# Dynamic Radius Jet Clustering Algorithm

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Based on JHEP 04 (2023) 019 [2301.13074]
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# Outline

- QCD Jets and boosted jets at collider.
- Fixed radius jet algorithms.
- Our proposal: dynamic radius jet algorithms.
- Some Illustrations and Usefulness.
- Summary and Outlook.

# Motivation

- QCD jets are ubiquitous at hadron colliders.

- High energy machines can also produce fat jets from boosted heavy particles.

- Study of these different objects are of huge interest to search for SM or BSM scenario.

- However, the current fixed radius jet algorithms ($k_t$, anti-$k_t$, C/A) are inadequate to capture these features in a single go.

- Variable radius jet clustering algorithm would thus be an important asset in our toolbox.

# Jets at Collider: QCD jets

- A QCD particle (q/g), produced at a collider, showers and then hadronize.
- These hadrons tend to be collimated, i.e. they appear within a small solid angle.

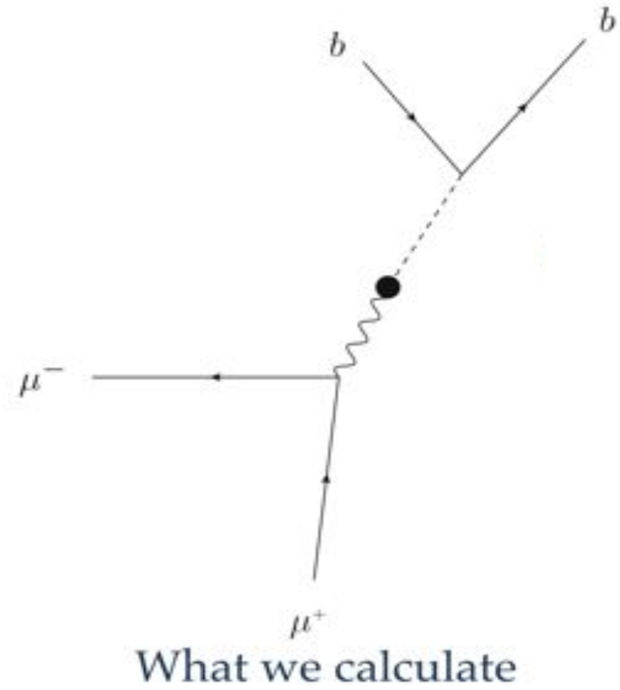

Fig.: **quantumdiaries.org**

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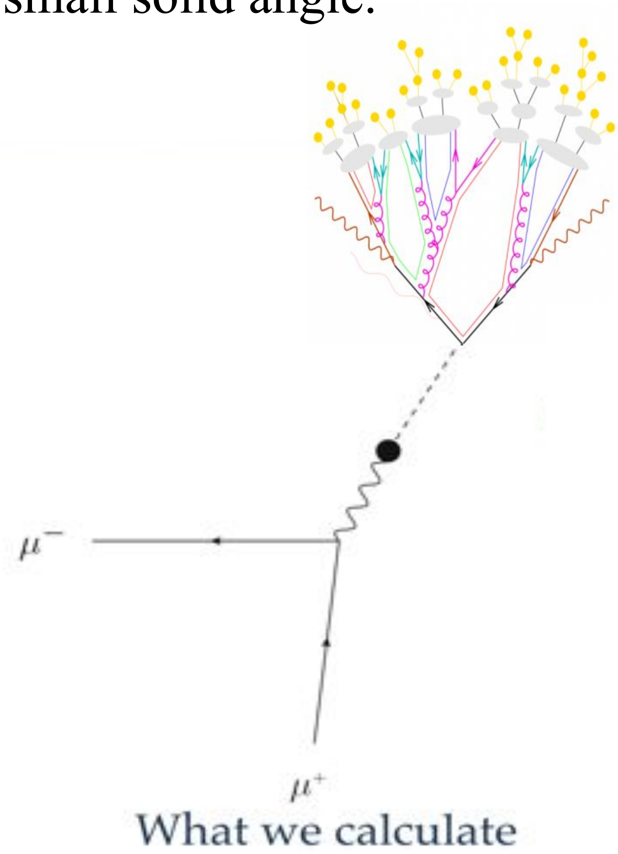

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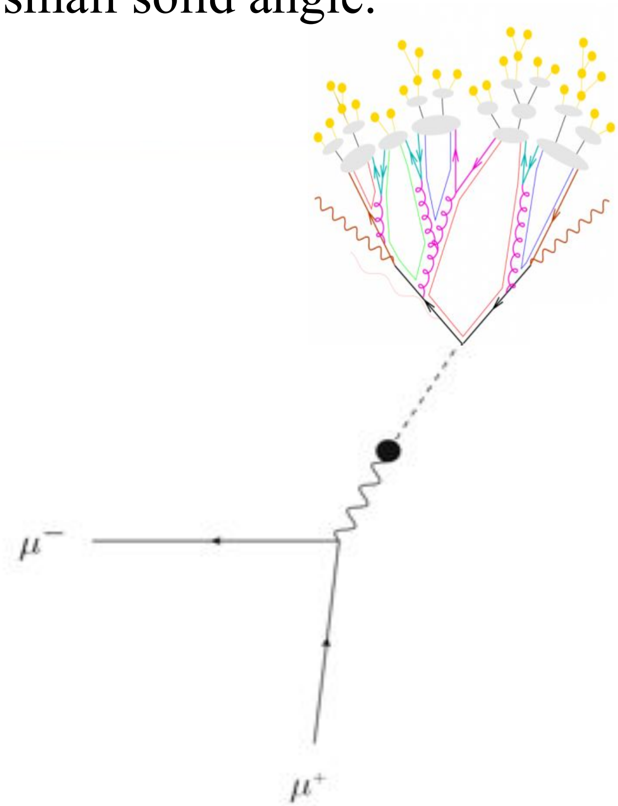
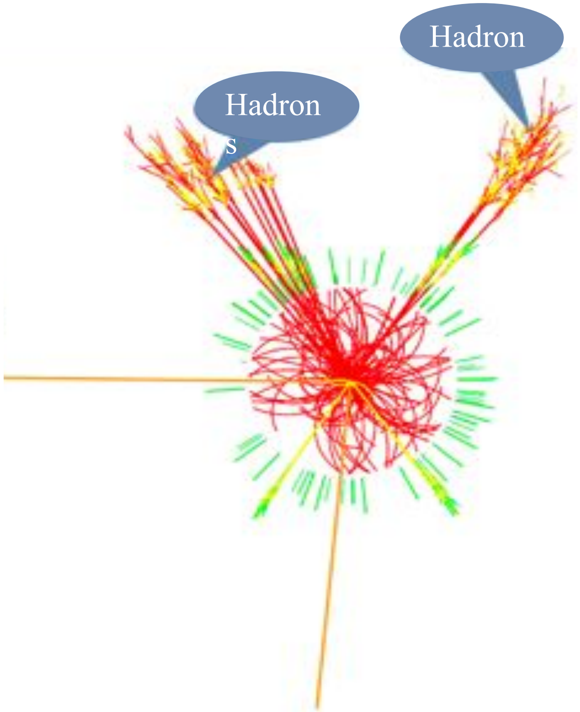


What we calculate



What we measure

Fig.: **quantumdiaries.org**

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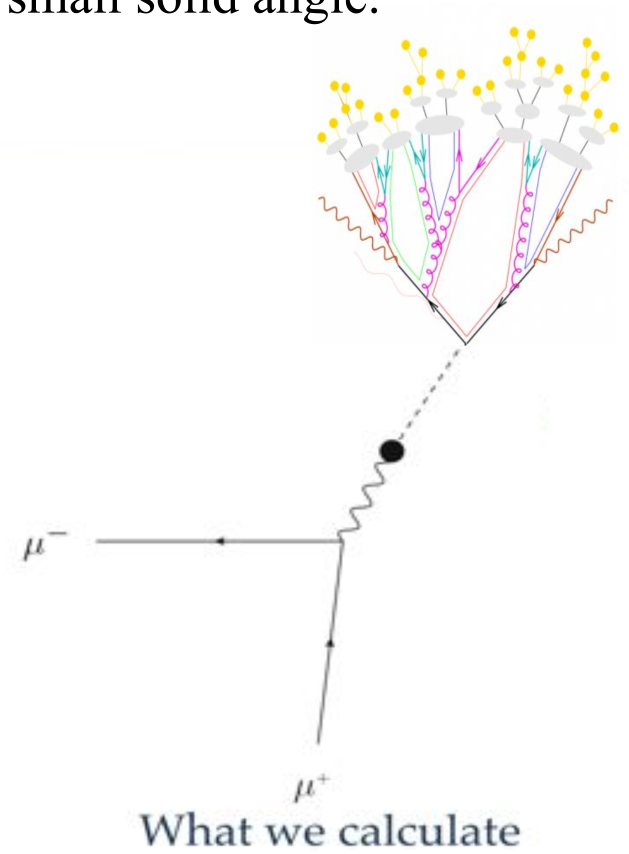


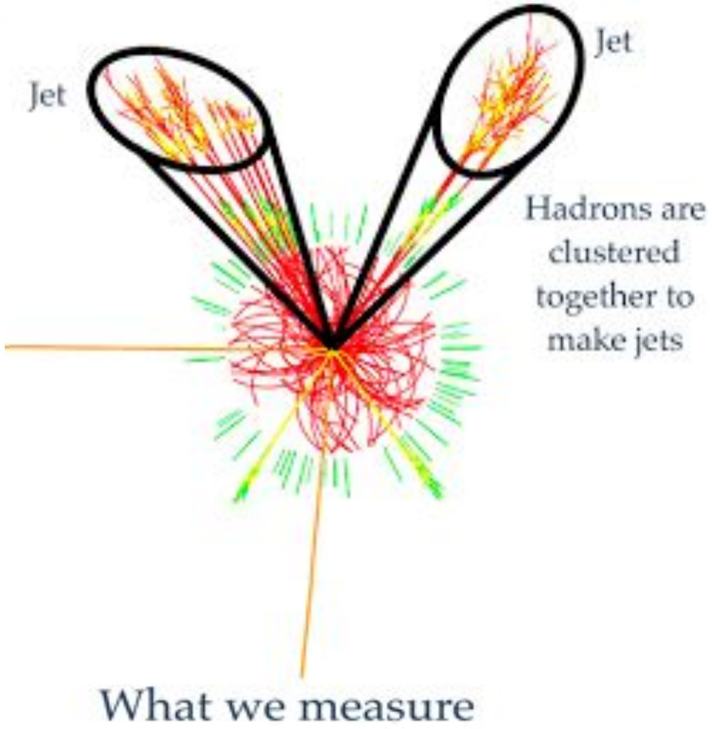


Fig.: **quantumdiaries.org**

# Jets at Collider: Fat Jets

- If a heavy particle (W, Z, top, etc.), are boosted enough, their decay products also come within a small solid angle.

- These jets, called fat jets, are wider than traditional QCD jets.

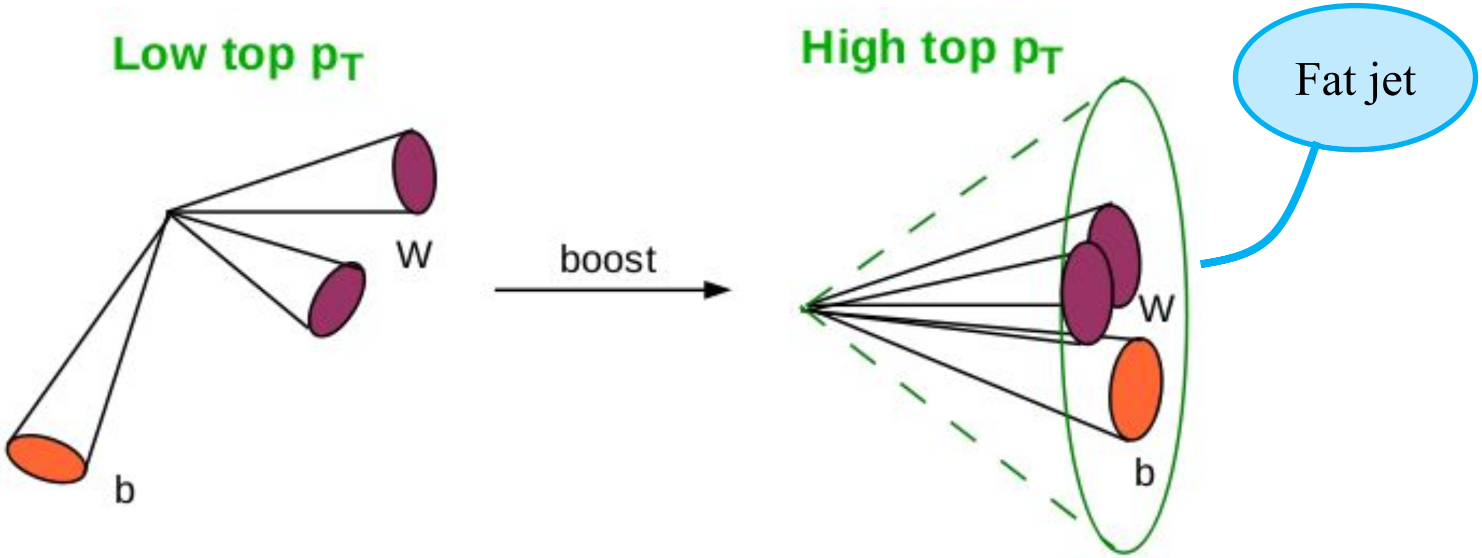
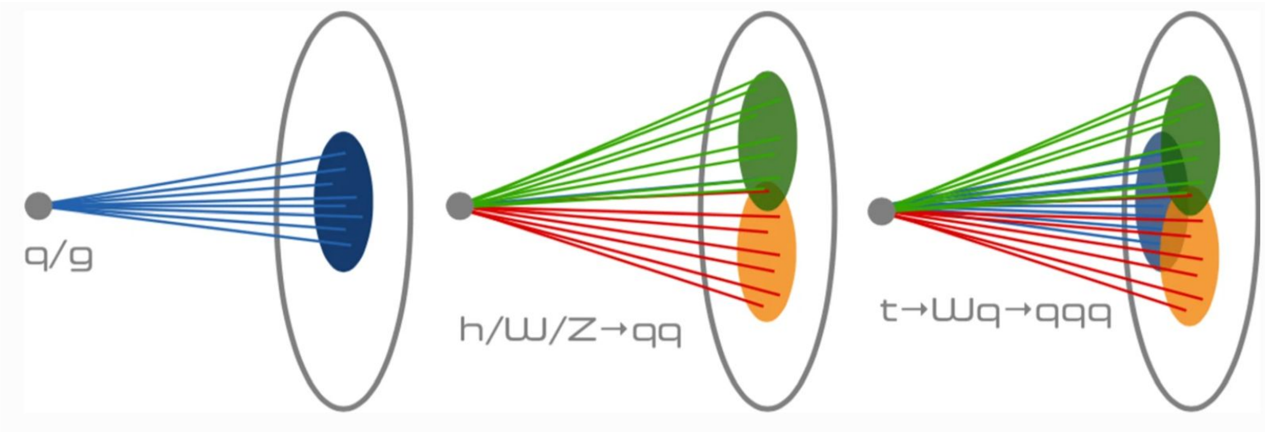


Fig.: **quantumdiaries.org**

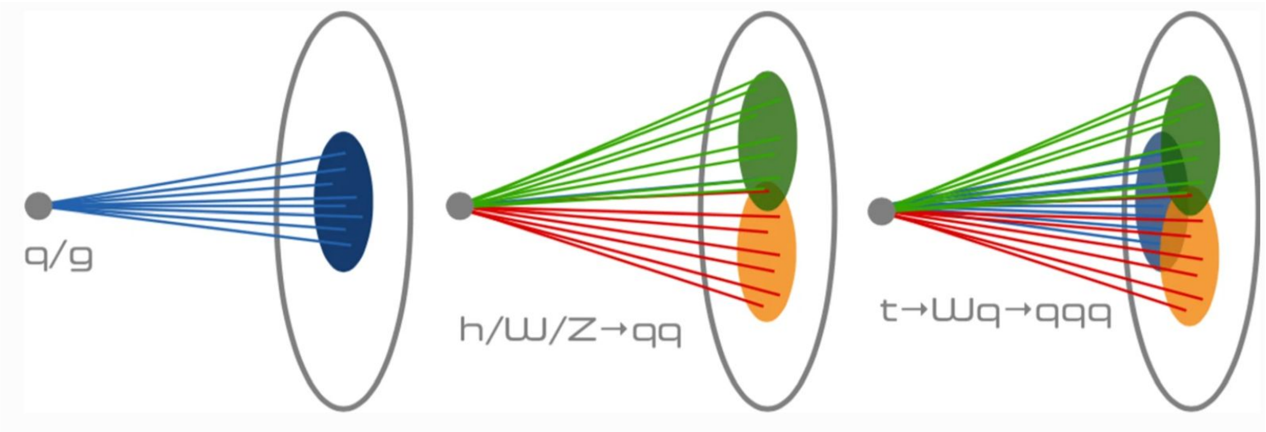
# Jet Clustering Algorithm



Be it a narrow QCD jet or a boosted fat jet, one needs an algorithm to find it by clustering the collimated sprays of hadrons.

Fig: *Eur. Phys. J. C* **80**, 58 (2020)

# Jet Clustering Algorithm



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**Jet Clustering Algorithm**

# Sequential Recombination Algorithm

Take all the four-momenta in a list, calculate all possible $d_{ij}$ and $d_{iB}$

$$d_{ij} = \min\left(p_{T_i}^{2p}, p_{T_j}^{2p}\right)\Delta\mathrm{R}_{ij}^2, \qquad \Delta R_{ij} = \left(y_i - y_j\right)^2 + \left(\phi_i - \phi_j\right)^2$$

$$d_{iB} = p_{T_i}^{2p} R_0^2$$

| **p** | **0** | **1** | **-1** |
|---|---|---|---|
| Name | CA | KT | AK |

1. Find the minimum of the $d_{ij}$ and $d_{iB}$.
2. Minimum is a $d_{ij}$: combine $i,\ j$ and add to the list, remove $i$ and $j$, return to step 1
3. Minimum is a $d_{iB}$: declare $i$ to be a jet (final), remove it from the list, return to step 1.
4. Stop when list gets empty.

Review: *Eur. Phys. J. C* 67 (2010) 637.

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Fixed radius $R_0$

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# Dynamic Radius Jet Algorithm

$$d_{ij} = \min\left(p_{T_i}^{2p}, p_{T_j}^{2p}\right) \Delta\mathrm{R}_{ij}^2, \qquad \Delta R_{ij} = \left(y_i - y_j\right)^2 + \left(\phi_i - \phi_j\right)^2$$

$$d_{iB} = p_{T_i}^{2p} \boxed{(R_0 + \sigma_i)^2} \rightarrow R_d^2$$

Radius modifier

$$\sigma_i^2 = \underbrace{\frac{\sum_{a<b} p_{T_a} p_{T_b} \Delta R_{ab}^2}{\sum_{a<b} p_{T_a} p_{T_b}}}_{\langle \Delta R^2 \rangle} - \underbrace{\left(\frac{\sum_{a<b} p_{T_a} p_{T_b} \Delta R_{ab}}{\sum_{a<b} p_{T_a} p_{T_b}}\right)^2}_{\langle \Delta R \rangle^2} ;\ a, b \in i$$

| **p** | **0** | **1** | **-1** |
|---|---|---|---|
| Algorithm | DR-CA | DR-KT | DR-AK |

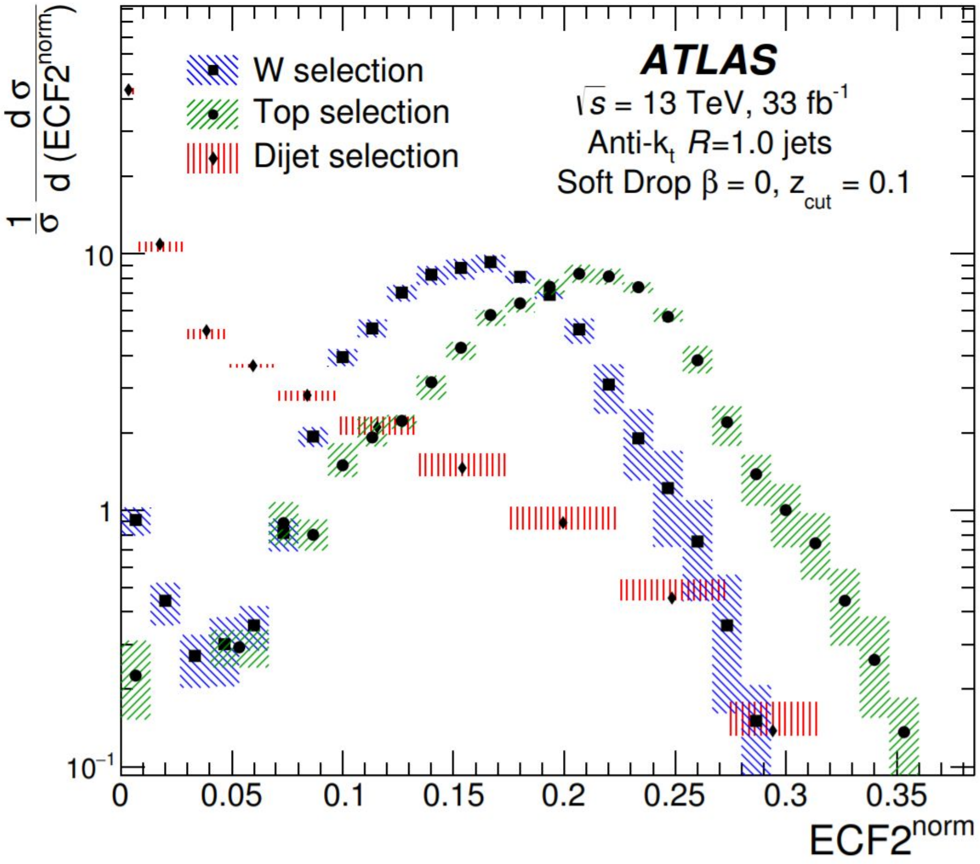
# Why $\sigma_i$?

- $\sigma_i^2 = \langle \Delta R^2 \rangle - \langle \Delta R \rangle^2$ (standard deviation) measures fuzziness of a jet.

- Fat jets are expected to be fuzzier than QCD jets

- Energy Correlation Function:

$$\mathrm{ECF2}_\beta = \sum_{a<b} p_{T_a} p_{T_b} \Delta R_{ab}^{\beta}$$



ECF: *JHEP* 06 (2013) 108.
Fig: *JHEP* 08 (2019) 033.

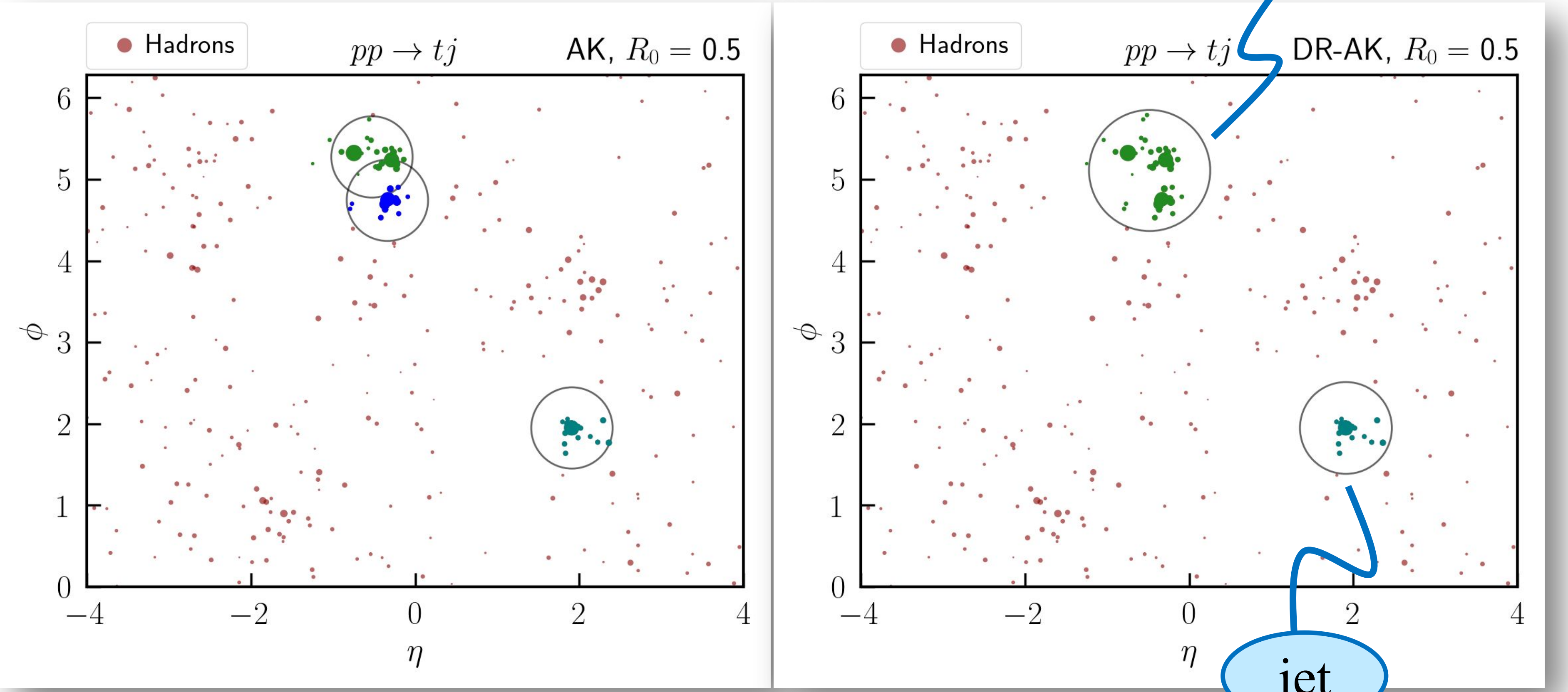
# Illustration (SM): $pp \to t\,j$

$$pp \to t\,j$$

13 TeV
LHC

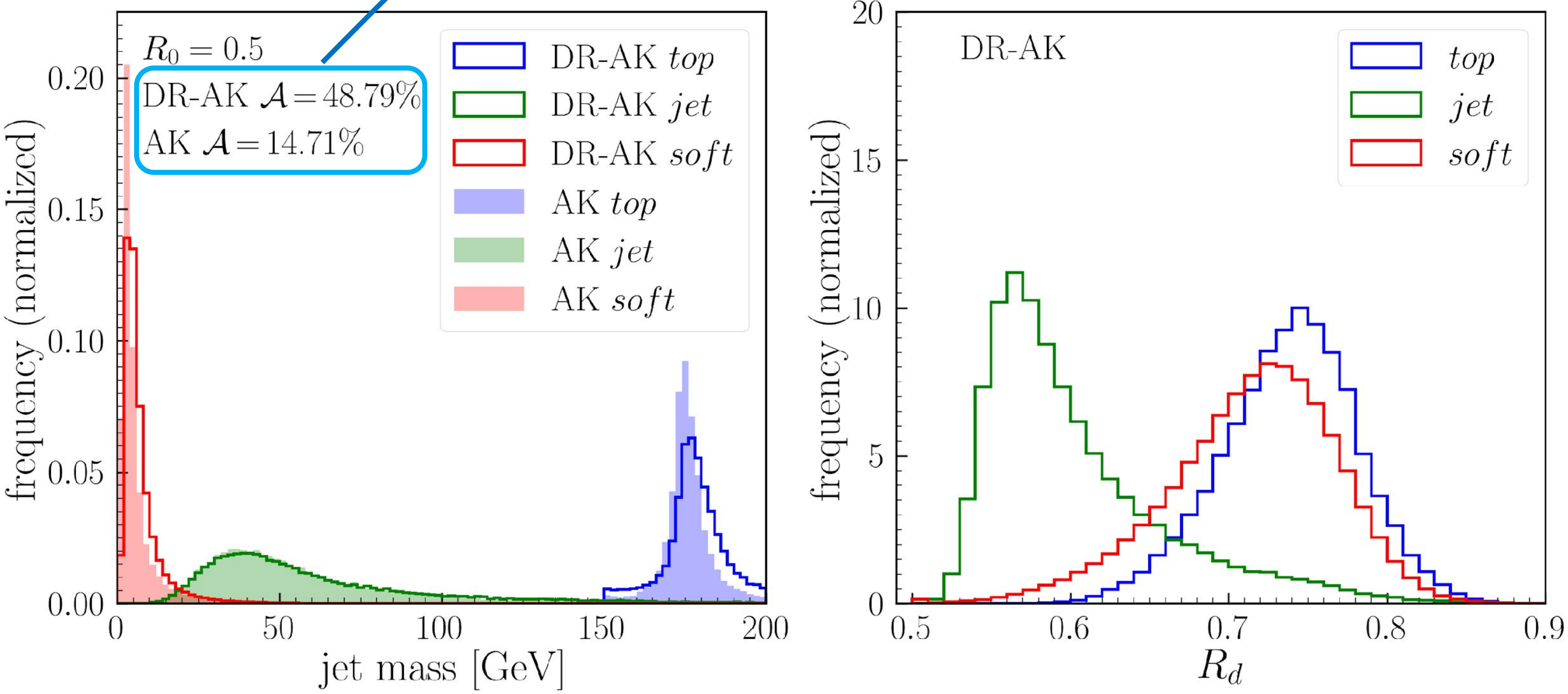
Fat jet

QCD jet



# Illustration (SM): $pp \to t\,j$

$$\text{Acceptance efficiency } \mathcal{A} = \frac{\text{No. top recontructed events}}{\text{No. of total events}}$$



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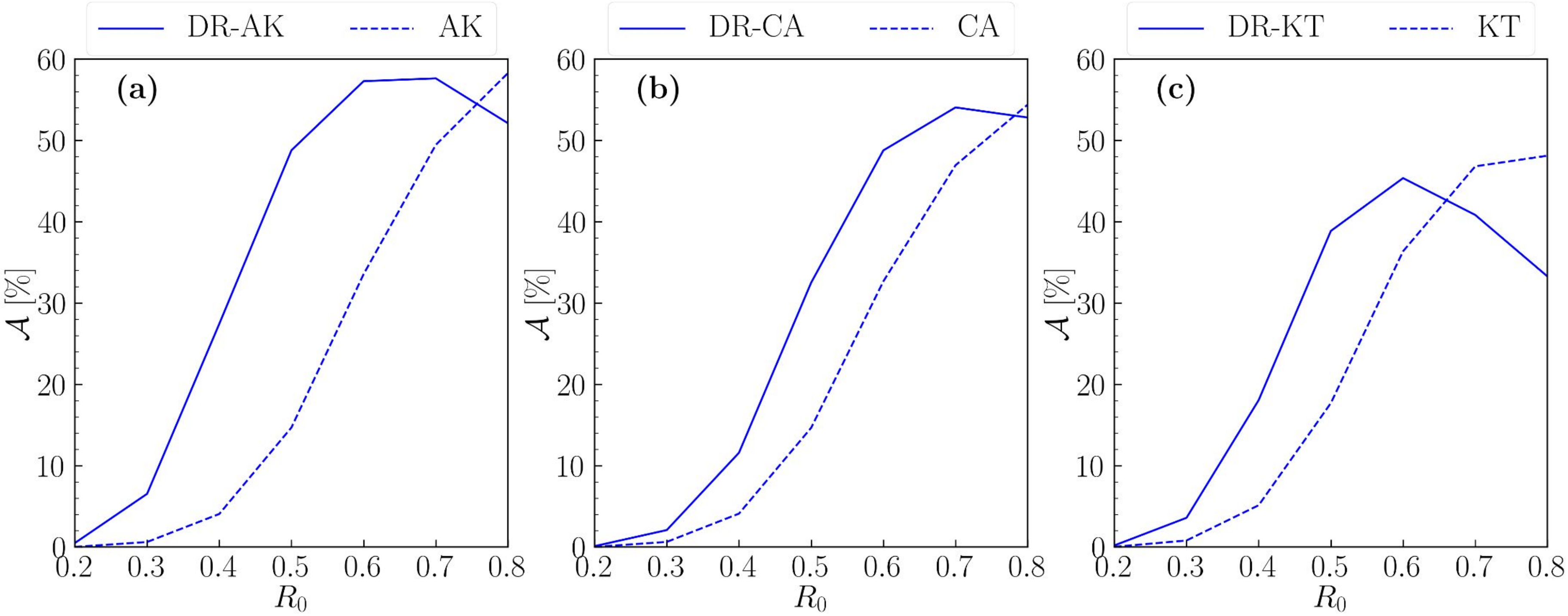
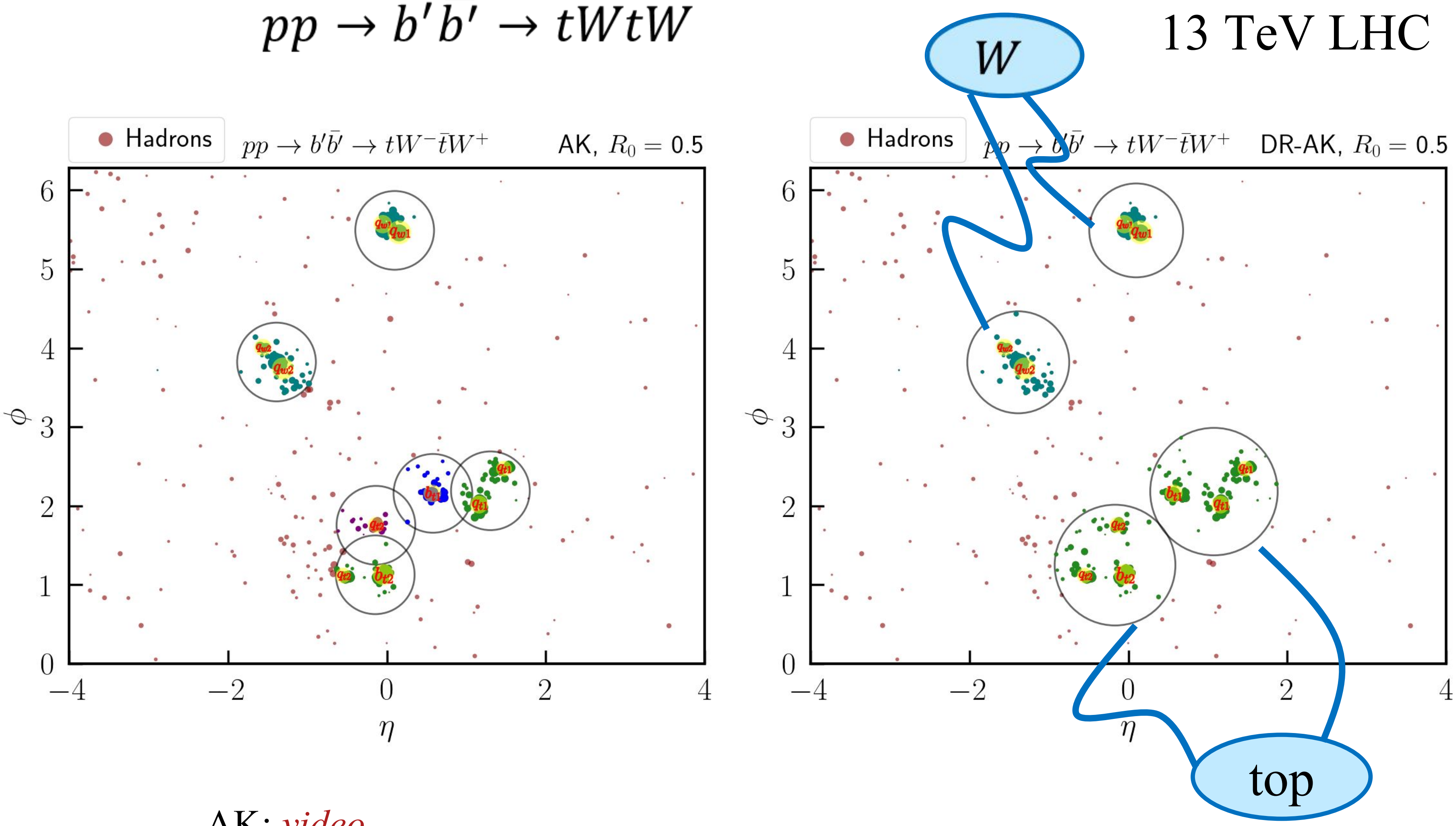


Acceptance efficiency ($\mathcal{A}$) vs. initial radius ($R_0$)

# Illustration (BSM): $pp \to b'\overline{b'} \to tWtW$

$$pp \to b'\overline{b'} \to tWtW$$

13 TeV LHC

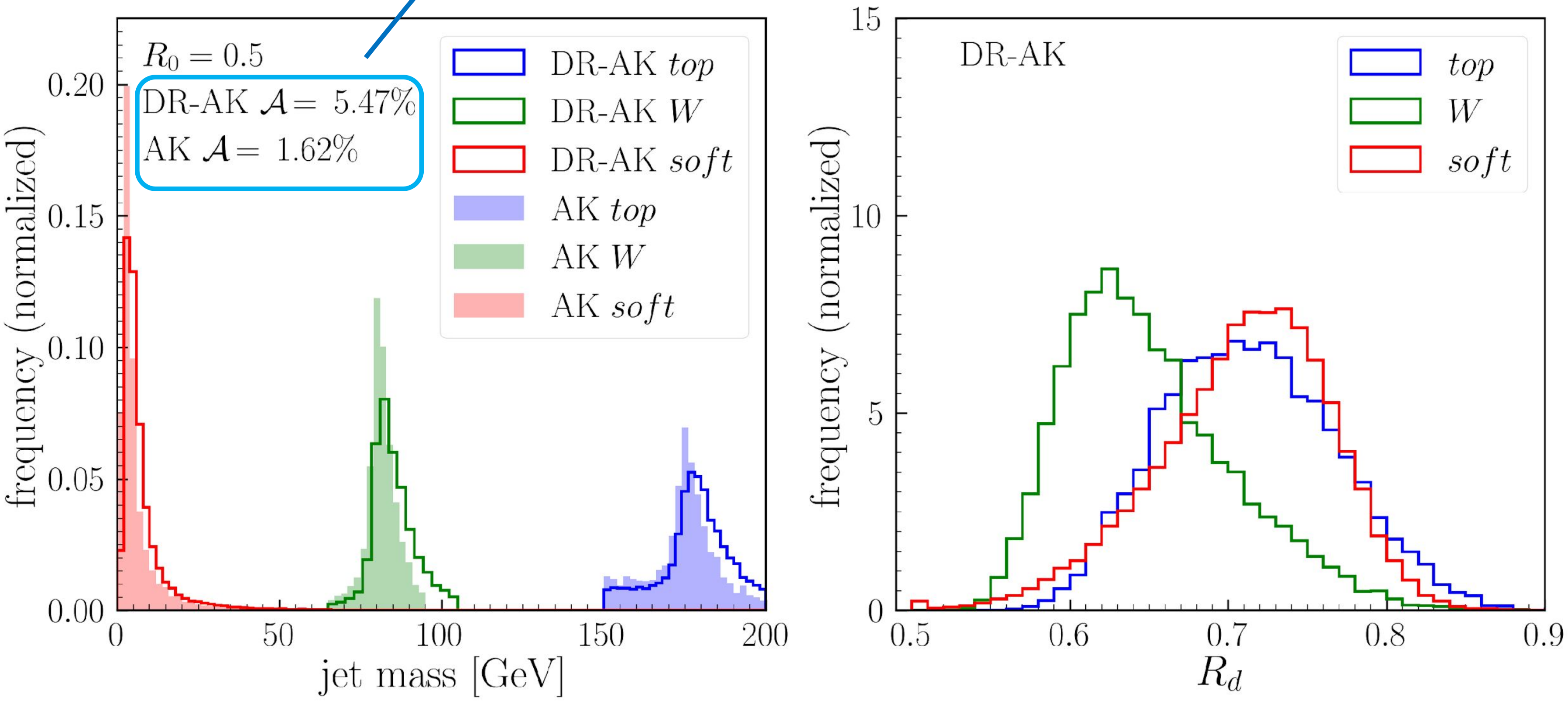


AK: *video*.

DR-AK: *video*.

# Illustration (BSM): $pp \to b'\overline{b'} \to tWtW$

$$\text{Acceptance efficiency } \mathcal{A} = \frac{\text{No. top and } W \text{ recontructed events}}{\text{No. of total events}}$$



# Summary and Outlook

- There is a need for variable radius jet algorithm. Current fixed-radius algorithms are inadequate for this purpose.
- We have proposed a jet algorithm with dynamic radius.
- The usefulness of the DR jet algorithm has presented in two process at 13 TeV LHC.
- Other studies related to this algorithm are ongoing.

**THANK YOU**

# BACKUP SLIDES

# IRC SAFETY

- **IR safety:** output of an algorithm should be unchanged with the introduction of a four-momentum $p^\mu \to 0$.
  - DR algorithm is IR safe.
- **C safety:** output of an algorithm should be unchanged with the collinear splitting of any four-momentum.
  - At its current version, DR algorithm is not exactly C safe. It is only approximately C safe.

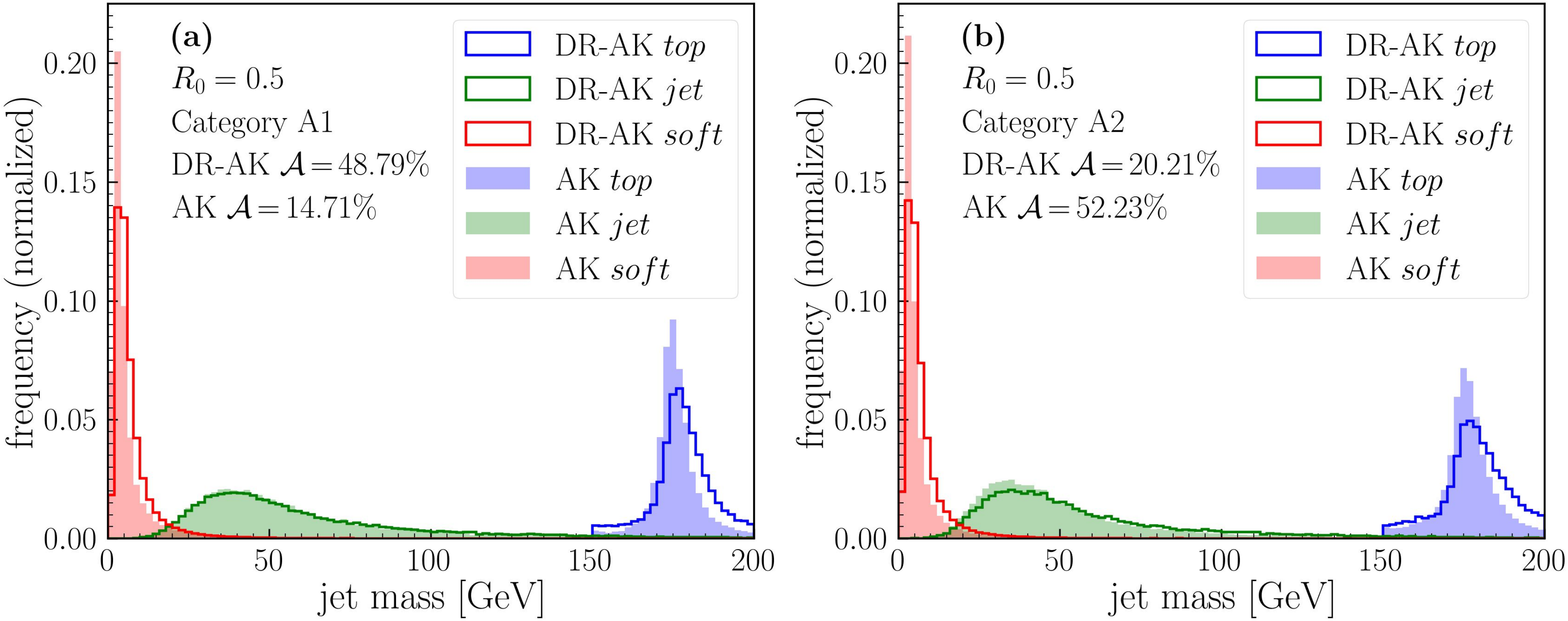
# COMPLEXITY

- $k_t$, anti-$k_t$, and C/A algorithms in FastJet has $N\log N$ complexity.
- In the current implementation, the complexity of the DR algorithm is $N^2$ at most.
- The worst case scenario occurs when the whole event is clustered as a jet. Actual complexity is hence lesser.

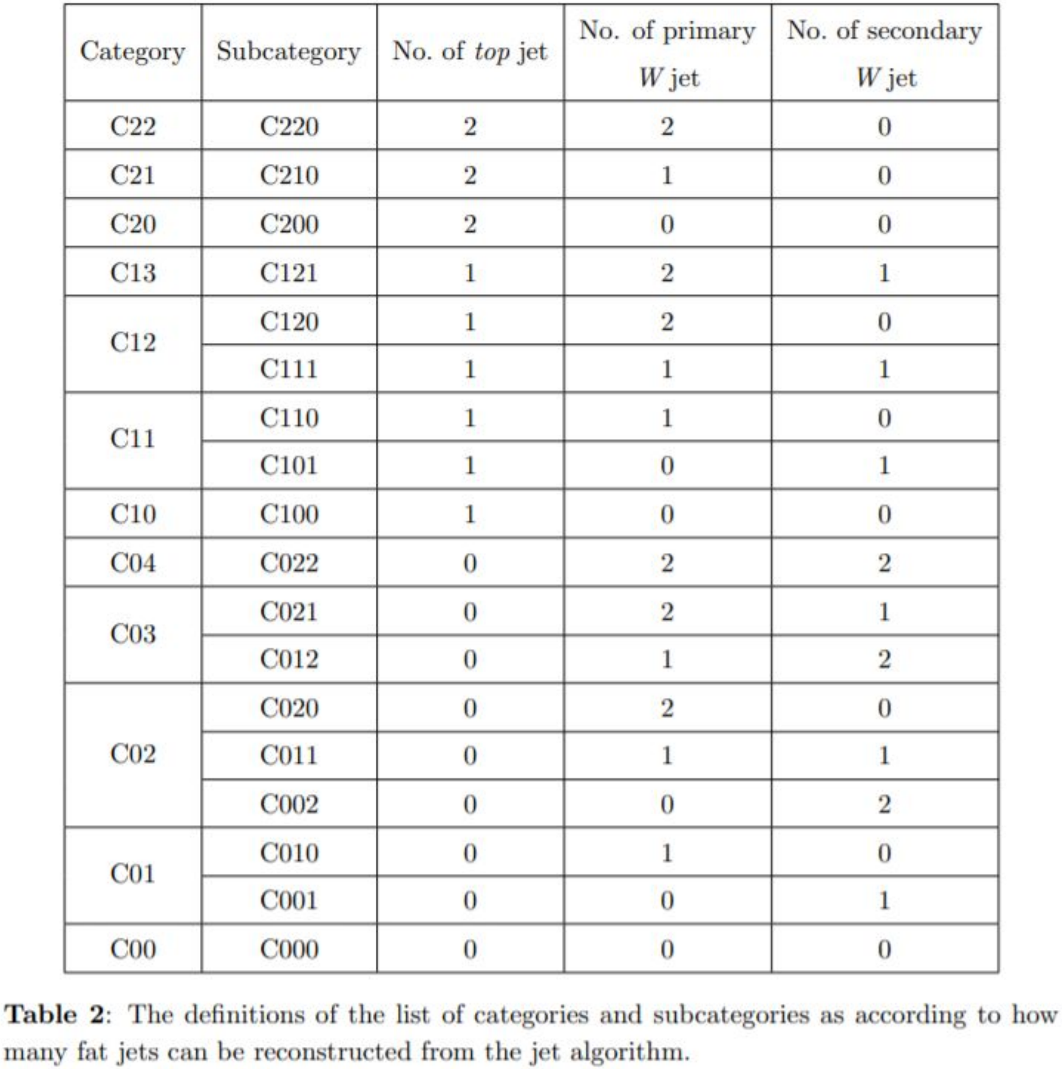
# PILEUP SENSITIVITY

- Pileup sensitivity study in ongoing.
- It seems that the pileup subtraction via PUPPI algorithm may help in addressing pileup sensitivity

# Jet Mass ($pp \to tj$)



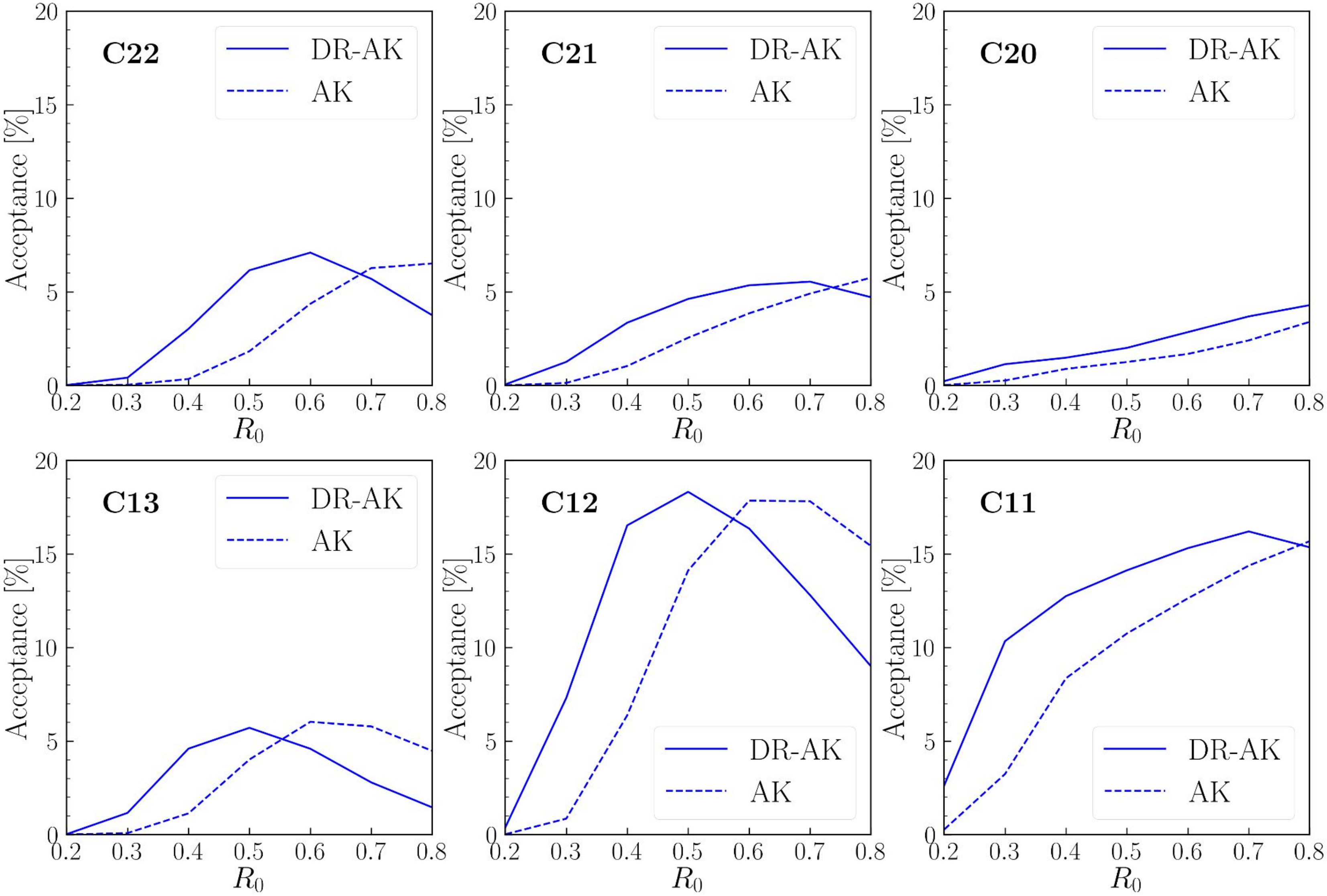
# Categories ($pp \to tWtW$)



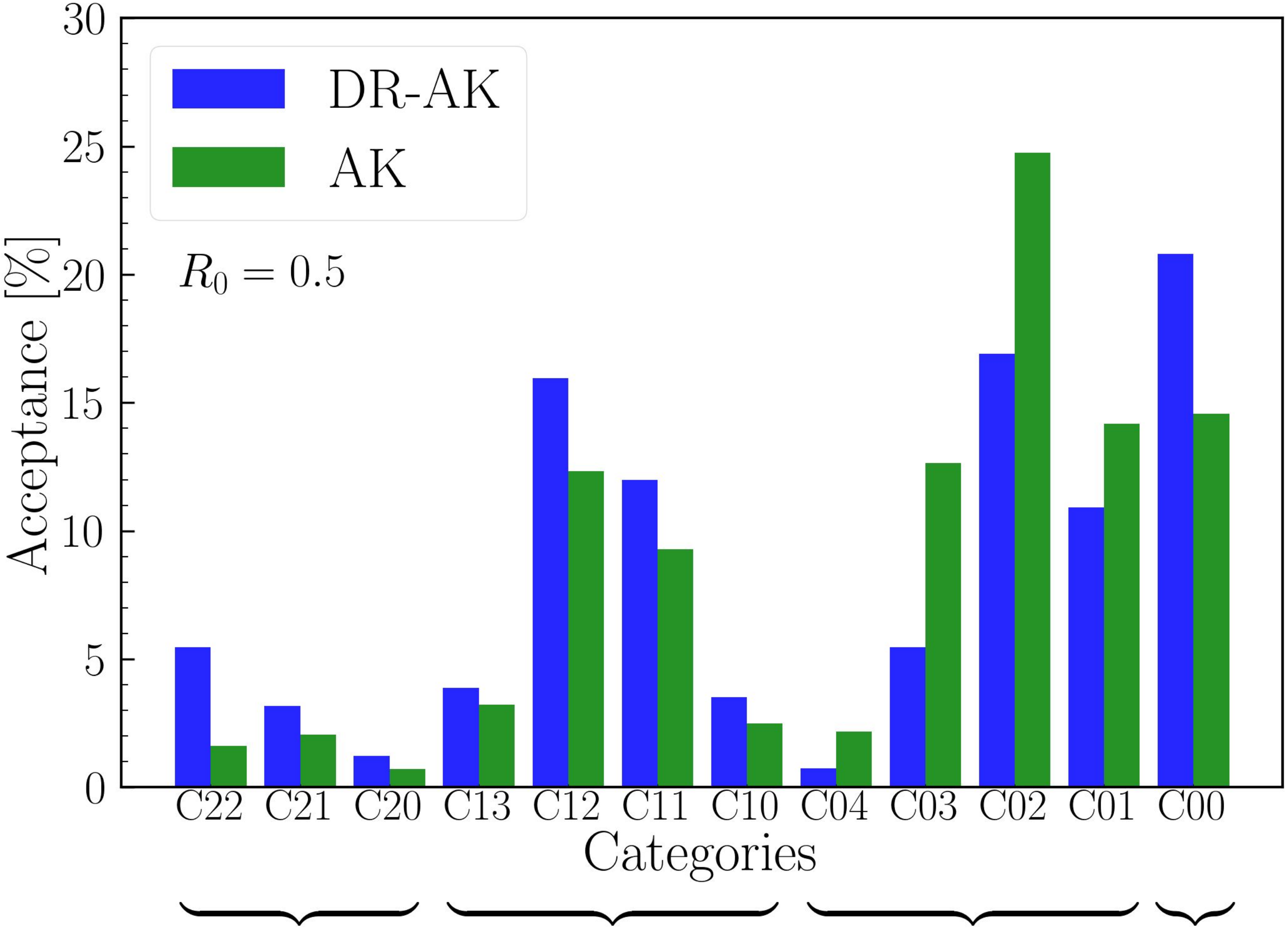
| Category | Subcategory | No. of *top* jet | No. of primary $W$ jet | No. of secondary $W$ jet |
|---|---|---|---|---|
| C22 | C220 | 2 | 2 | 0 |
| C21 | C210 | 2 | 1 | 0 |
| C20 | C200 | 2 | 0 | 0 |
| C13 | C121 | 1 | 2 | 1 |
| C12 | C120 | 1 | 2 | 0 |
| | C111 | 1 | 1 | 1 |
| C11 | C110 | 1 | 1 | 0 |
| | C101 | 1 | 0 | 1 |
| C10 | C100 | 1 | 0 | 0 |
| C04 | C022 | 0 | 2 | 2 |
| C03 | C021 | 0 | 2 | 1 |
| | C012 | 0 | 1 | 2 |
| C02 | C020 | 0 | 2 | 0 |
| | C011 | 0 | 1 | 1 |
| | C002 | 0 | 0 | 2 |
| C01 | C010 | 0 | 1 | 0 |
| | C001 | 0 | 0 | 1 |
| C00 | C000 | 0 | 0 | 0 |

**Table 2**: The definitions of the list of categories and subcategories as according to how many fat jets can be reconstructed from the jet algorithm.

# Efficiency ($pp \to tWtW$)



# Bar Plot ($pp \to tWtW$)



| | | | | |
|---|---|---|---|---|
| DR-AK: | 9.86% | 35.33% | 34.02% | 20.79% |
| AK: | 4.38% | 27.31% | 53.75% | 14.56% |