INCLUSIVE SEMILEPTONIC B,D DECAYS

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THE V_{cb} (and V_{ub}) PUZZLE

Since many years the inclusive and exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$ diverge

Recent burst of activity: new exp analyses, new lattice and pert calculations.

This plot is outdated!



The importance of $|V_{cb}|$

An important CKM unitarity test is the Unitarity Triangle (UT) formed by

$$1 + \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

 V_{cb} plays an important role in UT $\varepsilon_K \approx x |V_{cb}|^4 + \dots$

and in the prediction of FCNC: $\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2 \Big[1 + O(\lambda^2)\Big]$

where it often dominates the theoretical uncertainty. V_{ub}/V_{cb} constrains directly the UT

Our ability to determine precisely V_{cb} is crucial for indirect NP searches

angles



VIOLATION of LFU with TAUS

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_{\tau}\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_{\ell}\right)}$$

SM predictions based on same theory as V_{cb} extraction



INCLUSIVE DECAYS: BASICS



- Simple idea: inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows us to express it in terms of B meson matrix elements of local operators.
- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: **double series in** α_s , Λ/m_b
- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on $m_{b,c}$, two parameters at $O(\Lambda^2/m_b^2)$, 2 more at $O(\Lambda^3/m_b^3)$... Many higher order effects have been computed.

INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and α_s

$$M_{i} = M_{i}^{(0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(1)} + \left(\frac{\alpha_{s}}{\pi}\right)^{2} M_{i}^{(2)} + \left(M_{i}^{(\pi,0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(\pi,1)}\right) \frac{\mu_{\pi}^{2}}{m_{b}^{2}} + \left(M_{i}^{(G,0)} + \frac{\alpha_{s}}{\pi} M_{i}^{(G,1)}\right) \frac{\mu_{G}^{2}}{m_{b}^{2}} + M_{i}^{(D,0)} \frac{\rho_{D}^{3}}{m_{b}^{3}} + M_{i}^{(LS,0)} \frac{\rho_{LS}^{3}}{m_{b}^{3}} + \dots$$

Global shape parameters (first moments of the distributions, with various lower cuts on E_1) tell us about $m_{b,} m_c$ and the B structure, total rate about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks: they are useful in many applications (rare decays, inclusive $V_{ub...}$)

Reliability of the method depends on our control of higher order effects. Quark-hadron duality violation would manifest itself as inconsistency in the fit.

3LOOP CALCULATIONS

Fael, Schoenwald, Steinhauser, 2011.11655, 2011.13654, 2205.03410

3loop and 2loop charm mass effects in relation between kinetic and \overline{MS} b mass

 $m_b^{kin}(1\text{GeV}) = \left| 4163 + 259_{\alpha_s} + 78_{\alpha_s^2} + 26_{\alpha_s^3} \right| \text{MeV} = (4526 \pm 5) \text{MeV}$ Using FLAG $\overline{m}_b(\overline{m}_b) = 4.198(12)$ GeV one gets $m_b^{kin}(1$ GeV) = 4.565(19)GeV 3loop correction to total semileptonic width $\Gamma_{sl} = \Gamma_0 f(\rho) \Big[0.9255 - 0.1162_{\alpha_s} - 0.0350_{\alpha_s^2} - 0.0097_{\alpha_s^3} \Big] \Big] \begin{bmatrix} 0.9255 - 0.1162_{\alpha_s} - 0.0350_{\alpha_s^2} - 0.0097_{\alpha_s^3} \Big] \Big] \Big] \Big]$ in the kin scheme with $\mu = 1 \text{GeV}$ and $\overline{m}_c(3 \text{GeV}) = 0.987 \text{ GeV}, \mu_{\alpha_s} = m_b^k$ $\Gamma_{sl} = \Gamma_0 f(\rho) \left[0.9255 - 0.1140_{\alpha_s} - 0.0011_{\alpha_s^2} + 0.0103_{\alpha_s^3} \right]^{\frac{1}{2}}$ in the kin scheme with $\mu = 1 \text{GeV}$ and $\overline{m}_c(2 \text{GeV}) = 1.091 \text{ GeV}, \mu_{\alpha_c} = m_b^{kin}/2$ (f)**3**loop correction tends to lower Γ_{sl} and therefore pushes $|V_{cb}|$ slightly up (~0.5%)

RESIDUAL UNCERTAINTY on Γ_{sl}

Bordone, Capdevila, PG, 2107.00604



Similar reduction in μ_{kin} dependence. Purely perturbative uncertainty ±0.7 % (max spread), central values at $\mu_c = 2\text{GeV}, \mu_{\alpha_s} = m_b/2$.

 $O(\alpha_s/m_b^2, \alpha_s/m_b^3)$ effects in the width are known. Additional uncertainty from higher power corrections, soft charm effects of $O(\alpha_s/m_b^3m_c)$, duality violation.

Conservatively: 1.2% overall theory uncertainty in Γ_{sl} (a ~50% reduction) Interplay with fit to semileptonic moments, known only to $O(\alpha_s^2, \alpha_s \Lambda^2/m_h^2)$

HIGHER ORDER CORRECTIONS TO MOMENTS

- complete $O(\alpha_s)$ corrections to triple differential Aquila, Ridolfi, PG, Trott, Czarnecki, Jezabek, Kuhn, ...
- complete $O(\alpha_s^2)$ corrections to leptonic, hadronic (*partly numerical*), q^2 moments at arbitrary cuts Biswas, Melnikov, Czarnecki, Pak, PG, Fael, Herren
- $O(\alpha_s^3)$ corrections to leptonic, hadronic, q^2 moments without cuts Fael, Schoenwald, Steinhauser
- complete $O(\alpha_s \Lambda^2/m_b^2)$ corrections to triple differential, $O(\alpha_s \Lambda^3/m_b^3)$ to width and q^2 moments Alberti, Healey, Nandi, PG, Becher, Lunghi, Mannel, Moreno, Pivovarov
- power corrections of $O(\Lambda^2/m_b^2)$ and $O(\Lambda^3/m_b^3)$ to triple differential, $O(\Lambda^4/m_b^4)$ and $O(\Lambda^5/m^5)$ for moments Manohar, Wise, Blok, Koyrakh, Shifman, Vainshtein, Grimm, Kapustin, Mannel, Turzcyk, Uraltsev, Milutin, Vos

QED CORRECTIONS

Bigi, Bordone, Haisch, Piccione PG 2309.02849

In the presence of photons, **OPE valid only for total width** and moments that do not resolve charged lepton or hadron properties $(E_{\ell}, q^2, E_X...)$. Expect mass singularities and $O(\alpha \Lambda/m_b)$ corrections.

Leading logs $\alpha \ln m_e/m_b$ can be easily computed for simple observables using structure function approach, for ex the lepton energy spectrum

$$\left(\frac{d\Gamma}{dy}\right)^{(1)} = \frac{\alpha}{2\pi} \ln \frac{m_b^2}{m_\ell^2} \int_y^1 \frac{dx}{x} P_{\ell\ell}^{(0)} \left(\frac{y}{x}\right) \left(\frac{d\Gamma}{dx}\right)^{(0)}$$
$$P_{\ell\ell}^{(0)}(z) = \left[\frac{1+z^2}{1-z}\right]_+$$



QED Leading contributions

1. Collinear logs: captured by splitting functions



2. Threshold effects or Coulomb terms



3. Wilson Coefficient



 $\sim rac{4\pi lpha_e}{9}$ discontinuity at threshold

$$\sim \frac{\alpha_e}{\pi} \left[\ln \frac{M_Z^2}{\mu^2} - \frac{11}{6} \right]$$

M. Bordone

also at subleading power!

$$\sim \frac{\alpha_e}{\pi} \log \frac{m_b^2}{m_e^2}$$

COMPLETE $O(\alpha)$ EFFECTS IN LEPTONIC SPECTRUM

Typical measurements are completely inclusive, $B \to X_c \ell \nu(\gamma)$, but QED radiation is **subtracted** by experiments using **PHOTOS** (soft-collinear photon radiation to MC final states).

Small but non-negligible differences with PHOTOS in BaBar leptonic moments hep-ex/0403030

$E_{\rm cut}$	$\delta { m BR}_{ m incl}^{ m BaBar}$	$\delta \mathrm{BR}^{\mathrm{LL}}_{\mathrm{incl}}$	$\delta \mathrm{BR}^{\mathrm{NLL}}_{\mathrm{incl}}$	$\delta \mathrm{BR}^{lpha}_{\mathrm{incl}}$	$\delta \mathrm{BR}_\mathrm{incl}^{1/m_b^2}$	$\delta \mathrm{BR}_\mathrm{incl}$	σ
0.6	-1.26%	-1.92%	-1.95%	-0.54%	-0.50%	-0.45%	+0.34
0.8	-1.87%	-2.88%	-2.91%	-1.36%	-1.29%	-1.22%	+0.30
1.0	-2.66%	-4.03%	-4.04%	-2.38%	-2.26%	-2.15%	+0.25
1.2	-3.56%	-5.43%	-5.41%	-3.65%	-3.43%	-3.27%	+0.14
1.5	-5.22%	-8.41%	-8.26%	-6.37%	-5.73%	-5.39%	-0.09

~0.2% reduction in V_{cb}



The black curve corresponds to the correction obtained by BaBar using PHOTOS, while the red (green) curve corresponds to our QED prediction including the LL terms (all QED corrections). The grey band represents the systematic uncertainty on the PHOTOS bremsstrahlungs corrections that BaBar quotes, while the black error bars correspond to the total uncertainties of the QED corrected BaBar results.

A GLOBAL FIT

$m_b^{ m kin}$	$\overline{m}_c(2{ m GeV})$	μ_π^2	$\mu_G^2(m_b)$	$ ho_D^3(m_b)$	$ ho_{LS}^3$	$BR_{c\ell\nu}$	$10^{3} V_{cb} $
4.573	1.090	0.454	0.288	0.176	-0.113	10.63	41.97
0.012	0.010	0.043	0.049	0.019	0.090	0.15	0.48

Includes all leptonic, hadronic, and q^2 moments measured by BaBar, Belle, Belle II, Cleo, CDF, Delphi

Up to $O(\alpha_s^2)$, $O(\alpha_s/m_b^2)$, $O(1/m_b^3)$ for M_X , E_{ℓ} moments, up to $O(\alpha_s^2\beta_0)$, $O(\alpha_s/m_b^3)$ for q^2 moments (complete $O(\alpha_s^2)$ calculation by Fael and Herren 2403.03976 to be implemented)

Subtracts QED effects beyond those computed by PHOTOS (only BaBar BR and lept moments) $\delta V_{cb} \sim -0.2~\%$

Employs $\overline{m}_b(\overline{m}_b) = 4.203(11)$ GeV and $\overline{m}_c(3$ GeV) = 0.989(10)GeV (FLAG) $\chi^2_{min}/dof = 0.55$ $|V_{cb}| = (41.97 \pm 0.27_{exp} \pm 0.31_{th} \pm 0.25_{\Gamma}) \times 10^{-3} = (41.97 \pm 0.48) \times 10^{-3}$ consistent with analysis of q^2 moments by Bernlochner et al, 2205.10274

MINOR TENSIONS IN HIGHER q^2 MOMENTS



Finauri, PG 2310.20324

 $Q_1 = \langle q^2 \rangle, \qquad Q_2 = \langle (q^2 - \langle q^2 \rangle)^2 \rangle, \qquad Q_3 = \langle (q^2 - \langle q^2 \rangle)^3 \rangle$

Highly correlated data and theoretical predictions: we include only 5 Belle II and 4 Belle points for each moment and employ an ansatz for theory correlations

Schwanda, PG 1307.4551

comparison of different datasets

Finauri, PG 2310.20324



Theory correlations are no longer an issue

Finauri, PG 2310.20324



HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters starting $1/m^4$: 9 at dim 7, 18 at dim 8 In principle relevant: HQE contains $O(1/m_b^n 1/m_c^k)$ Mannel,Turczyk,Uraltsev 1009,4622

Lowest Lying State Saturation Approx (LLSA) truncating $\langle B|O_1O_2|B\rangle = \sum \langle B|O_1|n\rangle \langle n|O_2|B\rangle$

see also Heinonen, Mannel 1407.4384

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$\rho_D^3 = \epsilon \,\mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon \,\mu_G^2 \quad \epsilon \sim 0.4 \text{GeV}$$

 ϵ excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases 1206.2296

We use LLSA as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers.

still without q^2 moments!

$$|V_{cb}| = 42.00(53) \times 10^{-3}$$

Bordone, Capdevila, PG, 2107.00604 **Update of 1606.06174**

The advantage of q^2 moments

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177

- Standard lepton energy and hadronic mass moments are not RPI quantities
- New q^2 moments are RPI!

Reparametrization invariant quantities:

- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of v not unique: Reparametrization invariance (RPI)

$$v_{\mu}
ightarrow v_{\mu} + \delta v_{\mu}$$

$$\delta_{RP} v_{\mu} = \delta v_{\mu}$$
 and $\delta_{RP} iD_{\mu} = -m_b \delta v_{\mu}$

- links different orders in $1/m_b \rightarrow$ reduction of parameters
- up to $1/m_b^4$: 8 parameters (previous 13)
- q^2 moments enable (?) a full extraction up to $1/m_b^4$

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|_{ ext{incl}}^{q^2} = (41.69 \pm 0.63) imes 10^{-3}$$
 Important consistency check

Keri Vos (Maastricht)



PROSPECTS

- **Experiment**: more precise measurement of moments and BR at Belle II, new observables (A_{FB} , quantities computable on the lattice with optimal uncertainty), correlations between different kinds of moments, improved QED treatment
- **Theory**: analytic calculation of $O(\alpha_s^2)$ corrections to moments, inclusion of complete $O(\alpha_s^2)$ in q^2 -moments, $O(\alpha_s \rho_D^3/m_b^3)$ to lept and hadr moments, QED effects in q^2 -moments...
- Interplay with lattice calculations (this afternoon discussion): in the mid term look for complementarity with exp data, and new directions in parameters space (lattice as virtual lab: new observables, V, A, S, P currents, ...)



• While the lattice calculation of the spectral density of hadronic correlators is an **ill-posed problem**, the spectral density is δa ccessible after smearing, as provided by phase-space integration Hansen, Meyer, Robaina, Hansen, Lupo, Tantalo, Bailas, Hashimoto, Ishikawa $\mathcal{O}(\mathcal{U} - E_{\mathrm{thresh}}) \times (\text{phase space})$



A PRACTICAL APPROACH

Hashimoto, PG 2005.13730

4-point functions on the lattice are related to the hadronic tensor in euclidean

$$\sim \langle B | J^{\dagger}_{\mu}(\mathbf{x},t) J_{\nu}(\mathbf{0},0) | B \rangle \qquad d\Gamma \sim L^{\mu\nu}W_{\mu\nu}, \quad W_{\mu\nu} \sim \sum_{X} \langle B | J^{\dagger}_{\mu} | X \rangle \langle X | J_{\nu} | B \rangle$$

$$B \qquad \int_{t_{\rm src}} J^{\dagger}_{\mu} \qquad \int_{t_{2}} J^{\dagger}_{\nu} \qquad \int_{t_{2}} d^{3}x \frac{e^{i\mathbf{q}\cdot\mathbf{x}}}{2M_{B}} \langle B | J^{\dagger}_{\mu}(\mathbf{x},t) J_{\nu}(\mathbf{0},0) | B \rangle \sim \int_{0}^{\infty} d\omega W_{\mu\nu} e^{-t\omega}$$
smearing kernel $f(\omega) = \sum_{n} a_{n}e^{-na\omega}$

The necessary smearing is provided by phase space integration over the hadronic energy, which is cut by a θ function: sigmoid $1/(1 + e^{x/\sigma})$ can be used to replace kinematic $\theta(x)$. Larger number of polynomials needed for small σ The kernel must be reconstructed by taking $\sigma \to 0$



Two methods based on Chebyshev polynomials and Backus-Gilbert. Important:

 $\lim_{\sigma \to 0} \lim_{V \to \infty} \lim_{a \to 0} X_{\sigma}$

LATTICE VS OPE



m_b^{kin} (JLQCD)	2.70 ± 0.04
$\overline{m}_c(2 \text{ GeV}) \text{ (JLQCD)}$	1.10 ± 0.02
m_b^{kin} (ETMC)	2.39 ± 0.08
$\overline{m}_c(2 \text{ GeV}) \text{ (ETMC)}$	1.19 ± 0.04
μ_π^2	0.57 ± 0.15
$ ho_D^3$	0.22 ± 0.06
$\mu_G^2(m_b)$	0.37 ± 0.10
$ ho_{LS}^3$	-0.13 ± 0.10
$\alpha_s^{(4)}(2 \text{ GeV})$	0.301 ± 0.006

OPE inputs from fits to exp data (physical m_b), HQE of meson masses on lattice 1704.06105, J.Phys.Conf.Ser. 1137 (2019) 1,012005

We include $O(1/m_b^3)$ and $O(\alpha_s)$ terms Hard scale $\sqrt{m_c^2 + \mathbf{q}^2} \sim 1 - 1.5 \,\text{GeV}$ We do not expect OPE to work at high $|\mathbf{q}|$

Twisted boundary conditions allow for any value of \vec{q}^2 Smaller statistical uncertainties

MOMENTS

PG, Hashimoto, Maechler, Panero, Sanfilippo, Simula, Smecca, Tantalo, 2203.11762



smaller errors, cleaner comparison with OPE, individual channels AA, VV, parallel and perpendicular polarization, could help extracting OPE parameters

$D_{(s)}$ inclusive semileptonic decays



slow convergence (?) of HQE in charm decays

KERNEL RECONSTRUCTION





preliminary by ETMC

Backus-Gilbert

CONTINUUM AND INFINITE VOLUME LIMITS

$\sigma \rightarrow 0$ EXTRAPOLATION

preliminary by ETMC

∮ ∮

1250

1500

1750

2000

SIGMOID

ERF

NB we expect a quadratic dependence on σ

D_s inclusive semileptonic decays

Connected diagrams only

Weak Annihilation (disconnected diagrams) in progress

₽ ₽ fg = csfg = cd0.50.4 $\frac{|V_{fg}|^2}{m_{D_s}^3} \frac{d\Gamma_{fg}}{dq^2}}{\frac{d\Gamma_{fg}}{dq^2}}$ preliminary 0.10.0 0.20.30.40.00.10.50.60.70.8 $\mathbf{q}^2 \ [\text{GeV}]^2$

~3.5% uncertainty on total s.l. width

 $\mathcal{B}(D_s^+ \to X e^+ \nu_e) = (6.30 \pm 0.13 \text{ (stat.)} \pm 0.10 \text{ (syst.)})\%.$

BES-III

ETMC

Moments of the E_e distribution coming soon...

SUMMARY AND OUTLOOK

- Inclusive $b \rightarrow c$ appears OK: q^2 moments consistent with leptonic and hadronic ones, perturbation theory generally OK; higher powers look small. But I doubt one can go below 1% on V_{cb}
- Calculations of inclusive semileptonic meson decays on the lattice have started. Precision to be seen, but you can count they will soon contribute.
- Preliminary ETMC results for D_s inclusive semileptonic decays encouraging: they provide a validation of the method on experimental data. Next: move to b physics, including $b \rightarrow u$

$O(\alpha_s^2 \beta_0)$ corrections to q^2 moments

Finauri, PG 2310.20324

sizeable for 2nd and 3rd moments Belle and Belle II moments differ by ~ 2σ New $O(\alpha_s^2)$ calculation Fael and Herren 2403.03976

EXCLUSIVE DECAYS

There are I(2) and 3(4) FFs for D and D^{*} for light (heavy) leptons, for instance $\langle D(k)|\bar{c}\gamma^{\mu}b|\bar{B}(p)\rangle = \left[(p+k)^{\mu} - \frac{M_B^2 - M_D^2}{q^2}q^{\mu}\right]f_+^{B\to D}(q^2) + \frac{M_B^2 - M_D^2}{q^2}q^{\mu}f_0^{B\to D}(q^2)$

Information on FFs from LQCD (at high q^2), LCSR (at low q^2), HQE, exp, extrapolation, unitarity constraints, ...

A *model independent parametrization* is very useful. In particular BGL (Boyd, Grinstein, Lebed)

INCLUSIVE $|V_{ub}|$

Important Belle measurement 2102.00020

In my opinion, the cleanest measurement is the most inclusive one with $M_X < 1.7 \text{GeV}, E_{\ell} > 1 \text{GeV}$:

$|V_{ub}| = (3.97 \pm 0.08 \pm 0.16 \pm 0.16) \, 10^{-3}$

Framework	$ V_{ub} [10^{-3}]$
BLNP	$4.28 \pm 0.13^{+0.20}_{-0.21}$
DGE	$3.93 \pm 0.10^{+0.09}_{-0.10}$
GGOU	$4.19 \pm 0.12^{+0.11}_{-0.12}$
ADFR	$3.92\pm0.1^{+0.18}_{-0.12}$
BLL $(m_X/q^2 \text{ only})$	$4.62 \pm 0.20 \pm 0.29$

Not all approaches at the same level Some discrepancy hidden in the average Recent calculation of the $O(\alpha_s/m_b^2)$ effects in $B \rightarrow X_u \ell \nu$, Capdevila, Nandi, PG

 $\left| V_{ub}^{\text{excl.}} \right| / \left| V_{ub}^{\text{incl.}} \right| = 0.97 \pm 0.12$ 2303.17309

Look forward to validating approaches on Belle II data (SIMBA, NNVUB)!

NEW PHYSICS FOR THE V_{cb} PUZZLE?

Differential distributions constrain NP strongly, SMEFT interpretation incompatible with LEP data: Crivellin, Pokorski, Jung, Straub...

HEAVY QUARK MASSES AND THEORETICAL CORRELATIONS C. Schwanda, PG 2013

<(M²-<M²>)²>

Tests of Lepton Flavor Universality

KKV, Rahimi [2207.03432]; Ligeti, Tackmann [1406.7013];Bernlocner, Sevilla, Robinson, Wormser [2101.08326]

$${\sf R}_{e/\mu}(X)\equiv rac{\Gamma(B o X_c ear
u_e)}{\Gamma(B o X_c \muar
u_\mu)}$$

- Belle II result: $R_{e/\mu}(X) = 1.033 \pm 0.022$ prl131 [2023] [2301.08266]
- In agreement with new SM predictions: 1.006 \pm 0.001 at 1.2 σ
- New! Belle II result: $R_{\tau/\ell}(X) = 0.228 \pm 0.016 \pm 0.036$ @EPS 2311.07248
- In agreement with SM prediction:

$$R_{ au/\ell}(X) = 0.221 \pm 0.004$$

