

# INCLUSIVE DECAYS: SYNERGIES BETWEEN LATTICE AND CONTINUUM?

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Discussion session - Lattice@CERN 2024 - July 11th 2024



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# LATTICE VS HQE

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- Lattice can calculate  $\Gamma(B \rightarrow X_c l \bar{\nu}_l)$  and extract  $|V_{cb}|^{\text{inc}}$ .
- But before reaching 1-2% precision, **how do we use lattice** to improve the precision in the predictions obtained with the Heavy Quark Expansion (HQE)?
- The extraction of HQE parameters  $\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3$  are relevant not only for  $\Gamma(B \rightarrow X_c l \bar{\nu}_l)$  and  $|V_{cb}|$  but also for  $|V_{ub}|, \tau_B, \Gamma(B \rightarrow X_s \gamma), \Gamma(B \rightarrow X_s l \bar{l})$  etc.
- Which quantities should be **calculated on the lattice** to improve the precision in  $\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \dots$ ?

# POSSIBLE STRATEGIES

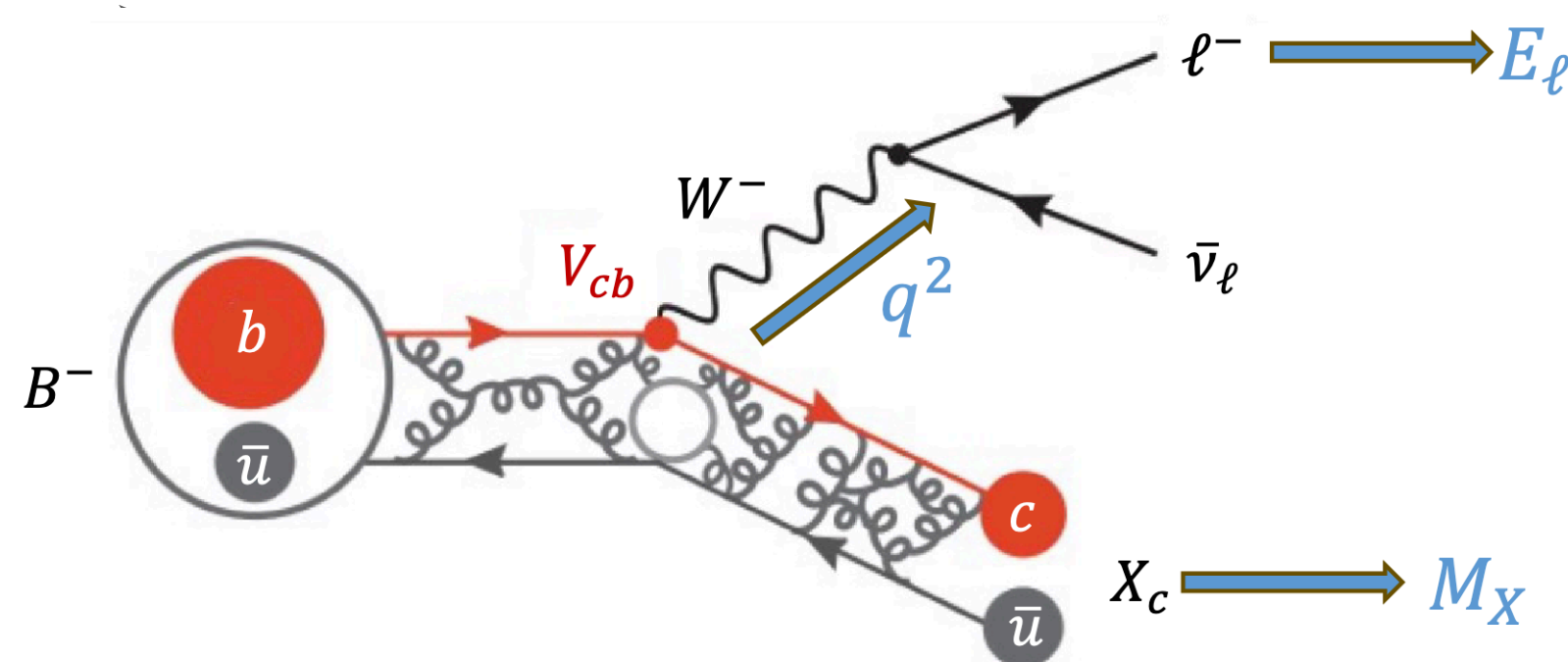
- In the OPE we usually consider moments of the differential distributions
- Perform global fits of semileptonic decays on lattice data

$$\langle (O)^n \rangle_{\text{cut}} = \int_{\text{cut}} (O)^n \frac{d\Gamma}{dO} dO \bigg/ \int_{\text{cut}} \frac{d\Gamma}{dO} dO$$

$$\langle E_\ell^n \rangle_{\text{cut}} \quad \langle (q^2)^n \rangle_{\text{cut}} \quad \langle (M_X^2)^n \rangle_{\text{cut}}$$

where we restrict the integration:  $E_\ell > E_{\text{cut}}$  or  $q^2 > q_{\text{cut}}^2$

$$\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \dots$$



- $O = E_\ell$  : energy of the charged lepton
- $O = M_X^2$  : hadronic invariant mass
- $O = q^2$  : leptonic invariant mass

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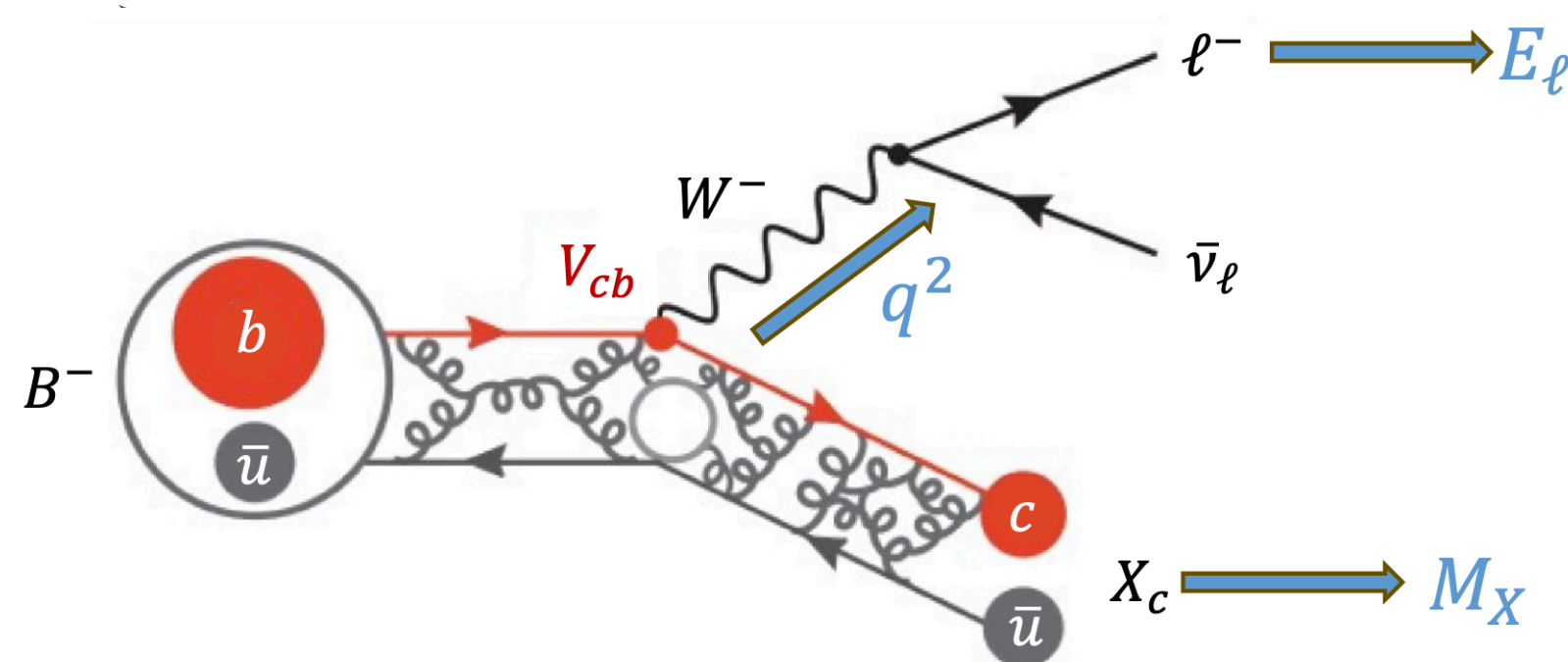
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$$\langle E_l^n \rangle_{\text{cut}} \quad \langle (q^2)^n \rangle_{\text{cut}} \quad \langle (M_X^2)^n \rangle_{\text{cut}}$$

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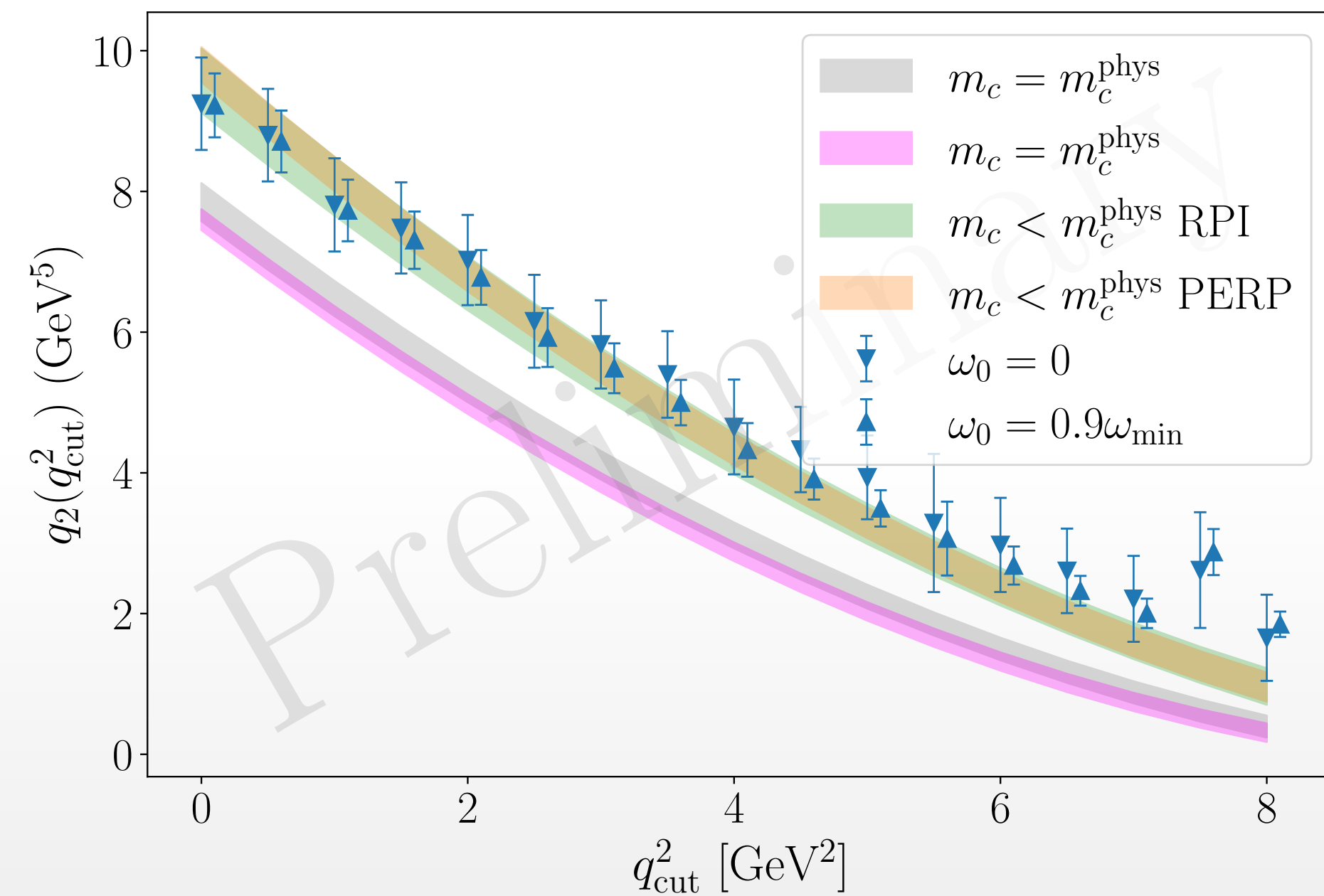
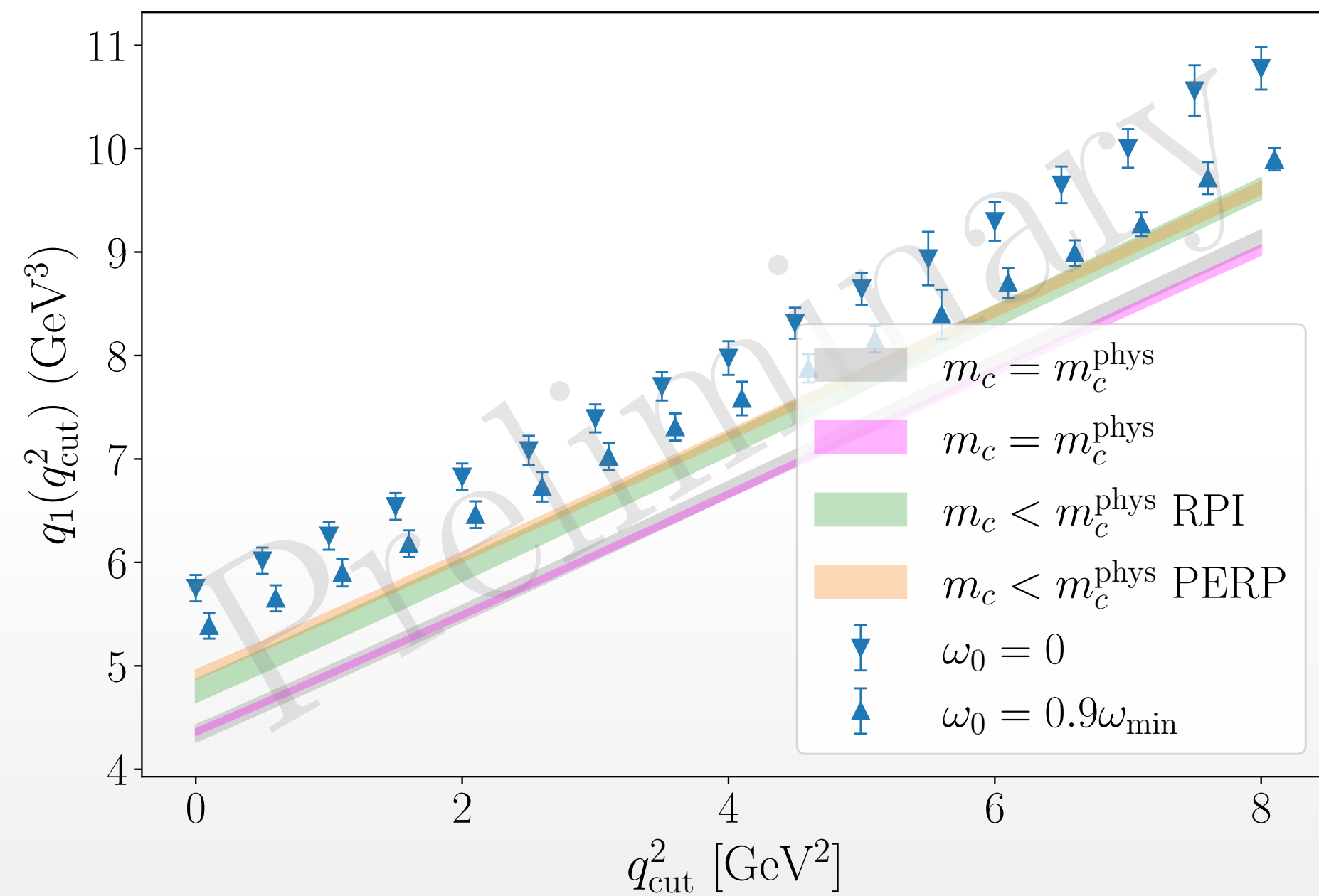
$$\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \dots$$



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# Centralized moments: lattice “VS” continuum

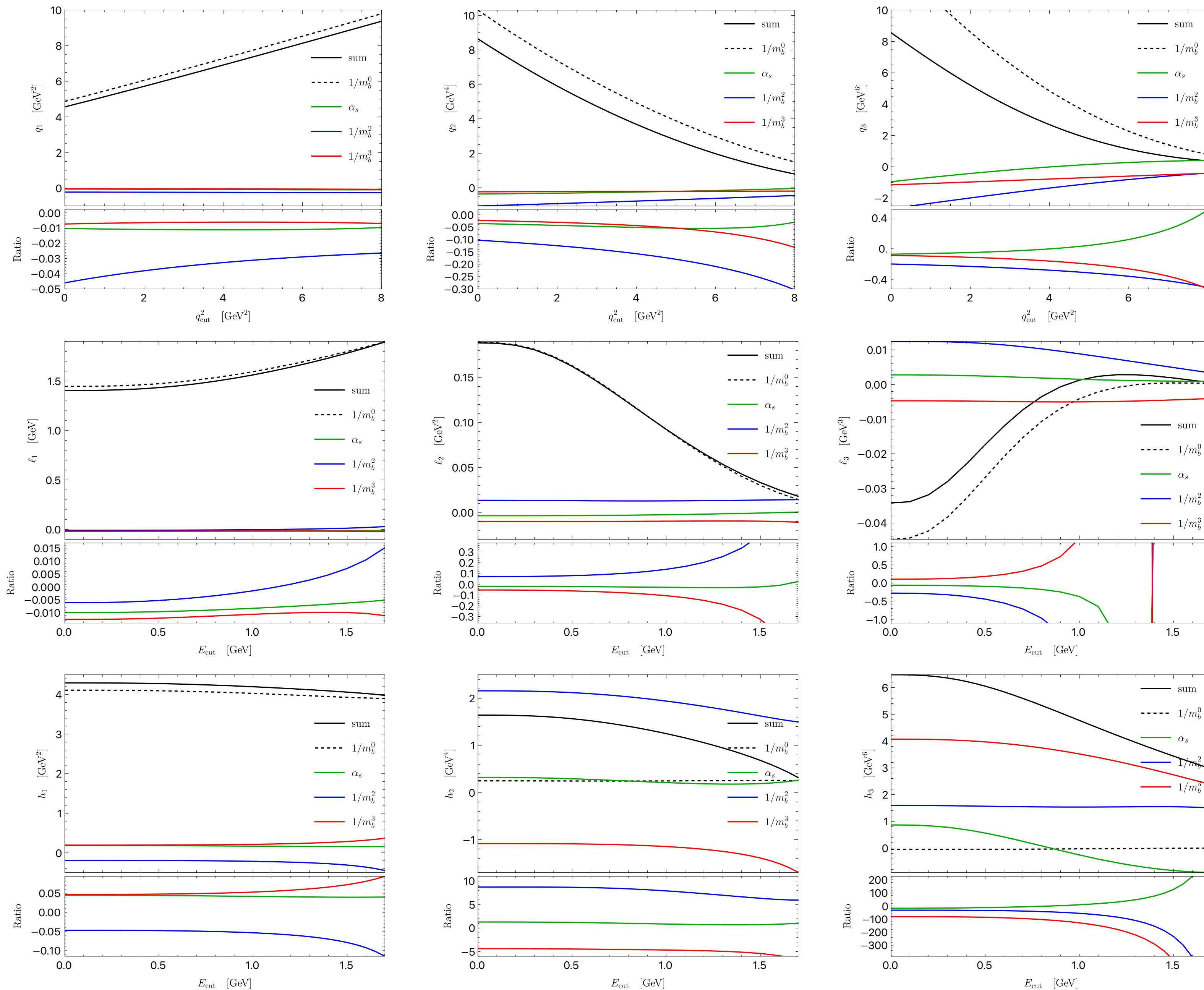
Preliminary results - feasibility study.

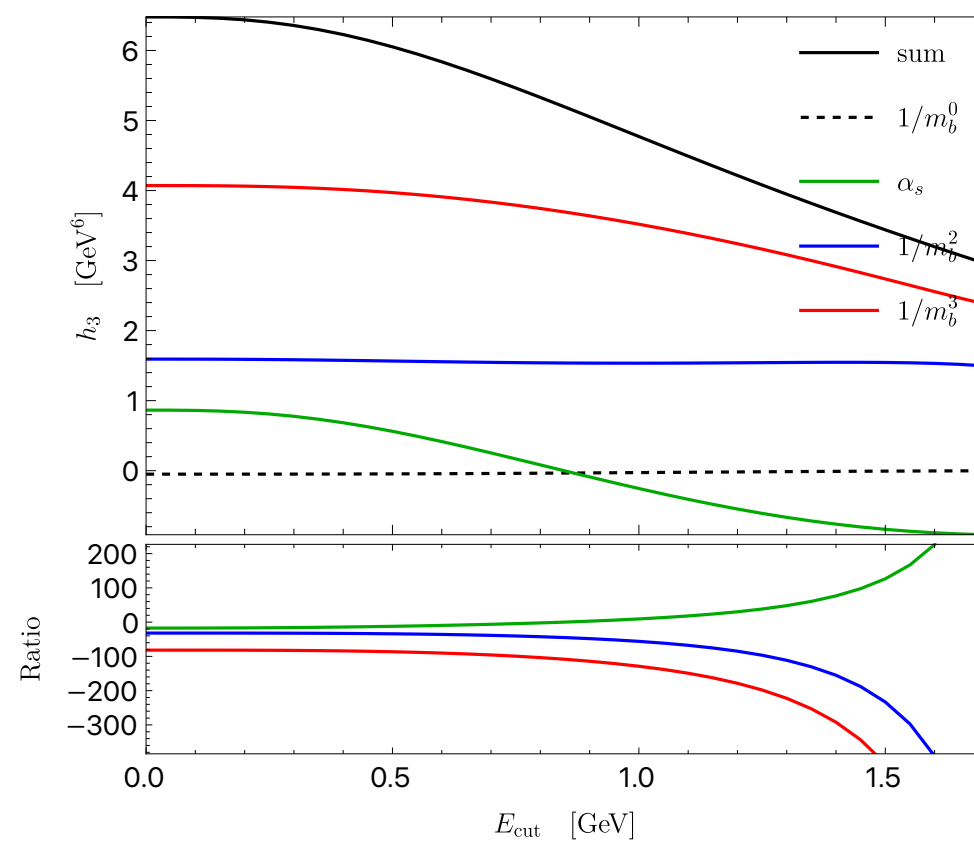
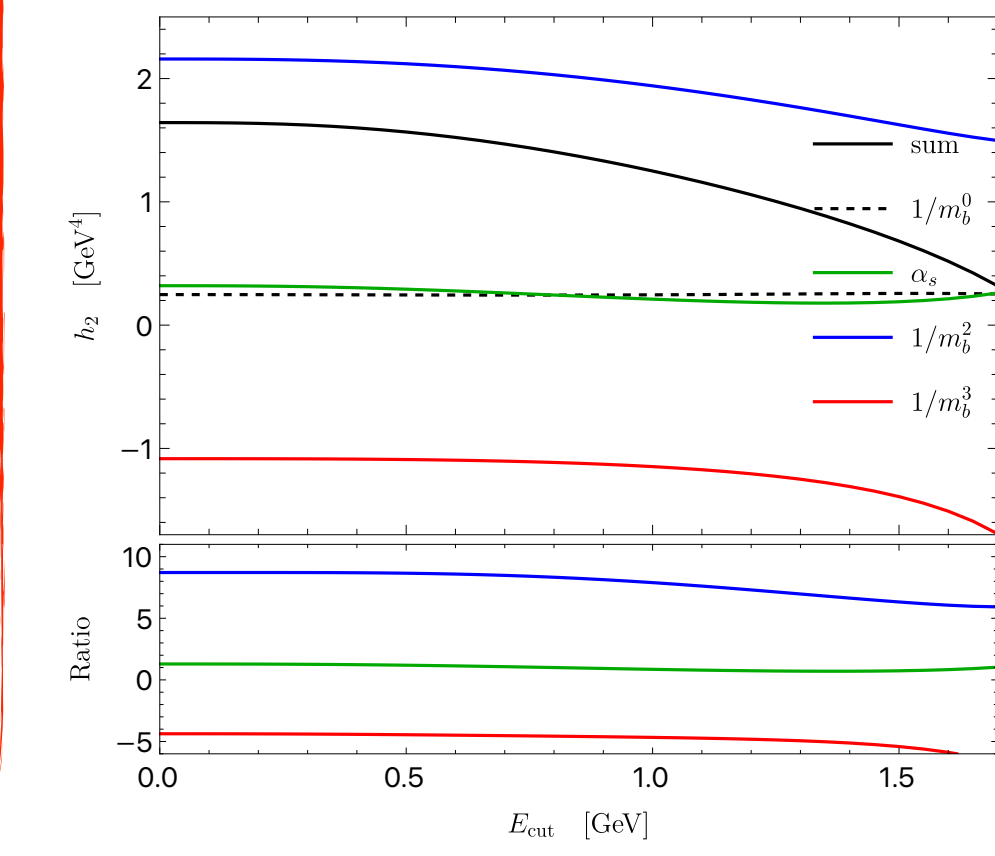
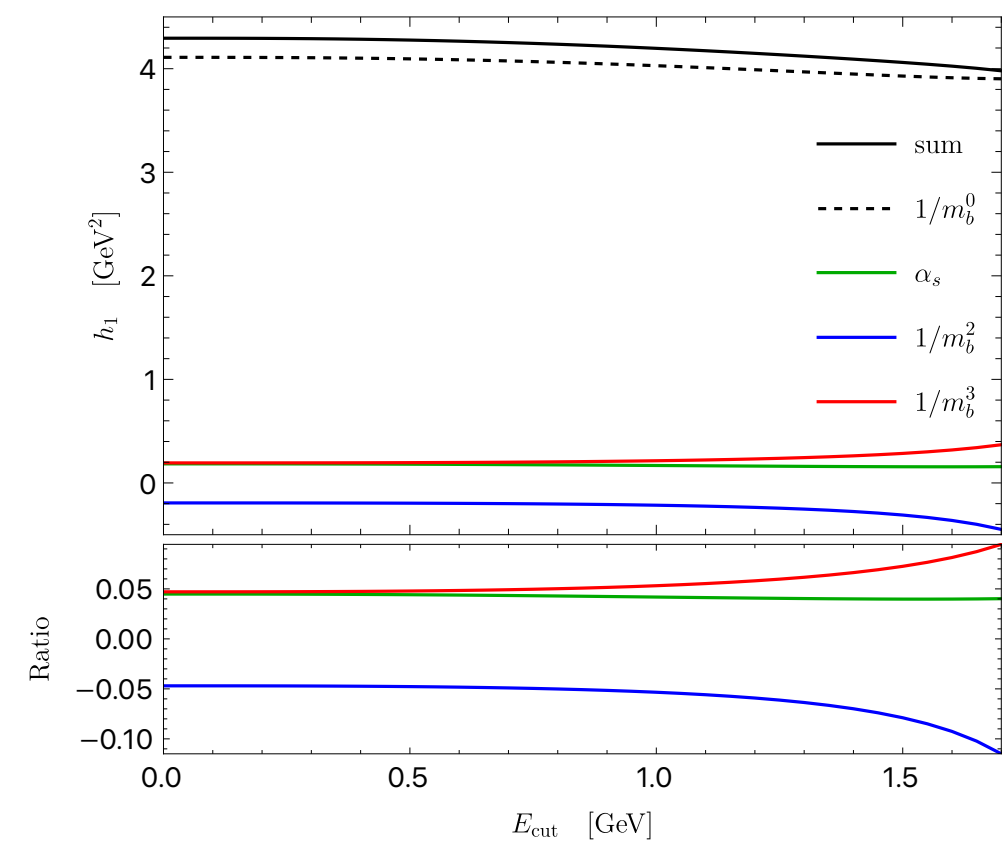
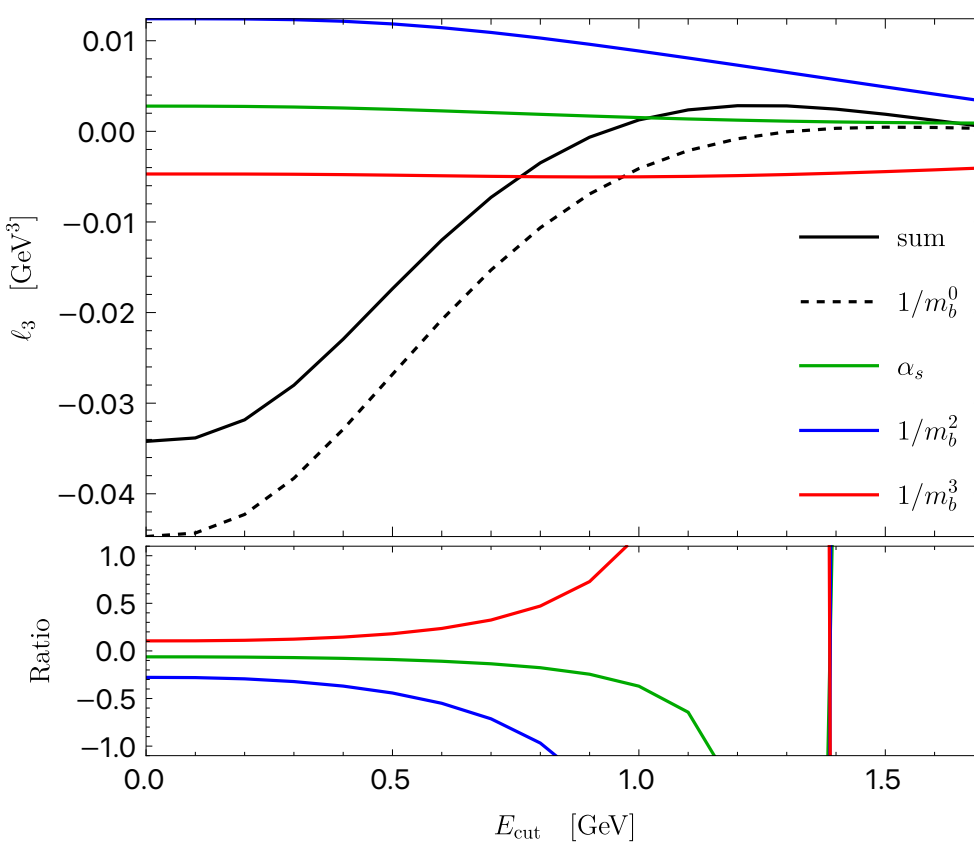
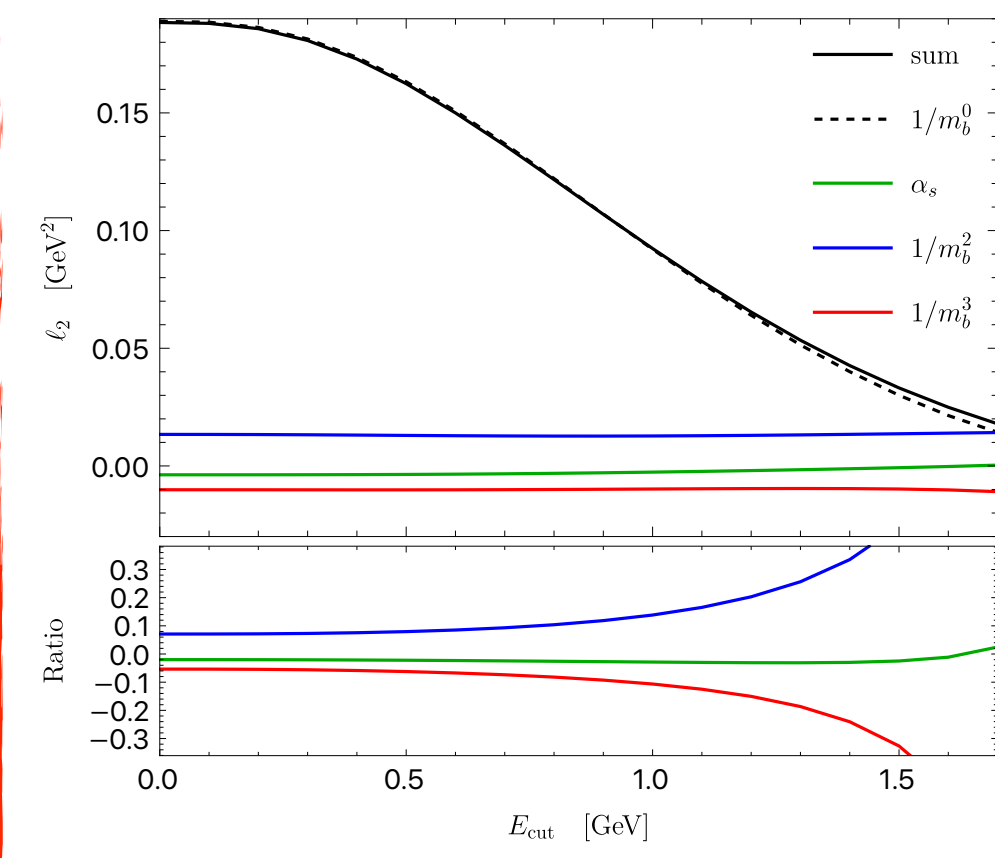
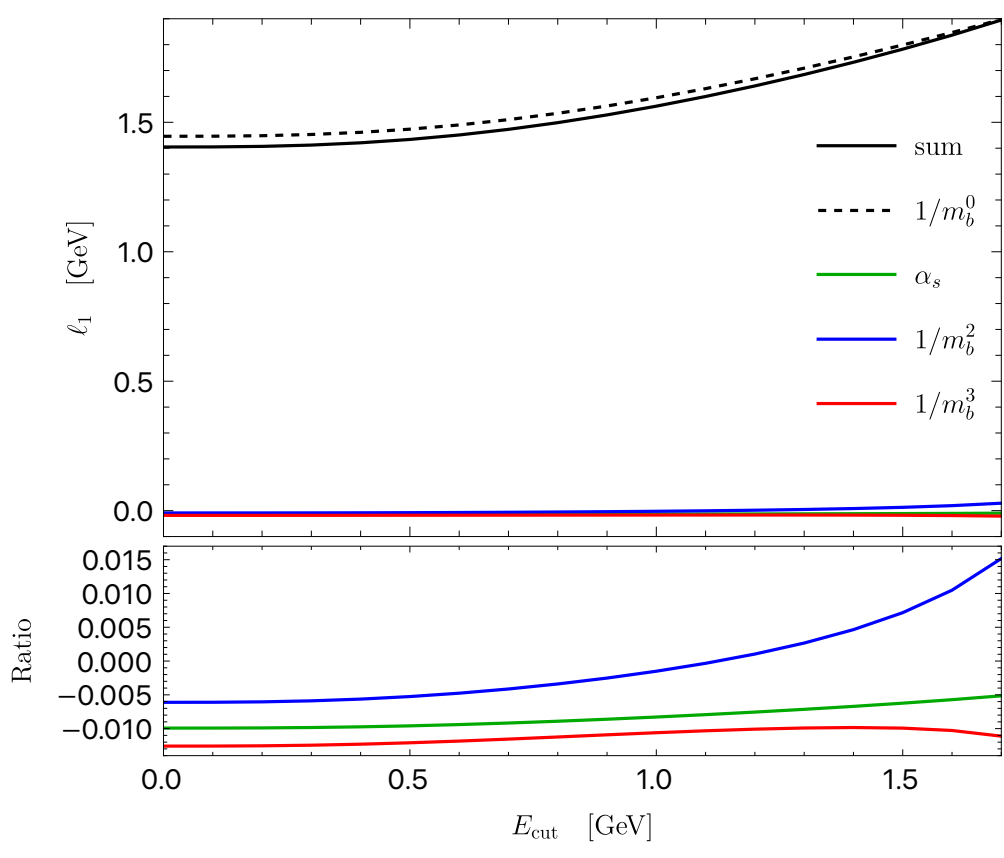
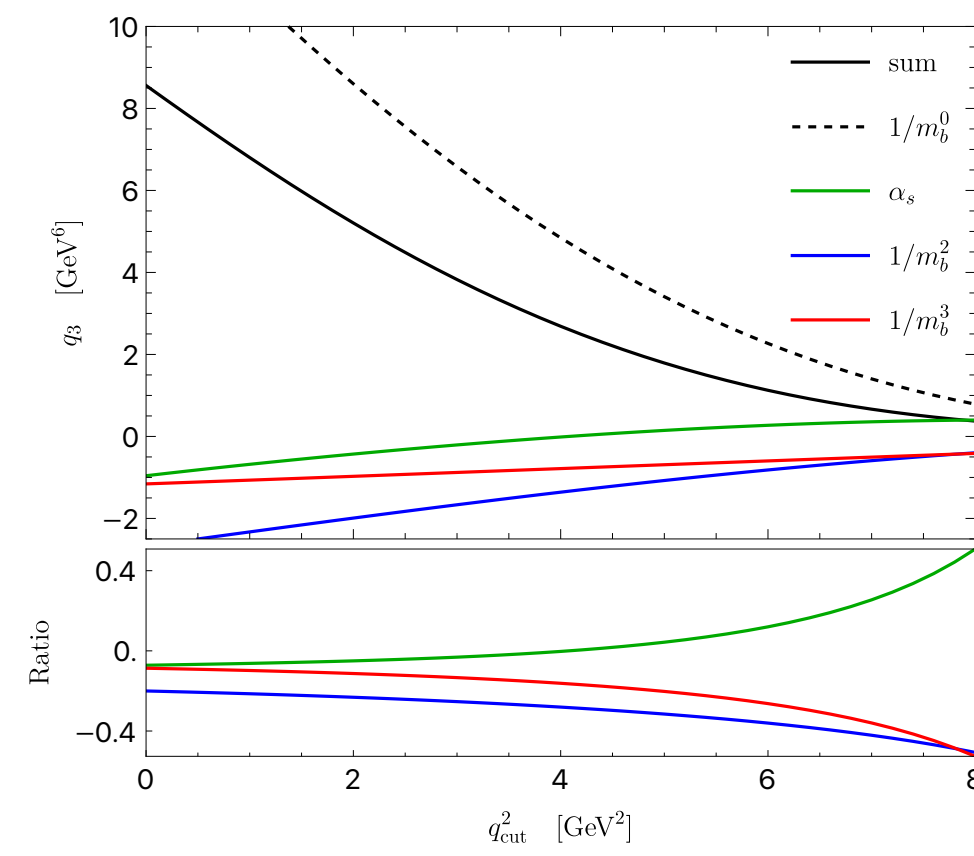
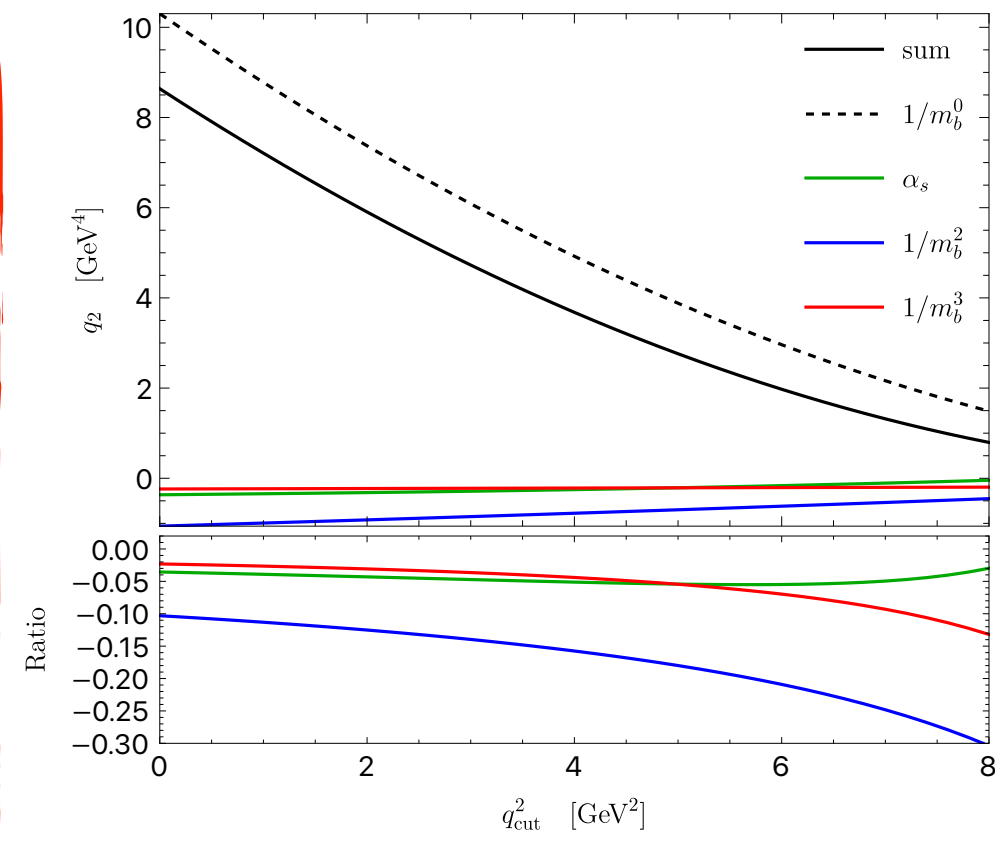
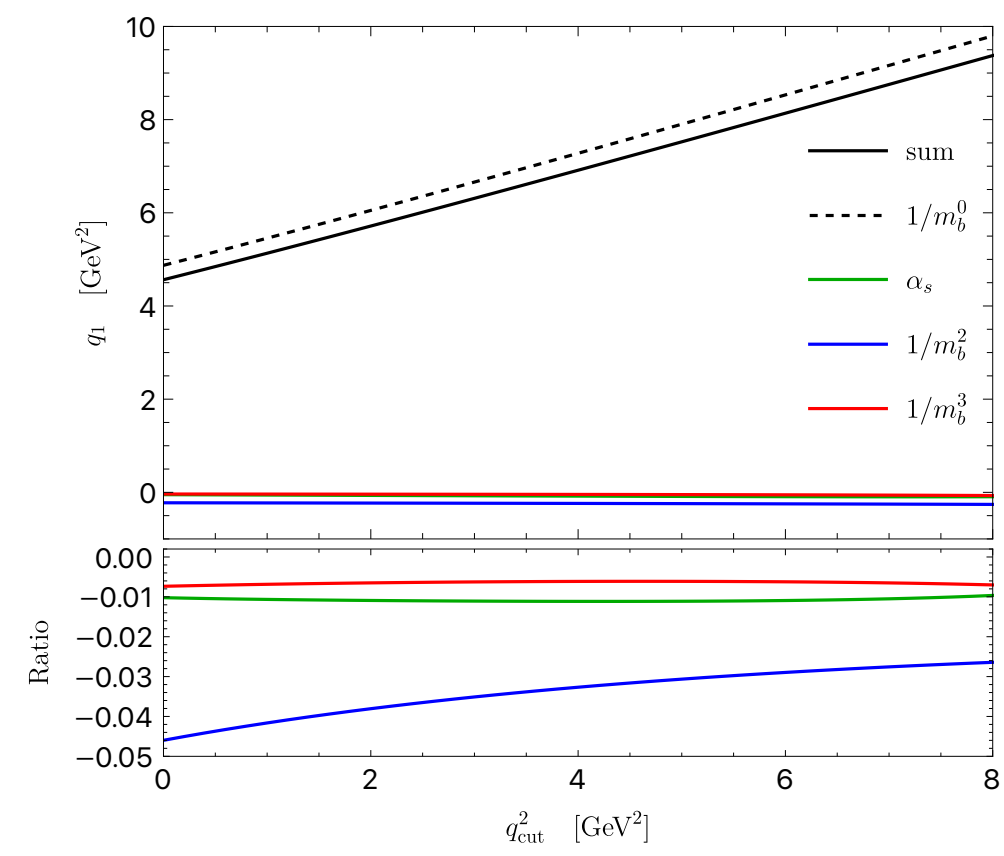


Lattice data (after extrapolation to the physical world) can be used to extract HQET parameters used in the OPE expansion

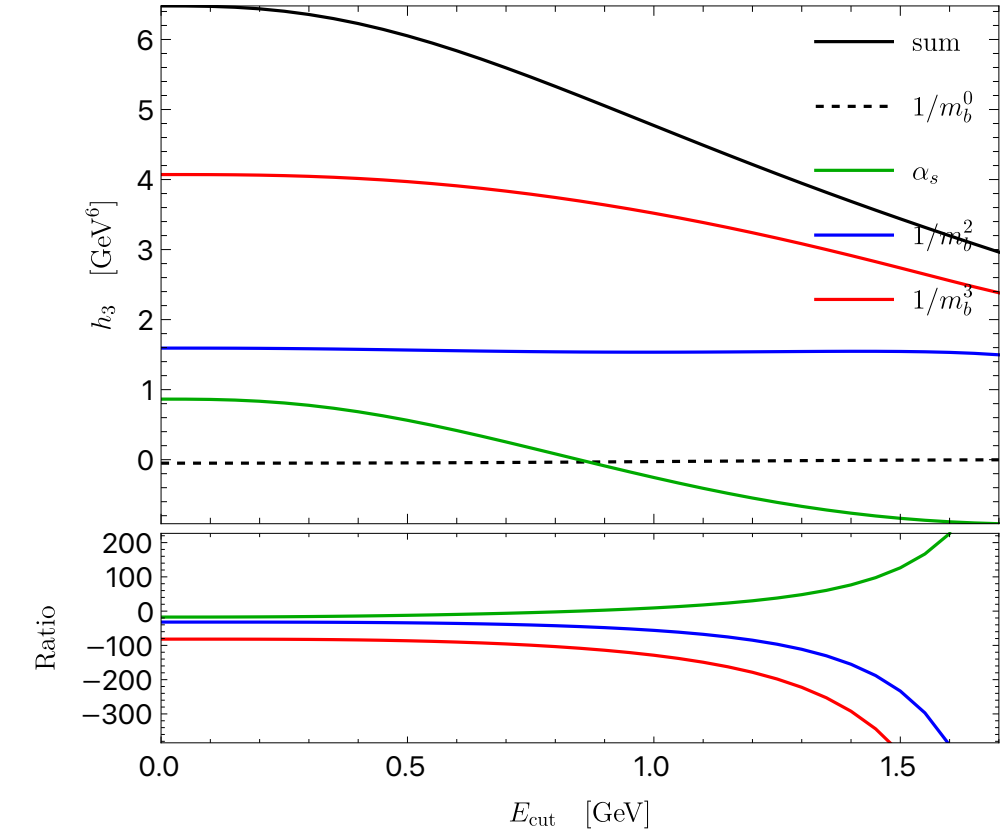
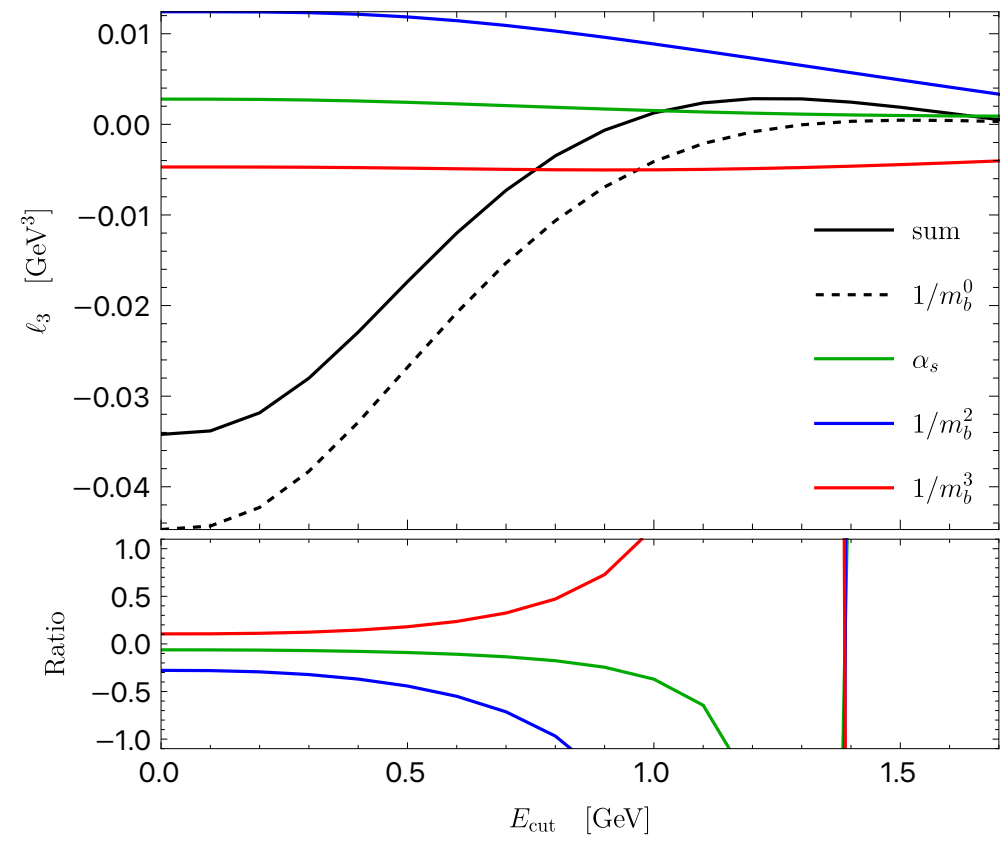
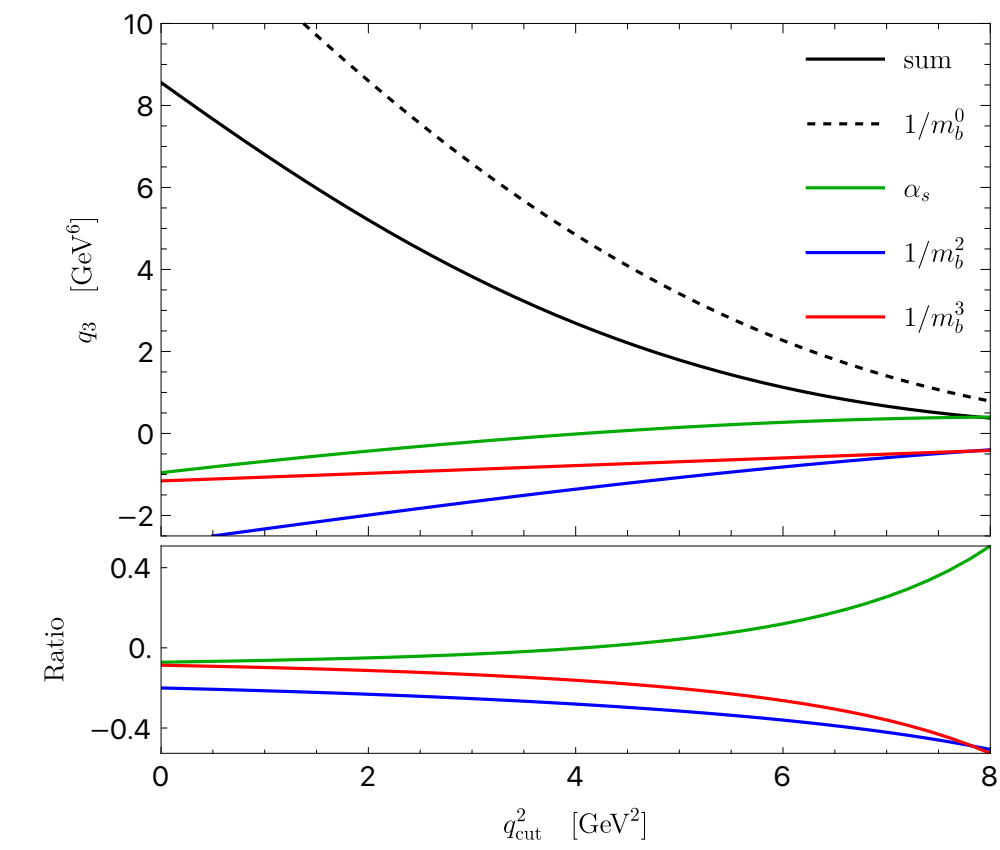
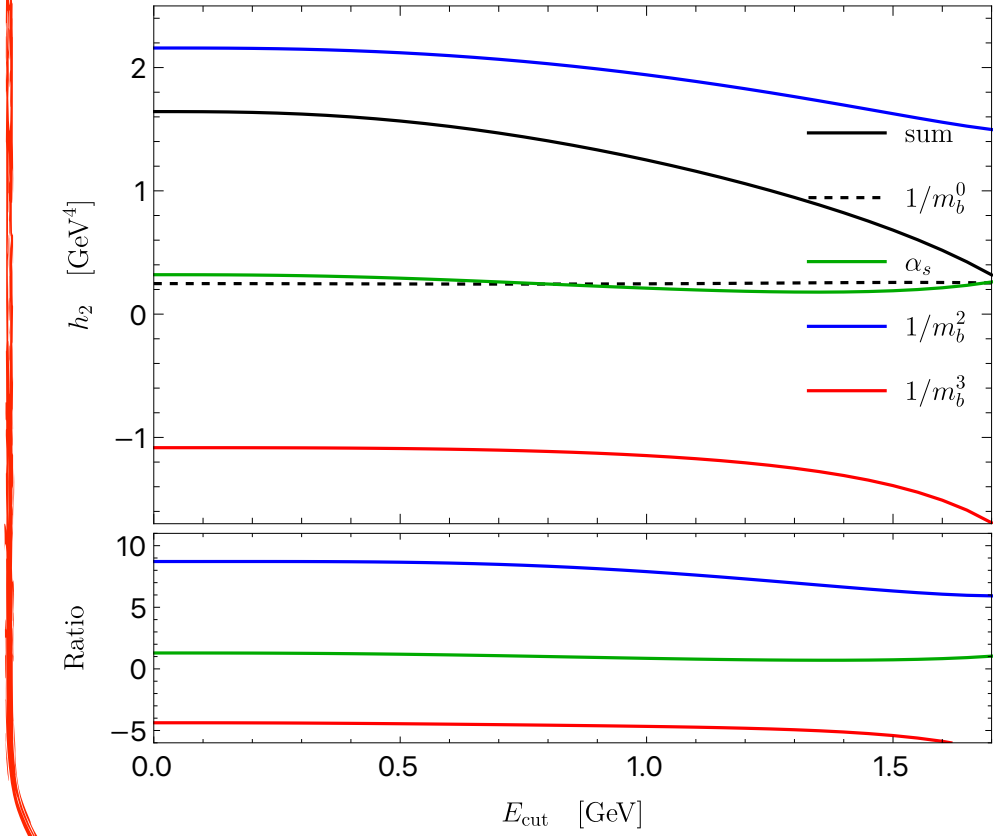
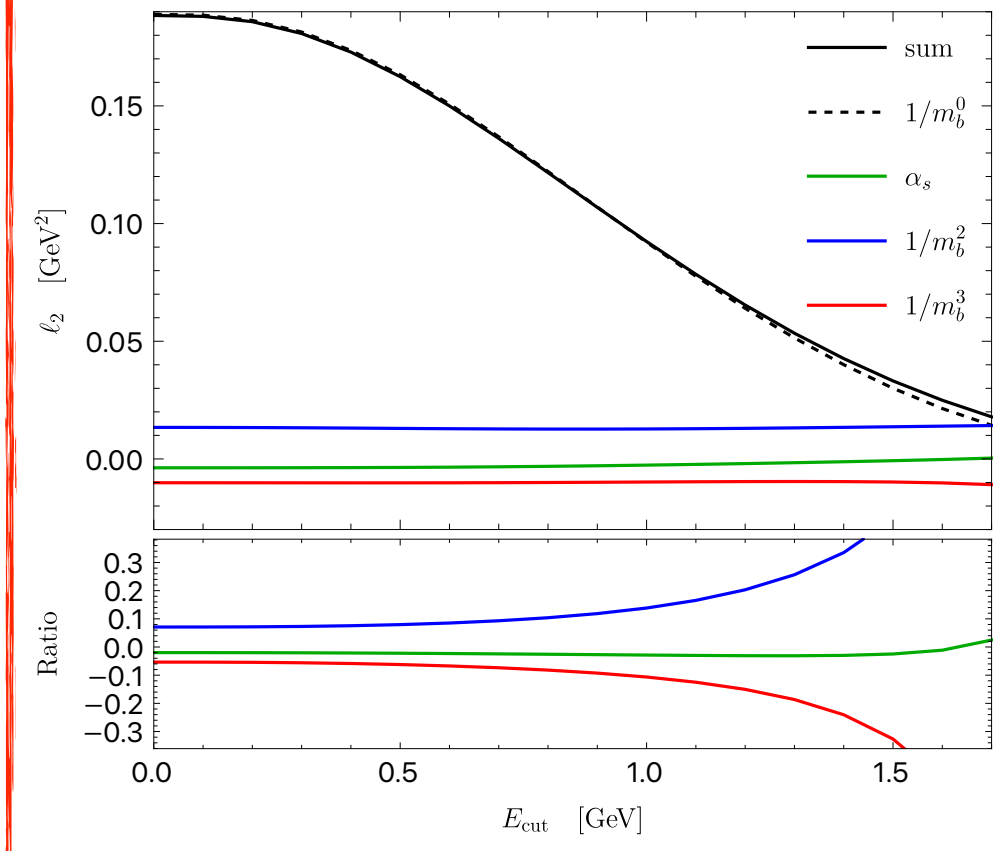
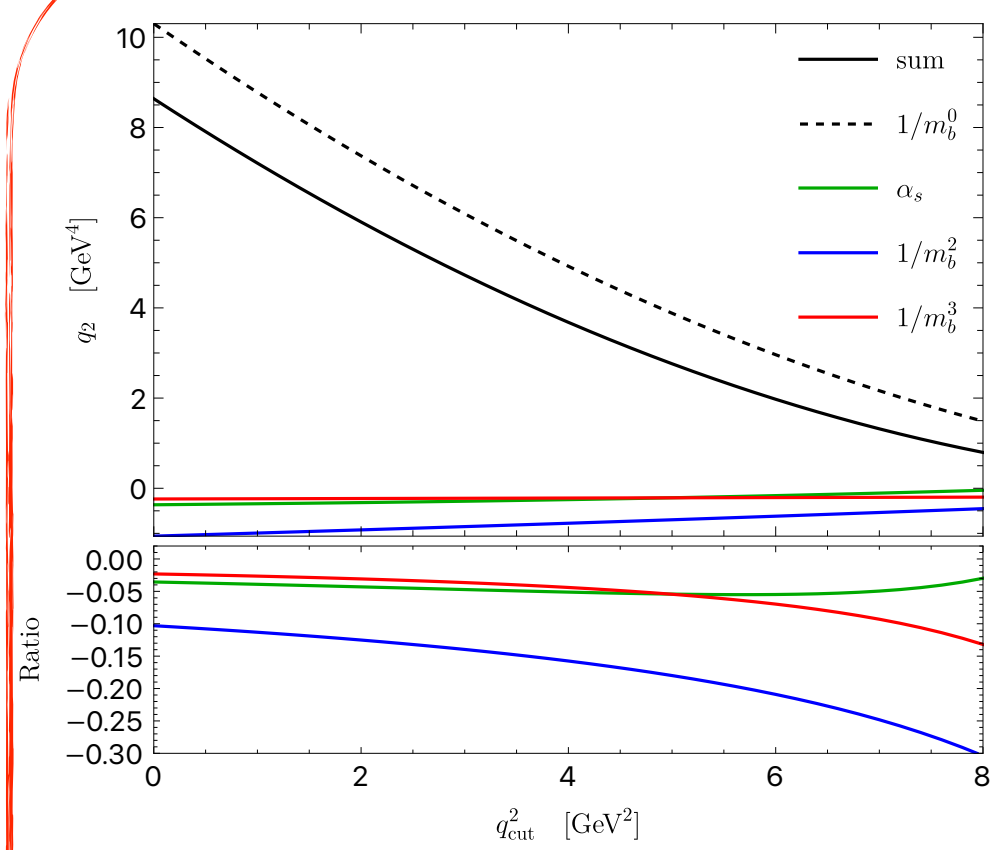
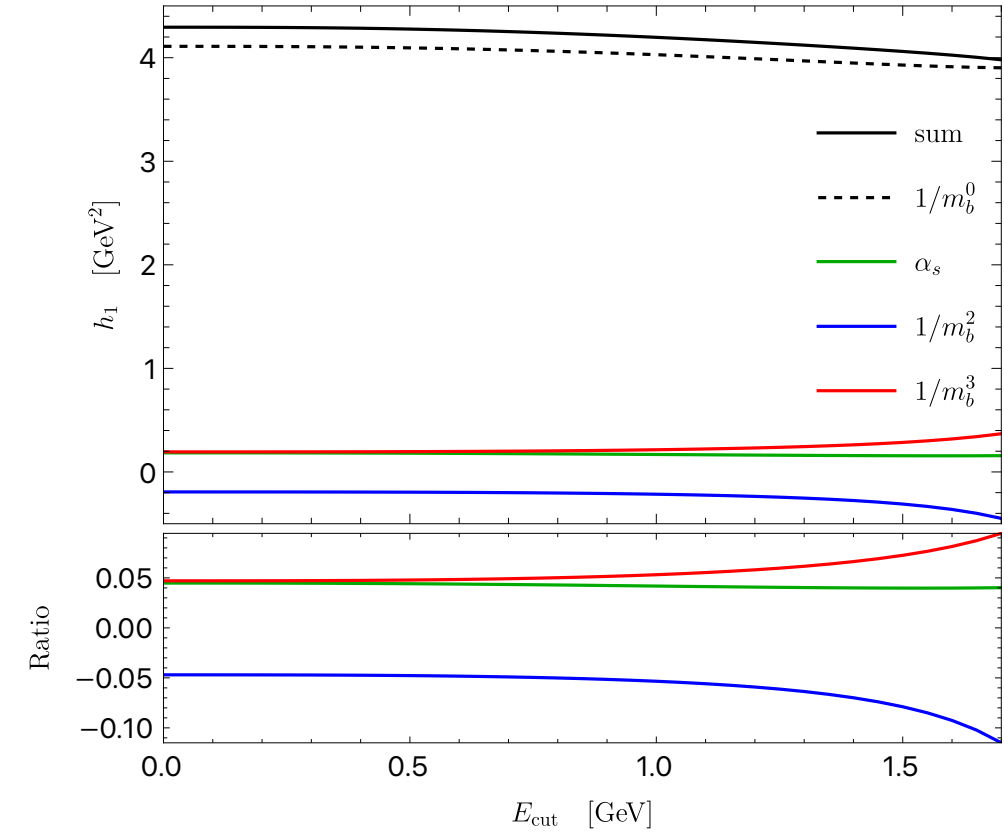
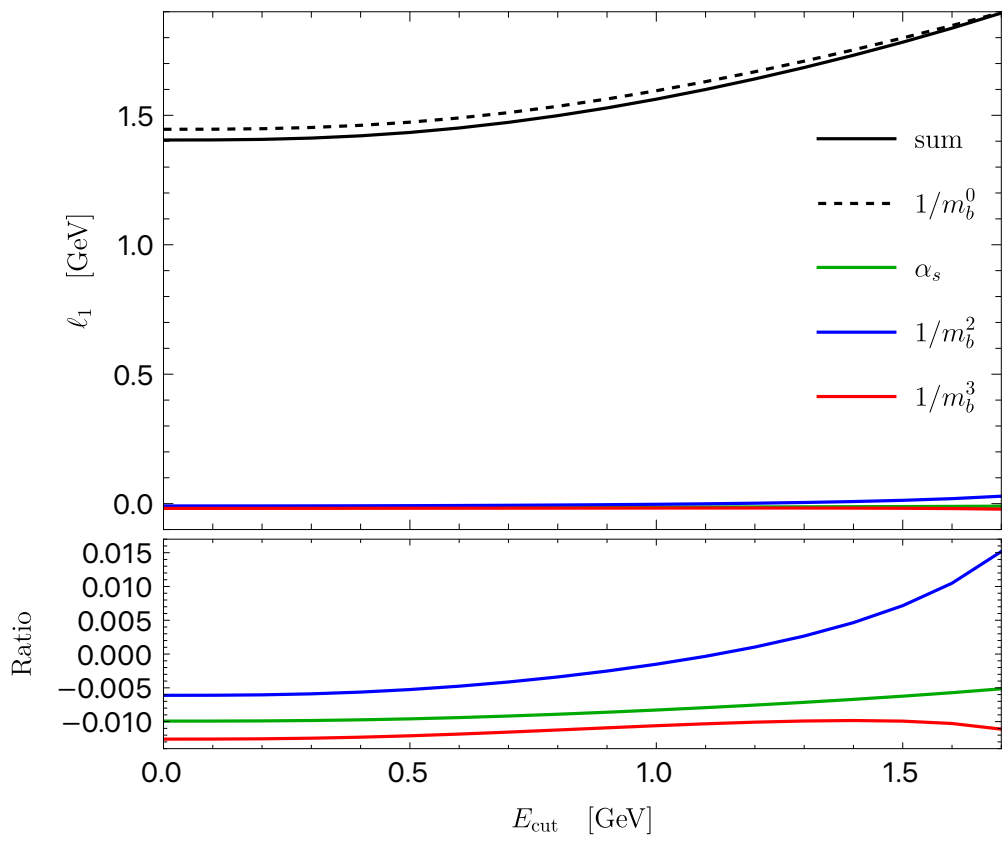
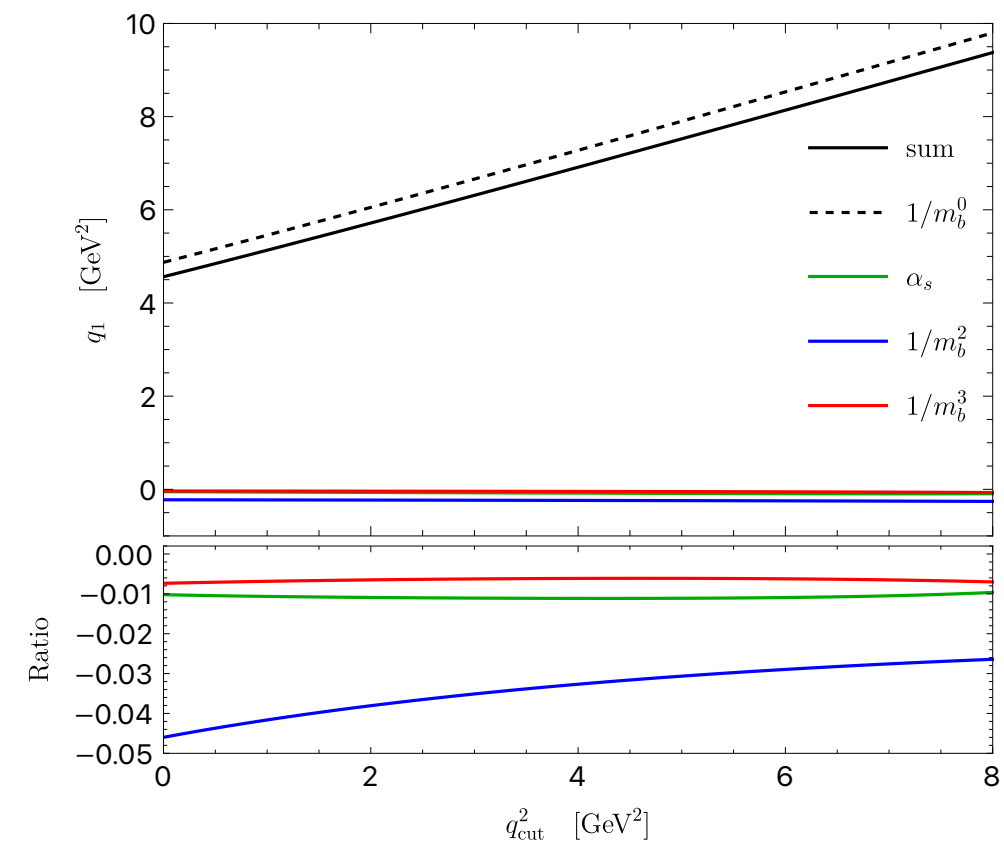


► Different moments have different sensitivities to the power corrections





➤ First moments are rather insensitive on power corrections

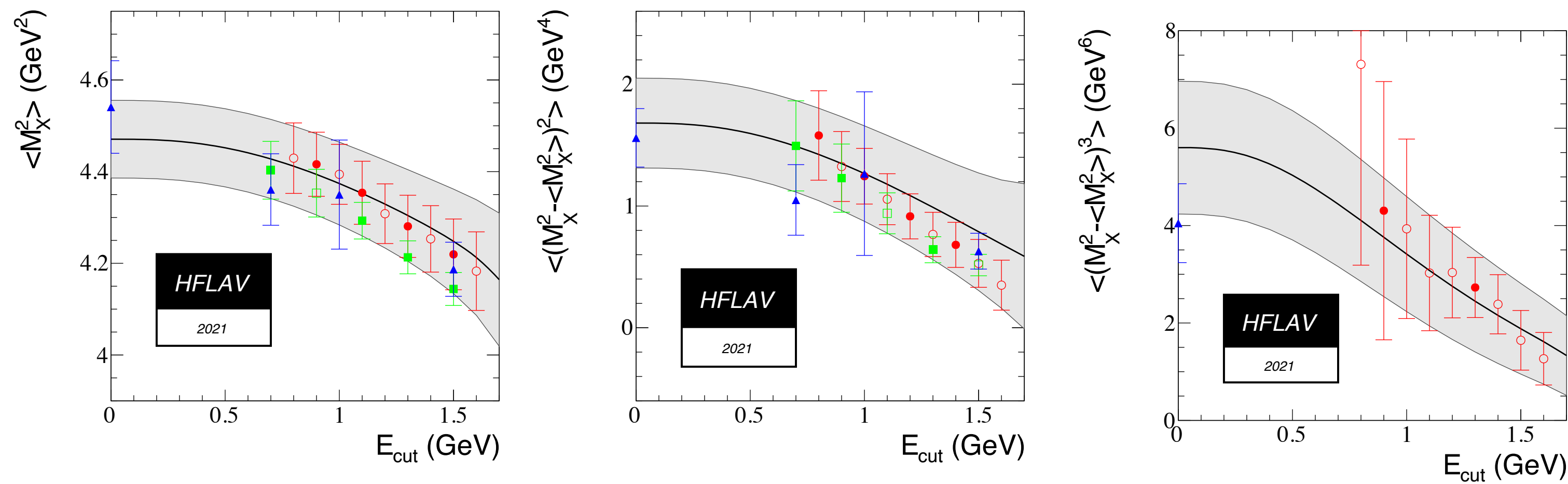


- First moments are rather insensitive on power corrections
- Higher moments are sensitive to power corrections

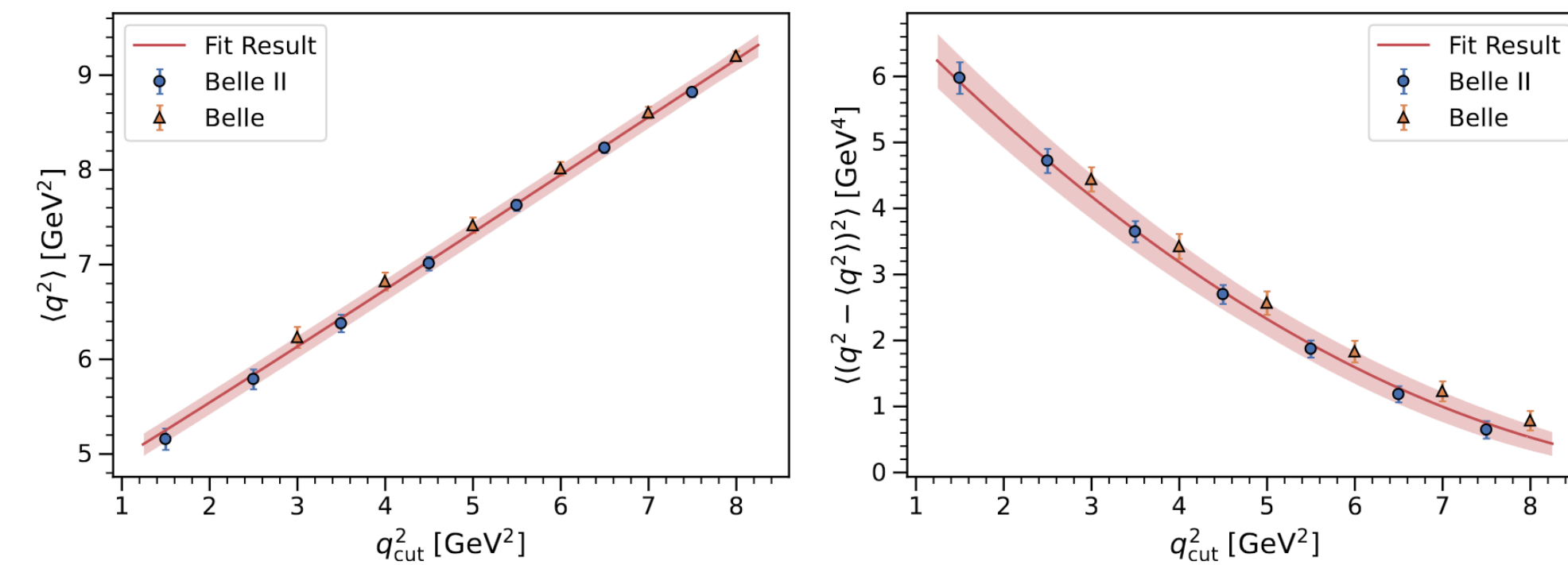


# HOW TO IMPROVE

- Some experimental data are very precise, some are not



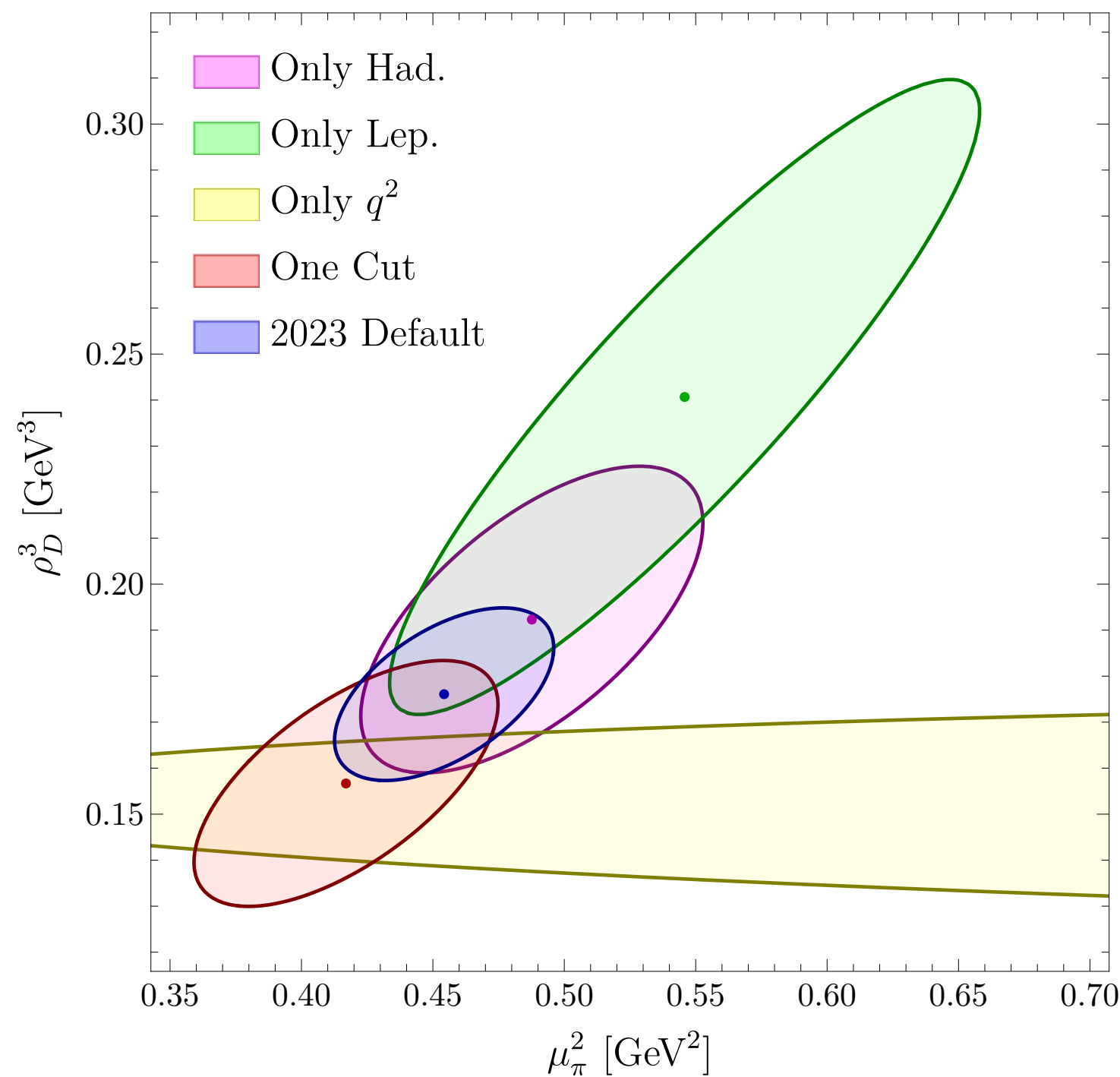
based on Gambino, Schwanda Phys.Rev.D 89 (2014) 1, 014022



Bernlochner, et al, *JHEP* 10 (2022) 068

- Can lattice substantially improve the precision?

# COMPLEMENTARITY BETWEEN OBSERVABLES



Gambino, Finauri, JHEP 02 (2024) 206

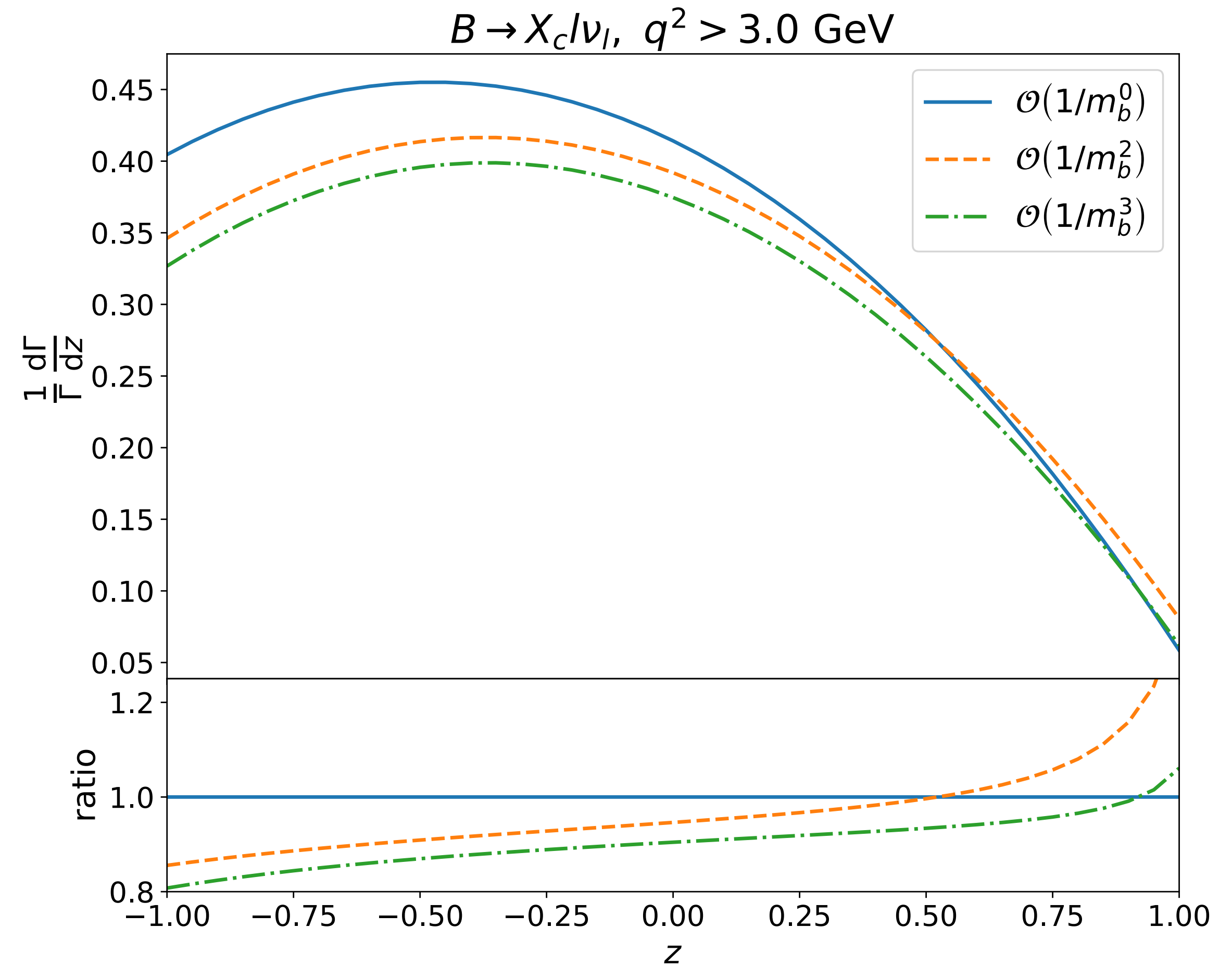
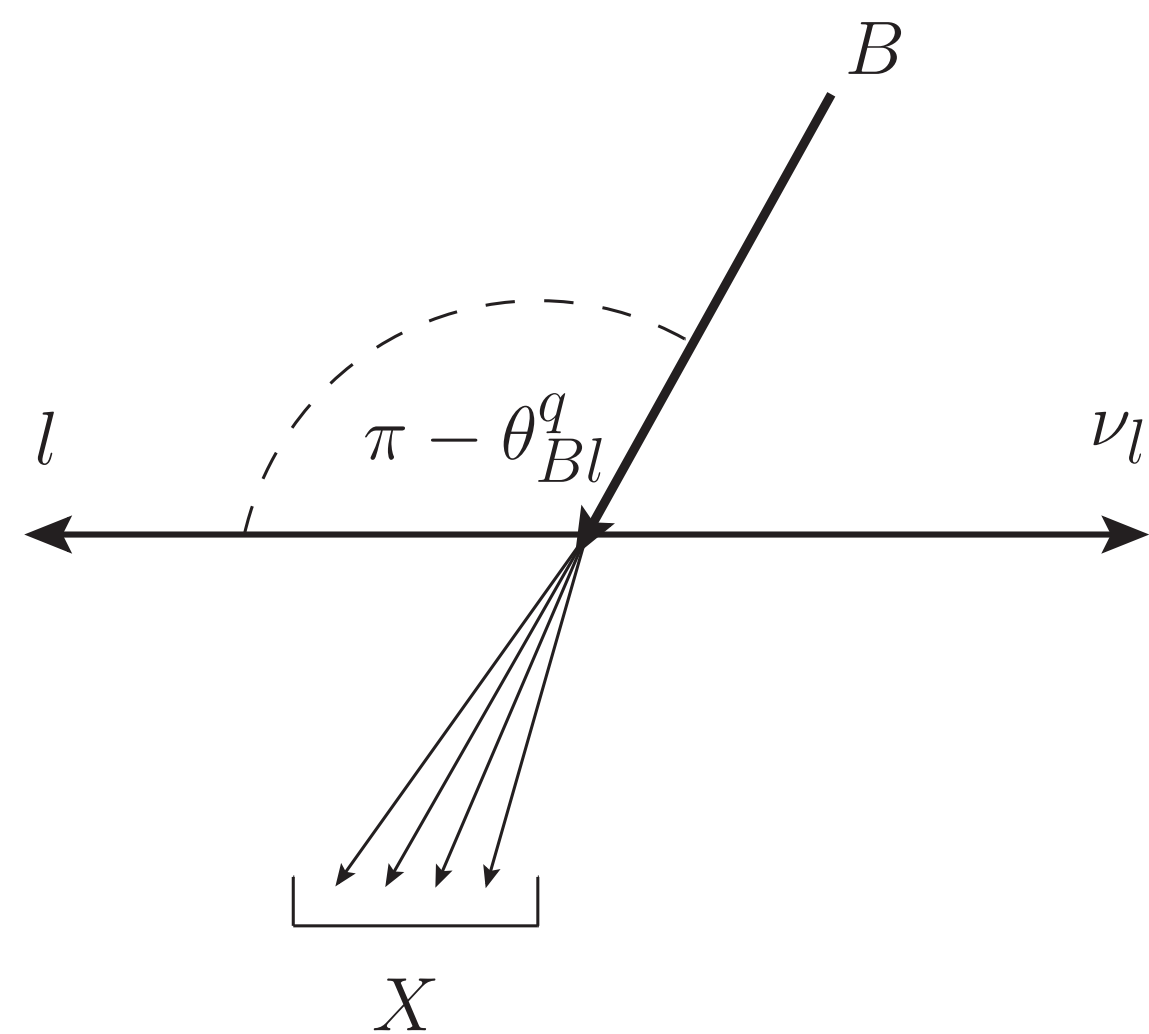
- Moments exhibit “flat directions” in the space of HQE parameters.
- Use “orthogonal observables” to break the degeneracy and improve the sensitivity of the fits on the HQE
- It can be beneficial to keep separated the V and A currents for all moments.
- Consider also scalar and pseudo-scalar currents.
- Expressions available up to  $1/m_b^3$  and  $O(\alpha_s)$  at  $1/m_b^0$

MF, Rahimi, Vos, 2208.04282 [hep-ph]

# FORWARD-BACKWARD ASYMMETRIES

$$A_{FB} = \frac{\int_{-1}^0 \frac{d\Gamma}{dz} - \int_{-1}^0 \frac{d\Gamma}{dz}}{\int_{-1}^0 \frac{d\Gamma}{dz} + \int_{-1}^0 \frac{d\Gamma}{dz}}$$

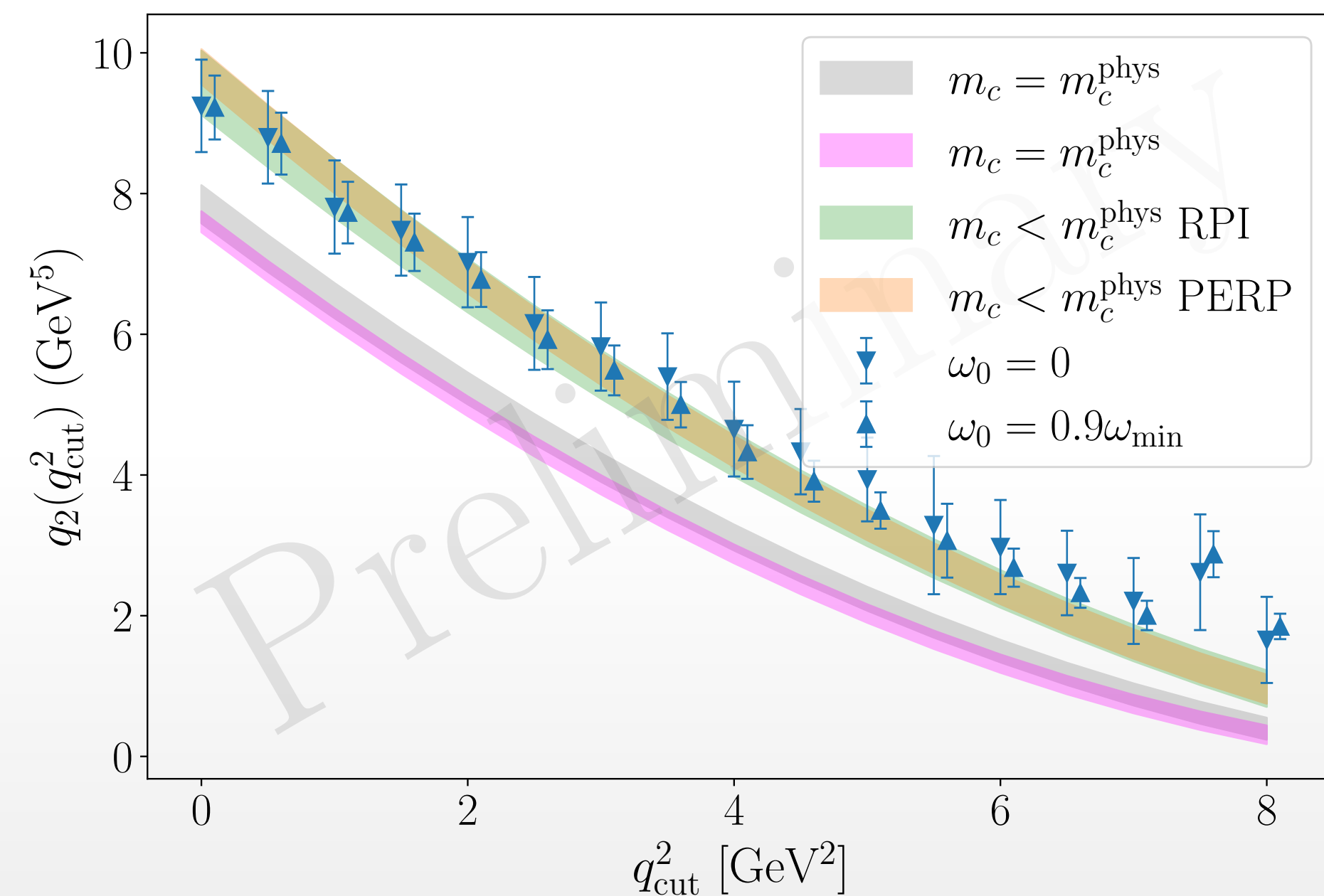
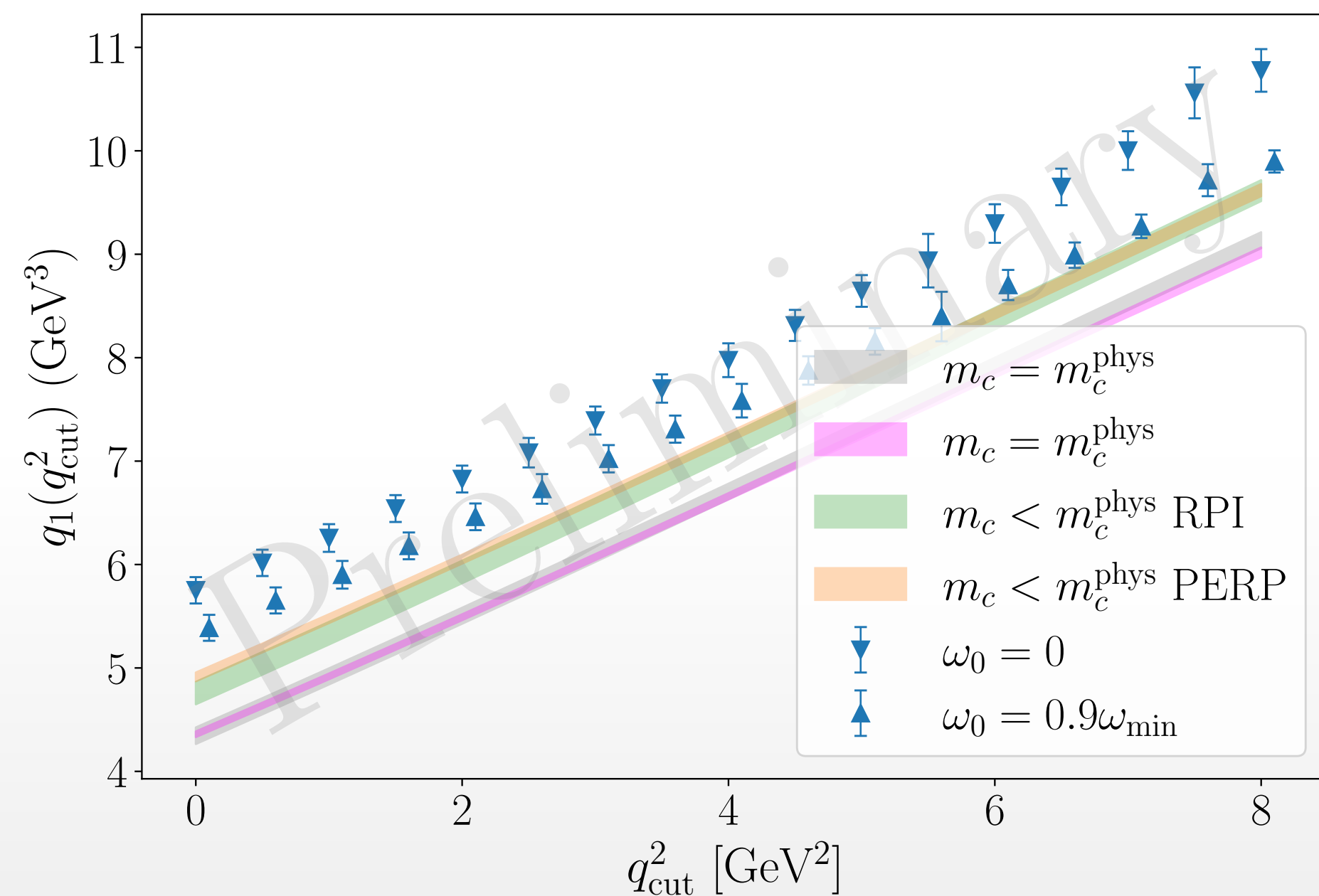
$$z = \cos \theta = \frac{v \cdot p_\nu - v \cdot p_\ell}{\sqrt{(v \cdot q)^2 - q^2}}$$



Turczyk, JHEP 04 (2016) 131  
 Herren, SciPost Phys. 14 (2023) 020

# Centralized moments: lattice “VS” continuum

Preliminary results - feasibility study.



Lattice data (after extrapolation to the physical world) can be used to extract HQET parameters used in the OPE expansion

# BOTTOM AND CHARM MASSES

e.g.  $\mu_\pi^2 = \frac{1}{2M_B} \langle B | \bar{b}_v (iD)^2 b_v | B \rangle$

- $m_b$  must be physical to extract the HQE parameters
- Physical or unphysical charm mass?

$\mu_\pi^2, \mu_G^2, \dots$  are independent on  $m_c$   
however they depend on  $M_B$

$$\langle O^n \rangle_{\text{cut}} = (m_b)^{mn} \left[ X_0^{(O,n)} + \frac{\alpha_s}{\pi} X_1^{(O,n)} + \left( \frac{\alpha_s}{\pi} \right)^2 X_2^{(O,n)} + \frac{\mu_\pi^2}{m_b^2} \left( p_0^{(O,n)} + \frac{\alpha_s}{\pi} p_1^{(O,n)} + \dots \right) \right. \\ \left. + \frac{\mu_G^2}{m_b^2} \left( g_0^{(O,n)} + \frac{\alpha_s}{\pi} g_1^{(O,n)} + \dots \right) + \frac{\rho_D^3}{m_b^3} \left( d_0^{(O,n)} + \frac{\alpha_s}{\pi} d_1^{(O,n)} + \dots \right) + \frac{\rho_{LS}}{m_b^2} \left( l_0^{(O,n)} + \frac{\alpha_s}{\pi} l_1^{(O,n)} + \dots \right) + O \left( \frac{1}{m_b^4} \right) \right]$$

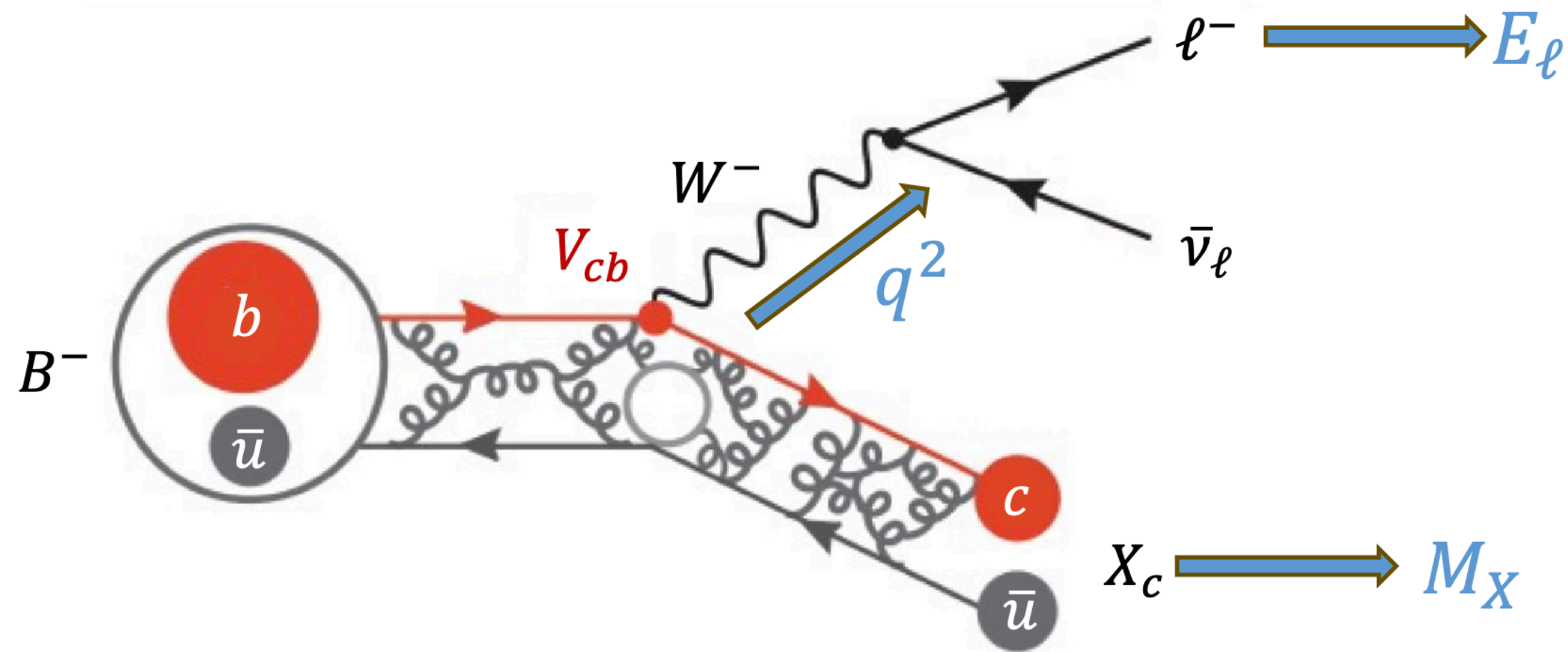
➤ HQET estimate:  $M_D = m_c + \bar{\Lambda} + \frac{\mu_\pi^2 - \frac{d_H}{3} \mu_G^2}{2m_c} + O \left( \frac{1}{m_c^2} \right)$

- With  $m_c < m_c^{\text{phys}}$ , lattice should provide reference value for  $\bar{m}_c$ !



# SMALL VELOCITY SUM RULES

Bigi, Shifman, Uraltsev, Vainshtein, *Phys.Rev.D* 56 (1997) 4017



Excitation energy of the  $X_c$  system

$$\omega_X = M_B - q_0 - \sqrt{M_D^2 + \mathbf{q}^2}$$

Maximum energy of the dilepton system

$$q_0^{\max} = M_B - \sqrt{M_D^2 + \mathbf{q}^2}$$

$$I_n(\mathbf{q}^2, \mu) = \int_{q_0^{\max} - \mu}^{q_0^{\max}} dq_0 \omega_X^n \frac{d^2\Gamma}{dq_0 d\mathbf{q}^2}$$

$$\mu \simeq 1 \text{ GeV}$$

# SMALL VELOCITY SUM RULES

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$$I_1(\mathbf{q}^2, \mu) = I_0(\mathbf{q}^2) \frac{\mathbf{q}^2}{2m_c^2} \bar{\Lambda}(\mu) + O(|\mathbf{q}|^3, \Lambda_{\text{QCD}}^2)$$

$$I_2(\mathbf{q}^2, \mu) = I_0(\mathbf{q}^2) \frac{\mathbf{q}^2}{3m_c^2} \mu_\pi^2(\mu) + O(|\mathbf{q}|^3, \Lambda_{\text{QCD}}^3)$$

$$I_3(\mathbf{q}^2, \mu) = I_0(\mathbf{q}^2) \frac{\mathbf{q}^2}{2m_c^2} \rho_D^3(\mu) + O(|\mathbf{q}|^3, \Lambda_{\text{QCD}}^4)$$

- SV sum rules are independent on the current (S,V,A)
- Can lattice verify these relations?
- Can one extract  $\mu_\pi^2, \rho_D^3$  by studying the limit  $|\mathbf{q}| \rightarrow 0$  and  $m_Q \rightarrow \infty$

**Note:** the HQE parameters extracted here are in  $m_Q \rightarrow \infty$  limit



**Funded by  
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