Pheno overview of $B \to D^*$

Marzia Bordone

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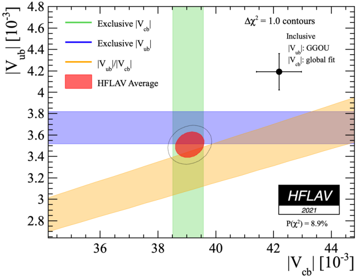
Long-standing puzzles in semileptonic decays



Two extraction methods:

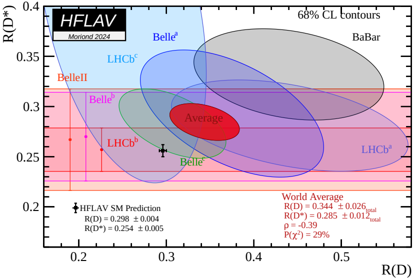
- From inclusive $B \to X_c \ell \bar{\nu}$ decays
- From exclusive decays

$\Rightarrow$ $B \to D^{(*)} \ell \bar{\nu}$
$\Rightarrow$ $\Lambda_b \to \Lambda_c \mu \bar{\nu} / \Lambda_b \to p \mu \nu$
$\Rightarrow$ $B_s \to D_s^{(*)} \ell \bar{\nu}$
$\Rightarrow$ $B_s \to K \mu \nu / B_s \to D_s \mu \nu$

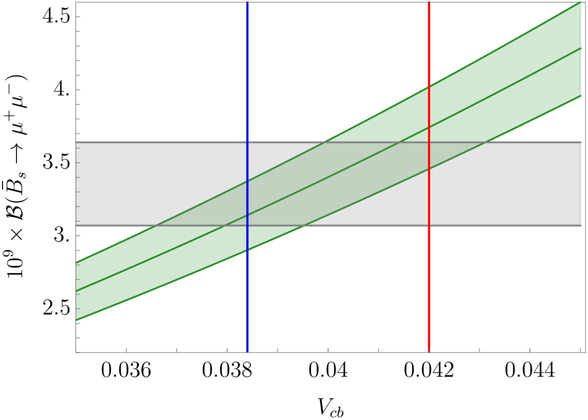
Lepton flavour universality

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)} \tau \bar{\nu})}{\mathcal{B}(B \to D^{(*)} \ell \bar{\nu})}$$

- Current discrepancy at the order of 3.3σ
- Theory prediction is the arithmetic average of before 2021 estimates

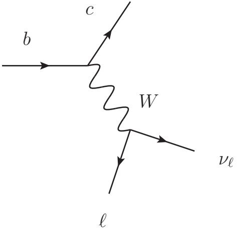


Why we need a better determination of V_{cb}?

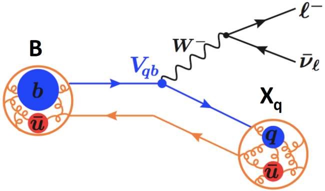


- The value of V_{cb} has a major impact on flavour observables like $\mathcal{B}(B_s \to \mu^+\mu^-)$ or ϵ_K
- A resolution of the puzzle is central given the perspective sensitivities at Belle II and LHCb

The theory drawback



$$\mu_{\text{partonic}} = m_b$$



$$\mu_{\text{hadronic}} = \Lambda_{\text{QCD}}$$

Fundamental challenge to match partonic and hadronic descriptions

Exclusive matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S^i_\mu \mathcal{F}_i$$

Exclusive matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S^i_\mu \mathcal{F}_i$$

← form factor

scale Λ_{QCD}

independent Lorentz structures

Exclusive matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S^i_\mu \mathcal{F}_i$$

form factor

scale $\Lambda_{\rm QCD}$

independent

Lorentz structures

Form factors determinations

- Lattice QCD
- QCD SR, LCSR

only points at specific kinematic points

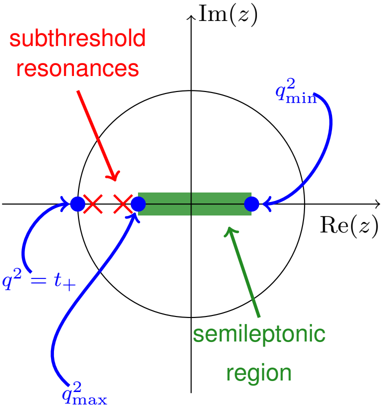
Form factors parametrisations

- HQET (CLN + improvements) $\Rightarrow$ reduce independent degrees of freedom
- Analytic properties $\rightarrow$ BGL

data points needed to fix the coefficients of the expansion

The z-expansion and unitarity

[Boyd, Grinstein, Lebed, '95, Caprini, Lellouch, Neubert, '98]



- in the complex plane form factors are real analytic functions
- q^2 is mapped onto the conformal complex variable z

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- q^2 is mapped onto a disk in the complex z plane, where $|z(q^2, t_0)| < 1$

$$F_i = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$

$$\sum_{k=0}^{n_i} |a_k^i|^2 < 1$$

How to apply unitarity

- Penalty function in the χ^2 or likelihood [P. Gambino, M. Jung, S. Schacht, '19]

$$\chi^2 \to \chi^2(a_k^i, a_k^i|_{\text{data}}) + w_i \theta \left(\sum_{k=0}^{n_i} |a_k^i|^2 - 1 \right)$$

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- Dispersive Matrix Method [M. Di Carlo, G. Martinelli, M. Naviglio, F. Sanfilippo, S. Simula, L. Vittorio, '21] [G. Martinelli, S. Simula, L. Vittorio, '21,'23]

$$\mathbf{M} = \begin{pmatrix} \chi & \phi f & \phi_1 f_1 & \phi_2 f_2 & \ldots & \phi_N f_N \\ \phi f & \frac{1}{1-z^2} & \frac{1}{1-zz_1} & \frac{1}{1-zz_2} & \ldots & \frac{1}{1-zz_N} \\ \phi_1 f_1 & \frac{1}{1-z_1 z} & \frac{1}{1-z_1^2} & \frac{1}{1-z_1 z_2} & \ldots & \frac{1}{1-z_1 z_N} \\ \phi_2 f_2 & \frac{1}{1-z_2 z} & \frac{1}{1-z_2 z_1} & \frac{1}{1-z_2^2} & \ldots & \frac{1}{1-z_2 z_N} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \phi_N f_N & \frac{1}{1-z_N z} & \frac{1}{1-z_N z_1} & \frac{1}{1-z_N z_2} & \ldots & \frac{1}{1-z_N^2} \end{pmatrix}$$

$$\det \mathbf{M} > 0 \Rightarrow \beta - \sqrt{\gamma} \leq f_0 \leq \beta + \sqrt{\gamma}$$

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$$\det \mathbf{M} > 0 \Rightarrow \beta - \sqrt{\gamma} \le f_0 \le \beta + \sqrt{\gamma}$$

- Bayesian inference [J. Flynn, A. Jüttner, T. Tsang, '23]

$$\langle g(\mathbf{a}) \rangle = \mathcal{N} \int d\mathbf{a}\, g(\mathbf{a})\, \pi(\mathbf{a}|\mathbf{f}, C_{\mathbf{f}}) \pi_{\mathbf{a}}$$

$\pi_{\mathbf{a}}$: $\theta(1 - |\mathbf{a}|^2)$

$\pi(\mathbf{a}|\mathbf{f}, C_{\mathbf{f}})$: contains the lattice χ^2

The Heavy Quark Expansion in a nutshell

The HQE exploits the fact that the b and c quarks are heavy

- Double expansion in $1/m_{b,c}$ and α_s
- The HQE symmetries relate $B^{(*)} \to D^{(*)}$ form factors
- At $1/m_{b,c}$ drastic reduction of independent degrees of freedom

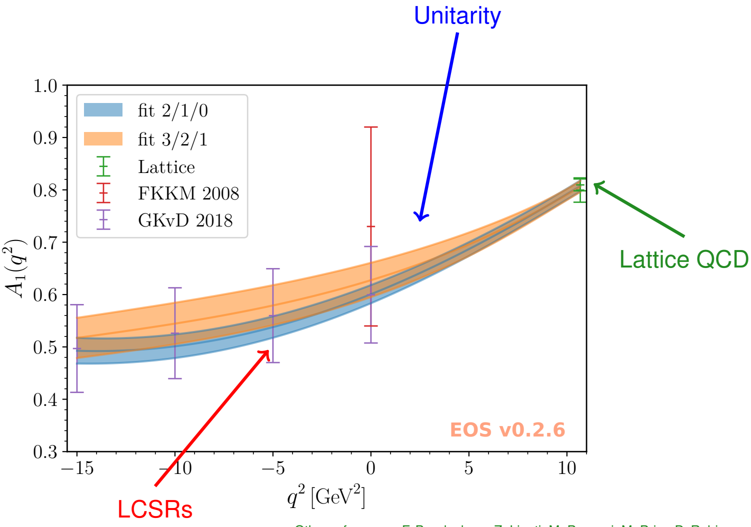
With current precision we know we have to go beyond the $1/m_{b,c}$ order and we use the following form

$$F_i = \left(a_i + b_i \frac{\alpha_s}{\pi}\right) \xi + \frac{\Lambda_{\mathrm{QCD}}}{2m_b} \sum_j c_{ij} \xi^j_{\mathrm{SL}} + \frac{\Lambda_{\mathrm{QCD}}}{2m_c} \sum_j d_{ij} \xi^j_{\mathrm{SL}} + \left(\frac{\Lambda_{\mathrm{QCD}}}{2m_c}\right)^2 \sum_j g_{ij} \xi^j_{\mathrm{SSL}}$$

- Total of 10 independent structures to be extracted from data
- We use the conformal mapping $q^2 \mapsto z(q^2)$ to include bounds and have a well-behaved series

$B \to D^*$ before 2021

[MB, Gubernari, Jung, van Dyk, '19]



Other references: F. Bernlochner, Z. Ligeti, M. Papucci, M. Prim, D. Robinson, '22
P. Gambino, M. Jung, S. Schacht, '19

Recent developments

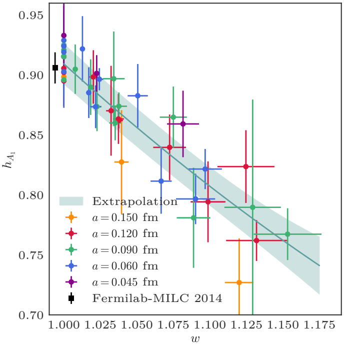
$B \to D^*$ after 2021

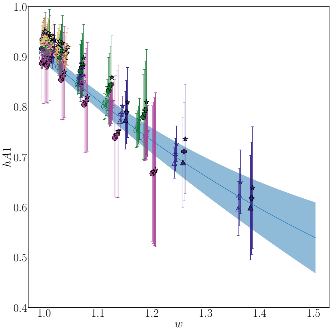
- In 2021, FNAL/MILC came out with the first, away from zero-recoil calculation of $B \to D^*$ form factors [2105.14019]

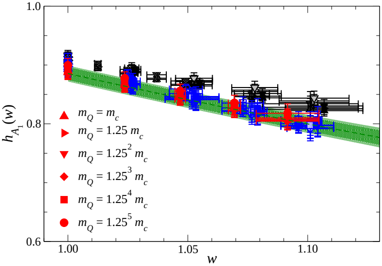
- In 2023, HPQCD and JLQCD released also $B \to D^*$ form factors away from zero-recoil [2304.03137, 2306.05657]

- For the first time, a lattice-only determination of $B \to D^*$ form factors is possible without using any assumption

- The three computations rely on different lattice actions, and different ensembles, so they are in principle independent

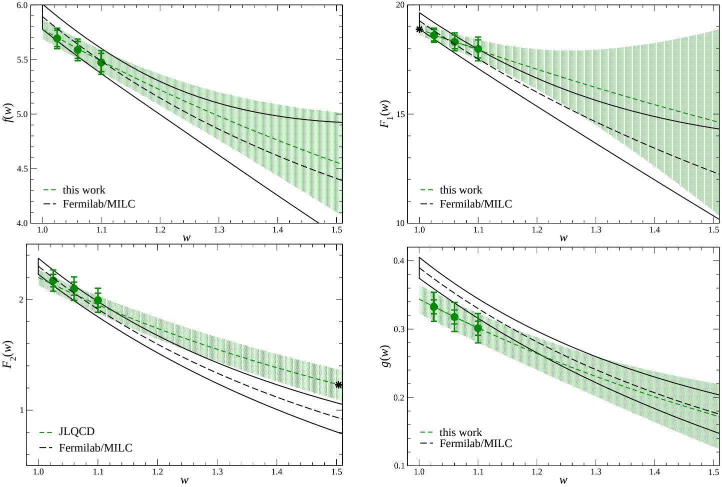






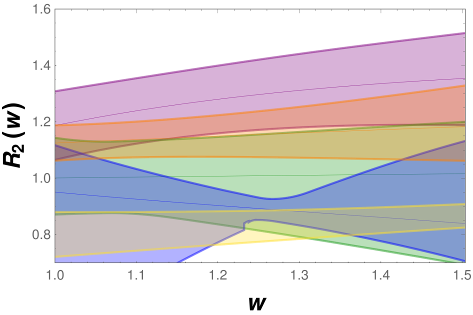
- Are these results compatible with each other?
- Are they compatible with experimental data?

Compatiblity of lattice data



- Similar results with HPQCD
- There are some differences in the slopes
- How good is the compatibility?
- Do the differences yield significant pheno consequences?

The HQET ratios

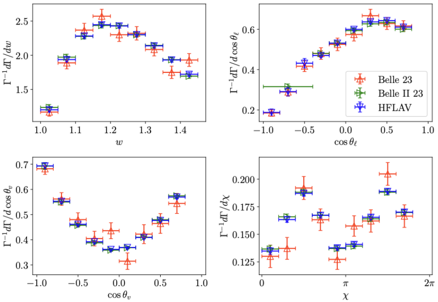


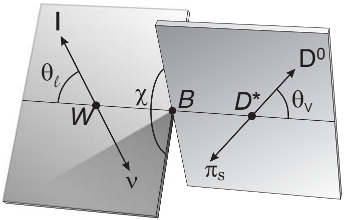
- FNAL/MILC '21
- HQE@$1/m_c^2$
- Exp data (BGL)
- JLQCD '23
- HPQCD '23

$$R_2(w) = \frac{r h_{A_2}(w) + h_{A_3}(w)}{h_{A_1}(w)}$$

- What's the source of the discrepancy with HQET? [MB, Harrison, Jung, ongoing]
- Why are experimental data (Belle 2018) so different from LQCD data?

New $B \to D^* \ell \bar{\nu}$ Belle and Belle II data





$$\frac{d\Gamma}{dw d\cos(\theta_\ell) d\cos(\theta_v) d\chi} = \frac{3G_F^2}{1024\pi^4} |V_{cb}|^2 \eta_{EW}^2 M_B r^2 \sqrt{w^2-1} q^2$$
$$\times \{(1-\cos(\theta_\ell))^2 \sin^2(\theta_v) H_+^2(w) + (1+\cos(\theta_\ell))^2 \sin^2(\theta_v) H_-^2(w)$$
$$+ 4\sin^2(\theta_\ell)\cos^2(\theta_v) H_0^2(w) - 2\sin^2(\theta_\ell)\sin^2(\theta_v)\cos(2\chi) H_+(w) H_-(w)$$
$$- 4\sin(\theta_\ell)(1-\cos(\theta_\ell))\sin(\theta_v)\cos(\theta_v)\cos(\chi) H_+(w) H_0(w)$$
$$+ 4\sin(\theta_\ell)(1+\cos(\theta_\ell))\sin(\theta_v)\cos(\theta_v)\cos(\chi) H_-(w) H_0(w)\}$$

- Between 7 to 10 bins per kinematic variable
- Available on HEPData with correlations
- Angular observables analysis are available, data just newly released

Fitting Strategies

Strategy A

- Fit BGL parametrisation to lattice data
- Compute theory predictions for $d\Gamma/dq^2/|V_{cb}|^2$
- Combine with experimental data
- "Final" extraction of $|V_{cb}|$ from a correlated weighted average of all binned-results

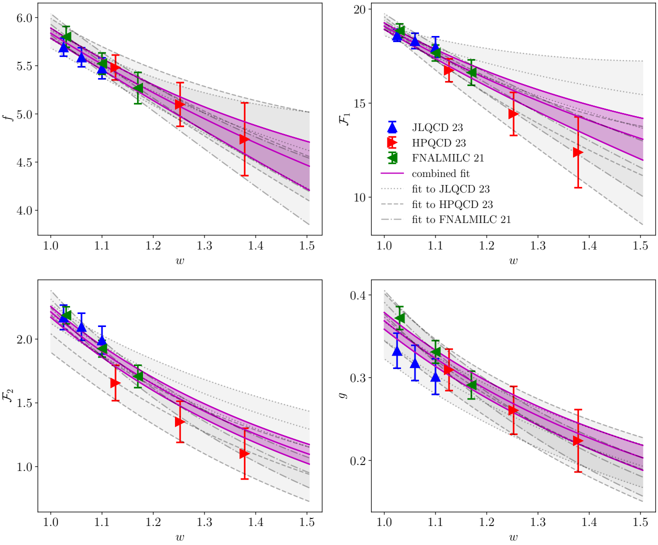
Strategy B

- Fit BGL parametrisation to lattice + experimental data at the same time
- Determine V_{cb} from such global fit

If the SM reproduces experimental data well,
there should be no tensions between A and B

Strategy A

[MB, A. Jüttner, '24]



Strategy A

[MB, A. Jüttner, '24]

Frequentist fit

K_f	$K_{\mathcal{F}_1}$	$K_{\mathcal{F}_2}$	K_g	$a_{g,0}$	$a_{g,1}$	$a_{g,2}$	$a_{g,3}$	p	χ^2/N_{dof}	N_{dof}
2	2	2	2	0.03138(87)	-0.059(24)	-	-	0.95	0.62	30
3	3	3	3	0.03131(87)	-0.046(36)	-1.2(1.8)	-	0.90	0.67	26
4	4	4	4	0.03126(87)	-0.017(48)	-3.7(3.3)	49.9(53.6)	0.79	0.75	22

- good fit quality
- lattice data are compatible
- no unitarity

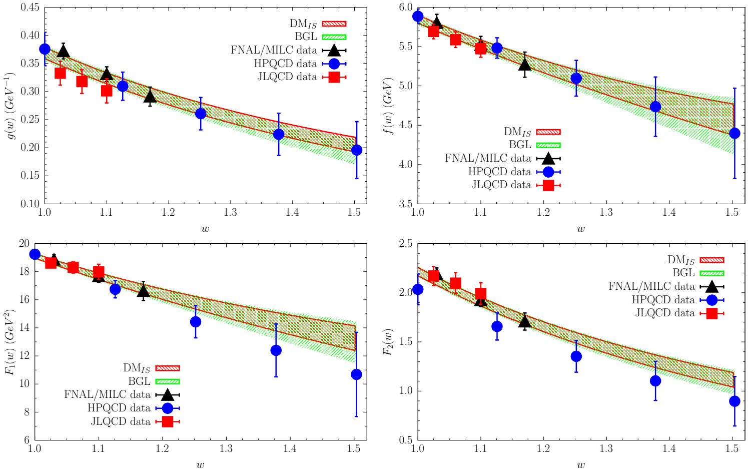
Bayesian Fit

K_f	$K_{\mathcal{F}_1}$	$K_{\mathcal{F}_2}$	K_g	$a_{g,0}$	$a_{g,1}$	$a_{g,2}$	$a_{g,3}$
2	2	2	2	0.03018(76)	-0.101(21)	-	-
3	3	3	3	0.03034(78)	-0.087(24)	-0.34(45)	-
4	4	4	4	0.03035(77)	-0.089(23)	-0.27(41)	-0.04(45)

- unitarity regulates higher orders
- truncation dependent

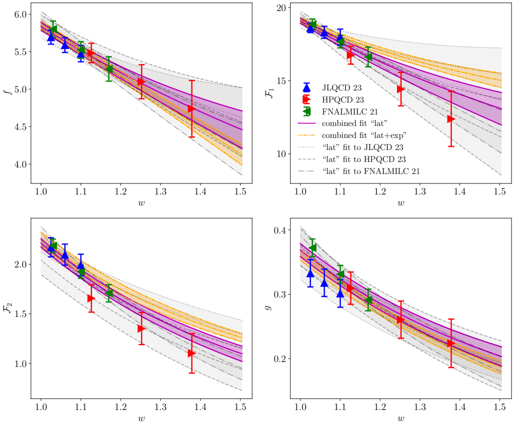
Comparison with DM

[G. Martinelli, S. Simula, L. Vittorio, '23]



Strategy B

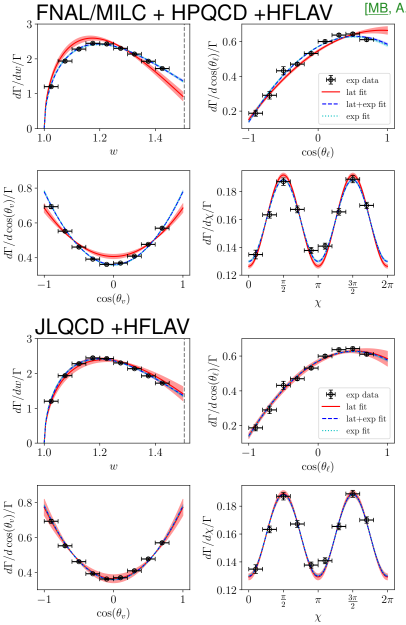
[MB, A. Jüttner, '24]



- Good fit quality for Strategy B (p-value $\sim 18\%$)
- Adding experimental data reduces the uncertainties, especially at large w
- Especially for $\mathcal{F}_1$ and $\mathcal{F}_2$, the shape changes between Strategy A and B

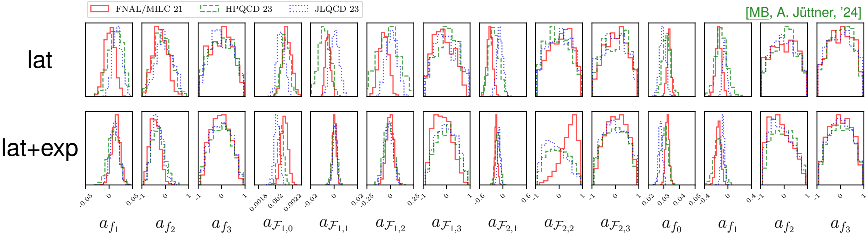
Strategy B

[MB, A. Jüttner, '24]



- Fit to HPQCD and FNAL/MILC misses experimental points
- BGL fit to experimental and lattice data has $p-$value $\sim 18\%$
- BGL coefficients shift of a few σ between Strategy A and B

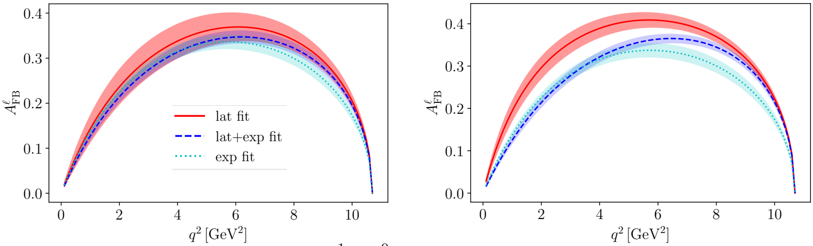
Posterior distribution



- Small shifts between lattice only and lattice + data
- Higher order coefficients well constrained by unitarity
- $a_{\mathcal{F}_{2,2}}$ has a strange behaviour, maybe kinematic constraints?

Differential observables

[MB, A. Jüttner, '24]

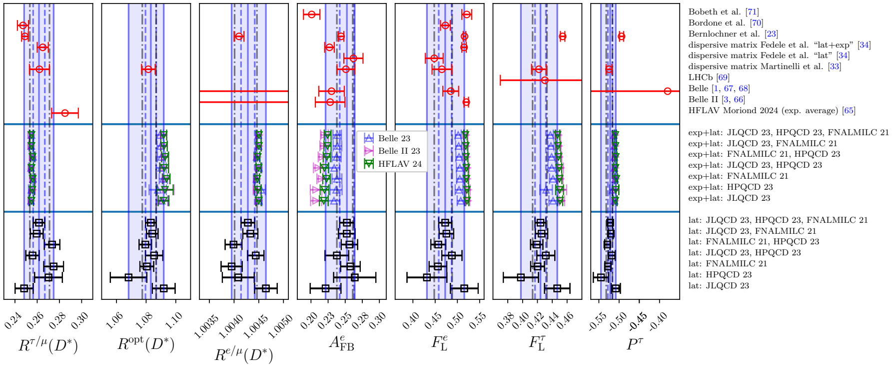


$$A^{\ell}_{\rm FB} = \frac{\int_0^1 - \int_{-1}^0 d\cos\theta_\ell d\Gamma/d\cos\theta_\ell}{\int_0^1 + \int_{-1}^0 d\cos\theta_\ell d\Gamma/d\cos\theta_\ell}$$

- The combined lattice + experimental precision makes it possible to study the differences in the shape
- It is clear that there is a distinct difference between JLQCD and FNAL/MILC+HPQCD
- Difficult to understand what is going on, JLQCD errors are also a bit larger

Integrated observables

[MB, A. Jüttner, '24]

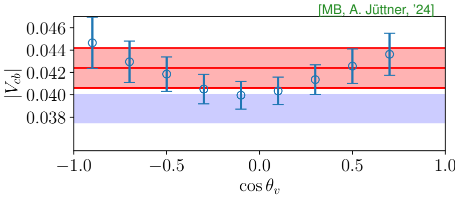


- Significant scatter between various combinations of lattice results
  - We apply a systematic error to account for the spread
- Consistent scatter of the experimental results independently of the lattice information

⇒ see also: Fedele et al, '23

$|V_{cb}|$ - Strategy A

$$|V_{cb}|_{\alpha,i} = \left(\Gamma_{\text{exp}} \left[\frac{1}{\Gamma}\frac{d\Gamma}{d\alpha}\right]^{(i)}_{\text{exp}} \Big/ \left[\frac{d\Gamma_0}{d\alpha}(\mathbf{a})\right]^{(i)}_{\text{lat}}\right)^{1/2}$$



Blue band

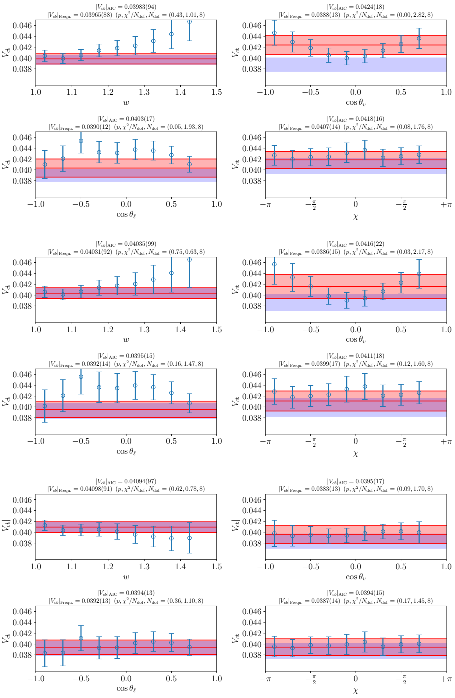
- Frequentist fit $p-$value $\sim 0\%$
- Affected by d'Agostini Bias

Red band

- Frequentist fit $p-$value $\sim 0\%$
- Akaike-Information-Criterion analysis: average over all possible fits with at least two data points and then weighted average

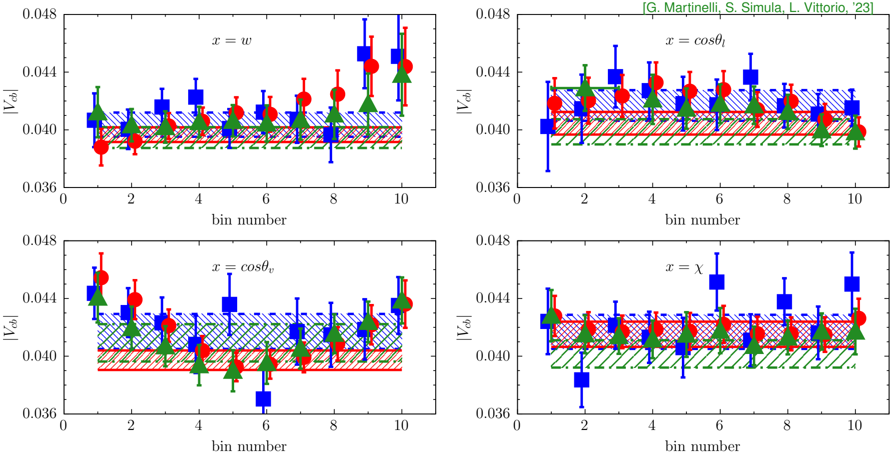
$$w_{\{\alpha,i\}} = \mathcal{N}^{-1}\exp\left(-\frac{1}{2}(\chi^2_{\{\alpha,i\}} - 2N_{\text{dof},\{\alpha,i\}})\right) \qquad \text{where} \quad \mathcal{N} = \sum_{\text{sets}\,\{\alpha,i\}} w_{\text{set}}$$

$$|V_{cb}| = \langle|V_{cb}|\rangle \equiv \sum_{\text{sets}\,\{\alpha,i\}} w_{\text{set}}|V_{cb}|_{\text{set}}$$



- Analysis based on Strategy A
- The AIC nicely reduces the d'Agostini bias
- Some lattice data behave strangely
- Would it be safer to discard the angular distributions?
- Combining the three lattice datasets doesn't help, shape driven by FNAL/MILC and HPQCD
- Good compatibility with Strategy B

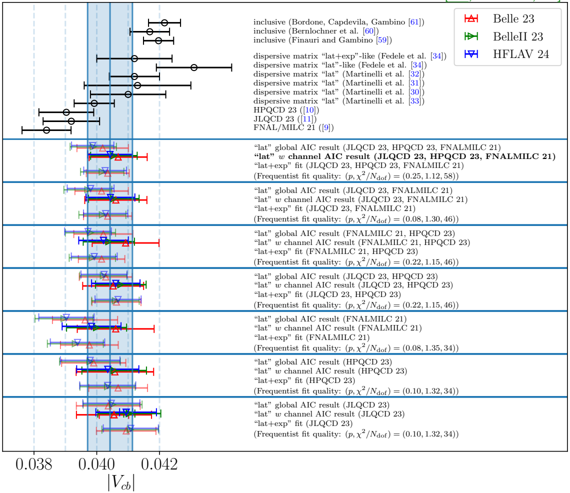
Results from the DM method



⇒ similar behaviour as we observe

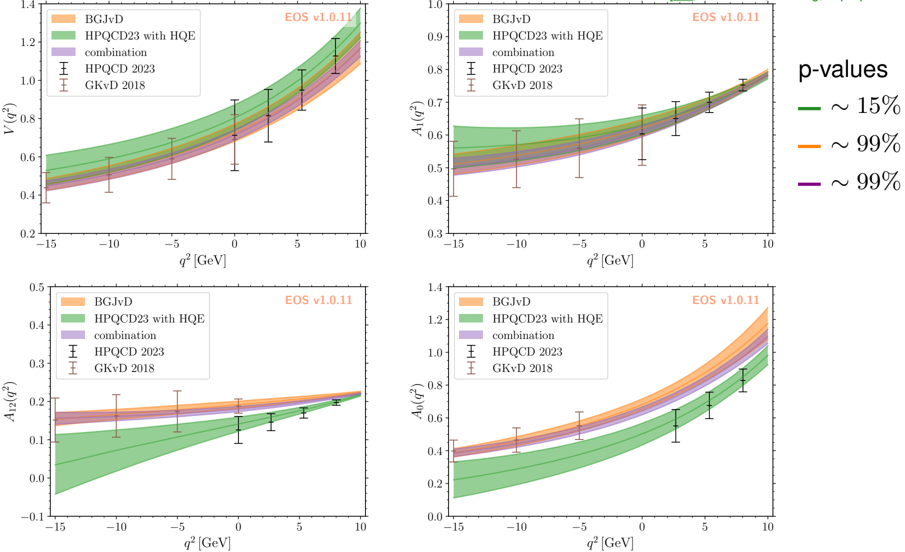
$|V_{cb}|$ - Summary

[MB, A. Jüttner, '24]



- Residual 2σ difference with inclusive
- The AIC produces slightly larger uncertainties, overall all results are quite consistent

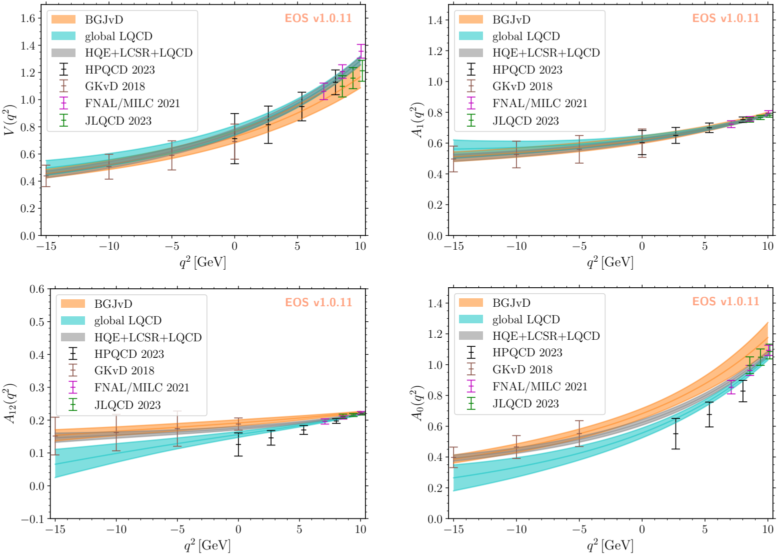
[MB, J. Harrison, M. Jung, in preparation]



- If for V and A_1 everything aligns well, for A_{12} and A_0 there is an evident shift in slopes
- In the combination difference is milder

Combining with other LQCD results

[MB, J. Harrison, M. Jung, in preparation]



p-values

- $\sim 70\%$
- $\sim 99\%$
- $\sim 98\%$

- All LQCD $B \to D^*$ results can be described in a global fit to the HQE
- R_D is in tension with $B \to D$ LQCD w/o LCSRs

Fitting HPQCD lattice data at multiple m_h with HQE

Lattice data includes $m_h = 1.5m_c$, $m_h = 0.9m_b$

- need to add $1/m_h^2$ terms to continuum HQE parameterisation.
- Also add generic α_s and α_s^2 contributions

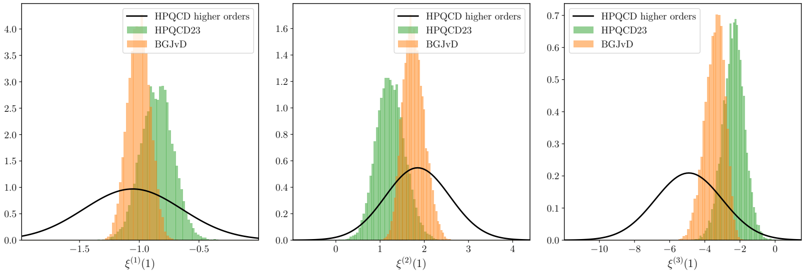
$$F_i = \left(a_i + b_i \frac{\alpha_s}{\pi} + k_i \frac{\alpha_s^2}{\pi^2}\right)\xi + \frac{\Lambda_{\rm QCD}}{2m_h}\sum_j c_{ij}\xi^j_{\rm SL} + \frac{\Lambda_{\rm QCD}}{2m_c}\sum_j d_{ij}\xi^j_{\rm SL}$$
$$+ \left(\frac{\Lambda_{\rm QCD}}{2m_c}\right)^2 \sum_j g_{ij}\xi^j_{\rm SSL} + \left(\frac{\Lambda_{\rm QCD}}{2m_h}\right)^2 \sum_j h_{ij}\xi^j_{\rm SSL}$$
$$+ \sum_{\substack{n,m=1,2\\ q=h,c}} \left(\frac{\Lambda_{\rm QCD}}{2m_q}\right)^n \left(\frac{\alpha_s}{\pi}\right)^m \zeta_i^{n,m,q}$$

with $\zeta = \zeta(1) + \zeta'(1)(w-1)$

- Use the same uniform prior widths for ξ, $\xi^j_{\rm SL}$ and $\xi^j_{\rm SSL}$, use gaussian priors of 0 ± 10 for $\zeta_i^{n,m,q(\prime)}(1)$, h_{ij}, k_i
- discretisation and chiral effects added analogously to 2304.03137

What changes in terms of parameters?

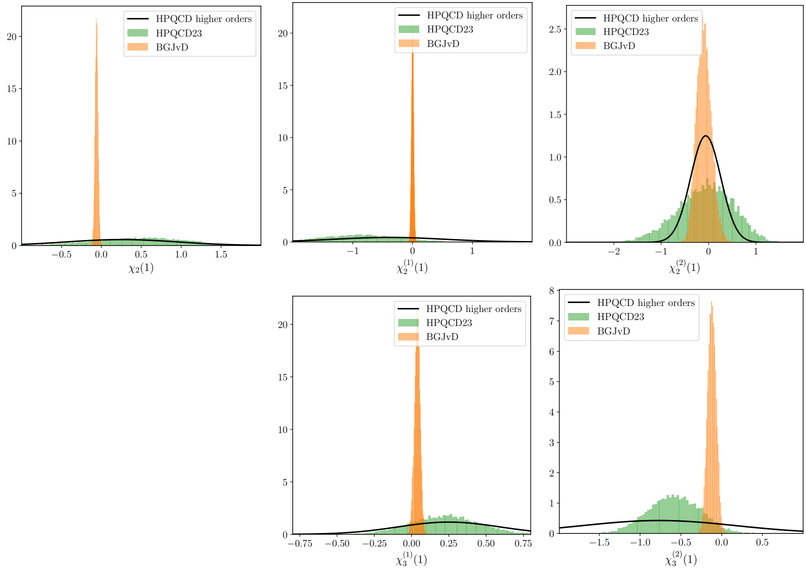
Isgur-Wise function: ξ



- Discrepancies arise in ξ at higher order in $(w-1)$, reflects difference seen in slope
- Similar situation comparing to combined fit posteriors

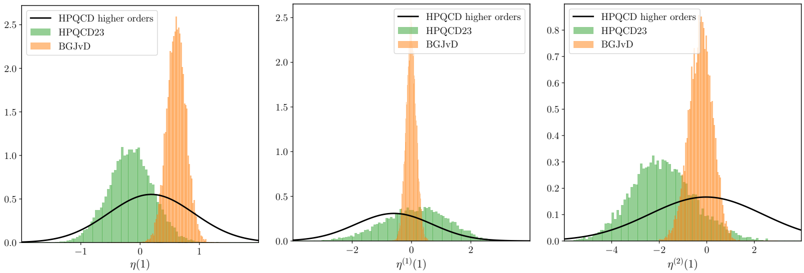
What changes in terms of parameters?

Sub-leading Isgur-Wise functions: χ_2, χ_3



What changes in terms of parameters?

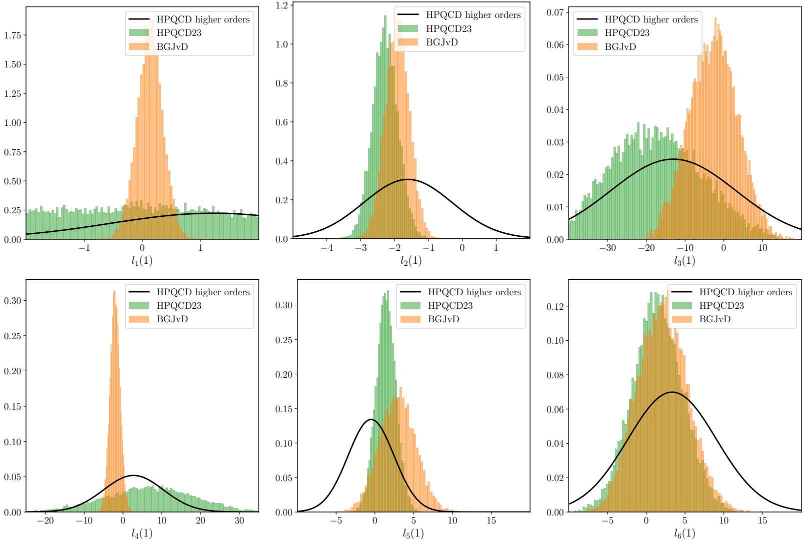
Sub-leading Isgur-Wise functions: η



- From QCDSR, $\eta(1)$ should be positive and different from zero

What changes in terms of parameters?

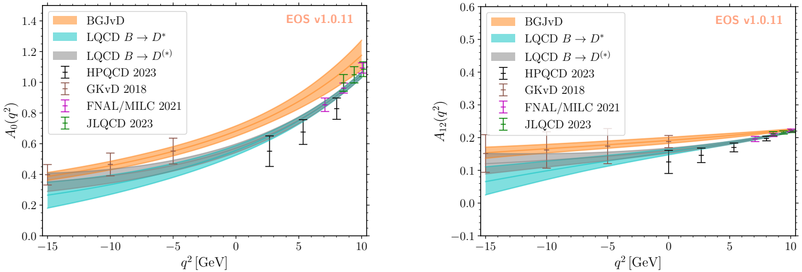
Sub-sub-leading Isgur-Wise functions: $l_i(1)$



Combining $B \to D$ and $B \to D^*$

- HQET allows to determine $B \to D$ and $B \to D^*$ form factors with a unique parametrisation

- Can $B \to D$ lattice data help?



- The inclusion of $B \to D$ data helps changing slightly the slope

- Motivated a correlated $B \to D$ and $B \to D^*$ lattice analysis

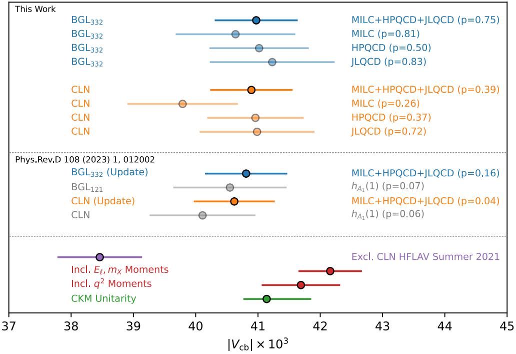
What else?

Measurements of angular observables:

$$\frac{\mathrm{d}\Gamma(\bar{B}\to D^*\ell\bar{\nu}_\ell)}{\mathrm{d}w\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_\mathrm{V}\,\mathrm{d}\chi} = \frac{2G_\mathrm{F}^2\eta_\mathrm{EW}^2|V_\mathrm{cb}|^2 m_B^4 m_{\mathrm{D}^*}}{2\pi^4}\times\Big(J_{1s}\sin^2\theta_\mathrm{V}+J_{1c}\cos^2\theta_\mathrm{V}$$
$$+(J_{2s}\sin^2\theta_\mathrm{V}+J_{2c}\cos^2\theta_\mathrm{V})\cos 2\theta_\ell + J_3\sin^2\theta_\mathrm{V}\sin^2\theta_\ell\cos 2\chi$$
$$+J_4\sin 2\theta_V\sin 2\theta_\ell\cos\chi + J_5\sin 2\theta_V\sin\theta_\ell\cos\chi + (J_{6s}\sin^2\theta_\mathrm{V}+J_{6c}\cos^2\theta_\mathrm{V})\cos\theta_\ell$$
$$+J_7\sin 2\theta_V\sin\theta_\ell\sin\chi + J_8\sin 2\theta_V\sin 2\theta_\ell\sin\chi + J_9\sin^2\theta_\mathrm{V}\sin^2\theta_\ell\sin 2\chi\Big)\,.$$

- Angular observables are a complete base
- It is not redundant

[2310.20286]

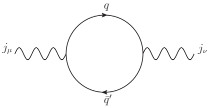


Conclusions

- $B \to D^*$ form factor are a crucial inputs to extract V_{cb} and ultimately in the search for new physics
- New Lattice results are a step forward
- Nevertheless, we notice a difference that have large impact on phenomenology
- Belle and Belle II have precise results for angular distributions and angular observables
- Belle II has the same precision of the untagged 2018 Belle analysis due to the inclusive tagging
- Angular distribution data still to be analysed
- Are all these experimental datasets consistent?

Appendix

Unitarity Bounds



$$= i\int d^4x\, e^{iqx}\langle 0|T\left\{j_\mu(x), j_\nu^\dagger(0)\right\}|0\rangle = (g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

- If $q^2 \ll m_b^2$ we can calculate $\Pi(q^2)$ via perturbative techniques $\Rightarrow \chi(0)$
- Dispersion relations link Im $\left(\Pi(q^2)\right)$ to sum over matrix elements

$$\sum_i |F_i(0)|^2 < \chi(0)$$

[Boyd, Grinstein,Lebed, '95
Caprini, Lellouch, Neubert, '97]

- The sum runs over **all** possible states hadronic decays mediated by a current $\bar{c}\Gamma_\mu b$
  - The unitarity bounds are more effective the most states are included in the sum
  - The unitarity bounds introduce correlations between FFs of different decays
  - $B_s \to D_s^{(*)}$ decays are expected to be of the same order of $B_{u,d} \to D_{u,d}^{(*)}$ decays due to $SU(3)_F$ simmetry