Neutrino Mass Sum Rules from Modular $\mathcal{A}_4$ Symmetry

(arXiv:2308.08981)

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In collab: **Salvador Centelles Chuliá, Oleg Popov, and Rahul Srivastava**

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phoeniX 2023

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- Flavor models $\rightarrow$ extra scalar particles, “Flavons”.

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- $\bar{\Gamma}$ is the modular group.
- γ is linear transformation of $\bar{\Gamma}$, which acts on τ and given by:

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}$$

where, $a, b, c, d \in \mathbb{Z}$, $ad - bc = 1$ and $\mathrm{Im}[\tau] > 0$.

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- The quotient groups $\Gamma_N \equiv \bar{\Gamma}/\bar{\Gamma}(N)$ known as finite modular group.
- **Groups Γ_N ($N = 2, 3, 4, 5$) are isomorphic to S_3, A_4, S_4 and A_5.**

Introduction-II

- A field $\phi^{(I)}$ transformation is given by:

$$\phi^{(I)} \to (c\tau + d)^{-k_I}\rho^{(I)}(\gamma)\phi^{(I)},$$

where, $-k_I$ is modular weight and $\rho^{(I)}(\gamma)$ signifies an unitary representation matrix of γ.

- The superpotential: $\mathcal{W} = \sum_n Y_{I_1...I_N}\phi^{(I_1)}...\phi^{(I_N)}$.
- For n-th order term to be invariant:
$Y_{I_1...I_N}(\gamma\tau) = (c\tau + d)^{k_Y(n)}\rho(\gamma)Y_{I_1...I_N}$ with $k_Y(n)$ and ρ such that:

$$(i) \quad k_Y(n) = k_{I_1} + ... + k_{I_N}.$$

$$(ii) \quad \text{product} \quad \rho \times \rho^{I_1} \times ...\rho^{I_N} \quad \text{forms singlet.}$$

Motivation

Neutrino Mass Sum Rules: Few of them are,

Barry and Rodejohann, 1007.5217

$$m_1 + m_2 = m_3$$
$$2m_2 + m_3 = m_1$$
$$\frac{1}{m_1} + \frac{1}{m_2} = \frac{1}{m_3}$$
$$\frac{2}{m_2} + \frac{1}{m_3} = \frac{1}{m_1}$$

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We have employed a modular framework based on finite modular group A_4, which is isomorphic to Γ_3.

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L_i	2	$-\frac{1}{2}$	**3**	-3
E_i^c	1	1	**1, 1′, 1″**	-1
H_u	2	$\frac{1}{2}$	**1**	0
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Table: The charge assignments of the superfields, where $-k$ is modular weight.

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Yukawas	$\Gamma_3 \simeq A_4$	$-k$
$Y_e = Y_3^{(4)}$	$\mathbf{3}$	4
$Y_{\nu,1} = Y_{3a}^{(6)}$	$\mathbf{3}$	6
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- Neutrino masses and mixing will be generated from the type-II seesaw mechanism.
- Modular Yukawas for charged lepton and neutrinos are of weight 4 and 6 respectively.

Mass Matrices

The superpotential of our model is given as follows:

$$\begin{aligned}\mathcal{W} &= \alpha_1 \left(\boldsymbol{Y_e} L\right)_1 E_1^c H_d + \alpha_2 \left(\boldsymbol{Y_e} L\right)_{1''} E_2^c H_d + \alpha_3 \left(\boldsymbol{Y_e} L\right)_{1'} E_3^c H_d \\ &+ \alpha \left(\boldsymbol{Y_{\nu,1}} \left(LL\right)_{3_S}\right)_1 \Delta + \beta \left(\boldsymbol{Y_{\nu,2}} \left(LL\right)_{3_S}\right)_1 \Delta \\ &+ \mu H_u H_d + \mu_\Delta H_d H_d \Delta\end{aligned}$$

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Mass Matrices for charged lepton and neutrino:

$$M_\ell = v_{H_d} \begin{pmatrix} Y_{3,1}^{(4)} & Y_{3,2}^{(4)} & Y_{3,3}^{(4)} \\ Y_{3,3}^{(4)} & Y_{3,1}^{(4)} & Y_{3,2}^{(4)} \\ Y_{3,2}^{(4)} & Y_{3,3}^{(4)} & Y_{3,1}^{(4)} \end{pmatrix} \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix},$$

$$M_\nu = v_\Delta \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ * & 2Y_2 & -Y_1 \\ * & * & 2Y_3 \end{pmatrix}.$$

where $Y_i \equiv \alpha\, \boldsymbol{Y_{3a,i}^{(6)}} + \beta\, \boldsymbol{Y_{3b,i}^{(6)}}$ with $i \in \{1,2,3\}$, v_Δ is the VEV of superfield Δ and $(*)$ represents the symmetric part of neutrino mass matrix.

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NO: $m_3 = m_1 + m_2 \quad \Delta m^2_{21} = 7.5 \times 10^{-5}\,\text{eV}^2, \quad \Delta m^2_{31} = 2.55 \times 10^{-3}\,\text{eV}^2$

$$\boxed{m_1 = 0.0282\,\text{eV}, \quad m_2 = 0.0295\,\text{eV}, \quad m_3 = 0.0578\,\text{eV}}.$$

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$$\boxed{m_3 = 7.5 \times 10^{-4}\,\text{eV}, \quad m_1 = 0.049\,\text{eV}, \quad m_2 = 0.050\,\text{eV}}.$$

Sum Rules Implications

- Effective mass of beta decay:

$$\langle m_{\nu_e}^{eff} \rangle = \sqrt{\sum_i \left|U_{ei}^2\right| m_i^2}$$
$$= \sqrt{c_{12}^2 c_{13}^2 m_1^2 + s_{12}^2 c_{13}^2 m_2^2 + s_{13}^2 m_3^2}\,.$$

NO (IO): $m_{\nu_e}^{\text{eff}} = 0.028\ (0.049)$ eV. It can be tested in KATRIN

Aker et al., 2105.08533

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- Considering 3σ ranges sum of neutrino masses:
$\sum_i m_i^{\text{NO}} \in [0.1138, 0.1176]$ eV,
$\sum_i m_i^{\text{IO}} \in [0.1007, 0.1041]$ eV.
✓ Consistent with the Planck 2018. Aghanim et al., 1807.06209

Neutrinoless Double Beta Decay

- Effective mass for neutrinoless double beta ($0\nu ee$) decay is given by

$$|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right|$$
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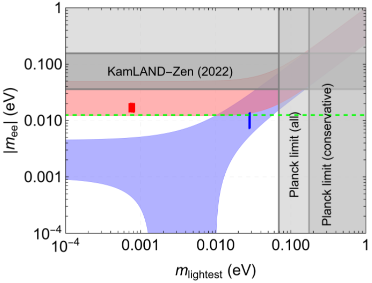

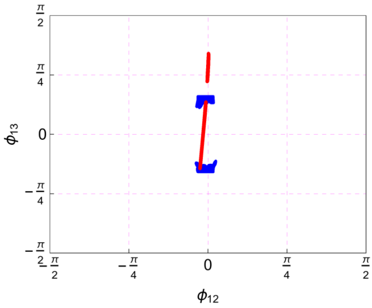

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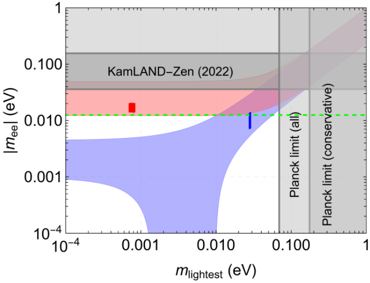

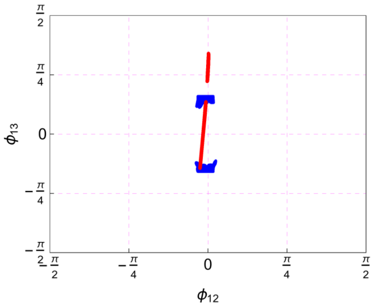

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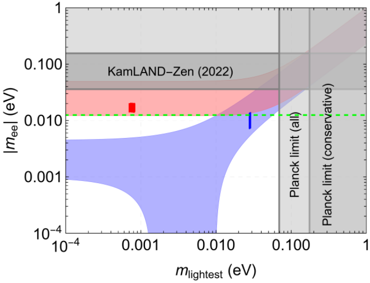
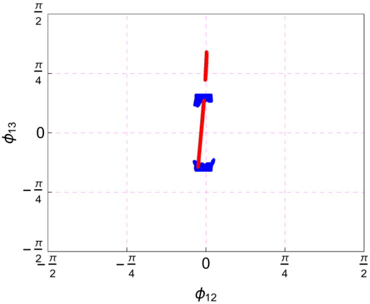
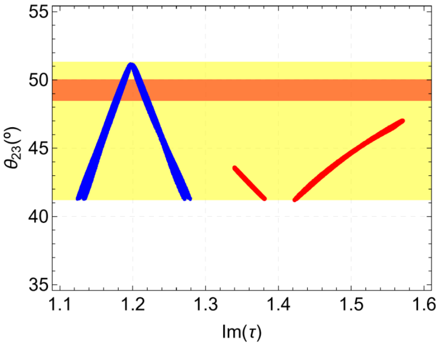




- ► Majorana phases are strongly correlated.
- ✓ **A precise prediction for neutrinoless double beta ($0\nu ee$) decay.**

Neutrino Oscillations Predictions

- In our proposal modular symmetry plays a crucial role in constraining the mixing angles.



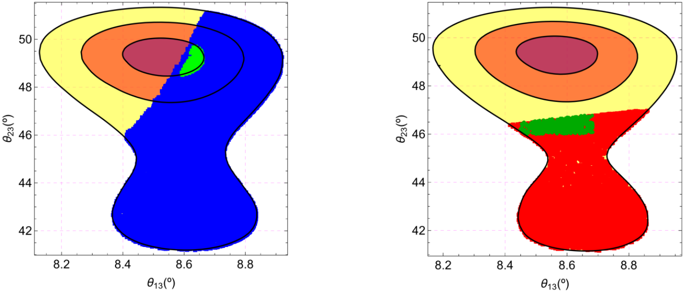
- Yukawas transform as modular forms, i.e. their values are controlled by modulus τ only.

✓ **Atmospheric angle θ_{23} is strongly correlated with imaginary part of modulus τ in both NO and IO.**

Neutrino Oscillations Predictions

AHEP global fit

de Salas, Forero, Gariazzo, Martínez-Miravé, Mena, Ternes, Tórtola, and Valle, 2006.11237



- ▶ We have precise predictions for mixing angles.
- ✓ **NO:** $\theta_{13} > 8.36^{\circ}$.
- ✓ **IO:** $\theta_{23} < 46.8^{\circ}$.

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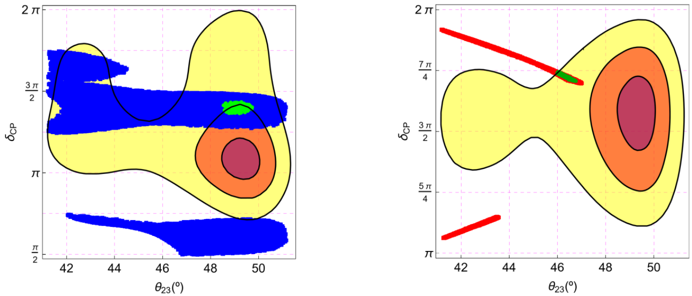

- In our model there is sharp correlation between θ_{23} and δ_{CP}.
- **Future experiments like DUNE can probe these correlations.**

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- ✓ **Neutrino mass structure leads to a sum rule for physical neutrino masses valid for both NO and IO.**
- ✓ **Sum rule fixes neutrino mass and provides a testable prediction for the sum of neutrino mass, neutrinoless double beta decay and beta decay.**
- ✓ **Model features correlation between modular symmetry parameter modulus τ and mixing angles.**
- ✓ **We also have sharp predictions for mixing angles and Dirac CP phase which can be tested in future experiments.**

Thank You

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