Comparison of Lattice QCD Results for

Judd Harrison (HPQCD)
Shoji Hashimoto (JLQCD)
Andreas Kronfeld & Alejandro Vaquero (Fermilab Lattice and MILC)

Lattice @ CERN
19 February 2024

Plan

- Summary of Fermilab-MILC, *Eur. Phys. J. C* **82** (2022) 1141 [arXiv:2105.14019], by A.S.K. with help from Alex Vaquero.
- Summary of HPQCD, *Phys. Rev. D* **109** (2024) 094515 [arXiv:2304.03137], by Judd Harrison.
- Summary of JLQCD, *Phys. Rev. D* 109 (2024) 074503 [arXiv:2306.05657], by Shoji Hashimoto.
- Responses to questions from the organizers, by all.

Challenges in Semileptonic B Decays

Sep 23 – 27, 2024
Europe/Zurich timezone

Enter your search term

- Overview
- Call for Abstracts
- Registration
- Venue
- How to reach the venue
- Public Transport

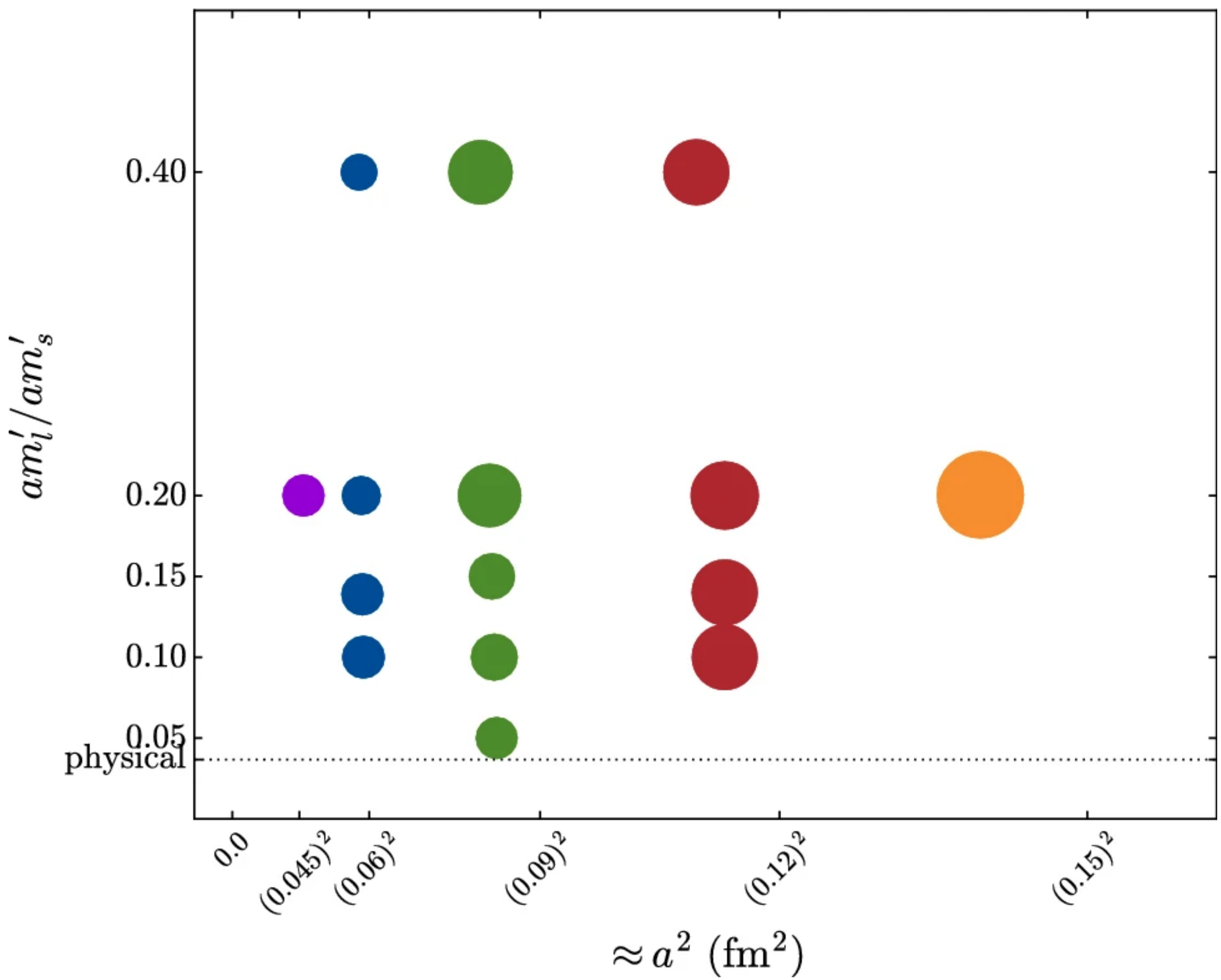
The aim of this workshop is to review recent progress in semileptonic B decays, both on the theory and the experimental front, and to form a consensus on how to extract the CKM matrix elements Vcb and Vub from inclusive and exclusive decays. Another topic of the workshop is the test of lepton flavour universality in semileptonic B decays in particular in the third generation.

This workshop follows previous workshops held at MITP (2015 and 2018) and in Barolo (2022). The venue of this edition is Campus Akademie, located in the center of Vienna, Austria.

Fermilab Lattice and MILC Collaborations

The Ensembles

- MILC asqtad 2+1 ensembles.
- asqtad spectator quark.
- b and c: clover action w/ “Fermilab” interpretation.
- area $\propto$ number of samples.



Form Factors from Ratios

$$h_{A_1}(w) = \rho_{A_1} \frac{2R_{A_1}}{w+1}$$

$$h_{A_2}(w) = \rho_{A_1} \frac{2R_{A_1}}{w^2-1} \left(wX_1 - \sqrt{w^2-1}\,\frac{\rho_{A_4}}{\rho_{A_1}} X_0 - 1 \right)$$

$$h_{A_3}(w) = \rho_{A_1} \frac{2R_{A_1}}{w^2-1} (w - X_1)$$

$$h_V(w) = \rho_{A_1} \frac{2R_{A_1}}{\sqrt{w^2-1}} \frac{\rho_{V_1}}{\rho_{A_1}} X_V$$

mostly nonperturbative matching;
remaining ρ_J to one loop;
blinded $\beta\rho_{A1}$ until the very end.

$$R_{A_1}^2 = \frac{\langle D^*(\boldsymbol{p}_\perp)|A_j|B(\mathbf{0})\rangle\langle B(\mathbf{0})|A_j|D^*(\boldsymbol{p}_\perp)\rangle}{\langle D^*(\mathbf{0})|V_4|B(\mathbf{0})\rangle\langle B(\mathbf{0})|V_4|D^*(\mathbf{0})\rangle}$$

really ratios of Euclidean
correlation functions;
excited states fitted

$$X_0 = \frac{\langle D^*(\boldsymbol{p}_\parallel)|A_4|B(\mathbf{0})\rangle}{\langle D^*(\boldsymbol{p}_\perp)|A_j|B(\mathbf{0})\rangle}, \qquad X_1 = \frac{\langle D^*(\boldsymbol{p}_\parallel)|A_j|B(\mathbf{0})\rangle}{\langle D^*(\boldsymbol{p}_\perp)|A_j|B(\mathbf{0})\rangle}$$

$$X_V = \frac{\langle D^*(\boldsymbol{p}_\perp)|V_j|B(\mathbf{0})\rangle}{\langle D^*(\boldsymbol{p}_\perp)|A_j|B(\mathbf{0})\rangle}, \qquad x_f = \frac{\langle D^*(\boldsymbol{p}_{\perp,\parallel})|V_j|D^*(\mathbf{0})\rangle}{\langle D^*(\boldsymbol{p}_{\perp,\parallel})|V_4|D^*(\mathbf{0})\rangle} \Rightarrow w$$

Chiral-Continuum Extrapolation

- Formula inspired by Symanzik EFT, XPT, HQET:

$$h_Y(a, m, m_s, w) = \left([1 \text{ or } 0]_Y + f_Y^{\text{NLO}} + \chi_Y + f_Y^{\text{NNLO}} + f_Y^w\right)\left(1 + f_Y^{\text{HQ}}\right)$$

$$f_Y^{\text{NLO}} \leftarrow \text{one-loop } \chi\text{PT for heavy mesons w/ staggered sea \& spectator}$$

$$\chi_Y \sim \left(\frac{\Lambda_\chi}{m_c}\right)^{1 \text{ or } 2} \quad \text{to cancel } \Lambda_\chi \text{ in } f_Y^{\text{NLO}} \text{ logs}$$

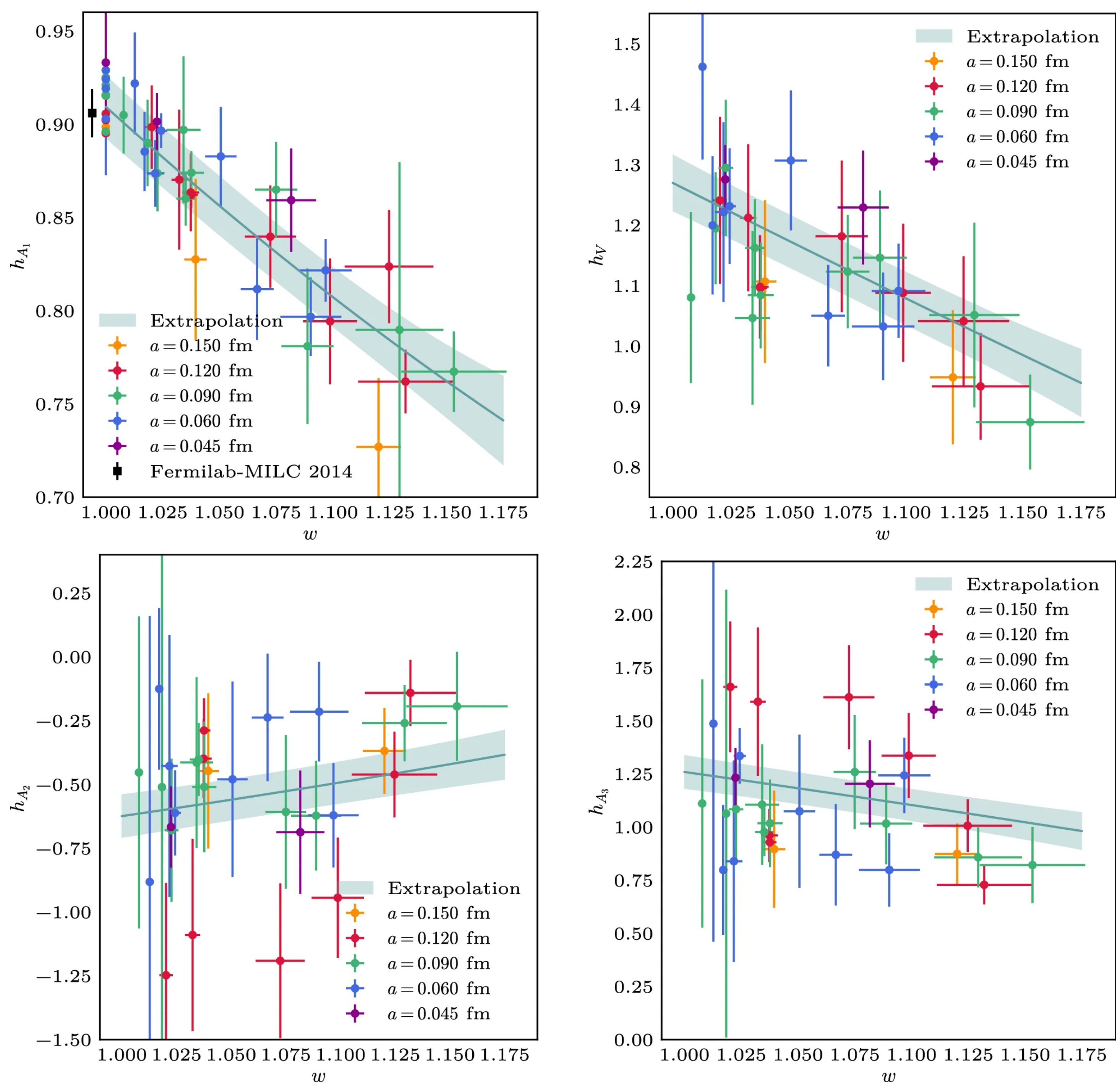
$$f_Y^{\text{NNLO}} = c_{c,Y} x_l x_{a^2} + c_{m_l^2,Y} x_l^2 + c_{a^2,Y} x_{a^2}^2$$

$$f_Y^w = -\rho_Y^2 (w-1) + \kappa_Y (w-1)^2$$

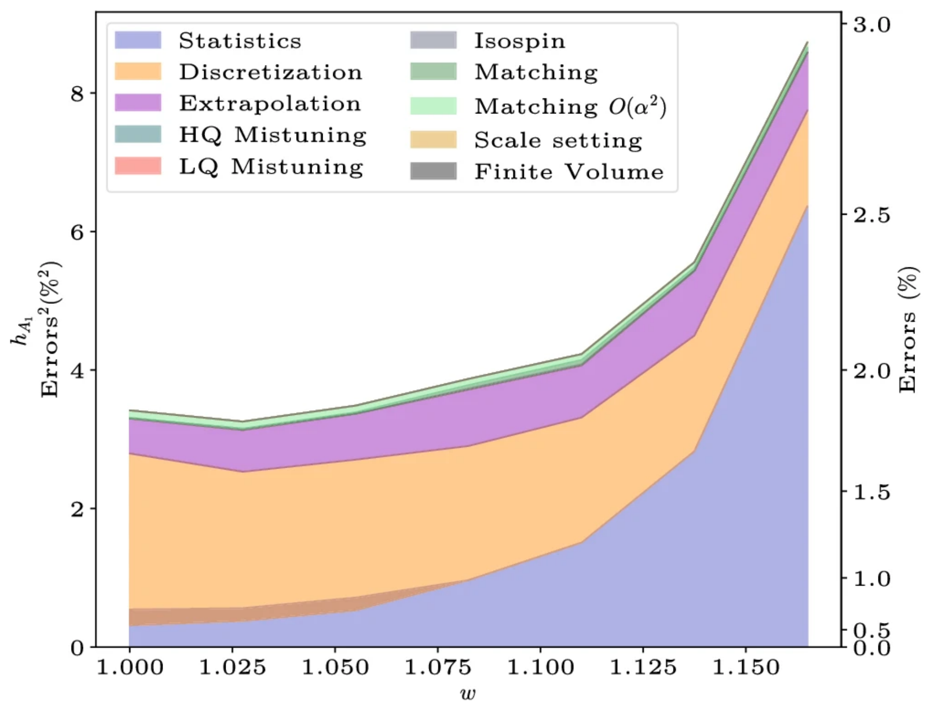
$$f_Y^{\text{HQ}} = \beta_Y^{\alpha_s a} \alpha_s a \Lambda_{\text{QCD}} + \beta_Y^{a^2} a^2 \Lambda_{\text{QCD}}^2 + \beta_Y^{a^3} a^3 \Lambda_{\text{QCD}}^3$$

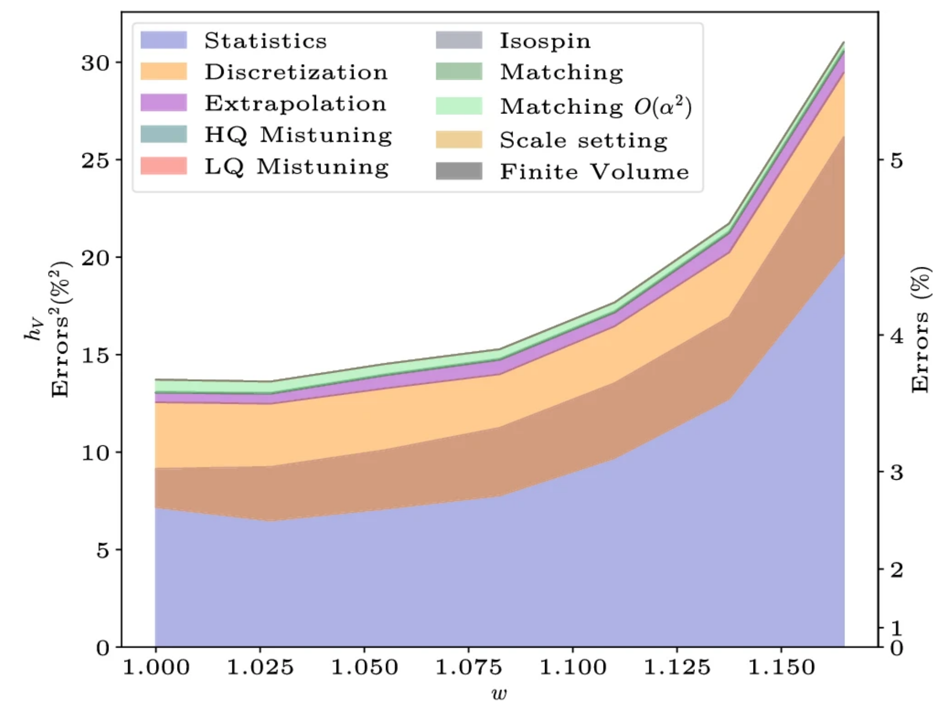
- Priors on all fit parameters with 0 central value, “reasonable” widths.

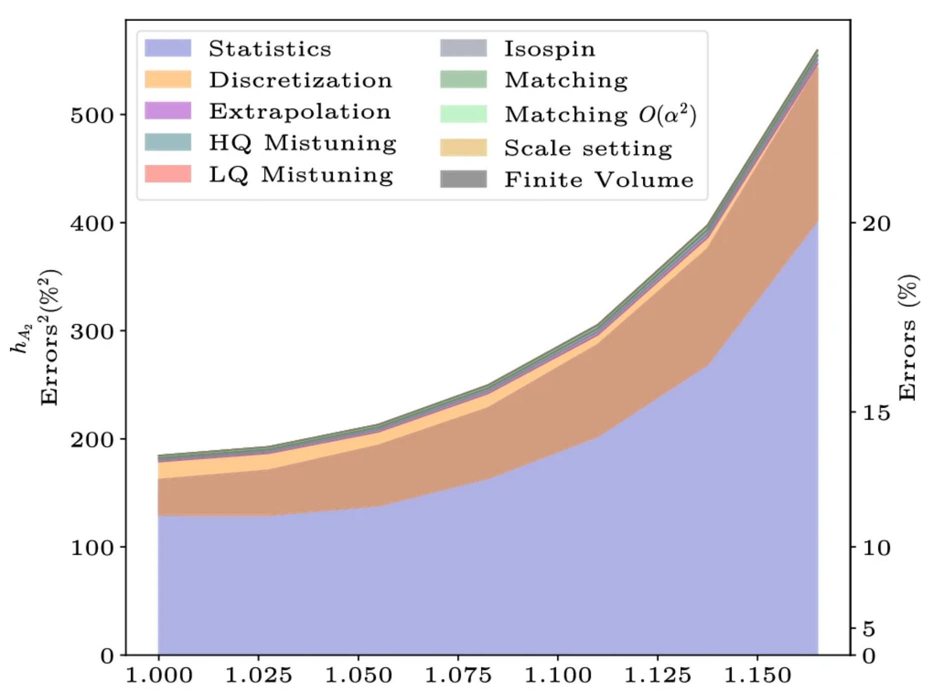
Results

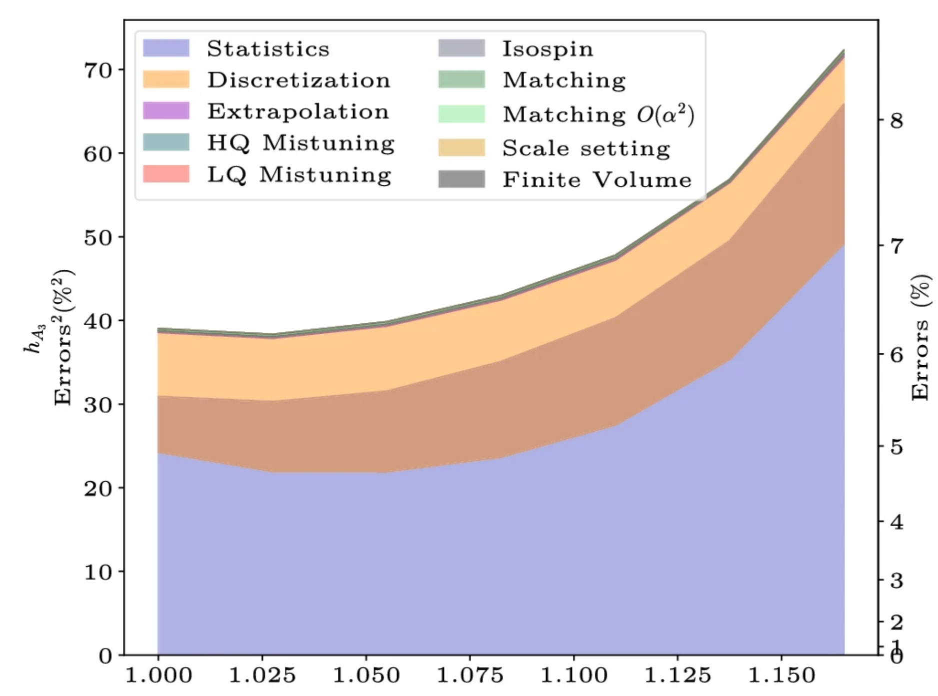

Extrapolation
a = 0.150 fm
a = 0.120 fm
a = 0.090 fm
a = 0.060 fm
a = 0.045 fm
Fermilab-MILC 2014
h_{A_1}
h_V
h_{A_2}
h_{A_3}
w

Systematics

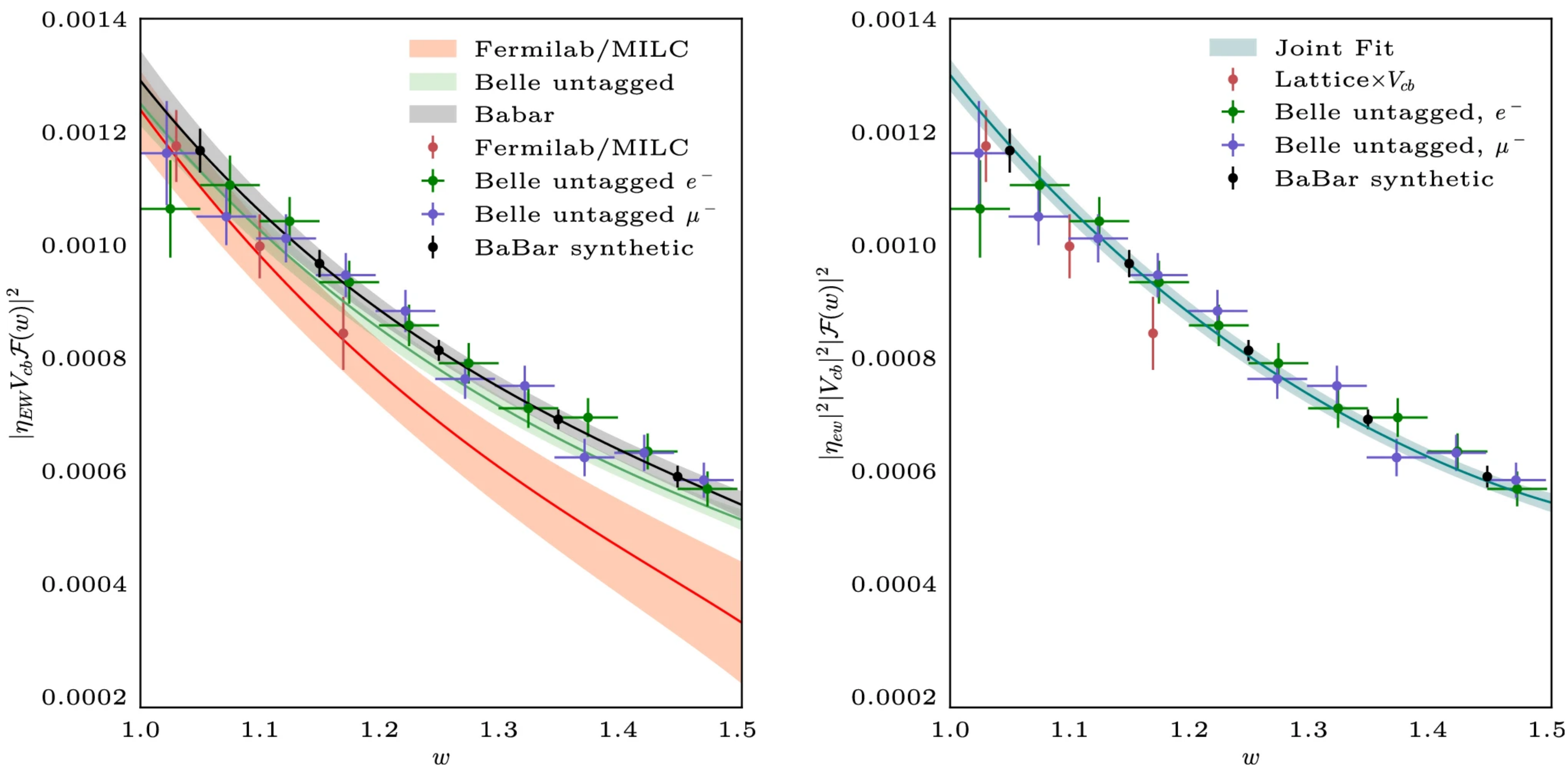


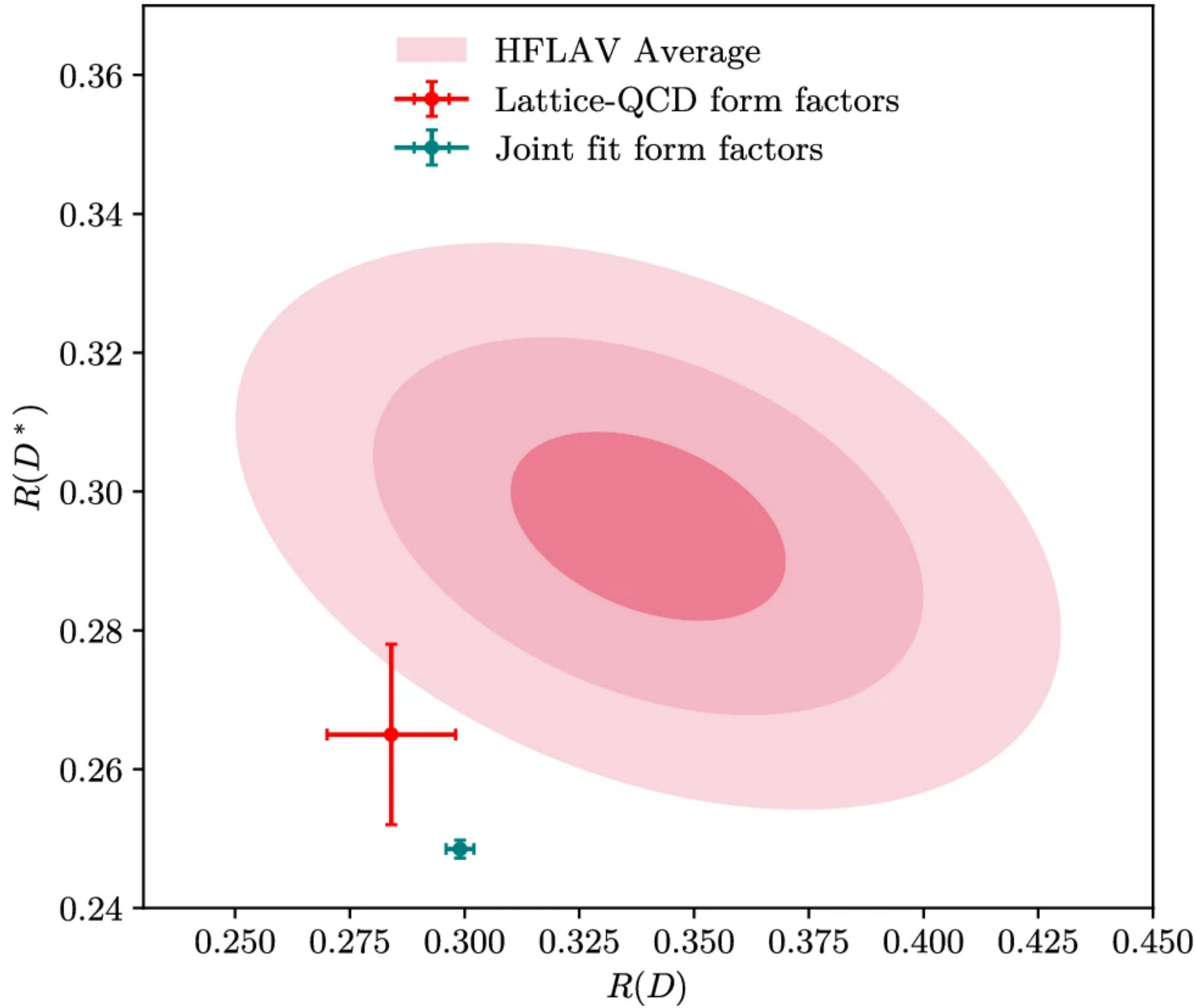






z Expansion & Phenomenology




HFLAV Average
Lattice-QCD form factors
Joint fit form factors
0.36
0.34
0.32
0.30
0.28
0.26
0.24
$R(D^*)$
0.250
0.275
0.300
0.325
0.350
0.375
0.400
0.425
0.450
$R(D)$

Comparison with Belle 2023

[arXiv:2301.07529](https://arxiv.org/abs/2301.07529)

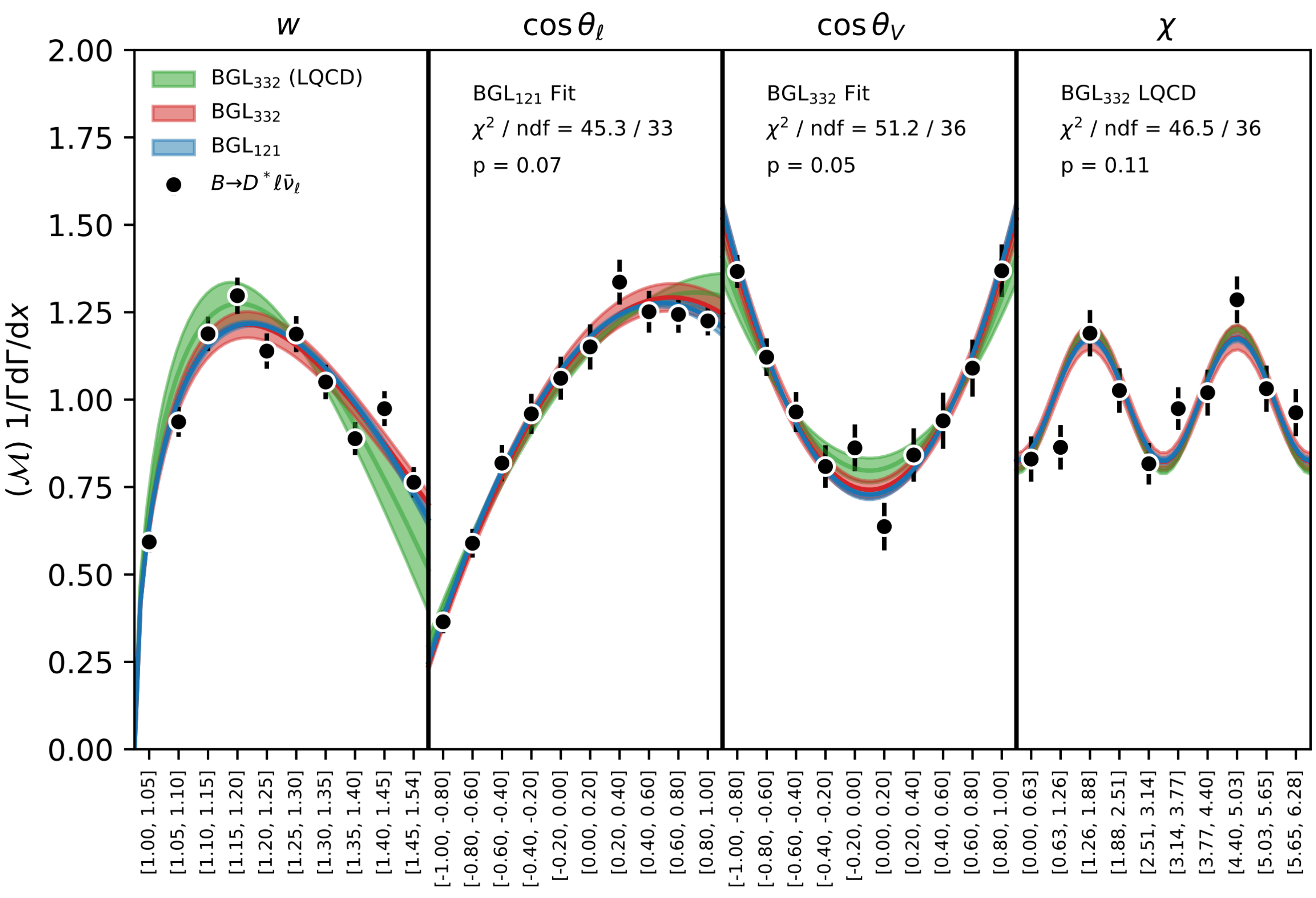


FIG. 12. The fitted shapes for our nominal BGL_{121} (blue) and CLN (orange) scenarios from the main text using the zero-recoil point only. The result of the BGL_{332} fit with the constraints from Ref. [16] on the BGL coefficients is shown in red.

HPQCD Collaboration

$B \to D^*$ from HPQCD

Judd Harrison

Lattice@CERN
Monday 15 July 2024

HPQCD 2304.03137

For $B \to D^*$, the matrix elements are parameterised in terms of form factors. The non-zero matrix elements (I will focus on the SM FFs here) are

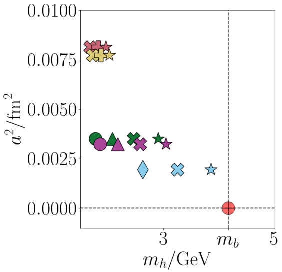
$$\langle D^*(\lambda, p')|\bar{c}\gamma^\mu b|\overline{B}(p)\rangle = i\sqrt{M_B M_{D^*}}\varepsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu(\lambda, p')v'_\alpha v_\beta h_V$$

$$\langle D^*(\lambda, p')|\bar{c}\gamma^\mu\gamma^5 b|\overline{B}(p)\rangle = \sqrt{M_B M_{D^*}}\big[h_{A_1}(w+1)\epsilon^{*\mu}(\lambda, p') - h_{A_2}(\epsilon^*(\lambda, p')\cdot v)v^\mu - h_{A_3}(\epsilon^*(\lambda, p')\cdot v)v'^\mu\big]$$

where $v' = p'/M_{D^*}$, $v = p/M_B$.

We compute matrix elements on the lattice for different combinations of λ and μ.

The HPQCD calculation uses data at multiple bottom quark masses, m_h, up to the physical value.



Some care should be taken when performing the continuum extrapolation, as the matrix elements for h_{A_2}, h_{A_3} and h_V vanish at $w = 1$

HPQCD 2304.03137

We use a HQET inspired function to describe the physical continuum form factors:

$$F^{Y^{(s)}}(w) = \sum_{n=0}^{10} a_n^{Y^{(s)}} (w-1)^n \mathcal{N}_n^{Y^{(s)}} + \frac{g^2_{D^*D\pi}}{16\pi^2 f_\pi^2}\left(\text{logs}^{Y^{(s)}}_{SU(3)} - \text{logs}^{Y^{(s)}}_{SU(3)\text{phys}}\right),$$

$$a_n^{Y^{(s)}} = \alpha_n^Y \times \left(1 + \sum_{j\neq 0}^{3} b_n^{Y,j}\Delta_h^{(j)} + \delta_\chi^{(s)} \sum_{j=0}^{3} \tilde{b}_n^{Y^{(s)},j}\Delta_h^{(j)}\right)$$

$$\delta_\chi^{(s)} = \left(\frac{M_{\pi(K)}}{\Lambda_\chi}\right)^2 - \left(\frac{M_\pi^{\text{phys}}}{\Lambda_\chi}\right)^2, \quad \Delta_h^{(j\neq 0)} = \left(\frac{\Lambda}{M_{H_s}}\right)^j - \left(\frac{\Lambda}{M_{B_s}^{\text{phys}}}\right)^j$$

Include $B_s \to D_s^*$ data, with $\delta_\chi^{(s)}$ term allowing for $\approx 25\%$ difference between $B \to D^*$ and $B_s \to D_s^*$. The α_n^Y are related to the BGL coefficients in the BGL form factor parameterisation:

$$\mathcal{F}^X(q^2) = \frac{1}{P^X(z)\phi^X(z)} \sum_{n=0}^{\infty} a_n^X z^n$$

$$\mathcal{F}^X(q^2) = L_{XY}(w)F^Y(w) \Rightarrow \alpha_n^Y = M_{nm,YX}\, a_m^X$$

by expanding z and $\frac{1}{P(z)\phi(z)}$ in powers of $(w-1)$, easy to compute $M_{nm,YX}$. We truncate the BGL expansion at order z^4, and use conservative priors of 0 ± 5 for a_n^X which we then convert to α_n^Y

HPQCD 2304.03137

Discretisation effects included at the level of matrix elements as

$$J^{\nu,\Gamma(s)}_{\rm latt} = J^{\nu,\Gamma(s)}_{\rm phys} + \sum_{j,n=0}^{3}\sum_{k,l\neq 0}^{3} c_n^{(\nu,\Gamma),jkl}\Delta_h^{(j)}(w-1)^n\left(\frac{am_c^{\rm val}}{\pi}\right)^{2k}\left(\frac{am_h^{\rm val}}{\pi}\right)^{2l}$$
$$+\sum_{j,n=0}^{3}\sum_{k,l\neq 0}^{3} \tilde{c}_n^{(\nu,\Gamma)(s),jkl}\Delta_h^{(j)}(w-1)^n\left(\frac{am_c^{\rm val}}{\pi}\right)^{2k}\left(\frac{am_h^{\rm val}}{\pi}\right)^{2l}\delta_\chi^{(s)}$$

This is important for the noisy form factors, such as h_{A_3}, where we have

$$h_{A_3} \sim \frac{J-J'}{p^2/M^2_{D^*}}$$

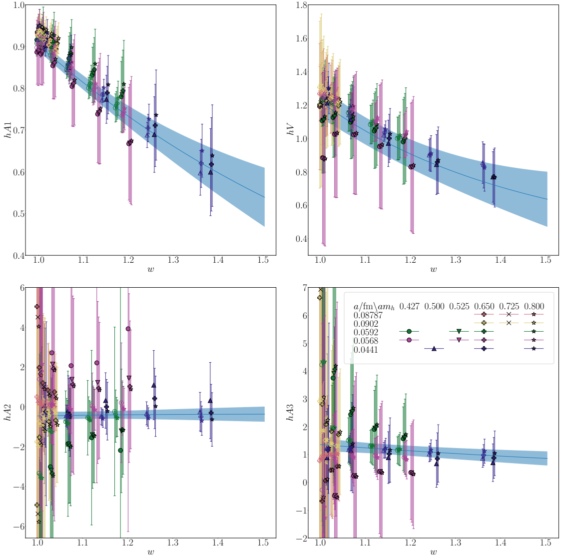
where J and J' are $\mathcal{O}(1)$ matrix elements.

In the continuum, with $m_h = m_b$, the $B \to D^*$ form factors in our parameterisation are given by

$$F^Y(w) = \sum_{n=0}^{10}\alpha_n^Y(w-1)^n.$$

HPQCD 2304.03137

SM and Tensor FFs with HISQ for c and h quarks, up to $m_h \approx 0.9 m_b$



HPQCD Future Prospects for $B \to D^{(*)}$ with heavy-HISQ

Our dominant uncertainty is statistics which may be reduced significantly on our currently available configurations by using:

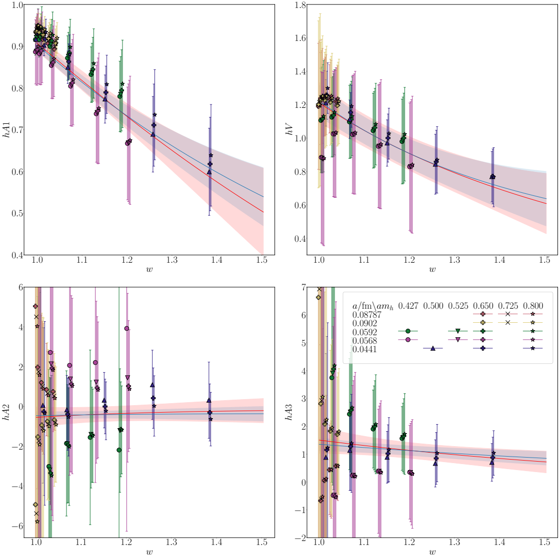
- More time sources
- Time-reversed correlation functions
- Rotationally equivalent correlation functions

Scope to also reduce systematic uncertainties:

- We can reach $m_h = m_b$ on several of the finest ensembles
- We can include a simultaneous lattice calculation of $B \to D$ form factors (this is currently in progress) $\Rightarrow$ include HQE relations at $\mathcal{O}(\alpha_s, 1/m_q)$
- Include dispersive bounds throughout $m_h \to m_b$ extrapolation, not just in the continuum limit.

Timeframe: 1-2 years

Fit to $B \to D^*$ only - what changes?



$$V_{cb} = 39.1(6)_{\mathrm{exp}}(9)_{\mathrm{latt}} \times 10^{-3},\ Q = 0.51$$

JLQCD Collaboration

B→D* from JLQCD (points to be improved)

Shoji Hashimoto (KEK, SOKENDAI)
on behalf of Takashi Kaneko

SOKENDAI

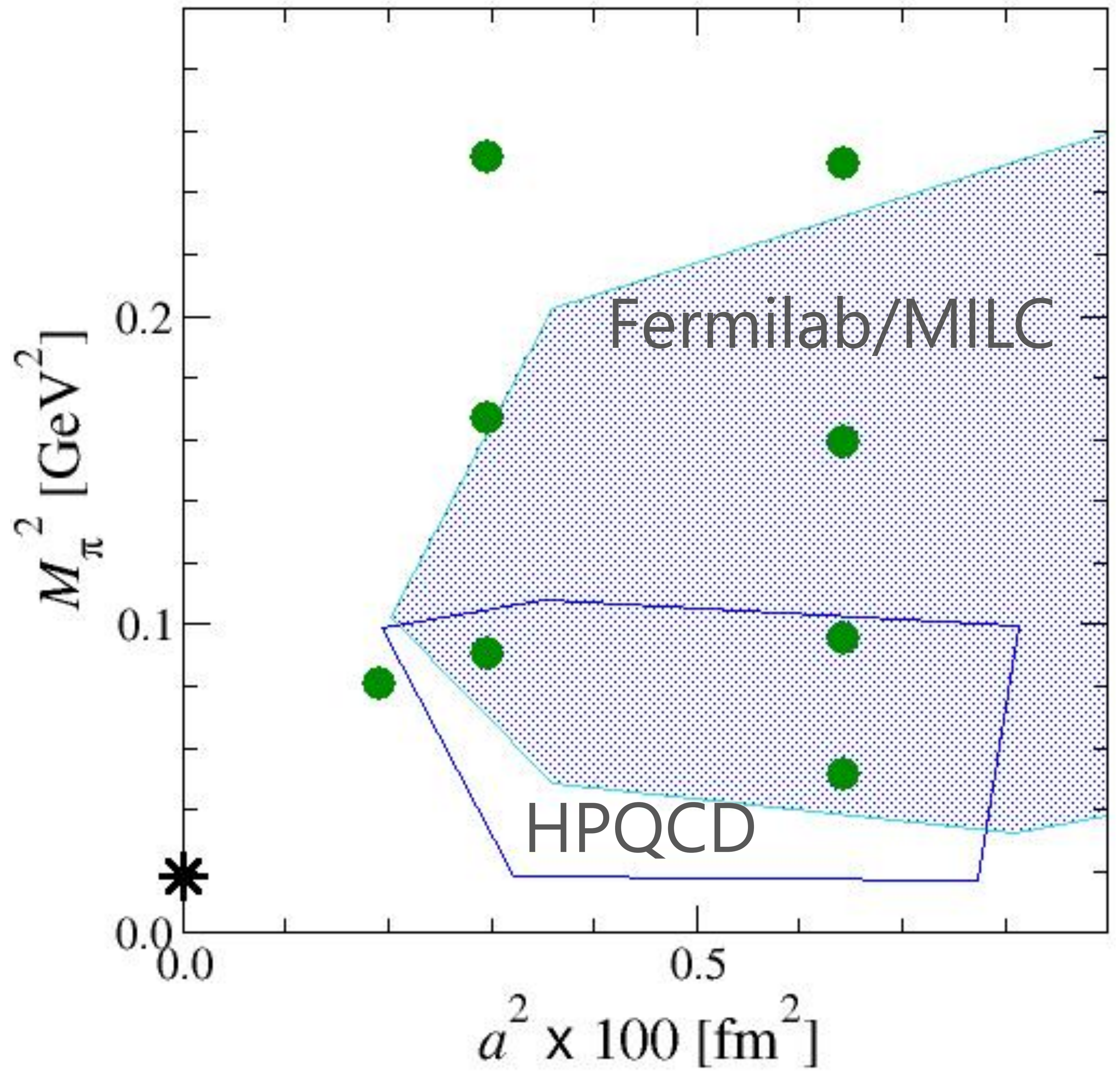
Jul 14, 2024

Lattice setup

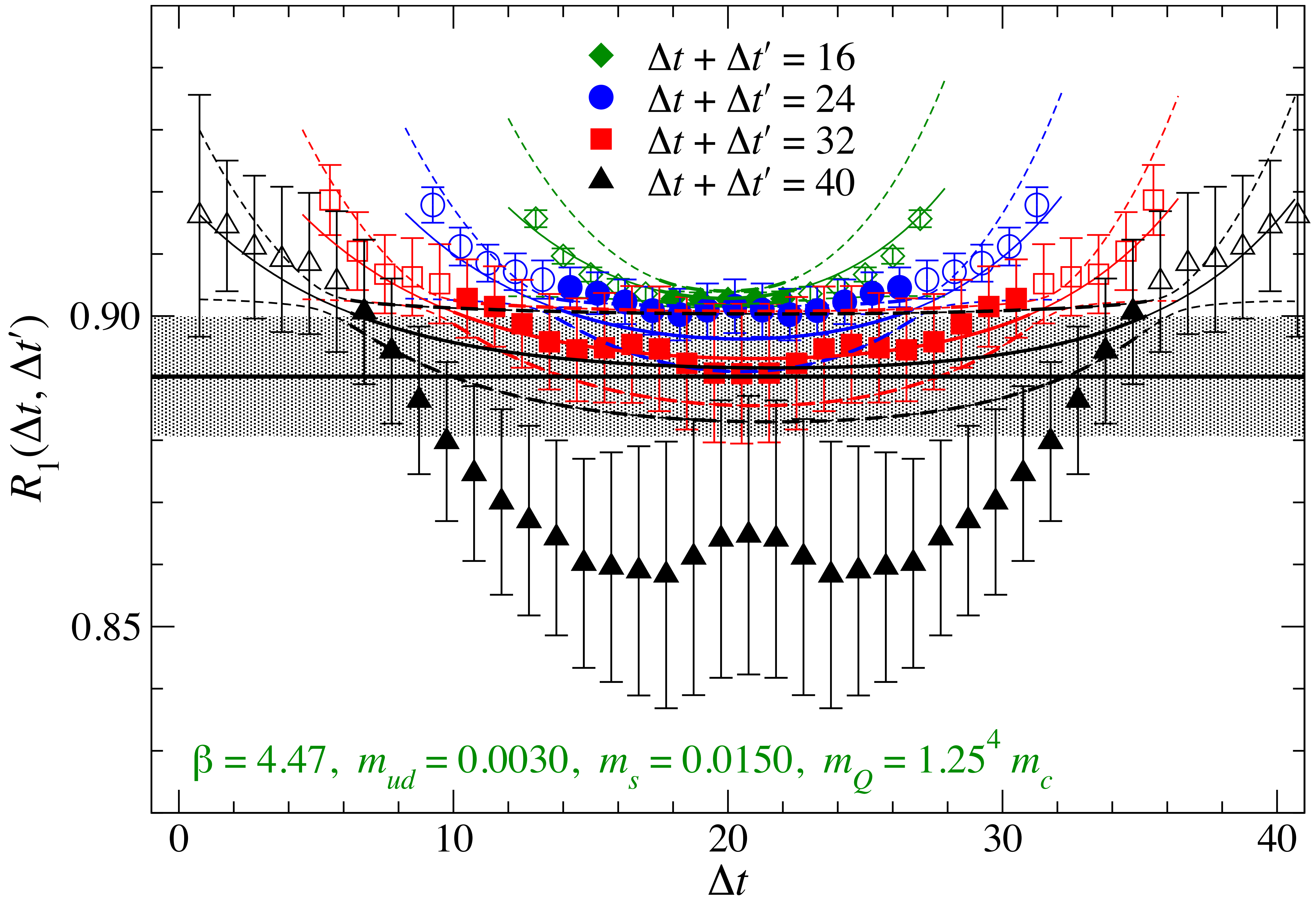
Mobius domain-wall fermion
for sea and valence quarks

- N_f=2+1
- 3 cutoffs $a^{-1} \lesssim$ 4.5 GeV ~ $m_{b,\mathrm{phys}}$
- $M_\pi \gtrsim$ 230 MeV
- ChPT to guide chiral extrapolation

- Scale from t_0.
- Charm quark mass tuned precisely.
- $m_b < 0.7/a$ to avoid too large disc effects, then extrapolate

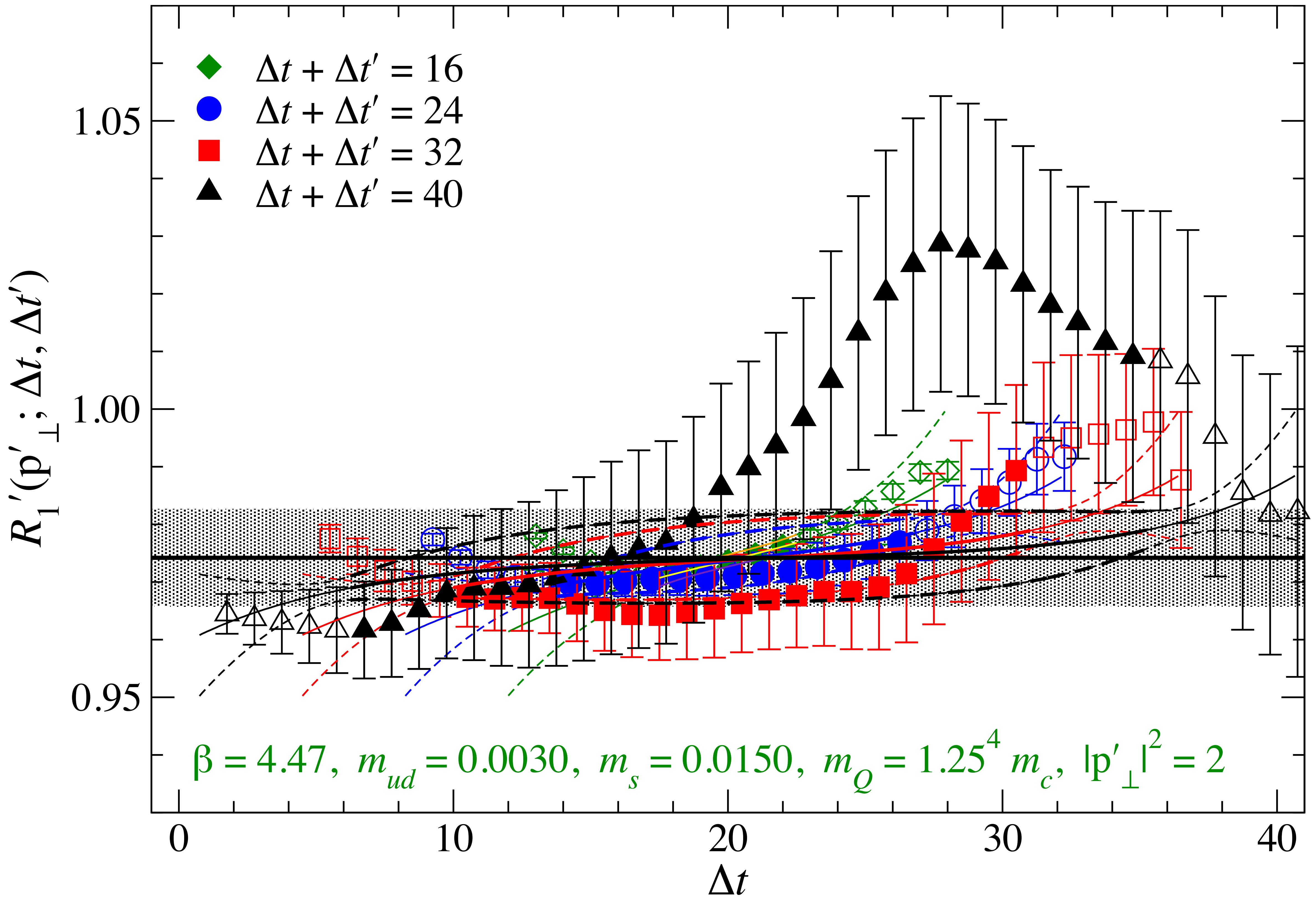


Excited-state contamination



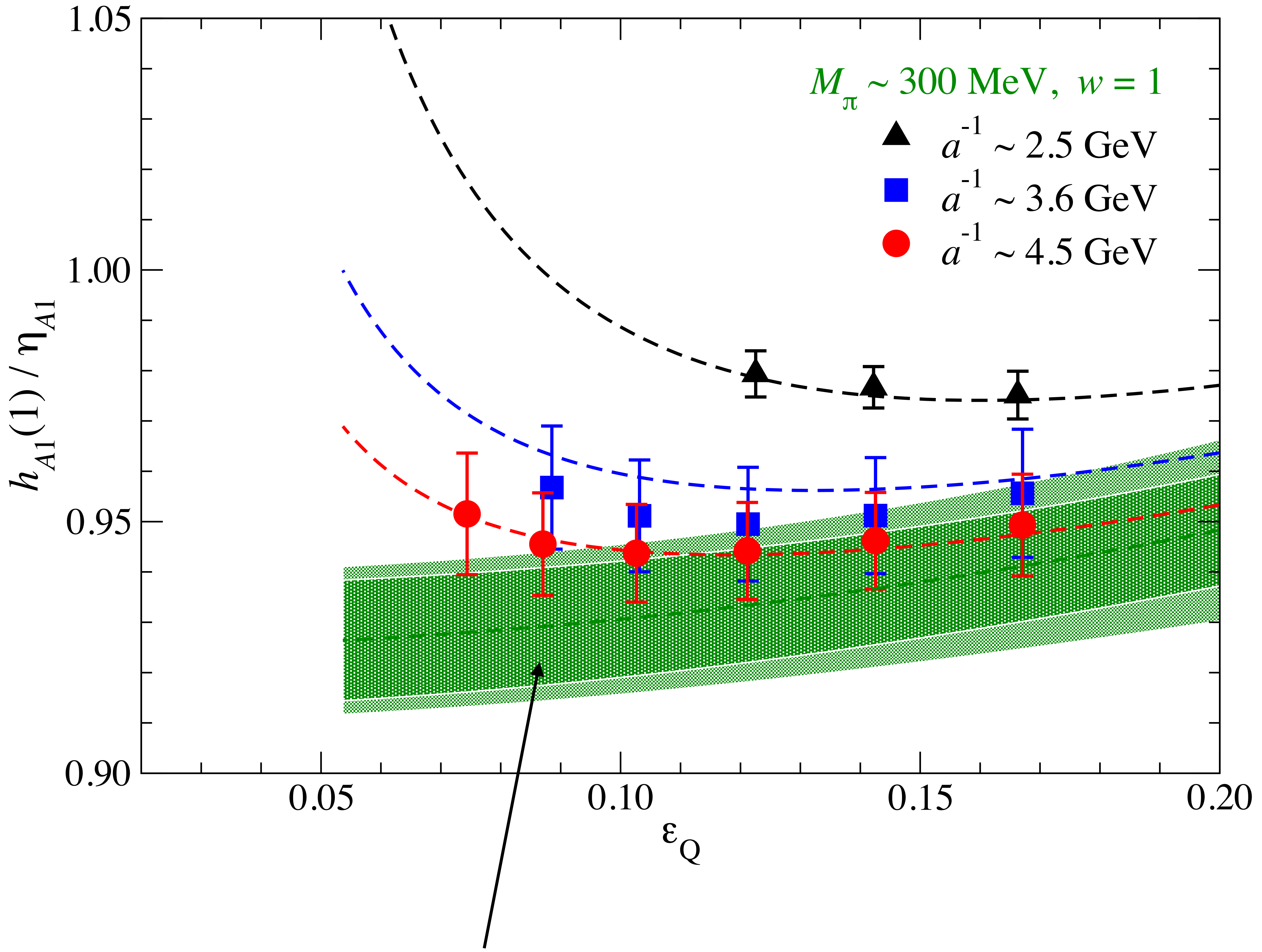
- Eliminated by a global fit involving the data at src-snk separation 0.7 ~ 1.6 fm. **Enough?**
- Better strategy to eliminate/ deplete excited state? Model with χPT, as in Bär et al.

Excited-state contamination



- Eliminated by a global fit involving the data at src-snk separation 0.7 ~ 1.6 fm. **Enough?**
- Better strategy to eliminate/ deplete excited state? Model with χPT, as in Bär et al.

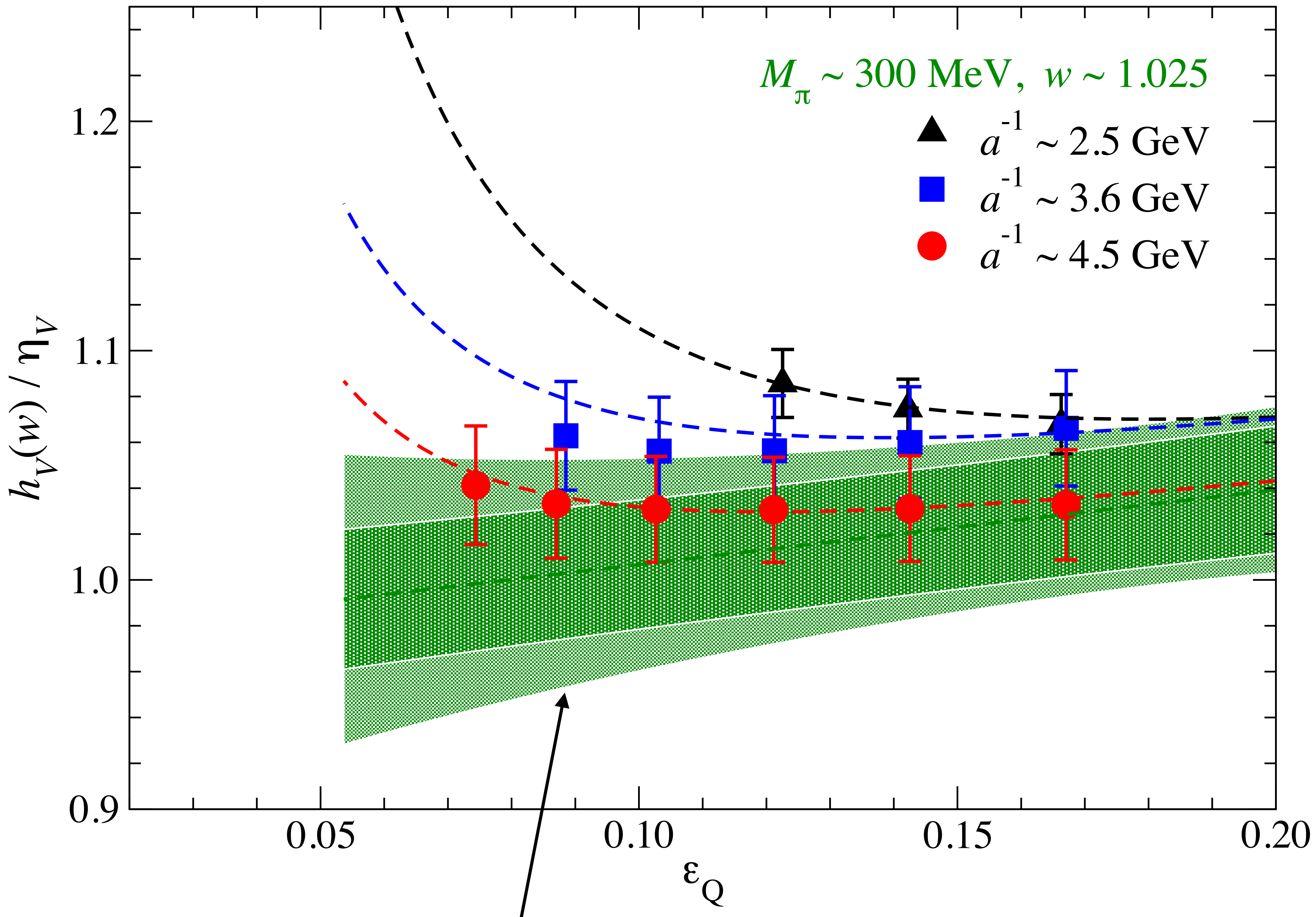
Control of 1/M effects



extrapolated to $a \to 0$

- Hard to distinguish $1/M$ from $(aM)^2$ by a global fit.
  - aM up to ~ 0.7 with domain-wall
- Plan to add a 5.x GeV lattice

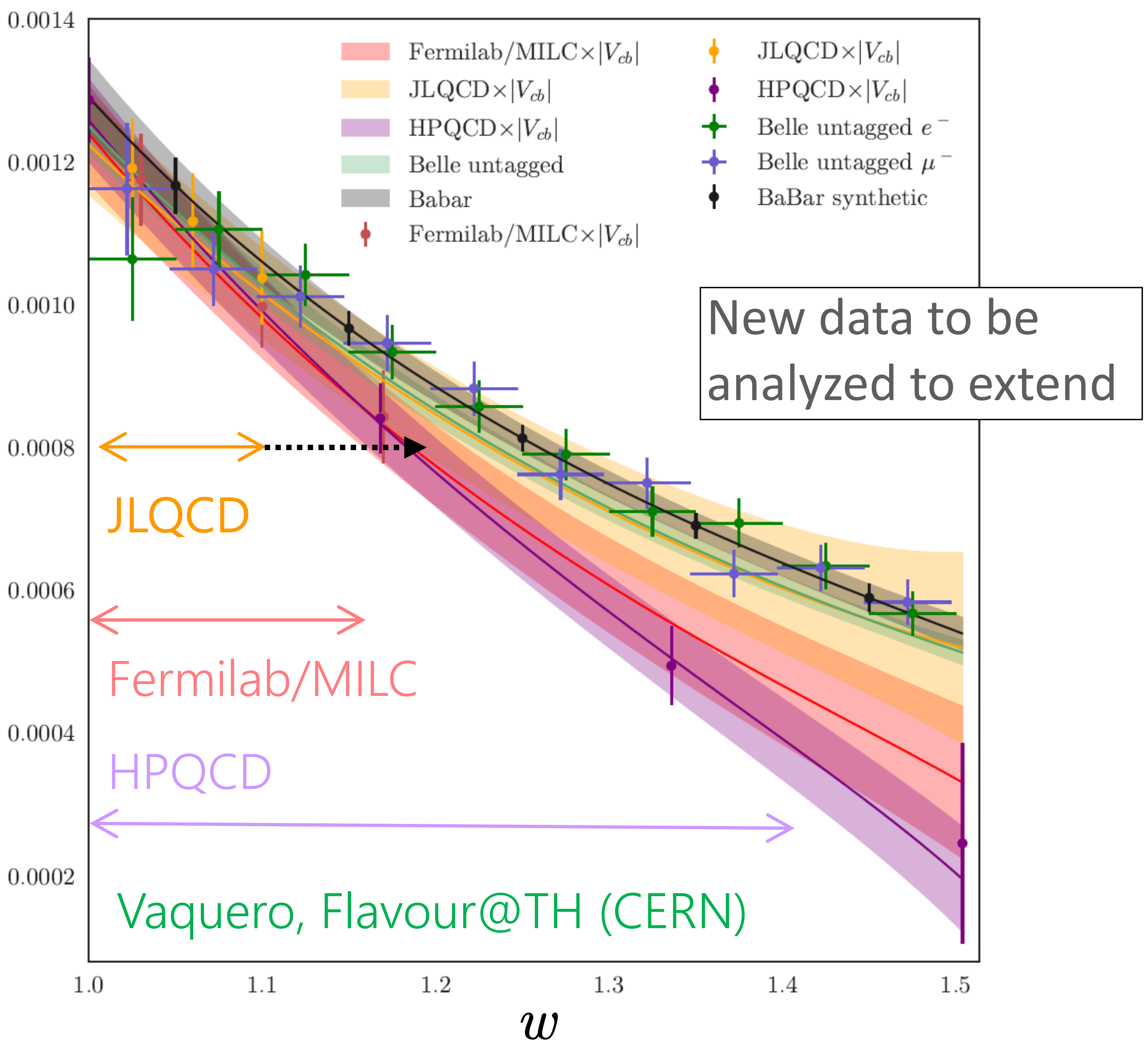
Control of 1/M effects

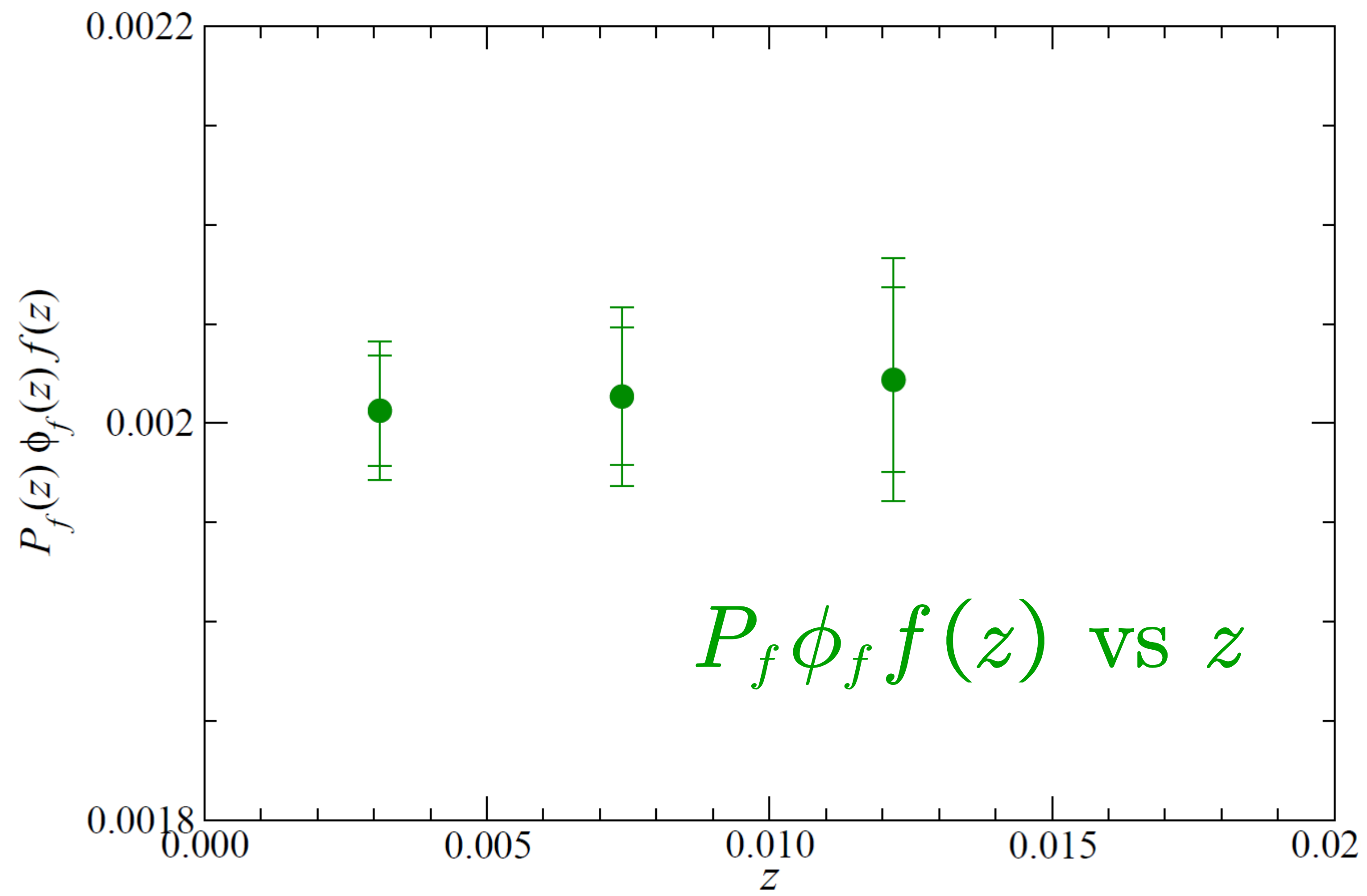


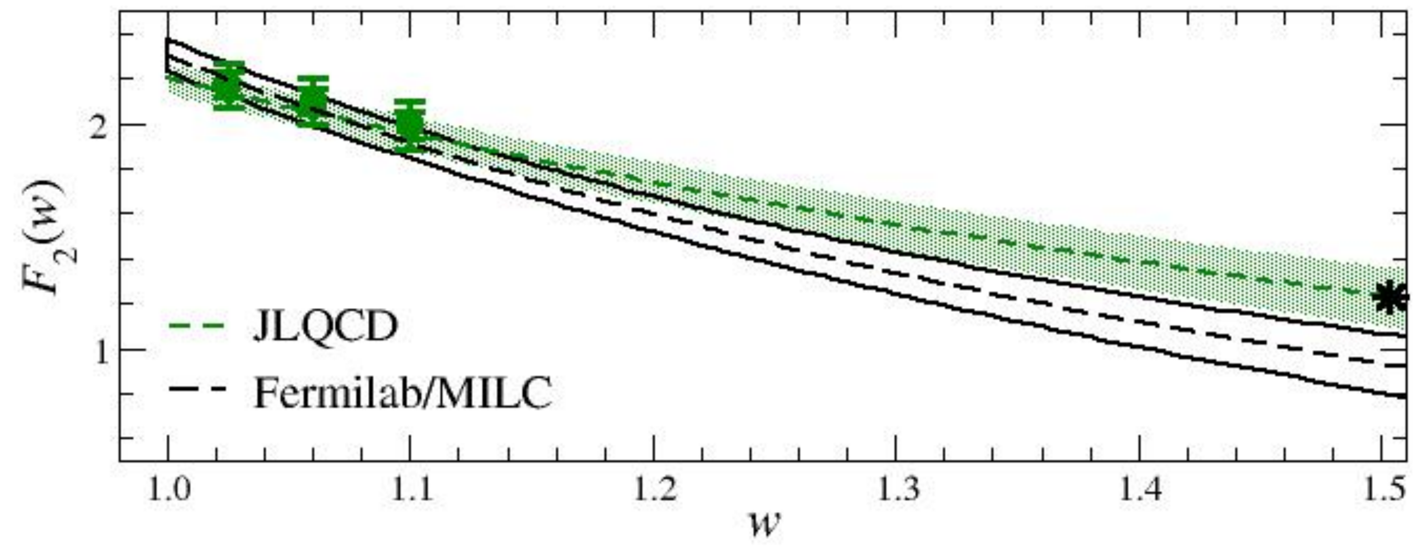
extrapolated to $a \rightarrow 0$

- Hard to distinguish $1/M$ from $(aM)^2$ by a global fit.
  - aM up to ~ 0.7 with domain-wall
- Plan to add a 5.x GeV lattice

Limited range of recoil momentum

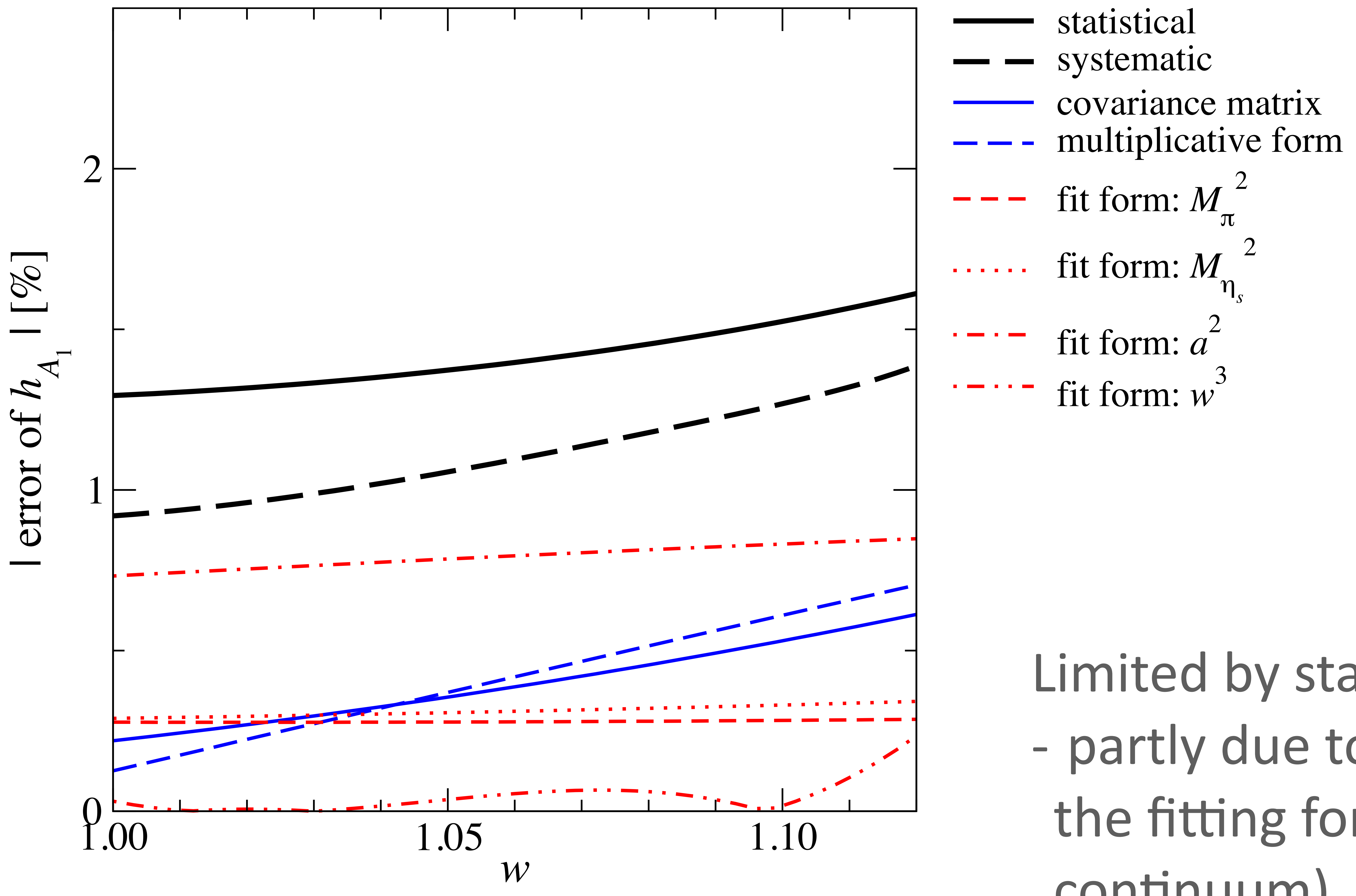






Essentially, no dependence seen in the z-expansion. But, let's see.

Systematic errors

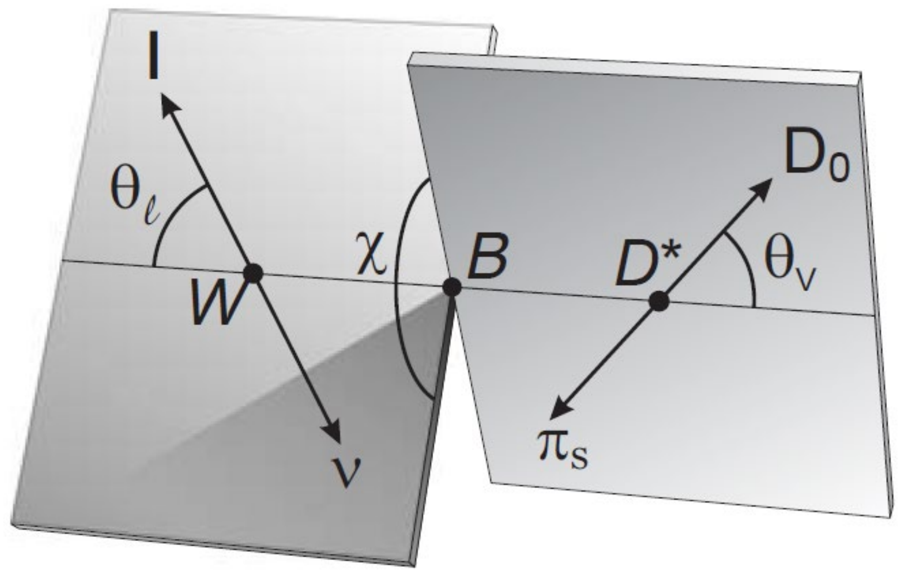


Limited by statistics:

- partly due to the conservative choice of the fitting form (correlator & chiral/ continuum)

$|V_{cb}|$ is not the only output!

Kaneko @ CCP2023

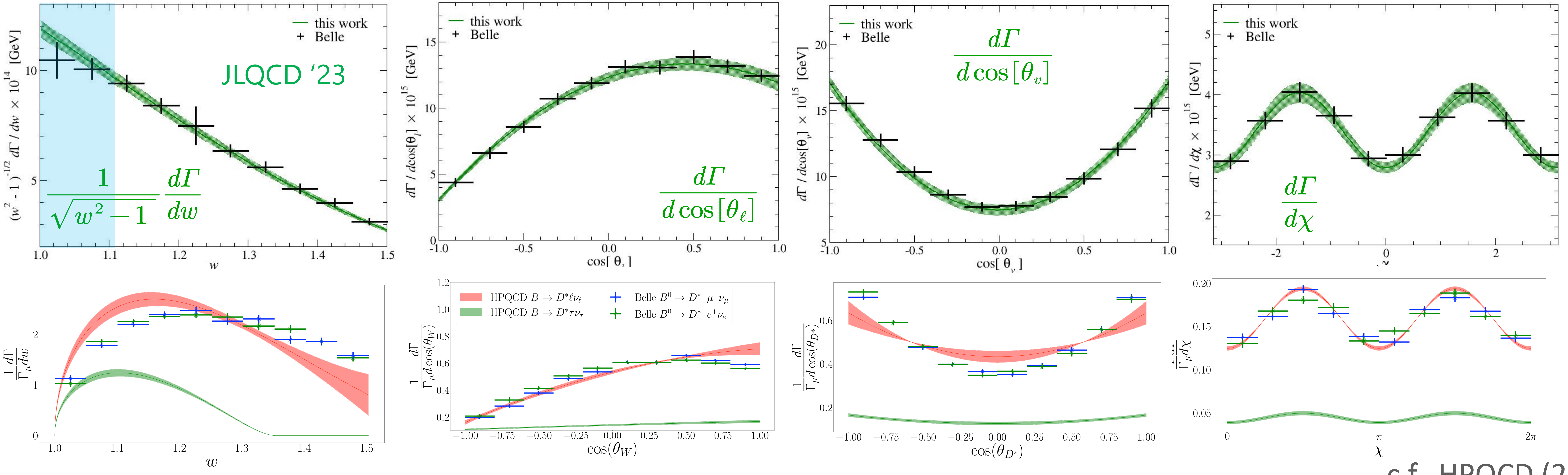


$$\frac{d\Gamma}{dw\; d\cos[\theta_\ell]\; d\cos[\theta_v]\; d\chi}$$

$$= |V_{cb}|^2 G_F^2 \eta_{\rm EW}^2 \sqrt{w^2-1}\, H(w, \cos[\theta_\ell], \cos[\theta_v], \chi, f, \mathcal{F}_1, g)$$

FFs in BGL → lattice data ⊕ differential decay rate from BGL → Belle data

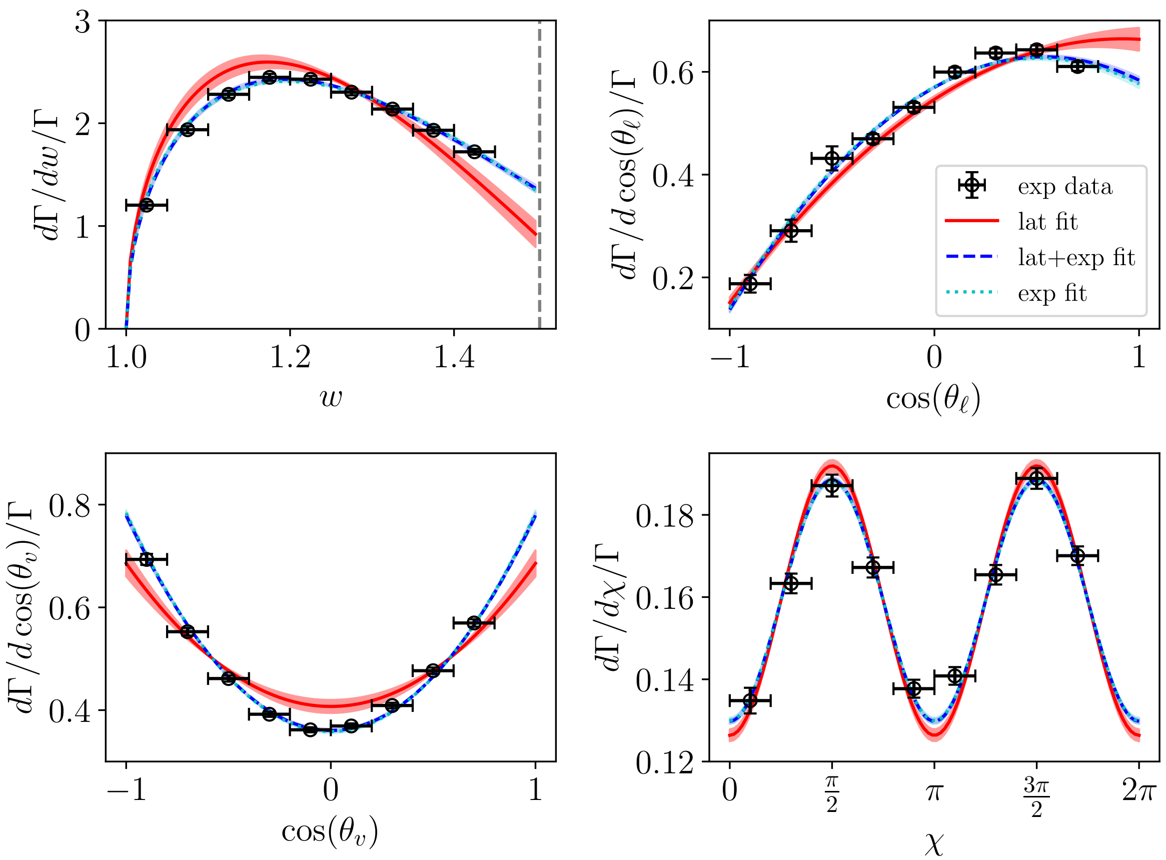
⇒ $|V_{cb}|$ as relative normalization ⊕ absolute kinematical distribution



c.f. HPQCD (2023)

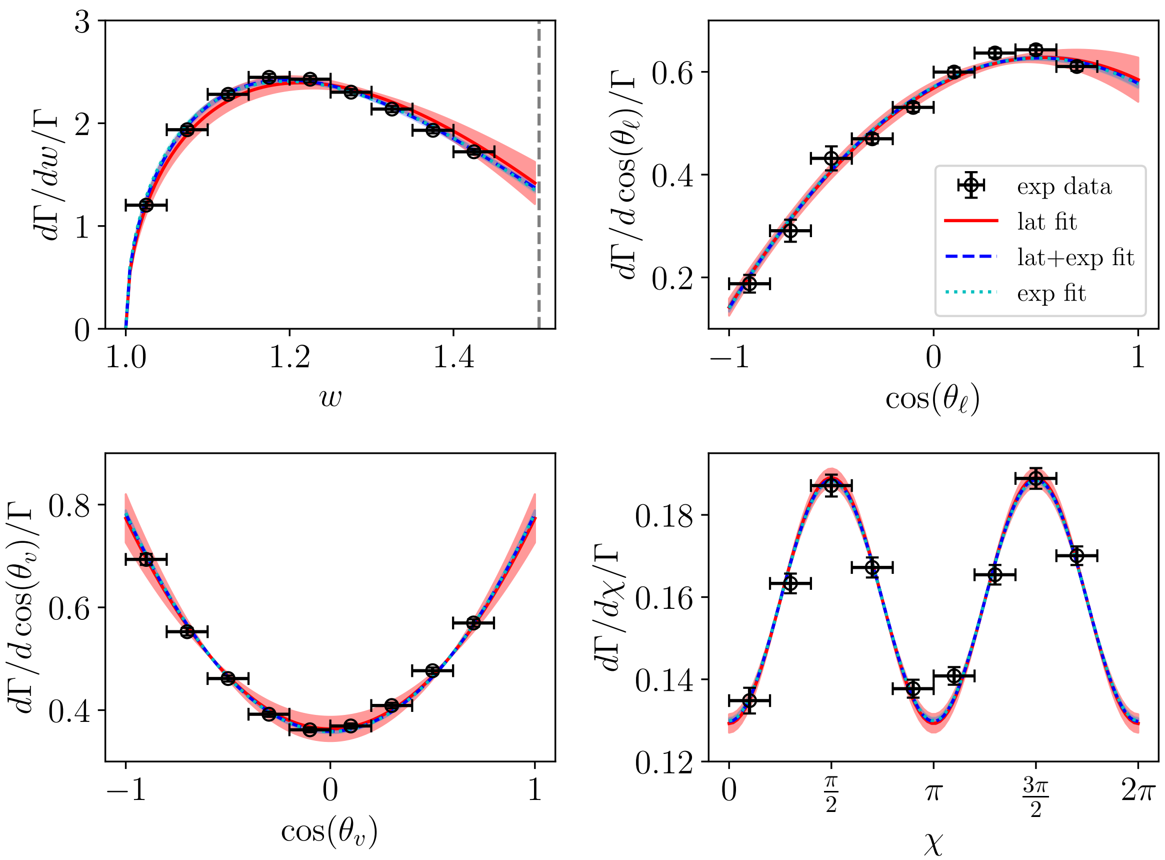
Questions from the organizers

- Comment on bin-by-bin result for $|V_{\mathrm{cb}}|$ in angular differential decay rate and large variation seen (or not). Is an experimental or lattice problem?



Fermilab-MILC ⊕ HPQCD

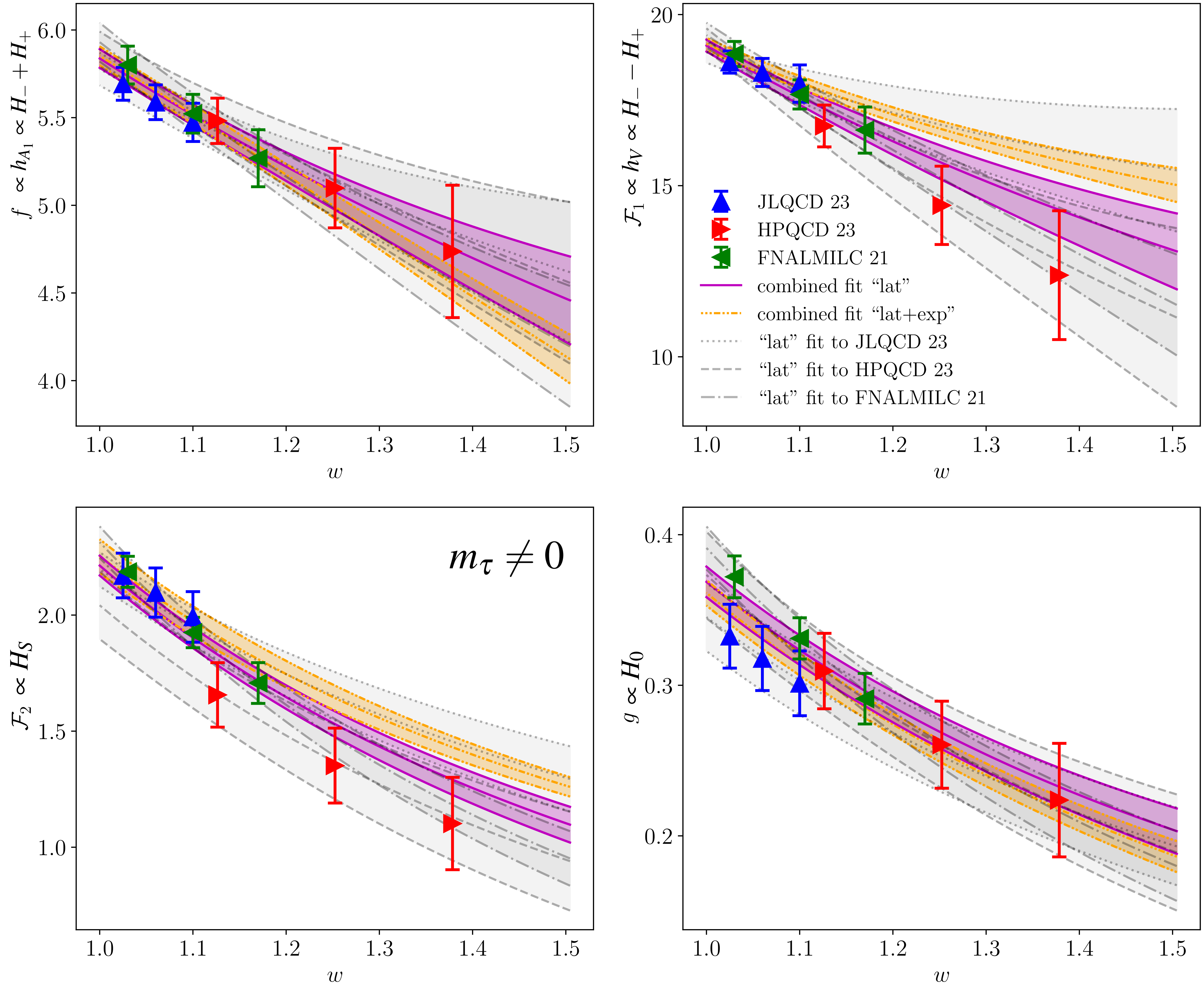
Belle 2301



JLQCD

Belle 2301

- Comment on bin-by-bin result for $|V_{cb}|$ in angular differential decay rate and large variation seen (or not). Is an experimental or lattice problem?
  - [AV] For me it is a clear sign that the shapes predicted by experiment and lattice are different.
  - [SH] It encourages us to reconsider the analysis steps once again. Correlator fits, global fit for chiral/continuum, etc are non-trivial.
  - [JH] Variations in bin-by-bin $|V_{cb}|$ are equivalent to the discrepancies seen in the normalized differential decay rates. Where there is a statistically significant disagreement in shape, it's not obvious that we should use those results to determine $|V_{cb}|$.
  - [AK] More worrisome is imperfect agreement of the three lattice-QCD calculations. LQCD vs. expt will clarify over time.



$f \propto h_{A_1} \propto H_- + H_+$
$\mathcal{F}_1 \propto h_V \propto H_- - H_+$
$\mathcal{F}_2 \propto H_S$
$g \propto H_0$
w
$m_\tau \neq 0$
JLQCD 23
HPQCD 23
FNALMILC 21
combined fit "lat"
combined fit "lat+exp"
"lat" fit to JLQCD 23
"lat" fit to HPQCD 23
"lat" fit to FNALMILC 21

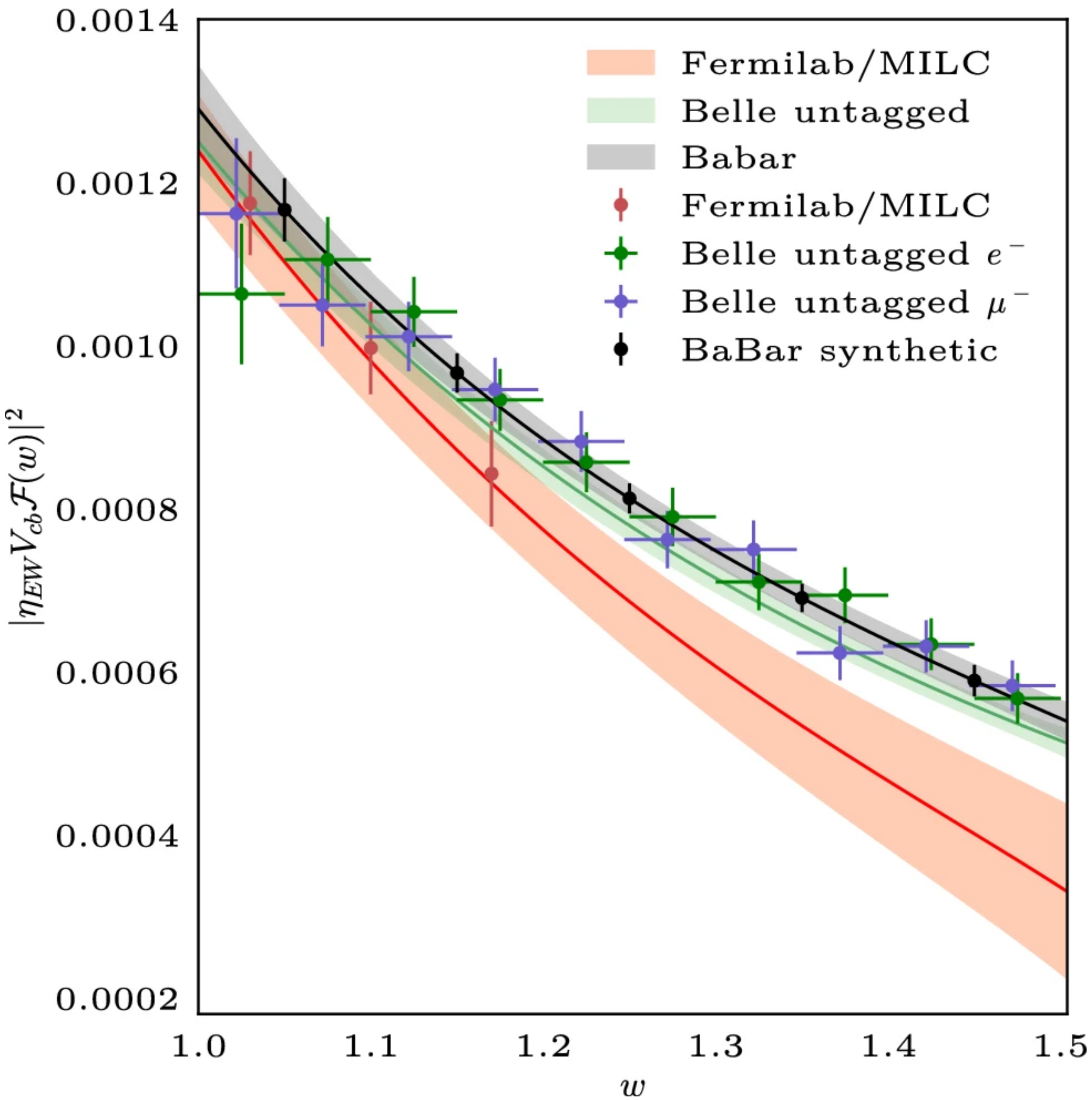
- What is the best/correct way to analyze lattice and experimental data to determine $|V_{cb}|$, $R(D^{(*)})$, etc? "Lat" (fit parameterization to lattice form factors and then do bin-by-bin analysis), "lat+exp" (simultaneous fit to lattice and experimental data), other?

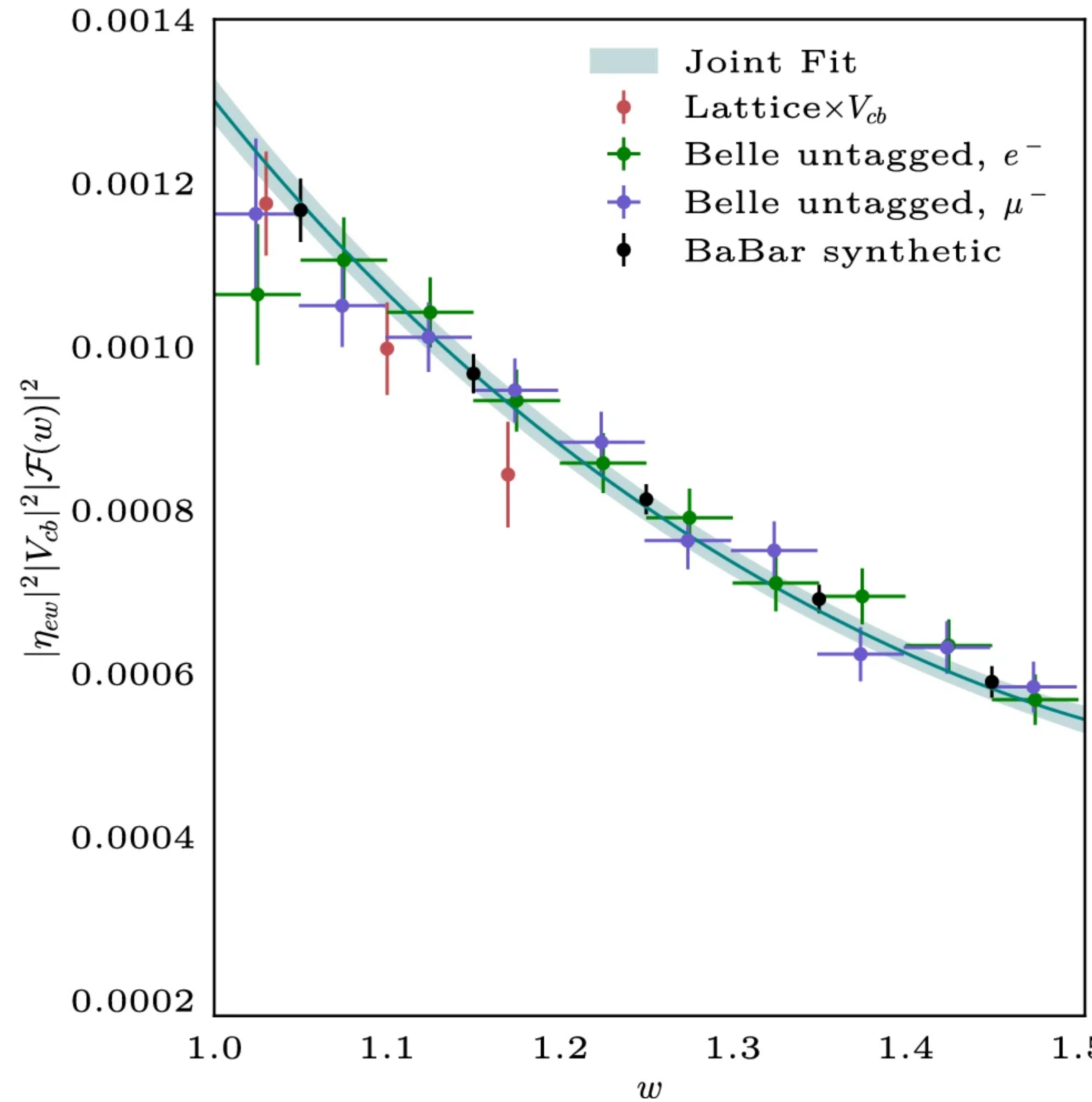
    - [AV] In lattice QCD we are more confident about the value of $h_{A1}(1)$, which should be subjected to less systematics, than to the rest. A simultaneous fit lat+exp results in a determination of $|V_{cb}|$ that is dominated by $h_{A1}(1)$, so this would be my choice.

    - [JH] For $R(D^{(*)})$, F_L etc. I would say the best approach is to use lattice + BGL without any experimental information (at least until questions about the shape are resolved). For $|V_{cb}|$, I think "lat" and "lat+exp" should give equivalent results provided the D'Agostini bias is correctly accounted for (for example by fitting normalized rates).

    - [AK] Can remove D'Agostini bias by, e.g., MCMC methods.

- What is the best/correct way to analyze lattice and experimental data to determine $|V_{cb}|$, $R(D^{(*)})$, etc? "Lat" (fit parameterization to lattice form factors and then do bin-by-bin analysis), "lat+exp" (simultaneous fit to lattice and experimental data), other?





- What is the best/correct way to analyze lattice and experimental data to determine $|V_{cb}|$, $R(D^{(*)})$, etc? "Lat" (fit parameterization to lattice form factors and then do bin-by-bin analysis), "lat+exp" (simultaneous fit to lattice and experimental data), other?

  - [AV] In lattice QCD we are more confident about the value of $h_{A1}(1)$, which should be subjected to less systematics, than to the rest. A simultaneous fit lat+exp results in a determination of $|V_{cb}|$ that is dominated by $h_{A1}(1)$, so this would be my choice.

  - [JH] For $R(D^{(*)})$, F_L etc. I would say the best approach is to use lattice + BGL without any experimental information (at least until questions about the shape are resolved). For $|V_{cb}|$, I think "lat" and "lat+exp" should give equivalent results provided the D'Agostini bias is correctly accounted for (for example by fitting normalized rates).

  - [AK] Can remove D'Agostini bias by, e.g., MCMC methods.

- Comment on the role of D'Agostini bias.
  - We [AV, JH, SH, AK] agree that D'Agostini bias [NIM A 346 (1994) 306] can arise in some ways of jointly fitting lattice and experiment.
  - Please note D'Agostini bias is *not* fit curves lying below or above correlated data; this is "normally" unbiased [NIM A 270 (1988) 110].
  - D'Agostini bias means normalization factors (here $|V_{cb}|$) enter χ^2 such that their value at χ^2_{min} lies lower their average in $\exp(-\chi^2)$.
  - Recent Belle paper finds D'Agostini bias in their joint fit ($|V_{cb}|$ via BR) is $30\times$ smaller than other uncertainties [arXiv:2301.07529].
  - Other biases to worry about, e.g., an unbiased sample estimate of the covariance leads to a biased estimate of its inverse—thinning, SVD, shrinkage, *etc*.

- Discuss tension in $R(D^*)$, A_{FB}, F_L, etc. between different lattice results.

  - [AV] I will discuss $R(D^*)$, because we did not calculate any of the others. The errors in [lattice-only] $R(D^*)$ are so large that everything is compatible with everything, so there isn't much that can be said right now. This is expected, since a calculation of $R(D^*)$ requires knowledge in the whole recoil region of all the form factors, and in order to get that information, we need to resort to a BGL extrapolation.

  - [JH] I don't think the 1–2σ tensions in $R(D^*)$, A_{FB}, F_L, etc. are surprising, as they just reflect (and potentially pick out) the disagreements seen in the form factors.

- Is p value good measure to indicate if lattice and/or experimental results mutually compatible?

  - [AV] According to my experience, yes. In fact, the p values of the different combinations of fits very accurately reflect the intuition one gets just by looking at the distribution of the data.

  - [AK] The frequentist p value is a good measure of a frequentist fit (of course), but "p" values obtained in Bayesian frameworks, because the priors influence whether χ^2_{min} follows a χ^2 distribution. They are probably fine for ranking fits, but there are likely better ways of ranking (e.g., a version of AIC that follows from the details of your choices).

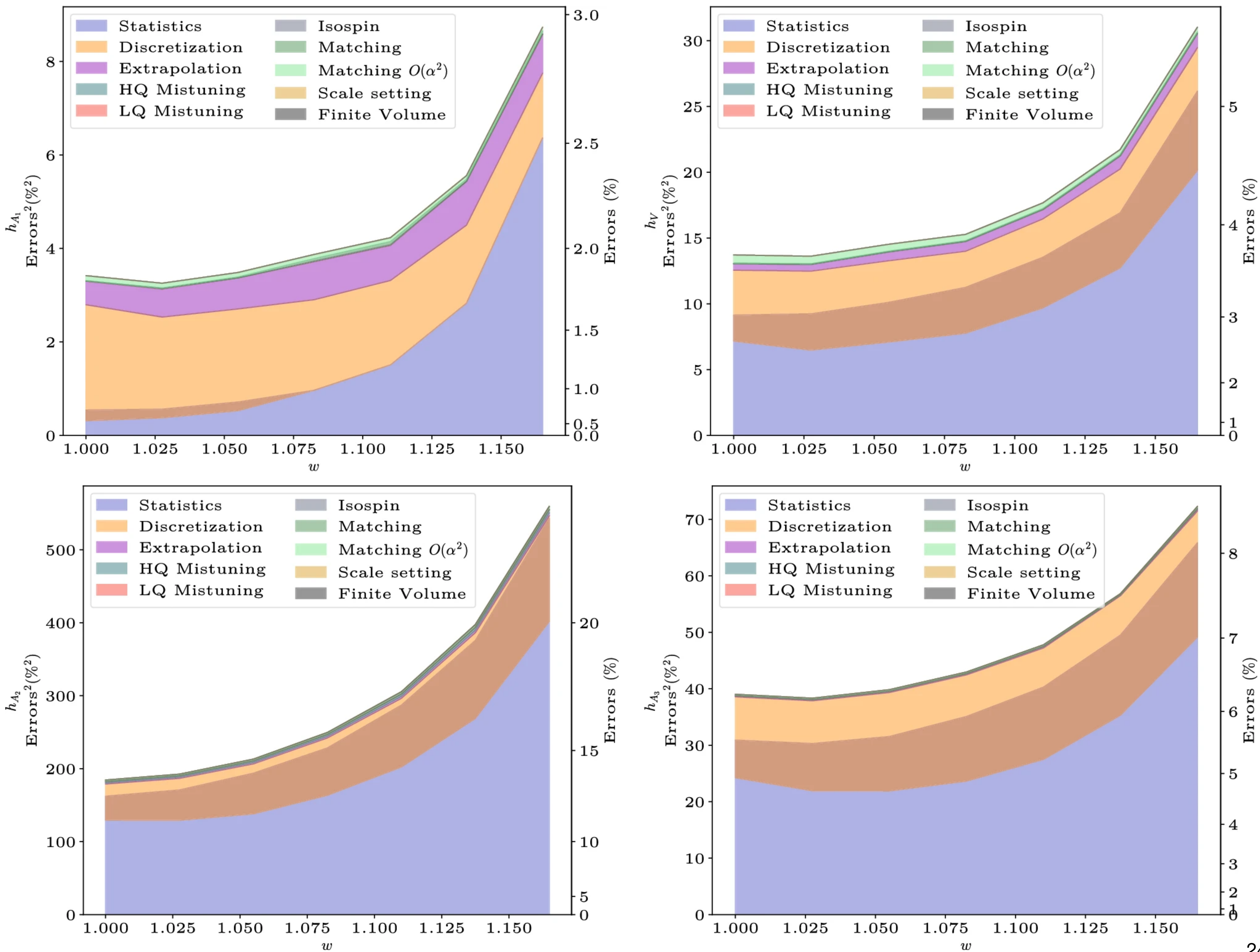
- Would you be willing to share this data with the community in digital format, to allow a joint fit of the three lattice results on the raw data before chiral continuum extrapolation?
  - [AV] How would you mix lattice data coming from different regularizations? I don't think that's wise.
  - [SH] I am positive about sharing the raw data in principle but am not sure if they are useful, though. The raw data contain those of different heavy/light quark masses, momentum insertions, lattice spacings. There are lots of choices of the global fit function, which becomes nearly prohibitive when combined with the data of other groups.
  - [JH] I am also positive about sharing the raw data, and I think it would improve the credibility of our calculations. I think a combined fit would potentially be useful where there are shared parameters (e.g., HPQCD and JLQCD should share physical $1/m_b$ power corrections, I think?) though there are clearly many dissimilar ingredients as well that would muddy the water.
  - [AK] Effort on chiral-continuum extrapolation should be spent on new data.

- What is your dominant source of uncertainty and how could/would/will you address this?

  - [AV] Our error budget shows that the discretization errors mainly come from heavy quarks. We are working on two new analyses that should be able to significantly reduce these systematics.

  - [JH] For HPQCD, the errors are laid out in Appendix C.2 of our paper. Statistics is dominant, followed by heavy quark discretisation effects. Statistics can be improved by computing time reversed and rotationally equivalent correlation functions as well as additional time sources.

  - [SH] Also for the JLQCD analysis, the largest error is statistical. It is partly due to a conservative choice of the correlator fit range. The chiral/continuum extrapolation fit-form might also be adjusted as statistics improve. We always want to lean to the safer (conservative) side by trading between the statistical and systematic errors.


Statistics
Discretization
Extrapolation
HQ Mistuning
LQ Mistuning
Isospin
Matching
Matching $O(\alpha^2)$
Scale setting
Finite Volume
h_{A_1} Errors^2(%^{2})
h_V Errors^2(%^{2})
h_{A_2} Errors^2(%^{2})
h_{A_3} Errors^2(%^{2})
Errors (%)
w
1.000 1.025 1.050 1.075 1.100 1.125 1.150

- What would you do differently in a future repeat/update of your calculation?
- [AV] Each calculation poses its own challenges—I would spend more time computing other interesting phenomenological observables and discussing the implications of the results.
- [AK] the HISQ ensembles have smaller staggered discretization effects and ensembles at physical pion mass: the chiral-continuum extrapolation will be under better control (chiral part will be interpolation).
- [JH] Besides more statistics, a future HPQCD calculation will include $B \to D$ and will include the LO HQE relations. Now that we have susceptibilities for all m_h, a fully dispersively bounded parameterisation can also be used for the physical-continuum extrapolation.
- [SH] The JLQCD plan is to increase the statistics and to add another lattice spacing to control the extrapolation in $1/M$.

- What's the time scale for updates of your calculations, and what will they entail?

  - [AV] Hopefully we can have another $B \to D^*$ result in 1–2 years (with Fermilab b and c quarks; others HISQ). The newest and most promising calculation (all HISQ on all 2+1+1 ensembles) might need to wait a bit more, since we are still generating data (~4-year time scale).

  - [JH] This will most likely take 1–2 years.

  - [SH] We hope to have updated results in a couple of years.

- Comment on the role of blinding.
  - Concern over the inclusive/exclusive tension leads to an effort to find the resolution—a huge temptation to analyst bias.
  - Blinding common in *other* B-factory analyses, but not mentioned in BaBar or Belle papers.
  - Fermilab-MILC blinded of course. No mention of blinding in HPQCD or JLQCD.
  - Explorations of new ways of combining lattice QCD and experiment would profit from blinding, to focus on soundness of new ideas rather than their potential to “solve” the mystery.

Coffee?