Exploring Axions through the Photon Ring of a Spherically Symmetric Black Hole

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in collaboration with

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Introduction

What is Axion ?

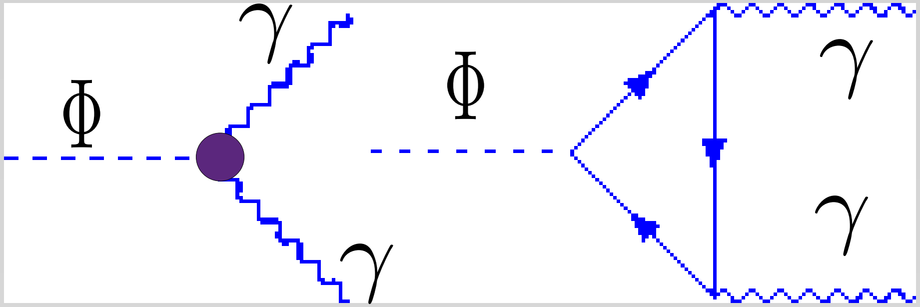
- A hypothetical elementary particle.
- **QCD Axions** Introduced to solve strong CP problem of QCD. → mass and coupling are proportional

Peccei and Quinn, Phys. Rev. Lett. 38(1977) 1440

- **Axion like particles(ALPs)** Light pseudoscalar fields predicted by many extensions of standard model.

- In the presence of magnetic field axions interact with photons through the coupling term,

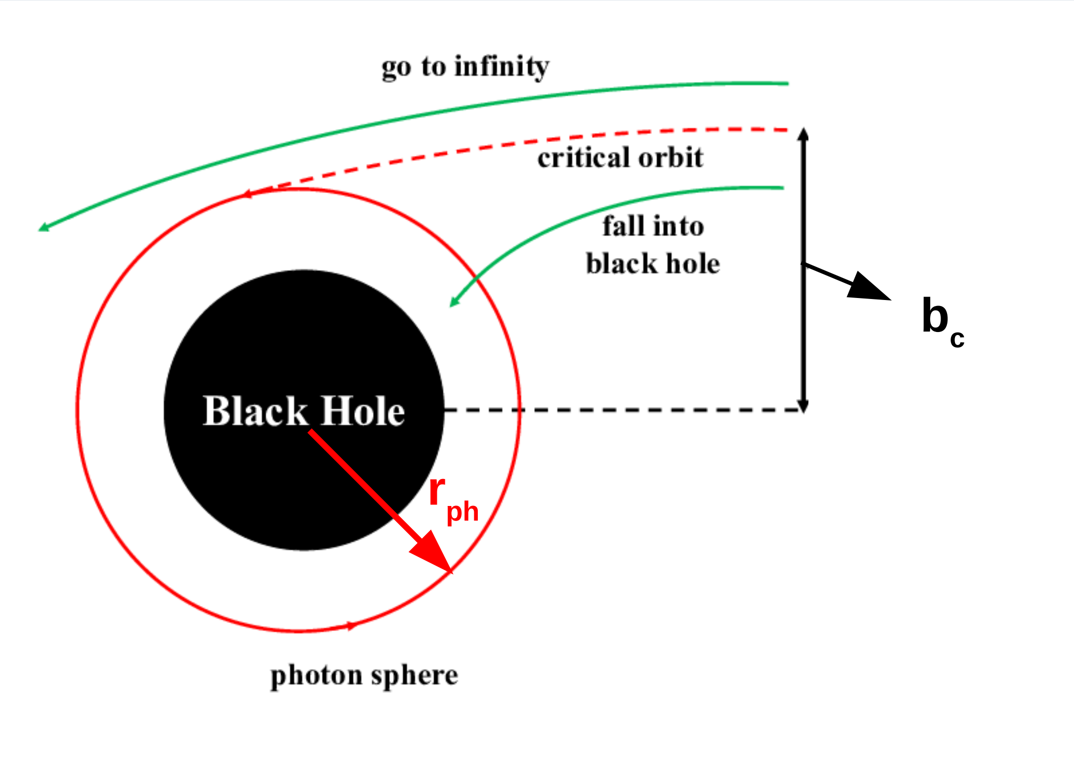
$$\mathcal{L}^{int} = -\frac{1}{4} g_{\Phi\gamma} \Phi F_{\mu\nu} \tilde{F}^{\mu\nu}$$

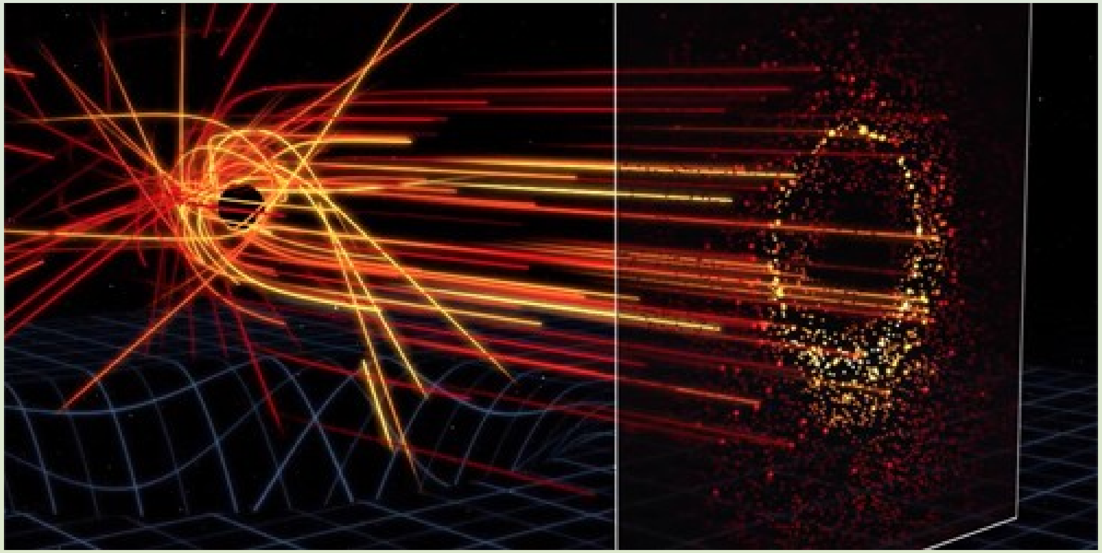


Objectives

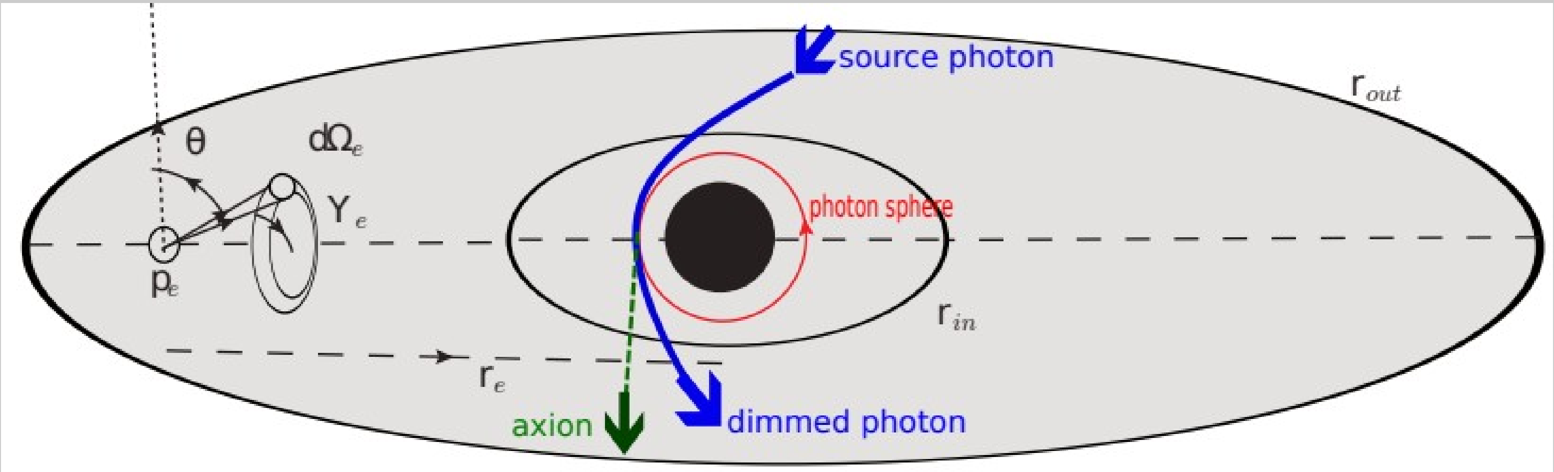
- Photon-axion conversion in black hole(BH) spacetime
- Generalizing to spherically symmetric non rotating BH
- Relevant ALP parameter space.
- Observation prospects .

Photon Ring


go to infinity
critical orbit
fall into
black hole
b_c
Black Hole
r_{ph}
photon sphere



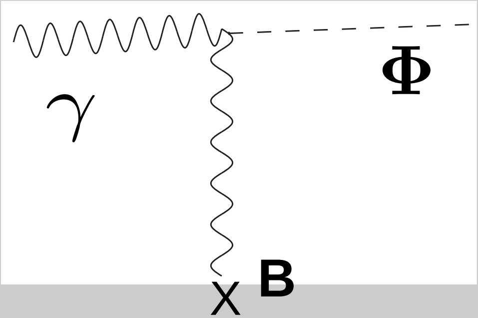
Motivation and Outline of the problem



- The Event Horizon Telescope observations of supermassive blackhole M87* centered in Messier galaxy discovered the potential existence of magnetic field of the order (1-30) gauss.

Akiyama et al. EHT collaboration Astrophys.J.Lett 875(2019)

- The Event Horizon Telescope achieved to get an image of the black hole photon sphere through radio observation.



- Photons can be converted to axion in presence of the magnetic field.

- The propagation length of photon-axion conversion is of the order of milli-parsec, comparable to the Schwarzschild radius of a supermassive black hole with a mass of $10^9 M_\odot$

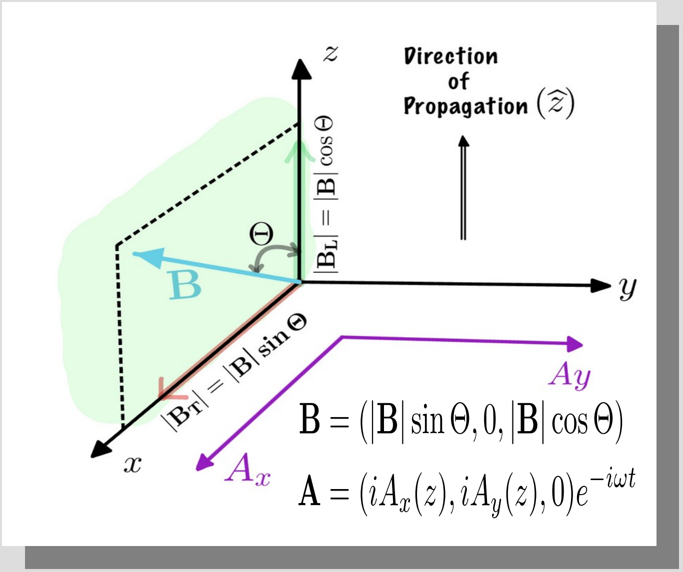
The Photon-Axion Conversion

We consider the following photon-axion system, Raffelt and Stodolsky , PRD 37(1988)1237

$$\mathcal{S} = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}m_\Phi^2\Phi^2 - \frac{1}{4}g_{\Phi\gamma}\Phi F_{\mu\nu}\widetilde{F}^{\mu\nu} \right)$$

With QED correction term to Maxwell equations, $$\mathcal{L}_{EH} = \frac{\alpha^2}{90m_e^4}\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2\right],$$

and also taking the effect of plasma as photon acquire effective mass in plasma, the equations of motion are,



$$\Box A_x - \omega_{pl}^2 A_x + 7\omega^2\frac{\alpha e^2}{45\pi m_e^4}|\mathbf{B}|^2\sin^2\Theta A_x + \omega g_{\Phi\gamma}|\mathbf{B}|\sin\Theta\Phi = 0$$

$$\Box A_y + 4\omega^2\xi\sin^2\Theta A_y = 0$$

$$(\Box - m_\Phi^2)\Phi + \omega g_{\Phi\gamma}|\mathbf{B}|\sin\Theta A_x = 0$$

The solutions of the fields are expressed as,

$$A_x(t,z) = \widetilde{A}(z)e^{-i(\omega t - kz)} + \mathrm{h.c}$$

$$\Phi(t,z) = \widetilde{\Phi}(z)e^{-i(\omega t - kz)} + \mathrm{h.c}$$

After few simplifications the equations of motion reduces to,

$$i\frac{d}{dz}\widetilde{A}(z) = \left(\frac{\omega_{\rm pl}^2}{2w} - \frac{28\alpha^2\omega}{90m_e^4}(|\mathbf{B}|\sin\Theta)^2\right)\widetilde{A}(z) - \frac{1}{2}g_{\Phi\gamma}|\mathbf{B}|\sin\Theta\widetilde{\Phi}(z)$$

$$i\frac{d}{dz}\widetilde{\Phi}(z) = \left(-\frac{1}{2}g_{\Phi\gamma}|\mathbf{B}|\sin\Theta\right)\widetilde{A}(z) + \frac{m_\Phi^2}{2\omega}\widetilde{\Phi}(z)$$

The Photon-Axion Conversion

It is convenient to rewrite the equations as,

$$i\frac{d}{dz}\begin{bmatrix}\widetilde{A}(z)\\ \widetilde{\phi}(z)\end{bmatrix} = \mathfrak{M}\begin{bmatrix}\widetilde{A}(z)\\ \widetilde{\phi}(z)\end{bmatrix}$$

Where, $\mathfrak{M} = \begin{bmatrix}\Delta_{\rm pl} - \Delta_{\rm vac} & -\Delta_{\rm M}\\ -\Delta_{\rm M} & \Delta_{\Phi}\end{bmatrix}$

The different contributions related to M87* are,

$n_e \sim 10^4 cm^{-3}$

$|B| sin\Theta = 30G$

$$\Delta_{\rm pl} \equiv \frac{\omega_{\rm pl}^2}{2\omega} = 6.9\times10^{-25}{\rm eV}\left(\frac{n_e}{\rm cm^{-3}}\right)\left(\frac{\rm keV}{\omega}\right)$$

$$\Delta_{\rm vac} \equiv \frac{28\alpha^2\omega}{90m_e^4}(|\mathbf{B}|\sin\Theta)^2 = 9.3\times10^{-29}{\rm eV}\left(\frac{\omega}{\rm keV}\right)\left(\frac{|\mathbf{B}|}{\rm Gauss}\right)^2\sin^2\Theta$$

$$\Delta_{\rm M} \equiv \frac{1}{2}g_{\Phi\gamma}|\mathbf{B}|\sin\Theta = 9.8\times10^{-23}{\rm eV}\left(\frac{g_{\Phi\gamma}}{10^{-11}{\rm GeV}^{-1}}\right)\left(\frac{|\mathbf{B}|}{\rm Gauss}\right)\sin\Theta$$

$$\Delta_{\Phi} \equiv \frac{m_\Phi^2}{2\omega} = 5\times10^{-22}{\rm eV}\left(\frac{m_\Phi}{\rm neV}\right)^2\left(\frac{\rm keV}{\omega}\right)$$

Assuming initial axion density is negligibly small with respect to that of photons, the probability of conversion of photon into axion as a function of distance z as

$$P_{\gamma\to\Phi}(z) = \left|\widetilde{\Phi}(z)\right|^2 = \left(\frac{\Delta_{\rm M}}{\Delta_{\rm osc}/2}\right)^2\sin^2\left(\frac{\Delta_{\rm osc}}{2}\,\boldsymbol{z}\right)$$

$$\Delta_{\rm osc} = \sqrt{(\Delta_\Phi - \Delta_{\rm pl} + \Delta_{\rm vac})^2 + (2\Delta_{\rm M})^2}\,.$$

Photon Pathlength

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Now, we will study the behaviour of photon geodesics near the photon sphere in perturbative approach. We define dimensionless fractional deviations around the radius of the photon sphere and the critical impact parameter as follows

$$r = r_{ph}(1+\delta r), \qquad b = b_c(1+\delta b)$$

Gralla, Holz and Wald,
PRD 100(2019) 024018

Expanding the impact parameter around its critical value we have

$$b = b_c\left[1 + \left(\frac{1}{2} - \frac{1}{4}b_c^2 f''(r_{ph})\right)\delta r^2 + \mathcal{O}(\delta r^3)\right]$$

Equation of null geodesics,

$$\frac{dr}{dz} = \pm\sqrt{\frac{b^2}{r^2}f(r)} = \pm\sqrt{\mathfrak{p}\delta r^2 - 2\delta b}$$

PS,Roy, Sau & Sengupta
Arxiv 2310.05908

where,

$$\mathfrak{p} \equiv 1 - \frac{1}{2}r_{ph}^2\frac{f''(r_{ph})}{f(r_{ph})}, \qquad \mathfrak{a} \equiv \left[\frac{1}{2} - \frac{1}{4}b_c^2 f''(r_{ph})\right]\left(\frac{M}{r_{ph}}\right)^2$$

M87*

M=$6.2\times10^9 M_\odot$

The pathlength of photon around the photon sphere,

$$\boldsymbol{Z} = -\frac{r_{ph}}{\sqrt{\mathfrak{p}}}\ln\left[\frac{2(b-b_c)}{\mathfrak{p}b_c} \times \frac{r_{ph}^2}{\epsilon^2 M^2}\right]$$

Photon number calculation :

The number of photons reaching the photon sphere per unit time t and unit frequency ω_c with impact parameter $(b, b+db)$ is given by

$$\left(\frac{d^3N}{dtd\omega_c db}\right) = 4\pi^2 \int_{r_{\rm in}}^{r_{\rm out}} dr_e J_e\left(\frac{\sqrt{f(r_{ph})}\omega_c}{\sqrt{f(r_e)}}, r_e\right) \times \frac{br_e\sqrt{f(r_e)}}{\sqrt{(r_e^2/f(r_e)) - b^2}}$$

Nomura, Saito and Soda, PRD 107(2023)123505

$$J_e^{(N)}(\omega_e, r_e) = \frac{1}{4\pi\omega_e}\left(\frac{2^4\alpha^3}{3m_e}\right)\left(\frac{2\pi}{3m_e}\right)^{1/2} T_e^{-1/2} n_e^2 e^{-\omega_e/T_e}\bar{g}_{ff}$$

$$\mathrm{T}_e = T_{e,c}\left(\frac{r_e}{r_{ph}}\right)^{-1} \qquad \mathrm{T}_{e,c} = 10^{11}K$$

$$n_e = n_{e,c}\left(\frac{r_e}{r_{ph}}\right)^{-3/2} \qquad \mathrm{n}_{e,c} = 10^4 cm^{-3}$$

To calculate the number of photons that transform to axions, near the photon sphere, per unit time t and unit frequency ω_c, the following equation can be used

$$\frac{d^2N_{\gamma\to\Phi}}{dtd\omega_c} = \int_{b_c}^{b_c\left(1+\mathfrak{a}\epsilon^2\right)} db\ \frac{1}{2}\left(\frac{d^3N}{dtd\omega_c db}\right) P_{\gamma\to\Phi}\left(\sqrt{f(r_{ph})}z(b)\right)$$

where

$$P_{\gamma\to\Phi}\left(\sqrt{f(r_{ph})}z(b)\right) = \left(\frac{2\Delta_{\rm M}}{\Delta_{\rm osc}}\right)^2 \times \sin^2\left(-\frac{\Delta_{\rm osc}}{2}\frac{r_{ph}}{\sqrt{\mathfrak{p}}\sqrt{f(r_{ph})}} \ln\left[\frac{2(b-b_c)}{b_c}\frac{r_{ph}^2}{\epsilon^2 M^2}\right]\right)$$

The fraction of photons entering the region near the photon sphere that are converted into axions is given as

$$\frac{d^2N_{\gamma\to\phi}}{dtd\omega_c} \Big/ \frac{d^2N}{dtd\omega_c} \simeq \frac{1}{4}\left(\frac{2\Delta_{\rm M}}{\Delta_{\rm osc}}\right)^2 \left[\frac{(r_{ph})^2(\Delta_{osc})^2}{(r_{ph})^2(\Delta_{osc})^2 + \mathfrak{p}f(r_{ph})}\right]$$

The maximal photon-axion dimming rate

For the efficient conversion, we have $\Delta_{\rm osc} \simeq 2\Delta_{\rm M}$. For such a scenario, the conversion factor (CF) can be given by,

$$CF \simeq \frac{1}{4}\left[\frac{\left(\frac{r_{ph}}{M}\right)^2 (2M\Delta_{\rm M})^2}{\left(\frac{r_{ph}}{M}\right)^2 (2M\Delta_{\rm M})^2 + f(r_{ph})}\right]$$

PS,Roy, Sau & Sengupta
Arxiv 2310.05908

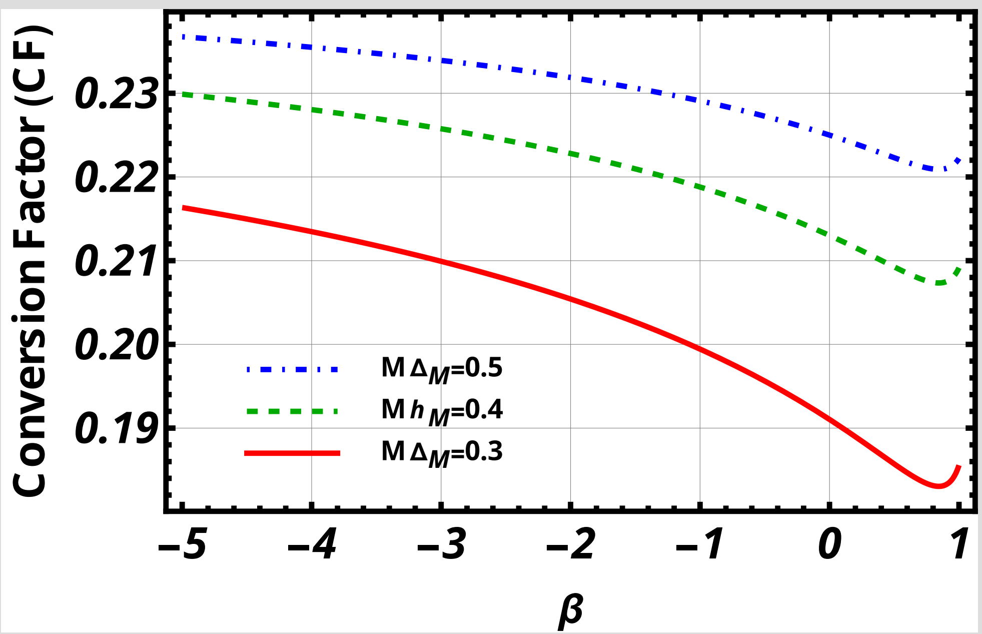
With

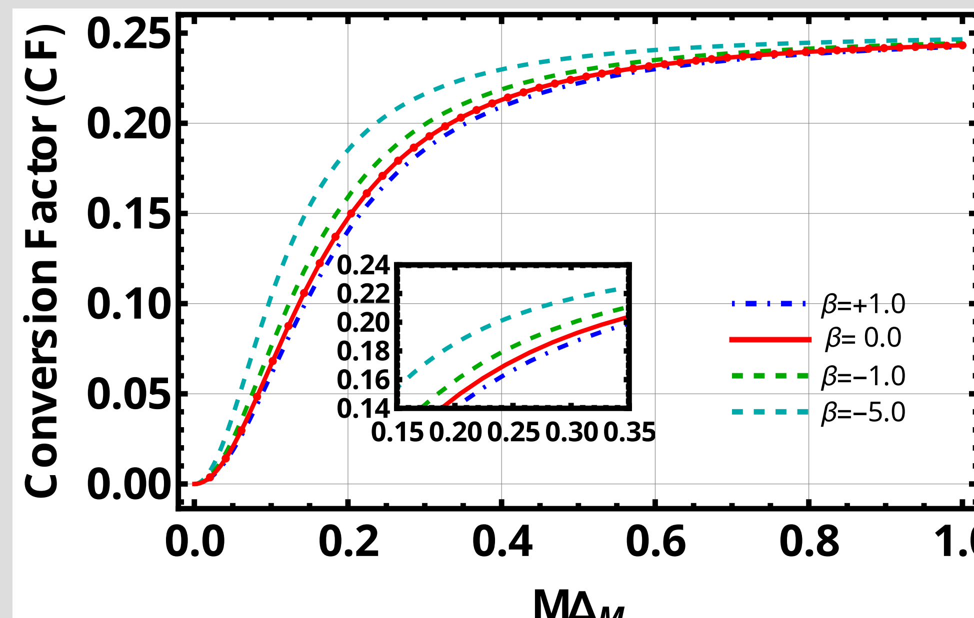
$$f(r) = \left(1 - \frac{2M}{r} + \frac{\beta M^2}{r^2}\right)$$

M87*

$M\Delta_{\rm M} = 0.13$

Special case : β=0 which corresponds to a Schwarzchild Black Hole





Expected normalized spectral luminosities

In the unit of $L^0_\omega = 6.48\times10^{27}\mathrm{erg}\cdot\mathrm{sec}^{-1}\cdot\mathrm{KeV}^{-1}\epsilon^2\left(\frac{M}{6.2\times10^9 M_\odot}\right)^3\left(\frac{T_{e,c}}{10^{11}K}\right)^{-1/2}\left(\frac{n_{e,c}}{10^4\mathrm{cm}^{-3}}\right)^2\bar{g}_{ff}$

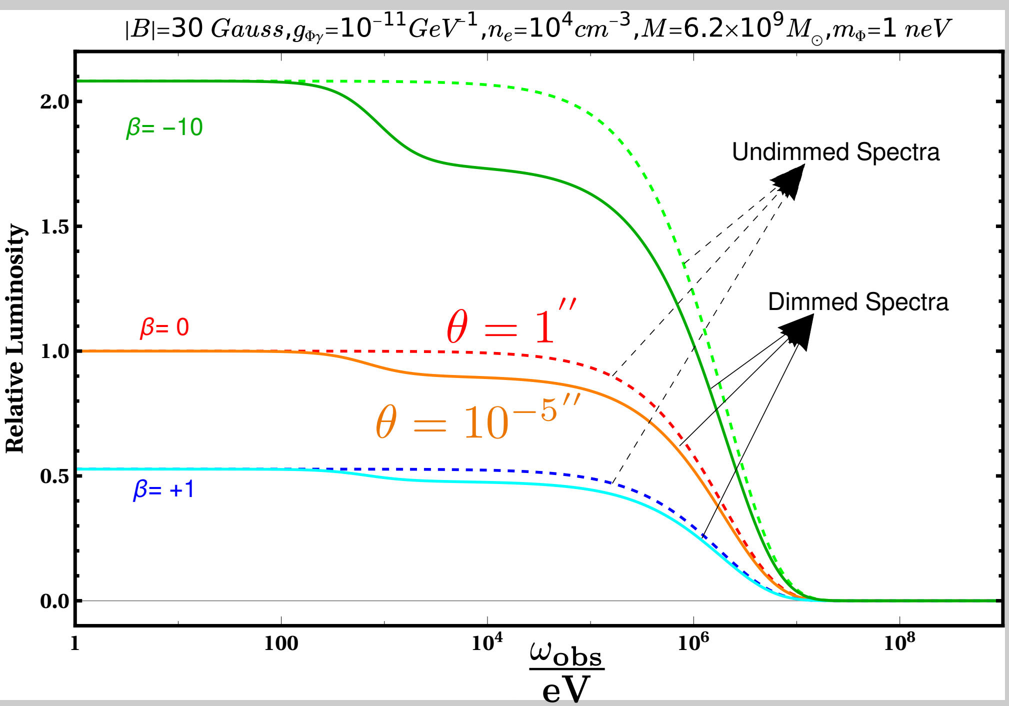
the spectras for different values of β in the metric

$$L_w^{\gamma\, undimmed} = \frac{1}{5.37L^0_w}\frac{d^2N}{dtdw_{obs}}\times w_{obs}$$

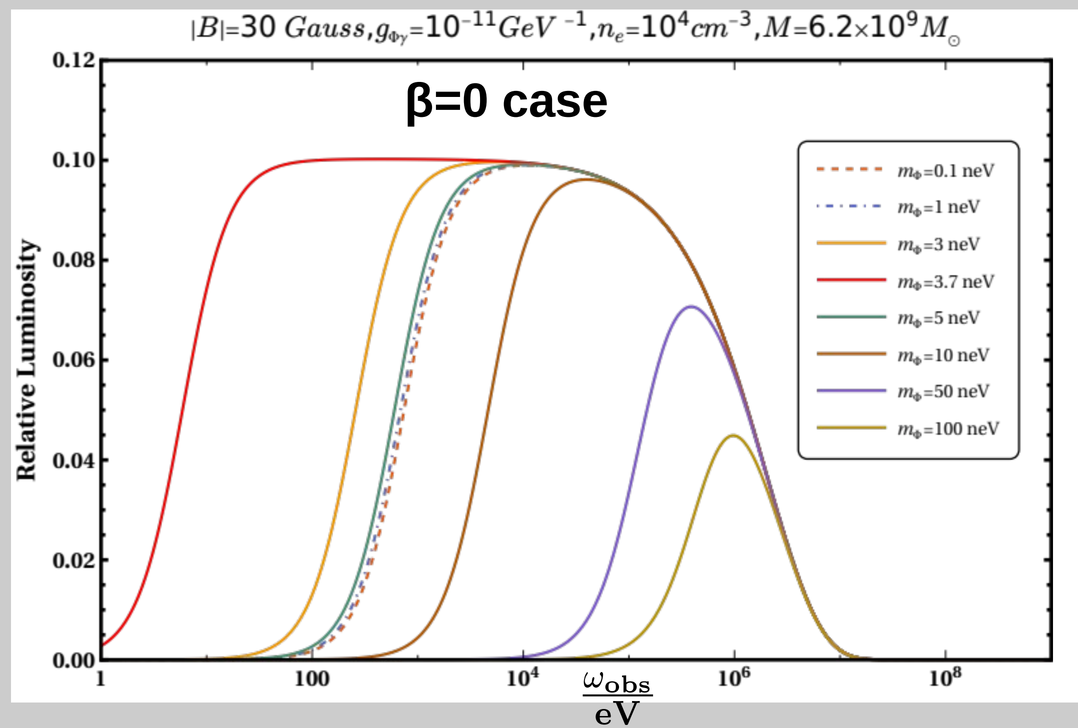
$$L_w^{\gamma\, dimmed} = \frac{1}{5.37L^0_w}(1-CF)\frac{d^2N}{dtdw_{obs}}\times w_{obs}$$

$$L_w^{\Phi} = \frac{1}{5.37L^0_w}\times CF\times\frac{d^2N}{dtdw_{obs}}\times w_{obs}$$

Photon Spectra



Axion Spectra

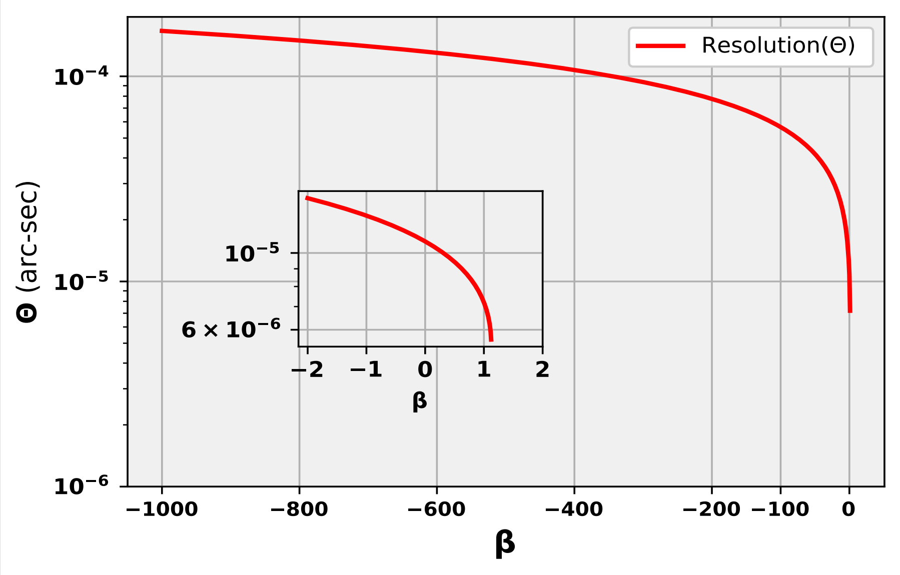


Required resolution :

In case of Schwarzchild black hole, the resolution

$$\theta = \frac{\Re}{D}\bigg|_{\beta=0} \lesssim 1.09 \times 10^{-5} \text{ arc-sec} \left(\frac{M}{6.2 \times 10^9 M_\odot}\right)\left(\frac{16.8 \text{ Mpc}}{D}\right)$$

$\Re$ denoting the size of photon sphere

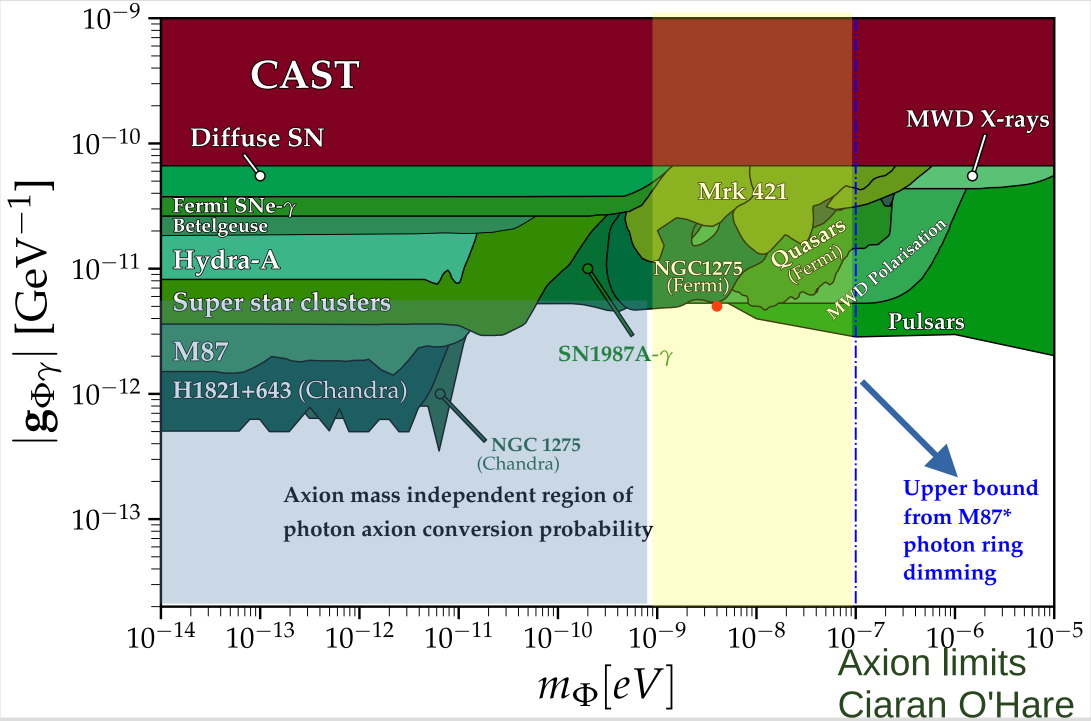


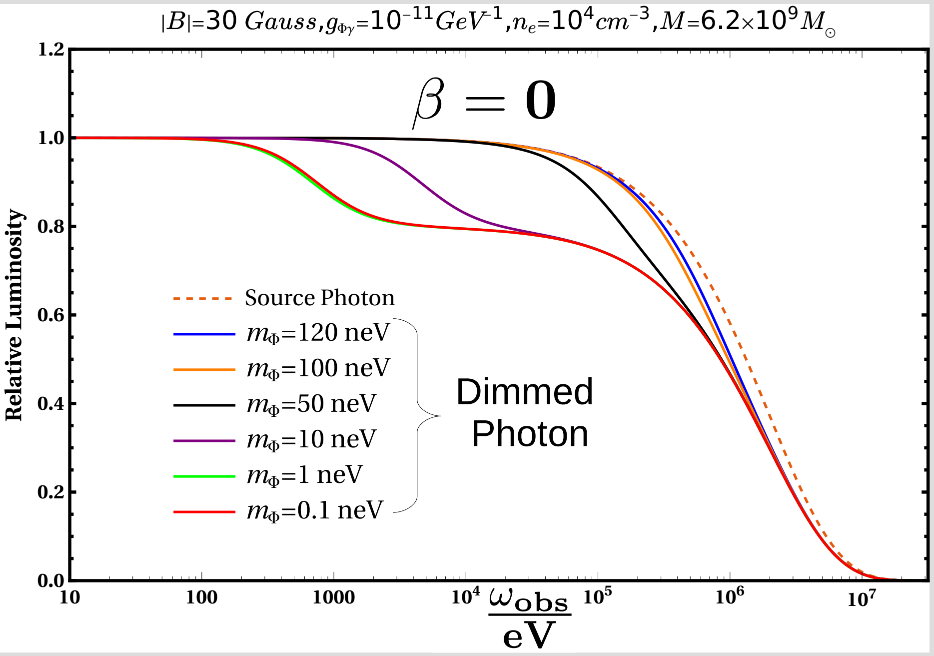
The resolution for non zero β parameter

Why interested with negative β ?

- Greater dimming
- Lesser resolution
- Signature of extra dimension

The axion mass window of expected photon-ring dimming:





Conclusion

- Photon-ring dimming is possible due to photon-axion oscillation with axion mass $\lesssim$ 100 neV and axion-photon coupling $\sim 10^{-11} GeV^{-1}$

- The dimming is expected to be observed in X-ray - Gamma ray band $(\approx (100 - 10^6) eV)$

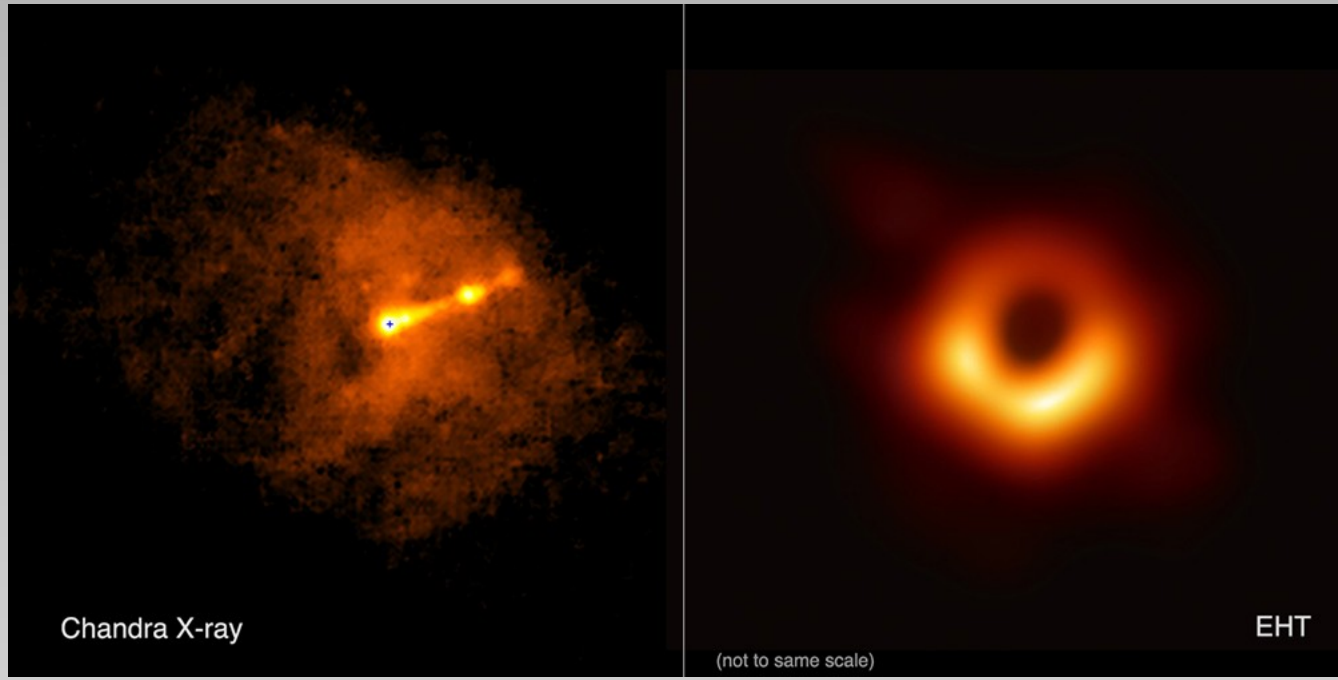
- The maximum possible dimming from our analysis of spherically symmetric black hole is about 25%.

- The expected resolution required for M87* to be a Schwarzchild BH is $\lesssim 10^{-5}$ arc-sec

- The extra dimensional signature can also be explored if the dimming rate and required resolution matches with observation.

- A similar approach can be used for a more massive black hole, along with stronger magnetic fields,resulting in increased dimming and subsequently higher flux levels.

https://chandra.cfa.harvard.edu/blackhole/



$\theta \approx 1''$

$\theta \approx 10^{-5''}$

$\nu \approx keV$

$\nu = 230GHz$

$\nu \approx keV - GeV$

$\theta \approx 10^{-5''}$


Thank You

$$\omega_{\rm pl} \equiv \sqrt{\frac{4\pi\alpha n_e}{m_e}} = 3.7\times 10^{-11}\,{\rm eV}\sqrt{\frac{n_e}{{\rm cm}^{-3}}}$$

$$\Box A_x(t,z) \simeq 2i\omega\partial_z\widetilde{A}(z)e^{-i(\omega t-kz)} + {\rm h.c.}$$

$$\Box\Phi(t,z) \simeq 2i\omega\partial_z\widetilde{\Phi}(z)e^{-i(\omega t-kz)} + {\rm h.c.}$$

$$\mathbf{O}^T\mathbf{M}\mathbf{O} = \begin{bmatrix}\lambda_+ & 0\\ 0 & \lambda_-\end{bmatrix}, \quad \mathbf{O} \equiv \begin{bmatrix}\cos\vartheta & \sin\vartheta\\ -\sin\vartheta & \cos\vartheta\end{bmatrix} \qquad \vartheta = \frac{1}{2}\arctan\left(\frac{2\Delta_{\rm M}}{\Delta_\Phi - \Delta_\parallel}\right).$$

$$i\frac{d}{dz}(\mathbf{O}^T\Psi(z)) = \begin{bmatrix}\lambda_+ & 0\\ 0 & \lambda_-\end{bmatrix}(\mathbf{O}^T\Psi(z)). \qquad \Psi(z) = \begin{bmatrix}\widetilde{A}(z)\\ \widetilde{\phi}(z)\end{bmatrix}$$

$$\widetilde{A}(z) = \left(\cos^2\vartheta e^{-i\lambda_+ z} + \sin^2\vartheta e^{-i\lambda_- z}\right)\widetilde{A}(0) + \sin\vartheta\cos\vartheta\left(e^{-i\lambda_+ z} - e^{-i\lambda_- z}\right)\widetilde{\Phi}(0)$$

$$\widetilde{\Phi}(z) = \sin\vartheta\cos\vartheta\left(e^{-i\lambda_+ z} - e^{-i\lambda_- z}\right)\widetilde{A}(0) + \left(\sin^2\vartheta e^{-i\lambda_+ z} + \cos^2\vartheta e^{-i\lambda_- z}\right)\widetilde{\Phi}(0)$$

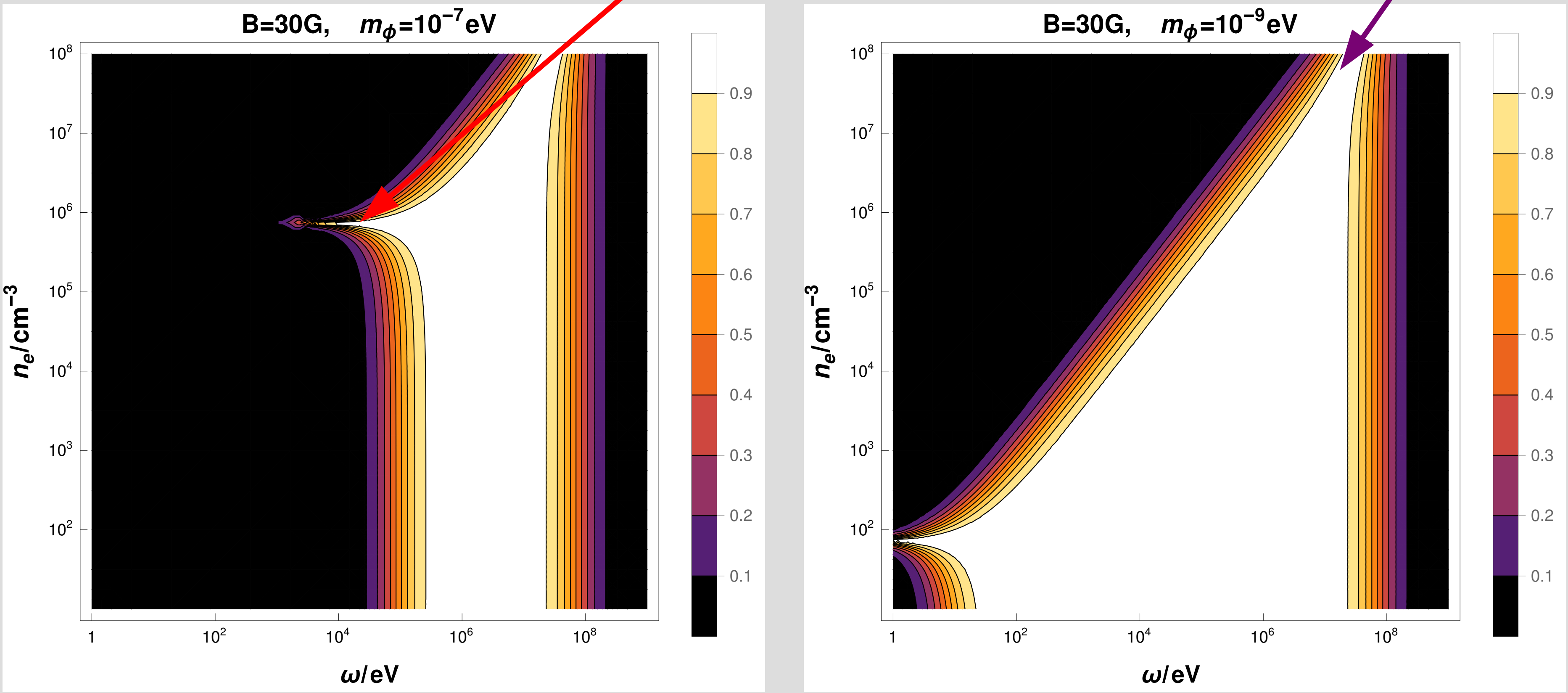
$$\triangle s = 2\times\frac{r_{ph}}{\sqrt{h(r_{ph})}}\int_{\sqrt{\frac{2}{\mathfrak{p}}\delta b}}^{\delta R}\frac{\delta r}{\sqrt{\mathfrak{p}\delta r^2 - 2\delta b}}$$

$$J_e^{(N)}(\omega_e, r_e) = \frac{1}{4\pi\omega_e}\left(\frac{2^4\alpha^3}{3m_e}\right)\left(\frac{2\pi}{3m_e}\right)^{1/2} T_e^{-1/2} n_e^2 e^{-\omega_e/T_e}\bar{g}_{ff}$$

ω-n_e plane for finite photon-axion conversion probability $\left(\frac{\Delta_M}{\Delta_{osc}/2}\right)^2$

Observationaly plasmatic electron number density has a range $n_e \sim 10^{4\text{-}7}\ \text{cm}^{-3}$

$\Delta_{plasma} = \Delta_{\Phi}$ $\Delta_{plasma} = \Delta_{vac}$



From the above region plots , we have set $n_e \sim 10^{4}\ \text{cm}^{-3}$ and the frequency band of finite Photon-axion conversion falls in ~ $(10^4 - 10^6)$ eV

The dimming rate with the change of coupling, mass of axion and the β parameter :

$g_{\phi\gamma}$	β	% of dim	β	% of dim	β	% of dim
10^{-11}	0	10	0	0.003	0	9.97
	-10	16.2	-10	0.003	-10	16.2
5×10^{-12}	0	3.56	0	8.6×10^{-6}	0	3.55
	-10	7.9	-10	8.6×10^{-6}	-10	7.89
10^{-12}	0	0.16	0	3.4×10^{-7}	0	0.16
	-10	0.45	-10	3.4×10^{-7}	-10	0.45
m_ϕ →		1 neV		100 neV		0.01 neV