Thermal Corrections to Dark Matter Annihilation Processes at NLO

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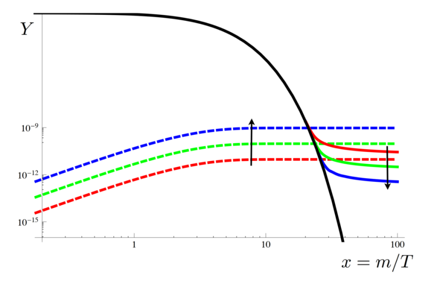
- Boltzmann Equation

$$\frac{dn_\chi}{dt} + 3Hn_\chi = \langle \sigma_{\chi\bar{\chi}\to f\bar{f}} . v_{rel} \rangle (n_\chi^{eq} . n_{\bar{\chi}}^{eq} - n_\chi . n_{\bar{\chi}})$$

- Dark Matter Freeze-out scenario



- Details

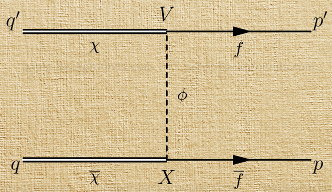
1. Relic abundance of Dark Matter $0.120 \pm 0.001 (:= \Omega_{DM} h^2)$
2. $\Omega = \rho/\rho_c$ (ρ_c : Critical Density for Our Universe)
3. $Y = n_\chi / T^3$
4. $x = m_\chi / T$
5. Coloured curves corresponds to different values of $\langle \sigma_{\chi\bar{\chi}\to f\bar{f}} . v_{rel} \rangle$

Courtesy (arXiv : 0911.1120)

Motivation

- Temperature dependence of dark matter annihilation cross-section σ due to thermal fluctuation utilizing Thermal Field Theory

$$\frac{dn_\chi}{dt} + 3Hn_\chi = \langle \sigma_{\chi\bar{\chi}\to f\bar{f}}.v_{rel} \rangle (n_\chi^{eq}.n_{\bar{\chi}}^{eq} - n_\chi.n_{\bar{\chi}})$$



- Details
  1. DM freezes-out in thermal plasma when expansion rate of the Universe dominates over the annihilation rate of DM particles
  2. This is determined by $\langle \sigma_{\chi\bar{\chi}\to f\bar{f}}.v_{rel} \rangle$
  3. Thermal corrections due to thermal fluctuations can be important

MODEL

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{f}(i\not{D} - m_f)f + \frac{1}{2}\bar{\chi}(i\not{\partial} - m_\chi)\chi$$
$$+ (D_\mu\phi)^\dagger(D_\mu\phi) - m_\phi^2\phi^\dagger\phi + (\lambda\bar{\chi}P_L f^-\phi^+ + h.c.)$$

Details of Model

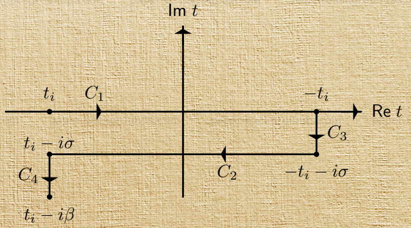
1. $f \equiv (f^0, f^-)^T$ are SM fermions, doublets under SU(2)
2. DM : χ (Majorana fermion), singlets under SU(2) $\times$ U(1)
3. $\phi = (\phi^+, \phi^0)^T$, massive scalars, SU(2) doublets, $m_\phi \gg m_\chi$
4. $m_\chi/T \sim 20$ at freeze-out

Thermal Field Theory [TFT]

- Photon Propagator in TFT

$$iD_{\mu\nu}^{t_a,t_b}(k) = -g_{\mu\nu}\left(\begin{bmatrix} i\Delta_k & 0 \\ 0 & i\Delta_k^* \end{bmatrix} + 2\pi\delta(k^2)N_B(|k^0|)\begin{bmatrix} 1 & e^{\frac{k^0}{2T}} \\ e^{\frac{k^0}{2T}} & 1 \end{bmatrix}\right)$$

- Time Path (Real Time Formalism)



- Details

1. $i\Delta_k \quad = i/(k^2 \pm i\epsilon)$

$$N_{\rm B}(|k^0|) \equiv \frac{1}{\exp\{|k^0|/T\}-1}$$

2. $S = i/(\not{p} - m + i\epsilon)$
   $S' = (\not{p} + m)$

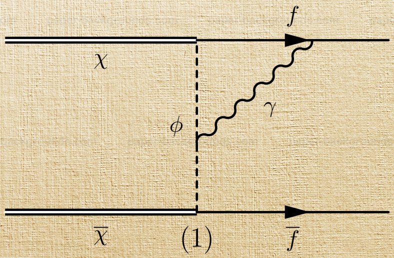
$$N_{\rm F}(|p^0|) \equiv \frac{1}{\exp\{|p^0|/T\}+1}$$

- Fermion Propagator in TFT

$$i\mathcal{S}_{\rm f}^{t_a,t_b}(p,m) = \begin{bmatrix} S & 0 \\ 0 & S^* \end{bmatrix} - 2\pi S'\delta(p^2 - m^2)N_{\rm F}(|p^0|)\begin{bmatrix} 1 & \epsilon(p_0)e^{\frac{|p^0|}{(2T)}} \\ -\epsilon(p_0)e^{\frac{|p^0|}{(2T)}} & 1 \end{bmatrix}$$

IR Divergence

- NLO scattering process



- Details
  1. Consider the sample diagram shown: $\chi(q')\,\overline{\chi}(q) \to f(p')\,\overline{f}(p)$
  2. Thermal virtual photon is inserted between fermion and scalar
  3. Photon Propagator is $iD^{t_\mu,t_\nu}_{\mu\nu,k} = -ig_{\mu\nu}\left[i/k^2 \pm 2\pi\delta(k^2)N_B(k)\right]$

- Soft and Collinear IR div. @ NLO

$$N_{\rm B}(|k^0|) \equiv \frac{1}{\exp\{|k^0|/T\}-1} \xrightarrow{k^0\to 0} \frac{T}{k^0}$$

$$-i\mathcal{M} = \int d^4k\{[\bar{u}_{p',m_f}(\gamma^\mu)(iS^{t_\mu,t_V}_{p'+k,m_f})(i\lambda^* P_R)u_{q',m_\chi}][i\Delta^{t_V,t_\nu}_{q-p+k,m_\phi}]$$

$$[(2q-2p+k)^\nu][i\Delta^{t_\nu,t_X}_{q-p,m_\phi}][\bar{v}_{q,m_\chi}(i\lambda P_L)v_{p,m_f}][iD^{t_\mu,t_\nu}_{\mu\nu,k}]\}$$

Cancellation of IR Divergence

- Rearrangement of Photon polarization sum [Virtual Photon]

$$-ig_{\mu\nu} \to -i\{g_{\mu\nu} - b_k(p_i, p_f)k_\mu k_\nu + b_k(p_i, p_f)k_\mu k_\nu\}$$

- Rearrangement of Photon polarization sum [Real Photon]

$$\sum_{\text{pol}} \epsilon^{*,\mu}(k)\,\epsilon^{\nu}(k) \to -g^{\mu\nu} \to [\widetilde{G}_k^{\mu\nu} + \widetilde{K}_k^{\mu\nu}]$$

- Structure of $b_k(p_f, p_i)$

$$\frac{1}{2}\left[\frac{(2p_f - k)\cdot(2p_i - k)}{((p_f - k)^2 - m^2)((p_i - k)^2 - m^2)} + (k \leftrightarrow -k)\right]$$

- Structure of $\widetilde{b}_k(p_f, p_i)$

$$\widetilde{b}_k(p_f, p_i) = b_k(p_f, p_i)|_{k^2 \to 0}$$

- IR divergence cancellation takes place order by order between $K_{\mu\nu}$ & $\widetilde{K}_{\mu\nu}$

- Details

1. $iD^{t_\mu, t_\nu}_{\mu\nu,k} = g_{\mu\nu}(iD^{t_\mu, t_\nu}_k)$
2. IR Finite Contribution
   $G_{\mu\nu} := g_{\mu\nu} - b_k(p_i, p_f)k_\mu k_\nu$
   $\widetilde{G}_{\mu\nu} := g_{\mu\nu} - \widetilde{b}_k(p_i, p_f)k_\mu k_\nu$
3. IR Divergent Contribution
   $K_{\mu\nu} := b_k(p_i, p_f)k_\mu k_\nu$
   $\widetilde{K}_{\mu\nu} := \widetilde{b}_k(p_i, p_f)k_\mu k_\nu$

LO scattering cross section [σ_{LO}]

$$\mathcal{M}^t_{LO} = \frac{i\lambda^2}{m_\phi{}^2} \left(\overline{v}(q, m_\chi) P_L\, v(p, m_f)\right) \left(\overline{u}(p', m_f) P_R\, u(q', m_\chi)\right) \;,$$

- LO scattering processes

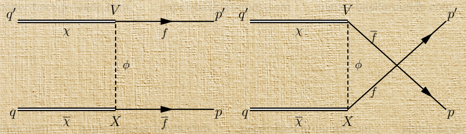


Figure: *The t-channel and u-channel dark matter annihilation processes at leading order (LO).*

- Details
  1. $P_L = \frac{1-\gamma^5}{2}$
  2. $P_R = \frac{1+\gamma^5}{2}$
  3. $H = \sqrt{s}/2$

$$\sigma_{LO} = \frac{1}{48\pi s} \frac{\lambda^4}{m_\phi^4} (8H^4 - 2H^2(4m_\chi^2 + m_f^2) + 5m_\chi^2 m_f^2) \;.$$

Effect of Thermal Fluctuations $[\sigma_{NLO(T)}]$

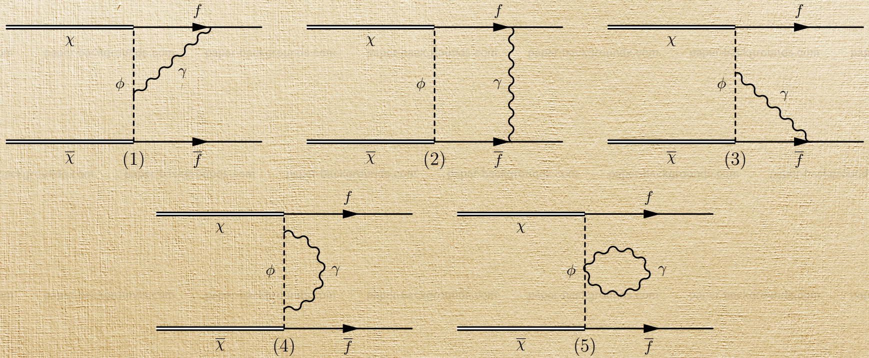
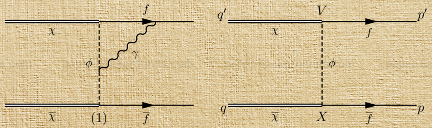


Figure: *The t-channel virtual photon corrections to the dark matter annihilation process at next to leading order (NLO). Diagrams are labelled from 1–5.* *Analogous contributions from the u-channel diagrams exist.*

Effect of Thermal Fluctuations $[\sigma_{NLO(T)}]$

$$-i\mathcal{M}^1_{t,FS} = [\bar{u}_{p',m_f}(-iQ_f\gamma^\mu)(iS^{t_\mu,t_V}_{p'+k,m_f})(i\lambda^* P_R)u_{q',m_\chi}][i\Delta^{t_V,t_\nu}_{q-p+k,m_\phi}]$$
$$[-iQ_\phi(2q-2p+k)^\nu][i\Delta^{t_\nu,t_X}_{q-p,m_\phi}][\bar{v}_{q,m_\chi}(i\lambda P_L)v_{p,m_f}][iD^{t_\mu,t_\nu}_{\mu\nu,k}]$$

- Feynman Diagram for Sample Calculation



1. Thermal contribution from ϕ is negligible $(m_\phi \gg m_\chi)$
2. ϕ Non-dynamical
   $i\Delta \to i/(-m_\phi^2)$
3. ϕ Dynamical
   $i\Delta_{q-p+k} \approx \frac{i}{\tilde{l}}\left(1 + \frac{2(q-p)\cdot k}{\tilde{l}}\right)$
   here $\tilde{l} = (q-p)^2 - m_\phi^2$

$$\sigma^{NLO} \propto \left[\frac{1}{4}\sum_{spins}(\mathcal{M}^t_{LO} - \mathcal{M}^u_{LO})^\dagger(\mathcal{M}^t_{NLO} - \mathcal{M}^u_{NLO}) + h.c.\right],$$

Effect of Thermal Fluctuations $[\sigma_{NLO(T)}]$

• It can be shown that the contribution with fermion and photon propagator simultaneously thermal, vanishes.

• Consider the contribution when the thermal part of the photon propagator is included.

• The photon propagator appearing in $\mathcal{M}^t_{NLO}$ term contains the $\delta - function$ term, which gives:

$$\begin{aligned}
\int \mathrm{d}^4k \left(2\pi\delta(k^2)\right) F(k) &= 2\pi \int_{-\infty}^{\infty} \mathrm{d}k^0 \int_0^{\infty} K^2 \mathrm{d}K \int \mathrm{d}\Omega_k \left[\delta((k^0)^2 - K^2)\right] F(k^0, K, \Omega_k)\ , \\
&= 2\pi \int \mathrm{d}k^0 \int \mathrm{d}\Omega_k \int K^2 \mathrm{d}K \, \frac{\left[\delta(k^0 - K) + \delta(k^0 + K)\right]}{|2k^0|} F(k^0, K, \Omega_k)\ , \\
&= \pi \int \mathrm{d}\Omega_k \left[\int_0^{\infty} |k^0| \mathrm{d}k^0 F(k^0, k^0, \Omega_k) + \int_{-\infty}^{0} |k^0| \mathrm{d}k^0 F(k^0, -k^0, \Omega_k)\right]\ , \\
&\equiv \pi \int_0^{\infty} \omega \mathrm{d}\omega \left[\int \mathrm{d}\Omega_k F_+(\omega, \omega, \Omega_k) + \int \mathrm{d}\Omega_k F_-(-\omega, \omega, \Omega_k)\right]\ ,
\end{aligned}$$

and the angular integrals can be performed analytically.

• If there is no explicit ω dependence in this angular integral, we have

$$\int_0^{\infty} \omega \mathrm{d}\omega \, N_B(\omega) = \frac{\pi^2 T^2}{6}\ .$$

Effect of Thermal Fluctuations $[\sigma_{NLO(T)}]$

- Result for Sample Case

$$\sigma^{t,1\gamma}_{NLO} = \frac{1}{128s(2\pi)^4}\,\frac{p'}{p}\int \omega d\omega\, n_B(\omega)\, Int^{t,1\gamma}_{NLO}\,,$$
$$= \frac{1}{128s(2\pi)^4}\,\frac{p'}{p}\,\frac{\pi^2 T^2}{6} \times Int^{t,1\gamma}_{NLO}\,,$$

$$Int^{tt+uu-tu,1\gamma}_{NLO} = \frac{256\pi e^2\lambda^4}{3{m_\phi}^6}\left(8H^4 - 2\,H^2\left(4{m_\chi}^2 + {m_f}^2\right) + 5{m_\chi}^2\,{m_f}^2\right)\,,$$

• Collinear IR Divergences get cancelled after inclusion of real emission and absorption of photon at NLO

$$Int^{tt+uu-tu,1\gamma}_{NLO,complete} = \frac{64\pi e^2\lambda^4}{3{m_\phi}^6\,p'}\Big[4p'\left(8H^4 - 2\,H^2\left(4{m_\chi}^2 + {m_f}^2\right) + 5{m_\chi}^2\,{m_f}^2\right)$$
$$+3\log\frac{H-p'}{H+p'}\left(8H^5 - 4H^3\left(2{m_\chi}^2 + {m_f}^2\right) + 5H{m_\chi}^2{m_f}^2\right)\Big]$$

Effect of Thermal Fluctuations [$\sigma_{NLO(T)}$]

- Contribution to σ_{NLO} from one NLO process (photon thermal)

$$Int_{NLO}^{tt+uu-tu,1\gamma} = \frac{64\pi e^2\lambda^4}{3{m_\phi}^6\ p'}4p'\left(8H^4 - 2\ H^2\left(4{m_\chi}^2 + {m_f}^2\right) + 5{m_\chi}^2\ {m_f}^2\right) \qquad (1)$$

- Contribution to σ_{NLO} from all NLO processes (photon thermal)

$$\begin{aligned} Int_{NLO}^{\gamma T} &= Int_{NLO}^{tt+uu-tu,1\gamma} + Int_{NLO}^{tt+uu-tu,2\gamma} + Int_{NLO}^{tt+uu-tu,3\gamma} + Int_{NLO}^{tt+uu-tu,4\gamma} + Int_{NLO}^{tt+uu-tu,5\gamma} \\ &= \frac{512\pi e^2\lambda^4}{15{m_\phi}^8}\left[216H^6 - 4H^4\left(68\ {m_\chi}^2 + 7{m_f}^2\right)\right. \\ &\quad \left. + H^2\left(56{m_\chi}^4 + 86{m_\chi}^2\ {m_f}^2 + 5{m_\phi}^4\right) - 28{m_\chi}^4{m_f}^2 - 5{m_\chi}^2\ {m_\phi}^4\right] \qquad (2) \end{aligned}$$

- NLO scattering cross section for process (photon / fermion thermal)

$$\sigma_{NLO} = \frac{1}{128s(2\pi)^4}\frac{\sqrt{H^2 - m_f^2}}{\sqrt{H^2 - m_\chi^2}}\frac{\pi^2 T^2}{6} \times \left[Int_{NLO}^{\gamma T} + \frac{1}{2}Int_{NLO}^{fT}\right] \qquad (3)$$

Conclusion

1. We investigated Thermal correction to DM annihilation process due to thermal fluctutation ,
2. We perfomed analytic calculations
   Utilizing Grammer and Yennie's approach for IR Div. Cancellation , and obtain Finite Remainder for σ_{NLO} ,
3. Calculations are done with approximations ,
   Non-Dynamical Scalar
   Dynamical Scalar (Long Expression not shown here) ,
4. Thermal fluctuation gives T^2 contribution to σ_{NLO} ,
5. Quadratic correction due to thermal fluctuation will affect $\langle \sigma v \rangle$ which will alter ρ_{DM}, in the solution of Boltzmann Equation.


Thank You

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