Status of exclusive (heavy-light) decays

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Outline

1. Introduction and Status
2. An overlooked systematic? [JTT, RBC/UKQCD'23: 2303.11280, Flynn, Jüttner, JTT: 2303.11285]
3. Suggestion of benchmark quantities [JTT, Della Morte: 2310.02705]
4. A preview: Reducing discretisation effects [JTT, RBC/UKQCD: 2407.XXXXX]
5. Conclusions

b-decays as "sweet spot" for experiments

Properties of b-decays [PDG'20]

1. $\overline{m}_b(\overline{m}_b) = 4.18(3)\,\mathrm{GeV} \gg \overline{m}_c(\overline{m}_c) = 1.27(2)\,\mathrm{GeV} \gg m_s, m_u, m_d$
   $\rightarrow$ many different decay products
2. b hadrons have *relatively long* lifetime of $\tau_b \sim 10^{-12} s$ ($\tau_t \sim 10^{-25} s$)
   $\rightarrow$ b hadronises and b-jets travel some distance before decaying
   $\rightarrow$ but not far enough to escape the detector
   $\rightarrow$ allows for $b-$**tagging**

$\Rightarrow$ **Plethora of <u>accessible</u> decay channels for hadrons with b-quarks**

Distinguish two categories:

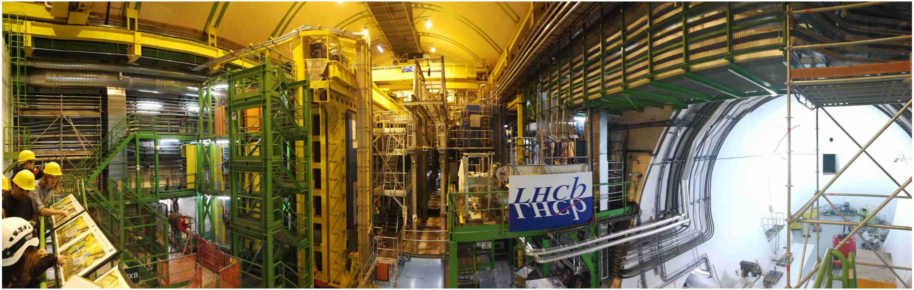
Charged currents

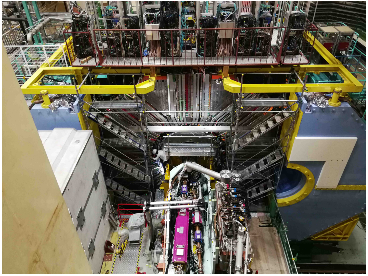
- Present at *tree level* in the SM
  e.g. $B^0 \rightarrow D^+ \ell^- \nu_\ell$
  $\Rightarrow$ Precision tests of the SM

Flavour changing neutral currents

- Only at *loop level* in the SM
  e.g. $B \rightarrow K \ell^+ \ell^-$
  $\Rightarrow$ Sensitive to NP searches

b-physics experiments





top: LHCb at LHC, CERN

left: Belle II at SuperKEKb, KEK

⇒ Huge experimental efforts!
+ BES-III and other LHC experiments

⇒ <u>*B*-**factory**</u> vs <u>**hadron machine**</u>
Very complementary

- "Old" data from BaBar, Belle, Cleo, . . .

CKM: Relating experiment and theory

- Experiment measures differential decay rates, mass differences, branching fractions, ...,
- In the SM predictions are parameterised as a sum of products between known/calculable coefficients and low energy matrix elements.

For example for tree-level PS $\to$ PS decays:

$$\frac{\mathrm{d}\Gamma(B_{(s)} \to P\ell\nu_\ell)}{\mathrm{d}q^2} = |V_{qb}|^2 \mathcal{K} \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) |f_+(q^2)|^2 + \mathcal{K}_2 m_\ell^2 |f_0(q^2)|^2 \right]$$

Knowledge of $|V_{qb}|$ depends on

- precision of experimental data
- precision of non-perturbative form factors

Many decays constrain the same V_{qb}. For example ($\ell \in \{e, \mu, \tau\}$)
$|V_{ub}|$: $B \to \ell\nu_\ell$, $B \to \pi\ell\nu_\ell$, $B_s \to K\ell\nu_\ell$, $\Lambda_b \to p\ell\nu_\ell$, ...

my perception from 2020 (Anomalies meeting, Hyderabad)

Semileptonic decays: form factors, PS vs V

Note: Pseudoscalars (PS) are QCD-stable, Vectors (V) are QCD-unstable

✓ Pseudoscalar to pseudoscalar at tree-level

- 2 form factors: f_+ and f_0

(✓) Pseudoscalar to pseudoscalar at loop-level ("rare decays")

- 3 form factors: f_+, f_0 and f_T
- Fewer results than at tree-level

(✗) Pseudoscalar to vector at tree-level

- 4 form factors: V, A_0, A_1, A_2
- $1 \to 2$ transitions (e.g. $D^* \to D\pi$) understood on the lattice, but more involved and technical
- In current studies V are treated as QCD-stable .

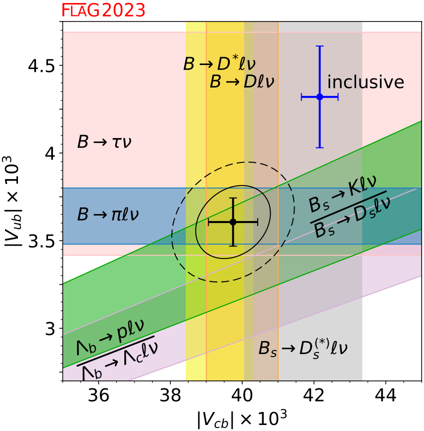
✗ Pseudoscalar to vector at loop-level ("rare decays")

- 7 form factors: V, A_0, A_1, A_2, T_1, T_2, T_3
- Single unquenched result for $B \to K^*\ell^+\ell^-$, $B_s \to \phi\ell^+\ell^-$ treating V as stable [PRD**89** 094501 (2014)]

18 / 31 J Tobias Tsang (CP^3-Origins, SDU) Results and challenges in the $B_{(s)}$-sector from lattice QCD

What is the status now?

What is the status of V_{ub} and V_{cb}? Let's look at FLAG!



Consistency between different determinations ✓(or is it?!)

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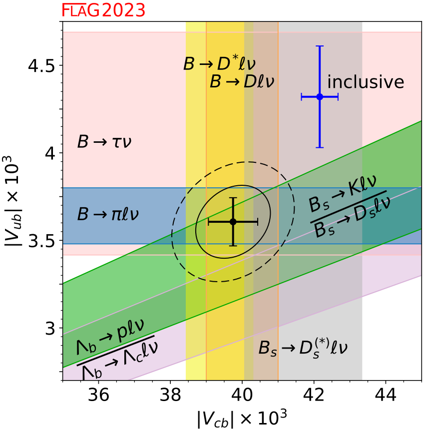

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$B \to \tau\nu$: good agreement! ✓

$\Lambda_b \to p$: Only a single result ✓(?)

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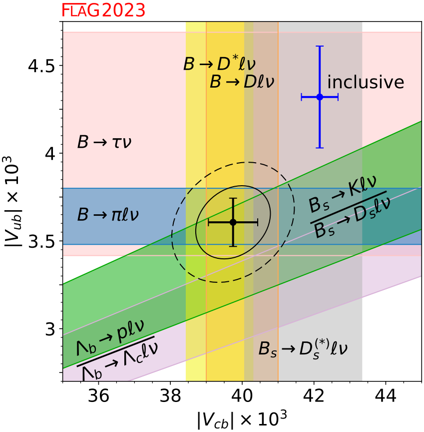

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$B \to \pi\ell\nu$: $p \sim 2 \times 10^{-5}$ ✗

Y. Aoki et al. *FLAG Review 2021* 2111.09849

$B \to \pi$ ($N_f = 2+1$)

	Central Values	Correlation Matrix				
a_0^+	0.423 (21)	1	-0.00466	-0.0749	0.402	0.0920
a_1^+	−0.507 (93)	-0.00466	1	0.498	-0.0556	0.659
a_2^+	−0.75 (34)	-0.0749	0.498	1	-0.152	0.677
a_0^0	0.561 (24)	0.402	-0.0556	-0.152	1	-0.548
a_1^0	−1.42 (11)	0.0920	0.659	0.677	-0.548	1

Table 46: Coefficients and correlation matrix for the $N^+ = N^0 = 3$ z-expansion fit of the $B \to \pi$ form factors f_+ and f_0. The coefficient a_2^0 is fixed by the $f_+(q^2=0) = f_0(q^2=0)$ constraint. The chi-square per degree of freedom is $\chi^2/\mathrm{dof} = 43.6/12$ and the errors on the z-parameters have been rescaled by $\sqrt{\chi^2/\mathrm{dof}} = 1.9$. The lattice calculations that enter this fit are taken from FNAL/MILC 15 [58], RBC/UKQCD 15 [59] and JLQCD 22 [60]. The parameterizations are defined in Eqs. (533) and (534).

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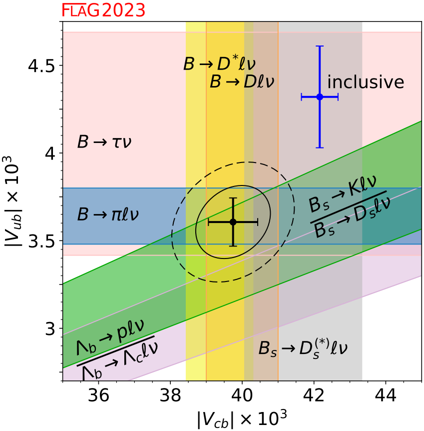

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$B_s \to K\ell\nu$: $p \sim 7 \times 10^{-6}$ ✗

(I counted 7 fit parameters and 19 datapoints ⇒ 12 dof's)

Y. Aoki et al. *FLAG Review 2021* 2111.09849

$B_s \to K$ ($N_f = 2+1$)

	Central Values	Correlation Matrix						
a_0^+	0.370(21)	1.	0.2781	−0.3169	−0.3576	0.6130	0.3421	0.2826
a_1^+	−0.68(10)	0.2781	1.	0.3672	0.1117	0.4733	0.8487	0.8141
a_2^+	0.55(48)	−0.3169	0.3672	1.	0.8195	0.3323	0.6614	0.6838
a_3^+	2.11(83)	−0.3576	0.1117	0.8195	1.	0.2350	0.4482	0.4877
a_0^0	0.234(10)	0.6130	0.4733	0.3323	0.2350	1.	0.6544	0.5189
a_1^0	0.135(86)	0.3421	0.8487	0.6614	0.4482	0.6544	1.	0.9440
a_2^0	0.20(35)	0.2826	0.8141	0.6838	0.4877	0.5189	0.9440	1.

Table 48: Coefficients and correlation matrix for the $N^+ = N^0 = 4$ z-expansion of the $B_s \to K$ form factors f_+ and f_0. The coefficient a_3^0 is fixed by the $f_+(q^2=0) = f_0(q^2=0)$ constrain. The chi-square per degree of freedom is $\chi^2/\mathrm{dof} = 3.82$ and the errors on the z-parameters have been rescaled by $\sqrt{\chi^2/\mathrm{dof}} = 1.95$.

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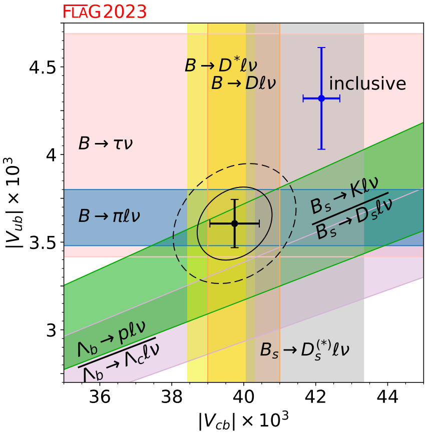

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$|V_{ub}|(B \to \pi)$: $p \sim 3 \times 10^{-5}$ ✗

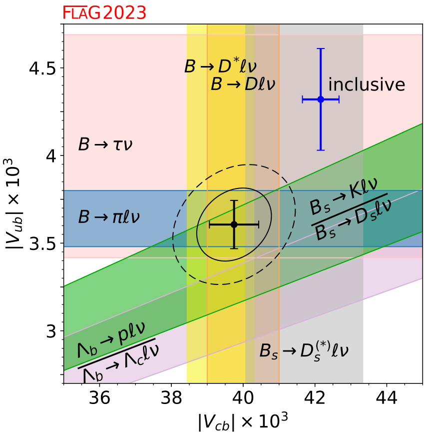
Y. Aoki et al. *FLAG Review 2021* 2111.09849

$B \to \pi\ell\nu$ ($N_f = 2+1$)

	Central Values	Correlation Matrix					
$\lvert V_{ub}\rvert \times 10^3$	3.64 (16)	1	-0.812	-0.108	0.128	-0.326	-0.151
a_0^+	0.425 (15)	-0.812	1	-0.188	-0.309	0.409	0.00926
a_1^+	−0.441 (39)	-0.108	-0.188	1	-0.498	-0.0343	0.150
a_2^+	−0.52 (13)	0.128	-0.309	-0.498	1	-0.190	0.128
a_0^0	0.560 (17)	-0.326	0.409	-0.0343	-0.190	1	-0.772
a_1^0	−1.346 (53)	-0.151	0.00926	0.150	0.128	-0.772	1

Table 57: $|V_{ub}|$, coefficients for the $N^+ = N^0 = N^T = 3$ z-expansion of the $B \to \pi$ form factors f_+ and f_0, and their correlation matrix. The chi-square per degree of freedom is $\chi^2/\text{dof} = 116.7/62 = 1.88$ and the errors on the fit parameters have been rescaled by $\sqrt{\chi^2/\text{dof}} = 1.37$. The lattice calculations that enter this fit are taken from FNAL/MILC [58], RBC/UKQCD [59] and JLQCD [60]. The experimental inputs are taken from BaBar [161, 162] and Belle [163, 164].

What is the status of V_{ub} and V_{cb}? Let's look at FLAG!



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$B \to \pi\ell\nu$: $p \sim 2 \times 10^{-5}$ ✗

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$|V_{ub}|\,(B \to \pi)$: $p \sim 3 \times 10^{-5}$ ✗

$B \to K\ell\ell$: $p \sim 0.046$ (✓)

(does not include HPQCD'23 $N_f = 2+1+1$ yet)

(I counted 8 fit parameters and 18 datapoints $\Rightarrow$ 10 dof's)

Y. Aoki et al. *FLAG Review 2021* 2111.09849

$B \to K$ ($N_f = 2+1$)

	Central Values	Correlation Matrix							
a_0^+	0.471 (14)	1	0.513	0.128	0.773	0.594	0.613	0.267	0.118
a_1^+	−0.74 (16)	0.513	1	0.668	0.795	0.966	0.212	0.396	0.263
a_2^+	0.32 (71)	0.128	0.668	1	0.632	0.768	-0.104	0.0440	0.187
a_0^0	0.301 (10)	0.773	0.795	0.632	1	0.864	0.393	0.244	0.200
a_1^0	0.40 (15)	0.594	0.966	0.768	0.864	1	0.235	0.333	0.253
a_0^T	0.455 (21)	0.613	0.212	-0.104	0.393	0.235	1	0.711	0.608
a_1^T	−1.00 (31)	0.267	0.396	0.0440	0.244	0.333	0.711	1	0.903
a_2^T	−0.9 (1.3)	0.118	0.263	0.187	0.200	0.253	0.608	0.903	1

Table 51: Coefficients and correlation matrix for the $N^+ = N^0 = N^T = 3$ z-expansion of the $B \to K$ form factors f_+, f_0 and f_T. The coefficient a_2^0 is fixed by the $f_+(q^2=0) = f_0(q^2=0)$ constraint. The chi-square per degree of freedom is $\chi^2/\text{dof} = 1.86$ and the errors on the z-parameters have been rescaled by $\sqrt{\chi^2/\text{dof}} = 1.36$.

What is the status of V_{ub} and V_{cb}? Let's look at FLAG!

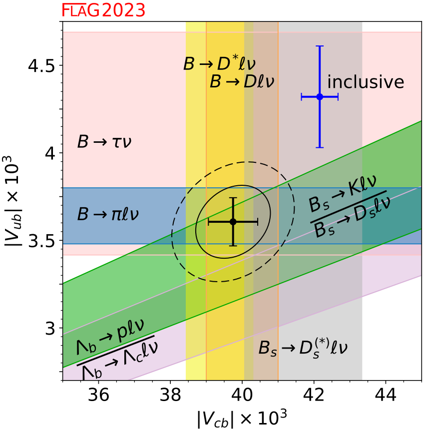

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Table 51: Coefficients and correlation matrix for the $N^+ = N^0 = N^T = 3$ z-expansion of the $B \to K$ form factors f_+, f_0 and f_T. The coefficient a_2^0 is fixed by the $f_+(q^2=0) = f_0(q^2=0)$ constraint. The chi-square per degree of freedom is $\chi^2/\mathrm{dof} = 1.86$ and the errors on the z-parameters have been rescaled by $\sqrt{\chi^2/\mathrm{dof}} = 1.36$.

We need to scrutinise this!

Example: Literature of $B \to \pi$ results

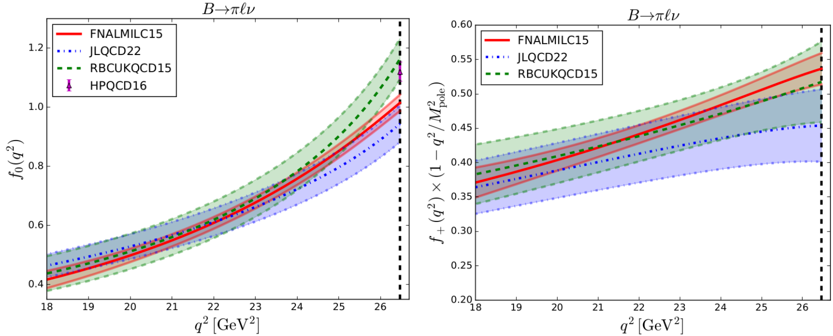
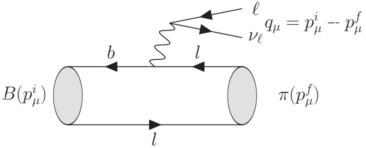


Fig. 3 Comparison plot of recent results for $B \to \pi$ form factors from Refs. [71–73]. The individual data point HPQCD16 [74] stems from a $N_f = 2+1+1$ calculation, but only provides as results $f_0(q^2_{\rm max})$.

[JTT and Della Morte: 2310.02705]

Form factors are shown for the range of available data

Challenges in computing $f_X(q^2)$: example $B \to \pi\ell\nu$



- $q^\mu = p^\mu_B - p^\mu_\pi$
- $M_B \approx 5.28\,\mathrm{GeV}$, $M_\pi \approx 0.14\,\mathrm{GeV}$
- Semileptonic region $q^2 \in [0, q^2_{\mathrm{max}}]$
- $q^2_{\mathrm{max}} \equiv (M_B - M_\pi)^2 \sim 26.4\,\mathrm{GeV}^2$

- physical kinematics in the B rest-frame: $q^2 = 0 \Leftrightarrow |p_\pi|^2 = 6.96\,\mathrm{GeV}^2$
- Assuming $M_\pi L = 4$ and physical pion masses implies:
  $\Rightarrow$ final state momentum of $\vec{p}_\pi = \frac{2\pi}{L}(7,7,7)$ to reach $q^2 \sim 0$.
- typical simulations cannot achieve (i.e. control) this
  $\Rightarrow$ compromise in at least one of the following:
  - $M_\pi > M_\pi^{\mathrm{phys}}$ ($\Rightarrow$ need chiral extrapolation)
  - $M_B < M_B^{\mathrm{phys}}$ ($\Rightarrow$ need heavy quark mass extrapolation)
  - $q^2_{\mathrm{min}} \gg 0$ ($\Rightarrow$ need kinematic extrapolation)

How to simulate the b-quark?

$m_b/M_\pi \approx 30$ and we want to be “far” away from IR and UV cut-offs.
$\Rightarrow$ Need to simultaneously satisfy: $(am_b)^{-1} \gg 1$, $M_\pi L \geq 4$
$\Rightarrow$ $(am_b)^{-1} M_\pi L \gg 4$, so we require $L/a \gg 120$ (for multiple choices of a!)
Currently computationally impossible at physical quark masses!

Effective action for b

- Can tune to $m_b \sim m_b^{\text{phys}}$
- comes with systematic errors which are hard to estimate/reduce

(NRQCD, Fermilab, RHQ,...)

Relativistic action for b

- Theoretically cleaner and systematically improvable
- $m_b < m_b^{\text{phys}}$: control extrapolation to m_b^{phys}

(HISQ, DWF, TM, Wilson,...)

- relativistic will win in the long term
- for now, settle on a compromise.
- different systematics but should produce complementary results ($\Rightarrow$ reminiscent of (light) fermion discretisations...)

From approximations to the physical world

Extrapolations are based on theoretical foundations...

- $M_\pi^{\rm phys}$ (chiral) extrapolation guided by heavy meson chiral perturbation theory (HMχPT)
- $M_B^{\rm phys}$ (heavy quark) extrapolation guided by heavy quark effective theory (HQET) [$\Rightarrow$ Simon's talk before lunch]
- $q^2 = 0$ (kinematic) extrapolation guided by model independent z-expansion (BGL) [or $(w-1)$ for heavy to heavy $\Rightarrow$ yesterday]
  - Physical q^2 dependence can be mapped to interval $z(q^2) \in [-z_{\rm max}, z_{\rm max}]$ with $0 < z_{\rm max} \ll 1$
  - BGL expansion: $f_X(z) = \frac{1}{B_X \phi_X} \sum_i a_i z^i$, unitarity bounds $\sum_i |a|_i^2 < 1$.
- $a \to 0$ (continuum limit) extrapolation guided by Symanzik effective theory [$\Rightarrow$ Rainer's discussion session this afternoon]

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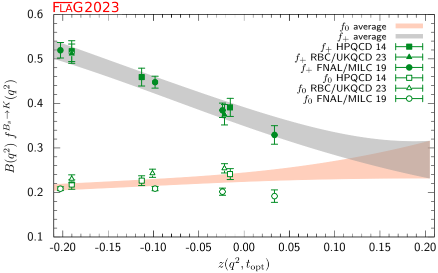
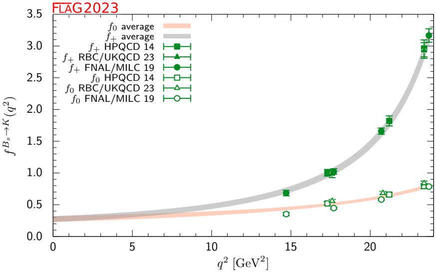
... but they are intertwined and difficult

and all of them come with systematic uncertainties - are they controlled?

FLAG's summary of $B_s \to K$



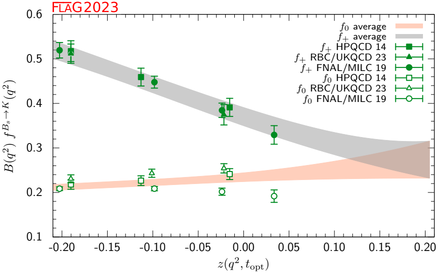


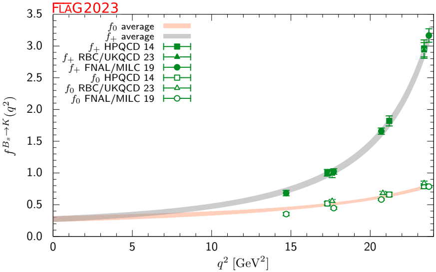
- f_+ looks fine, f_0 shows some tensions
- Most experimental data obtained for $\ell \in \{e, \mu\}$, so $m_\ell \sim 0$ and recall:

$$\frac{\mathrm{d}\Gamma(B_{(s)} \to P\ell\nu_\ell)}{\mathrm{d}q^2} = |V_{qb}|^2 \mathcal{K} \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) \left|f_+(q^2)\right|^2 + \mathcal{K}_2 m_\ell^2 \left|f_0(q^2)\right|^2 \right]$$

Does that mean V_{ub} should be fine?

FLAG's summary of $B_s \to K$





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Does that mean V_{ub} should be fine? ✗

- kinematic extrapolation (z-expansion) stabilised by kinematic constraint $f_0(0) = f_+(0)$, so f_0 does impact CKM determinations!

Example of a calculation (and systematics): $B_s \to K$

PHYSICAL REVIEW D **107,** 114512 (2023)

Exclusive semileptonic $B_s \to K\ell\nu$ decays on the lattice

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Semileptonic $B_s \to K\ell\nu$ decays provide an alternative b-decay channel to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$ and to obtain a R-ratio to investigate lepton-flavor-universality violations. Results for the CKM matrix element may also shed light on the discrepancies seen between analyses of inclusive or exclusive decays. We calculate the decay form factors using lattice QCD with domain-wall light quarks and a relativistic b-quark. We analyze data at three lattice spacings with unitary pion masses down to 268 MeV. Our numerical results are interpolated/extrapolated to physical quark masses and to the continuum to obtain the vector and scalar form factors $f_+(q^2)$ and $f_0(q^2)$ with full error budgets at q^2 values spanning the range accessible in our simulations. We provide a possible explanation of tensions found between results for the form factor from different lattice collaborations. Model- and truncation-independent z-parametrization fits following a recently proposed Bayesian-inference approach extend our results to the entire allowed kinematic range. Our results can be combined with experimental measurements of $B_s \to D_s$ and $B_s \to K$ semileptonic decays to determine $|V_{ub}| = 3.8(6) \times 10^{-3}$. The error is currently dominated by experiment. We compute differential branching fractions and two types of R ratios, the one commonly used as well as a variant better suited to test lepton-flavor universality.

- light and strange: domain wall fermions
- bottom: relativistic heavy quarks
- $M_\pi \gtrsim 270\,\mathrm{MeV}$, 3 lattice spacings

Choice of ff basis: (f_0, f_+) vs $(f_\parallel, f_\perp)$

- Interested in matrix elements of the vector current $\langle B_s | \mathcal{V}^\mu | K \rangle$
- Can be decomposed as

$$\langle K|\mathcal{V}^\mu|B_s\rangle = \sqrt{2M_{B_s}}\left[v^\mu f_\parallel(E_K) + p_\perp^\mu f_\perp(E_K)\right]$$

- trivially related to f_+, f_0 via

$$f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 - M_K^2}\left[(M_{B_s} - E_K)f_\parallel(E_K) + (E_K^2 - M_K^2)f_\perp(E_K)\right]$$

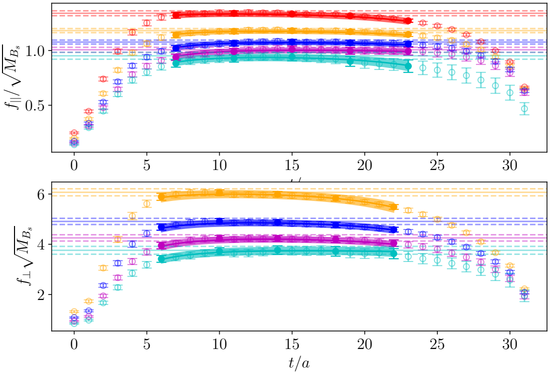
$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}}\left[f_\parallel(E_K) + (M_{B_s} - E_K)f_\perp(E_K)\right]$$

$\Rightarrow$ Convenient: $\mathcal{V}_0$ vs $\mathcal{V}_i$ to isolate $f_\parallel$ and $f_\perp$ from correlators

$O(a)$ improvement and renormalisation

- Mixed action current (DWF-RHQ) so no automatic $O(a)$-improvement. Done via 1-loop computation of improvement operators + non-perturbative operator insertions
- "Mostly non-perturbative" renormalisation:
  - Z_V^{ll} and Z_V^{hh} computed non-perturbatively
  - $Z_V^{lh} \approx \rho^{hl}\sqrt{Z_V^{ll} Z_V^{hh}}$
  - ρ computed at one loop
- Expect residual discretisation effects of size: $(a\Lambda)^2$, $(a\vec{p})^2$, $\alpha_s a m_q$, $(a m_q)^2$ (the first two enter the chiral-continuum limit fit, the last two are small)

Correlation function fits



$M_\pi \sim 270\,\mathrm{MeV}, a^{-1} \sim 2.8\,\mathrm{GeV}$

- jointly fitting two point functions and ratios which converge to desired matrix elements
- frequentist fit
- jointly fit multiple momenta (imposing lattice dispersion relation)
- single source-sink sep., fit inc. excited states
- thinned data (to stabilise covariance matrix)

Extrapolation: lattice → real world

Parameterise chiral, kinematic and discretisation effects via HMχPT:

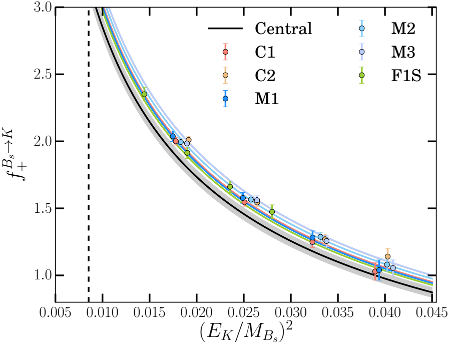
$$f_X^{B_s \to K}(M_\pi, E_K, a^2) = \frac{\Lambda}{E_K + \Delta_X} \times \left[c_{X,0}\left(1 + \frac{\delta f(M_\pi^s) - \delta f(M_\pi^p)}{(4\pi f_\pi)^2}\right) + c_{X,1}\frac{\Delta M_\pi^2}{\Lambda^2} + c_{X,2}\frac{E_K}{\Lambda} + c_{X,3}\frac{E_K^2}{\Lambda^2} + c_{X,4}(a\Lambda)^2 \right]$$

- Δ_X: relevant pole mass
- δf: chiral logs
- ΔM_π^s: $M_\pi^{2^{\rm sim}} - M_\pi^{2^{\rm phys}}$

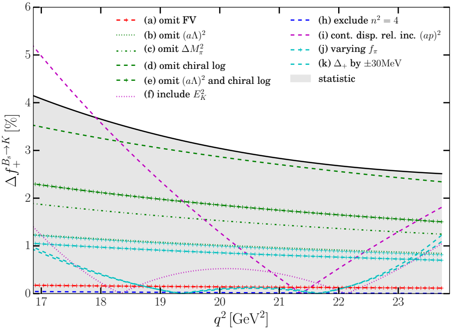
Then vary fit ansatz, estimate missing/H.O terms

Fit results f_+

HMχPT fit to lattice data

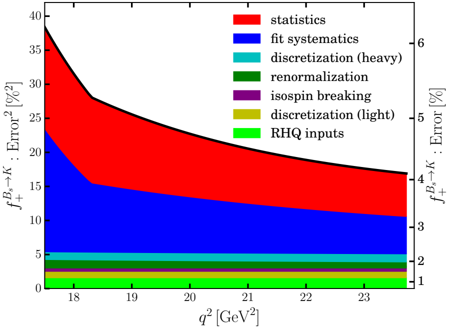


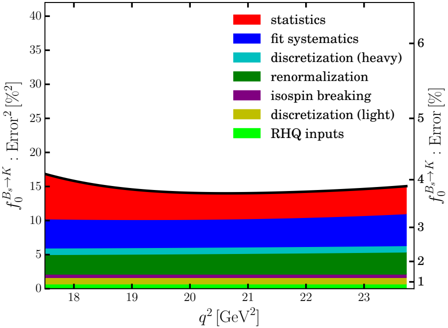
Fit systematics



$\Rightarrow$ take maximal deviation between the chosen fit and fit variation as fit-systematic value.

Assembling the error budget





- Dominated by statistical and fit systematic uncertainties $\Rightarrow$ both improvable!
- Most precise near q^2_{max}
- Data covers range $q^2 > 17\,\mathrm{GeV}^2$

Caveat: HMχPT in terms of f_+, f_0 or $f_\parallel, f_\perp$? (1)

Recall $\Lambda/(E_K + \Delta_X)$-term in HMχPT

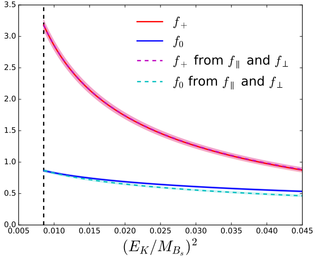
$$\Delta_+ = M_{B^*(1^-)} - M_{B_s}, \;\; M_{B^*(1^-)} = 5.32471\,\mathrm{GeV} \quad \text{(exp.)}$$
$$\Delta_0 = M_{B^*(0^+)} - M_{B_s}, \;\; M_{B^*(0^+)} = 5.63\,\mathrm{GeV} \quad \text{(the.)}$$

RBC/UKQCD'15 and FNAL/MILC'19 strategy:

1. Assume $f_\parallel$ dominated by f_0 and $f_\perp$ dominated by f_+.
2. HMχPT fit to $f_\parallel$, $f_\perp$ using $\Delta_\parallel \sim \Delta_0$, $\Delta_\perp \sim \Delta_+$
3. converting to f_+, f_0 in the continuum

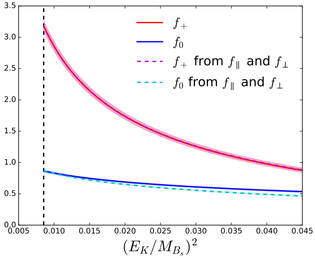
Is this justified?

Caveat: HMχPT in terms of f_+, f_0 or $f_\parallel, f_\perp$? (2)

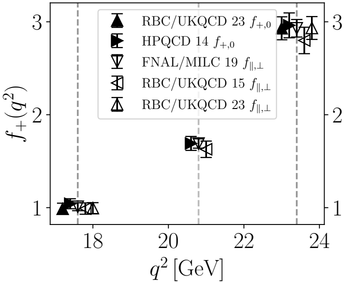


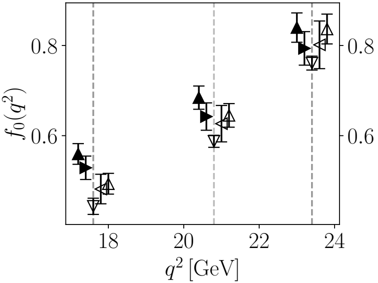
- ← All fine for f_+ (red vs magenta)✓
- ← Several (stat) sigmas difference for f_0 ✗!!
- ← Discrepancy gets worse with increasing energy ⇒ easy to miss!

Caveat: HMχPT in terms of f_+, f_0 or $f_\parallel, f_\perp$? (2)



- $\leftarrow$ All fine for f_+ (red vs magenta)✓
- $\leftarrow$ Several (stat) sigmas difference for f_0 ✗!!
- $\leftarrow$ Discrepancy gets worse with increasing energy $\Rightarrow$ easy to miss!
- $\downarrow$ picture persists with full error budget





$\Rightarrow$ Not unique to $B_s \to K$, same strategy was used for $B \to \pi$

Extracting $|V_{ub}|$ from $B_s \to K\ell\nu$ – in practice

$$\frac{d\Gamma(B_s \to K\ell\nu_\ell)}{dq^2} \approx |V_{ub}|^2 \times \left[\left| f_+(q^2) \right|^2 \mathcal{K}_1 + \left| f_0(q^2) \right|^2 \mathcal{K}_2 \right]$$

- Two bins for LHCb measurement

$$R_{BF} \equiv \frac{\mathcal{B}(B_s^0 \to K^- \mu^+ \nu_\mu)}{\mathcal{B}(B_s^0 \to D_s^- \mu^+ \nu_\mu)}$$

$$q^2 \leq 7\,\mathrm{GeV}^2: \quad R_{BF}^{\mathrm{low}} = 1.66(80)(86) \times 10^{-3}\,,$$
$$q^2 \geq 7\,\mathrm{GeV}^2: \quad R_{BF}^{\mathrm{high}} = 3.25(21)(^{+18}_{-19}) \times 10^{-3}\,.$$

- Lattice: Controlled uncertainties for $17\,\mathrm{GeV}^2 \lesssim q^2$.

Way out: Model Independent z-expansion!

Extrapolating over the full kinematic range: z-expansion

- Lattice data typically limited to $q^2 \in [q^2_{\mathrm{min,\,sim}}, q^2_{\mathrm{max}}]$.
- Want form factors over <u>**full** range $[0, q^2_{\mathrm{max}}]$</u>.
- Ff's satisfy kinematic constraint $f_+(0) = f_0(0)$.
- Map $q^2 \in [0, q^2_{\mathrm{max}}]$ to $z \in [z_{\mathrm{min}}, z_{\mathrm{max}}]$ with $|z| < 1$ and branch cut t_*.

$$z(q^2; t_0) = \frac{\sqrt{t_* - q^2} - \sqrt{t_* - t_0}}{\sqrt{t_* - q^2} + \sqrt{t_* - t_0}}$$

- Form factor is a polynomial in z after poles have been removed

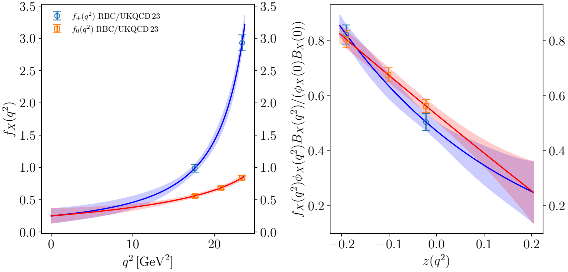
e.g. BGL: Boyd, Grinstein, Lebed [PRL **74** 4603]:

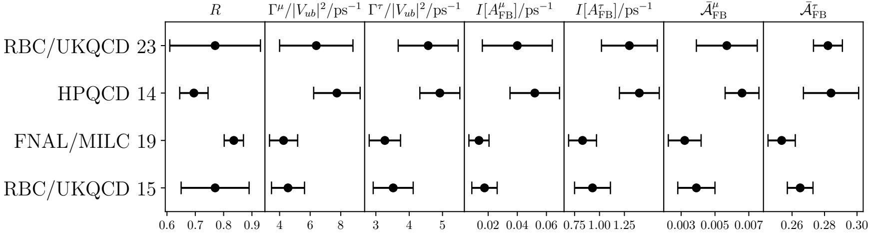
$$f_X(q^2) = \left(\prod_{\mathrm{poles}} \frac{1}{B_X(q^2)}\right) \frac{1}{\phi_X(q^2)} \sum_{n \geq 0} a_{X,n} z^n \quad \text{with} \quad \sum_n |a_{X,n}|^2 \leq 1$$

Goal: Determine some un-truncated number of coefficients $a_{X,n}$ to obtain model independent parameterisation [Flynn, Jüttner, JTT: 2303.11285]

z-expansion results and pheno

- B.I. converges from z-exp order $(K_+, K_0) \sim (5,5)$ onward
- $10^3 \times |V_{ub}| = 3.78(61)$
- $R_{B_s \to K} = 0.77(16)$
- $R^{\rm impr}_{B_s \to K} = 1.72(11)$





How to scrutinise results?

- these analyses are **hard** and very time consuming!
- many dependencies (sources of systematics) to consider:
  - excited states (particularly when approaching M_π^{phys} ensembles
  - chiral (M_π)
  - heavy quark (m_b)
  - kinematic (q^2)
  - discretisation, improvement and renormalisation (a)
- limited data to control all of these
- many choices to make and/or parameters to fit

⇒ **easy to miss something!** (and from looking at the comparison of different results we clearly do!)

Furthermore different works have

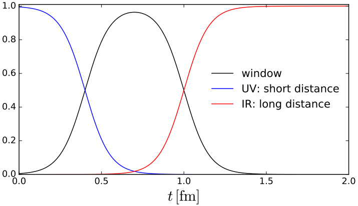
- data sets with different parameter coverage
- data sets with different statistical and systematic properties
- different approaches

Learning from our community: Similarities with $g-2$

- the stakes are high ✓
- if final results disagree, it is very hard to pin down why ✓
- potential for an "analyst bias" ✓
- "easy" to blind ✗
  (normalisation vs shapes!)
  but we should still do it!

"Resolved" by **simpler quantities**, less susceptible to some systematics

["windows" - JTT, RBC/UKQCD 18: 1801.07224]



is there something similar we can do for exclusive decays?

Suggestion for benchmarks and checks [JTT, Della Morte: 2310.02705]

- **Full error budget** for $f_0(q^2_{\rm max})$ for PS $\to$ PS or $f(w=1)$ for PS $\to$ V **solely based on the zero momentum data points**.
  $\Rightarrow$ only most precise data points enter.
  $\Rightarrow$ no interpolation in final state energy required.
- $f_X(q^2_{\rm ref})$ **based only on data in the vicinity** of $q^2_{\rm ref}$.
  $\Rightarrow$ more direct comparisons, relying less on analysis strategy.
  $\Rightarrow$ highlights differences: modified-z vs. $w-1$ vs. E_P expansions?
  $\Rightarrow$ Illustrates information content of simulated data near $q^2_{\rm ref}$.
- For $m_h < m_b$: perform M_π–a–q^2– extrapolation at **fixed** m_h.
  $\Rightarrow$ Eliminates heavy-quark extrapolation dimension
  $\Rightarrow$ disentangles $m_h \to m_b$ and $a \to 0$.
    $\Rightarrow$ Better control over the continuum limit.
  $\Rightarrow$ 'canonical choices' e.g. $2m_c$ or $m_b/2$ helps comparisons.
- Publish reference q^2 value data **before** z-expansion
  $\Rightarrow$ no unitarity imposed yet, no error reduction from z-expansion
- **publish fit coefficients** & **correlations** (for all fits)

Moving to massive schemes: RI/mSMOM

- RI/MOM and RI/SMOM are defined in the chiral limit of QCD

$\Rightarrow$ mass independent, i.e. all renormalisation constants Z independent of the fermion masses.

$\Rightarrow$ introduces discretisation effects scaling with $(am_q)^n$.

$\Rightarrow$ on typical lattices $am_c \sim 0.2$ $am_b \lesssim 1$. Large cut-off effects!

- extension of RI/SMOM away from chiral limit: renormalisation conditions at finite renormalised mass $\overline{m}$ suggested in [Boyle et al., 2016],

ADVANTAGE: Different masses at which the scheme is defined different approaches to the continuum limit. Possible to choose this to reduce cut-off effects?

Preliminary! – first numerical implementation of mSMOM, computing the charm quark mass [Del Debbio, Erben, Flynn, Mukherjee, JTT - in preparation, but near final]

Renormalisation conditions (RI/SMOM)

Evaluated for SMOM momentum configuration

$$1 = \lim_{m_R \to 0} \frac{1}{12p^2} \operatorname{Tr}\left[-iS_R(p)^{-1}\not{p}\right],$$

$$1 = \lim_{m_R \to 0} \frac{1}{12m_R}\left\{ \operatorname{Tr}\left[S_R(p)^{-1}\right] + \frac{1}{2}\operatorname{Tr}\left[\left(iq \cdot \Lambda_{\mathrm{A},R}\right)\gamma_5\right]\right\},$$

$$1 = \lim_{m_R \to 0} \frac{1}{12q^2} \operatorname{Tr}\left[\left(q \cdot \Lambda_{\mathrm{V},R}\right)\not{q}\right],$$

$$1 = \lim_{m_R \to 0} \frac{1}{12q^2} \operatorname{Tr}\left[q \cdot \Lambda_{\mathrm{A},R}\gamma_5\not{q}\right],$$

$$1 = \lim_{m_R \to 0} \frac{1}{12i} \operatorname{Tr}\left[\Lambda_{\mathrm{P},R}\gamma_5\right],$$

$$1 = \lim_{m_R \to 0} \frac{1}{12} \operatorname{Tr}\left[\Lambda_{\mathrm{S},R}\right].$$

ensures continuum Ward Identities hold, yielding

$$Z_V = Z_A = 1 \qquad Z_P = Z_S \qquad Z_m Z_P = 1$$

Renormalisation conditions (RI/mSMOM) [Boyle et al., 2016]

$$1 = \lim_{m_R \to \overline{m}} \frac{1}{12p^2} \mathrm{Tr}\left[-iS_R(p)^{-1} \not{p}\right],$$

$$1 = \lim_{m_R \to \overline{m}} \frac{1}{12m_R} \left\{ \mathrm{Tr}\left[S_R(p)^{-1}\right] + \frac{1}{2}\mathrm{Tr}\left[(iq \cdot \Lambda_{\mathrm{A},R})\gamma_5\right] \right\},$$

$$1 = \lim_{m_R \to \overline{m}} \frac{1}{12q^2} \mathrm{Tr}\left[(q \cdot \Lambda_{\mathrm{V},R}) \not{q}\right],$$

$$1 = \lim_{m_R \to \overline{m}} \frac{1}{12q^2} \mathrm{Tr}\left[(q \cdot \Lambda_{\mathrm{A},R} + 2m_R \Lambda_{P,R})\gamma_5 \not{q}\right],$$

$$1 = \lim_{m_R \to \overline{m}} \frac{1}{12i} \mathrm{Tr}\left[\Lambda_{\mathrm{P},R}\gamma_5\right],$$

$$1 = \lim_{m_R \to \overline{m}} \left\{ \frac{1}{12} \mathrm{Tr}\left[\Lambda_{\mathrm{S},R}\right] + \frac{1}{6q^2} \mathrm{Tr}\left[2m_R \Lambda_{P,R}\gamma_5 \not{q}\right] \right\}.$$

- evaluated at arbitrary mass scale $m_R = \overline{m}$, which defines the scheme.
- constructed so that continuum Ward Identities still hold.
- linear system of equations for Z_q, Z_m, Z_A, Z_V, Z_S, Z_P.
- $m_R \to 0$ limit reproduces RI/SMOM.

Illustration of RI/mSMOM strategy: the charm-quark mass

$$\hat{m}_R^{\mathrm{RI/mSMOM}}(\overline{\mu}, \overline{m}) = \lim_{a\to 0} Z_m(a, \overline{\mu}, a\overline{m}_0) \underbrace{(a\hat{m}_q + a\hat{m}_{\mathrm{res}})}_{a\hat{m}_0} a^{-1}$$

1. Simulate at a range of mass points am_q on several ensembles.
2. Determine ren. constants $Z_i(a, a\mu, am_q)$, hadron mass $M(a, am_q)$ as well as $am_{\mathrm{res}}(a, am_q)$ (additive quark mass renormalisation).
3. Define scheme: Fix $\overline{\mu}$ (ren. scale) and $\overline{M}$ (to set $\overline{m}$ in ren. conds.).
4. Fix $\hat{M}$ to set the quark mass $\hat{m}$ we want to determine (e.g. η_c to determine m_c).
5. Interpolate $Z_i(a, a\mu, am_0)$ to fixed μ and $a\overline{m}_0$ on all ensembles.
6. Interpolate to determine $a\hat{m}_0$ on all ensembles.
7. Take the continuum limits of $\hat{m}_0$ and $\overline{m}_0$.
8. (optional) Convert to RI/SMOM or $\overline{\mathrm{MS}}$ and/or run to desired scale μ.

RI/mSMOM - First numerical implementation

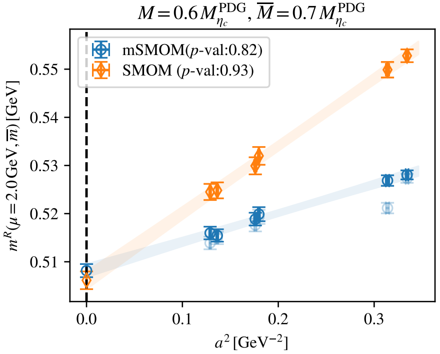
Charm quark mass good testing ground (cheap and precise):

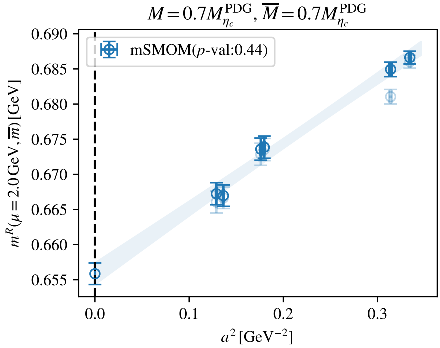
- only 2pt-functions and NPR bilinears needed
- charm-quark can be computed fully relativistically [JTT et al. JHEP 04 (2016) 037, JHEP 12 (2017) 008].

Domain-wall fermion ensembles at 3 lattice spacings (C, M, F) with the Möbius (M) and Shamir (S) kernels

name	L/a	T/a	$a^{-1}[\mathrm{GeV}]$	$m_\pi[\mathrm{MeV}]$	am_l	am_s
C1M	24	64	1.7295(38)	276	0.005	0.0362
C1S	24	64	1.7848(50)	340	0.005	0.04
M0M	64	128	2.3586(70)	139	0.000678	0.02661
M1M	32	64	2.3586(70)	286	0.004	0.02661
M1S	32	64	2.3833(86)	304	0.004	0.03
F1M	48	96	2.708(10)	232	0.002144	0.02144
F1S	48	96	2.785(11)	267	0.002144	0.02144

modified approach to the continuum - **PRELIMINARY**

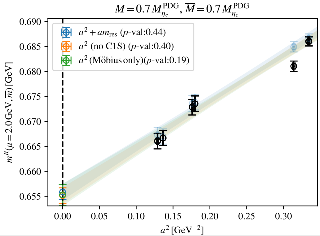




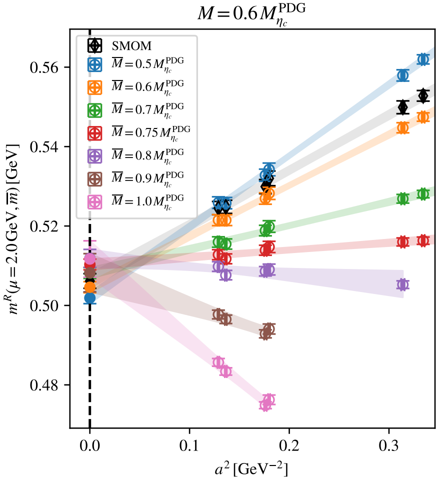
↑ continuum limit of desired m (in scheme defined by $\overline{m}$)

→↑ continuum limit to determine definition of the scheme defined by $\overline{m}$)

→ variations



modified approach to the continuum - **PRELIMINARY**



← continuum limit results still in different schemes! Values cannot be directly compared from plot.

- Very different CL approaches,
- conversion factor is known and close to 1, consistent results after conversion to same scheme (take my word for it for now)
- full details plus value of the charm quark mass to appear soon (arXiv:2407.XXXXX)

(somewhat sobering) Conclusions

- "bread and butter" quantities not in the best shape - various niggling tensions
- current results require scrutiny by our community
- suggestion for benchmark quantities to help this

- resolved some issues for $B_s \to K$ (and possibly $B \to \pi$)

- Many decay channels not covered here since there are not enough results to make comparisons. Often only a single result for a given channel.
- ...but these are difficult observables with many hard to control sources of systematic uncertainties. $\Rightarrow$ very human-time intensive!

- First numerical implementation of massive NPR scheme promising and to appear soon!