INCLUSIVE DECAYS:
SYNERGIES BETWEEN LATTICE AND CONTINUUM?

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Discussion session - Lattice@CERN 2024 - July 11th 2024

MARIE CURIE ACTIONS
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LATTICE VS HQE

- Lattice can calculate $\Gamma(B \to X_c l \bar{\nu}_l)$ end extract $|V_{cb}|^{\text{inc}}$.

- But before reaching 1-2% precision, how do we use lattice to improve the precision in the predictions obtained with the Heavy Quark Expansion (HQE)?

- The extraction of HQE parameters $\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3$ are relevant not only for $\Gamma(B \to X_c l \bar{\nu}_l)$ and $|V_{cb}|$ but also for $|V_{ub}|$, $\tau_B, \Gamma(B \to X_s \gamma), \Gamma(B \to X_s l \bar{l})$ etc.

- Which quantities should be calculated on the lattice to improve the precision in $\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \ldots$?

POSSIBLE STRATEGIES

- In the OPE we usually consider moments of the differential distributions
- Perform global fits of semileptonic decays on lattice data

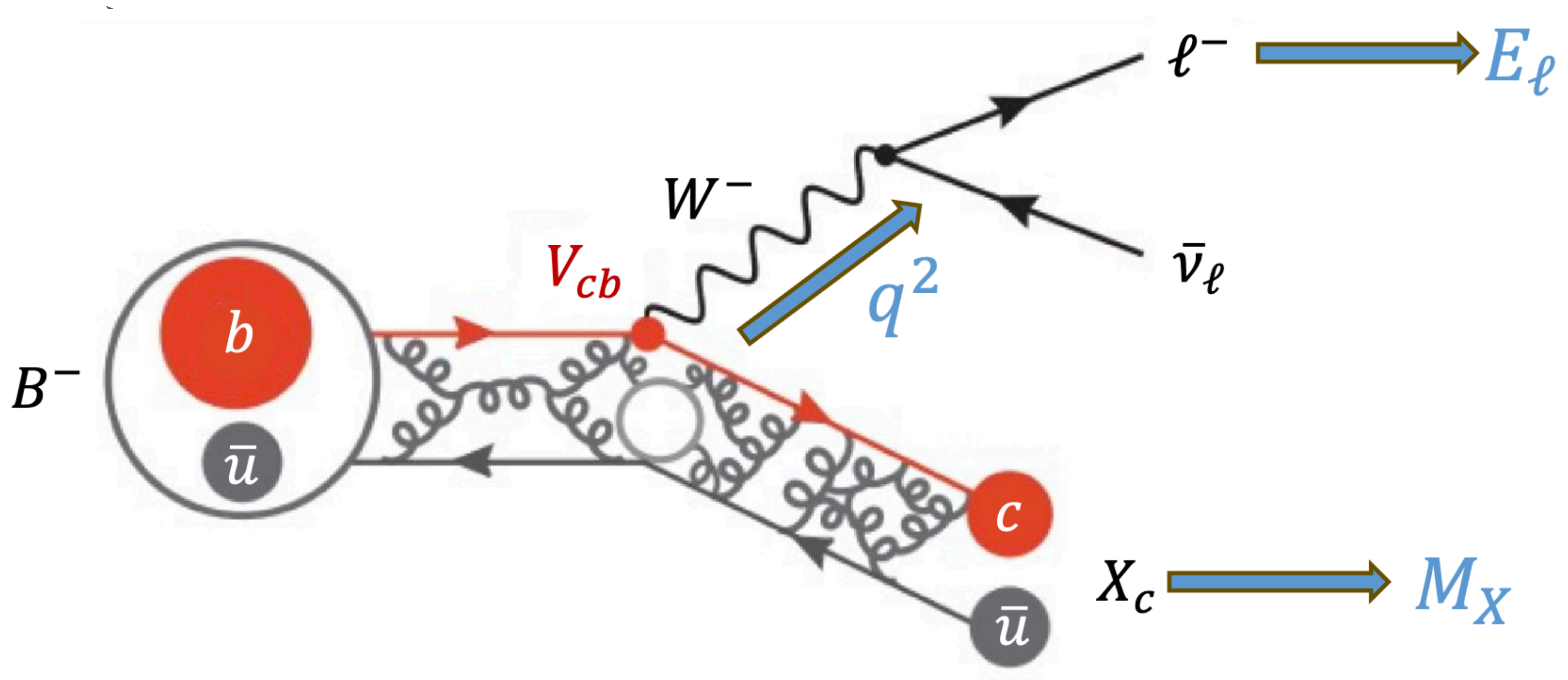
$$\langle (O)^n \rangle_{\text{cut}} = \int_{\text{cut}} (O)^n \frac{\mathrm{d}\Gamma}{\mathrm{d}O}\,\mathrm{d}O \Bigg/ \int_{\text{cut}} \frac{\mathrm{d}\Gamma}{\mathrm{d}O}\,\mathrm{d}O$$

where we restrict the integration: $E_l > E_{\text{cut}}$ or $q^2 > q^2_{\text{cut}}$

$\langle E_l^n \rangle_{\text{cut}} \quad \langle (q^2)^n \rangle_{\text{cut}} \quad \langle (M_X^2)^n \rangle_{\text{cut}}$

↓

$\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \ldots$



- $O = E_\ell$: energy of the charged lepton
- $O = M_X^2$: hadronic invariant mass
- $O = q^2$: leptonic invariant mass

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$\langle E_l^n \rangle_{\rm cut} \quad \langle (q^2)^n \rangle_{\rm cut} \quad \langle (M_X^2)^n \rangle_{\rm cut}$

$\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \ldots$

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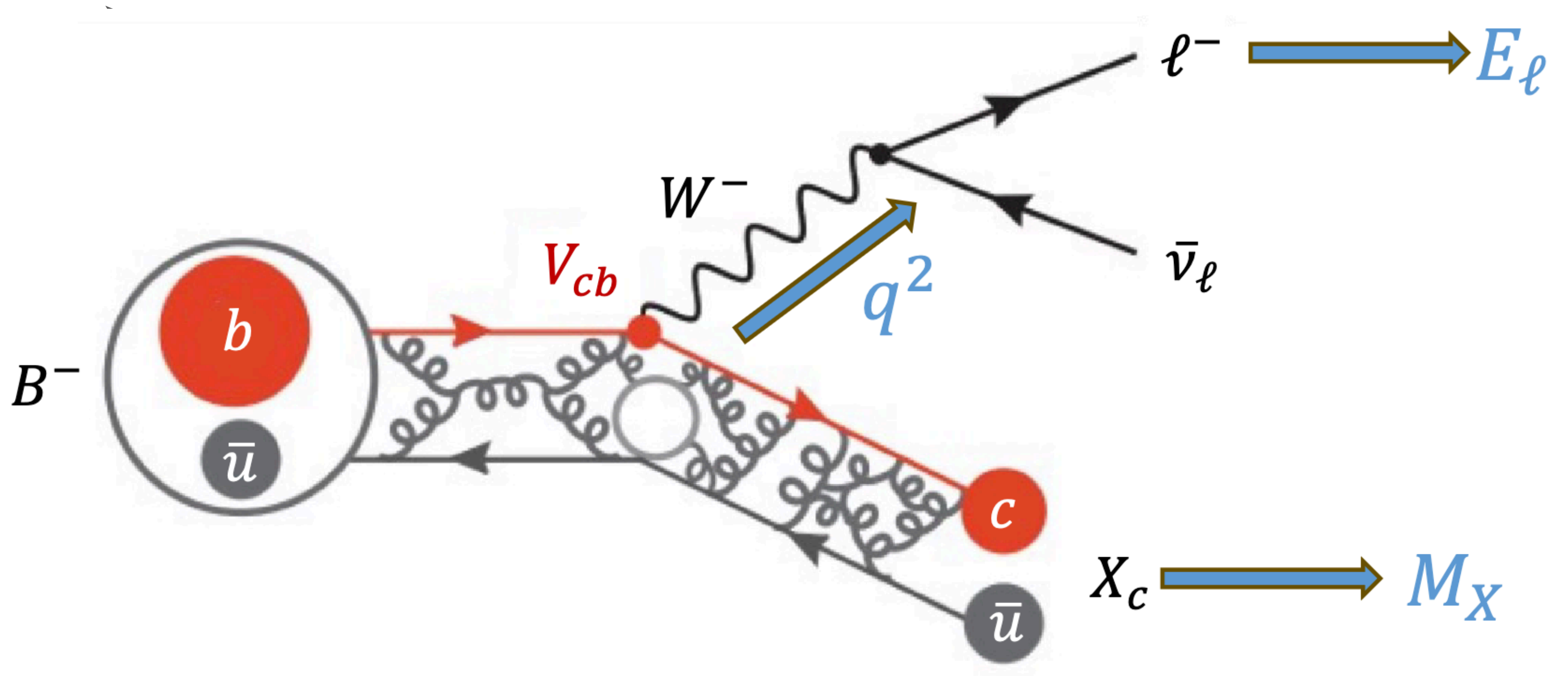

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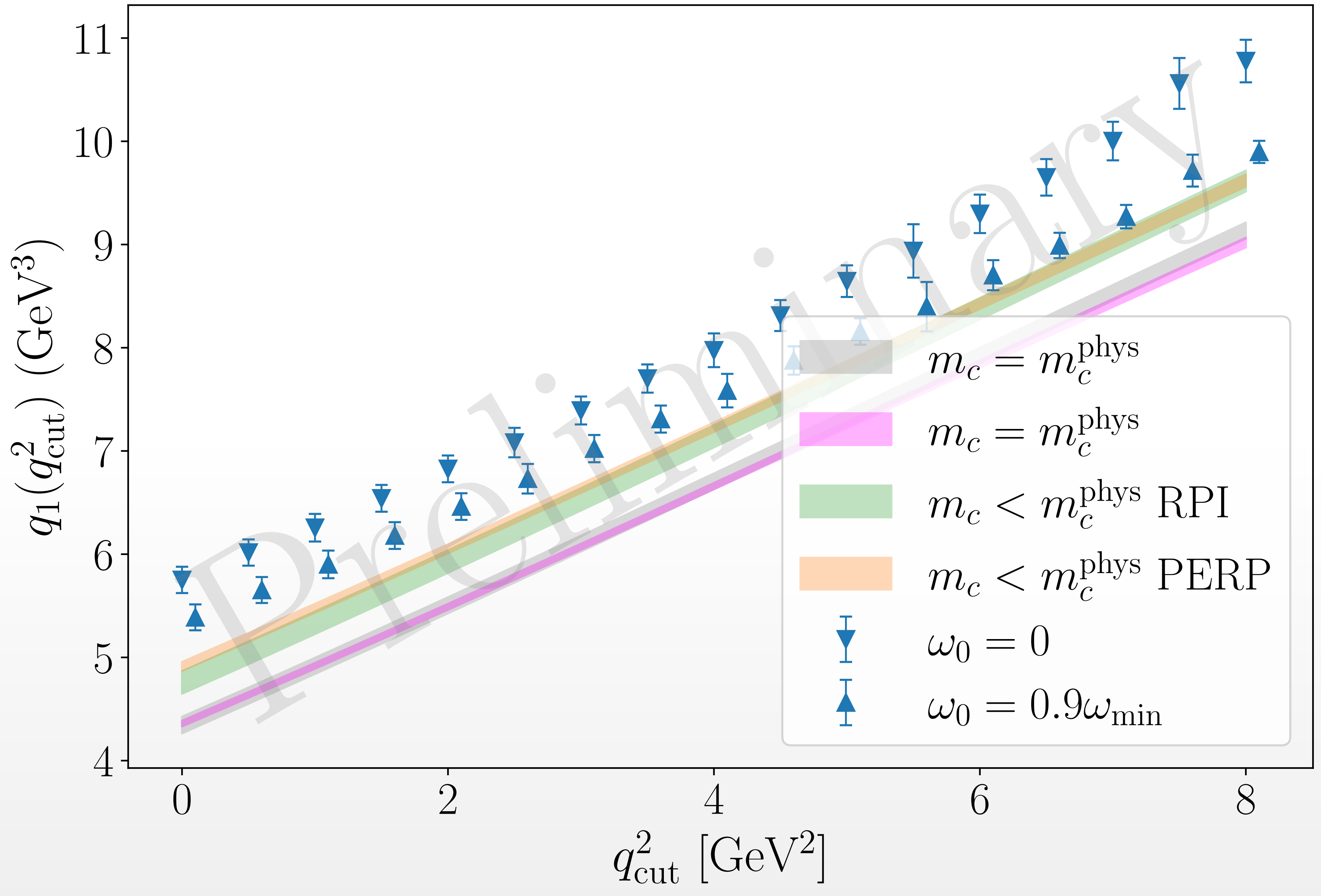
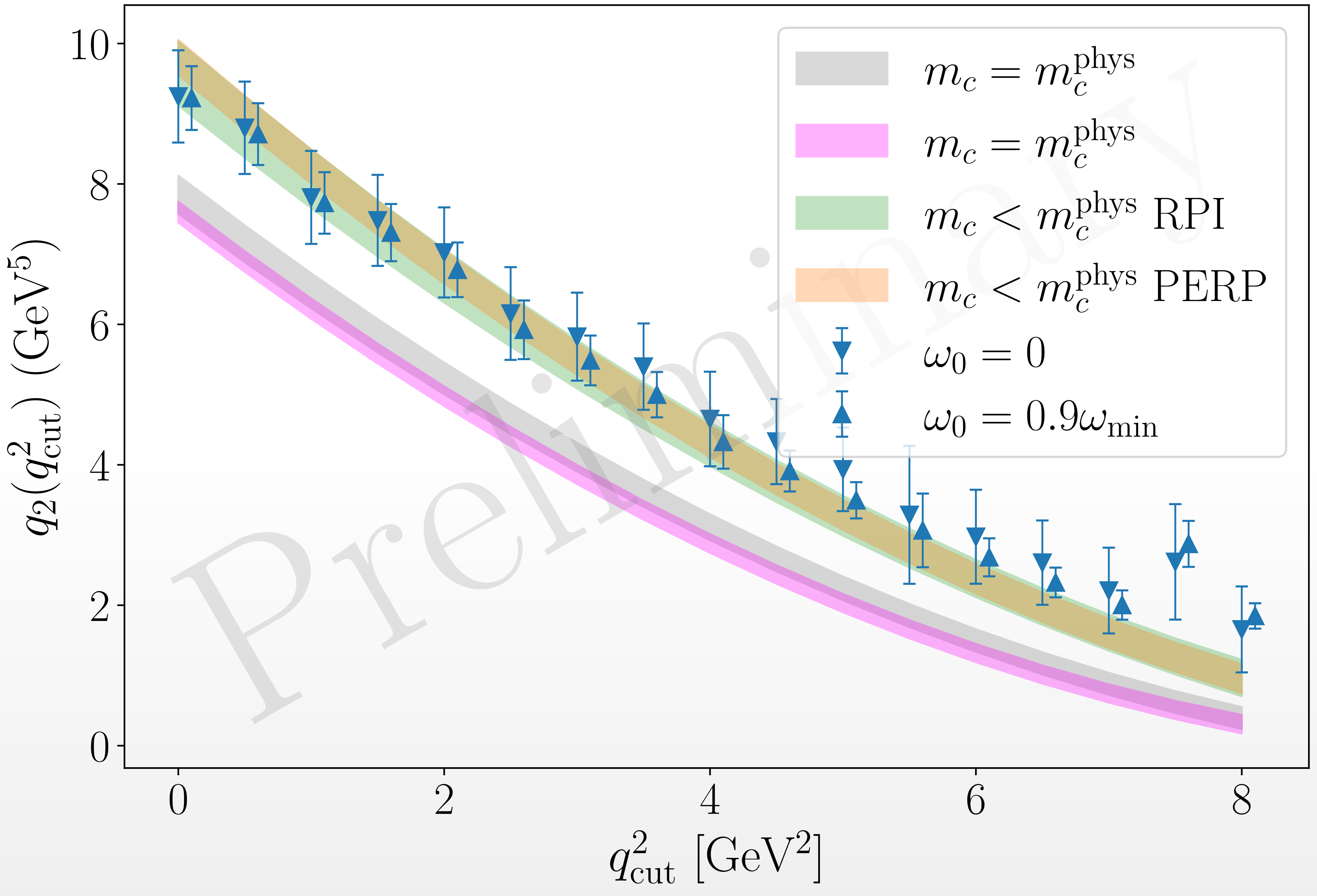
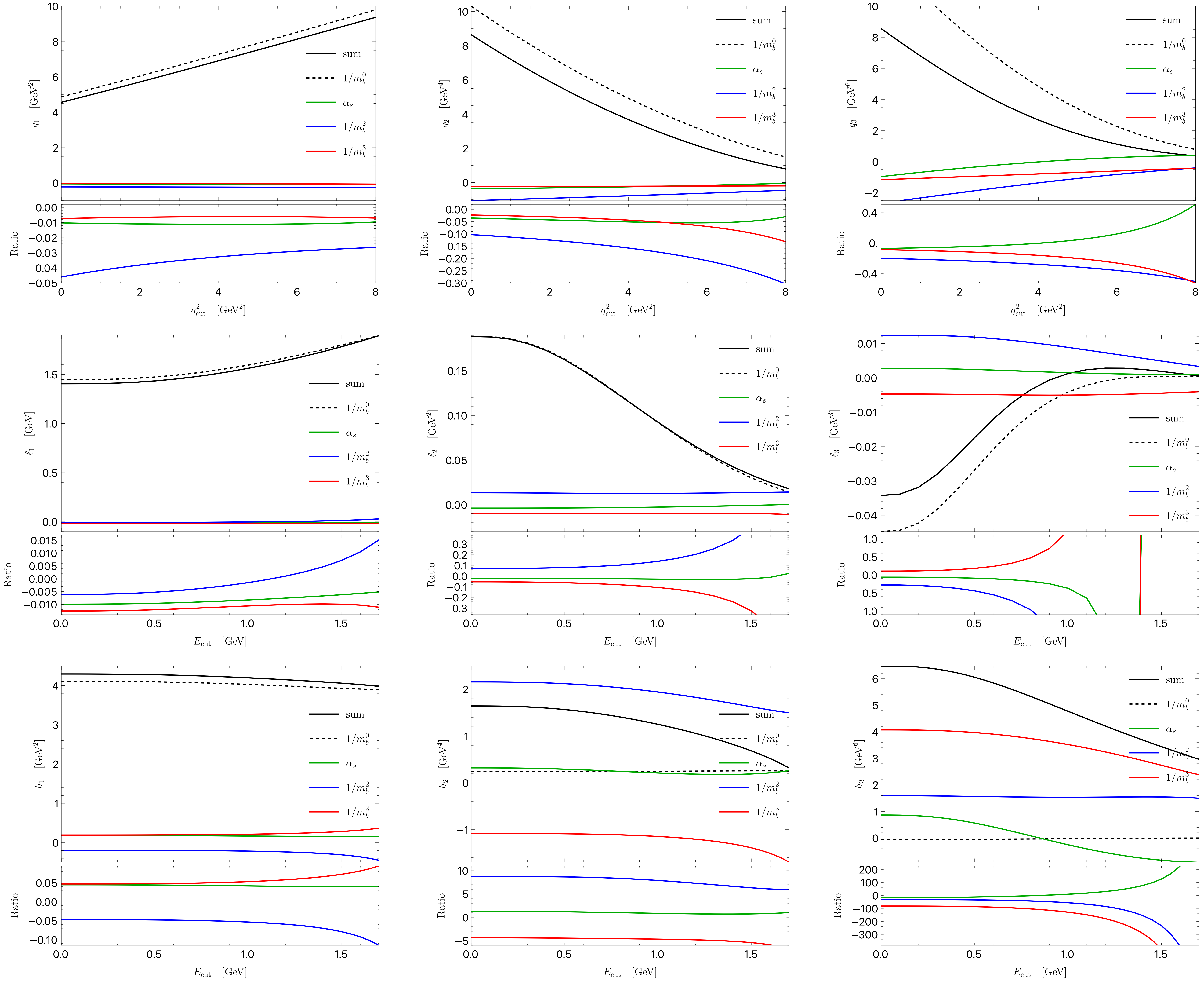
Centralized moments: lattice “VS” continuum

Preliminary results - feasibility study.



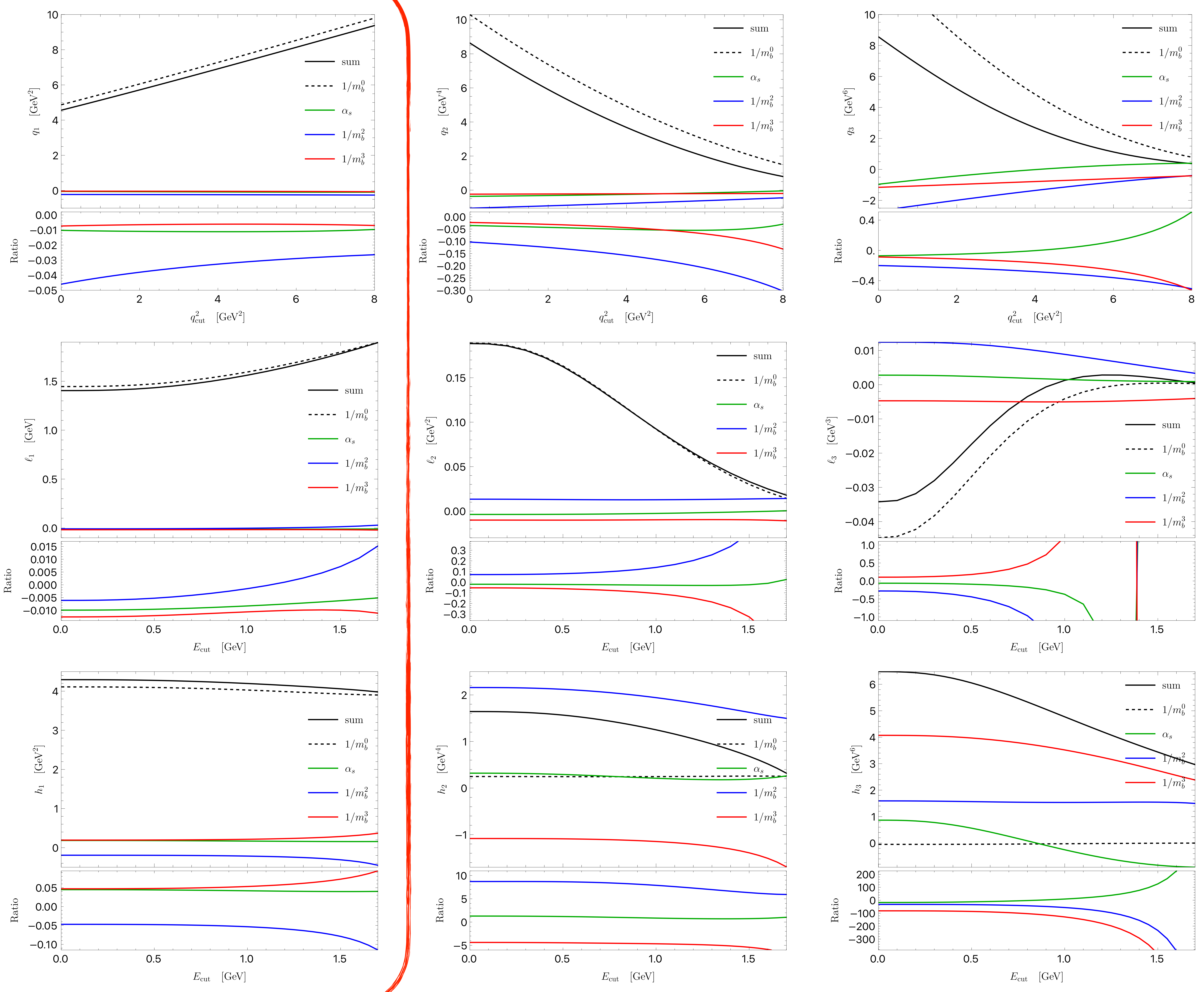


Lattice data (after extrapolation to the physical world) can be used to extract HQET parameters used in the OPE expansion



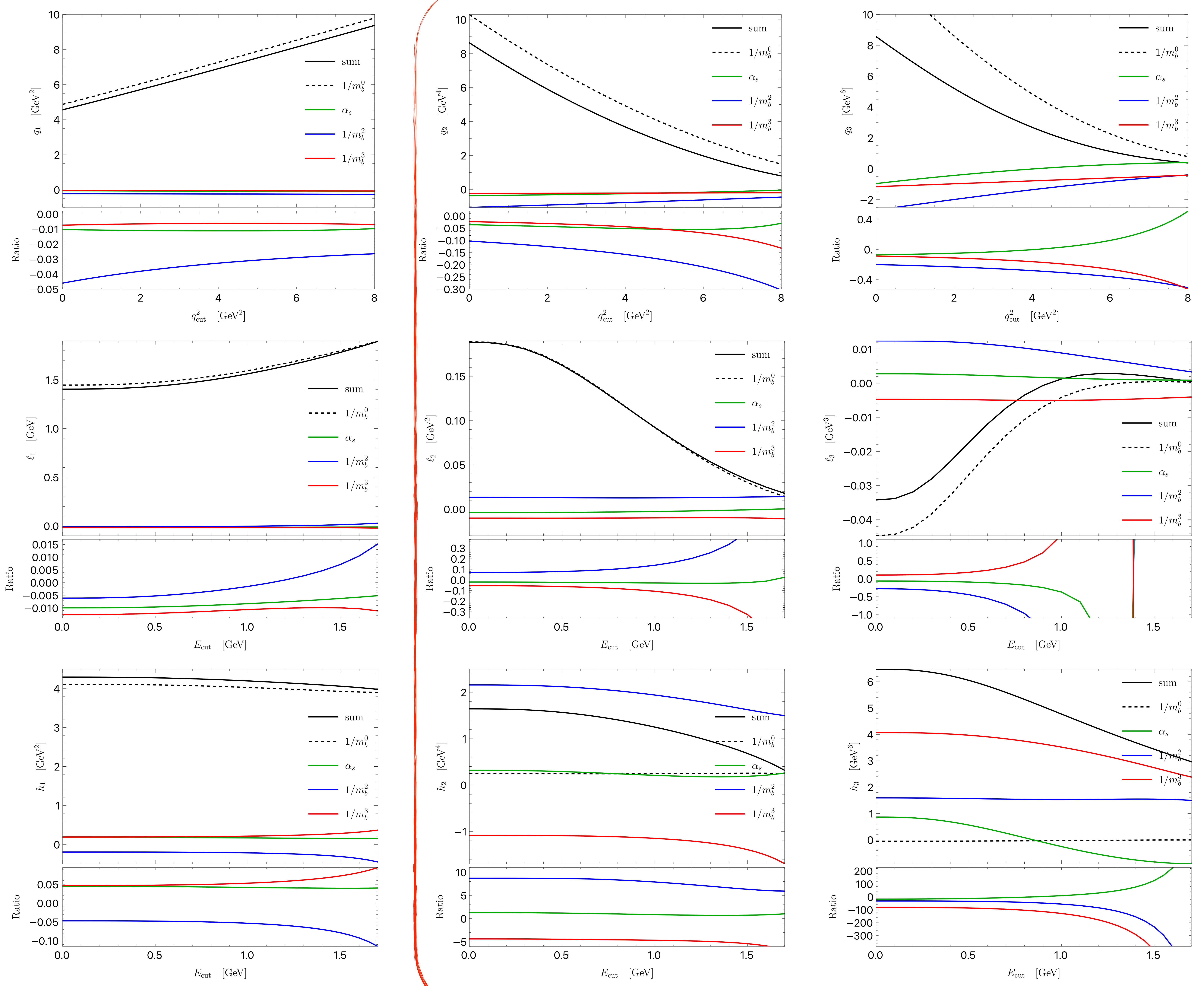
- Different moments have different sensitivities to the power corrections

MF, Prim, Vos, Eur.Phys.J.ST 233 (2024) 2, 325



- First moments are rather insensitive on power corrections

MF, Prim, Vos, Eur.Phys.J.ST 233 (2024) 2, 325

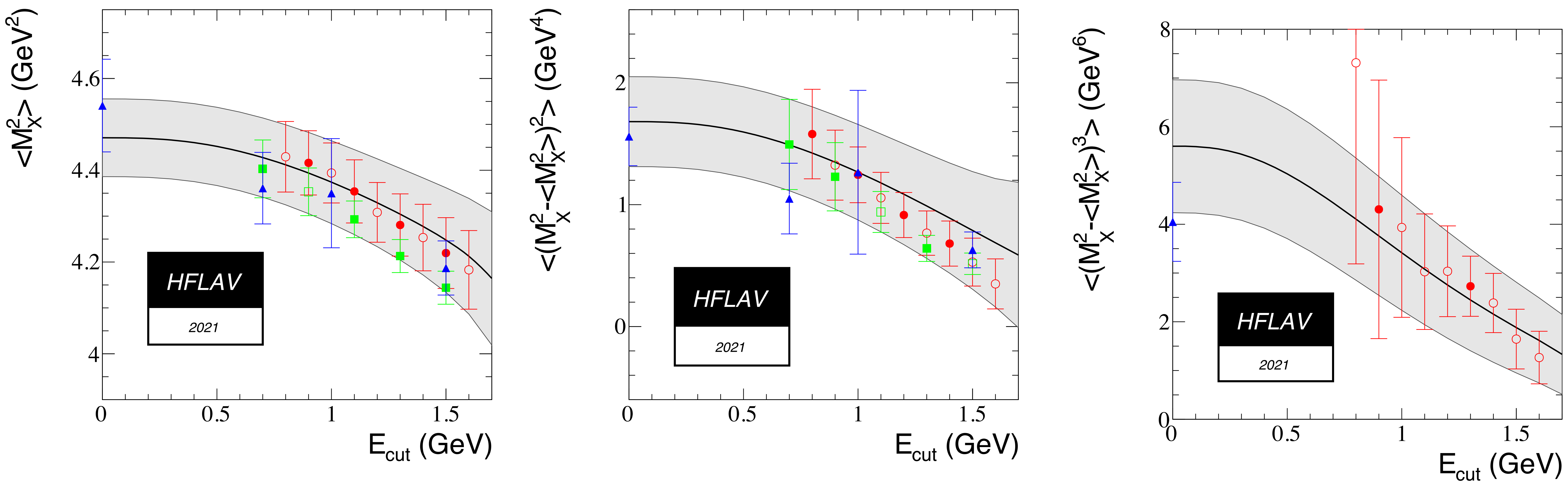


- ➤ First moments are rather insensitive on power corrections
- ➤ Higher moments are sensitive to power corrections

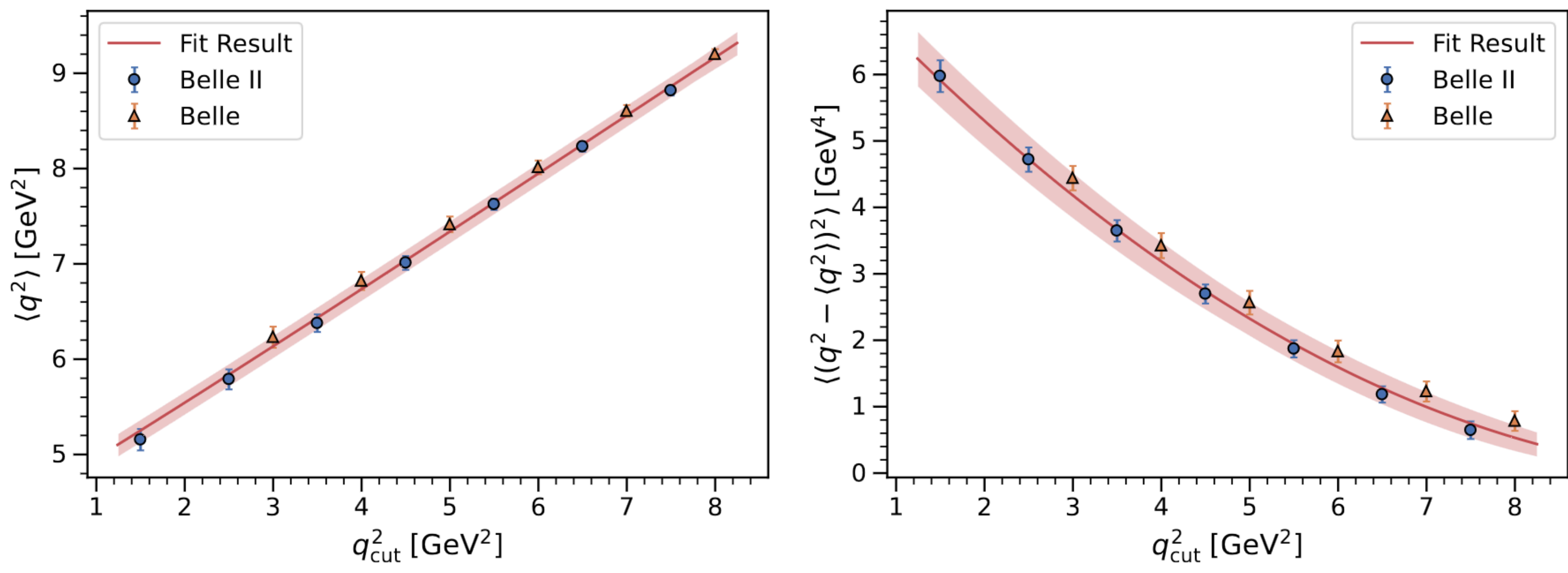
MF, Prim, Vos, Eur.Phys.J.ST 233 (2024) 2, 325

HOW TO IMPROVE

- Some experimental data are very precise, some are not



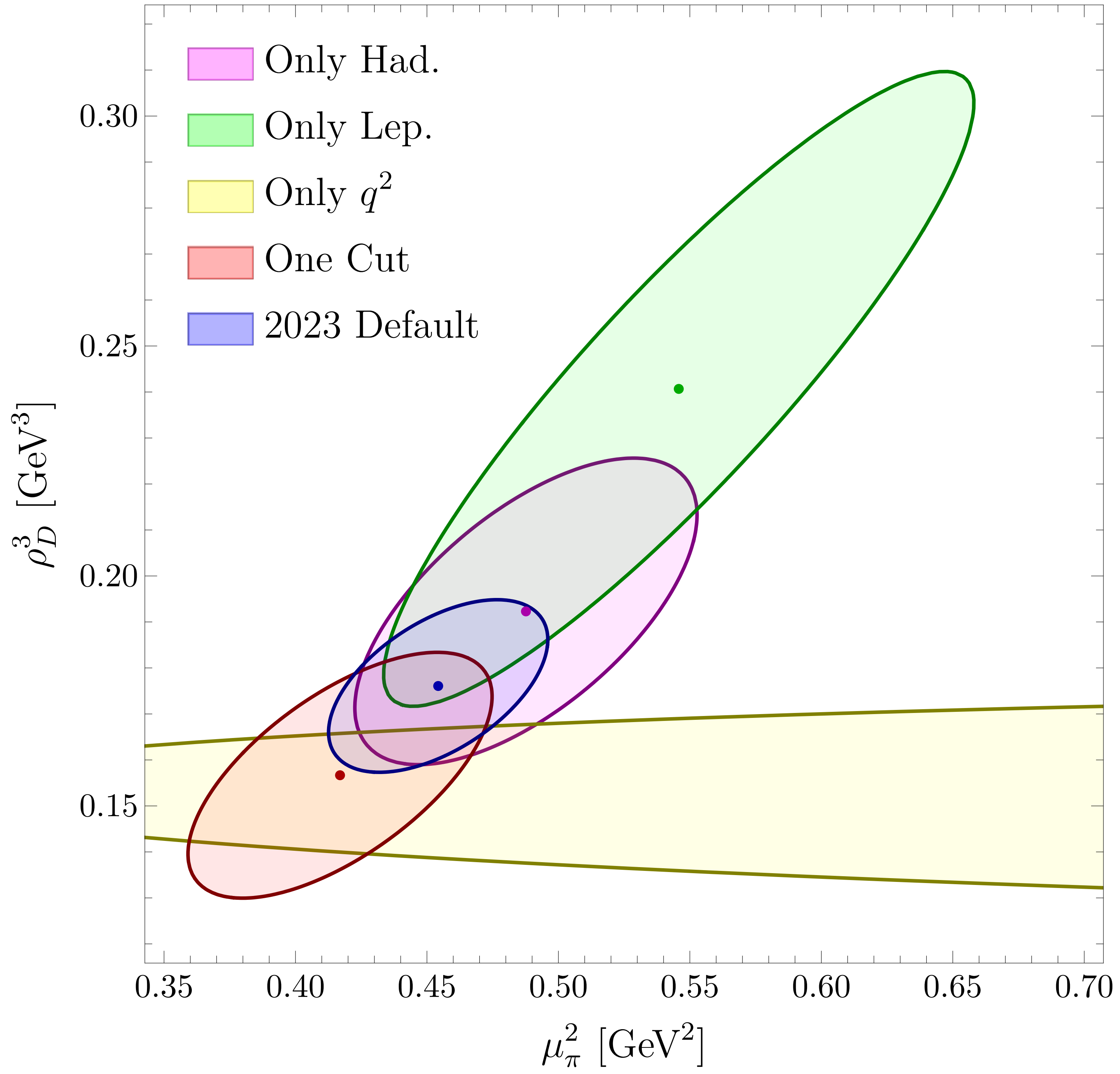
based on Gambino, Schwanda Phys.Rev.D 89 (2014) 1, 014022



Bernlochner, et al, *JHEP* 10 (2022) 068

- Can lattice substantially improve the precision?

COMPLEMENTARITY BETWEEN OBSERVABLES



Gambino, Finauri, JHEP 02 (2024) 206

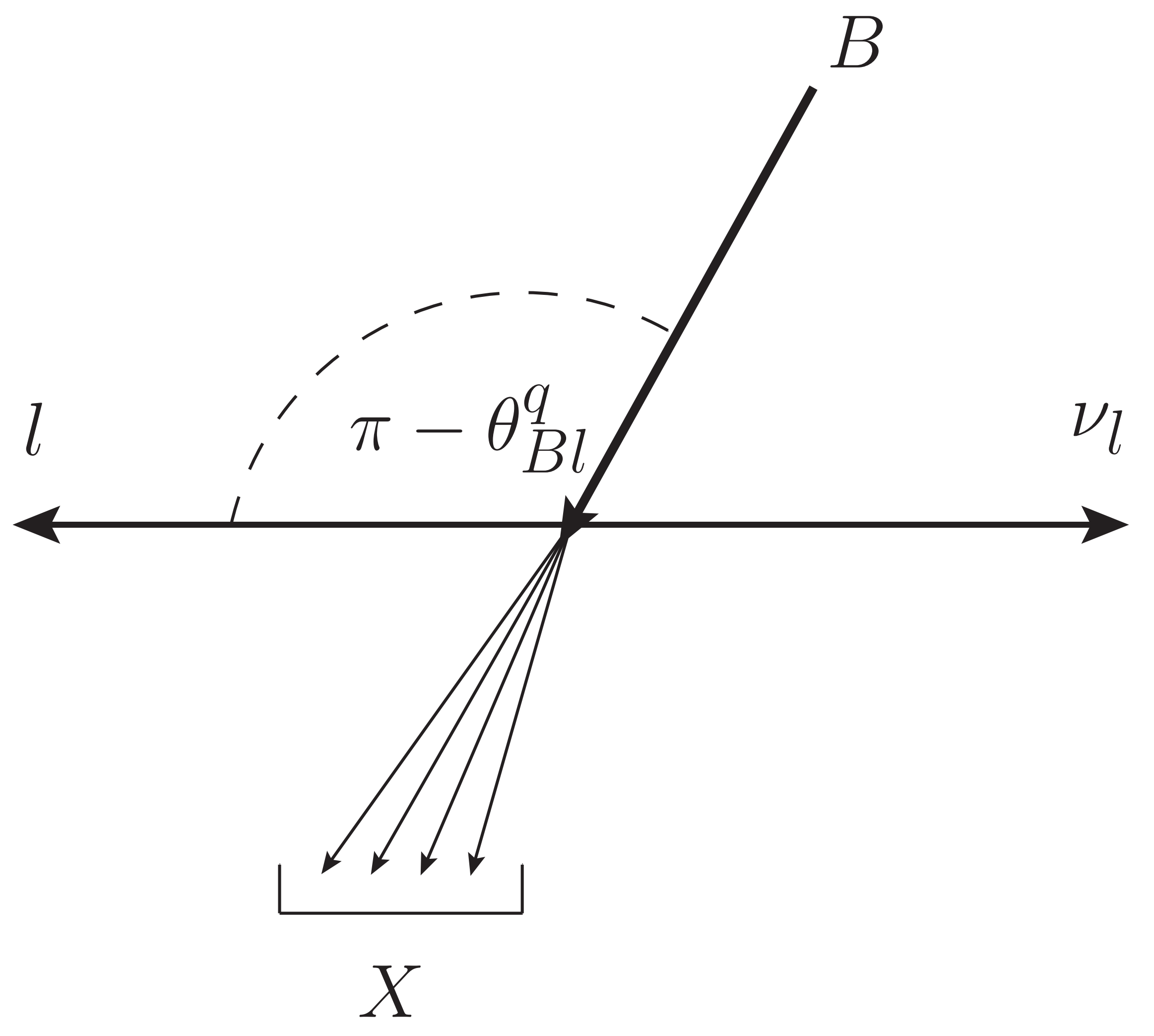
- Moments exhibit "flat directions" in the space of HQE parameters.
- Use "orthogonal observables" to break the degeneracy and improve the sensitivity of the fits on the HQE
- It can be beneficial to keep separated the V and A currents for all moments.
- Consider also scalar and pseudo-scalar currents.
- Expressions available up to $1/m_b^3$ and $O(\alpha_s)$ at $1/m_b^0$

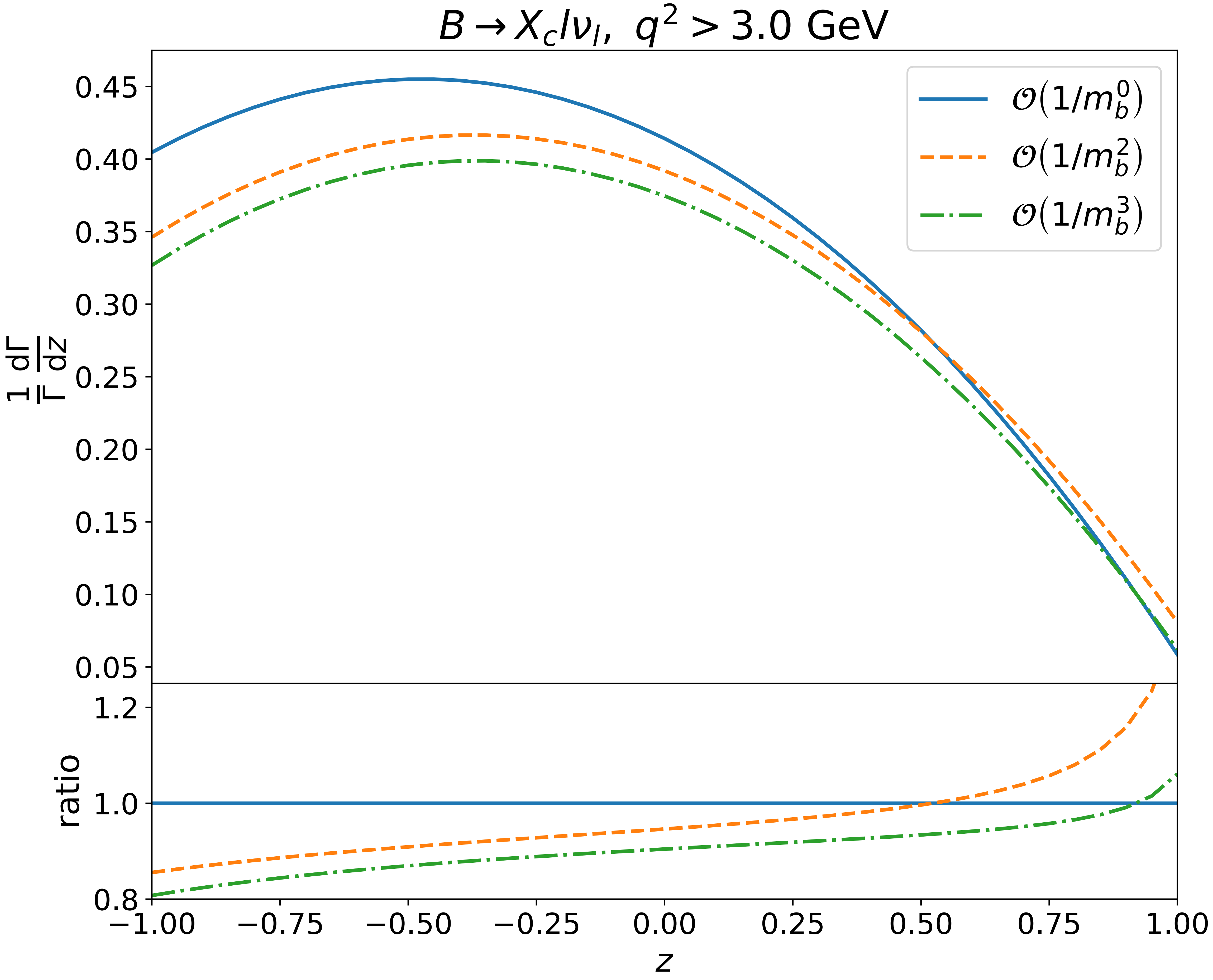
MF, Rahimi, Vos, 2208.04282 [hep-ph]

FORWARD-BACKWARD ASYMMETRIES

$$A_{FB} = \frac{\int_{-1}^{0} \frac{d\Gamma}{dz} - \int_{-1}^{0} \frac{d\Gamma}{dz}}{\int_{-1}^{0} \frac{d\Gamma}{dz} + \int_{-1}^{0} \frac{d\Gamma}{dz}}$$

$$z = \cos\theta = \frac{v \cdot p_\nu - v \cdot p_\ell}{\sqrt{(v \cdot q)^2 - q^2}}$$

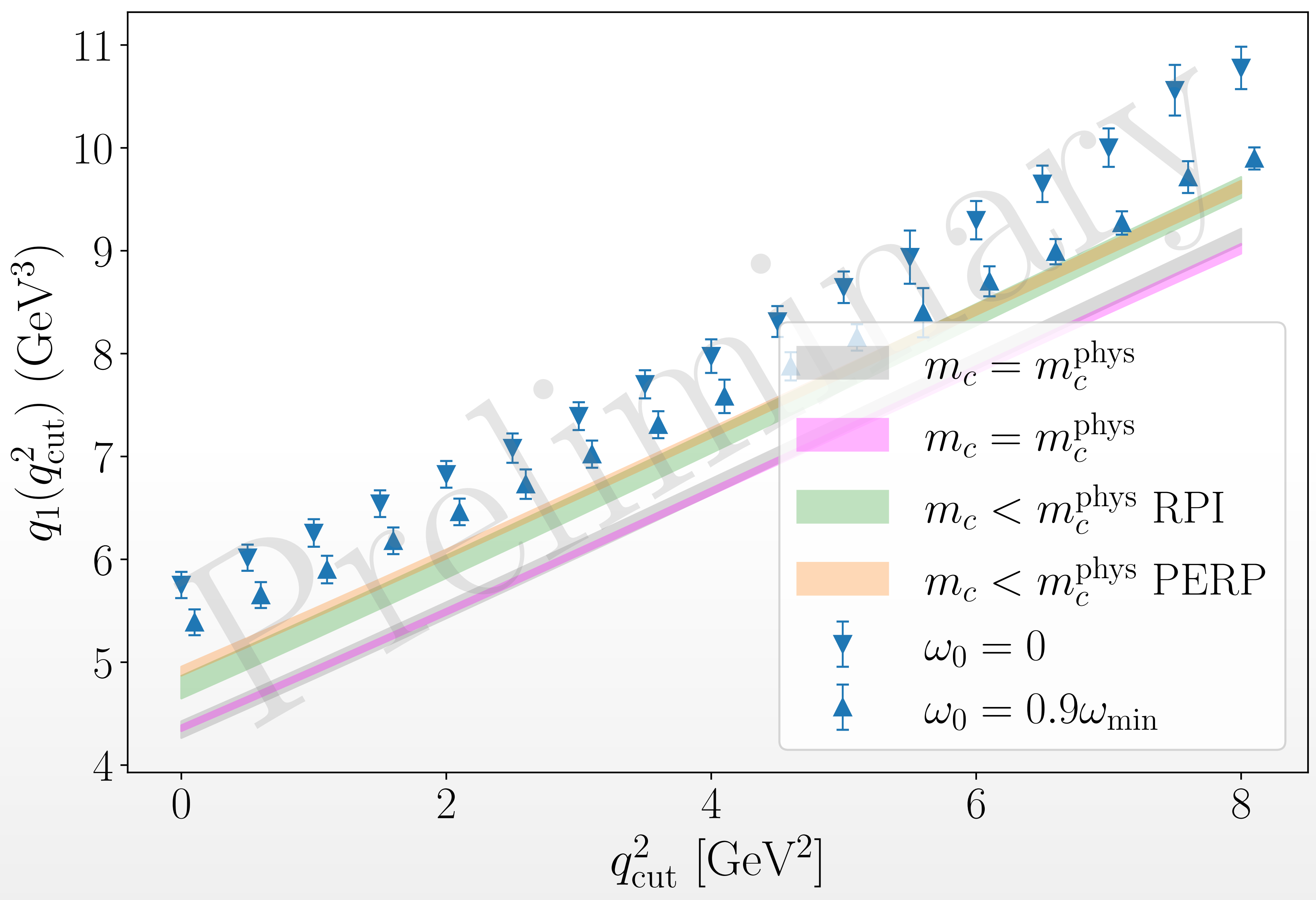


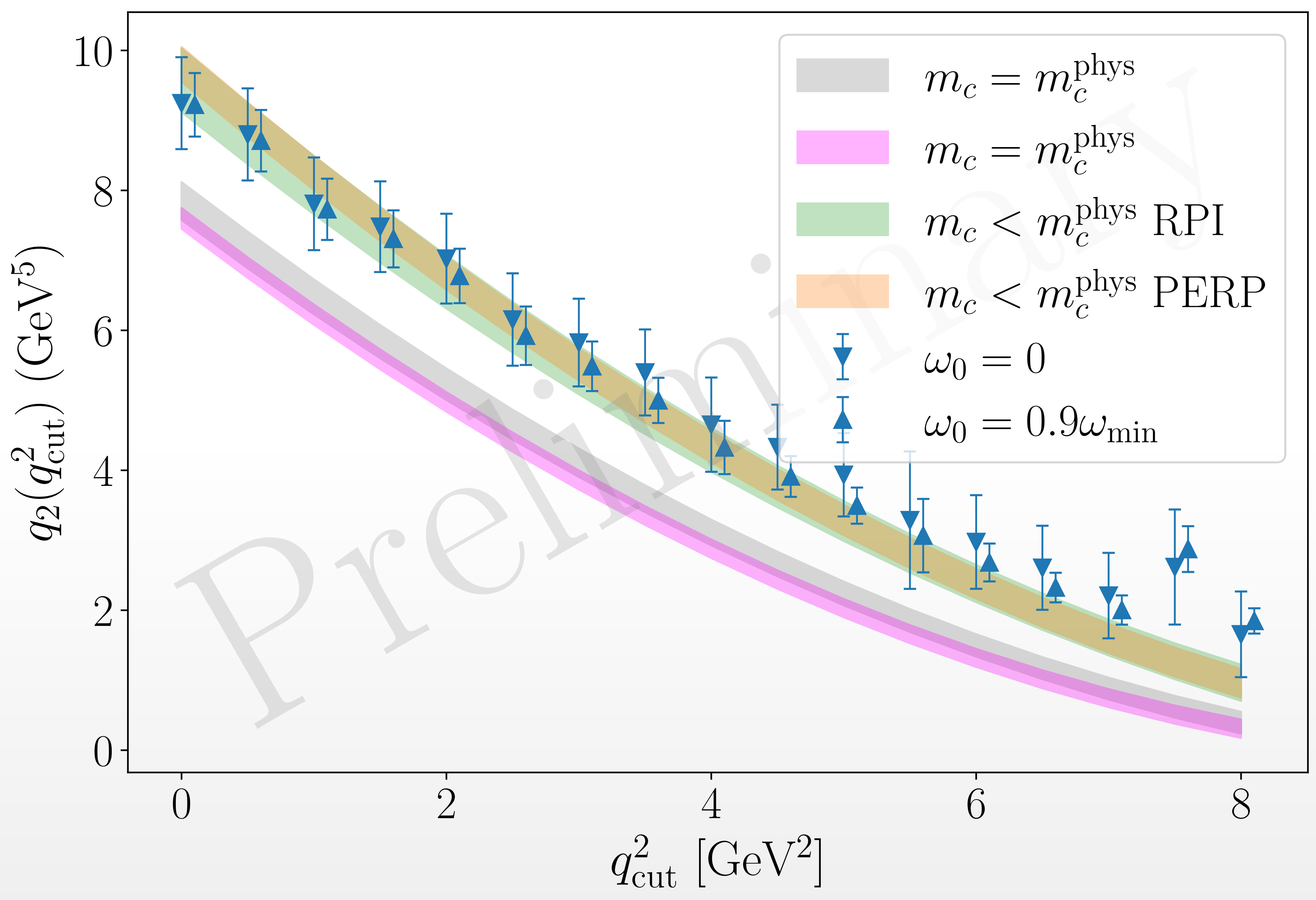


Turczyk, JHEP 04 (2016) 131
Herren, SciPost Phys. 14 (2023) 020

Centralized moments: lattice “VS” continuum

Preliminary results - feasibility study.





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BOTTOM AND CHARM MASSES

e.g. $\mu_\pi^2 = \frac{1}{2M_B}\langle B|\bar{b}_v(iD)^2 b_v|B\rangle$

- m_b must be physical to extract the HQE parameters
- Physical or unphysical charm mass?

$\mu_\pi^2, \mu_G^2, \ldots$ are independent on m_c
however they depend on M_B

$$\langle O^n\rangle_{\text{cut}} = (m_b)^{mn}\Bigg[X_0^{(O,n)} + \frac{\alpha_s}{\pi}X_1^{(O,n)} + \left(\frac{\alpha_s}{\pi}\right)^2 X_2^{(O,n)} + \frac{\mu_\pi^2}{m_b^2}\left(p_0^{(O,n)} + \frac{\alpha_s}{\pi}p_1^{(O,n)} + \ldots\right)$$

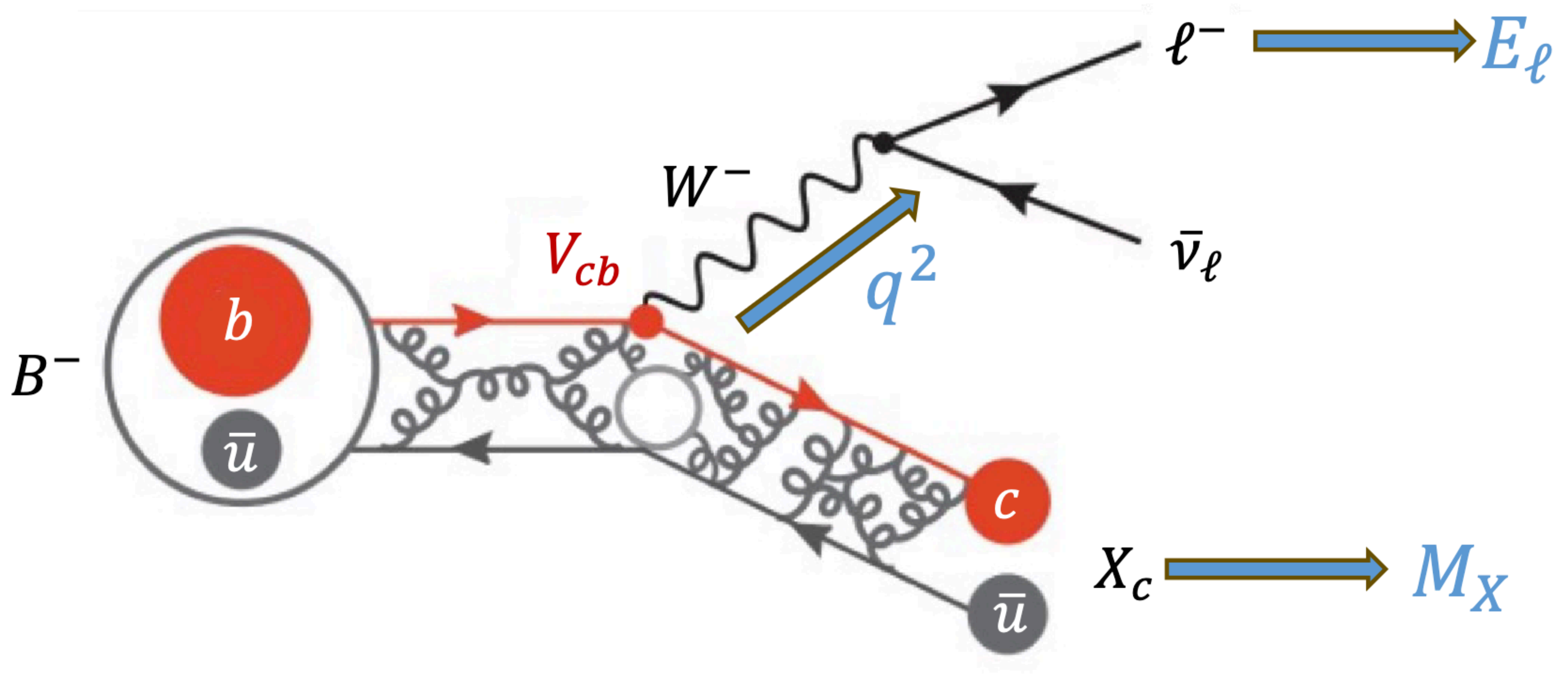
$$+\frac{\mu_G^2}{m_b^2}\left(g_0^{(O,n)} + \frac{\alpha_s}{\pi}g_1^{(O,n)} + \ldots\right) + \frac{\rho_D^3}{m_b^3}\left(d_0^{(O,n)} + \frac{\alpha_s}{\pi}d_1^{(O,n)} + \ldots\right) + \frac{\rho_{LS}}{m_b^2}\left(l_0^{(O,n)} + \frac{\alpha_s}{\pi}l_1^{(O,n)} + \ldots\right) + O\left(\frac{1}{m_b^4}\right)\Bigg]$$

- HQET estimate: $M_D = m_c + \bar{\Lambda} + \frac{\mu_\pi^2 - \frac{d_H}{3}\mu_G^2}{2m_c} + O\left(\frac{1}{m_c^2}\right)$

- With $m_c < m_c^{\text{phys}}$, lattice should provide reference value for $\overline{m}_c$!

SMALL VELOCITY SUM RULES

Bigi, Shifman, Uraltsev, Vainshtein, *Phys.Rev.D* 56 (1997) 4017



Excitation energy of the X_c system

$$\omega_X = M_B - q_0 - \sqrt{M_D^2 + \mathbf{q}^2}$$

Maximum energy of the dilepton system

$$q_0^{\max} = M_B - \sqrt{M_D^2 + \mathbf{q}^2}$$

$$I_n(\mathbf{q}^2, \mu) = \int_{q_0^{\max}-\mu}^{q_0^{\max}} dq_0\, \omega_X^n \frac{d^2\Gamma}{dq_0\, d\mathbf{q}^2}$$

$\mu \simeq 1\,\mathrm{GeV}$

SMALL VELOCITY SUM RULES

$$I_1(\mathbf{q}^2, \mu) = I_0(\mathbf{q}^2)\frac{\mathbf{q}^2}{2m_c^2}\overline{\Lambda}(\mu) + O(|\mathbf{q}|^3, \Lambda_{\mathrm{QCD}}^2)$$

$$I_2(\mathbf{q}^2, \mu) = I_0(\mathbf{q}^2)\frac{\mathbf{q}^2}{3m_c^2}\mu_\pi^2(\mu) + O(|\mathbf{q}|^3, \Lambda_{\mathrm{QCD}}^3)$$

$$I_3(\mathbf{q}^2, \mu) = I_0(\mathbf{q}^2)\frac{\mathbf{q}^2}{2m_c^2}\rho_D^3(\mu) + O(|\mathbf{q}|^3, \Lambda_{\mathrm{QCD}}^4)$$

- SV sum rules are independent on the current (S,V,A)
- Can lattice verify these relations?
- Can one extract μ_π^2, ρ_D^3 by studying the limit $|\mathbf{q}| \to 0$ and $m_Q \to \infty$

Note: the HQE parameters extracted here are in $m_Q \to \infty$ limit