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Astrophysical Q-balls and their gravitational microlensing signature

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Outline

- Theoretical framework for Q-balls
- Maximum radius using Jeans Stability criteria
- Constraints on parameters from non-gravitational condition
- Gravitational lensing of astrophysical Q-balls
- f_{DM} constraint on astrophysical Q-balls

Q-balls

- Q-balls are non-topological solitons which arise in theories with conserved global charge and there potential satisfies certain conditions. Coleman (1985)

- We consider U(1) theory whose Lagrangian is given by,

$$\mathcal{L} = \partial_\mu \Phi \partial^\mu \Phi^\dagger - U(\Phi^\dagger \Phi) \ .$$

- Q-balls exist when $U(\Phi^\dagger\Phi)/\Phi^\dagger\Phi$ has minimum at $\Phi_{\rm Q}$,

$$0 \leqslant \left| \frac{U(\Phi_{\rm Q})}{\Phi_{\rm Q}^2} \right| \equiv \omega_{\rm Q}^2 < m^2 \ .$$

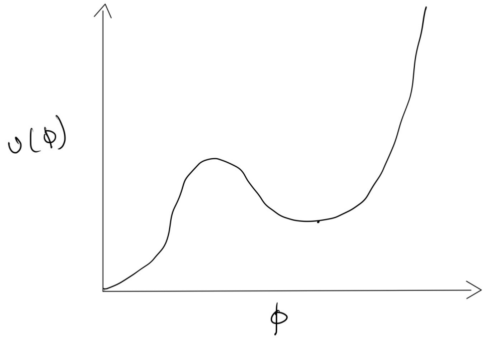

Fig. 1: Potential for Q-balls.

$$\Phi(r,t) = \phi(r) e^{i\omega t} \ .$$

- Energy density of profile ,

$$\boxed{\rho_{\rm Q}^{\rm M}(r) = 2\omega^2 \phi(r)^2 + \frac{2}{3}\phi(r)'^2} \ .$$

Q-balls

- We consider the sextic potential,

$$U(\phi) = m^2\phi^2 + \lambda\phi^4 + \zeta\phi^6 \, . \qquad \lambda < 0 \, .$$ Heeck (2021)

- Local minima at ,

$$\phi = \phi_{\text{Q}} \equiv \sqrt{\frac{|\lambda|}{2\zeta}} \, .$$

- Q-balls existence criteria,

$$\omega_{\text{Q}} = m\sqrt{1 - \frac{\lambda^2}{4m^2\zeta}} < m \, .$$

- Equation of motion,

$$\phi'' + \frac{2}{r}\phi' = -\frac{\partial}{\partial\phi}\left[\frac{1}{2}\omega^2\phi^2 - U(\phi)\right] \, .$$

- Q-balls exist for,

$$\boxed{\omega_{\text{Q}} < \omega < m} \, .$$

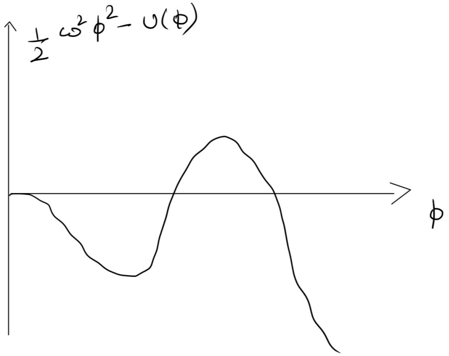

Fig. 2: Effective Potential for Q-balls.

Q-balls Profile

- Coleman solution (Thin wall Q-Balls),

$$\phi^{\rm TW}(r) = \begin{cases} \phi_{\rm Q} & r \leqslant R_{\rm Q}\ , \\ 0 & r > R_{\rm Q}\ . \end{cases}$$

Coleman (1985)

- Beyond Thin wall profile,

$$\phi(r) = \frac{\phi_*}{\sqrt{1 + 2\exp\left[2\sqrt{m^2 - \omega_{\rm Q}^2}\,(r - R_{\rm Q})\right]}}\ .$$

Heeck (2021)

where,

$$\phi_*^2 = \frac{\phi_{\rm Q}^2}{3}\left[2 + \sqrt{1 + 3\left(\frac{\omega^2 - \omega_{\rm Q}^2}{m^2 - \omega_{\rm Q}^2}\right)}\ \right]\ .$$

- Radius ,

$$R_{\rm Q} = \frac{\sqrt{m^2 - \omega_{\rm Q}^2}}{\omega^2 - \omega_{\rm Q}^2}\ .$$

Jeans instabilitiy criteria for Q-balls

- The astrophysical Q-balls should be stable against gravitational collapse.

- If Q-ball is compressed, the internal outward pressure will counteract the compression.

- The jeans length is given by,

$$d_{\rm Q}^{\rm J} = v_{\rm Q}^{\rm s}\, t_{\rm Q}^{\rm g} \simeq \frac{v_{\rm Q}^{\rm s}}{\sqrt{G_{\rm N}\, \rho_{\rm Q}^{\rm M}}}\ .$$

  where, the velocity of sound is

$${v_{\rm Q}^{\rm s}}^2 \equiv \frac{dP_{\rm Q}}{d\rho_{\rm Q}^{\rm M}} = \frac{\lambda^2}{4m^2\zeta}\ .$$

- The jeans length is,

$$\boxed{d_{\rm Q}^{\rm J} = \sqrt{\frac{|\lambda|}{4m^4 G_{\rm N}\left(1 - \frac{\lambda^2}{4m^2\zeta}\right)}} \equiv R_{\rm Q,\,max}^{\rm J}}\ .$$

- For stability of astrophysical Q-balls,

$$\boxed{R_{\rm Q}\ <\ d_{\rm Q}^{\rm J}}\ .$$

Non- gravitational criteria for Q-balls

- The effects due to gravity are negligible compared to the attractive self-interactions.
- Weak field limit metric is given by

$$\eta_{\mu\nu} \to g_{\mu\nu}(x) = (1 + 2\varphi^{\mathrm{G}}, -1, -1, -1) \ .$$

- Consider the field as ,

$$\Phi(r,t) = (2E/N)^{-1/2}\psi(r)e^{-iEt} \ .$$

- The equation of motion becomes ,

$$E_B\,\psi = -\frac{1}{2m}\nabla^2\psi + m\varphi^{\mathrm{G}}\psi + \frac{N\lambda}{2m^2}\psi^3 + \frac{3\zeta N^2}{8m^3}\psi^5 \ .$$

- Comparing the attractive self-interactions and gravity,

$$|m\varphi^{\mathrm{G}}| < \frac{N\lambda}{2m^2}\psi^2 \simeq \frac{M_{\mathrm{Q}}\lambda}{2\omega m^2}\psi^2 \ .$$

- Criteria of non-gravitational astrophysical Q-balls,

$$\boxed{\frac{G_{\mathrm{N}} m^3 \omega\lambda}{2\zeta\left(\omega^2 - m^2 + \frac{\lambda^2}{4\zeta}\right)^2} < 1} \ .$$

Gravitational lensing

- Light passing near a heavy objects bends due to gravity, the phenomenon is called gravitational lensing.

- For point mass, deflection angle is given by

$$\hat{\theta}_{\mathrm{D}} = \frac{2}{c^2}\int \nabla_{\chi}\varphi^{\mathrm{G}}\, dz = \frac{4G_{\mathrm{N}}M}{c^2|\vec{\chi}|}\hat{\chi}\ .$$

- For extended mass source ,

$$\hat{\theta}_{\mathrm{D}}(\chi) = \frac{4G_{\mathrm{N}}\widetilde{M}(\chi)}{c^2\chi}\ .$$

where ,

$$\widetilde{M}(\chi) = \int_0^{\chi} d^2\chi'\sigma(\chi')\ , \quad \sigma(\chi) = \int_{-\infty}^{\infty}\rho(\chi, z)dz\ .$$

- Lens equation,

$$\boxed{\theta_{\mathrm{S}} = \theta_{\mathrm{I}} - \frac{d_{\mathrm{LS}}}{d_{\mathrm{S}}d_{\mathrm{L}}}\frac{4G_{\mathrm{N}}\widetilde{M}(\theta_{\mathrm{I}})}{c^2\theta_{\mathrm{I}}}}\ .$$

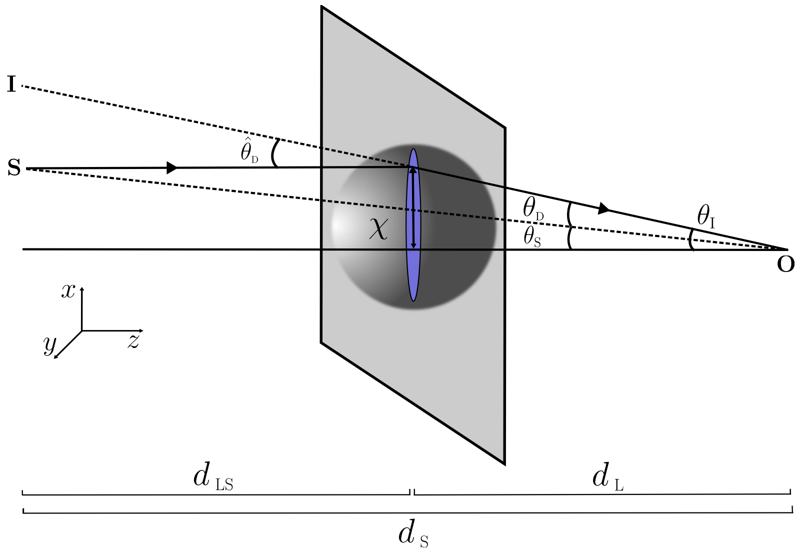

Fig.1 A diagrammatic representation of a lens, observer and source lensing system showing the various relevant quantities involved.

Gravitational lensing

- For point lens,

$$\theta_{\rm S} = 0, \qquad \theta_{\rm I} = \theta_{\rm E} \equiv \frac{R_{\rm E}}{d_{\rm L}} , \qquad \theta_{\rm I\pm} = \frac{1}{2}\left(\theta_{\rm S} \pm \sqrt{\theta_{\rm S}^2 + 4\theta_{\rm E}^2}\right) .$$

where , $R_{\rm E}$ is the Einstein radius

$$R_{\rm E} = \left(\frac{4G_{\rm N}M}{c^2}\frac{d_{\rm L}d_{\rm LS}}{d_{\rm S}}\right)^{\frac{1}{2}} .$$

- Magnification of the image,

$$\mathfrak{m}_{\rm I} = \frac{\theta_{\rm I}}{\theta_{\rm S}}\frac{d\theta_{\rm I}}{d\theta_{\rm S}} .$$

- Total magnification,

$$\mathfrak{m} = \sum_{\rm I} |\mathfrak{m}_{\rm I}| .$$

- For point lens,

$$\mathfrak{m}_{\rm point} = \frac{\theta_{\rm S}^2/\theta_{\rm E}^2 + 2}{\theta_{\rm S}/\theta_{\rm E}\,(\theta_{\rm S}^2/\theta_{\rm E}^2 + 4)^{1/2}} .$$

For $\theta_{\rm S} = \theta_{\rm E}$,

$$\mathfrak{m}_{\rm point} \approx 1.34 .$$

Threshold impact parameter for Astrophysical Q-Balls

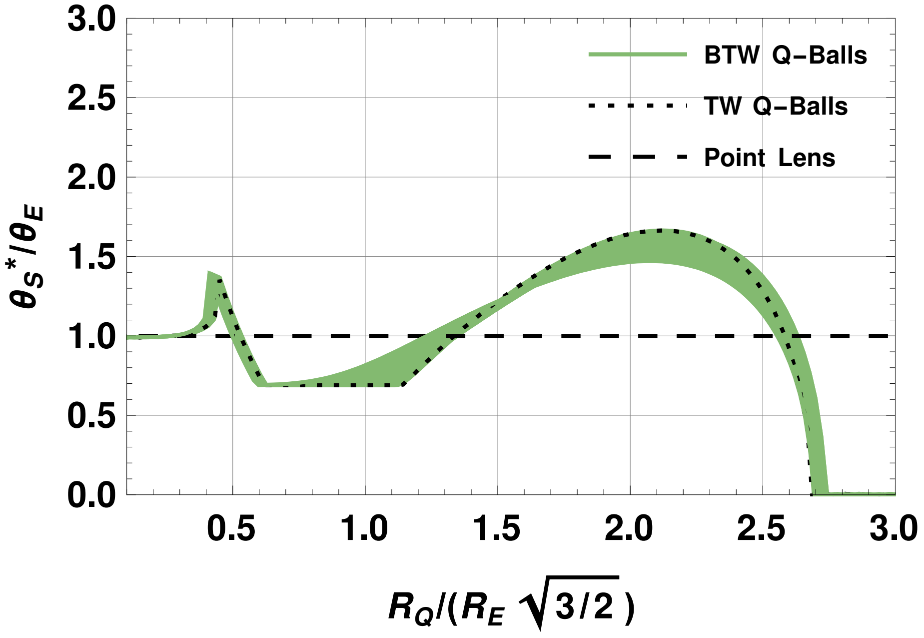

Fig. 1: The threshold value of the source position, below which one obtains a total magnification 1.34 as a function of Q- ball radius.

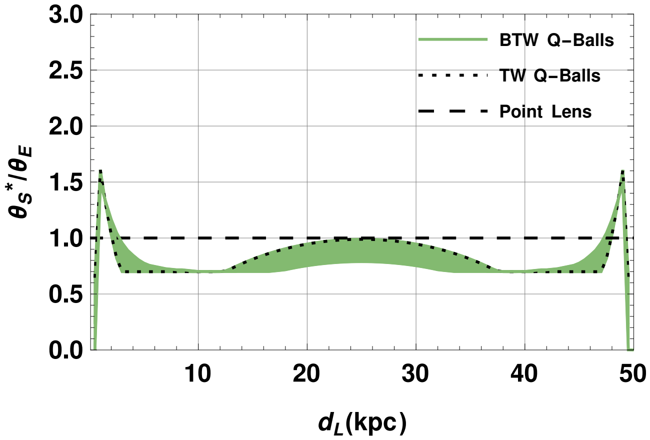

Fig. 2a: Large Magllenic Cloud (for EROS)

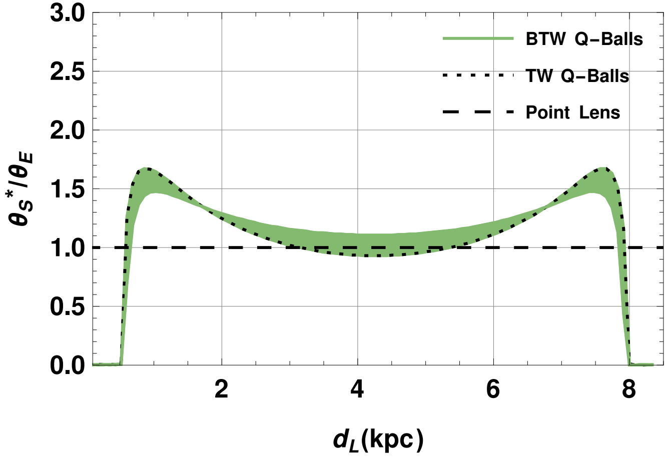

Fig. 2b: Milky Way Galaxy (for OGLE and WFIRST)

f_{DM} constraint for astrophysical Q-balls

- For a single background source, for differential event rate for unit exposure time is given by

$$\frac{d^2\Gamma}{d\gamma d\tau} = \frac{2d_S e(\tau)}{v_\odot^2 M_Q} f_{DM}\rho_{DM}(\gamma) v_Q^4(\gamma) e^{-v_Q^2(\gamma)/v_\odot^2} .$$

Griest (1991)

where,

$$v_Q \equiv \frac{2\theta_S^*(\gamma) d_L}{\tau}, \quad \gamma \equiv d_L/d_S .$$

- The total no. of expected astrophysical Q-balls events are,

$$N_{exp} = N_s T_o \int dR_* \int_0^1 d\gamma \int_{\tau_{min}}^{\tau_{max}} d\tau \frac{dn}{dR_*} \frac{d^2\Gamma}{d\gamma d\tau} .$$

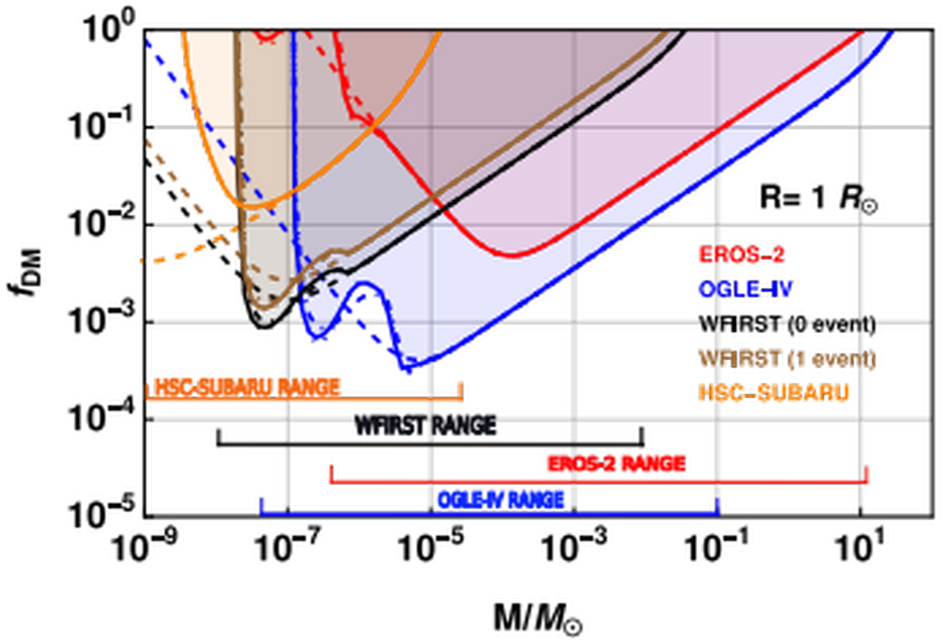

Fig. 1: Bounds on the fraction of dark matter comprised of astrophysical Q-balls are shown from various microlensing surveys. The point lens and TW astrophysical Q-balls are shown by the dashed and dotted lines, respectively. While the solid and dashed-dotted lines represent the two BTW Q-balls .

Summary

- Astrophysical Q-balls
  - Based on gravitational stability, we have found condition for maximum radius of Q-balls, which in directly constrains the coupling parameters and mass.
  - Furthermore, we have found condition on coupling and mass for non-gravitational Q-balls.
  - From various gravitational microlensing survey we have put bounds on the fraction of dark matter composed of Q-balls

Dr. Arun M. Thalapillil

Arhum Ansari

Thank You

Q-balls (Reveiw)

- Q-balls are non-topological solitons which arise in theories with conserved global charge and there potential satisfies certain conditions. Coleman (1985)

- We consider U(1) theory whose Lagrangian is given by,

$$\mathcal{L} = \partial_\mu \Phi \partial^\mu \Phi^\dagger - U(\Phi^\dagger \Phi) \ .$$

- Q-balls exist when $U(\Phi^\dagger\Phi)/\Phi^\dagger\Phi$ has minimum at $\Phi_{\rm Q}$,

$$0 \leqslant \left| \frac{U(\Phi_{\rm Q})}{\Phi_{\rm Q}^2} \right| \equiv \omega_{\rm Q}^2 < m^2 \ .$$

- The Q-balls have the time dependence,

$$\Phi(r,t) = \phi(r) e^{i\omega t} \ .$$

- Energy density of profile ,

$$\rho_{\rm Q}^{\rm M}(r) = 2\omega^2 \phi(r)^2 + \frac{2}{3}\phi(r)'^2 \ .$$

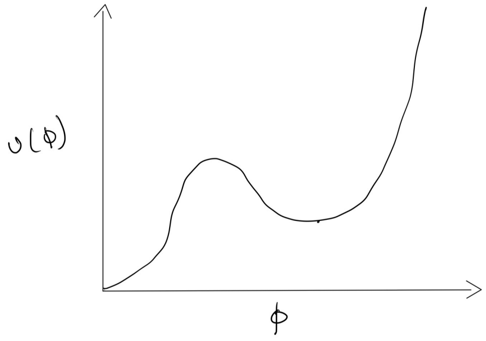

Fig. 1: Potential for Q-balls.

Q-balls (Reveiw cont.)

- We consider the sextic potential,

$$U(\phi) = m^2\phi^2 + \lambda\phi^4 + \zeta\phi^6 \,. \qquad \lambda < 0 \,.$$

Heeck (2021)

- Local minima at ,

$$\phi = \phi_{\rm Q} \equiv \sqrt{\frac{|\lambda|}{2\zeta}} \,.$$

- Q-balls existence criteria,

$$\omega_{\rm Q} = m\sqrt{1 - \frac{\lambda^2}{4m^2\zeta}} < m \,.$$

- Equation of motion,

$$\phi'' + \frac{2}{r}\phi' = -\frac{1}{2}\frac{\partial}{\partial\phi}\left[\omega^2\phi^2 - U(\phi)\right] \,.$$

Q-balls (Review cont.)

- Coleman solution (Thin wall Q-Balls),

Coleman (1985)

$$\phi^{\mathrm{TW}}(r) = \begin{cases} \phi_{\mathrm{Q}} & r \leqslant R_{\mathrm{Q}} \, , \\ 0 & r > R_{\mathrm{Q}} \, , \end{cases} \, .$$

- Beyond Thin wall profile,

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$$\phi(r) = \frac{\phi_*}{\sqrt{1 + 2\exp\left[2\sqrt{m^2 - \omega_{\mathrm{Q}}^2}\,(r - R_{\mathrm{Q}})\right]}} \, .$$

where,

$$\phi_*^2 = \frac{\phi_{\mathrm{Q}}^2}{3}\left[2 + \sqrt{1 + 3\left(\frac{\omega^2 - \omega_{\mathrm{Q}}^2}{m^2 - \omega_{\mathrm{Q}}^2}\right)}\,\right] \, .$$

- Radius ,

$$R_{\mathrm{Q}} = \frac{\sqrt{m^2 - \omega_{\mathrm{Q}}^2}}{\omega^2 - \omega_{\mathrm{Q}}^2} \, .$$

Jeans instabilitiy Criteria

- The astrophysical Q-balls should be stable against gravitational collapse.
- If Q-ball is compressed, the internal outward pressure will counteract the compression.
- The jeans length is given by,

$$d_{\rm Q}^{\rm J} = v_{\rm Q}^{\rm s}\, t_{\rm Q}^{\rm g} \simeq \frac{v_{\rm Q}^{\rm s}}{\sqrt{G_{\rm N}\, \rho_{\rm Q}^{\rm M}}} \;.$$

where, the velocity of sound is

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$$R_{\rm Q} \;<\; d_{\rm Q}^{\rm J} \;.$$

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- The effects due to gravity are negligible compared to the attractive self-interactions.
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- Consider the field as ,

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- The equation of motion becomes ,

$$E_B\,\psi = -\frac{1}{2m}\nabla^2\psi + m\varphi^{\rm G}\psi + \frac{N\lambda}{2m^2}\psi^3 + \frac{3\zeta N^2}{8m^3}\psi^5 \ .$$

- Comparing the attractive self-interactions and gravity,

$$|m\varphi^{\rm G}| < \frac{N\lambda}{2m^2}\psi^2 \simeq \frac{M_{\rm Q}\lambda}{2\omega m^2}\psi^2 \ . \qquad \varphi^{\rm G} \sim -\frac{G_{\rm N}M_{\rm Q}}{R_{\rm Q}} \ . \qquad \psi^2 \sim 1/R_{\rm Q}^3 \ .$$

- Criteria of non-gravitational astrophysical Q-balls,

$$\frac{G_{\rm N}m^3\omega\lambda}{2\zeta\left(\omega^2 - m^2 + \frac{\lambda^2}{4\zeta}\right)^2} < 1 \ .$$

Gravitational lensing (for point source)

- Light passing near a heavy objects bends due to gravity, the phenomenon is called gravitational lensing.

- For point mass, deflection angle is given by

$$\hat{\theta}_{\rm D} = \frac{2}{c^2}\int \nabla_\chi \varphi^{\rm G}\, dz = \frac{4G_{\rm N}M}{c^2|\vec{\chi}|}\hat{\chi}\ .$$

- For extended mass source ,

$$\hat{\theta}_{\rm D}(\chi) = \frac{4G_{\rm N}\widetilde{M}(\chi)}{c^2\chi}\ .$$

where ,

$$\widetilde{M}(\chi) = \int_0^\chi d^2\chi'\sigma(\chi')\ ,\quad \sigma(\chi) = \int_{-\infty}^{\infty}\rho(\chi, z)dz\ .$$

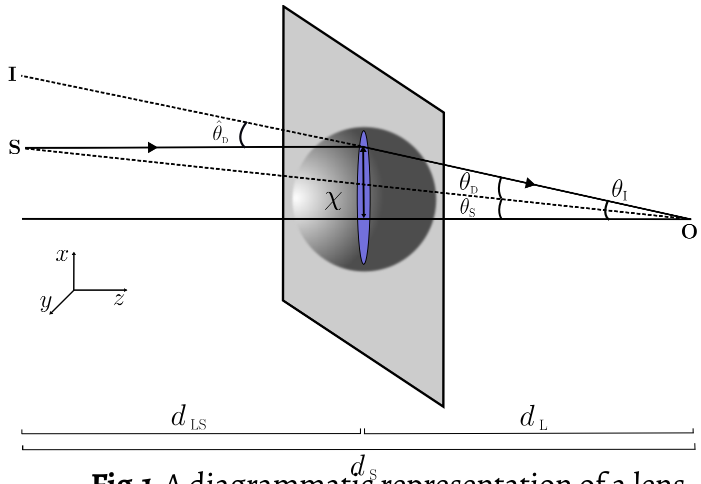


Fig.1 A diagrammatic representation of a lens, observer and source lensing system showing the various relevant quantities involved.

- The angular source position and angular image positions are related,

$$d_{\rm S}\theta_{\rm I} = d_{\rm S}\theta_{\rm S} + d_{\rm LS}\hat{\theta}_{\rm D}\ .$$

- Lens equation,

$$\theta_{\rm S} = \theta_{\rm I} - \frac{d_{\rm LS}}{d_{\rm S}d_{\rm L}}\frac{4G_{\rm N}\widetilde{M}(\theta_{\rm I})}{c^2\theta_{\rm I}}\ .$$

Gravitational lensing (for point source cont.)

- For point lens,

$$\theta_{\rm S} = 0, \qquad \theta_{\rm I} = \theta_{\rm E} \equiv \frac{R_{\rm E}}{d_{\rm L}}\,, \qquad \theta_{\rm I\pm} = \frac{1}{2}\left(\theta_{\rm S} \pm \sqrt{\theta_{\rm S}^2 + 4\theta_{\rm E}^2}\right)\,.$$

where , $R_{\rm E}$ is the Einstein radius

$$R_{\rm E} = \left(\frac{4G_{\rm N}M}{c^2}\frac{d_{\rm L}d_{\rm LS}}{d_{\rm S}}\right)^{\frac{1}{2}}\,.$$

- Magnification of the image,

$$\mathfrak{m}_{\rm I} = \frac{\theta_{\rm I}}{\theta_{\rm S}}\frac{d\theta_{\rm I}}{d\theta_{\rm S}}\,.$$

- Total magnification,

$$\mathfrak{m} = \sum_{\rm I} |\mathfrak{m}_{\rm I}|\,.$$

- For point lens,

$$\mathfrak{m}_{\rm point} = \frac{\theta_{\rm S}^2/\theta_{\rm E}^2 + 2}{\theta_{\rm S}/\theta_{\rm E}\left(\theta_{\rm S}^2/\theta_{\rm E}^2 + 4\right)^{1/2}}\,.$$

For $\theta_{\rm S} = \theta_{\rm E}$,

$$\mathfrak{m}_{\rm point} \approx 3.14\,.$$

Gravitational lensing (for extended source)

- For extended circular source, we project the source onto lens plane

$$R_{\rm S} = \frac{d_{\rm L} R_*}{d_{\rm S}} \ .$$

- The angular position of a point on the edge of this circle is ,

$$\theta_{\rm S}(\phi) = \sqrt{\theta_{\rm S,0}^2 + 2r_{\rm S}^2 + \theta_{\rm S,0} r_{\rm S} \cos\phi} \ . \qquad r_{\rm S} \equiv \frac{R_{\rm S}}{R_{\rm E}},$$

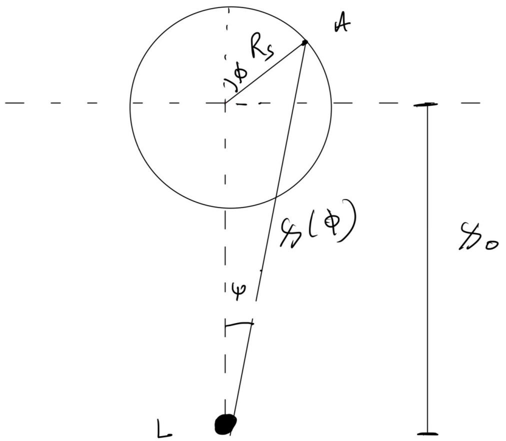


Fig. 1: Schematic for extended lens source.

- Lens equation,

$$\theta_{\rm S}(\phi) = \theta_{\rm I}(\phi) - \frac{d_{\rm LS}}{d_{\rm S} d_{\rm L}} \frac{4G_{\rm N} \widetilde{M}(\theta_{\rm I}(\phi))}{c^2 \theta_{\rm I}(\phi)} \ .$$

- Total magnification,

$$\mathfrak{m} = \sum_{\rm i} \frac{\eta}{2\pi r_{\rm S}^2} \int_{\phi=0}^{\phi=2\pi} d\Psi(\phi) \ \ y_{\rm I,i}^2(\phi) \ .$$

Witt (1994)

$$\Psi(\phi) = \tan^{-1}\left(\frac{r_{\rm S}\sin\phi}{\theta_{\rm S,0} + r_{\rm S}\cos\phi}\right) \ .$$

Threshold impact parameter for Astrophysical Q-Balls(for point source)

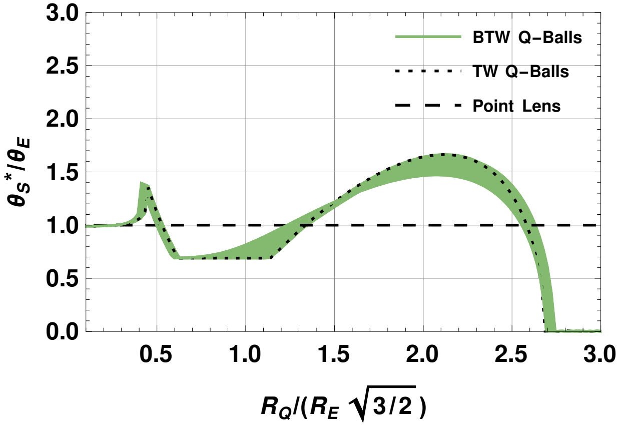


Fig. 1: The threshold value of the source position, below which one obtains a total magnification 3.14 as a function of Q- ball radius.

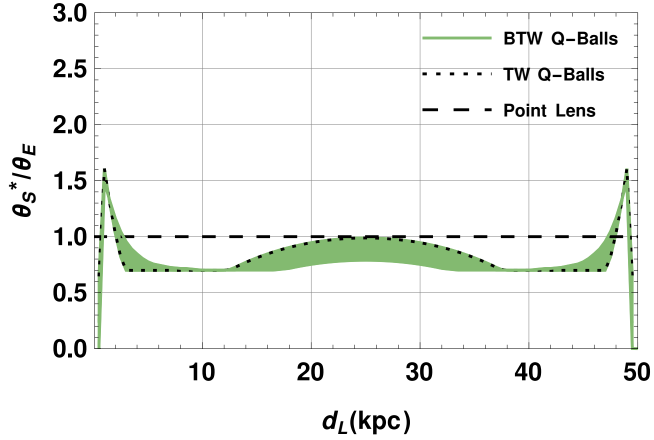
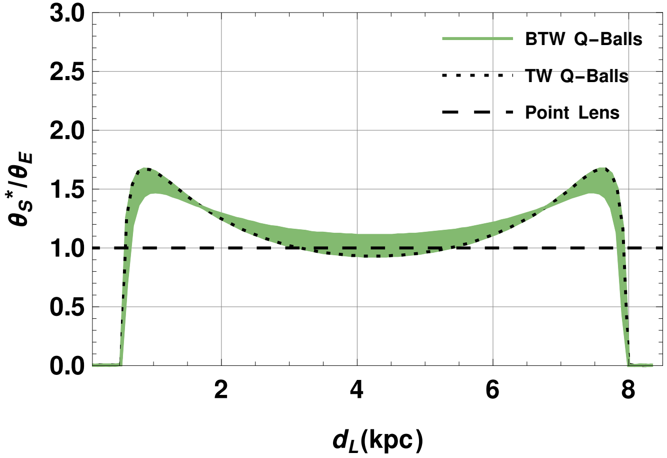


Fig. 2a: Large Magllenic Cloud (for EROS)

Fig. 2: The threshold value of the source position, below which one obtains a total magnification 3.14 as a function of Q- ball distance.



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Threshold impact parameter for Astrophysical Q-Balls (for extended source)

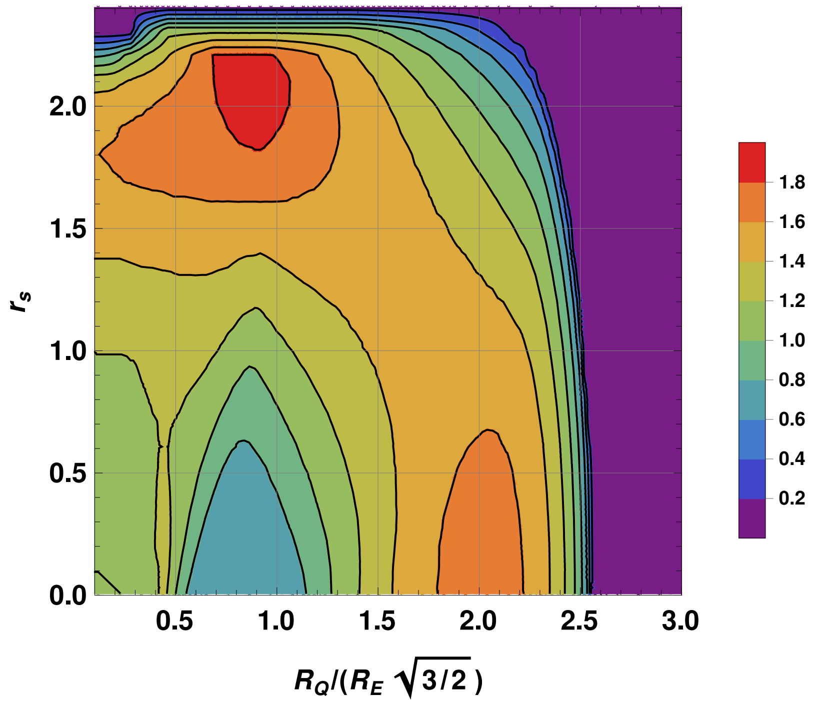


Fig. 1: The threshold value of the source position, below which one obtains a total magnification 3.14 as a function of Q- ball radius and source radius.

Constraints on astrophysical Q-balls

- For a single background source, for differential event rate for unit exposure time is given by

$$\frac{d^2\Gamma}{d\gamma d\tau} = \frac{2d_{\rm S}e(\tau)}{v_\odot^2 M_{\rm Q}} f_{\rm DM}\rho_{\rm DM}(\gamma)v_{\rm Q}^4(\gamma)e^{-v_{\rm Q}^2(\gamma)/v_\odot^2} \, .$$

Griest (1991

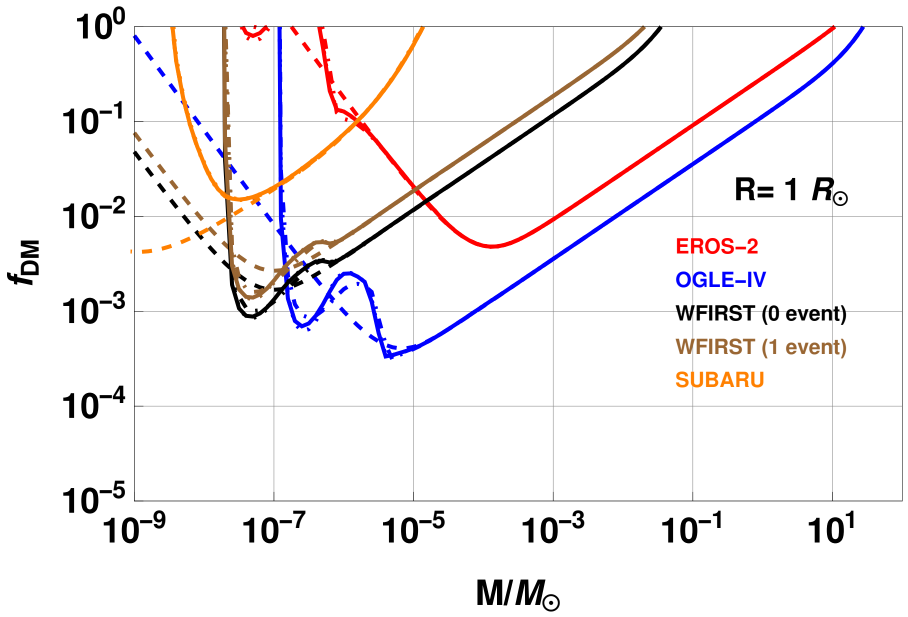
where,

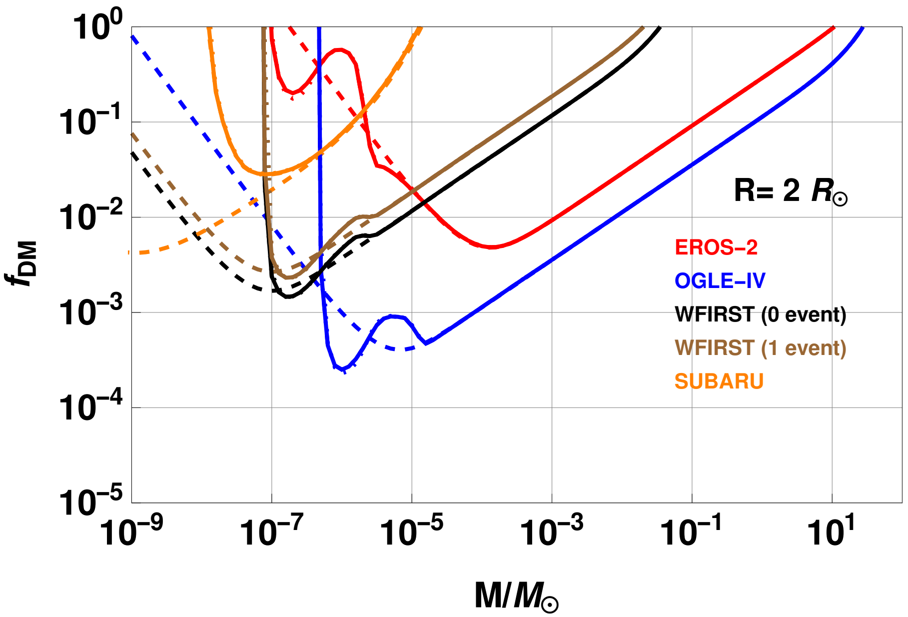
$$v_{\rm Q} \equiv \frac{2\theta_{\rm S}^*(\gamma)d_{\rm L}}{\tau}, \quad \gamma \equiv d_{\rm L}/d_{\rm S} \, .$$

- The total no. of expected astrophysical Q-balls events are,

$$N_{\rm exp} = N_s T_o \int dR_* \int_0^1 d\gamma \int_{\tau_{\rm min}}^{\tau_{\rm max}} d\tau \, \frac{dn}{dR_*} \frac{d^2\Gamma}{d\gamma d\tau} \, .$$

f_{DM} constraint plots for astrophysical Q-balls





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