RH W's goes to zero for $x \to 1$ faster than LH W's, since $P_{V+f_L}(z) = (1-z)/z$ while $P_{V-f_L}(z) = 1/z$.

Photon - Z mixing

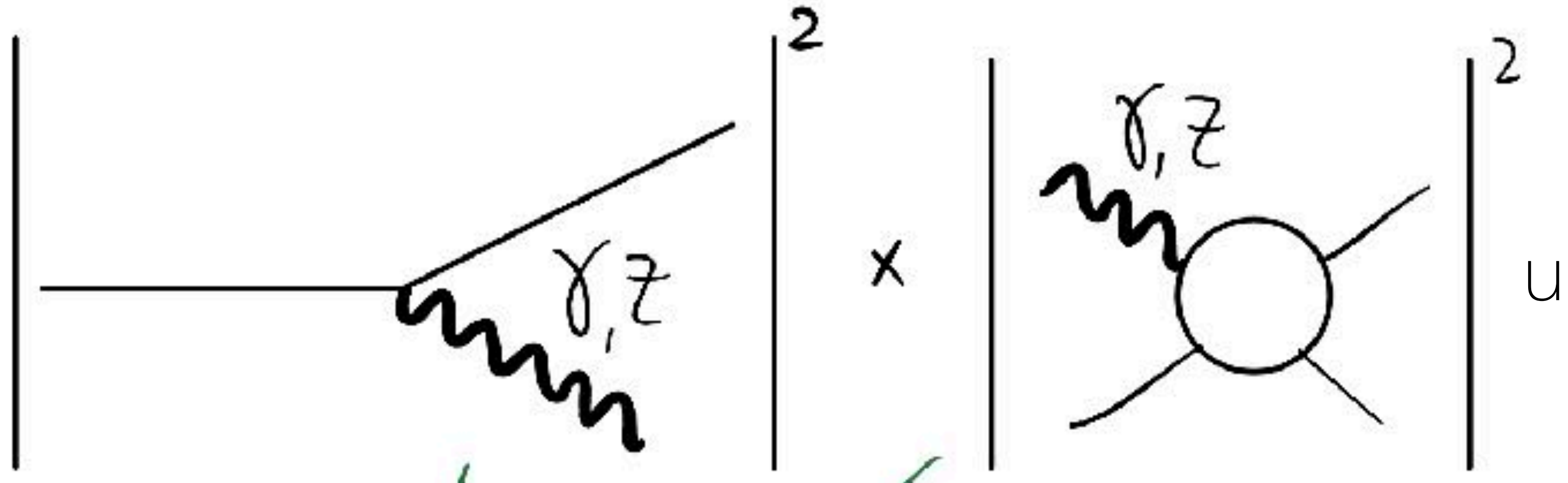
Photon and Z bosons can interfere



In the collinear limit this can be described by a **mixed Z/γ PDF**.
Similarly for Z_L and H.

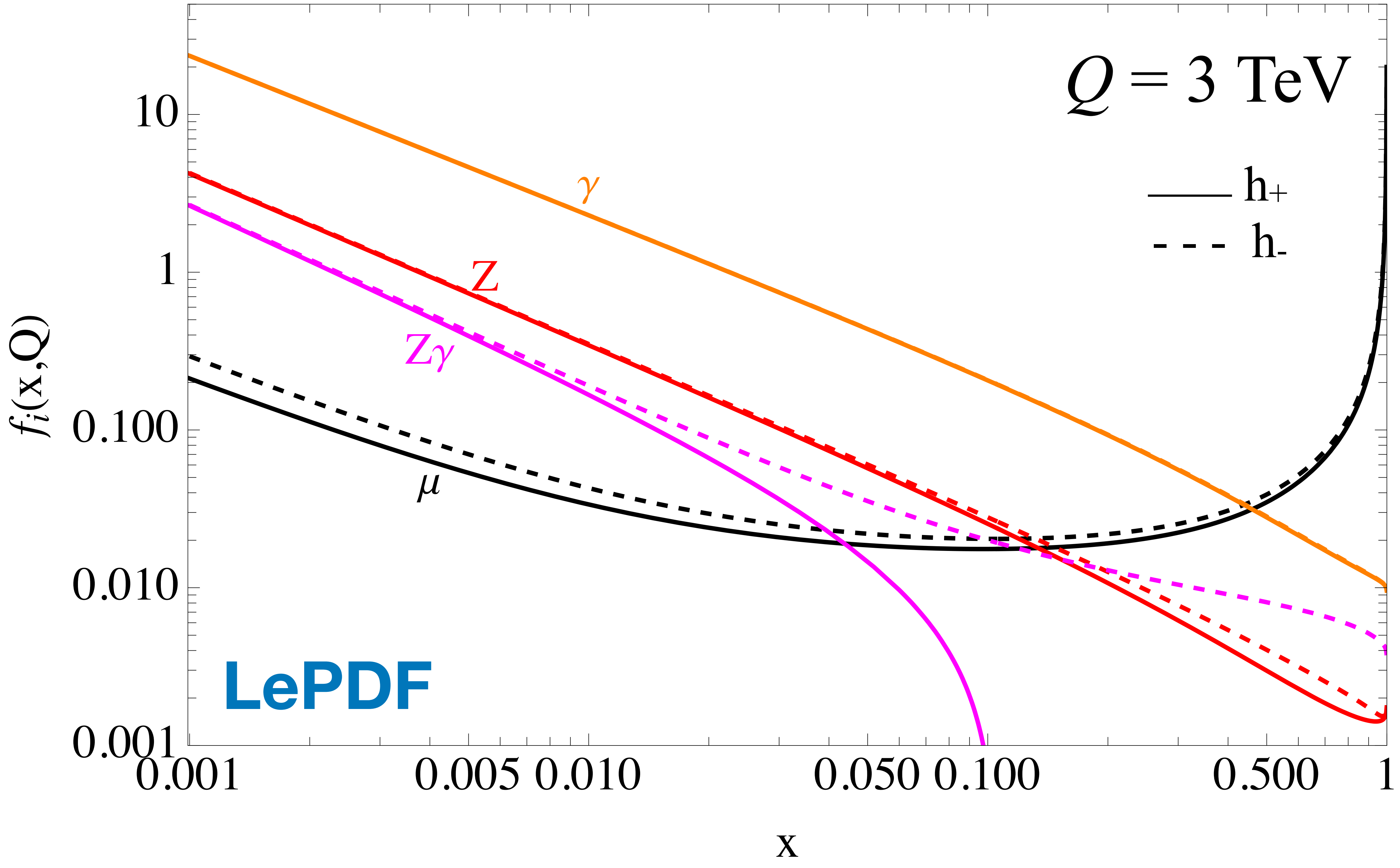
P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0505047]
Chen, Han, Tweedie [1611.00788]

The splitting function must be generalised to a splitting matrix.
The rate is computed by tracing against the matrix of the hard scattering process



up to $O(k_T^2/E^2, m^2/E^2)$

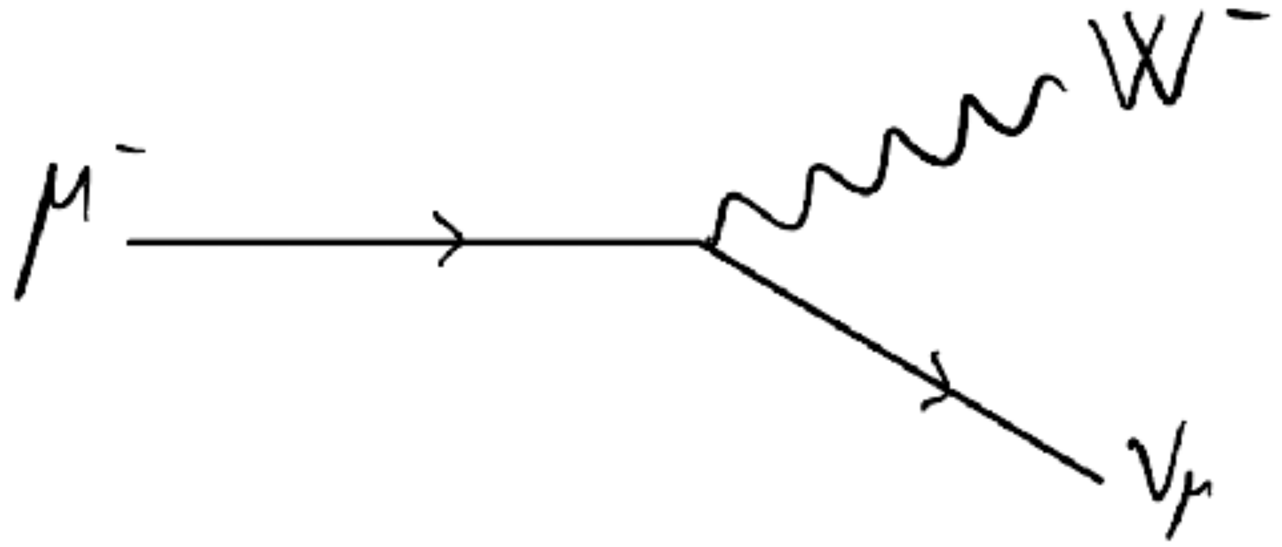
$$\sigma \propto \left[\frac{dP_{A\to BC}}{dz\, dk_T^2}\right]_{ij} \hat{\sigma}_{ij}\,, \quad i,j = \gamma, Z$$



Effective Vector Boson Approximation

At energies **above the EW scale**, collinear emission of EW gauge boson can be described at fixed log with the **Effective Vector Boson Approximation**

Fermi ('24) Weizsacker, Williams ('34) Landau, Lifschitz ('34) Kane, Repko, Rolnik; Dawson; Chanowitz, Gaillard '84, See also Borel et al. [1202.1904], Costantini et al. [2005.10289] Ruiz et al. [2111.02442], etc…



With W-mass effects:

$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) = \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x)\left(\log\frac{Q^2+(1-x)m_W^2}{m_\mu^2+(1-x)m_W^2} - \frac{Q^2}{Q^2+(1-x)m_W^2}\right)$$

$$f^{(\alpha)}_{W^-_L}(x,Q^2) = \frac{\alpha_2}{4\pi}\frac{1-x}{x}\frac{Q^2}{Q^2+(1-x)m_W^2}$$

(similar expressions also for Z_T, Z_L, Z/γ)

For $Q \gg m_W$:

$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) \approx \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x)\log\frac{Q^2}{m_W^2}$$

← This one is now implemented in `MadGraph5_aMC@NLO` [Ruiz, Costantini, Maltoni, Mattelaer 2111.02442]

NOTE: mass effects remain of O(1) also at TeV scale! Chen, Han, Tweedie [1611.00788]

Do we need SM/EW PDFs?

Collinear factorisation works if $p_T,\ m_W \ll E_{hard}$, so it can be **viable for a 3 TeV MuC**.

Particularly **useful for processes well below threshold** $E_{hard} \ll E_{collider}$ (e.g. production of EW final states).

The $W,\ Z$ PDFs are suppressed compared to the photon one by a factor $\sim \log m_W^2/m_\mu^2 \sim O(10)$.
Nevertheless, they induce the **dominant contribution in a large class of processes** (vector boson collider).

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Why not just EVA?

- ★ For **QCD (gluon and quarks) DGLAP resummation is required** since α_s is large at small scales.
- ★ The expected **relative corrections to the LO EVA** result are proportional to (*Sudakov double logs*) $\alpha_2 \left(\log \frac{Q^2}{M_W^2}\right)^2 \sim 1$ for $Q \sim 1.5$ TeV. still sizeable at lower Q.

For precise vector boson PDFs at the TeV scale it is important to re-sum the EW double logs.

→ **PDF approach** M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0111109]
Bauer, Ferland, Webber [1703.08562]

LePDF vs. EVA

EVA$_{\mathbf{LO}}$: $f^{(\alpha)}_{W^-_\pm}(x,Q^2) = \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x)\left(\log\frac{Q^2+(1-x)m_W^2}{m_\mu^2+(1-x)m_W^2} - \frac{Q^2}{Q^2+(1-x)m_W^2}\right)$

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We can expect large deviations from EVA, since $\alpha_2\left(\log\frac{Q^2}{m_W^2}\right)^2 \sim 1$ for $Q \sim 1.5$ TeV.

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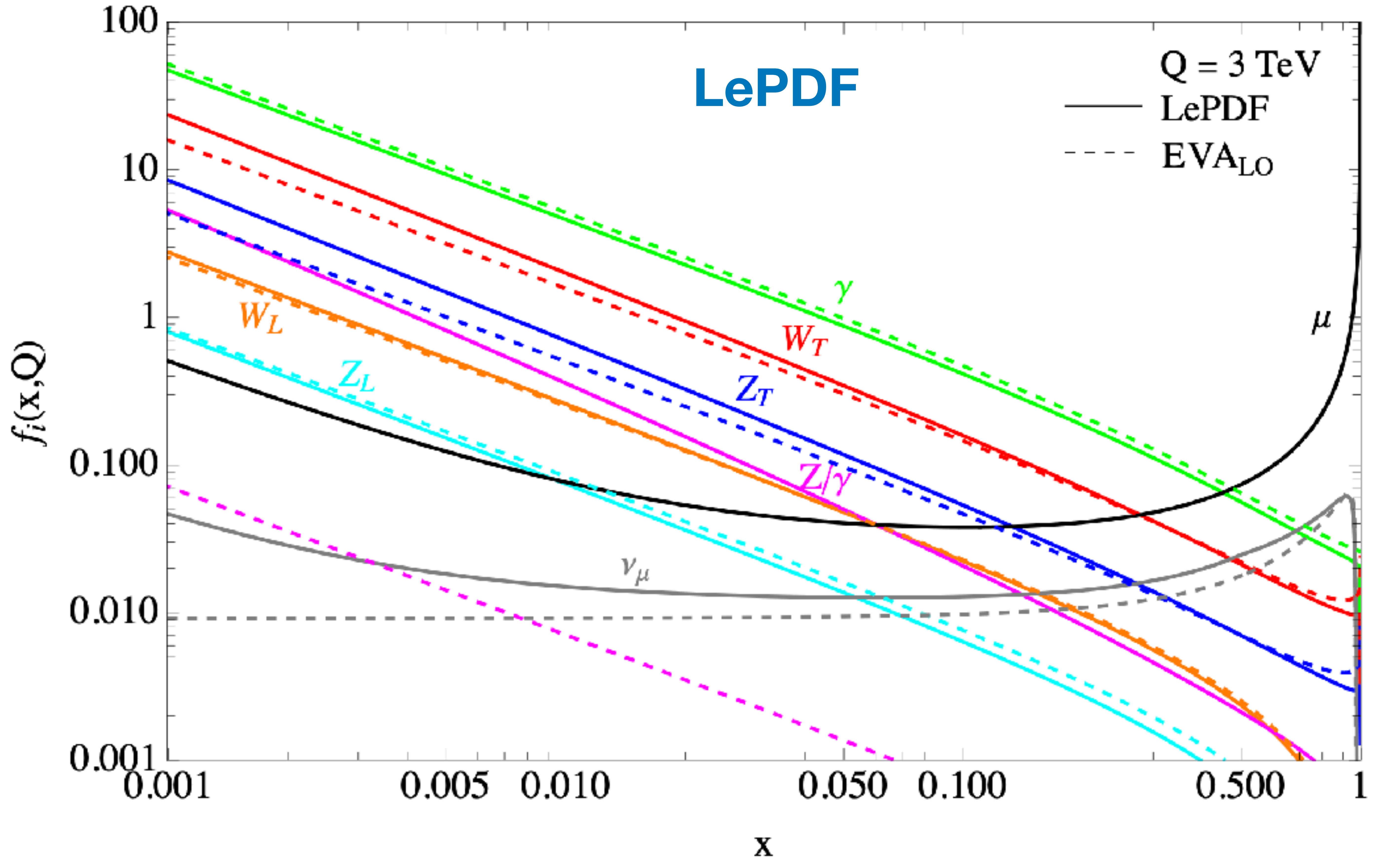

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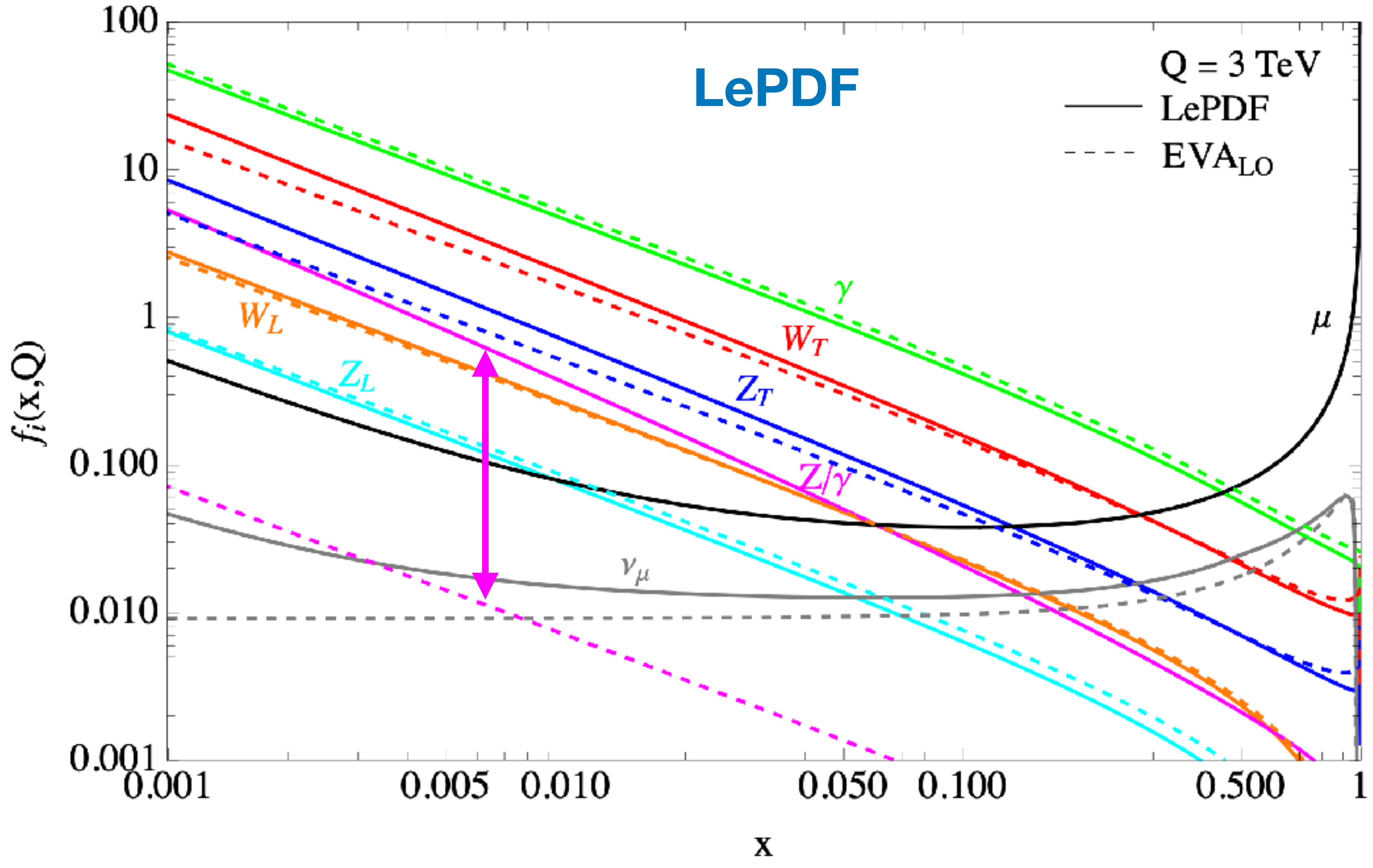

- The **EVA Z/γ PDF is off by ~10²**, due to the fact that in EVA the muon is taken unpolarised and

$$Q^Z_{\mu_L} + Q^Z_{\mu_R} = -\tfrac{1}{2} + 2s_W^2 \ll 1$$

Instead, the muon gains a O(1) polarisation, so the actual Z/γ PDF is much larger.

$$f^{(\alpha)}_{Z/\gamma_\pm}(x,Q^2) = -\frac{\sqrt{\alpha_\gamma\alpha_2}}{2\pi c_W}\left(P^f_{V_\pm f_L}(x)Q^Z_{\mu_L} + P^f_{V_\pm f_R}(x)Q^Z_{\mu_R}\right)\log\frac{Q^2+(1-x)m_Z^2}{m_\mu^2+(1-x)m_Z^2}$$

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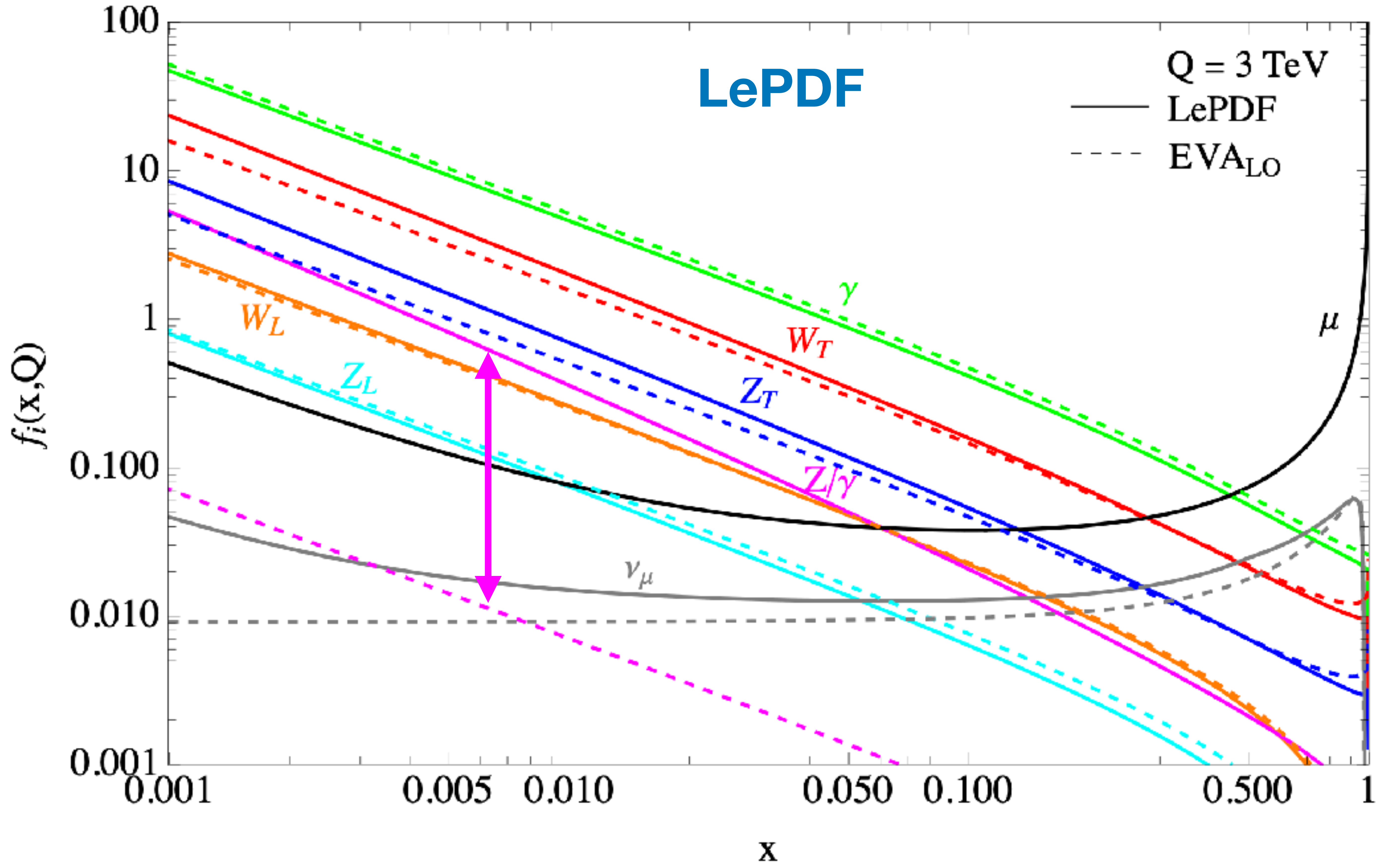

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- We can also see a **sizeable deviation** (in this log-log plot) for the **W_T** and **Z_T** PDF. Mostly due to the double-log arising at O(α^2) from VV interactions.

How to use SM PDFs?

For each parton i, we export the numerical values of $x\, f_i(x, Q)$ for a grid in x and Q.
These are typically saved in a .dat file with the standard **LHAPDF6** format.

LHAPDF6: Buckley et al. [1412.7420]

These can also be loaded into Mathematica for semi-analytical studies.

MonteCarlo Generators (e.g. Madgraph) can read this and interpolate the PDFs using LHAPDF.

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The problems

1) **LHAPDF6** classifies particles according to the PDG index: **no helicity dependence**.

Vs.

Polarisation is important in our case.

2) The use of **interference PDFs like Z/γ or Z_L/h** is not implemented in today's generators.

1) Polarisation & generators

Our solution

In **LePDF** **we modified LHAPDF6** format by **including helicity dependence**: adding a line on the file with the helicity of each state. E.g.

μ_L	muL	13	-
μ_R	muR	13	+
ν_μ	numu	14	-

W^+_+	Wpp	24	+
W^+_-	Wpm	24	-
W^+_L	WpL	24	0

LePDF_mu_6FS_0000.dat

```
1   PdfType: central
2   Format: lhagrid1
3   ---
4   1.0000000e-006  1.1877330e-006  1.4088890e-006  1.6690720e-006  1.9747630e-006  2.3334480e-006
```

(x Grid)

```
... 1.0000000e+000
5   1.0984270e+001  1.3403190e+001  1.6354800e+001  1.9956410e+001  2.4351160e+001  2.9713710e+001
```

(Q Grid)

```
... 4.6895550e+004  5.7222760e+004
6   eL  eR  nue muL muR numu    taL taR nuta    eLb eRb nueb    muLb    muRb    numub   taLb
... taRb    nutab   dL  dR  uL  uR  sL  sR  cL  cR  bL  bR  tL  tR  dLb dRb uLb uRb sLb sRb cLb cRb
... bLb bRb tLb tRb gp  gm  gap gam Zp  Zm  ZL  Zgap    Zgam    Wpp Wpm WpL Wmp Wmm WmL h   hZL
7   11  11  12  13  13  14  15  15  16  -11 -11 -12 -13 -13 -14 -15 -15 -16 1   1   2   2   3   3
... 4   4   5   5   6   6   -1  -1  -2  -2  -3  -3  -4  -4  -5  -5  -6  -6  21  21  22  22  23  23
... 23  2223    2223    24  24  24  -24 -24 -24 25  2523
8   -   +   -   -   +   -   -   +   -   +   -   +   +   -   +   +   -   +   -   +   -   +   -   +
... -   +   -   +   -   +   +   -   +   -   +   -   +   -   +   -   +   -   +   -   +   -   +   -
... 0   +   -   +   -   0   +   -   0   0   0
```

In this way we can use these files to load **LePDF** for simple phenomenological analyses in Mathematica.

Using them in Madgraph, however, would *require changes in both LHAPDF and Madgraph* codes.

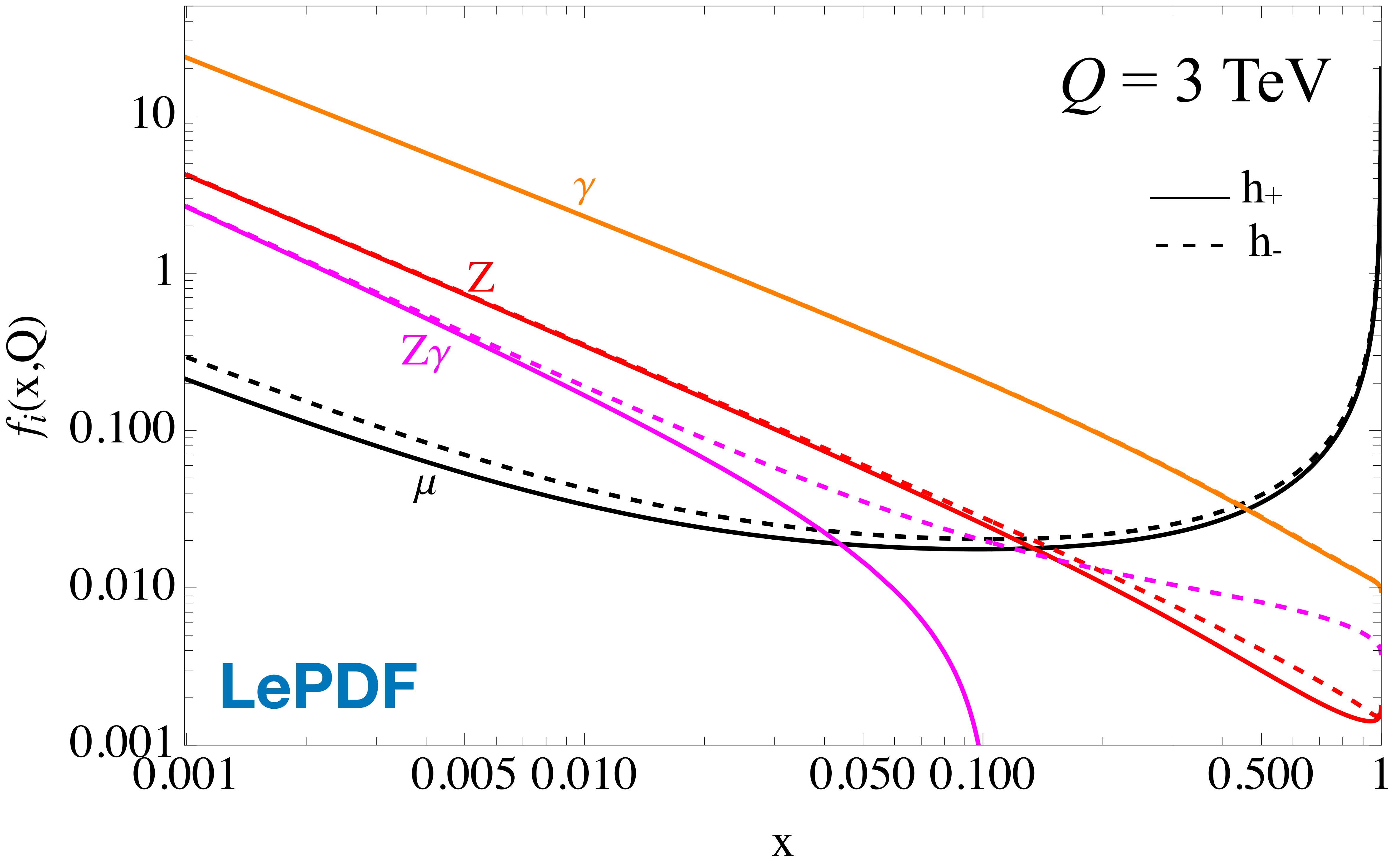
An alternative

We can also **use to the standard LHAPDF6 format**, creating one **separate file for each helicity** state:

LePDF_mu_6FS_+_0000.dat LePDF_mu_6FS_0_0000.dat LePDF_mu_6FS_-_0000.dat

Would this help implementation in Madgraph? No change needed in LHAPDF.

2) Z/γ & generators



Implementing this effect in generators probably requires deep changes (mixed states).

$$f_{Z/\gamma} \sim f_Z \ll f_\gamma$$

However, the suppression of the Z and Z/γ PDFs w.r.t. the photon one could imply that the mixed PDF contribution is suppressed compared to the diagonal ones.

This should be quantified with explicit examples.

This might be a secondary issue, compared to the polarisation.

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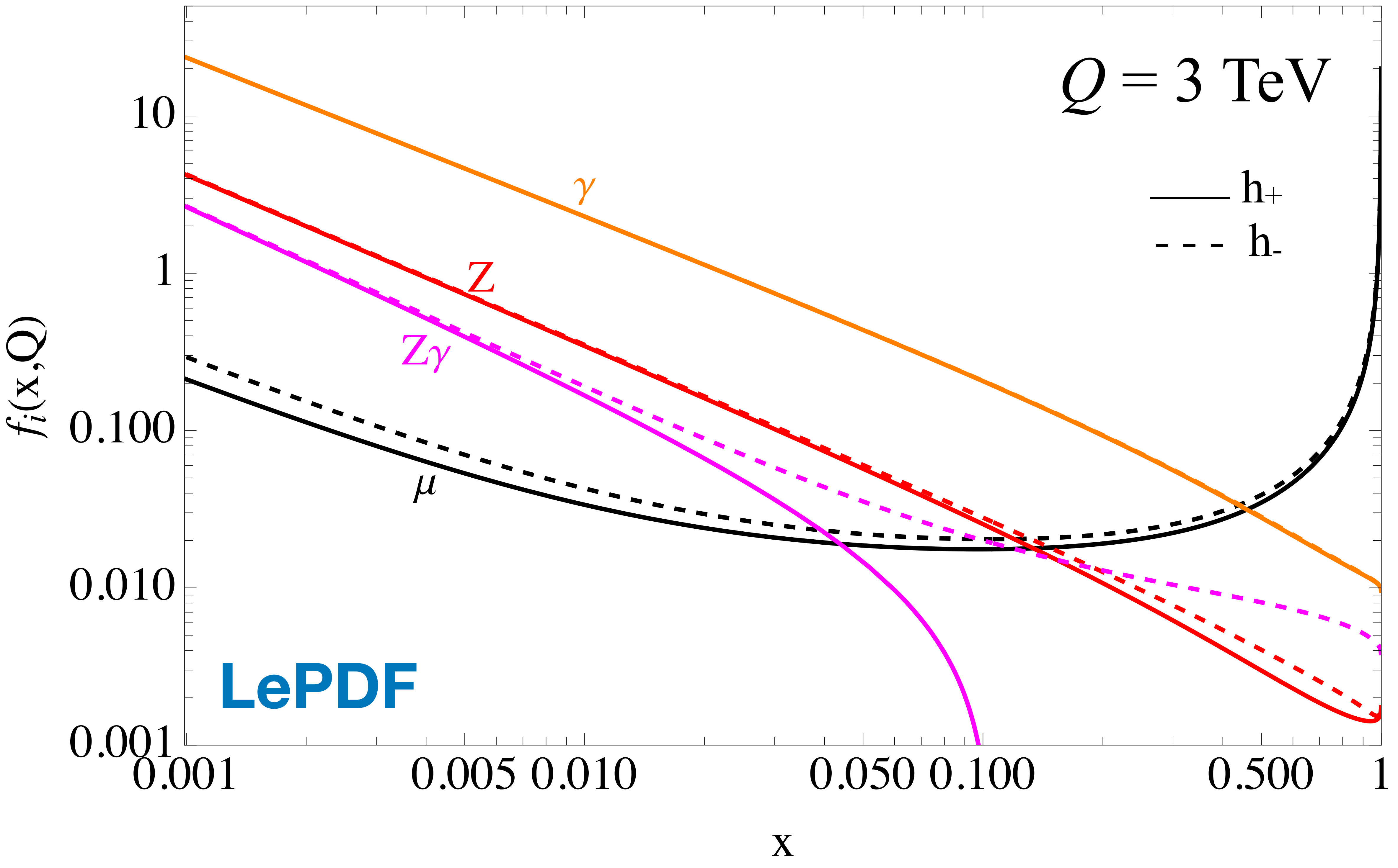

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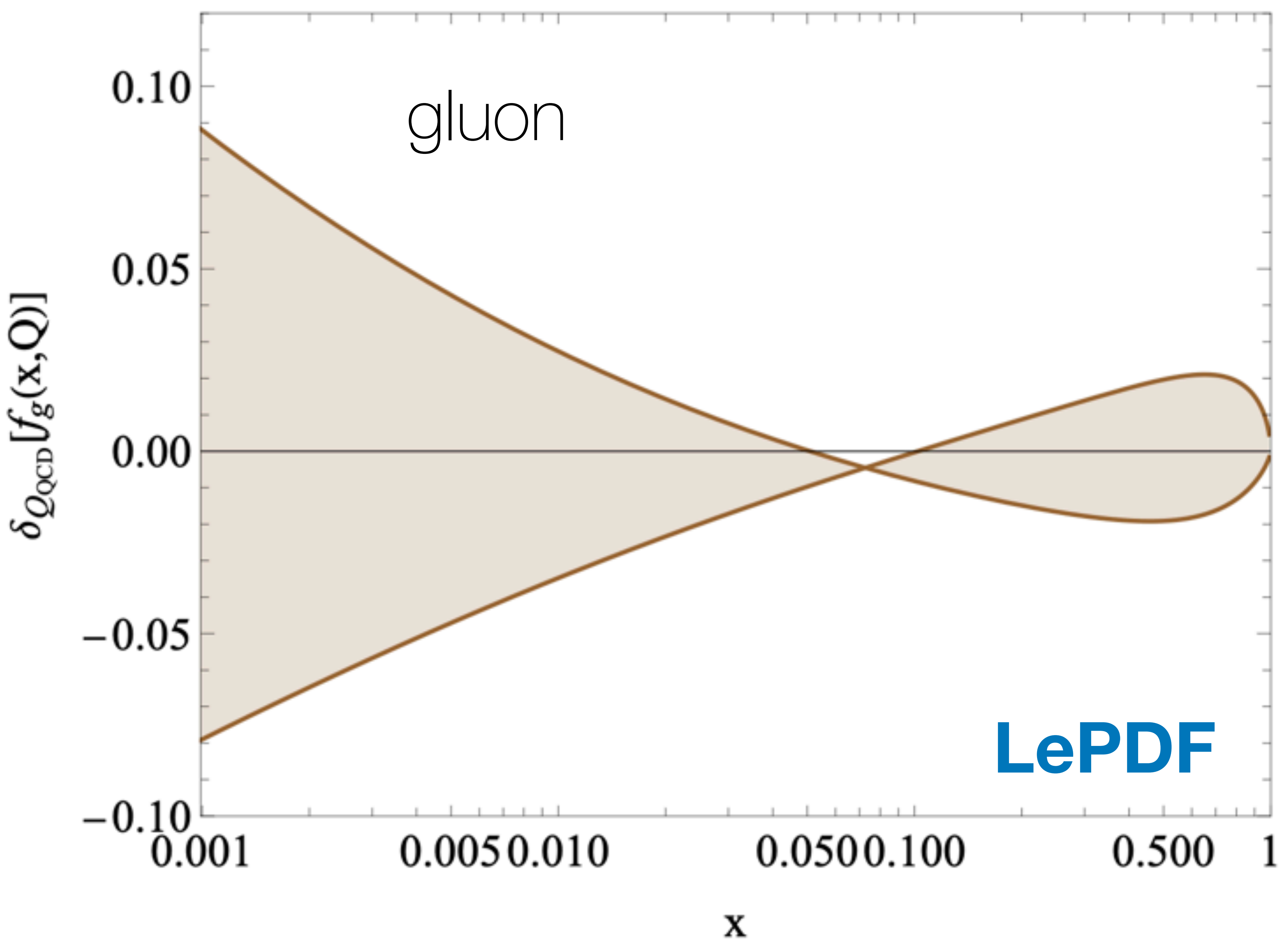
Discussion point: how can these PDFs be implemented to allow for their use in Madgraph?

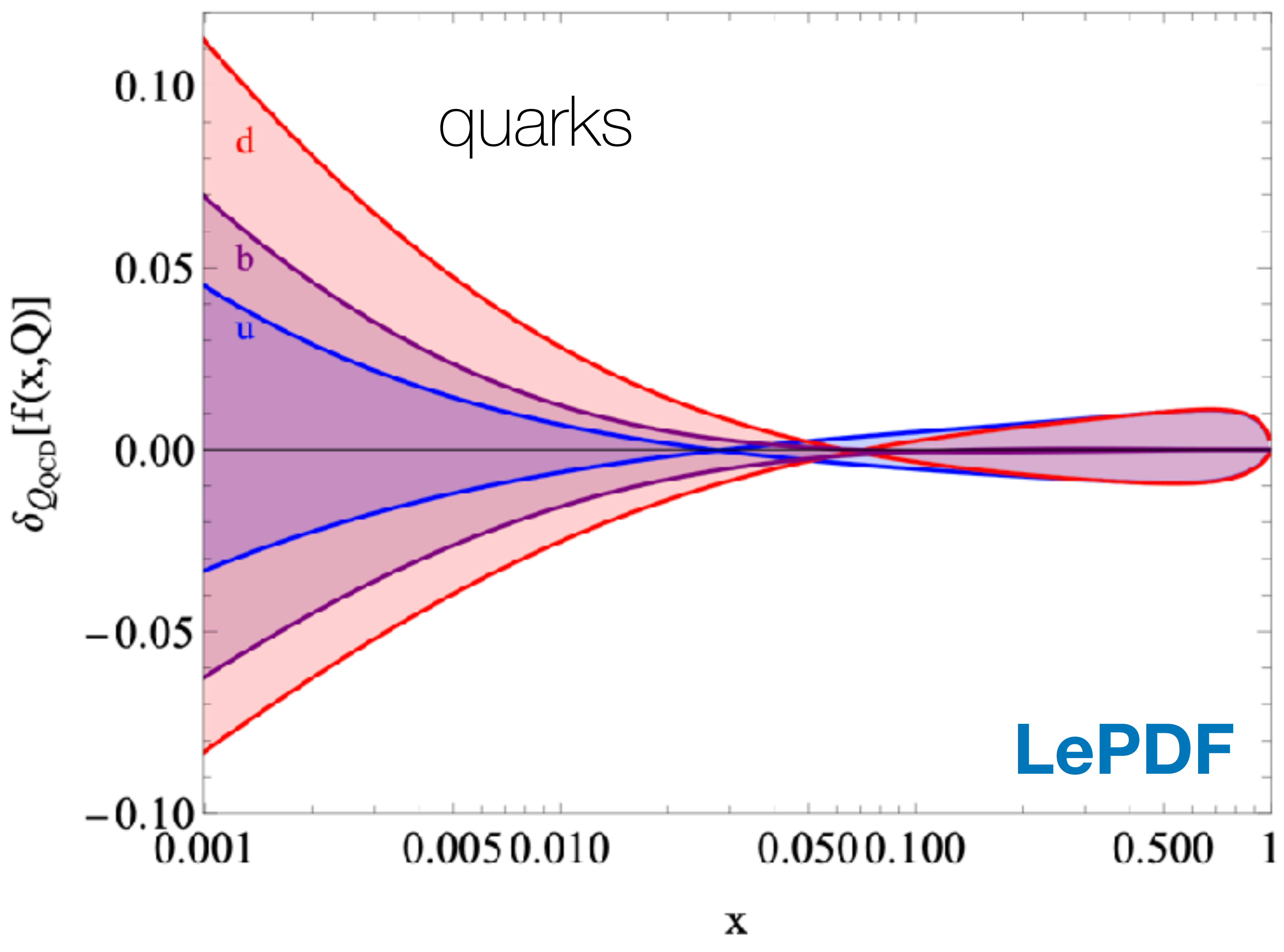
Backup

Uncertainty due to choice of Q_{QCD}

Changing the scale in the interval $Q_{QCD} = [0.5 - 1]$ GeV

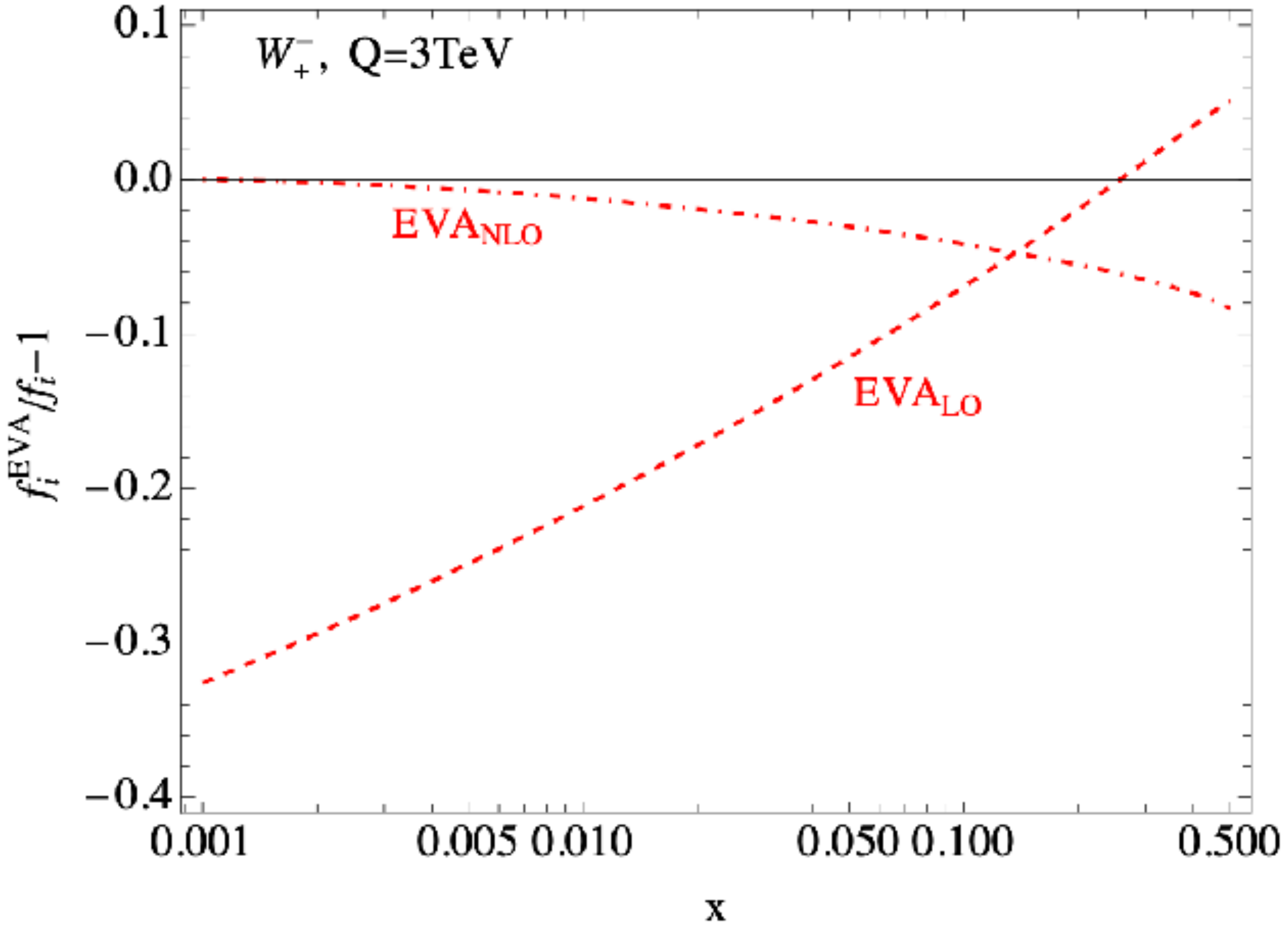
Relative variation in the PDFs, evaluated at the $\boldsymbol{m_W}$ **scale**.

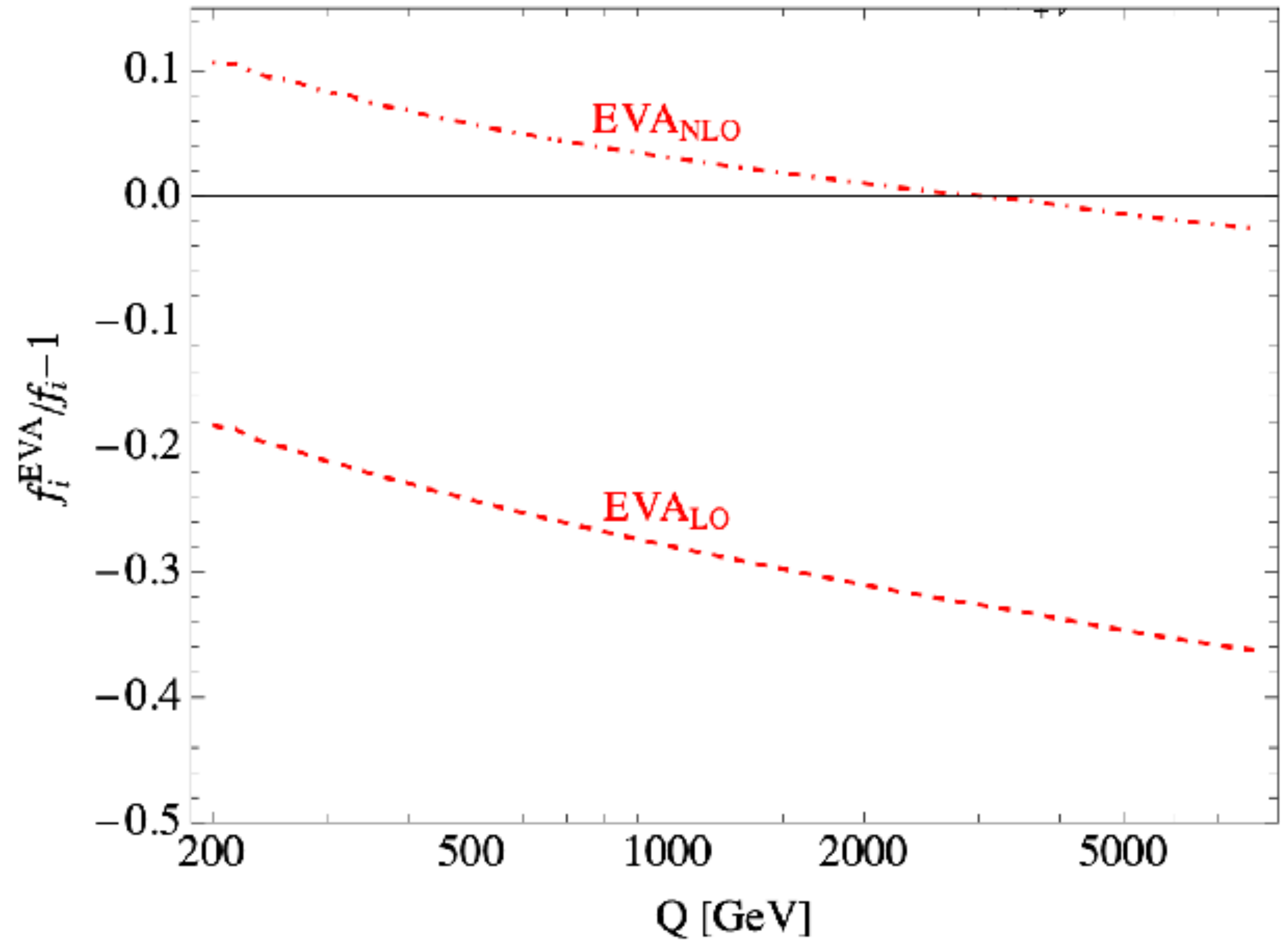




For **leptons** and the **photon**, relative variations are **smaller than** $\mathbf{10^{-5}}$.

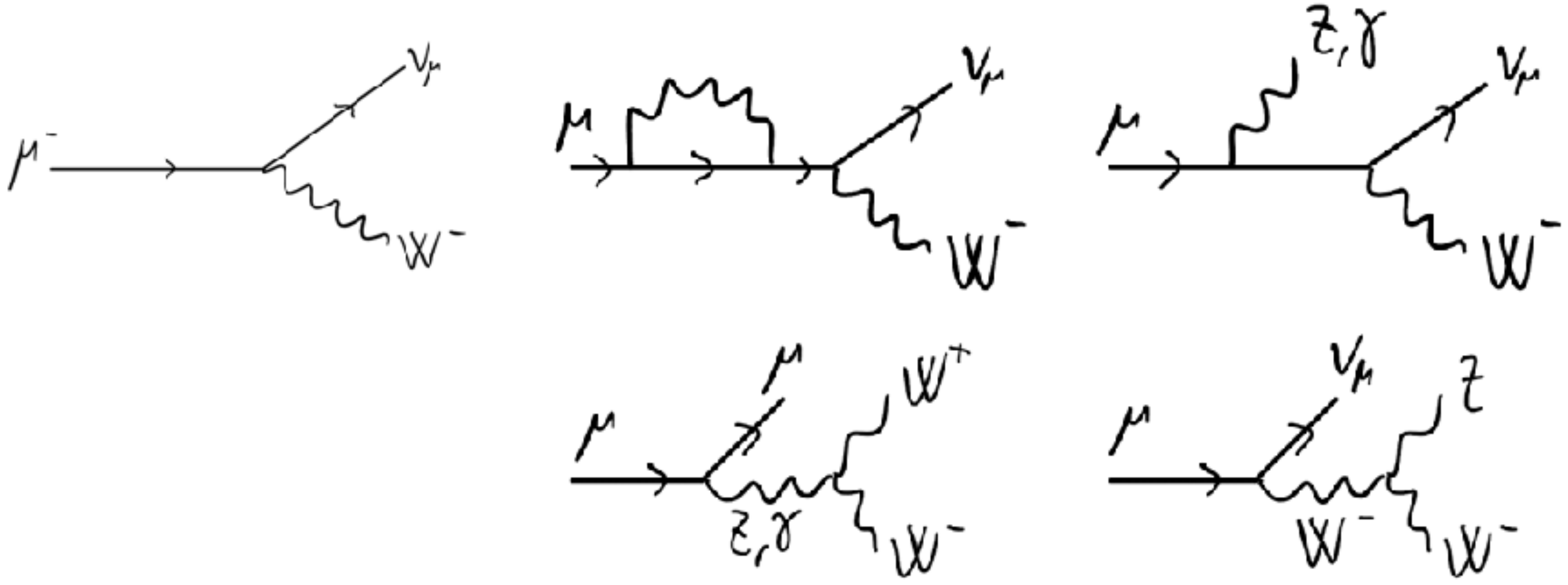
LePDF vs. EVA





The **deviation becomes larger at small x and at large scales** (Sudakov double logs are absent in EVA).

We improve EVA by computing iteratively the $\boldsymbol{W^-_+}$ **PDF at** $\mathbf{O(\alpha^2)}$. *



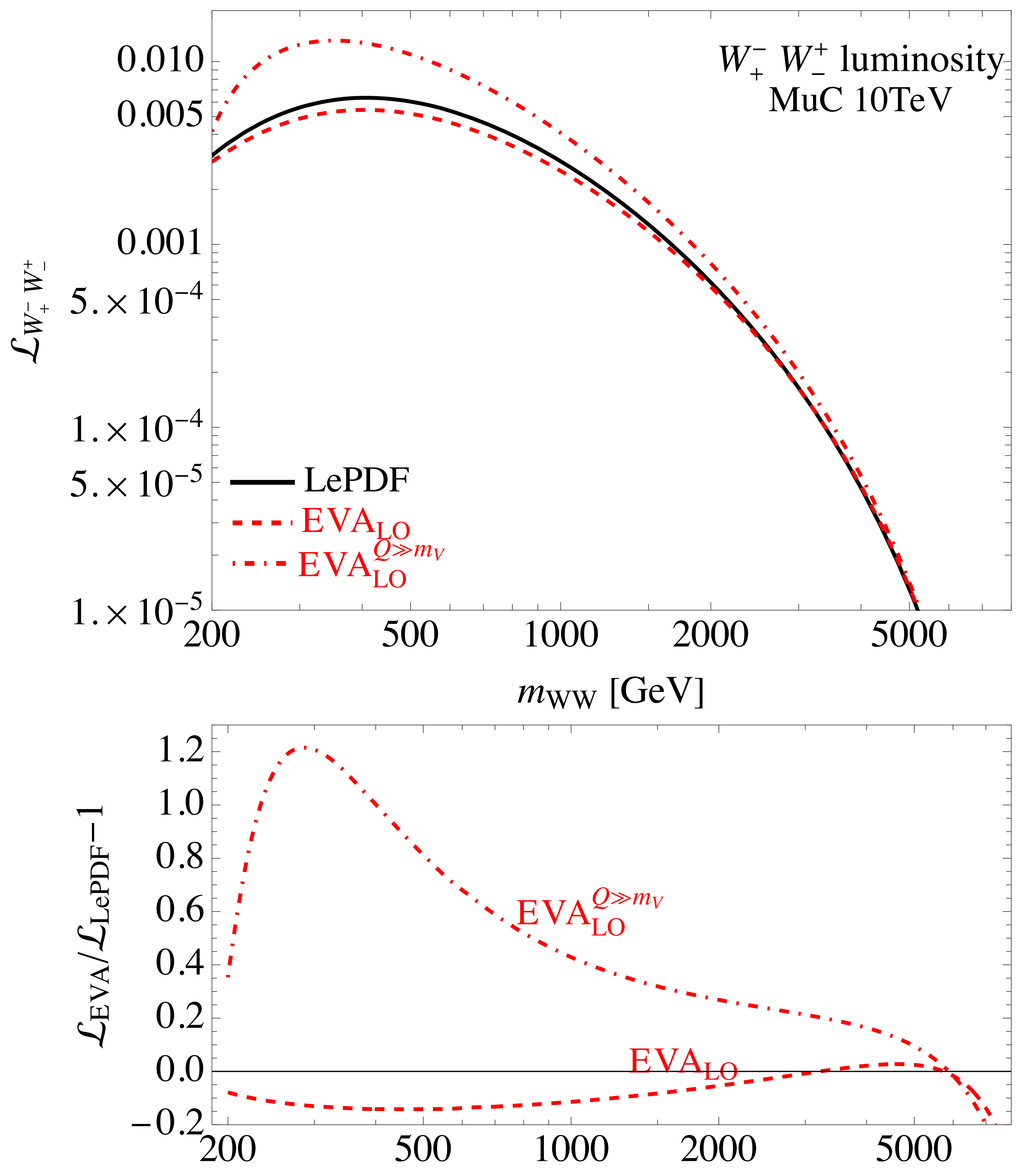
$$f^{(\alpha)}_{\mu_L}(x,t) \simeq \int_{t_{m_W}}^{t} dt' \left(\frac{1}{2}P^{v}_{\mu_L}(t')\delta(1-x) + \frac{\alpha_\gamma}{4\pi}P^{V}_{ff}(x) + \frac{\alpha_2}{4\pi c_W^2}(Q^Z_{\mu_L})^2 P^V_{ff}(x)\right),$$

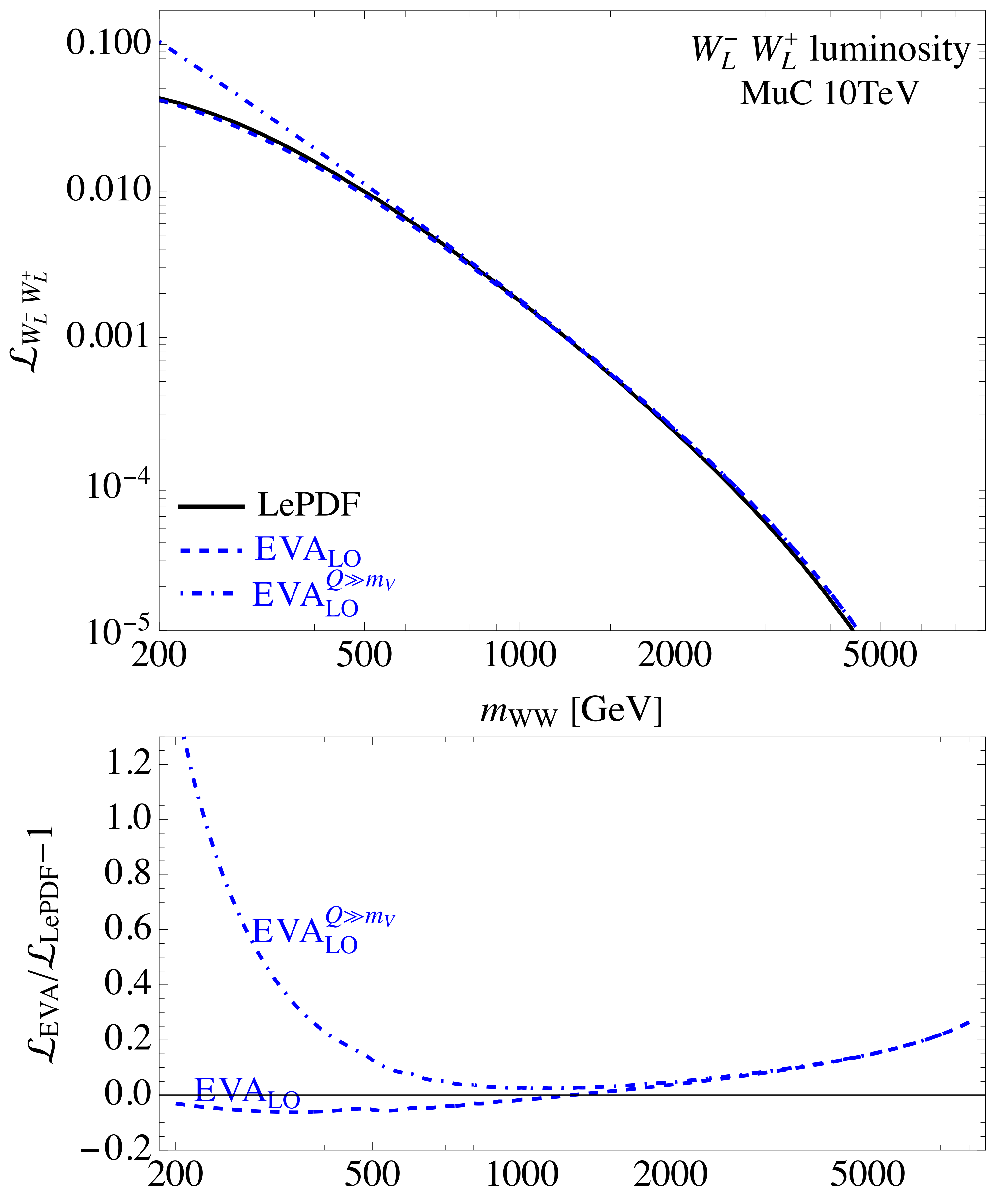
$$f^{(\alpha^2)}_{W^-_+}(x,t) \simeq \int_{t_{m_W}}^{t} dt' \Big(P^v_{W^-_+} f^{(\alpha)}_{W^-_+} + \frac{\alpha_2}{4\pi}P^f_{V_+ f_L}\otimes f^{(\alpha)}_{\mu_L} + \frac{\alpha_2}{2\pi}c_W^2 P_{V_+V_s}\otimes(f^{(\alpha)}_{W^-_s}+f^{(\alpha)}_{Z_s}) + \frac{\alpha_\gamma}{2\pi}P_{V_+V_s}\otimes(f^{(\alpha)}_{W^-_s}+f^{(\alpha)}_{\gamma_s}) + \frac{\sqrt{\alpha_\gamma\alpha_2}}{2\pi}c_W P_{V_+V_s}\otimes f^{(\alpha)}_{Z/\gamma_s}\Big).$$

* for simplicity, in the NLO part we take the Q » m_W and x « 1 limit in the LO EVA expression.

Several **double logs appear at this order**, we find a **much improved agreement** with the LePDF resummation.

LePDF vs. EVA: WW Luminosity





At the level of **parton luminosity**:

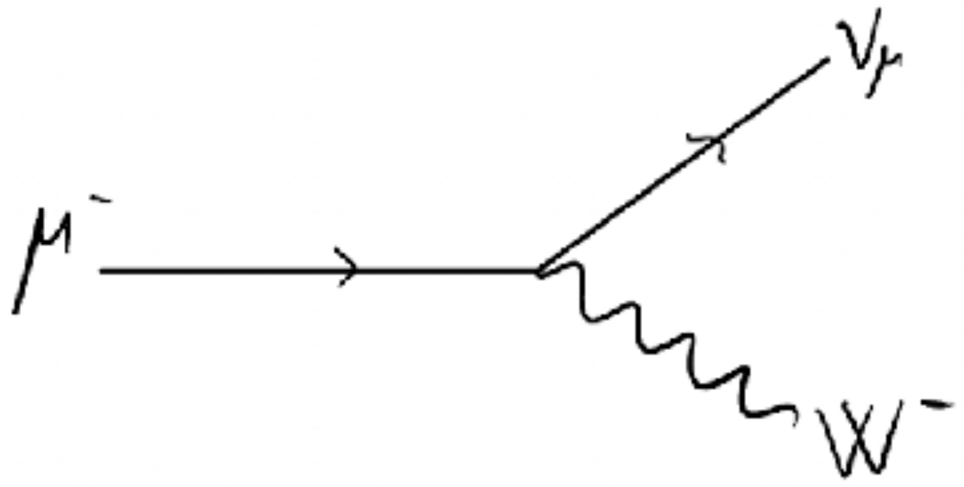
- for $\mathbf{W_T W_T}$: $\mathrm{EVA_{LO}}$ is accurate to **~15%**
- for $\mathbf{W_L W_L}$: $\mathrm{EVA_{LO}}$ is accurate to **~5%**
- The **Q≫m$_V$** approximation does not reproduce well the complete result, with **O(1) differences** up to large scales (particularly for transverse modes).

$\mathrm{EVA_{LO}}$
$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) = \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x)\left(\log\frac{Q^2+(1-x)m_W^2}{m_\mu^2+(1-x)m_W^2} - \frac{Q^2}{Q^2+(1-x)m_W^2}\right)$$

$\mathrm{EVA_{LO}}^{mV\to 0}$
$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) \approx \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x)\log\frac{Q^2}{m_W^2}$$

Implemented in `MadGraph5_aMC@NLO`
Ruiz, Costantini, Maltoni, Mattelaer [2111.02442]

Muon Neutrino PDF



Emission of **collinear W^- from the muon** generates a large content of muon neutrinos inside the muon.

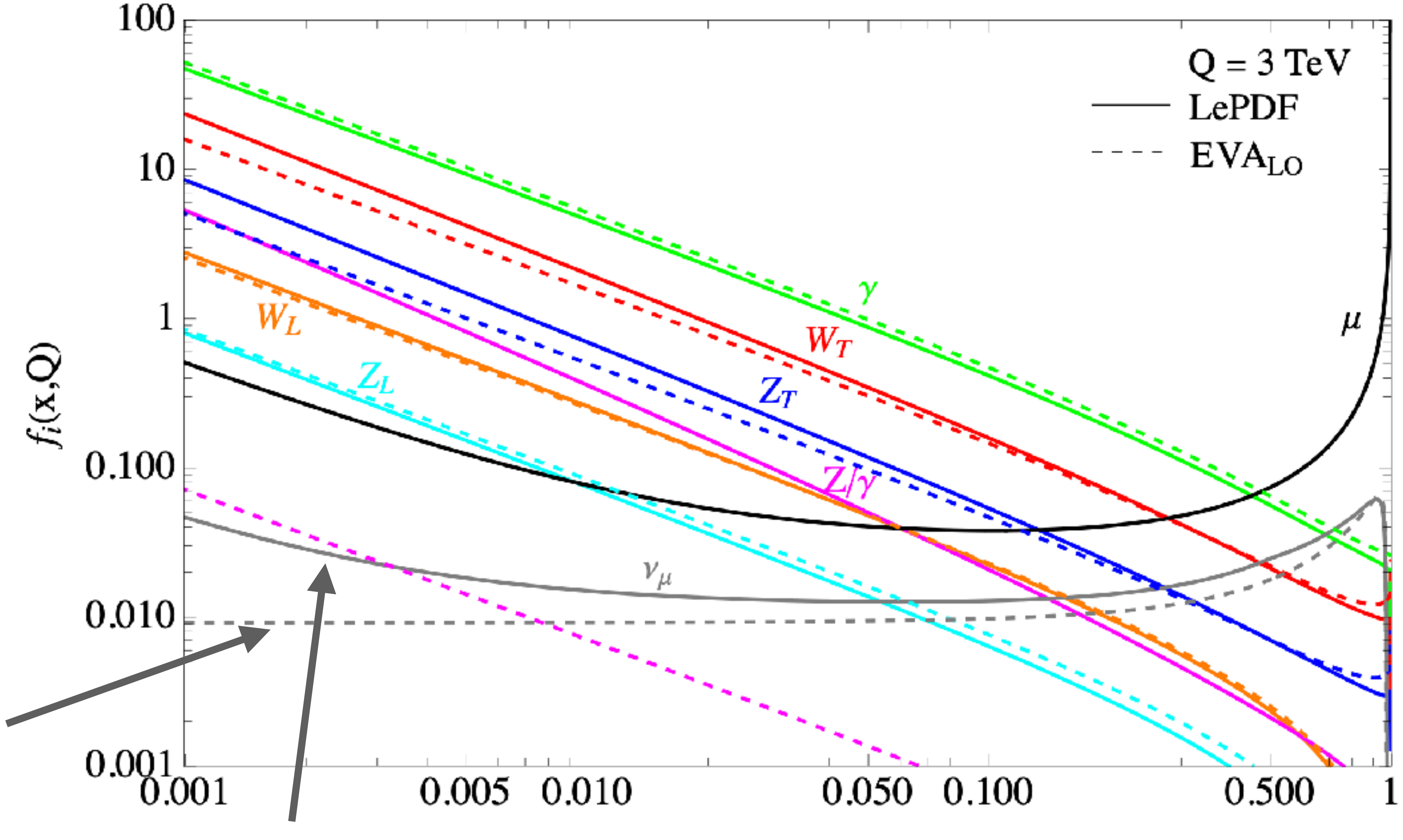
We can compute the **ν_μ PDF at $O(\alpha)$** as did for EVA:

$$\frac{df_{\nu_\mu}}{d\log Q^2} = \frac{\alpha_2}{4\pi}\int_x^{1-m_W/Q}\frac{dz}{z}\frac{Q^4}{(Q^2+zm_W^2)^2}P_{ff}^V(z)\frac{1}{2}\delta\left(1-\frac{x}{z}\right)+\mathcal{O}(\alpha^2) =$$

$$= \frac{\alpha_2}{8\pi}\frac{Q^4}{(Q^2+xm_W^2)^2}P_{ff}^V(x)\,\theta\left(1-\frac{m_W}{Q}-x\right)+\mathcal{O}(\alpha^2)\,.$$

$$f_{\nu_\mu}^{(\alpha)}(x,Q^2) = \frac{\alpha_2}{8\pi}\,\theta\left(Q^2-\frac{m_W^2}{(1-x)^2}\right)P_{ff}^V(x)\left(\log\frac{Q^2+xm_W^2}{m_W^2}+\right.$$

$$\left.+\log\frac{(1-x)^2}{1+x(1-x)^2}+\frac{xm_W^2}{Q^2+xm_W^2}+\frac{1}{1+x(1-x)^2}-1\right)$$



Here $Z \to \bar{\nu}_\mu\, \nu_\mu$ dominates

The Sudakov double log does not appear at this order because the muon PDF is just a δ at LO.
It will however appear in the xsec computation upon integration of the PDF, due to the x→ 1 divergence inside P^V_{ff}.

Top quark PDF

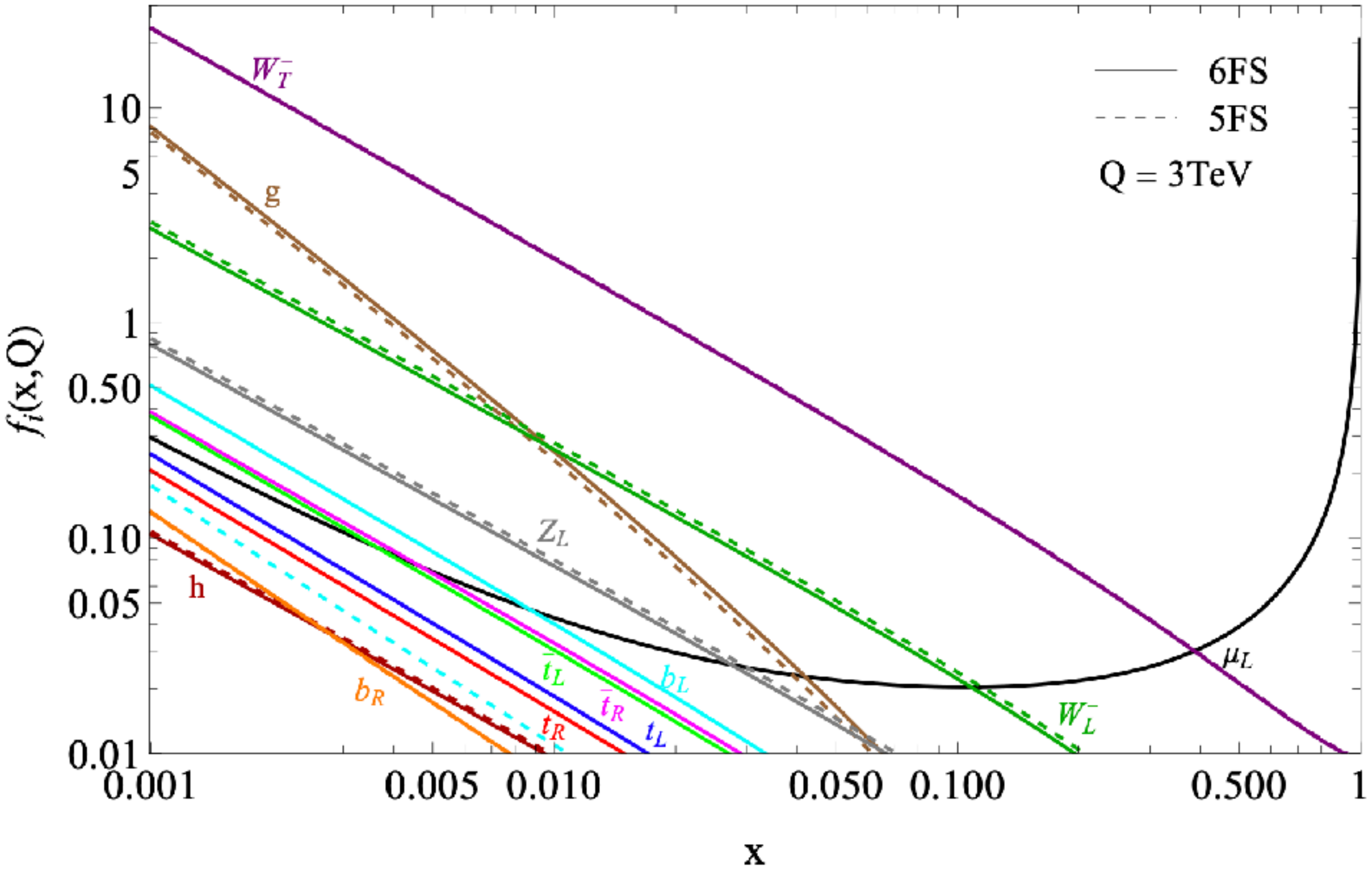
For hard scattering energies $E \gg m_t$, terms with $\log E/m_t$ due to collinear emission of top quarks can arise.

These can be resummed by including the **top quark PDF** within the DGLAP evolution, in a **6FS**.

Barnett, Haber, Soper '88; Olness, Tung '88

Whether or not this is useful depends on the process under consideration.

Dawson, Ismail, Low [1405.6211]
Han, Sayre, Westhoff [1411.2588]



We provide two version of the codes: **5FS** and **6FS**.
In the 6FS we keep **finite top quark mass** effects, like we do for other heavy SM states.

Above the EW scale

All SM interactions and fields must be considered and

several new effects must be taken into account:

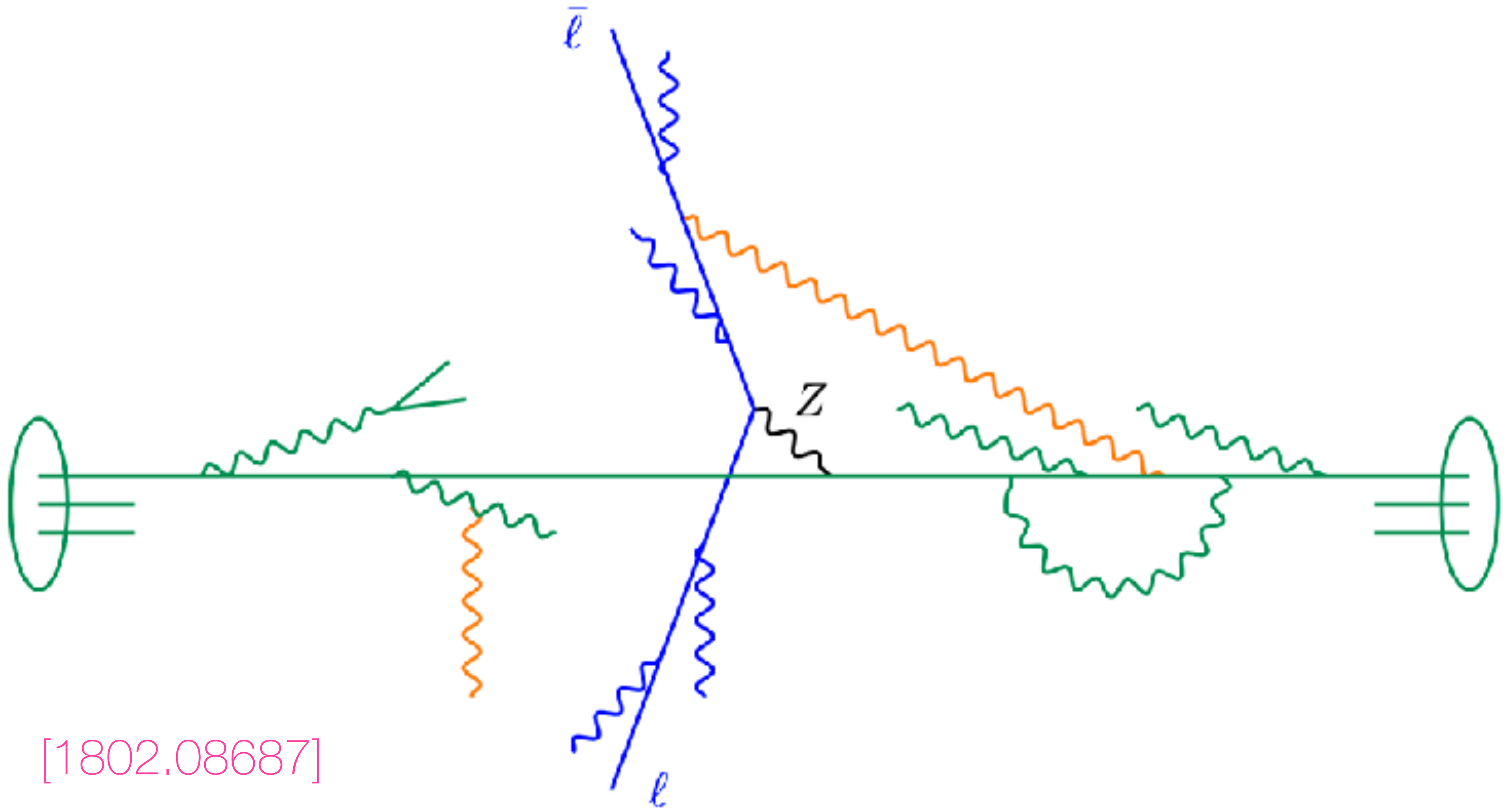
- **PDFs become polarised**, since EW interactions are chiral. Bauer, Webber [1808.08831]
- At high energies **EW Sudakov double logarithms** are generated. P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0001142, hep-ph/0505047], Bauer, Webber [1703.08562, 1808.08831], Chen, Han, Tweedie [1611.00788], Han, Ma, Xie [2103.09844], F. Garosi, D.M., S. Trifinopoulos [2303.16964]
- Neutral bosons interfere with each other: **Z/γ and h/Z_L PDFs mix**. P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0505047] Chen, Han, Tweedie [1611.00788]
- **Mass effects** of partons with EW masses (W, Z, h, t) become relevant and remain so even at multi-TeV scale.
- EW symmetry is broken. Another set of splitting functions, proportional to v^2 instead of p_T^2, arise: **ultra-collinear splitting functions**. Chen, Han, Tweedie [1611.00788]

EW Sudakov double logs from ISR

The Bloch-Nordsieck theorem is violated for non-abelian gauge theories
→ IR divergencies are not cancelled in inclusive processes, since the initial state is EW non-singlet
→ We are often interested in exclusive processes, since we measure the SU(2) charge (W vs Z, t vs b, etc…)

The **EW Sudakov double logs** arises as a non-cancellation of the IR soft divergences $(z \to 1)$ between real emission and virtual corrections.

P. Ciafaloni, Comelli [hep-ph/9809321], Fadin et al. [hep-ph/9910338], M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0001142, hep-ph/0103315]
see also Denner, Pozzorini [hep-ph/0010201], Pozzorini [hep-ph/0201077], Manohar [1409.1918], Pagani, Zaro [2110.03714], …
Manohar, Waalewijn [1802.08687], Chen, Glioti, Rattazzi, Ricci, Wulzer [2202.10509]



[1802.08687]

Here I am interested in **resumming the EW double logs** related to the **initial-state radiation**.
At the leading-log level we can neglect soft radiation

Manohar, Waalewijn [1802.08687]

EW Sudakov double logs from ISR

In case of collinear W emission they can be implemented (and resummed) at he **Double Log** level equations by putting an **explicit IR cutoff** $z_{max} = 1 - Q_{EW}/Q$ $(Q_{EW} = m_W)$

M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0111109]
Bauer, Ferland, Webber [1703.08562]
see Manohar, Waalewijn [1802.08687] for a different approach

$$\frac{\alpha_{ABC}(Q)}{2\pi}\int_x^1 \frac{dz}{z} P_{BA}^C(z) f_A\left(\frac{x}{z}, Q^2\right) \rightarrow \frac{\alpha_{ABC}(Q)}{2\pi}\int_x^{z_{\max}^{ABC}(Q)} \frac{dz}{z} P_{BA}^C(z) f_A\left(\frac{x}{z}, Q^2\right)$$

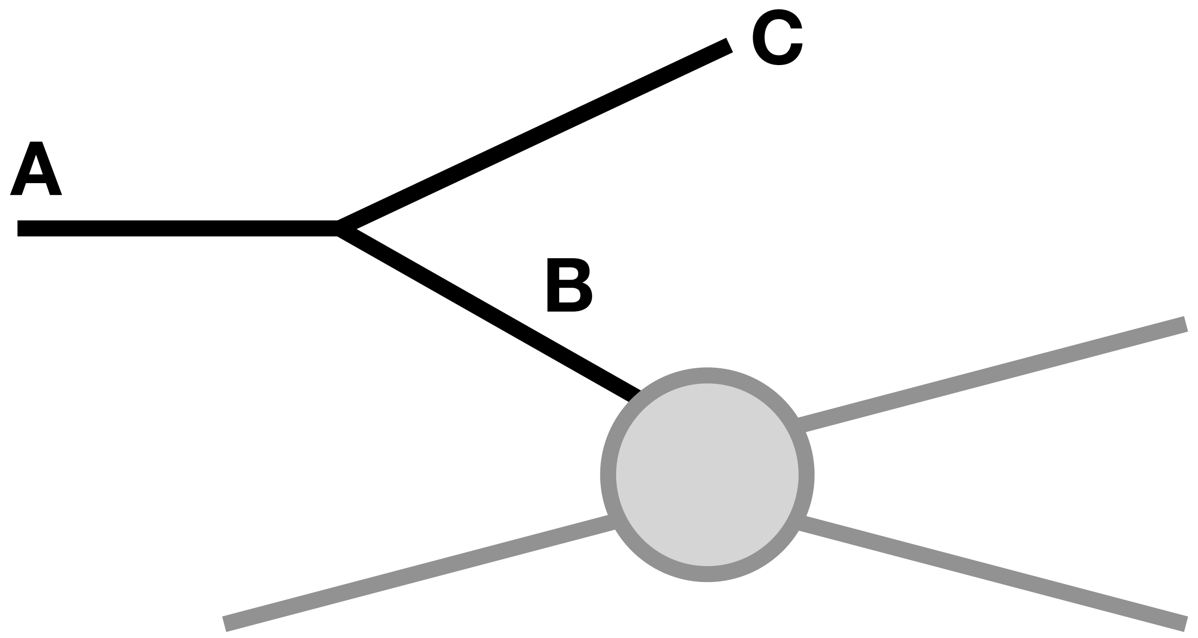
This modifies also the **virtual corrections** as:

$$P_A^v(Q) \supset -\sum_{B,C} \frac{\alpha_{ABC}(Q)}{2\pi}\int_0^{z_{\max}^{ABC}(Q)} dz\, z\, P_{BA}^C(z)$$

The non-cancellation of the z_{max} dependence between emission and virtual corrections generates the double logs.

This happens if $P_{BA}^C,\ U_{BA}^C \propto \frac{1}{1-z}$ and $A \neq B$ otherwise we set $z_{max}=1$ and use the +-distribution.

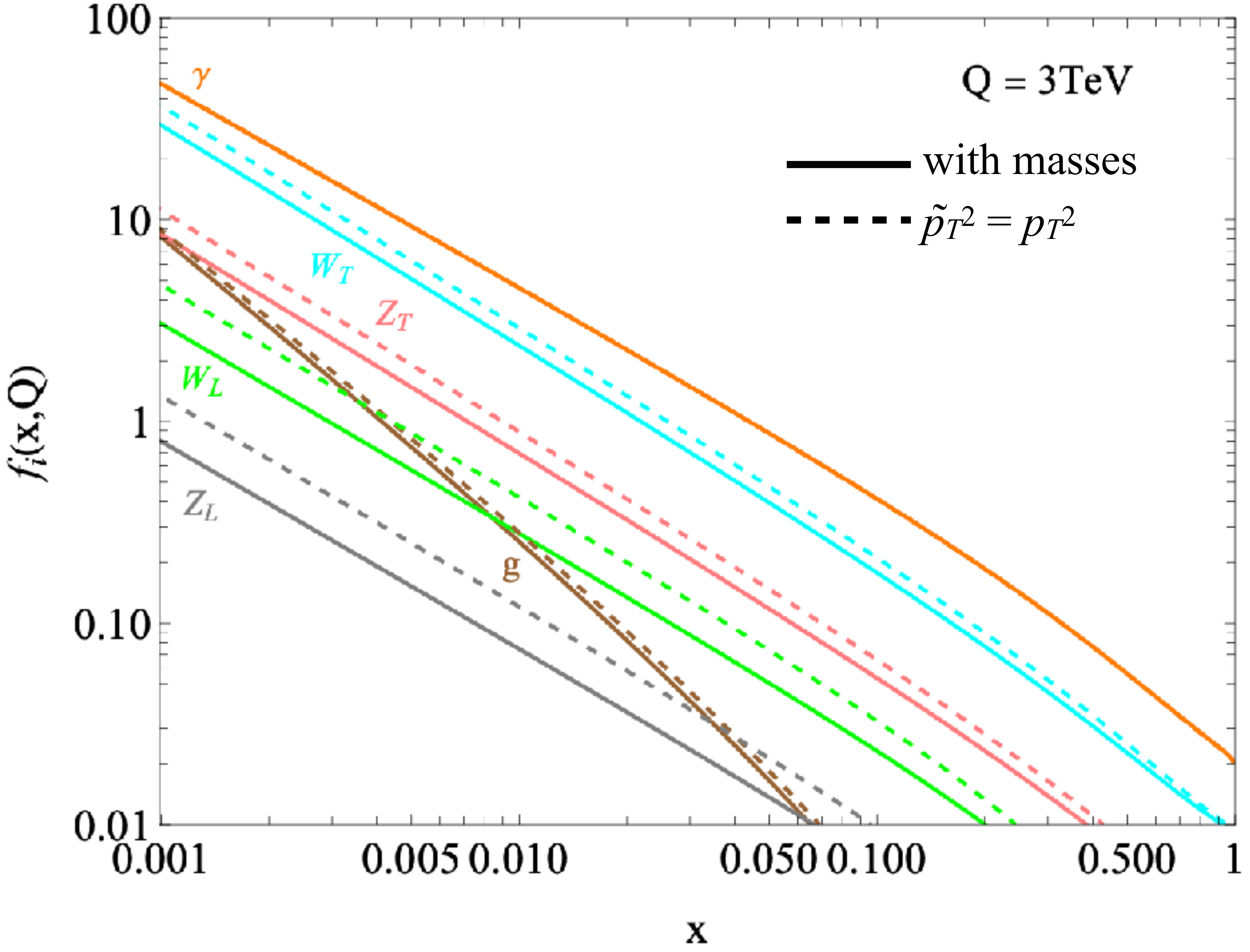
Mass effect



The mass modifies the propagator of the off-shell parton which then enters the hard scattering:

$$\widetilde{p}_T^2 \equiv \bar{z}(m_B^2 - q^2) = p_T^2 + z m_C^2 + \bar{z} m_B^2 - z\bar{z} m_A^2 + \mathcal{O}\left(\frac{m^2}{E^2}, \frac{p_T^2}{E^2}\right)$$

Chen, Han, Tweedie [1611.00788]



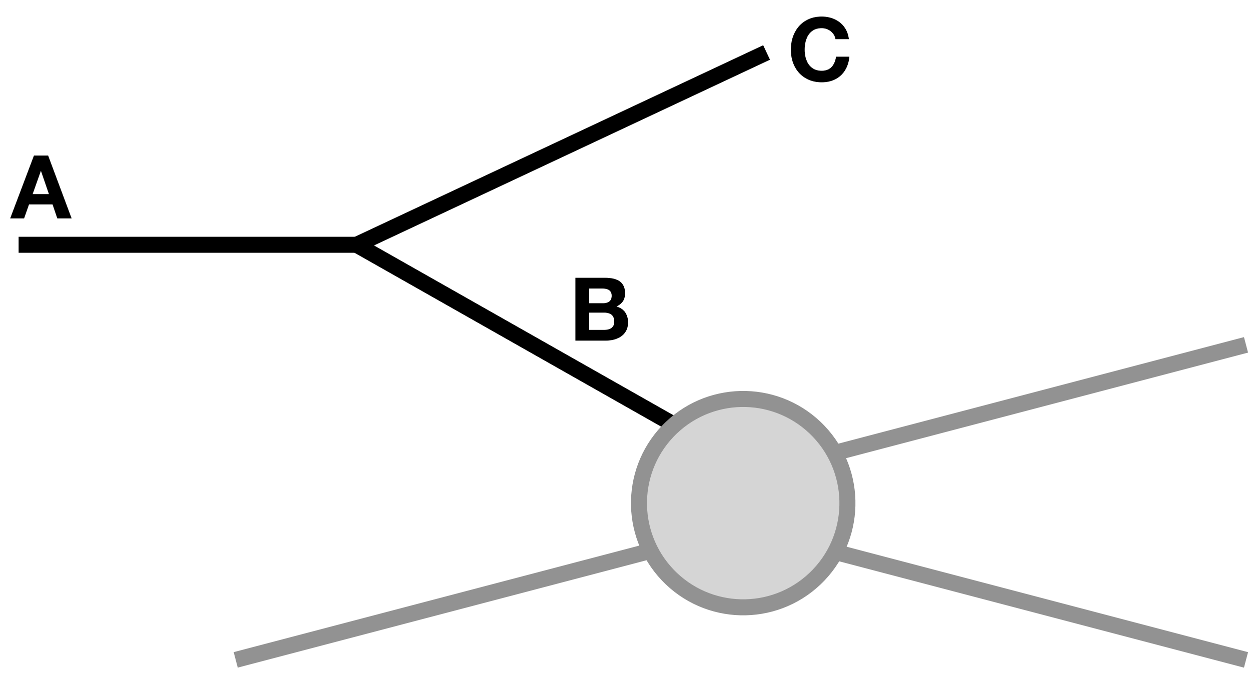
The **effect of finite EW masses is sizeable** even at TeV scales.

The kinematical effect of the mass of particle C is instead negligible in the collinear limit

$E_C = (z\text{-}x)\, E > m_C \qquad z \geq x + \frac{m_C}{E}$

For $E \gg p_T, m$, **we can neglect this effect**.

Ultracollinear splittings



In the unbroken phase, splitting matrix elements are proportional to p_T^2

$$|\mathcal{M}(A \to B + C)|^2 \equiv 8\pi\alpha_{ABC}\frac{p_T^2}{z\bar{z}}P_{BA}^C(z)$$

Ultra-collinear splitting function Chen, Han, Tweedie [1611.00788]

Upon EWSB, further splittings proportional to v^2 are generated.
They generalise the EWA splitting $f \to W_L f'$

$$|\mathcal{M}_{A\to B+C}|^2 \equiv \frac{v^2}{z\bar{z}}P_{BA,C}^{u.c.}(z)$$

For example: $$P^{u.c.}_{f_L^{(2)} f_L^{(1)}, W_L}(z) = \left(y_{f_1}^2 z\bar{z} - y_{f_2}^2 \bar{z} - g_2^2 z\right)^2 \frac{1}{2\bar{z}_+}$$

coupling of massless fermions to W_L, with no chirality flip
(via coupling to remainder gauge field W_n in GEG)

The missing p_T^2 factor removes the log enhancement at high scales, making the **u.c. terms approach a constant value**.

The DGLAP equations are generalised as:

$$Q^2\frac{df_B(x,Q^2)}{dQ^2} = P_B^v f_B(x,Q^2) + \sum_{A,C}\frac{\alpha_{ABC}}{2\pi}\widetilde{P}_{BA}^C \otimes f_A + \frac{v^2}{16\pi^2 Q^2}\sum_{A,C}\widetilde{U}_{BA}^C \otimes f_A$$

Implementation

We work in the **mass eigenstate basis**,
same numerical method used below the EW scale.

After identifying PDFs which are identical because of flavour symmetry, we remain with **42 independent PDFs**:

$$f_{e_L} = f_{\tau_L}\,, \quad f_{\bar\ell_L} = f_{\bar e_L} = f_{\bar\mu_L} = f_{\bar\tau_L}\,,$$
$$f_{e_R} = f_{\tau_R}\,, \quad f_{\bar\ell_R} = f_{\bar e_R} = f_{\bar\mu_R} = f_{\bar\tau_R}\,,$$
$$f_{\nu_e} = f_{\nu_\tau}\,, \quad f_{\bar\nu_\ell} = f_{\bar\nu_e} = f_{\bar\nu_\mu} = f_{\bar\nu_\tau}\,,$$
$$f_{u_L} = f_{c_L}\,, \quad f_{\bar u_L} = f_{\bar c_L}\,, \quad f_{u_R} = f_{c_R}\,, \quad f_{\bar u_R} = f_{\bar c_R}\,,$$
$$f_{d_L} = f_{s_L}\,, \quad f_{\bar d_L} = f_{\bar s_L}\,, \quad f_{d_R} = f_{s_R}\,, \quad f_{\bar d_R} = f_{\bar s_R}\,.$$

Leptons	μ_L	μ_R	e_L	e_R	ν_μ	ν_e	$\bar\ell_L$	$\bar\ell_R$	$\bar\nu_\ell$
Quarks	u_L	d_L	u_R	d_R	t_L	t_R	b_L	b_R	$+$ h.c.
Gauge Bosons	$\gamma_\pm$	$Z_\pm$	$Z\gamma_\pm$	$W^\pm_\pm$	$G_\pm$				
Scalars	h	Z_L	hZ_L	$W^\pm_L$					

Starting from $Q_{\rm EW} = m_W$, heavy states are added at the corresponding mass threshold.

DGLAP equations: $$Q^2\frac{df_B(x,Q^2)}{dQ^2} = P^v_B\, f_B(x,Q^2) + \sum_{A,C}\frac{\alpha_{ABC}}{2\pi}\widetilde{P}^C_{BA}\otimes f_A + \frac{v^2}{16\pi^2 Q^2}\sum_{A,C}\widetilde{U}^C_{BA}\otimes f_A$$

$$\widetilde{P}^C_{BA}(z,p_T^2) = \left(\frac{p_T^2}{\widetilde{p}_T^2}\right)^2 P^C_{BA}(z)$$

$$\widetilde{p}_T^2 \equiv \bar z(m_B^2 - p_B^2) = p_T^2 + z m_C^2 + \bar z m_B^2 - z\bar z m_A^2 + \mathcal{O}\left(\frac{m^2}{E^2}, \frac{p_T^2}{E^2}\right)$$

Ultra-collinear splittings

LePDF: Numerical Implementation

We solve the DGLAP numerically in x space. Due to the sharp behaviour of the muon PDF near x=1, the typical interpolation techniques used for PDFs of proton do not work.

We discretise x interval $[x_{min}=10^{-6},1]$ in N_x small intervals, denser for $x \approx 1$:

$$x_\alpha = 10^{-6((N_x-\alpha)/N_x)^{2.5}}$$

$$\alpha = 0, 1, \ldots, N_x$$

For the splitting functions divergent in $z \to 1$ we us the "+" distribution

$$\int_x^1 dz \frac{f(z)}{(1-z)_+} = \int_x^1 dz \frac{f(z)-f(1)}{1-z} - f(1)\int_0^x \frac{dz}{1-z} = \int_x^1 dz \frac{f(z)-f(1)}{1-z} + f(1)\log(1-x)$$

The differential evolution is done in $t = \log Q^2/m_\mu^2$ with 4th order Runge-Kutta.

At x=1 we fix $$f_{iN_x}(t) = \begin{cases} \frac{L(t)}{\delta x_{N_x}} & i = \mu \\ 0 & i \neq \mu \end{cases}$$

where L(t) is fixed imposing momentum conservation:

Han, Ma, Xie [2103.09844]

$$L(t) = 1 - \sum_{i=1}^{N_f} \sum_{\alpha=1}^{N_x-1} \delta x_\alpha x_\alpha f_{i\alpha}(t)$$

The uncertainties due to x and t discretisation are estimated to be of ~1% and ~0.1%, respectively, for N_x=1000.