Thermal field theory of dark matter and thermal corrections to dark matter annihilation cross sections

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based on
Prabhat Butola, D. Indumathi, Pritam Sen, 2023, in preparation;
Pritam Sen, D. Indumathi, D. Choudhury, Eur.Phys.J.C 80 (2020) 10, 972;
Pritam Sen, D. Indumathi, D. Choudhury, Eur.Phys.J.C 79 (2019) 6, 532;
D. Indumathi, Annals Phys. 263 (1998) 310-339

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Overview

1. Introduction and Motivation
2. The theory of interacting scalars and fermions
3. Calculation of IR divergent piece
4. IR finiteness of $\chi\overline{\chi} \to f\overline{f}$

- The existence of dark matter (DM) is widely accepted today, consistent with many astronomical and cosmological observations.
- Believed that DM particles thermally produced in early Universe
- At high temperature, a typical DM particle can stay in equilibrium with the standard model (SM) sector via

$$\chi + \overline{\chi} \leftrightarrow f_{SM} + \overline{f}_{SM} \ , \quad \chi + f_{SM} \leftrightarrow \chi + f_{SM} \ .$$

- This relic density $\sim$ the correct order as per "WIMP paradigm"
- In this age of precision cosmology, an order of magnitude estimate no longer acceptable
- Both higher order corrections and thermal effects need to be included
- First calculation of thermal corrections to above cross sections to NLO, including thermal effects *(M Beneke et al., JHEP 1410 (2014) 45; JHEP 09 (2016) 031)*
- They showed the processes are IR finite to NLO
- Here we provide an all-order proof of IR divergence cancellation. We have also computed the finite remainder to NLO.

Typical class of DM models

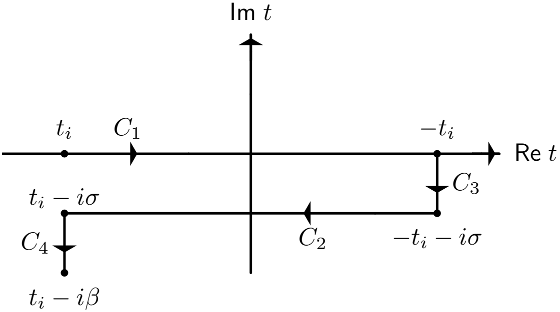
- The relevant Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{f}\left(i\not{D} - m_f\right) f + \frac{1}{2}\overline{\chi}\left(i\not{\partial} - m_\chi\right)\chi$$
$$+ \left(D^\mu\phi\right)^\dagger\left(D_\mu\phi\right) - m_\phi^2\phi^\dagger\phi + \left(\lambda\,\overline{\chi}P_L f^-\phi^+ + \mathrm{h.c.}\right) .$$

- Here $f = (f^0, f^-)^T$ is left-handed fermion doublet; $\phi = (\phi^+, \phi^0)^T$ is an additional scalar doublet; χ is $[SU(2) \times U(1)]$ singlet Majorana fermion.
- Assume bino to be TeV-scale DM, so freeze-out occurs *after* electro-weak phase transition; so only EM interactions relevant at IR.
- So we have a theory of fermions and scalars interacting with photons in a heat bath at finite temperature.

Real time formulation of thermal field theory

- The generating functional $Z_C(\beta; j)$ and time ordered path C from t_i to $t_i - i\beta$, where β is the inverse temperature of the heat bath, $\beta = 1/T$.
- Type-1 and type-2 thermal fields "live" on the C_1 and C_2 paths.



- Hence periodic boundary conditions,

$$\varphi(t_0) = \pm\varphi(t_0 - i\beta) \ ,$$

where ± 1 correspond to boson and fermion fields respectively.

Thermal Feynman rules

- The Scalar propagator

$$i\mathcal{S}^{t_a,t_b}_{\rm scalar}(p,m) = \begin{pmatrix} \Delta(p) & 0 \\ 0 & \Delta^*(p) \end{pmatrix} + 2\pi\delta(p^2-m^2)N_{\rm B}(|p^0|)\begin{pmatrix} 1 & e^{|p^0|/(2T)} \\ e^{|p^0|/(2T)} & 1 \end{pmatrix}$$

$$\Delta(p) = i/(p^2 - m^2 + i\epsilon);\ t_a, t_b = 1, 2.$$

- The Photon Propagator

$$i\mathcal{D}^{t_a,t_b}_{\mu\nu}(k) = -ig_{\mu\nu} D^{t_a,t_b}(k) = -ig_{\mu\nu}\, S^{t_a,t_b}_{\rm scalar}(k,0)\ .$$

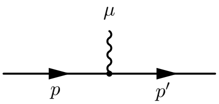
- The Fermion propagator (zero chemical potential)

$$i\mathcal{S}^{t_a,t_b}_{\rm ferm}(p,m) = \begin{pmatrix} S & 0 \\ 0 & S^* \end{pmatrix} - 2\pi S'\delta(p^2-m^2)N_{\rm F}(|p^0|)\begin{pmatrix} 1 & \epsilon(p_0)e^{|p^0|/(2T)} \\ -\epsilon(p_0)e^{|p^0|/(2T)} & 1 \end{pmatrix}$$

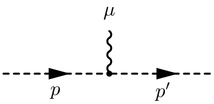
$$\equiv (\not{p} + m)\begin{pmatrix} F_p^{-1} & G_p^{-1} \\ -G_p^{-1} & F_p^{*-1} \end{pmatrix} ,$$

where $S = i/(\not{p} - m + i\epsilon)$, and $S' = (\not{p} + m)$.

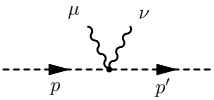
Thermal Feynman rules: 2



a. Fermion-Photon Vertex



b. Scalar-Photon Vertex



c. Seagull Vertex

$(-ie\gamma_\mu)(-1)^{t_\mu+1}$ $\qquad$ $[-ie(p_\mu + p'_\mu)](-1)^{t_\mu+1}$ $\qquad$ $[+2ie^2 g_{\mu\nu}](-1)^{t_\mu+1}$.

- The fermionic number operator

$$N_{\rm F}(|p^0|) \equiv \frac{1}{\exp\{|p^0|/T\}+1} \xrightarrow{p^0\to 0} \frac{1}{2} ,$$

- The bosonic number operator

$$N_{\rm B}(|k^0|) \equiv \frac{1}{\exp\{|k^0|/T\}-1} \xrightarrow{k^0\to 0} \frac{T}{|k^0|} .$$

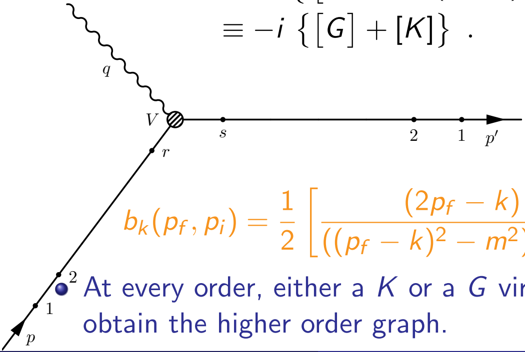
$$i\mathcal{D}^{ab}(k) = \left[\frac{i}{k^2+i\epsilon} \pm 2\pi\delta(k^2)N_B(|k^0|)D_T^{ab}\right] .$$

- So leading IR divergence in the finite temperature part is linear rather than logarithmic (as at $T=0$). Consequently, there is a residual logarithmic sub-divergence at finite temperature.

IR behaviour of QED: Grammer, Yennie Phys. Rev. D8, 4332 (1973)

- Consider a generic $n = (r+s)^{th}$ order graph. An additional virtual photon insertion can be made on the final or initial lines or with one vertex on each.
- To separate out the IR finite and divergent parts, rearrange the photon propagator into K and G parts:

$$
\begin{aligned}
-ig^{\mu\nu} &= -i \left\{ \left[g^{\mu\nu} - b_k(p_f, p_i)\, k^\mu k^\nu \right] + \left[b_k(p_f, p_i) k^\mu k^\nu \right] \right\} \ , \\
&\equiv -i \left\{ [G] + [K] \right\} \ .
\end{aligned}
$$

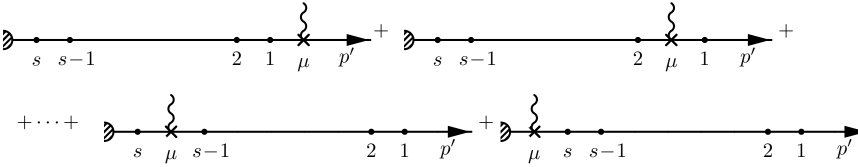


Here b_k is a function of k as well as the momenta, depending on whether the vertices of the photon were inserted on the p' or p leg.

$$
b_k(p_f, p_i) = \frac{1}{2} \left[\frac{(2p_f - k)\cdot(2p_i - k)}{((p_f - k)^2 - m^2)((p_i - k)^2 - m^2)} + (k \leftrightarrow -k) \right] \ .
$$

- At every order, either a K or a G virtual photon can be inserted to obtain the higher order graph.

- Consider a generic n^{th} order graph. Photon insertions can be made on the fermion or scalar lines or with one vertex on each.
- Start with all possible photon insertions on p' fermion line.



- A K-photon insertion at vertices (μ, ν) has a factor $(b_k k^\mu k^\nu)$, where the vertices have a factor $-ie\gamma_\mu$ and $-ie\gamma_\nu$. Hence, when the vertex μ is inserted to the right of the vertex q, we get $k^\mu \gamma_\mu = \not{k}$:

$$\mathcal{M}_{n+1,K}^{\text{right of } q} \propto \bar{u}_{p'} \gamma_{\mu_1} \cdots \gamma_{\mu_{q-1}} \left[S^{t_{q-1},t_\mu}_{p'+\sum_{q-1}} \not{k} \, S^{t_\mu,t_q}_{p'+\sum_{q-1}+k} \right] \cdots \Gamma_V \cdots \ ,$$

$$= (-1)^{t_\mu+1} \ \bar{u}_{p'} \gamma_{\mu_1} \cdots \gamma_{\mu_{q-1}} \left[S^{t_{q-1},t_q}_{p'+\sum_{q-1}} \delta_{t_\mu,t_q} - S^{t_{q-1},t_q}_{p'+\sum_{q-1}+k} \delta_{t_\mu,t_{q-1}} \right] \cdots \Gamma_V \cdots$$

$$\equiv M_q - M_{q-1} \ .$$

$T = 0$ example

$$\mathcal{M}_{n+1,K}^{\text{right of } q} \propto \bar{u}_{p'}\,\gamma_{\mu_1}\cdots\gamma_{\mu_{q-1}}\left[S^{t_{q-1},t_\mu}_{p'+\sum_{q-1}}\,\not{k}\,S^{t_\mu,t_q}_{p'+\sum_{q-1}+k}\right]\cdots\Gamma_V\cdots\;,$$

$$\xrightarrow{T=0}\;\frac{\bar{u}_{p'}\gamma_{\mu_1}\cdots\gamma_{\mu_{q-1}}}{(P'+k)^2-m^2)(P'^2-m^2)}\left[(\not{P}'+m)\not{k}(\not{P}'+\not{k}+m)\right]\cdots\Gamma_V\cdots\;,$$

with $P' = p' + \sum_{q-1}$ and

$$\left[(\not{P}'+m)\not{k}(\not{P}'+\not{k}+m)\right] = \left[(\not{P}'+m)\left\{2P'\cdot k + k^2 - (\not{P}'-m)\not{k}\right\}\right]\;,$$

$$= \left[(P'+k)^2-m^2)(\not{P}'+m) - (P'^2-m^2)(\not{P}'+\not{k}+m)\right]$$

so that

$$\mathcal{M}_{n+1,K}^{\text{right of } q} \propto \bar{u}_{p'}\,\gamma_{\mu_1}\cdots\gamma_{\mu_{q-1}}\left[S^{t_{q-1},t_\mu}_{p'+\sum_{q-1}}\,\not{k}\,S^{t_\mu,t_q}_{p'+\sum_{q-1}+k}\right]\cdots\Gamma_V\cdots\;,$$

$$\xrightarrow{T=0}\;\bar{u}_{p'}\gamma_{\mu_1}\cdots\gamma_{\mu_{q-1}}\left[S_{P'} - S_{P'+k}\right]\cdots\Gamma_V\cdots\;,$$

In the thermal case, we have, for insertions on both fermions and scalars,

$$S^{t_q,t_\mu}_{p'+\sum_q}\,\not{k}\,S^{t_\mu,t_{q+1}}_{p'+\sum_q+k} = (-1)^{t_\mu+1}\left[S^{t_q,t_{q+1}}_{p'+\sum_q}\delta_{t_\mu,t_q+1} - S^{t_q,t_{q+1}}_{p'+\sum_q+k}\delta_{t_\mu,t_q}\right]\;.$$

K photon insertions on p' leg

- The total matrix element obtained when the vertex is inserted in s possible ways (excluding the first graph)

$$\mathcal{M}_{n+1}^{s \text{ terms}} = (M_2 - M_1) + (M_3 - M_2) + \cdots + (M_{s+1} - M_s) \; ,$$
$$= - M_1 + M_{s+1} \; .$$

- The first term/graph gives

$$\mathcal{M}_{n+1}^{1} \propto \bar{u}_{p'} \not{k} \, S_{p'+k}^{t_\mu, t_1} \gamma_{\mu_1} S_{p'+\sum_1 + k}^{t_1, t_2} \cdots \Gamma_V \cdots \; ,$$
$$\equiv M_1 \; ,$$

where we have used $\overline{u}(p')\not{k} S_{p'+k}^{t_\mu, t_1} = \overline{u}(p')(-1)^{t_\mu+1}\delta_{t_\mu, t_1}$, and the on-shell condition $\not{p}' u(p') = m\, u(p')$.

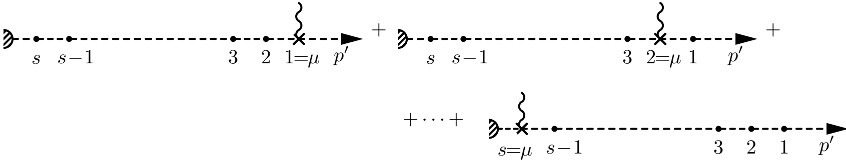
- Thus, the total contribution from ALL $(s+1)$ terms :

$$\mathcal{M}_{n+1}^{p', \mu} = (-1)^{t_\mu + 1} \delta_{t_\mu, t_V} \left[\overline{u}_{p'} \, \gamma_{\mu_1} \cdots \gamma_{\mu_{q-1}} S_{p'+\sum_{q-1}}^{t_{q-1}, t_q} \cdots S_{p'+\sum_s}^{t_s, t_V} \Gamma_V \cdots \right] \; ,$$

which is proportional to the lower order matrix element, $\mathcal{M}_n$.

Insertions on the scalar lines

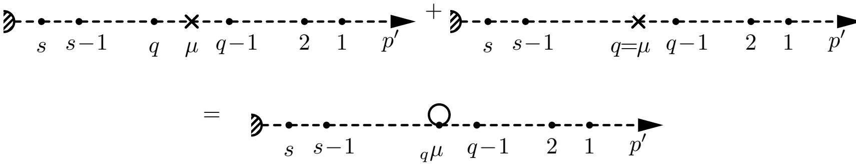
- Two possibilities: (i) a new insertion, just as for fermions, (ii) insertion at an already existing vertex (4-point vertex, does not exist for fermions)



- The diagrams can be combined to get a simplification, although not as clean as the case of fermions.
- Each insertion at an old vertex q, and the insertion μ at q, can be combined to get a *circled vertex*. Set of such terms can be combined to get similar results as the fermionic case.

The circled vertex

- Circled vertex:



- Analogous identities:

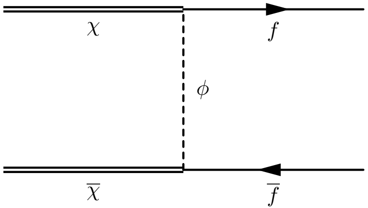
$$S^{t_q, t_\mu}_{p' + \sum_q} \left[(2p' + 2\Sigma_q + k)\cdot k\right] S^{t_\mu, t_{q+1}}_{p' + \sum_q + k} = (-1)^{t_\mu + 1} \left[S^{t_q, t_{q+1}}_{p' + \sum_q} \delta_{t_\mu, t_q + 1} - S^{t_q, t_{q+1}}_{p' + \sum_q + k} \delta_{t_\mu, t_q} \right] ,$$

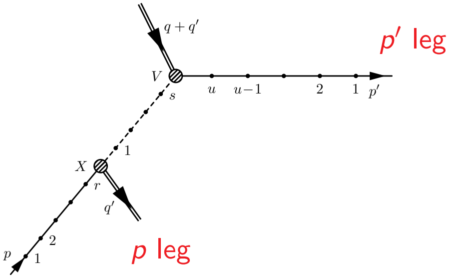
Insertion right of '1': $[(2p' + k)\cdot k] S^{t_\mu, t_1}_{p'+k} = (-1)^{t_\mu + 1} \delta_{t_\mu, t_1}$, since $p'^2 = m^2$.

- We now apply these results to calculate $\sigma(\chi\overline{\chi} \to f\overline{f})$.

$\chi\overline{\chi} \to f\overline{f}$

- A generic process in the DM model can also be identified via a p and p' leg:





$$\mathcal{M}_n^{p_f,p_i} = \mathcal{C}_u^{\text{fermion}\ p'} \times \left(\mathcal{C}_{s+r}^{p}\right) = \mathcal{C}_u^{\text{fermion}\ p'} \times \left(\mathcal{C}_s^{\text{scalar}} \times \mathcal{C}_r^{\text{fermion}\ p}\right) \ ,$$

- We need to consider $(n+1)$th photon insertion with (i) both vertices on p' leg, (ii) both on p leg, and (iii) one on each.

Both K photon vertices on p' leg

This is the same as the fermion case. The $\mathcal{M}_{n+1}$ matrix element with K photon insertion in all possible ways on the p' leg is proportional to the $\mathcal{M}_n$ matrix element.

K photon insertion with vertices on p and p' leg

$$\mathcal{M}_{n+1}^{p',p} \sim \mathcal{C}_{u+1}^{\text{fermion } p',\mu} \times \mathcal{C}_{s+r+1}^{p,\nu} ,$$

$$\begin{aligned} \mathcal{C}_{s+r+1}^{p,\nu} = & \left[\mathcal{C}_{s+1}^{\text{scalar},\nu} \times \mathcal{C}_{r}^{\text{fermion } p}\right] + \left[\mathcal{C}_{s}^{\text{scalar}} \times \mathcal{C}_{r+1}^{\text{fermion } p,\nu}\right] , \\ \equiv & \ \mathcal{C}_{s+r+1}^{\text{scalar},\nu} + \mathcal{C}_{s+r+1}^{\text{fermion},\nu} . \end{aligned}$$

This is a combination of pure fermion (p') and fermion-scalar (p).

- The $\mathcal{C}_{s+r+1}^{\text{scalar},\nu}$ has pair-wise cancellation, with one term left over, from insertion just above X.
- The $\mathcal{C}_{s+r+1}^{\text{fermion},\nu}$ term, which should have only one term left over after pair-wise cancellation has two terms. One is the expected term, proportional to the lower order matrix element. The other (arising from insertions just below X), cancels the first contribution.
- Hence, $\mathcal{M}_{n+1}$ matrix element with K photon insertion in all possible ways on the p and p' leg is proportional to the $\mathcal{M}_n$ matrix element.
- This double cancellation is independent of the nature of vertex X.

Both K photon vertices on p leg

- This is the most complicated. There are three contributions: (i) both vertices on scalar, (ii) both on fermion and (iii) one on each. The first is most complicated, with many diagrams:

- Very complicated structure. In short, there is again a double cancellation. Here, not only is there a cancellation across vertex X, it is also needed to observe that vertices V and X have external particles and thus can only be of type-1 thermal.

Schematic of K photon insertion p',p' leg

Linear in k with k dependence in denominator

Linear in k with no k dependence in denominator

$$\mathcal{M}_{n+1} \propto \mathcal{M}_n + \mathcal{O}_d(k) + \mathcal{O}(k) + \mathcal{O}(k^2)$$

odd under integral Hence vanishes

Quadratic in k

Exactly Cancels

+ Seagull

+ Tadpole

Leaving us with exact factorisation leading to resummasion

Courtesy: Pritam Sen

Total K photon matrix element $\mathcal{M}_{n+1}$

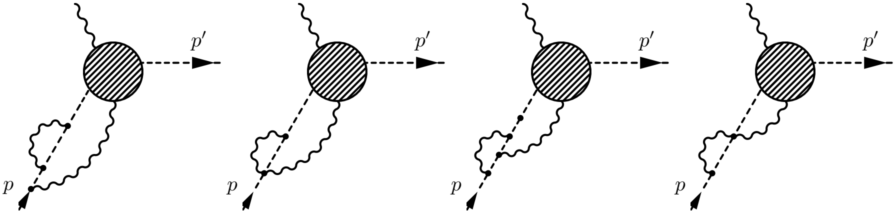
$$\mathcal{M}_{n+1}^{K\gamma,\mathrm{tot}} = \frac{ie^2}{2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \left\{ \delta_{t_\mu,t_1}\, \delta_{t_\nu,t_1}\, D^{t_\mu,t_\nu}(k) \left[b_k(p',p') + b_k(p,p) \right] \right.$$
$$\left. + \, \delta_{t_\mu,t_V}\, \delta_{t_\nu,t_V}\, D^{t_\mu,t_\nu}(k) \left[-2 b_k(p',p) \right] \right\} \mathcal{M}_n \, ,$$
$$\equiv [B]\, \mathcal{M}_n \, ,$$

$$B = \frac{ie^2}{2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} D^{11}(k) \left[b_k(p',p') - 2 b_k(p',p) + b_k(p,p) \right] \, ,$$
$$\equiv \frac{ie^2}{2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} D^{11}(k) \left[J^2(k) \right] \, ,$$

- Universal nature of B; D^{11} contains thermal dependence.

Remarks

- Disallowed diagrams at higher order: diagrams which are disallowed at one order may be allowed at higher order.



- Contribution computes to zero.
- Virtual G photon insertions are IR finite. This is a difficult calculation since there is a linear and sub-leading log divergence and both must be shown to cancel in G photon insertions in graphs which have arbitrary mix of K and G insertions (and real photon insertions).

Factorisation and resummation of K photon contributions

Summing over all orders, we get

$$\sum_{n=0}^{\infty} \frac{1}{n!}\mathcal{M}_n = \sum_{n=0}^{\infty}\sum_{n_K=0}^{n} \frac{1}{n_K!}\frac{1}{n-n_K!}\mathcal{M}_{n_G,n_K} ,$$

$$= \sum_{n_K=0}^{\infty}\sum_{n_G=0}^{\infty} \frac{1}{n_K!}\frac{1}{n_G!}\mathcal{M}_{n_G,n_K} , \tag{1}$$

Since the K photon contribution is proportional to the lower order matrix element we have,

$$\mathcal{M}_{n_G,n_K} = (B)^{n_K} M_{n_G,0} , \tag{2}$$

$$\sum_{n=0}^{\infty} \frac{1}{n!}\mathcal{M}_n = \sum_{n_K=0}^{\infty} \frac{(B)^{n_K}}{n_K!}\sum_{n_G=0}^{\infty} \frac{1}{n_G!}\mathcal{M}_{n_G} ,$$

$$= \mathrm{e}^B \sum_{n_G=0}^{\infty} \frac{1}{n_G!}\mathcal{M}_{n_G} . \tag{3}$$

Cancellation of IR divergence to all orders

- As is the case with $T = 0$ field theories, the IR cancellation occurs between the virtual and real photon contributions.
- The real photon contributions can also be written as polarisation sums:

$$
\begin{aligned}
\sum_{\mathrm{pol}} \epsilon^*_\mu(k)\, \epsilon_\nu(k) &= -g_{\mu\nu} \;, \\
&= -\left\{ \left[g_{\mu\nu} - \tilde{b}_k(p_f, p_i) k_\mu k_\nu \right] + \left[\tilde{b}_k(p_f, p_i) k_\mu k_\nu \right] \right\} \;, \\
&\equiv -\left\{ \left[\widetilde{G}_{\mu\nu} \right] + \left[\widetilde{K}_{\mu\nu} \right] \right\} \;,
\end{aligned} \tag{4}
$$

- The phase space factor for real photons includes both emission into and absorption from heat bath: crucial for IR divergence cancellation.

$$
\mathrm{d}\phi_i = \frac{\mathrm{d}^4 k_i}{(2\pi)^4} \, 2\pi\delta(k_i^2) \, \left[\theta(k_i^0) + N_{\mathrm{B}}(|k_i^0|) \right] \;. \tag{5}
$$

- We will see that the $\widetilde{K}$ contribution to the cross section cancels the IR divergent part of the virtual photon contribution.

Real and Virtual K photon contributions

- The total cross section:

$$\mathrm{d}\sigma^{\rm tot} = \int \mathrm{d}^4 x \, e^{-i(p+q-p')\cdot x} \mathrm{d}\phi_{p'} \mathrm{d}\phi_{q'} \exp\left[B + B^*\right] \exp\left[\widetilde{B}\right] \times \sum_{n_G=0}^{\infty} \frac{1}{n_G!} \times$$

$$\prod_{j=0}^{n_G} \int \mathrm{d}\phi_j e^{\pm i k_j \cdot x} \left[-G_{\mu\nu} \mathcal{M}^{\dagger\mu}_{n_G} \mathcal{M}^{\nu}_{n_G} \right]$$

- Here $= \int \mathrm{d}^4 x \, e^{-i(p+q-p')\cdot x} \, \mathrm{d}\phi_{p'} \mathrm{d}\phi_{q'} \exp\left[B + B^* + \widetilde{B}\right] \, \sigma^{\rm finite}(x)$.

$$(B+B^*)+\widetilde{B} = e^2 \int \mathrm{d}\phi_k \Big[J^2(k)(1+2N_0) - \widetilde{J}^2(k) \Big\{ (1+N_0)\, e^{ik\cdot x} + N_0 \, e^{-ik\cdot x} \Big\} \Big]$$

$$\stackrel{k\to 0}{\longrightarrow} \; 0 + \mathcal{O}(k^2) \; , \text{ with } \widetilde{J}^2 = J^2(k^2 = 0) \; .$$

- Both the linear ($\mathcal{O}(k^0)$) and logarithmic ($\mathcal{O}(k^1)$) terms cancel order by order, to all orders in the thermal field theory. Latter not easy to see explicitly, but due to integrand being odd in k.

Proof of IR finiteness of G photon contributions

- We began with the Grammer-Yennie technique of replacing the term $g^{\mu\nu}$ in the photon propagator by K and G terms:

$$
\begin{aligned}
-ig^{\mu\nu} &= -i\left\{\left[g^{\mu\nu} - b_k(p_f,p_i)\,k^\mu k^\nu\right] + \left[b_k(p_f,p_i)k^\mu k^\nu\right]\right\} \;, \\
&\equiv -i\;\left\{\left[G\right] + [K]\right\} \;.
\end{aligned}
$$

- We showed that the entire IR divergence was contained in the K-photon contribution and that this could be factorised and exponentiated.
- Further, a similar process occurs in real photon emission/absorption: $\chi\overline{\chi} \to f\overline{f}\gamma$ and $\chi\overline{\chi}\gamma \to f\overline{f}$, separable into $\widetilde{K}$ and $\widetilde{G}$ contributions: former contain the IR divergences.
- The IR divergences cancel between the K and $\widetilde{K}$ contributions. Both photon emission into andabsorption from the heat bath are required for this to occur. Both linear and logarithmic divergences cancel order by order, to all orders in the theory.
- Hence need to show that the G and $\widetilde{G}$ contributions are IR finite.

The proof is ...

The proof is ...

- $\cdots$ very tedious!
- In summary,
  - Linearly divergent terms in G cancel because of *structure* of b_k giving terms proportional to

$$[g^{\mu\nu} - b_k(p_f, p_i) k^\mu k^\nu] \times p_f^\mu p_i^\nu \to 0 + \mathcal{O}(k^1) \ , \tag{6}$$

  - Log divergent terms in G vanish because of symmetry of both measure and $N_B(|k^0|)$ in $k \leftrightarrow -k$.
- Hence G photon contributions are IR finite.

The IR finite cross section at NLO

- The cross section $\sigma(\chi\overline{\chi} \to f\bar{f})$ which occurs in the Boltzmann equation for the dark matter relic density calculation is now proven to be IR finite in the thermal field theory.
- The IR finite piece and its thermal contribution can now be calculated explicitly. While the thermal contribution may be small, it can significantly impact the thermal evolution at early times.
- Details of the calculation will be presented by Prabhat Butola in his talk.

Conclusions

- Thermal theories of charged scalars and fermionic QED are IR safe at all orders in perturbation theoy.
- Using these results, typical theories of thermal dark matter are found to be IR safe at all orders in perturbation theory.
- The seagull and tadpole diagrams were crucial to obtain a neat factorization leading to resummation.
- Both the emission to and absorption from heat bath was crucial for the IR divergence cancellation.
- The whole procedure does not depend on the exact interaction term in the Lagrangian, and hence can be easily applied to any theory of charged scalars and fermions.
- The IR finiteness of DM is a generic requirement, which has important implications in cosmology. The IR finite piece has been calculated to NLO in the thermal field theory (Beneke 2014, Butola 2023).

Conclusions

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- Thank you