Phenomenology of Dirac Scotogenic Model

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Outline

- Introduction (SM and its Shortcomings)
- Scotogenic Model
- Dirac Scotogenic Model
- Dark Matter
- Higgs Vacuum Stability
- CDF II W Anomaly
- Conclusion

The Standard Model (SM) and its Shortcomings

- A quantum field theory which provides a very good understanding of three of the four fundamental forces: Strong, Weak, and EM.
- It is based on the "Gauge Principle". $SU(3)_C \times SU(2)_L \times U(1)_Y$
- It contains 3 generations of leptons and quarks with Gauge bosons and Higgs boson.
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- Neutrino Mass:
  1. The SM predicts neutrinos to be massless.
  2. Neutrino oscillation experiments **[Super-Kamiokande:1998kpq]** predict masses for neutrinos.
  3. At least two neutrinos are massive.
- Dark Matter:
  1. Discrepancy in galactic rotation curve $\Rightarrow$ One possible solution is DM.
  2. Flat galactic rotation curves seem to suggest that each galaxy is surrounded by significant amounts of non-visible matter known as dark matter.
  3. There is no candidate for dark matter within SM.

Scotogenic Model

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- Two possible DM candidates:
  1. Neutral Scalar η^0
  2. Neutral Fermion N

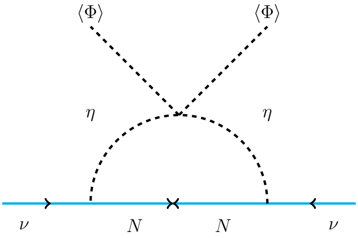

Figure 1: Leading neutrino mass generation diagram.

Dirac Scotogenic Model:

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  $U(1)_{B-L} \to \mathcal{Z}_m \equiv \mathcal{Z}_{2n+1}$with $n \in \mathbb{Z}^+ \Rightarrow$ neutrinos are Dirac particles
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$$\text{Lepton doublet} \quad L_i \begin{cases} \nsim \omega^n & \text{under } \mathcal{Z}_{2n} & \Rightarrow \text{Dirac neutrinos} \\ \sim \omega^n & \text{under } \mathcal{Z}_{2n} & \Rightarrow \text{Majorana neutrinos} \end{cases} \tag{1}$$

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  2. Protect the Dirac nature of the neutrinos and the smallness of neutrino masses by forbidding the tree-level coupling with the Higgs field.
  3. It also predicts that the lightest neutrino is massless.

The Model Setup

- $(-4, -4, 5)$ Chiral solutions to $U(1)_{B-L}$ anomaly cancellation conditions (forbidding the tree-level neutrino Yukawa couplings).

	Fields	$SU(2)_L \otimes U(1)_Y$	$U(1)_{B-L}$	$\mathcal{Z}_6$
Fermions	L_i	$(\mathbf{2}, -1/2)$	-1	ω^4
	ν_{R_i}	$(\mathbf{1}, 0)$	$(-4, -4, 5)$	$(\omega^4, \omega^4, \omega^4)$
	N_{L_l}	$(\mathbf{1}, 0)$	$-1/2$	ω^5
	N_{R_l}	$(\mathbf{1}, 0)$	$-1/2$	ω^5
Scalars	H	$(\mathbf{2}, 1/2)$	0	1
	η	$(\mathbf{2}, 1/2)$	$1/2$	ω
	ξ	$(\mathbf{1}, 0)$	$7/2$	ω

Table 1: Charge assignment for all the fields.

- 0 B-L charge for Higgs is zero to preserve Yukawa terms for fermions that give mass to them.

- The $U(1)_{B-L} \to Z_6$ breaking happens because of the presence of the soft term $(\kappa\,\eta^\dagger H \xi + h.c.)$

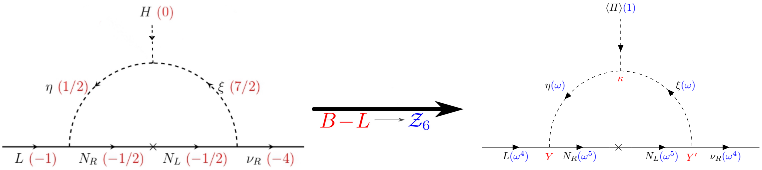

Figure 2: Charge assignment and symmetry breaking pattern for $U(1)_{B-L} \to Z_6$.

- This residual Z_6 symmetry simultaneously protects the Dirac nature of neutrinos and the stability of DM.

The general form of the scalar potential is given by

$$V = -\mu_H^2 H^\dagger H + \mu_\eta^2 \eta^\dagger \eta + \mu_\xi^2 \xi^* \xi + \frac{1}{2}\lambda_1 (H^\dagger H)^2 + \frac{1}{2}\lambda_2 (\eta^\dagger \eta)^2 + \frac{1}{2}\lambda_3 (\xi^* \xi)^2$$
$$+\lambda_4 (H^\dagger H)(\eta^\dagger \eta) + \lambda_6 (\eta^\dagger \eta)(\xi^* \xi) + \lambda_7 (H^\dagger \eta)(\eta^\dagger H) + \lambda_8 (H^\dagger H)(\xi^* \xi)$$
$$+(\kappa\, \eta^\dagger H \xi + h.c.) \quad (2)$$

The Scalar Potential

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$$\begin{aligned} V &= -\mu_H^2 H^\dagger H + \mu_\eta^2 \eta^\dagger \eta + \mu_\xi^2 \xi^* \xi + \frac{1}{2}\lambda_1 (H^\dagger H)^2 + \frac{1}{2}\lambda_2 (\eta^\dagger \eta)^2 + \frac{1}{2}\lambda_3 (\xi^* \xi)^2 \\ &+\lambda_4 (H^\dagger H)(\eta^\dagger \eta) + \lambda_6 (\eta^\dagger \eta)(\xi^* \xi) + \lambda_7 (H^\dagger \eta)(\eta^\dagger H) + \lambda_8 (H^\dagger H)(\xi^* \xi) \\ &+(\kappa\, \eta^\dagger H \xi + h.c.) \end{aligned} \tag{2}$$

Bounded from below scalar potential, ensured by the following conditions [Kannike:2016fmd]

$$\begin{aligned} &\lambda_1, \lambda_2, \lambda_3 \geq 0; \qquad \lambda_4 > -\sqrt{\lambda_1 \lambda_2}, \quad \lambda_6 > -\sqrt{\lambda_2 \lambda_3}, \quad \lambda_8 > -\sqrt{\lambda_1 \lambda_3}, \\ &\sqrt{\frac{\lambda_3}{2}}\lambda_4 + \sqrt{\frac{\lambda_1}{2}}\lambda_6 + \sqrt{\frac{\lambda_2}{2}}\lambda_8 + \sqrt{\frac{\lambda_1\lambda_2\lambda_3}{8}} \\ &> -\sqrt{(\lambda_4 + \sqrt{\lambda_1\lambda_2})(\lambda_8 + \sqrt{\lambda_1\lambda_3})(\lambda_6 + \sqrt{\lambda_2\lambda_3})} \end{aligned} \tag{3}$$

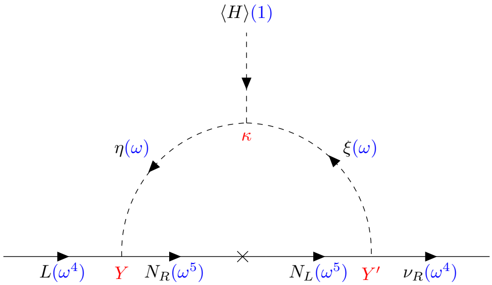

Figure 3: Leading neutrino mass generation diagram.

The relevant Yukawa Lagrangian for neutrino masses is given by

$$-\mathcal{L}_Y \supset Y_{il}\bar{L}_i\tilde{\eta}N_{R_l} + Y'_{li}\bar{N}_{L_l}\nu_{R_i}\xi + M_{lm}\bar{N}_{R_l}N_{L_m} + h.c. \quad (4)$$

We can calculate neutrino masses from the diagram Fig.3 as

$$(M_\nu)_{ij} = \frac{1}{16\pi^2}\sum_{k=1}^{3} Y_{ik}Y'_{kj}\frac{\kappa v}{m_\xi^2 - m_\eta^2}M_k\sum_{l=1}^{2}(-1)^l B_0(0, m_l^2, M_k^2). \quad (5)$$

- We performed a detailed numerical scan for the model parameters with various experimental and theoretical constraints.
- We have implemented the model in SARAH-4.14.5 [Staub:2015kfa] and SPheno-4.0.5 [Porod:2011nf] to calculate all the vertices, mass matrices and tadpole equations.
- Thermal component to the DM relic abundance as well as the DM nucleon scattering cross sections are determined by micrOMEGAS-5.2.13 [Belanger:2014vza]

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we have also imposed the following additional conditions when generating the allowed points:

- Neutrino oscillation parameters.
- Bounded from below scalar potential, ensured by the vacuum stability constraints.
- Perturbativity of Yukawas and quartic couplings.
- If η^0 is the DM particle its mass must be smaller than the charged counterpart η^+. This implies $\lambda_7 < 0$ in the small mixing limit.
- Finally, we impose the LEP constraint on the light-neutral component of a doublet. This limit is actually simply $m_{\eta R} + m_{\eta I} > m_Z$ which in our case translates to $m_{\eta^0} > m_Z/2 \approx 45.6$ GeV.

Dark Matter

Doublet DM Case:

- Relic density computation and direct detection prospects for doblet DM case involve the exchange of a Higgs or Z boson.
- Our analysis shows that magenta points cover three mass regions in the relic density plot:
  1. The low mass region from 10 GeV to around 30 GeV (Ruled out by LEP constraints).
  2. The medium mass region from 58 GeV to around 122 GeV.
  3. The high mass region from 200 GeV to around 4.8 TeV.

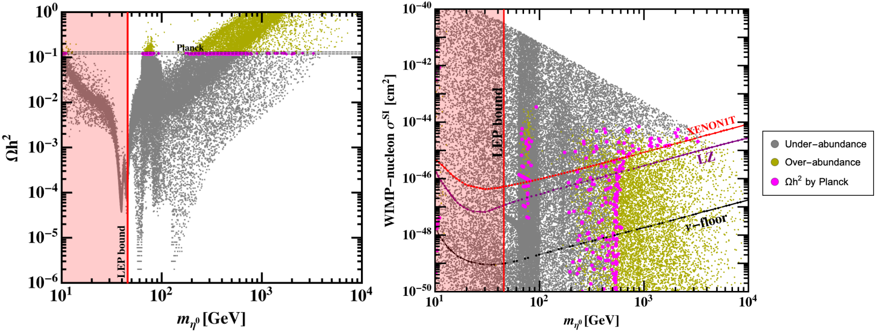

Figure 4: **Left:** Relic density plot for η^0 dominated DM. **Right:** Spin-independent WIMP-nucleon cross section for the η^0 dominated DM candidate case.

Singlet DM Case:

- Relic density computation and direct detection prospects involve just a Higgs portal in the singlet case.
- Magenta points cover a broad mass region up to around 5 TeV that satisfy all theoretical and experimental constraints.

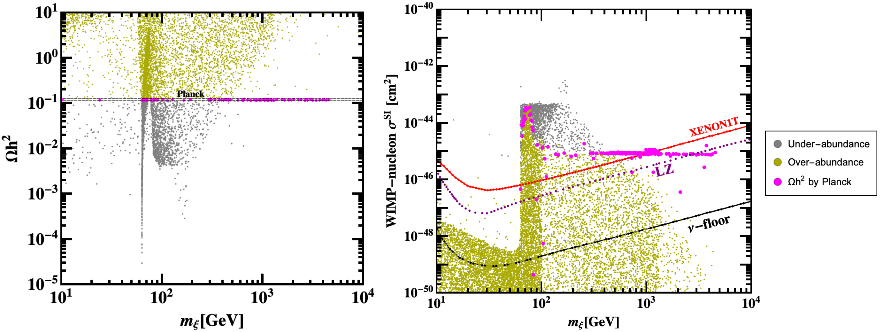

Figure 5: **Left:** Relic density vs singlet DM mass. **Right:** Spin-independent WIMP-nucleon cross section for the ξ dominated DM candidate vs DM mass.

Fermionic DM Case:

- Fermionic DM candidate satisfies all the constraints up to around 2 TeV.
- In the relic density computation for fermionic DM, coannihilation channels between dark fermion and doublet scalar become important if the relative dark fermion-scalar mass difference is below 10 GeV.

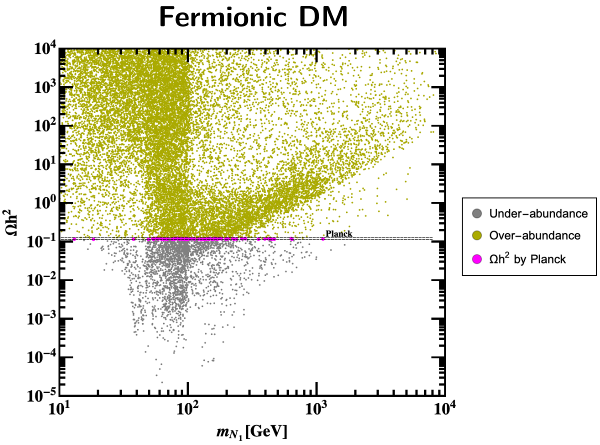

Figure 6: Relic density vs fermionic DM mass.

- In SM, there exists a problem with the stability of the electroweak vacuum since the electroweak vacuum becomes unstable at large scale ($\sim 10^{10}\,GeV$).
- At this elevated scale, the quartic coupling of the SM Higgs, denoted as λ_{HH}, undergoes a transition to a negative value as dictated by the evolution of the renormalization group equations (RGE).

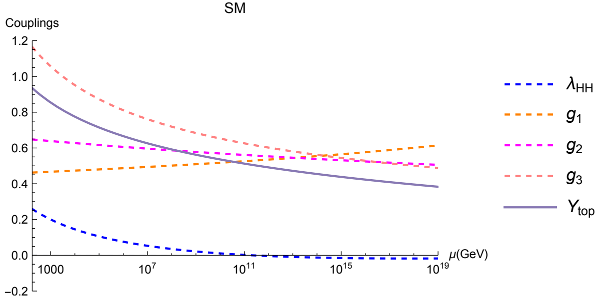

Figure 7: The RG evolution of the SM gauge couplings g_1, g_2, g_3, the top quark Yukawa coupling Y_{top} and the quartic Higgs boson self-coupling λ_{HH} in the Standard Model.

Vacuum Stability in Dirac Scotogenic Model

- The beta functions for various gauge, quartic and Yukawa couplings in the model are evaluated up to the two-loop level.

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- As we explore the parameter space, we find that the values of these couplings within the range of 0.15 to 0.50 yield significant corrections to λ_{HH}.

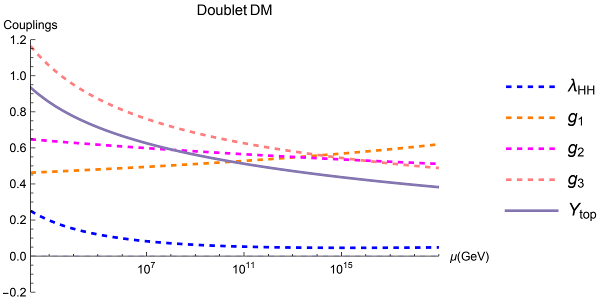

Figure 8: The RG evolution of the SM gauge couplings g_1, g_2, g_3, the top quark Yukawa coupling Y_{top} and the quartic Higgs boson self-coupling λ_{HH}in the Dirac Scotogenic Model.

CDF-II W anomaly

- In 2022, the CDF-II collaboration reported a 7σ excess on the mass of the W boson with respect to the SM prediction.
- In the Dirac Scotogenic model, the doublet dark scalar leads to radiative corrections to W boson mass. [CentellesChulia:2022vpz] arXiv:2206.11903

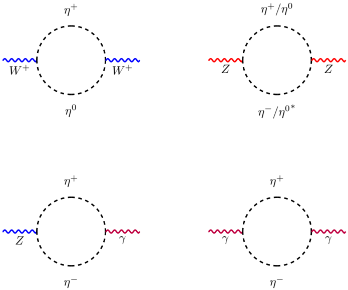

Figure 9: One loop polarization diagrams that contribute to the oblique S, T and U parameters.

CDF-II mW analysis for doublet scalar DM:

- Doublet scalar mass is constrained to the medium mass region (58-86 GeV) after applying CDF-II mW constraints.

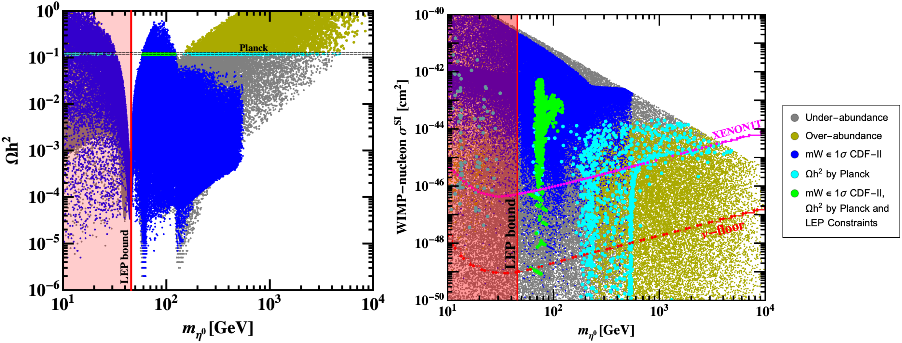

Figure 10: **Left:** Relic density plot for η^0 dominated DM. **Right:** Spin-independent WIMP-nucleon cross section for the η^0 dominated DM candidate case.

Singlet Scalar DM

CDF-II mW analysis for singlet scalar DM:

- Singlet scalar mass is constrained to up to around 500 GeV after applying CDF-II mW constraints.

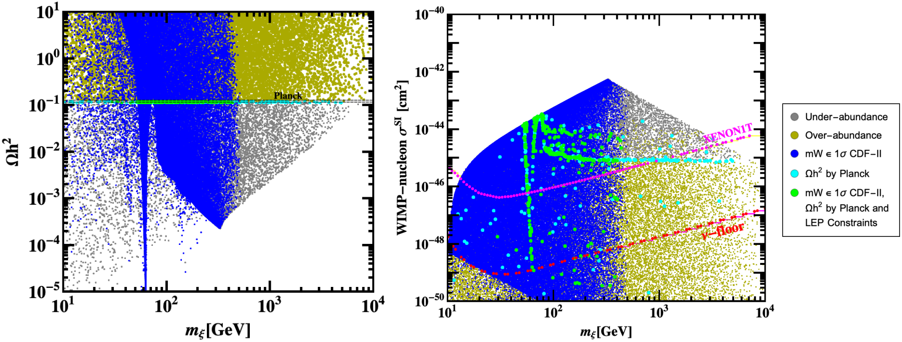

Figure 11: **Left:** Relic density vs singlet DM mass. **Right:** Spin-independent WIMP-nucleon cross section for the ξ dominated DM candidate vs DM mass.

Conclusion

- We have shown that the Dirac scotogenic model can reproduce the neutrino masses and mixing, the DM relic abundance and explain the CDF-II W boson mass anomaly while also ensuring the stability of electroweak vacuum up to the Planck energy scale.
- We find that the case of a mainly scalar doublet DM is constrained for the mass range of 58-86 GeV by the combination of the requirements that W boson mass remains within 1σ of the CDF-II measurement and the constraints coming from DM relic density, direct detection and invisible Z boson decays.
- We showed that if the singlet scalar is the DM candidate then all the above constraints are simultaneously satisfied along with W boson mass within 1σ range of the CDF-II measurement, where the singlet DM mass is constrained up to around 500 GeV.
- Fermionic DM is permissible within a mass range extending from 10 GeV to approximately 2000 GeV.
- Moreover, we find that we have to take some scalar couplings within the range of 0.15 to 0.50 to get enough correction to Higgs self-coupling so that electroweak vacuum will remain stable up to the Planck energy scale

Thank you for your attention!

Fleshing out the $SU(2)_L$ components of the scalars, we can write the doublets as

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \qquad \eta = \begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix} \tag{6}$$

$$H^0 = \frac{1}{\sqrt{2}}(v + h + iA), \qquad \eta^0 = \frac{1}{\sqrt{2}}(\eta_R + i\eta_I), \qquad \xi = \frac{1}{\sqrt{2}}(\xi_R + i\xi_I) \quad . \tag{7}$$

We can now compute the masses of the physical scalar states after symmetry breaking

$$m_h^2 = \lambda_1 v^2, \tag{8}$$

$$m^2_{\eta^\pm} = \mu_\eta^2 + \frac{\lambda_4}{2} v^2, \tag{9}$$

The real part of ξ will mix with the real part of η^0 and similarly the imaginary part of ξ will mix with the imaginary part of η^0 with the same mixing matrix.

$$m^2_{(\xi_R,\eta_R)} = m^2_{(\xi_I,\eta_I)} = \begin{pmatrix} \mu_\xi^2 + \lambda_8 \frac{v^2}{2} & \kappa \frac{v}{\sqrt{2}} \\ \kappa \frac{v}{\sqrt{2}} & \mu_\eta^2 + (\lambda_4 + \lambda_7) \frac{v^2}{2} \end{pmatrix} \tag{10}$$

We can compute the mixing angle

$$\tan 2\theta = \frac{\sqrt{2}\kappa\, v}{(\mu_\xi^2 - \mu_\eta^2) + (\lambda_8 - \lambda_4 - \lambda_7)\frac{v^2}{2}}, \tag{11}$$

and the mass eigenstates for the real/imaginary part of neutral scalars η^0 and ξ are given by

$$m_{1R}^2 = m_{1I}^2 = \left(\mu_\xi^2 + \lambda_8 \frac{v^2}{2}\right)\cos^2\theta + \left(\mu_\eta^2 + (\lambda_4 + \lambda_7)\frac{v^2}{2}\right)\sin^2\theta - 2\kappa v \sin\theta\cos\theta = m_\xi^2 \tag{12}$$

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We can calculate the BSM contributions to S, T and U of the model as

$$S = \frac{1}{12\pi}\log\frac{m^2_{\eta^0}}{m^2_{\eta^+}} \tag{14}$$

$$T = \frac{G_F}{4\sqrt{2}\pi^2\alpha_{em}}\left(\frac{m^2_{\eta^0}+m^2_{\eta^+}}{2} - \frac{m^2_{\eta^0}m^2_{\eta^+}}{m^2_{\eta^+}-m^2_{\eta^0}}\log\frac{m^2_{\eta^+}}{m^2_{\eta^0}}\right) \tag{15}$$

$$U = \frac{1}{12\pi}\left(\frac{(m^2_{\eta^0}+m^2_{\eta^+})\left(m^4_{\eta^0}-4m^2_{\eta^0}m^2_{\eta^+}+m^4_{\eta^+}\right)\log\left(\frac{m^2_{\eta^+}}{m^2_{\eta^0}}\right)}{(m^2_{\eta^+}-m^2_{\eta^0})^3} - \frac{5m^4_{\eta^0}-22m^2_{\eta^0}m^2_{\eta^+}+5m^4_{\eta^+}}{3(m^2_{\eta^+}-m^2_{\eta^0})^2}\right)$$

In terms of the oblique S, T and U parameters, the corrections to the W boson mass are given by

$$m^2_W = m^{2\,(\mathrm{SM})}_W + \frac{\alpha_{em}\cos^2\theta_w}{\cos^2\theta_w - \sin^2\theta_w}m^2_Z\left[-\frac{1}{2}S + \cos^2\theta_w T + \frac{(\cos^2\theta_w - \sin^2\theta_w)}{4\sin^2\theta_w}U\right] \tag{16}$$

where θ_w is the weak angle, α_{em} is the fine-structure constant and $m_W^{(\mathrm{SM})}$ is the Standard Model prediction for m_W.