

# $W$ -mass and lepton $g-2$ in extended inert 2HDM

Based on JHEP 11 (2021) 056 and ongoing study

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# Introduction

## Anomaly in lepton magnetic dipole moment

- ▶ Muon anomalous magnetic moment

$$a_{\mu}^{\text{SM}} = 116591810(43) \times 10^{-11} \text{ [Aoyama et. al., 2020]}$$

$$a_{\mu}^{\text{BNL}} = 116592089(63) \times 10^{-11} \text{ [PRD 73 (2006) 072003]}$$

$$a_{\mu}^{\text{FNAL}} = 116592040(54) \times 10^{-11} \text{ [PRL. 126 (2021) 141801]}$$

$$a_{\mu}^{\text{FNAL}} = 116592055(24) \times 10^{-11} \text{ [PRL 131 (2023) no.16, 161802]}$$

$$\text{New world average: } a_{\mu}^{\text{exp.}} = 116592059(22) \times 10^{-11}$$

- ▶ Discrepancy in MDM

$$\Delta a_{\mu} = a_{\mu}^{\text{exp.}} - a_{\mu}^{\text{SM}} = (249 \pm 48) \times 10^{-11}$$

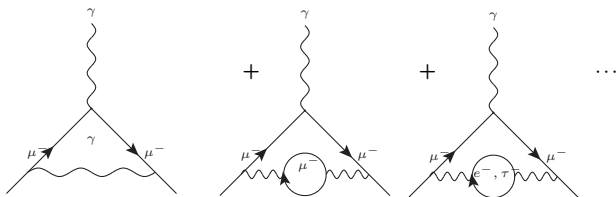
$$\Delta a_e = a_e^{\text{exp.}} - a_e^{\text{SM}} = (-88 \pm 28 \pm 23 \pm 2) \times 10^{-14} \text{ [Science 360 (2018) 191]}$$

# 1. Muon $g - 2$ (SM)

$a_\mu$  in SM

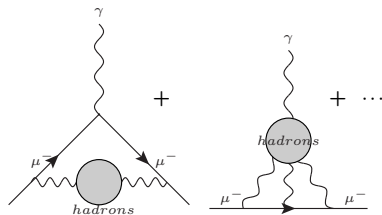
$$a_\mu(\text{SM}) = a_\mu(\text{QED leptonic}) + a_\mu(\text{QED hadronic}) + a_\mu(\text{EW})$$

## 1. QED Contributions (leptonic)



# 1. Muon $g - 2$ (SM)

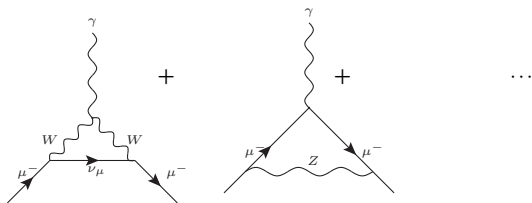
## QED Hadronic contributions



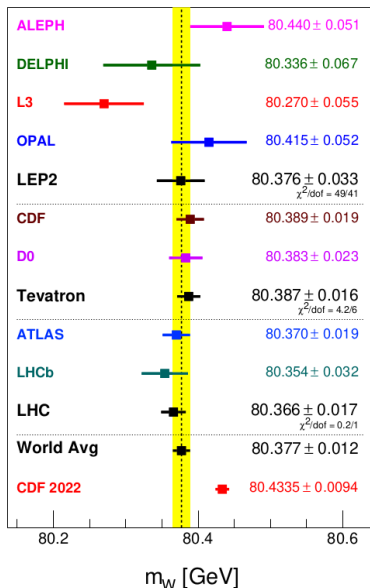
Hadronic vacuum polarization (HVP)

Hadronic light-by-light scattering (HLbL)

## Electroweak contributions



# W mass problem



- The pre-2022 CDF result of  $W^-$  mass [PTEP 2022, 083C01 (2022) (PDG)]

$$m_W^{\text{SM}} = 80.377 \pm 0.006 \text{ GeV}$$

- CDF-2022 result

[Science 376, 170 (2022)]

$$m_W^{\text{CDF}} = 80.4335 \pm 0.0094 \text{ GeV}$$

# Model

- SM augmented by 2HDM + complex scalar singlet + vector-like lepton

$$\mathcal{L} \supset \mathcal{L}_{\text{scalar}} + \mathcal{L}_Y + \mathcal{L}_{\text{VL}}$$

$$\Phi_1 = \begin{bmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}} (v_{\text{SM}} + \phi_1^0 + i\eta_1^0) \end{bmatrix}; \quad \Phi_2 = \begin{bmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}} (\phi_2^0 + i\eta_2^0) \end{bmatrix}$$

$$\Phi_3 = \frac{1}{\sqrt{2}} (v_s + \phi_3^0 + i\eta_3^0); \quad \chi^\pm$$

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$$\Phi_3 = \frac{1}{\sqrt{2}} (v_s + \phi_3^0 + i\eta_3^0); \quad \chi^\pm$$

- Quantum numbers

Fields	$Q_l$	$l_L$	$u_R$	$d_R$	$e_R$	$\Phi_1$	$\Phi_2$	$\Phi_3$	$\chi_L$	$\chi_R$	$V^\mu$
$SU(3)_c$	3	1	3	3	1	1	1	1	1	1	$G^\mu$
$SU(2)_L$	2	2	1	1	1	2	2	1	1	1	$W_i^\mu$
$U(1)_Y$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	-1	-1	$B^\mu$
$Z_2$	+	+	+	+	+	+	-	-	-	+	+



# Model

## ► Scalar Lagrangian

$$\begin{aligned}
 \mathcal{L}_{\text{scalar}} &= (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) + (D_\mu \Phi_3)^* (D^\mu \Phi_3) - V_{\text{scalar}} \\
 V_{\text{scalar}} &= -\frac{1}{2} m_{11}^2 (\Phi_1^\dagger \Phi_1) - \frac{1}{2} m_{22}^2 (\Phi_2^\dagger \Phi_2) + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 \\
 &\quad + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{1}{2} \left[ \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + h.c. \right] \\
 &\quad - \frac{1}{2} m_{33}^2 \Phi_3^* \Phi_3 + \frac{\lambda_8}{2} (\Phi_3^* \Phi_3)^2 + \lambda_{11} |\Phi_1|^2 \Phi_3^* \Phi_3 + \lambda_{13} |\Phi_2|^2 \Phi_3^* \Phi_3 \\
 &\quad - i \kappa \left[ (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) (\Phi_3 - \Phi_3^*) \right]
 \end{aligned}$$

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 &\quad - \frac{1}{2} m_{33}^2 \Phi_3^* \Phi_3 + \frac{\lambda_8}{2} (\Phi_3^* \Phi_3)^2 + \lambda_{11} |\Phi_1|^2 \Phi_3^* \Phi_3 + \lambda_{13} |\Phi_2|^2 \Phi_3^* \Phi_3 \\
 &\quad - i \kappa \left[ (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) (\Phi_3 - \Phi_3^*) \right]
 \end{aligned}$$

## ► Yukawa terms

$$\begin{aligned}
 \mathcal{L}_Y &= -y_u \bar{Q}_L \widetilde{\Phi}_1 u_R - y_d \bar{Q}_L \Phi_1 d_R - y_l \bar{L} \Phi_1 e_R - y_1 \bar{L} \Phi_2 e_R + h.c. \\
 \mathcal{L}_{VL} &= \bar{\chi} i \left( \not{\partial} - ig' \frac{Y}{2} \not{B} \right) \chi - m_\chi \bar{\chi} \chi - y_2 \bar{\chi}_L \chi_R \Phi_3 - y_3 \bar{\chi}_L e_R \Phi_3
 \end{aligned}$$

# Model

## Scalar mass eigen states

- $\Phi_1 - \Phi_3$  mixing  $\rightarrow$  CP even states

$$M_{\phi_1^0 \phi_3^0}^2 = \frac{1}{2} \begin{pmatrix} \phi_1^0 & \phi_3^0 \end{pmatrix} \begin{pmatrix} 1 v_{SM}^2 & 11 v_{SM} v_s \\ 11 v_{SM} v_s & 8 v_s^2 \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_3^0 \end{pmatrix}$$

$$m_{h_1}^2 = \cos^2 \theta_{13} \lambda_1 v_{SM}^2 + \sin(2\theta_{13}) v_s \lambda_{11} v_{SM} + \sin^2 \theta_{13} v_s^2 \lambda_8$$

$$m_{h_3}^2 = \sin^2 \theta_{13} \lambda_1 v_{SM}^2 - \sin(2\theta_{13}) v_s \lambda_{11} v_{SM} + \cos^2 \theta_{13} v_s^2 \lambda_8$$

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- $\Phi_2 - \Phi_3$  mixing  $\rightarrow$  CP odd states

$$\frac{1}{2} \begin{pmatrix} \eta_2^0 & \eta_3^0 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} m_{22}^2 + \frac{1}{2} \lambda_{345} v_{SM}^2 + \frac{1}{2} v_s^2 \lambda_{13} & -\sqrt{2} \kappa v_{SM} \\ -\sqrt{2} \kappa v_{SM} & 0 \end{pmatrix} \begin{pmatrix} \eta_2^0 \\ \eta_3^0 \end{pmatrix}$$

$$m_{A^0}^2 = \frac{1}{2} \left( \bar{\lambda}_{345} v_{SM}^2 - m_{22}^2 + 13 v_s^2 \right) \cos^2 \theta_{23} - \sqrt{2} \kappa v_{SM} \sin 2\theta_{23}$$

$$m_{P^0}^2 = \frac{1}{2} \left( \bar{\lambda}_{345} v_{SM}^2 - m_{22}^2 + 13 v_s^2 \right) \sin^2 \theta_{23} + \sqrt{2} \kappa v_{SM} \sin 2\theta_{23}$$

# Model

## Scalar mass eigen states

$$\phi_2^0 \rightarrow h_2$$

$$\eta_1^0 \rightarrow G^0 \text{ (massless Nambu-Goldstone Boson)}$$

$$\phi_1^\pm \rightarrow G^\pm \text{ (massless Nambu-Goldstone Boson)}$$

$$\phi_2^\pm \rightarrow H^\pm$$

$$m_{h_2}^2 = \frac{1}{2} [-m_{22}^2 + (\lambda_3 + \lambda_4 + \lambda_5) v_{\text{SM}}^2 + 13 v_s^2]$$

$$m_{H^\pm}^2 = -m_{22}^2 + \lambda_3 v_{\text{SM}}^2 + 13 v_s^2$$

**Model parameters:**  $m_{11}^2, m_{22}^2, m_{33}^2, \lambda_{i=1,\dots,5}, \lambda_8, \lambda_{11}, \lambda_{13}$  and  $\kappa$



**Physical parameters:**  $v_{\text{SM}}, v_s, m_{h_1}^2, m_{H^\pm}^2, m_{h_2}^2, m_{H^\pm}^2, m_{A^0}^2, m_{P^0}^2, \theta_{13}, \theta_{23}$  and  $m_{22}^2$

## Model (Positivity and minimisation Conditions)

### ► Positivity Conditions

$$\mathcal{H} = \begin{vmatrix} \lambda_1 & \lambda_3 + \lambda_4 - |\lambda_5| & \lambda_{11} \\ \lambda_3 + \lambda_4 - |\lambda_5| & \lambda_2 & \lambda_{13} \\ \lambda_{11} & \lambda_{13} & \lambda_8 \end{vmatrix} > 0$$

along with  $\lambda_1, \lambda_2$  and  $\lambda_8 > 0$ . This leads to the following co-positivity conditions:

$$\lambda_1, \lambda_2, \lambda_8 > 0,$$

$$\bar{\lambda}_{12} \equiv \lambda_3 + \Theta[|\lambda_5| - \lambda_4] (\lambda_4 - |\lambda_5|) + \sqrt{\lambda_1 \lambda_2} > 0,$$

$$\bar{\lambda}_{13} \equiv \lambda_{11} + \sqrt{\lambda_1 \lambda_8} > 0, \quad \bar{\lambda}_{23} \equiv \lambda_{13} + \sqrt{\lambda_2 \lambda_8} > 0 \text{ and}$$

$$\sqrt{\lambda_1 \lambda_2 \lambda_8} + [\lambda_3 + \Theta[|\lambda_5| - \lambda_4] (\lambda_4 - |\lambda_5|)] \sqrt{\lambda_8} + \lambda_{11} \sqrt{\lambda_2} + \sqrt{2 \bar{\lambda}_{12} \bar{\lambda}_{13} \bar{\lambda}_{23}} > 0$$

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$$\sqrt{\lambda_1 \lambda_2 \lambda_8} + [\lambda_3 + \Theta[|\lambda_5| - \lambda_4] (\lambda_4 - |\lambda_5|)] \sqrt{\lambda_8} + \lambda_{11} \sqrt{\lambda_2} + \sqrt{2 \bar{\lambda}_{12} \bar{\lambda}_{13} \bar{\lambda}_{23}} > 0$$

### ► Minimisation Conditions

$$m_{11}^2 = \lambda_1 v_{SM}^2 + \lambda_{11} v_s^2$$

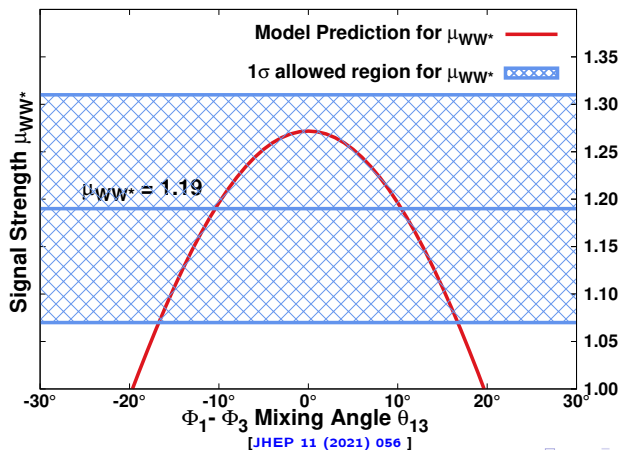
$$m_{33}^2 = \lambda_8 v_s^2 + \lambda_{11} v_{SM}^2$$

The  $m_{22}^2$  parameter remains unconstrained by the extremum condition.

## Constraints from Higgs decay

Signal strength:

$$\mu_{XY} = \frac{\sigma(pp \rightarrow h_1 \rightarrow XY)}{\sigma(pp \rightarrow h \rightarrow XY)^{SM}} = \frac{\Gamma(h_1 \rightarrow gg)}{\Gamma(h^{SM} \rightarrow gg)} \frac{BR(h_1 \rightarrow XY)}{BR(h^{SM} \rightarrow XY)} = \cos^4 \theta_{13} \frac{\Gamma(h^{SM} \rightarrow \text{all})}{\Gamma(h_1 \rightarrow \text{all})}$$

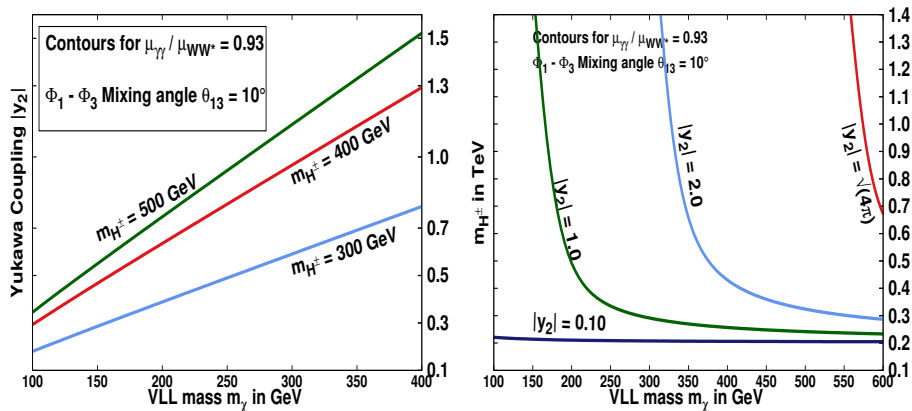




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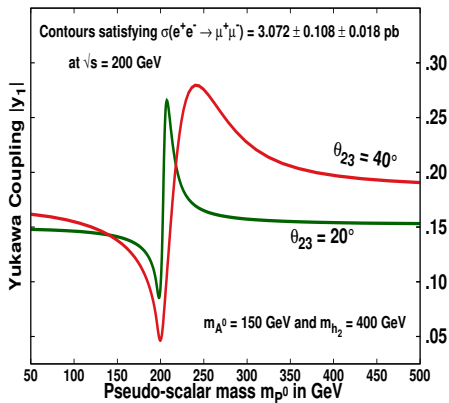
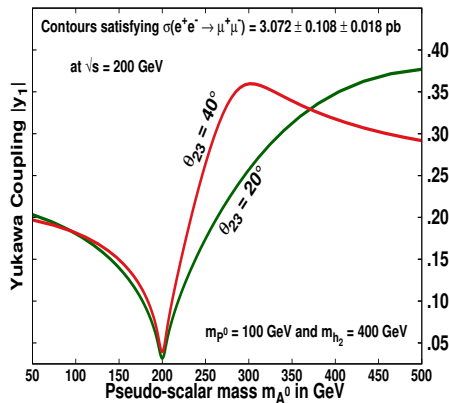


[JHEP 11 (2021) 056]

## Constraints from LEP II

$\sigma(e^+ e^- \rightarrow \mu^+ \mu^-) = 3.072 \pm 0.108 \pm 0.018$  pb at  $\sqrt{s} = 200$  GeV [Phys. Rept. 532 (2013) 119]

$$\sigma_{\mu^+\mu^-}^{\text{NP}} \simeq \frac{s}{64\pi} \sqrt{\frac{1 - 4\frac{m_\mu^2}{s}}{1 - 4\frac{m_e^2}{s}}} y_1^4 \left[ \left\{ \frac{\cos^2 \theta_{23}}{s - m_{A^0}^2} + \frac{\sin^2 \theta_{23}}{s - m_{P^0}^2} \right\}^2 + \frac{1}{(s - m_{h_2}^2)^2} \right]$$

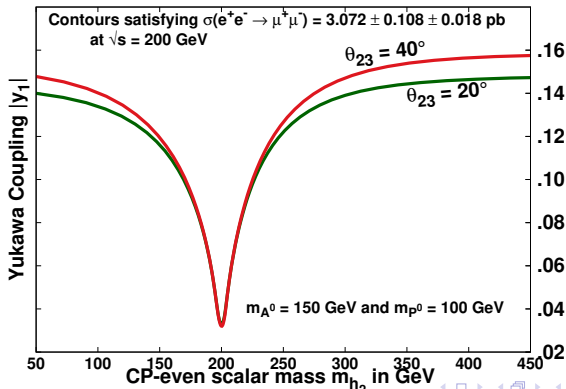


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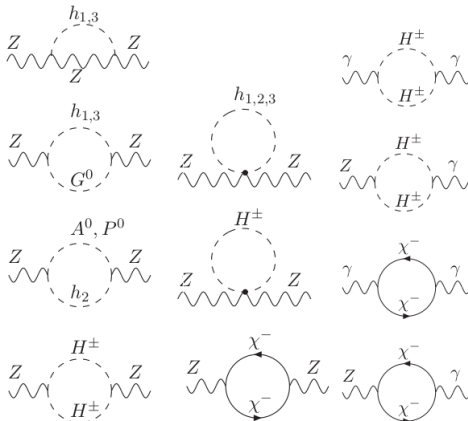


## Constraints from electroweak precision observables

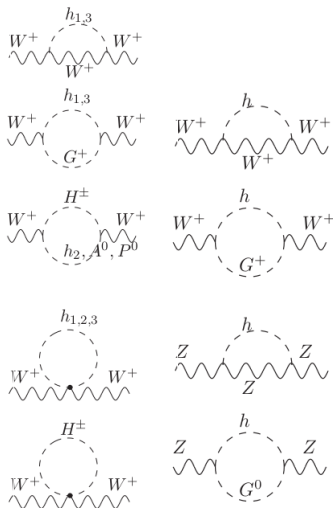
- ▶ The deviation in the theoretical predictions from the electroweak precision measurements [PTEP 2022, 083C01 (2022) (PDG)]

$$\Delta S = S_{\text{expt.}} - S_{\text{SM}} = 0.01 \pm 0.10$$

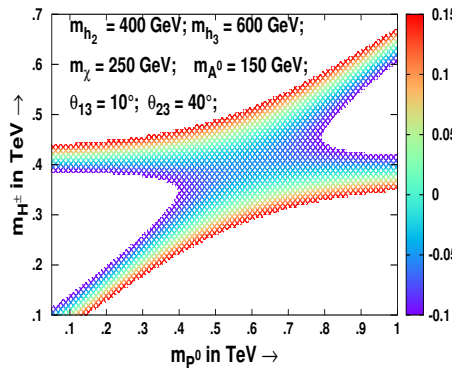
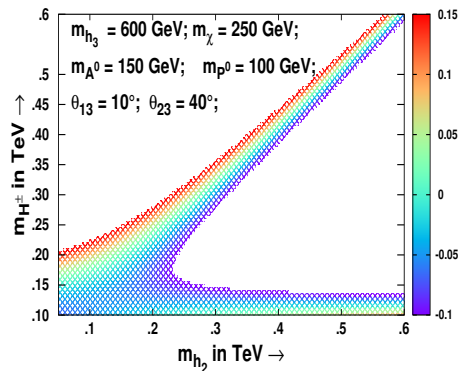
$$\Delta T = T_{\text{expt.}} - T_{\text{SM}} = 0.03 \pm 0.12$$



## Constraints from electroweak precision observables

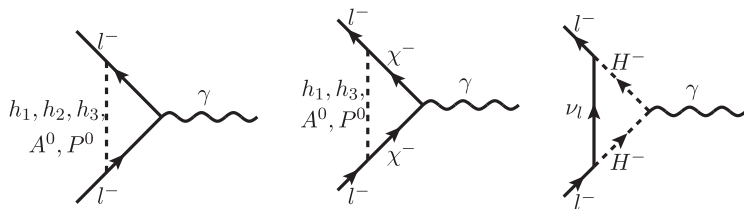


## Constraints from electroweak precision observables



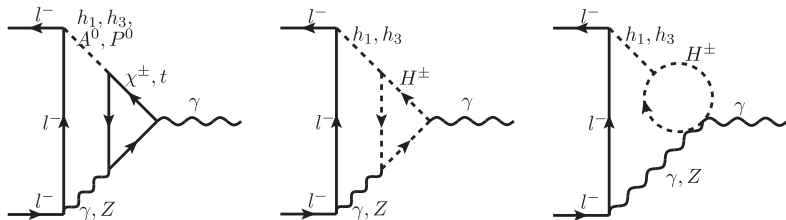
[JHEP 11 (2021) 056]

## Anomalous Magnetic dipole moment of leptons (1-loop)



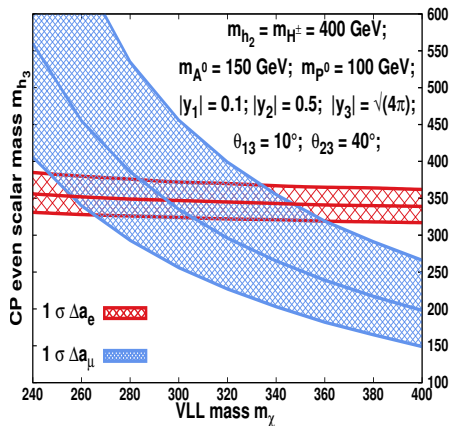
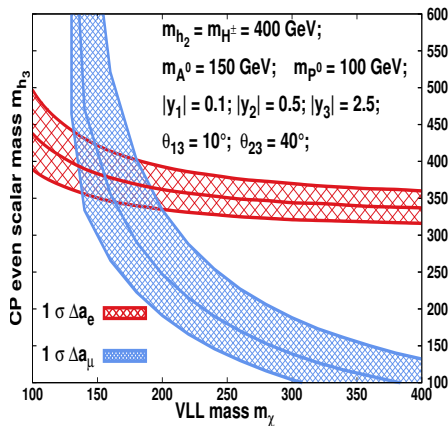
$$\Delta a_l^{1\text{-loop}} = \frac{1}{16\pi^2} \left[ 2 \frac{m_l^4}{v_{SM}^2} \left( \frac{\cos^2 \theta_{13}}{m_{h_1}^2} + \frac{\sin^2 \theta_{13}}{m_{h_3}^2} - \frac{1}{m_{h^{SM}}^2} \right) I_1 + m_l^2 \left( \frac{\cos^2 \theta_{23}}{m_{A^0}^2} + \frac{\sin^2 \theta_{23}}{m_{P^0}^2} \right) y_1^2 I_2 \right. \\ \left. + \frac{m_l^2}{m_{h_2}^2} y_1^2 I_1 + \sum_{s_i=h_1, h_3, A^0, P^0} |y_l \chi_{s_i}|^2 \frac{m_l^2}{m_{s_i}^2} I_3 + |y_1|^2 \frac{m_l^2}{m_{H^\pm}^2} I_4 \right]$$

## Anomalous Magnetic dipole moment of leptons (2-loop)

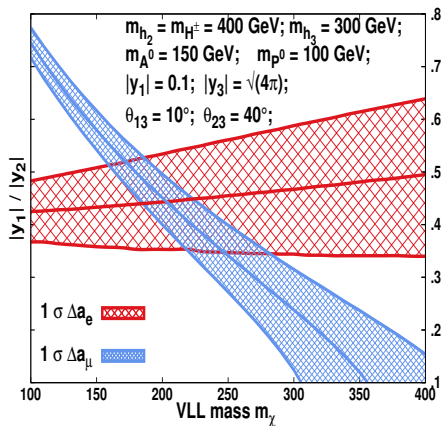
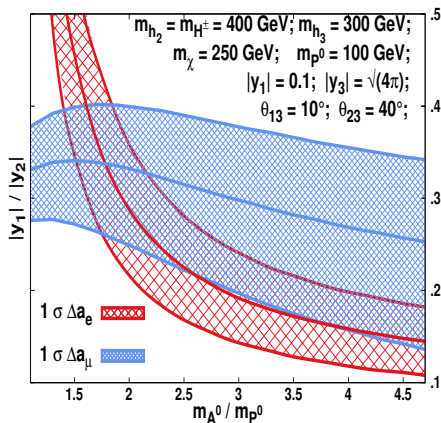


$$\begin{aligned}
 \Delta a_l^{2-loop} = & \frac{\alpha_{em}}{4 \pi^3} \left[ \frac{m_l}{v_{SM}} \frac{y_2}{\sqrt{2}} \sin \theta_{13} \cos \theta_{13} \left\{ f \left( \frac{m_\chi^2}{m_{h_3}^2} \right) - f \left( \frac{m_\chi^2}{m_{h_1}^2} \right) \right\} \right. \\
 & + \frac{m_l}{v_{SM}} \frac{m_t}{v_{SM}} \left\{ \sin^2 \theta_{13} f \left( \frac{m_t^2}{m_{h_3}^2} \right) - \cos^2 \theta_{13} f \left( \frac{m_t^2}{m_{h_1}^2} \right) + f \left( \frac{m_t^2}{m_{h_{SM}}^2} \right) \right\} \\
 & + \frac{y_1 y_2}{2} \frac{m_l}{m_\chi} \sin \theta_{23} \cos \theta_{23} \left\{ g \left( \frac{m_\chi^2}{m_{A^0}^2} \right) - g \left( \frac{m_\chi^2}{m_{P^0}^2} \right) \right\} \\
 & \left. - \frac{m_l^2}{4} \frac{m_l}{v_{SM}^2} \left\{ \frac{\cos \theta_{13}}{m_{h_1}^2} g_{h_1 H^+ H^-} \tilde{f} \left( \frac{m_{H^\pm}^2}{m_{h_1}^2} \right) - \frac{\sin \theta_{13}}{m_{h_3}^2} g_{h_3 H^+ H^-} \tilde{f} \left( \frac{m_{H^\pm}^2}{m_{h_3}^2} \right) \right\} \right]
 \end{aligned}$$

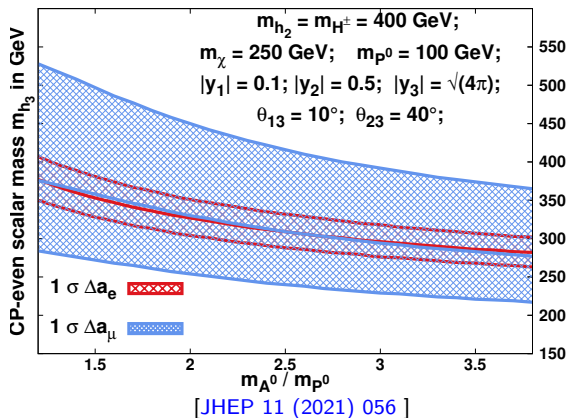


Parameter scan for  $\Delta a_\mu$  and  $\Delta a_e$ 

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Parameter scan for  $\Delta a_\mu$  and  $\Delta a_e$ 

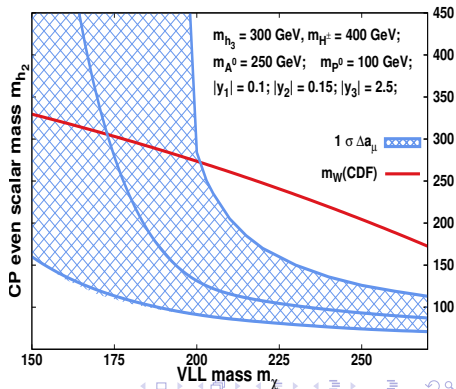
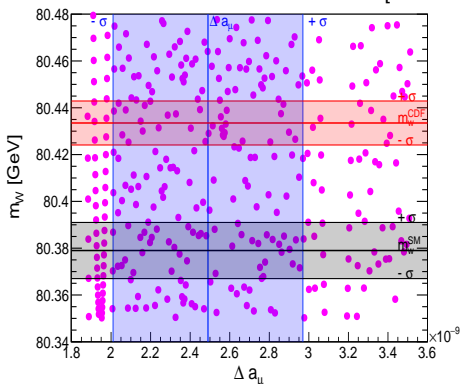
## W-mass Anomaly

$$m_W^{\text{SM}} = 80.377 \pm 0.006 \text{ GeV [PTEP 2022, 083C01 (2022)]}$$

$$m_W^{\text{CDF}} = 80.4335 \pm 0.0094 \text{ GeV [Science 376, 170 (2022)]}$$

$$\frac{\Delta m_W^2}{m_W^2} = \frac{2\Delta m_W}{m_W} = \frac{\alpha_{em}}{c_W^2 - s_W^2} \left( -\frac{\Delta S}{2} + c_W^2 \Delta T + \frac{c_W^2 - s_W^2}{4 s_W^2} \Delta U \right)$$

[Chin. Phys. C 47 (2023) no.4, 043102 ]



*Thank You*

## Backup slides

## Yukawa Couplings

$y_{ffh_1}$	$(\sqrt{2}m_f/v_{SM}) \cos \theta_{13}$	$y_{llh_2}$	$y_1$
$y_{ffh_3}$	$-(\sqrt{2}m_f/v_{SM}) \sin \theta_{13}$	$y_{llP^0}$	$-i y_1 \sin \theta_{23}$
$y_{\chi\chi h_1}$	$y_2 \sin \theta_{13}$	$y_{llA^0}$	$i y_1 \cos \theta_{23}$
$y_{\chi\chi h_3}$	$y_2 \cos \theta_{13}$	$y_{l\chi h_1}$	$y_3 \sin \theta_{13}$
$y_{\chi\chi P^0}$	$i y_2 \cos \theta_{23}$	$y_{l\chi h_3}$	$y_3 \cos \theta_{13}$
$y_{\chi\chi A^0}$	$i y_2 \sin \theta_{23}$	$y_{l\chi P^0}$	$i y_3 \cos \theta_{23}$
$y_{l\nu H^-}$	$y_1$	$y_{l\chi A^0}$	$i y_3 \sin \theta_{23}$

## Backup slides

S, T, U

$$S \equiv \frac{1}{g^2} \left( 16\pi \cos^2 \theta_W \right) \left[ F_{ZZ}(m_Z^2) - F_{\gamma\gamma}(m_Z^2) + \left( \frac{2 \sin^2 \theta_W - 1}{\sin \theta_W \cos \theta_W} \right) F_{Z\gamma}(m_Z^2) \right]$$

$$T \equiv \frac{1}{\alpha_{em}} \left[ \frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2} \right]$$

$$U \equiv \frac{1}{g^2} (16\pi) \left[ F_{WW}(m_W^2) - F_{\gamma\gamma}(m_W^2) - \frac{\cos \theta_W}{\sin \theta_W} F_{Z\gamma}(m_W^2) \right] - S$$

# Constraints from electroweak precision observables

$$\begin{array}{c}
 h_{1,3} \\
 \text{Z} \text{---} \text{---} \text{---} \text{Z} \\
 \text{Z}
 \end{array}
 = -\frac{g^2 M_Z^2}{16\pi^2 c_W^2} [\cos^2 \theta_{13} B_0(q^2; m_Z^2, m_{h_1}^2) + \sin^2 \theta_{13} B_0(q^2; m_Z^2, m_{h_3}^2)]$$

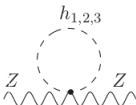
$$\begin{array}{c}
 h_{1,3} \\
 \text{Z} \text{---} \text{---} \text{---} \text{Z} \\
 G^0
 \end{array}
 = \frac{g^2}{16\pi^2 c_W^2} [\cos^2 \theta_{13} B_{22}(q^2; m_Z^2, m_{h_1}^2) + \sin^2 \theta_{13} B_{22}(q^2; m_Z^2, m_{h_3}^2)]$$

$$\begin{array}{c}
 A^0, P^0 \\
 \text{Z} \text{---} \text{---} \text{---} \text{Z} \\
 h_2
 \end{array}
 = \frac{g^2}{16\pi^2 c_W^2} [\cos^2 \theta_{23} B_{22}(q^2; m_{h_2}^2, m_{A^0}^2) + \sin^2 \theta_{23} B_{22}(q^2; m_{h_2}^2, m_{P^0}^2)]$$

$$\begin{array}{c}
 H^\pm \\
 \text{Z} \text{---} \text{---} \text{---} \text{Z} \\
 H^\pm
 \end{array}
 = \frac{g^2}{16\pi^2 c_W^2} c_W^2 B_{22}(q^2; m_{H^\pm}^2, m_{H^\pm}^2)$$

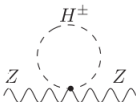


# Constraints from electroweak precision observables



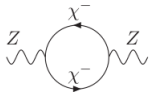
A Feynman diagram showing a Z boson (wavy line) entering from the left and exiting to the right. A loop of a scalar particle  $h_{1,2,3}$  (dashed line) is attached to the Z boson line. The loop is connected to the Z boson line at two vertices, each marked with a black dot.

$$= -\frac{1}{2} \frac{g^2}{16\pi^2} \sec^2 \theta_W [\cos^2 \theta_{13} A_0(m_{h_1}^2) + \sin^2 \theta_{13} A_0(m_{h_3}^2) + A_0(m_{h_2}^2)]$$



A Feynman diagram showing a Z boson (wavy line) entering from the left and exiting to the right. A loop of a charged scalar particle  $H^{\pm}$  (dashed line) is attached to the Z boson line. The loop is connected to the Z boson line at two vertices, each marked with a black dot.

$$= -\frac{1}{2} \frac{g^2}{16\pi^2} [(-2s_W^2 + c_W^2 + s_W^2 t_W^2) A_0(m_{H^{\pm}}^2)]$$



A Feynman diagram showing a Z boson (wavy line) entering from the left and exiting to the right. A loop of a fermion particle  $\chi^{-}$  (solid line) is attached to the Z boson line. The loop is connected to the Z boson line at two vertices, each marked with a black dot. Arrows on the fermion line indicate the direction of the loop.

$$= \frac{-e^2}{4\pi^2} \frac{s_W^2}{c_W^2} [m_{\chi}^2 B_0(q^2; m_{\chi}^2, m_{\chi}^2) - 2 B_{22}(q^2; m_{\chi}^2, m_{\chi}^2)]$$

# Constraints from electroweak precision observables

$$\begin{array}{c} \gamma \\ \text{---} \\ \gamma \end{array} \circlearrowleft \begin{array}{c} H^\pm \\ H^\pm \end{array} = 4 \frac{g^2}{16\pi^2} s_W^2 B_{22}(m_Z^2; m_{H^\pm}^2, m_{H^\pm}^2)$$

$$\begin{array}{c} Z \\ \text{---} \\ \gamma \end{array} \circlearrowleft \begin{array}{c} H^\pm \\ H^\pm \end{array} = \frac{2g^2}{16\pi^2 c_W} s_W c_{2W} B_{22}(m_Z^2; m_{H^\pm}^2, m_{H^\pm}^2)$$

$$\begin{array}{c} \gamma \\ \text{---} \\ \gamma \end{array} \circlearrowleft \begin{array}{c} \chi^- \\ \chi^- \end{array} = \frac{-e^2}{4\pi^2} [m_\chi^2 B_0(q^2; m_\chi^2, m_\chi^2) - 2 B_{22}(q^2; m_\chi^2, m_\chi^2)]$$

$$\begin{array}{c} Z \\ \text{---} \\ \gamma \end{array} \circlearrowleft \begin{array}{c} \chi^- \\ \chi^- \end{array} = \frac{e^2}{4\pi^2} \frac{s_W}{c_W} [m_\chi^2 B_0(q^2; m_\chi^2, m_\chi^2) - 2 B_{22}(q^2; m_\chi^2, m_\chi^2)]$$



