Hadronic matrix elements with the gradient flow

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Gradient flow

Narayanan, Neuberger:2006

Lüscher 2010
Lüscher, Weisz 2011

$x_\mu = (\mathbf{x}, x_4) \quad t \to \text{flow-time} \quad [t] = -2$

$A_\mu(x) = A_\mu^a(x) T^a \quad \to \quad \text{gluon fields}$

$$\partial_t B_\mu(x,t) = D_\nu G_{\nu\mu}(x,t)$$
$$B_\mu(x,t)|_{t=0} = A_\mu(x)$$

$D_\nu = \partial_\nu + [B_\nu(x,t), \cdot]$

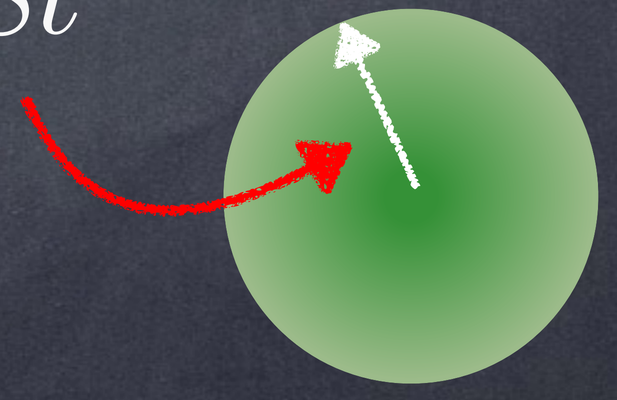
$G_{\mu\nu}(x,t) = \partial_\mu B_\nu(x,t) - \partial_\nu B_\mu(x,t) + [B_\mu, B_\nu]$

$$\partial_t B_\mu = \partial_\nu \partial_\nu B_\mu$$

$$B_\mu(x,t) = \int d^4y \; K(x-y;t) A_\mu(y)$$

$$K(x;t) = \int \frac{d^4p}{(2\pi)^4} \mathrm{e}^{ipx} \mathrm{e}^{-tp^2} = \frac{\mathrm{e}^{-x^2/4t}}{(4\pi t)^2}$$

- Gaussian damping at large momenta
- Smoothing at short distance over a range $\sqrt{8t}$



$B_\mu(x,t) \qquad t > 0 \qquad$ finite

Continuum limit is finite

Lüscher, Weisz: 2011

Gradient flow

Lüscher: 2013

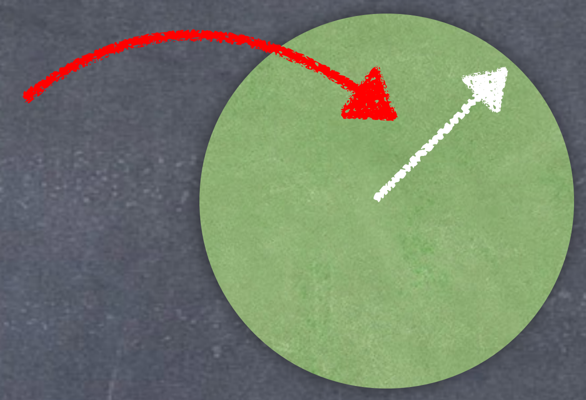
$x_\mu = (\mathbf{x}, x_4) \quad t \to \text{flow-time} \quad [t] = -2$

$$\partial_t \chi(x,t) = \Delta \chi(x,t)$$
$$\partial_t \bar{\chi}(x,t) = \bar{\chi}(x,t)\overleftarrow{\Delta}$$
$$\chi(x, t=0) = \psi(x)$$
$$\bar{\chi}(x, t=0) = \bar{\psi}(x)$$

$\Delta = D_{\mu,t} D_{\mu,t} \qquad D_{\mu,t} = \partial_\mu + B_{t,\mu}$

$$\chi(x,t) = \int d^4y K(x-y,t)\psi(y) \quad K(x,t) = \frac{1}{4\pi t^2} \mathrm{e}^{-\frac{x^2}{4t}}$$

- Smoothing over a range $\sqrt{8t}$



- Gaussian damping at large momenta

$$\chi_R(x,t) = Z_\chi^{1/2} \chi(x,t)$$

$$\mathcal{O}(x,t) = \overline{\chi}(x,t)\Gamma(x,t)\chi(x,t) \qquad \mathcal{O}_R = Z_\chi \mathcal{O}$$

$$\Sigma_t = \langle \overline{\chi}(x,t)\chi(x,t) \rangle \qquad \Sigma_{t,R} = Z_\chi \Sigma_t$$

No additive divergences

Continuum limit finite after normalizing fermion fields

Strategy - Short flow-time expansion

Lüscher: 2013

$$[\mathcal{O}_i(t)]_{\mathrm{R}} = \sum_i c_{ij}(t,\mu)\,[\mathcal{O}_i(t=0,\mu)]_{\mathrm{R}} + O(t)$$

LQCD PT - LQCD

$$c_{ij}(t,\mu) = \delta_{ij} + \frac{\alpha_s(\mu)}{4\pi} c_{ij}^{(1)}(t,\mu) + O(\alpha_s^2)$$

- Calculation of matrix elements with flowed fields
  - Multiplicative renormalization (no power divergences and no mixing)
- Calculation of Wilson coefficients
  - Insert OPE in off-shell amputated 1PI Green's functions
- Power divergences subtracted non-perturbatively (LQCD)
- Determination of the physical renormalized matrix element at zero flow-time

A.S., Luu, de Vries: 2014-2015
Dragos, Luu, A.S. de Vries: 2018-2019
Rizik, Monahan, A.S.: 2018-2020
A.S.: 2020
Kim, Luu, Rizik, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer: 2021
Monahan, Rizik, A.S., Stoffer: 2023
A.S.: 2023

PDF

Moments of parton distribution functions of any order from lattice QCD

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(Dated: December 1, 2023)

We describe a procedure to determine moments of parton distribution functions of any order in lattice QCD. The procedure is based on the gradient flow for fermion and gauge fields. The flowed matrix elements of twist-2 operators renormalize multiplicatively, and the matching with the physical matrix elements can be obtained using continuum symmetries and the irreducible representations of Euclidean 4-dimensional rotations. We calculate the matching coefficients at one-loop in perturbation theory for moments of any order in the flavor non-singlet case. We also give specific examples of operators that could be used in lattice QCD computations. It turns out that it is possible to choose operators with identical Lorentz indices and still have a multiplicative matching. One can thus use twist-2 operators exclusively with temporal indices, thus substantially improving the signal-to-noise ratio in the computation of the hadronic matrix elements.

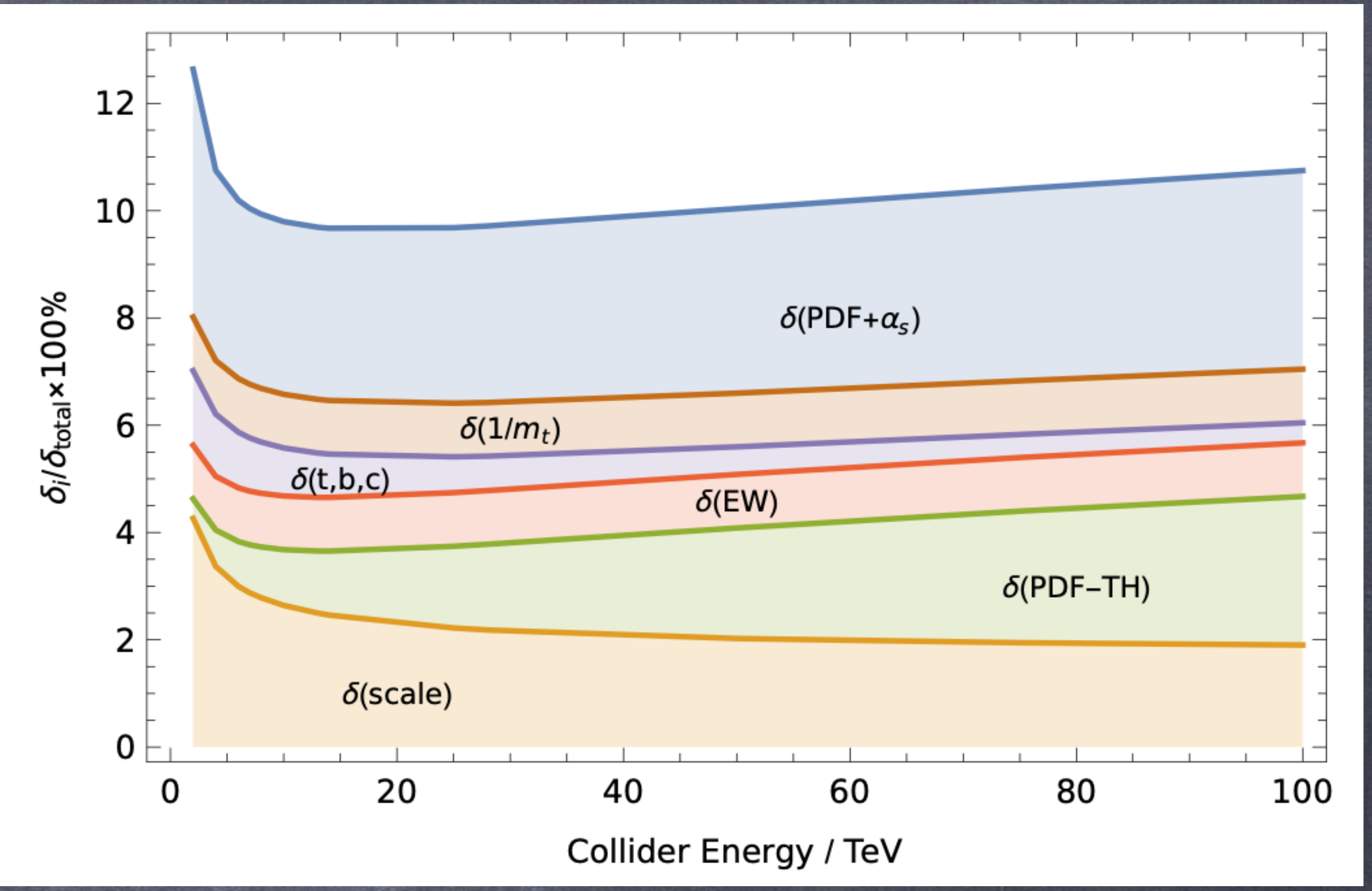
2311.18704 [hep-lat]
(PRD)

Motivations

- ## Higgs boson production

PDF uncertainty is still one of the largest sources of theoretical uncertainty affecting the predictions for Higgs boson production

LHC Higgs Cross section Working Group : 2016
Snowmass 2021 White paper



- ## SM parameters

PDFs contribute to precise extraction of the SM parameters from the LHC data ->

- strong coupling constant
- W boson mass

  for which different PDF sets yield results that are significantly different as compared to the size of statistical and systematic uncertainties

- ## New physics searches

PDF uncertainties become sizeable at large-x due to the lack of precise experimental constraints in that region
Large uncertainty in the predictions for the high-energy tails of the measured distributions, where programs of indirect new physics searches focus

Ball, Candido, Forte, Hekhorn, Nocera, Rojo, Schwan: 2022

PDF and Lattice QCD

- Connection between PDFs and hadronic matrix elements, which are calculable in lattice QCD, is established through the moments of the PDFs <x^n>

Curci, Furmanski, Petronzio: 1980
Collins, Soper: 1982

- Lattice QCD calculations of the moments of the PDFs, provide, in principle, a means for the complete reconstruction of the PDFs.

Kronfeld, Photiadis: 1985
Martinelli, Sachrajda: 1987 - 1988

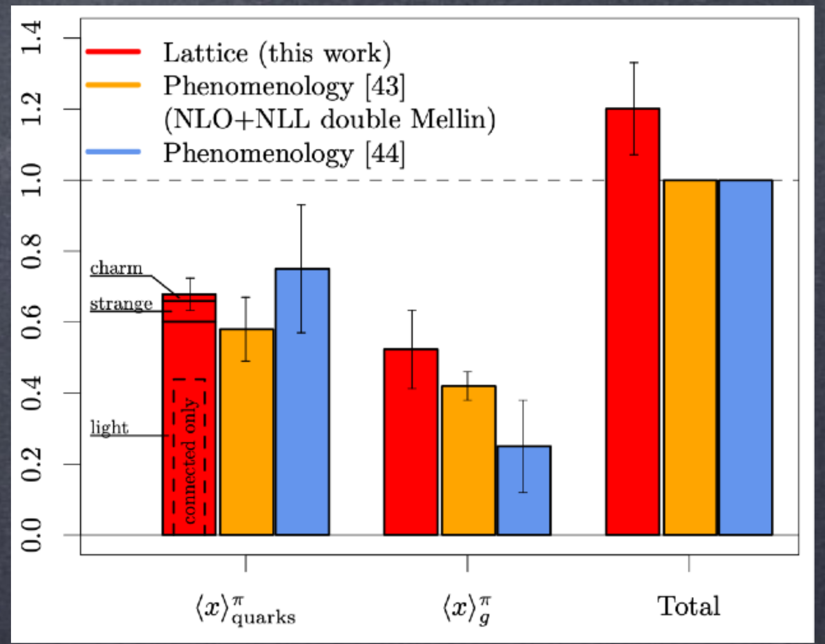
- This possibility has remained impractical due to the theoretical and numerical challenges associated with computing high moments
    - Continuum limit too difficult for <x^n> for n>3
    - For n=2,3 the need of non-vanishing external spatial momenta degrades the signal-to-noise ratio

$$\langle x^2\rangle^u_{\pi^+} = 0.110(7)(12)\,,\quad \langle x^2\rangle^u_{K^+} = 0.096(2)(2)\,,\quad \langle x^2\rangle^s_{K^+} = 0.139(2)(1)\,,$$

$$\langle x^3\rangle^u_{\pi^+} = 0.024(18)(2)\,,\quad \langle x^3\rangle^u_{K^+} = 0.033(6)(1)\,,\quad \langle x^3\rangle^s_{K^+} = 0.073(5)(2)\,,$$

Alexandrou et al. (ETMC): 2021

	this work	[20]	[44]	[45]
$\langle x\rangle^R_l$	$0.601(28)(_{-21})$	–	–	–
$\langle x\rangle^R_s$	$0.059(13)(_{-10})$	–	–	–
$\langle x\rangle^R_c$	$0.019(05)(_{-10})$	–	–	–
$\langle x\rangle^R_g$	$0.52(11)(^{+02})$	–	0.42(4)	0.25(13)
$\sum_f \langle x\rangle^R_f$	$0.68(05)(_{-03})$	0.220(207)	0.58(9)	0.75(18)
$\langle x\rangle^R_{u+d-2s}$	0.48(01)	0.344(28)	–	–
$\langle x\rangle^R_{u+d+s-3c}$	0.60(03)	–	–	–



Alexandrou et al. (ETMC): 2021

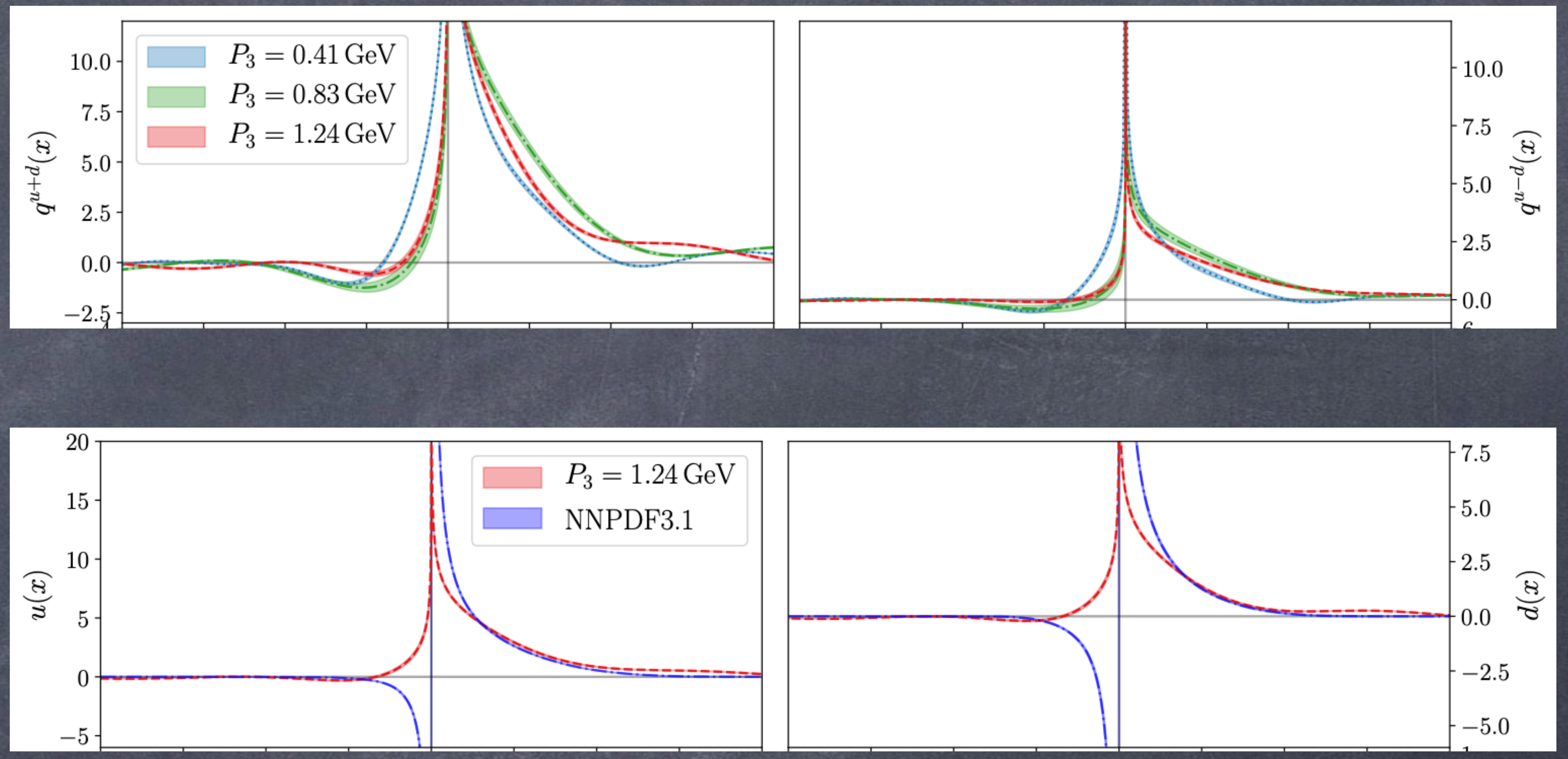
reviews of Refs. [37, 38]). Direct calculations of distribution functions on a Euclidean lattice have not been feasible due to the time dependence of these quantities. A way around this limitation is the calculation on the lattice of moments of distribution functions (historically for PDFs and GPDs) and the physical PDFs can, in principle, be obtained from operator product expansion (OPE). Realistically, only the lowest moments of PDFs and GPDs can be computed (see e.g. [39–44]) due to large gauge noise in high moments, and also unavoidable power-divergent mixing with lower-dimensional operators. Combination of the two prevents a reliable and accurate calculation of moments beyond the second or third, and the reconstruction of the PDFs becomes unrealistic.

Cichy, Constantinou: 2019

PDF and Lattice QCD

Approaches have been developed to determine the x-dependence of the PDFs

- Auxiliary scalar field — Aglietti et al.: 1998
- quasi-PDF (LaMET) — Ji: 2013
- pseudo-PDF — Radyushkin: 2017
- Fictitious heavy quark — Detmold, Lin: 2005
- Auxiliary scalar quark — Braun, Müller: 2008
- Compton amplitude + OPE — Chambers et al.: 2017
- Good Lattice Cross Sections — Ma, Qiu: 2018
- hadron tensor method — Lian et al.: 2019



Alexandrou et al. (ETMC): 2021

Method that addresses both the theoretical and numerical challenges faced in the past, which hindered the direct calculation of moments of any order from lattice QCD

Moments of the PDF: standard method

$$O_n^{rs}(x) = O_{\mu_1\cdots\mu_n}^{rs}(x) = \overline{\psi}^r(x)\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}}\psi^s(x)$$

$$\widehat{O}_n^{rs}(x) = Z_n^{\mathrm{MS}}\widehat{O}_{n,\mathrm{B}}^{rs}(x)$$

- Calculate matrix elements using lattice QCD
  - Rotational group symmetry is broken into the hypercubic group H(4)
- Irreducible representations of O(4) generally become reducible representations of H(4) inducing unwanted mixings under renormalization
  - Irreps of H(4) allow mixing with lower dimensional operators and complicate mixings with operators of the same dimension

Beccarini et al.: 1995
Gockeler et al.: 1996

- Operators with different index combinations belong to different irreps of H(4)

O_3 $\quad \mu_1 = \mu_2 = \mu_3 \quad \Rightarrow \quad 1/a^2\delta_{\mu_i\mu_j}\cos(ap_{\mu_j})$ Kronfeld, Photiadis: 1985

$\mu_1 \neq \mu_2 = \mu_3 \quad \Rightarrow \quad O_{411} - O_{433}$ Martinelli, Sachrajda: 1987

$\mu_1 \neq \mu_2 \neq \mu_3 \quad \Rightarrow \quad \left\langle h(p)|\widehat{O}_n|h(p)\right\rangle = p_{\mu_1}\cdots p_{\mu_n}A_n^h(\mu)$

Strategy

- Calculate matrix elements of flowed twist-2 fields -> renormalize multiplicatively
- Renormalize flowed fermion fields or build appropriate ratios
- Construct fields based on irreps of O(4). Perform continuum limit
- Compute matching factors in perturbation theory. Matching is multiplicatively in the continuum

Flowed twist-2 operators

$$O_n^{rs}(x,t) = \overline{\chi}^r(x,t)\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}}\chi^s(x,t)$$

$$O_n^{rs}(t) = Z_n O_{n,B}^{rs}(t)\,, \quad Z_n = Z_\chi$$

$$\left\langle \mathring{\overline{\chi}}_r(x,t)\overleftrightarrow{\not{D}}\mathring{\chi}_r(x,t)\right\rangle = -\frac{N_c}{(4\pi)^2 t^2}$$

Makino, Suzuki: 2014

$$\chi^{r,\mathrm{MS}}(x,t) = (8\pi t)^{\epsilon/2}\,\zeta_\chi^{1/2}\mathring{\chi}^r(x,t)$$

$$\overline{\chi}^{r,\mathrm{MS}}(x,t) = (8\pi t)^{\epsilon/2}\,\zeta_\chi^{1/2}\mathring{\overline{\chi}}_r(x,t)$$

$$\zeta_\chi = 1 - \frac{\overline{g}^2}{(4\pi)^2}C_F\left(3\log(8\pi\mu^2 t) - \log(432)\right)$$

$$D = 4 - 2\epsilon$$

$$C_F = \frac{N_c^2-1}{2N_c}$$

$$\log\mu^2 = \log\overline{\mu}^2 + \gamma_E - \log 4\pi$$

NNLO

Harlander, Kluth, Lange: 2018

Artz et al.: 2019

O(4) irreducible representations

GL(4) irrep $$T_{\{\mu_1\cdots\mu_n\}} = \frac{1}{n!} \sum_{\substack{\sigma\in\text{all}\\ \text{permutations}}} T_{\mu_{\sigma(1)}\cdots\mu_{\sigma(n)}}$$

In O(4) an additional operation is allowed that commutes with orthogonal trafo: contraction of 2 indices

$$T^{(12)}_{\mu_1\cdots\mu_n} = T_{\alpha\alpha\mu_3\cdots\mu_n} = \delta_{\mu_1\mu_2} T_{\mu_1\cdots\mu_n}$$ rank n-2 tensor

Subspace of traceless tensors is invariant under O(4), i.e. the traceless rank n tensors are transformed among themselves under O(4)

Always possible to decompose $$T_{\mu_1\cdots\mu_n} = \widehat{T}_{\mu_1\cdots\mu_n} + \underbrace{\delta_{\mu_1\mu_2} T^{(12)}_{\mu_1\cdots\mu_n} + \cdots}_{\text{n(n-1)/2 terms}}$$ Invariant under O(4)

E.g. $$\widehat{T}_{\mu_1\mu_2} = T_{\mu_1\mu_2} - \frac{1}{4}\delta_{\mu_1\mu_2} T_{\alpha\alpha} \qquad \widehat{T}_{\mu_1\mu_2\mu_3} = T_{\mu_1\mu_2\mu_3} - \frac{1}{6}\left[\delta_{\mu_1\mu_2} T_{\alpha\alpha\mu_3} + \delta_{\mu_1\mu_3} T_{\alpha\mu_2\alpha} + \delta_{\mu_2\mu_3} T_{\mu_1\alpha\alpha}\right]$$

Traceless tensors invariant under vector index permutations —> starting point to construct all the irreducible representations of O(4) (Young symmetrizers)

Traceless and symmetrized rank-n tensors are an irreducible representation of O(4)

Matching coefficients

- Consider flowed twist-2 operators $O_n^{rs}(x,t) = \overline{\chi}^r(x,t)\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}}\chi^s(x,t)$

- Renormalize flowed twist-2 operators —> renormalization is ALWAYS multiplicative

$$O_n^{rs}(t) = Z_n O_{n,B}^{rs}(t)\,, \quad Z_n = Z_\chi \qquad \left\langle \mathring{\overline{\chi}}_r(x,t)\overleftrightarrow{\not{D}}\mathring{\chi}_r(x,t)\right\rangle = -\frac{N_c}{(4\pi)^2 t^2}$$

- Perform a short flow time expansion —> consider O(4) irreps —> traceless operators

$$\widehat{\mathring{O}}_n^{rs}(x,t) = \mathring{\overline{\chi}}^r(x,t)\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}}\mathring{\chi}^s(x,t) - \text{terms with } \delta_{\mu_i\mu_j}$$

Continuum limit is finite for any n

$$\left\langle h(p)|\widehat{\mathring{O}}_n(t)|h(p)\right\rangle = p_{\mu_1}\cdots p_{\mu_n}\left\langle x^{n-1}\right\rangle_h(t)$$

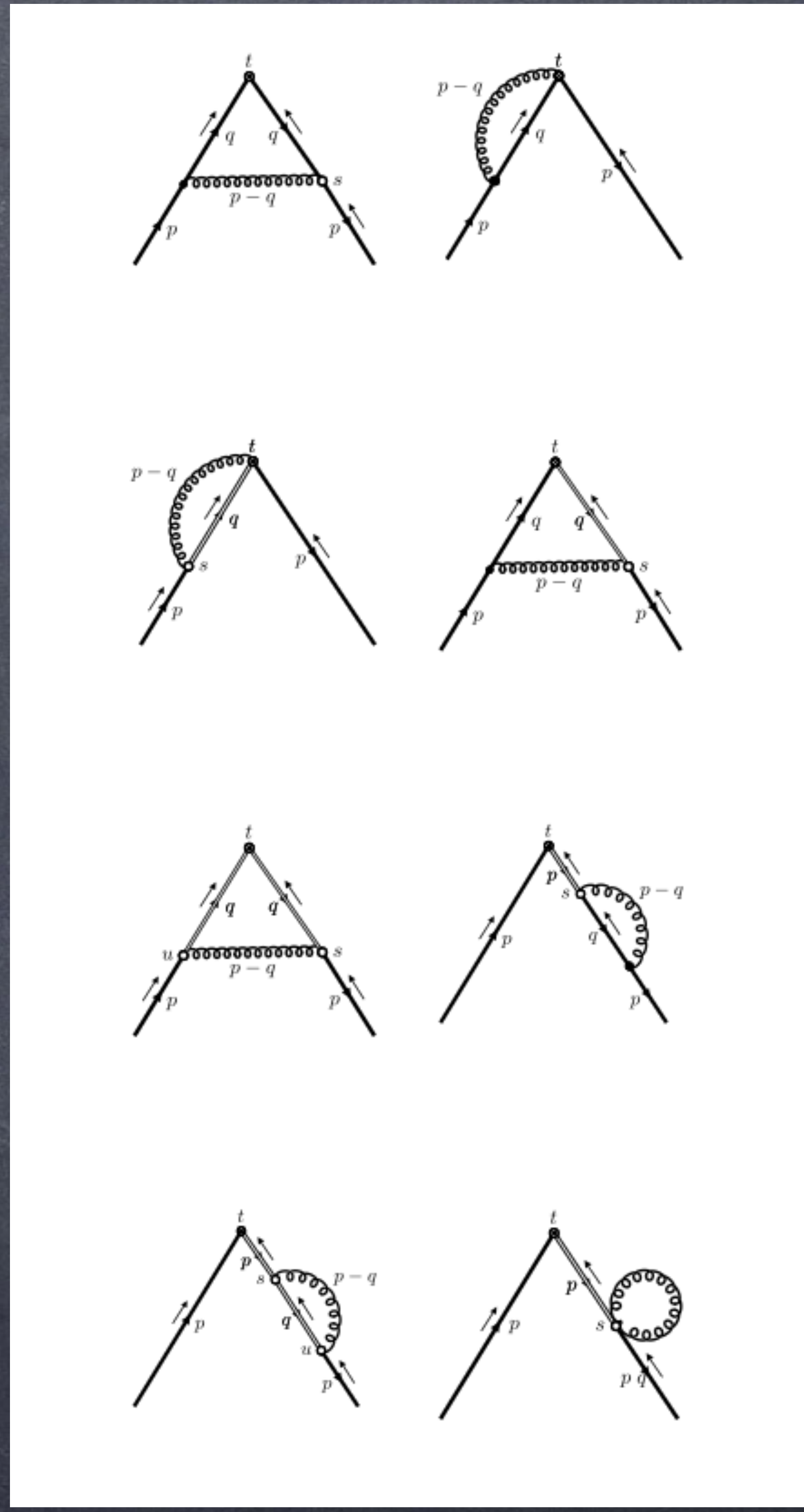
- Matching is multiplicative for traceless operators $\widehat{\mathring{O}}_n^{rs}(t) = c_n(t,\mu)\widehat{O}_n^{rs,\mathrm{MS}}(\mu) + O(t)$

- Calculate matching coefficients in PT

Matching coefficients

Matching equations $\left\langle \psi^r \mathring{\hat{O}}_n^{rs}(t)\overline{\psi}^s \right\rangle = c_n(t,\mu) \left\langle \psi^r \hat{O}_n^{rs,\mathrm{MS}}(t=0,\mu)\overline{\psi}^s \right\rangle$

A.S.: 2023



- Expand integrands of loop integrals in all scales excluding t
  - Analytic structure altered —> distortion of IR structure
  - in matching equation the IR modification drops out in the difference
  - Expanding loop integrals in the RHS vanish in DR —> UV and IR are identical
  - The LHS is UV-finite, beside the renormalization of the bare parameters and flowed fermion fields
  - The IR singularities on the LHS exactly match the UV MS counterterms

$$c_n(t,\mu) = 1 + \frac{\overline{g}^2(\mu)}{(4\pi)^2} c_n^{(1)}(t,\mu) + O(\overline{g}^4) \qquad c_n^{(1)}(t,\mu) = C_F\left[\gamma_n \log\left(8\pi\mu^2 t\right) + B_n\right]$$

$$\gamma_n = 1 + 4\sum_{j=2}^{n}\frac{1}{j} - \frac{2}{n(n+1)}$$

$$B_n = \frac{4}{n(n+1)} + 4\frac{n-1}{n}\log 2 + \frac{2-4n^2}{n(n+1)}\gamma_E - \frac{2}{n(n+1)}\psi(n+2) + \\ + \frac{4}{n}\psi(n+1) - 4\psi(2) - 4\sum_{j=2}^{n}\frac{1}{j(j-1)}\frac{1}{2^j}\phi(1/2,1,j) - \log(432)$$

Gross, Wilczek: 1974

$$\phi(z,s,a) = \sum_{k=0}^{\infty}\frac{z^k}{(k+a)^s}$$

$n=2$ Makino, Suzuki: 2014

O(a) improvement

$$O_n^{rs}(x) = O_{\mu_1\cdots\mu_n}^{rs}(x) = \overline{\psi}^r(x)\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}}\psi^s(x) \qquad \left\langle h(p)|\widehat{O}_n|h(p)\right\rangle = p_{\mu_1}\cdots p_{\mu_n}\left\langle x^{n-1}\right\rangle_h$$

- Beside the O(a) from the lattice theory twist-2 fields are affected by specific O(a) that depend on n
- Improvement coefficients are known only for n=2 and only in PT
- Only GW fermions or Wtm at maximal twist removes these O(a)

$$\widehat{\mathring{O}}_n^{rs}(x,t) = \overline{\mathring{\chi}}^r(x,t)\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}}\mathring{\chi}^s(x,t) - \text{terms with } \delta_{\mu_i\mu_j} \qquad \left\langle h(p)|\widehat{\mathring{O}}_n(t)|h(p)\right\rangle = p_{\mu_1}\cdots p_{\mu_n}\left\langle x^{n-1}\right\rangle_h(t)$$

- Hadronic matrix elements of flowed operators beside the O(a) from the lattice theory are only affected by O(am)
- The O(am) are independent on n (depend only on the fermion content)
- With ratios discretization effects are $O(a^2)$ —> clover fermions are back in the game

$$\frac{\left\langle x^{n-1}\right\rangle_h(t)}{\left\langle x^{m-1}\right\rangle_h(t)} \quad n \neq m \quad n \geq 3,\ m \geq 2$$

→ Finite continuum limit and O(a) improved

Strategy

$$\left\langle h(p)|\mathring{\widehat{O}}_n(t)|h(p)\right\rangle = p_{\mu_1}\cdots p_{\mu_n}\left\langle x^{n-1}\right\rangle_h(t)$$

Continuum limit is finite for any n

$$\left\langle x^{n-1}\right\rangle_h^{\mathrm{MS}}(\mu) = c_n(t,\mu)^{-1}\left\langle x^{n-1}\right\rangle_h(t) + O(t)$$

Matching is multiplicative for any n

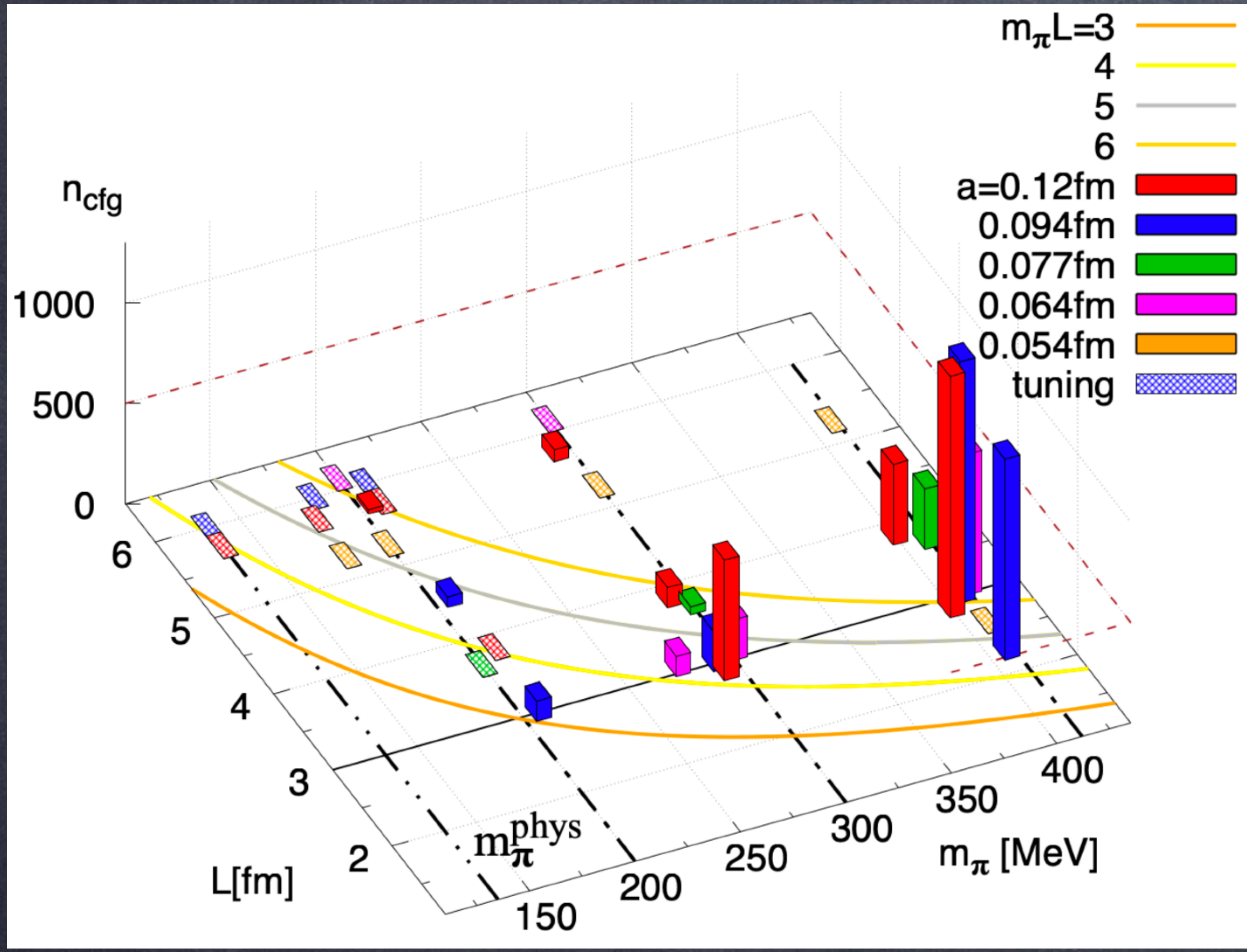
n=4 $\widehat{O}_{4444} = O_{4444} - \frac{3}{4}O_{\{\alpha\alpha 44\}} + \frac{1}{16}O_{\{\alpha\alpha\beta\beta\}}$

Vanishing spatial momenta for any n

$$\left\langle x^{n-1}\right\rangle_h^{\mathrm{MS}}(\mu) = \left\langle x^{m-1}\right\rangle_h^{\mathrm{MS}}(\mu)\frac{c_m(t,\mu)}{c_n(t,\mu)}\frac{\left\langle x^{n-1}\right\rangle_h(t)}{\left\langle x^{m-1}\right\rangle_h(t)}\,, \quad m\neq n \quad n\geq 3\; m\geq 2$$

Future with Open Science

- OpenLat: open science initiative. Gauges with SWF open to the whole community

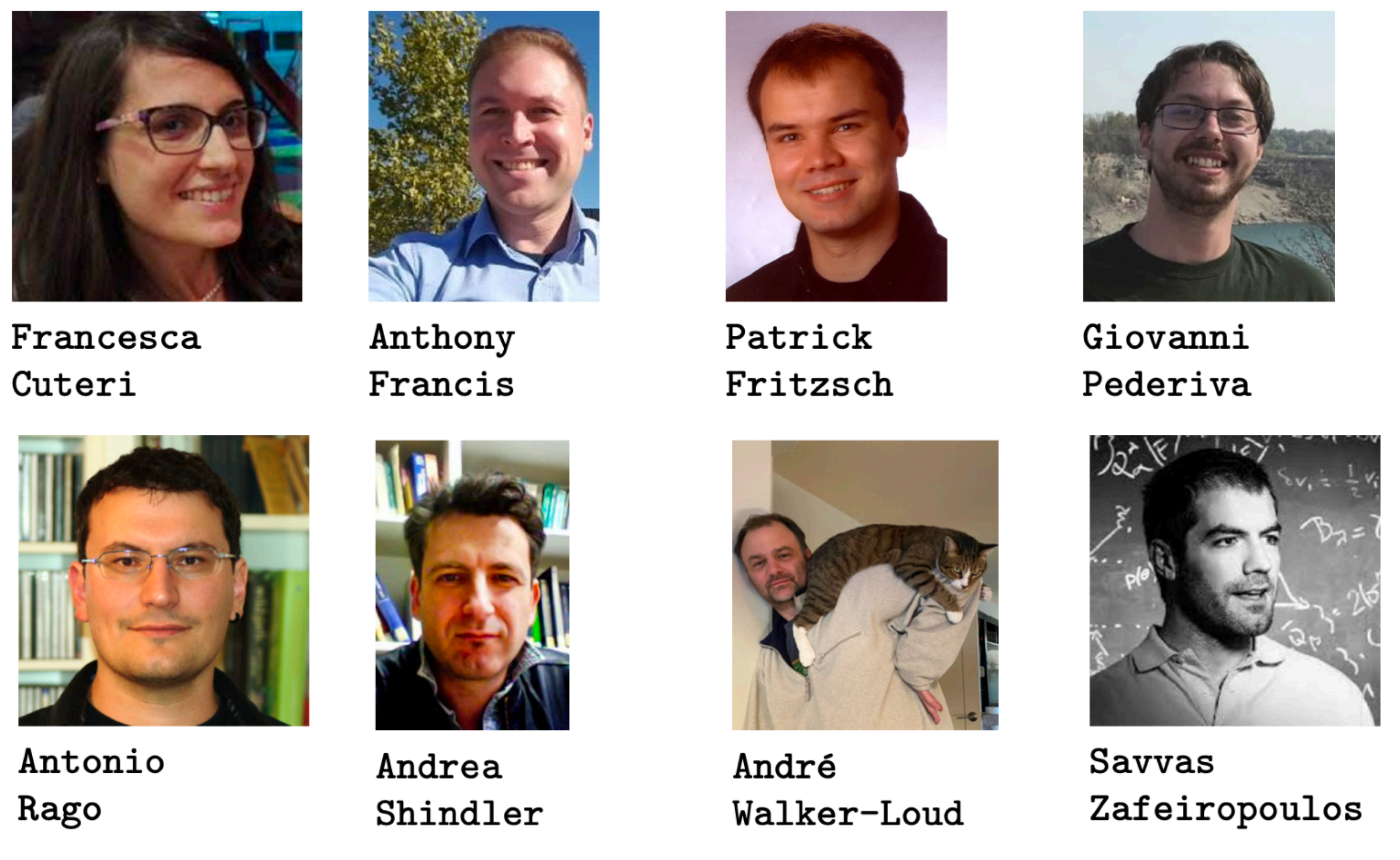


PRACE

EuroHPC

Neutron EDM with Stabilized Wilson Fermions: the theta term

Hadron structure with stabilized Wilson fermions



https://openlat1.gitlab.io

Flowed moments n=3

Jangho Kim (FZJ)

Dimitra Pefkou (LBL)

Lattice parameters (OpenLat)

$a \simeq 0.12$ fm $\quad L \simeq 2.9$ fm $\quad m_{PS} \simeq 410$ MeV

Statistics (sources x gauges)

$1 \times 200 = 200 \qquad \sim 4\%$

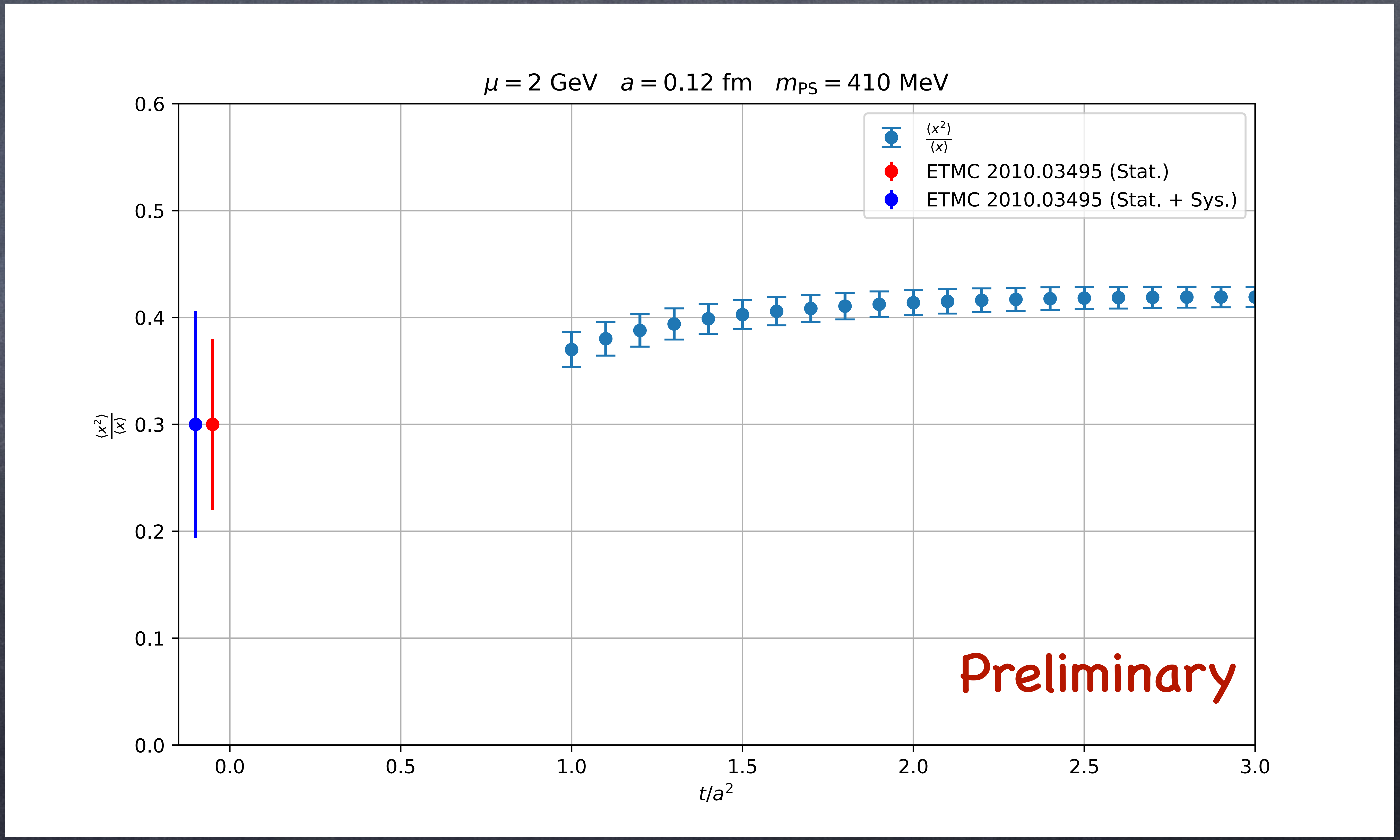
Lattice parameters (ETMC)

$a \simeq 0.093$ fm $\quad L \simeq 3$ fm $\quad m_\pi \simeq 260$ MeV

Statistics (sources x gauges)

$32 \times 122 = 3904 \qquad \sim 27\%$

$$\frac{\langle x^2 \rangle_{\overline{MS}}(\mu)}{\langle x \rangle_{\overline{MS}}(\mu)}$$



Flowed moments n=4

Lattice parameters (OpenLat)

$a \simeq 0.12$ fm $\quad L \simeq 2.9$ fm $\quad m_{PS} \simeq 410$ MeV

Statistics (sources x gauges)

$1 \times 200 = 200 \qquad \sim 5 - 7\%$

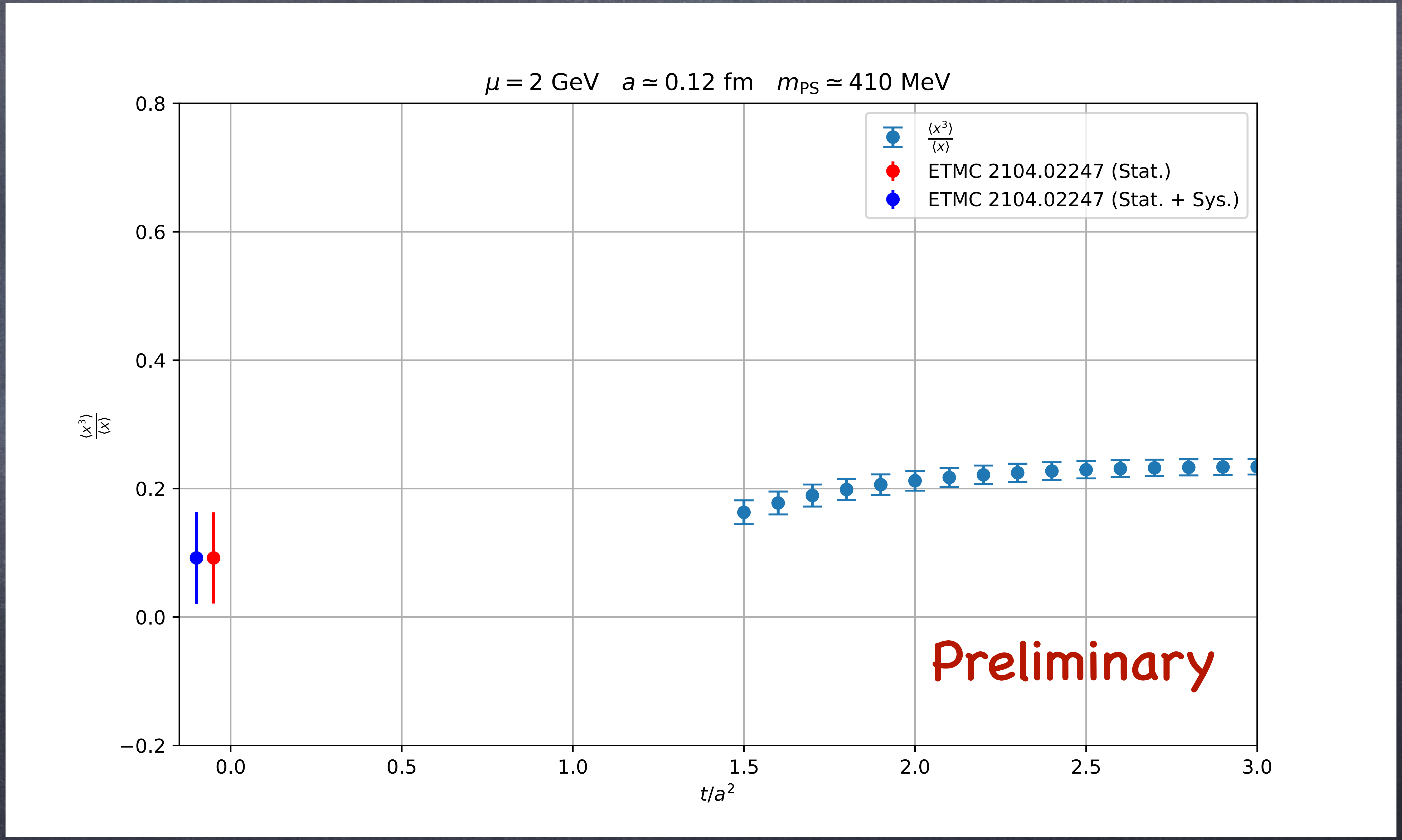
Lattice parameters (ETMC)

$a \simeq 0.093$ fm $\quad L \simeq 3$ fm $\quad m_{\pi} \simeq 260$ MeV

Statistics (sources x gauges)

$72 \times 122 = 8784 \qquad \sim 75\%$

$$\frac{\langle x^3 \rangle_{\overline{MS}}(\mu)}{\langle x \rangle_{\overline{MS}}(\mu)}$$



Potential systematics

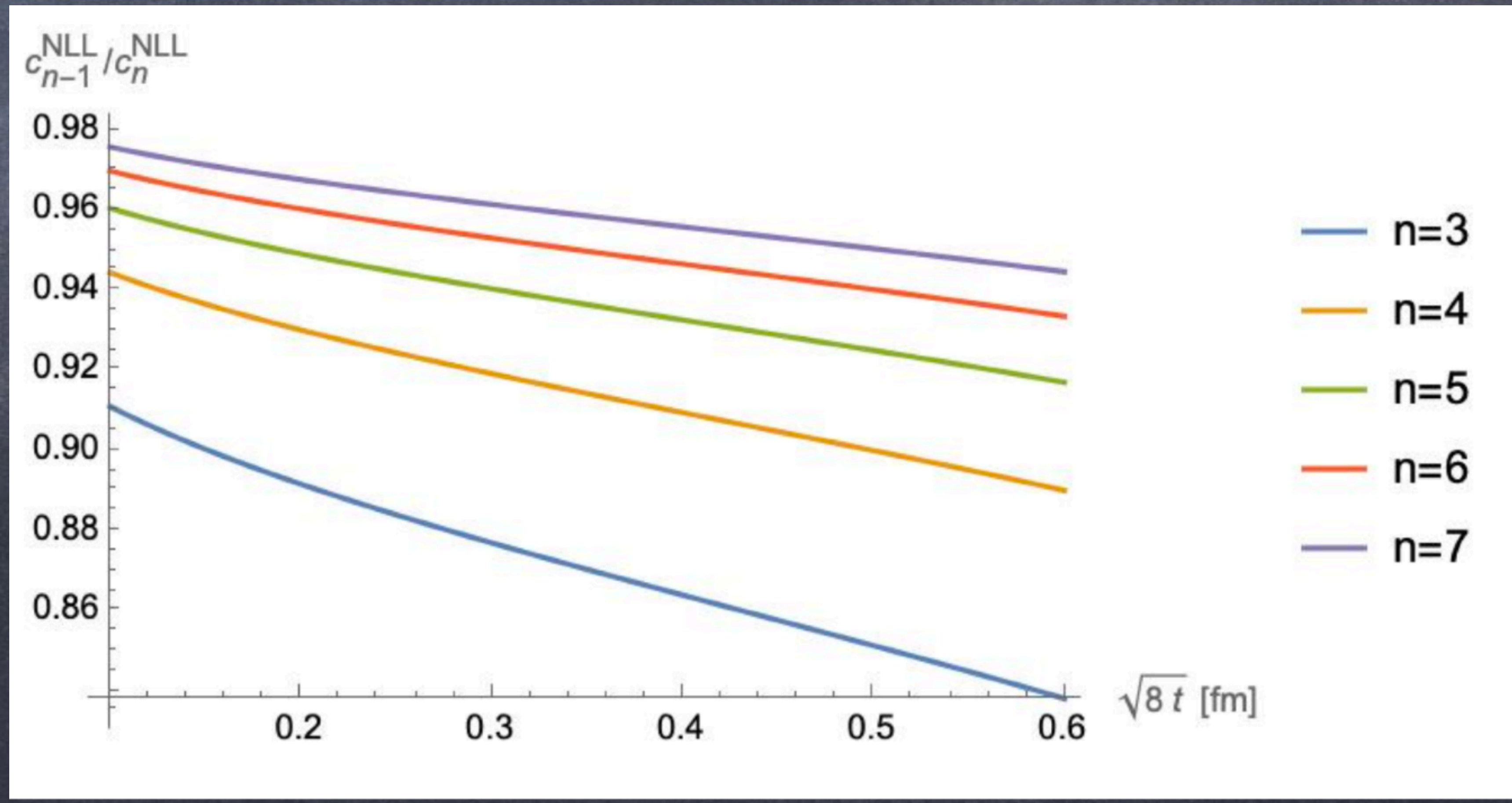
$$\langle x^{n-1}\rangle_h^{\rm MS}(\mu) = \langle x^{m-1}\rangle_h^{\rm MS}(\mu)\frac{c_m(t,\mu)}{c_n(t,\mu)}\frac{\langle x^{n-1}\rangle_h(t)}{\langle x^{m-1}\rangle_h(t)}\,, \quad m\neq n \quad n\geq 3\ m\geq 2$$

Finite volume effects at finite a the extension of the local operators is (n-1)a $\quad n\sim 10-12$

Discretization errors $\sqrt{8t}\gtrsim na$

Perturbative matching $\mu = 2\ \mathrm{GeV}$

$$c_n^{\rm NLL}(t,\mu,\overline{g}(\mu)) = c_n(t,q,\overline{g}(q))\exp\left\{-\int_{\overline{g}(\mu)}^{\overline{g}(q)} dx\frac{\gamma_n(x)}{\beta(x)}\right\}$$



n	$\langle x^{n-1}\rangle$	$\delta_{\rm PT}$
2	$\langle x\rangle$	2%
3	$\langle x^2\rangle$	11%
4	$\langle x^3\rangle$	8%
5	$\langle x^4\rangle$	7%
6	$\langle x^5\rangle$	6%
7	$\langle x^6\rangle$	6%

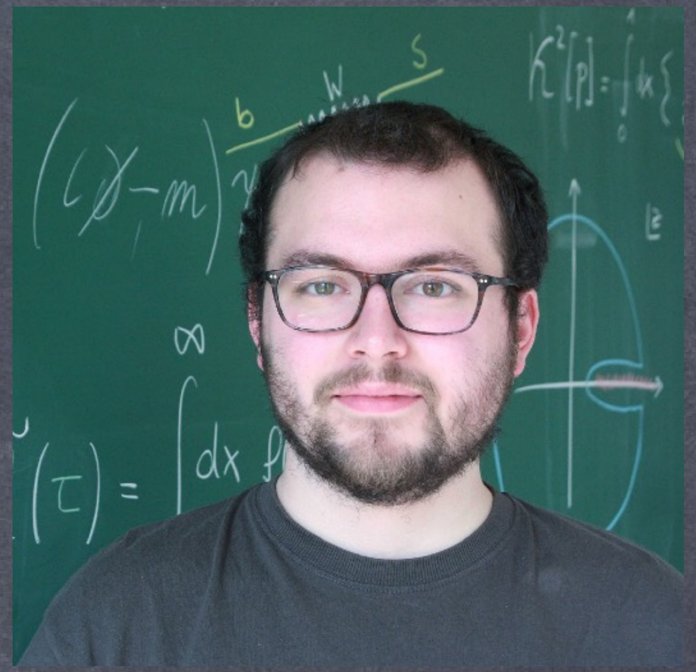
$$\sqrt{8t} = 2(n-1)a$$
$$a = 0.05\ \mathrm{fm}$$

Summary

- New method to calculate moments of any order from lattice QCD
- Method is general and can be used with any lattice action
- We make use of an intermediate regulator (GF) that simplifies the continuum limit
- After recovering O(4) symmetry the matching is done using continuum PT
- Matrix elements can be all calculated with vanishing external momenta
- Ratios of matrix elements improve further continuum limit and S/N

Heavy-quark physics

Black, Harlander, Lange, Rago, A.S., Witzel:
2310.18059 [hep-lat]
in progress



Matthew Black
(Siegen Uni)

Introduction

- Four-quark $\Delta B = 0$ and $\Delta B = 2$ matrix elements can be determined from lattice QCD simulations
- $\Delta B = 2$ well-studied by several groups -> precision increasing
  - preliminary $\Delta K = 2$ for Kaon mixing study with gradient flow

HPQCD: 1907.01025
FNAL/MILC: 1602.03560
RBC: 1406.6192
ETM,C:1308.1851

Taniguchi: 2019 Suzuki et al. :2020

- $\Delta B = 0$ -> exploratory studies from ~20 years ago + new developments for lifetime ratios
  - contributions from disconnected diagrams
  - mixing with lower dimensional operators

Lin, Detmold, Meinel: 2022

DiPierro, Sachrajda: 9805028
DiPierro, Sachrajda, Michael: 9906031
Becirevic: 0110124

1. Establish gradient flow renormalisation procedure with $\Delta B = 2$ matrix element
2. Pioneer $\Delta B = 0$ matrix elements calculations
3. Tackle disconnected contributions

Lattice details

- Gauge configurations RBC/UKQCD's 2+1 flavour Shamir DWF + Iwasaki gauge action ensembles

  Shamir: 1993 Iwasaki, Yoshie: 1984 Iwasaki: 1985

- To remove additional extrapolations in valence sector, we simulate at physical charm and strange
  - "neutral Ds" meson mixing

	L	T	L_s	a^{-1}/GeV	am_l^{sea}	am_s^{sea}	am_s^{val}	M_π/MeV	# cfgs	# sources
C1	24	64	16	1.785	0.005	0.040	0.03224	340	101	32
C2	24	64	16	1.785	0.010	0.040	0.03224	433	101	32
M1	32	64	16	2.383	0.004	0.030	0.02477	302	79	32
M2	32	64	16	2.383	0.006	0.030	0.02477	362	89	32
M3	32	64	16	2.383	0.008	0.030	0.02477	411	68	32
F1S	48	96	12	2.785	0.002144	0.02144	0.02167	267	98	24

Allton et al.: 2008
Aoki et al.: 2010
Blum et al.: 2014
Boyle et al.: 2017

- Fully relativistic DWF for valence quarks
- Strange tuned to physical value (Shamir DWF)
- Heavy c quarks tuned for physical Ds mass (Möbius DWF)
  -> am_c < 0.7 with stout smearing of gauge fields Morningstar, Peardon: 2003
- Z2 wall sources for all quark propagators Boyle et al. :2018
  - Sources for strange propagators are also Gaussian smeared Allton et al.: 1993

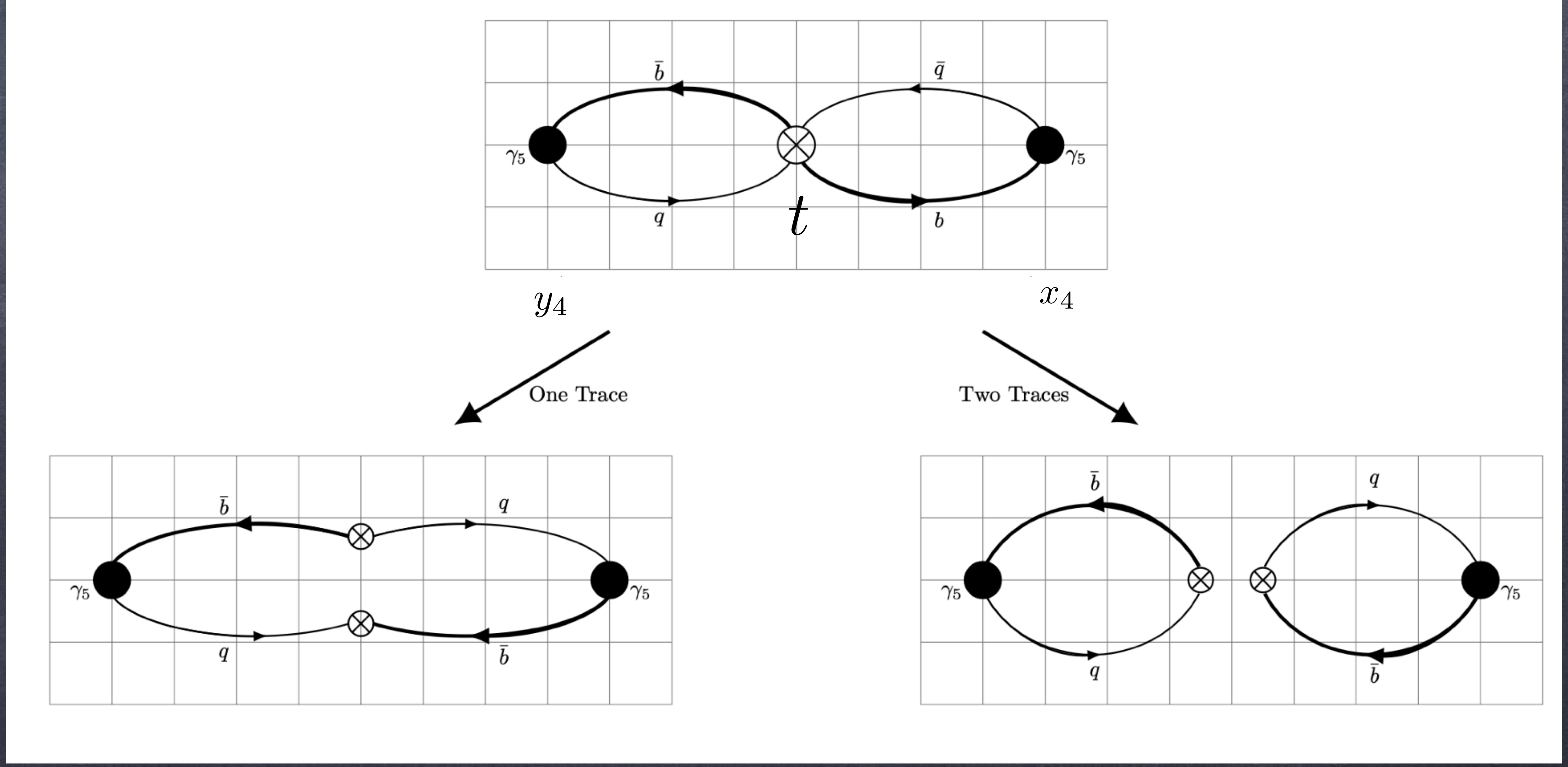
Matrix elements

$$O_1 = (\bar{b}_i \gamma_\mu (1-\gamma_5) q_i)(\bar{b}_j \gamma_\mu (1-\gamma_5) q_j) \qquad \langle O_1(\mu) \rangle = \frac{8}{3} M^2_{B_q} f^2_{B_q} B_1(\mu)$$

$$C^{\rm 2pt}_{\Gamma_I \Gamma_J}(x_4) = a^3 \sum_{\mathbf{x}} \left\langle \Gamma_J(\mathbf{x}, x_4) \Gamma_I^\dagger(0,0) \right\rangle \qquad C^{\rm 2pt}_{\Gamma_I \Gamma_J}(x_4) = \sum_n \frac{\langle 0|\Gamma_J|n\rangle \langle n|\Gamma_I^\dagger|0\rangle}{2M^{(n)}} \left(e^{-M^{(n)} x_4} \pm e^{-M^{(n)}(T-x_4)} \right)$$

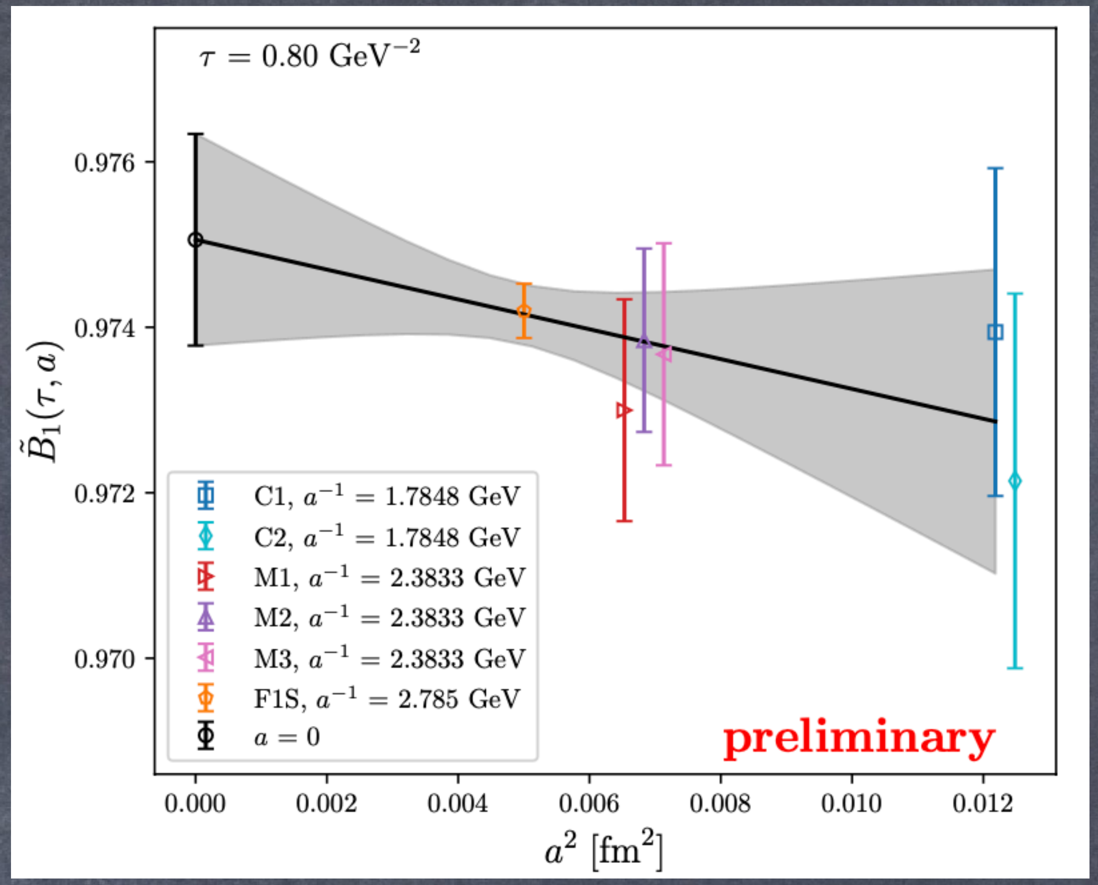
$$C^{\rm 3pt}_{O_1}(x_4, y_4, t) = a^6 \sum_{\mathbf{x},\mathbf{y}} \langle P(\mathbf{x}, x_4) O_1(0,t) P^\dagger(\mathbf{y}, y_4) \rangle$$

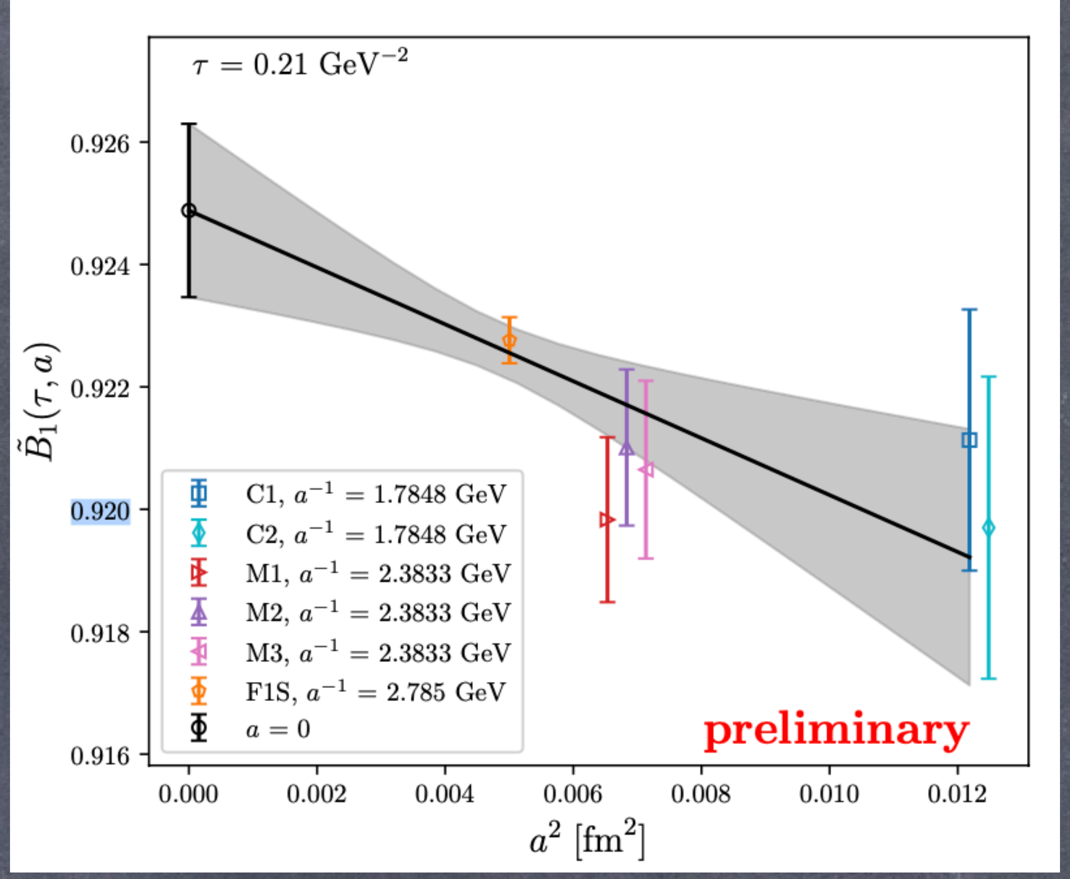
$$R^{\rm Bag}_{O_1}(x_4, y_4, t) = \frac{C^{\rm 3pt}_{O_1}(x_4, y_4, t)}{\frac{8}{3} C^{\rm 2pt}_{AP}(y_4, t) C^{\rm 2pt}_{AP}(x_4, t)}$$



Continuum limit & matching

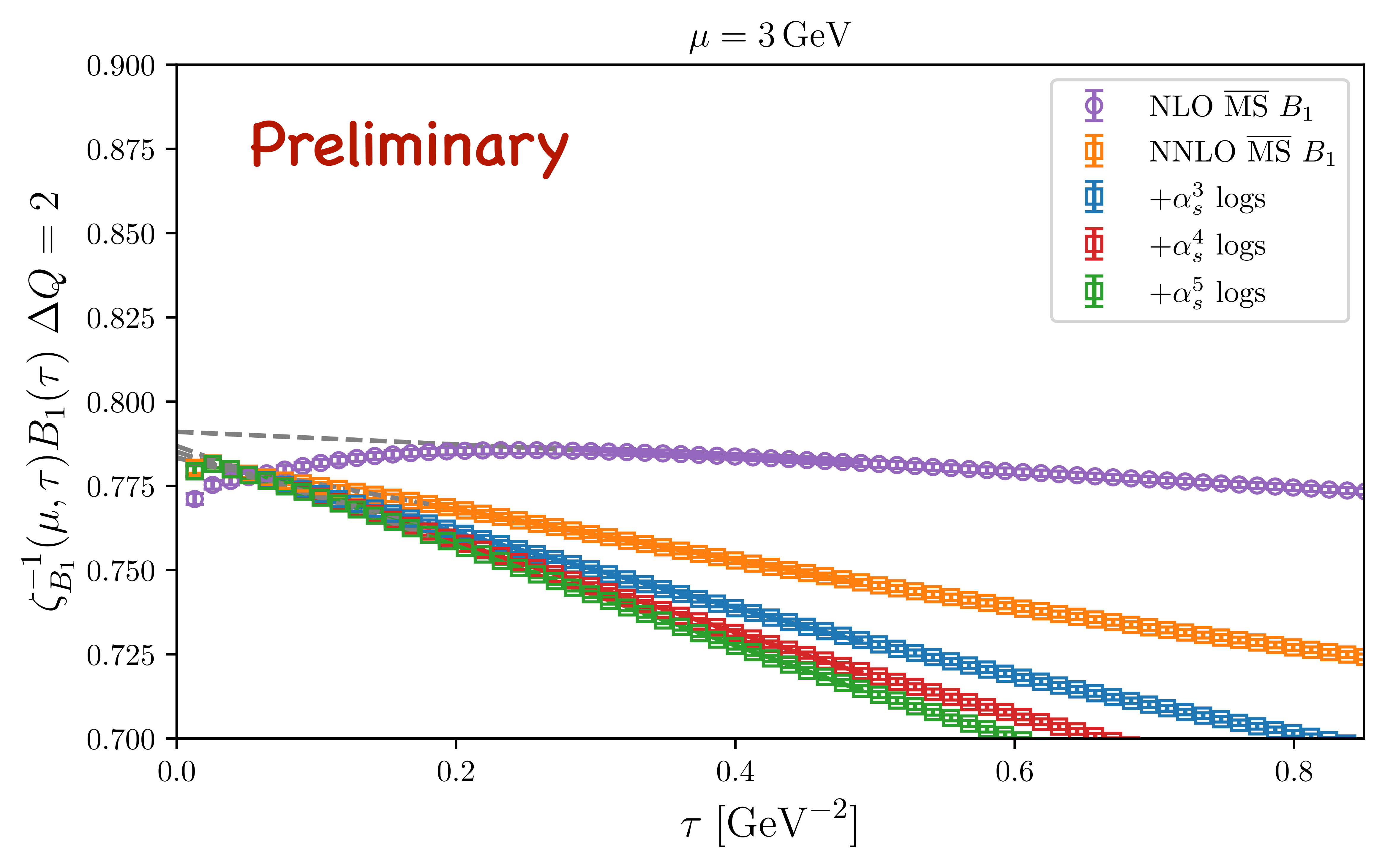
Continuum limit





$$c_{B_1}^{-1}(\mu, t) = 1 + \frac{\alpha_s}{4\pi}\left(-\frac{11}{3} - 2\log(2\mu^2 t) + \gamma_E\right) + O(\alpha_s^2)$$

Harlander, Lange: 2022



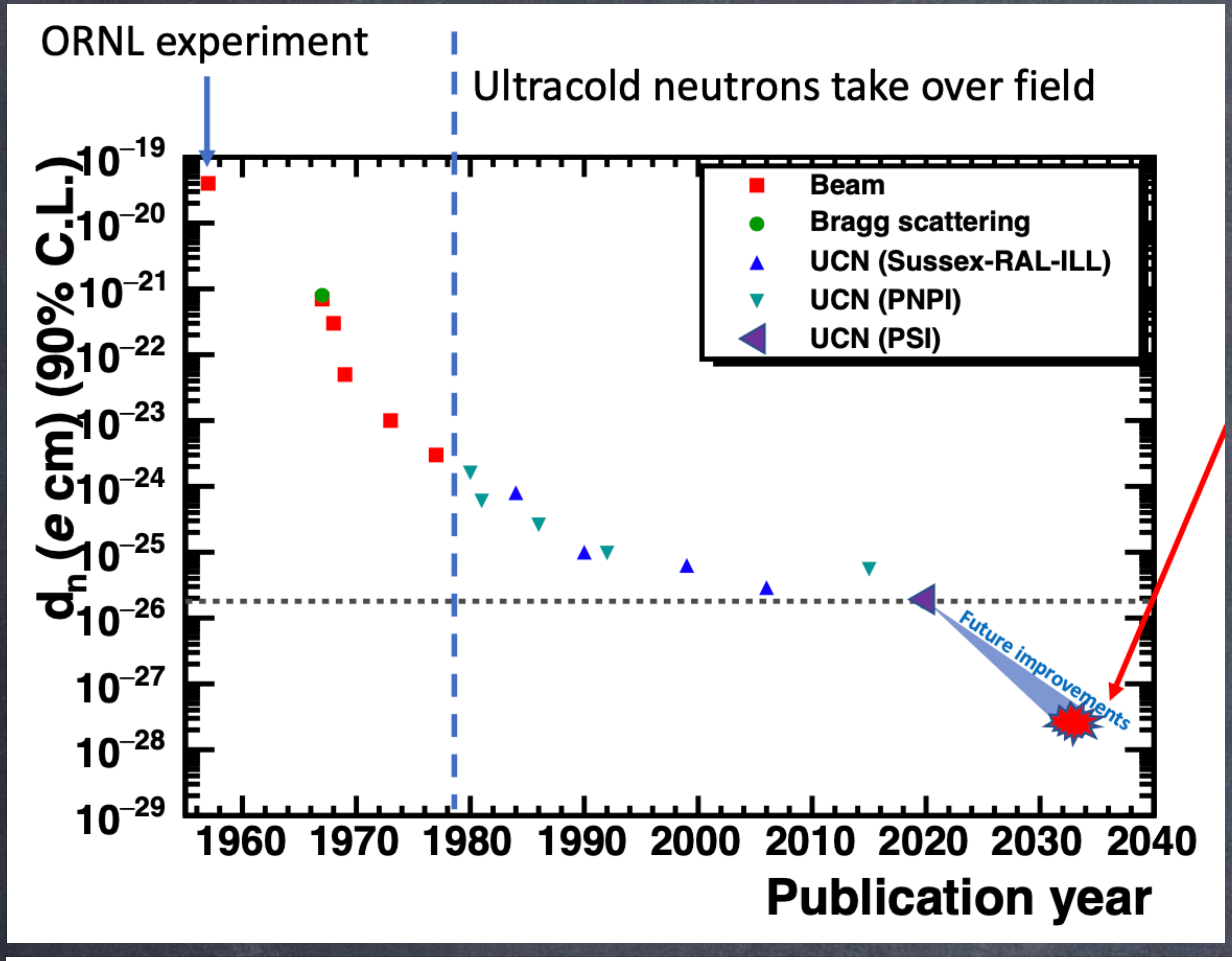
- $\Delta B = 0$ four-quark matrix elements are the final target
- Standard renormalization introduces mixing with lower dimensional operators -> Use the gradient flow
- Testing method with ΔC=2

Summary

- The gradient flow provides a powerful to resolve complicated renormalization patterns
- Application on matrix elements: PDF, EDM, scalar content, bag parameters,...
- Intermediate regulator that allows to recover continuum symmetries
- Reduce noise, but be careful on the exposed autocorrelation time
- Logarithmic matching coefficients are calculable in perturbation theory
- Power divergences can be eliminated non-perturbatively (case by case)
- Aspects to be understood: limit t -> 0 - window

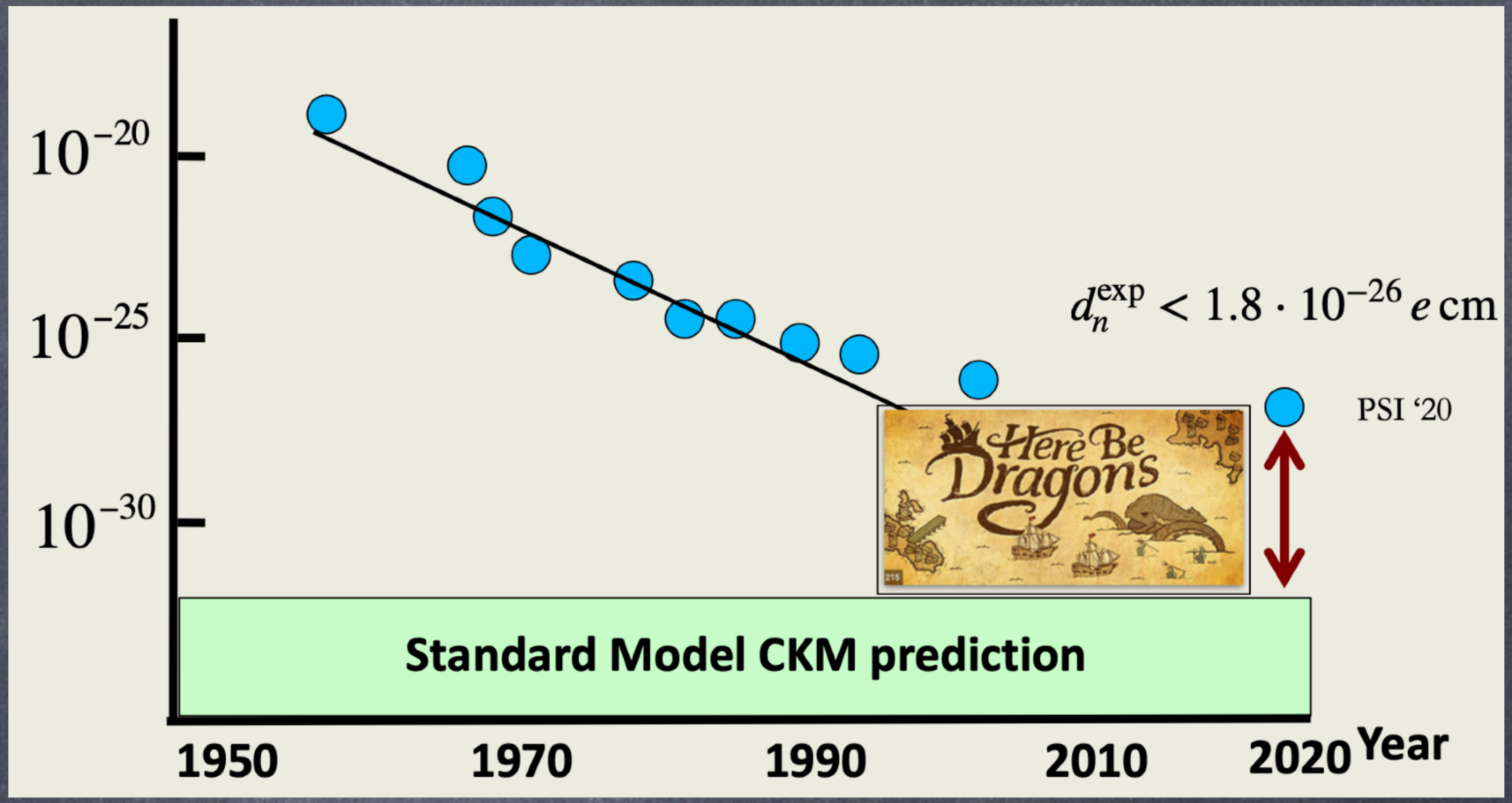
EDM

Neutron EDM



$$|d_n| < 1.8 \times 10^{-26}\ e\ \mathrm{cm}\ (90\%\ \mathrm{C.L.})$$

Abel et al.: 2020 (PSI)



Alarcon et al.: 2022
Snowmass Summer Study Report

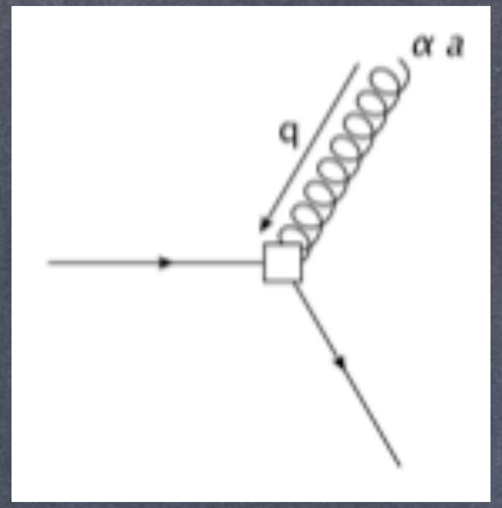
Experiment: Facility	Neutron Source	Measurement Cell	Measurement Techniques	90% C.L. (10^{-28} e-cm) With 300 Live Days	Year 90% C.L. Data Acquired
Crystal: JPARC	Cold Neutron Beam	Solid	Crystal Diffraction (High Internal $\vec{E}$)	< 100	Development
Beam: ESS	Cold Neutron Beam	Vacuum	Pulsed Beam	< 50	~ 2030
PNPI: ILL	ILL Turbine (UCN) PNPI/LHe (UCN)	Vacuum	Ramsey Technique, $\vec{E} = 0$ Cell for Magnetometry	Phase 1 < 100 < 10	Development Development
n2EDM: PSI	Solid D_2 (UCN)	Vacuum	Ramsey Technique, External Cs Magnetometers, Hg Co-Magnetometer	< 15	~ 2026
PanEDM ILL/Munich	Superfluid ^{4}He (UCN), Solid D_2 (UCN)	Vacuum	Ramsey Technique, Hg Co-External ^{3}He and Cs Magnetometers	< 30	~ 2026
TUCAN: TRIUMF	Superfluid ^{4}He (UCN)	Vacuum	Ramsey Technique, Hg Co-Magnetometer, External Cs Magnetometers	< 20	~ 2027
nEDM: LANL	Solid D_2 (UCN)	Vacuum	Ramsey Technique, Hg Co-Magnetometer, Hg External Magnetometer, OPM	< 30	~ 2026
nEDM@SNS: ORNL	Superfluid ^{4}He (UCN)	^{4}He	Cryogenic High Voltage, ^{3}He Capture for ω, ^{3}He Co-Magnetometer with SQUIDs, Dressed Spins, Superconducting Magnetic Shield	< 20 < 3	~ 2029 ~ 2031

CP-violating sources

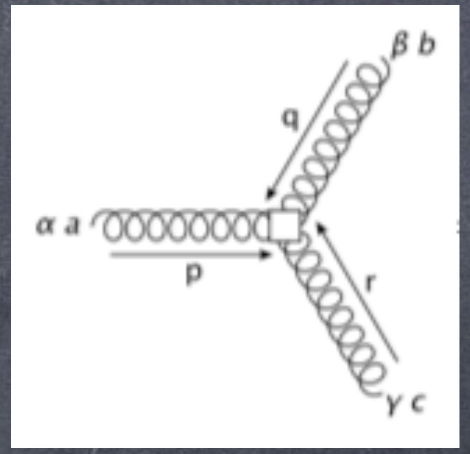
- Full list of dimension 5 and 6 operators is known

Buchmuller, Wyler: 1986
de Rujula et al.: 1991
Grzadkowski et al: 2010

$$\mathcal{O}_{\rm CE} = \sum_{f=u,d,s,\dots} \overline{\psi}_f(x)\gamma_5\sigma_{\mu\nu}G^a_{\mu\nu}T^a\psi_f(x)$$



$$\mathcal{O}_{\rm 3g}(x) = \frac{1}{6} i f^{abc} G^a_{\mu\rho}(x) G^b_{\nu\rho}(x) G^c_{\lambda\sigma}(x)\epsilon_{\mu\nu\lambda\sigma}$$



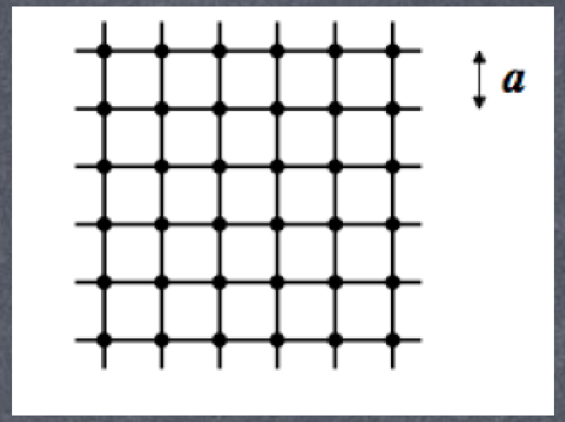
Weinberg: 1989

$$\mathcal{O}_{\rm q} = \sum_{f=u,d,s,\dots} \overline{\psi}_f(x)\gamma_5\sigma_{\mu\nu}\psi_f(x)F_{\mu\nu}$$

$$\mathcal{L}_{{\rm QCD}+\theta} = \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \sum_{f=u,d,s,\dots} \overline{\psi}_f(x)\left\{\gamma_\mu\left[\partial_\mu + gA^a_\mu T^a\right] + m_f\right\}\psi_f(x) - i\bar{\theta}q(x)$$

$$q(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}{\rm tr}\left\{G_{\mu\nu}(x)G_{\rho\sigma}(x)\right\}$$

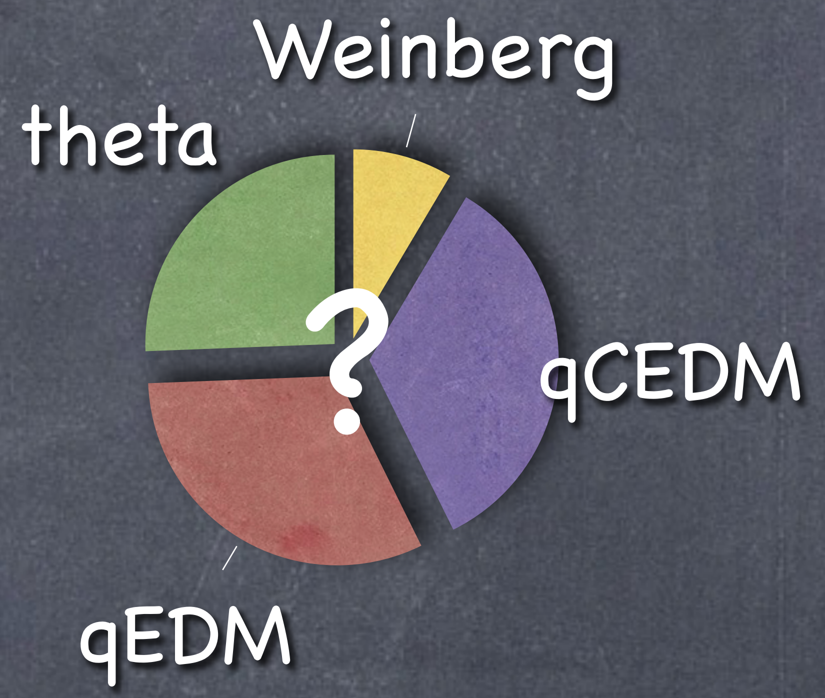
The role of lattice QCD



$$d_{\rm N} = M_{\rm N}^{\theta}\bar{\theta} + \left(\frac{v}{\Lambda}\right)^2 \sum_i M_{\rm N}^{(i)} \tilde{d}_i \qquad \langle N | J_\mu \mathcal{O}_{\not{CP}} | N \rangle \to d\, \underline{E} \cdot \underline{S}$$

$M_{\rm N}^{\theta}$ → Hadronic matrix element topological charge

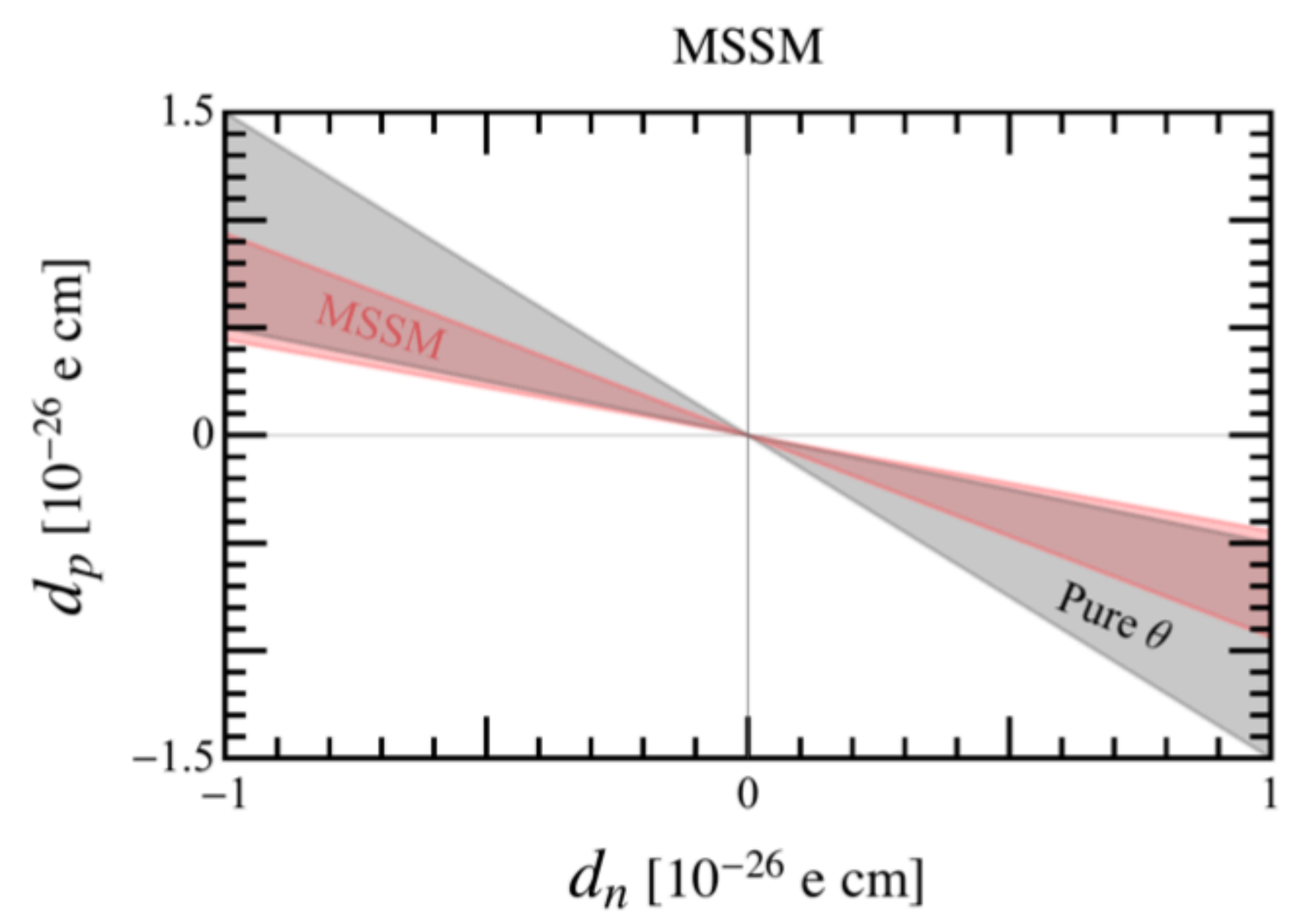
$M_{\rm N}^{(i)}$ → Hadronic matrix element CP odd operators

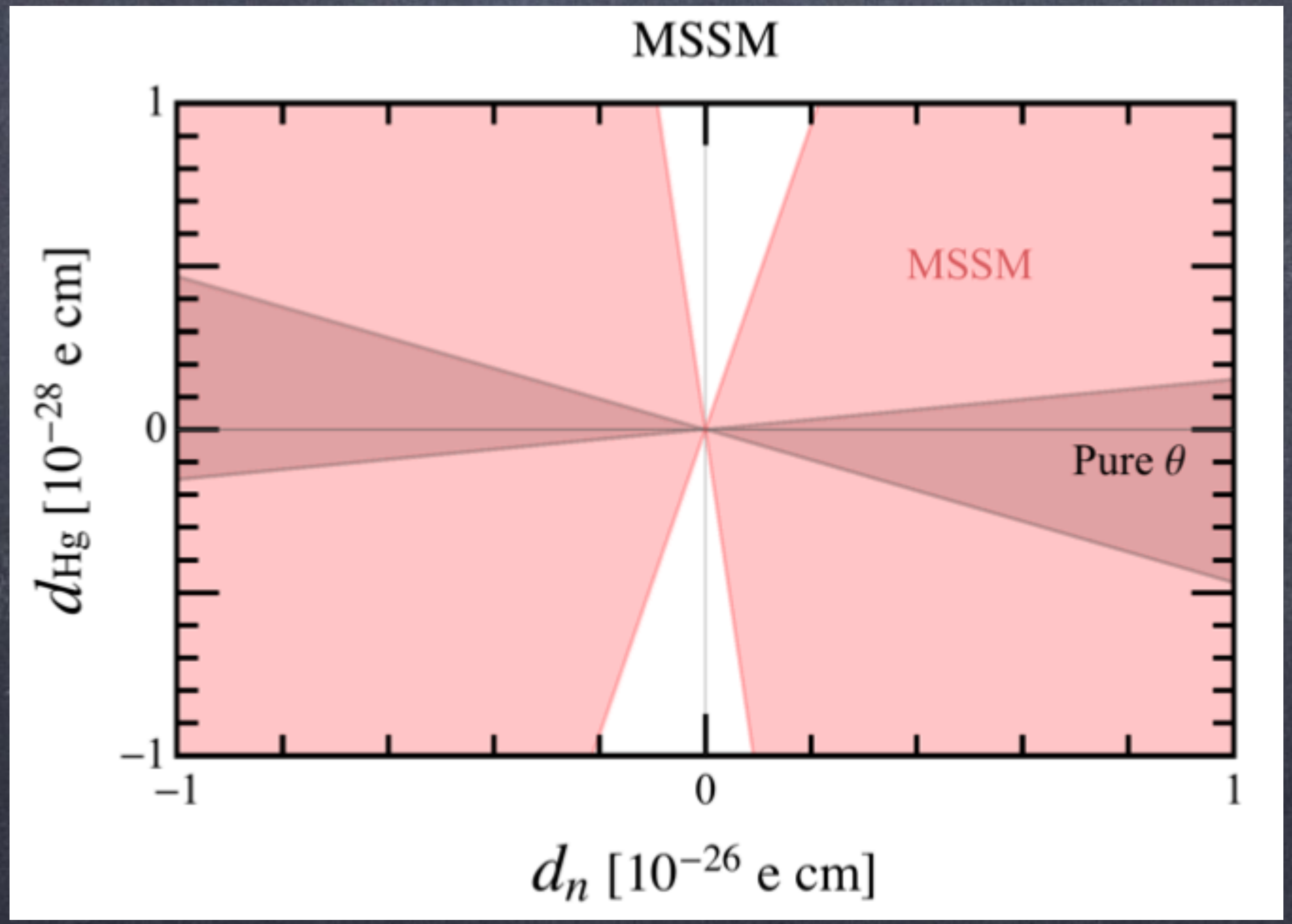


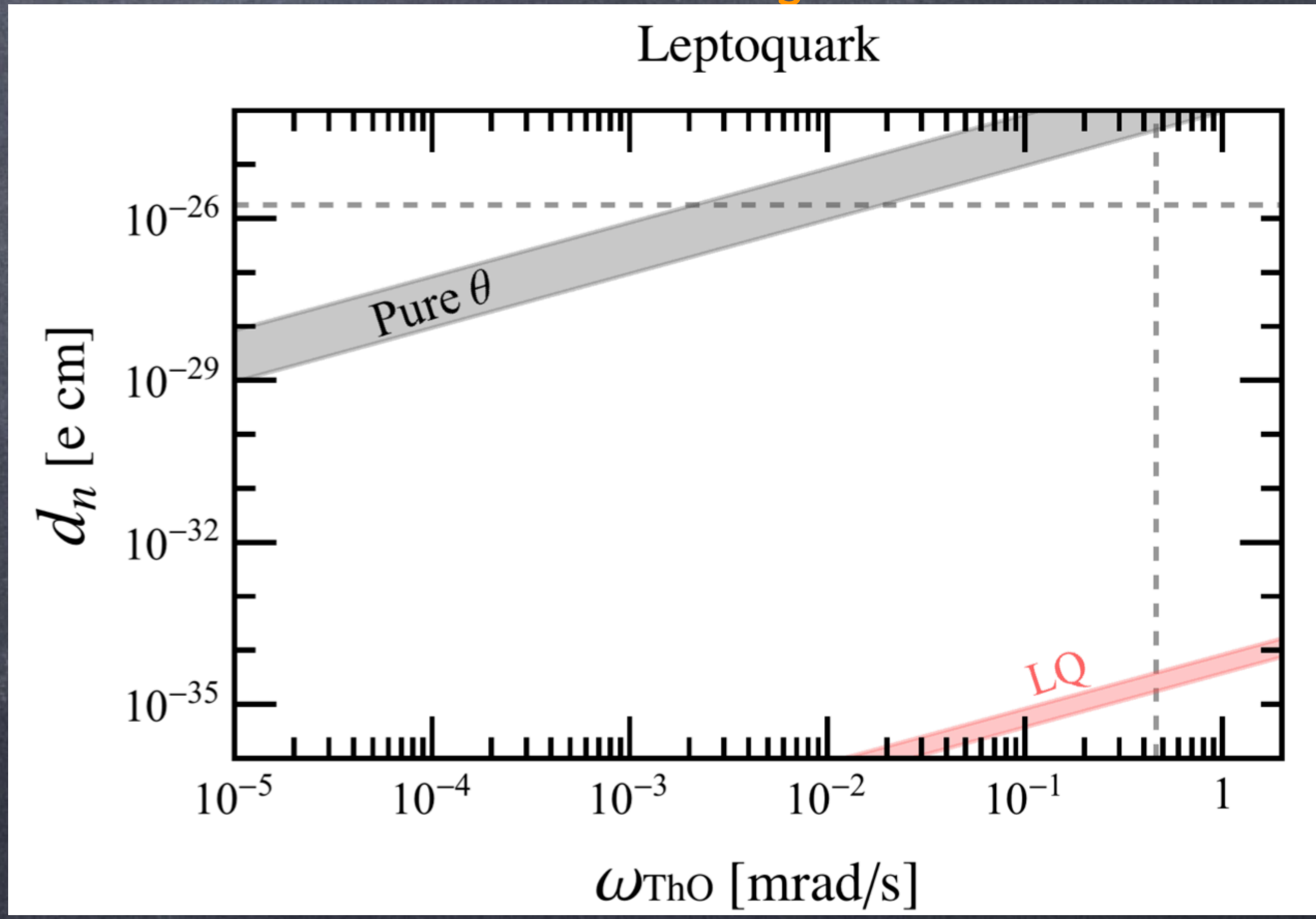
Alarcon et al.: 2022
Snowmass Summer Study Report

$$\begin{aligned} d_n = \; & - (1.5 \pm 0.7) \cdot 10^{-3} \bar{\theta} e \text{ fm} \\ & - (0.2 \pm 0.01) d_u + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.016) d_s \\ & - (0.55 \pm 0.28) e\tilde{d}_u - (1.1 \pm 0.55) e\tilde{d}_d + (50 \pm 40) \text{MeV} e\tilde{d}_G \end{aligned}$$

Shintani et al.: 2005
Berruto, Blum, Orginos, Soni 2006







de Vries et al.: 2021

Renormalization

Bhattacharya, Cirigliano, Gupta, Mereghetti, Yoon: 2015

$$\mathcal{O}_{\rm CE}(x) = \overline{\psi}(x)\gamma_5\sigma_{\mu\nu}G_{\mu\nu}t^a\psi(x)$$

$$[\mathcal{O}_{\rm CE}]_R = Z_{\rm qcEDM}\left[\mathcal{O}_{\rm CE} - \frac{C}{a^2}P\right] + \cdots$$

$$P(x) = \overline{\psi}(x)\gamma_5 t^a\psi(x)$$

RI-MOM Off-shell

$\frac{1}{a}$ d=4 –> 2 operators + 3 O(m)

$\log a$ d=5 –> 3 operators + (7 + 5) O(m,m^2) + 4 “nuisance”

Power divergences need to be subtracted non-perturbatively

Maiani, Martinelli, Sachrajda: 1992

Strategy

Kim, Luu, Rizik, A.S.:2020

$$\mathcal{O}_{\mathrm{CE,R}}(t) = \frac{c_{\mathrm{OP}}(t,\mu)}{t} P_{\mathrm{R}}(0,\mu) + \sum_i c_{\mathrm{CE},i}(t,\mu)\mathcal{O}_{i,\mathrm{R}}(0,\mu)$$

$$\Delta(\bar{g}^2) = t\frac{\langle \mathcal{O}_{\mathrm{R}}(t)P(0)\rangle}{\langle P_{\mathrm{R}}(t)P(0)\rangle}$$

$$\Delta_P(t,\mu) = \frac{\langle P_{\mathrm{R}}(t)P(0)\rangle}{\langle P_{\mathrm{R}}(0,\mu)P(0)\rangle}$$

$$t\frac{\langle \mathcal{O}_{\mathrm{CE,R}}(t)P(0)\rangle}{\langle P_{\mathrm{R}}(0,\mu)P(0)\rangle} = t\frac{\langle \mathcal{O}_{\mathrm{CE,R}}(t)P(0)\rangle}{\langle P_{\mathrm{R}}(t)P(0)\rangle}\frac{\langle P_{\mathrm{R}}(t)P(0)\rangle}{\langle P_{\mathrm{R}}(0,\mu)P(0)\rangle} = \Delta(\bar{g}^2)\Delta_P(t,\mu)$$

$$t\frac{\langle \mathcal{O}_{\mathrm{CE,R}}(t)P(0)\rangle}{\langle P_{\mathrm{R}}(0,\mu)P(0)\rangle} = c_{\mathrm{OP}}(t,\mu) + O(t) \qquad c_{\mathrm{OP}}(t,\mu) = \Delta(\bar{g}^2)\Delta_{\mathrm{P}}(t,\mu) + O(t)$$

$$\langle \mathcal{O}_{\mathrm{CE,R}}(t)\Phi\rangle = \frac{1}{t}\Delta(\bar{g}^2)\Delta_P(t,\mu)\,\langle P_{\mathrm{R}}(0,\mu)\Phi\rangle + F\left[c_{\mathrm{CE},i}(t,\mu), \langle P_{\mathrm{R}}(0,\mu)\Phi\rangle, \langle \mathcal{O}_{i,\mathrm{R}}(0,\mu)\Phi\rangle\right]$$

Strategy

Kim, Luu, Rizik, A.S.:2020

$$\langle \mathcal{O}_{\mathrm{CE,R}}(t)\Phi\rangle = \frac{1}{t}\Delta(\bar{g}^2)\Delta_P(t,\mu)\langle P_{\mathrm{R}}(0,\mu)\Phi\rangle + F\left[c_{\mathrm{CE},i}(t,\mu), \langle P_{\mathrm{R}}(0,\mu)\Phi\rangle, \langle \mathcal{O}_{i,\mathrm{R}}(0,\mu)\Phi\rangle\right] + O(t)$$

$\langle \mathcal{O}_{\mathrm{CE,R}}(t)\Phi\rangle$ ⟶ Z_χ 🚧 LQCD matrix element

$\Delta(\bar{g}^2) = t\dfrac{\langle \mathcal{O}_{\mathrm{R}}(t)P(0)\rangle}{\langle P_{\mathrm{R}}(t)P(0)\rangle}$ ⟶ LQCD finite ✅

$\Delta_P(t,\mu) = \dfrac{\langle P_{\mathrm{R}}(t)P(0)\rangle}{\langle P_{\mathrm{R}}(0,\mu)P(0)\rangle}$ ⟶ Z_χ 🚧 Z_{P} LQCD

$c_{\mathrm{CE},i}(t,\mu)$ ⟶ Perturbation theory 1-loop ✅ 2-loop 🚧

$\langle P_{\mathrm{R}}(0,\mu)\Phi\rangle$ ⟶ LQCD

$\langle \mathcal{O}_{\mathrm{CE,R}}(0,\mu)\Phi\rangle$ ⟶ output

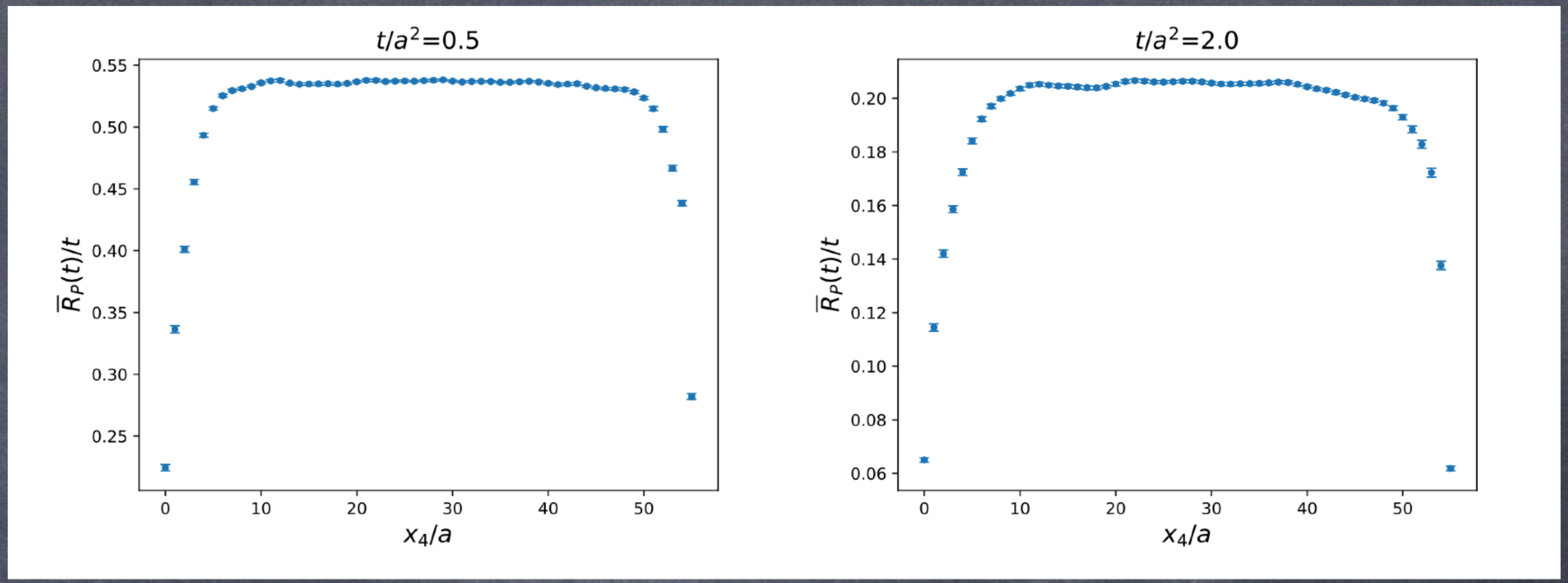
Quark-Chromo EDM: power divergences

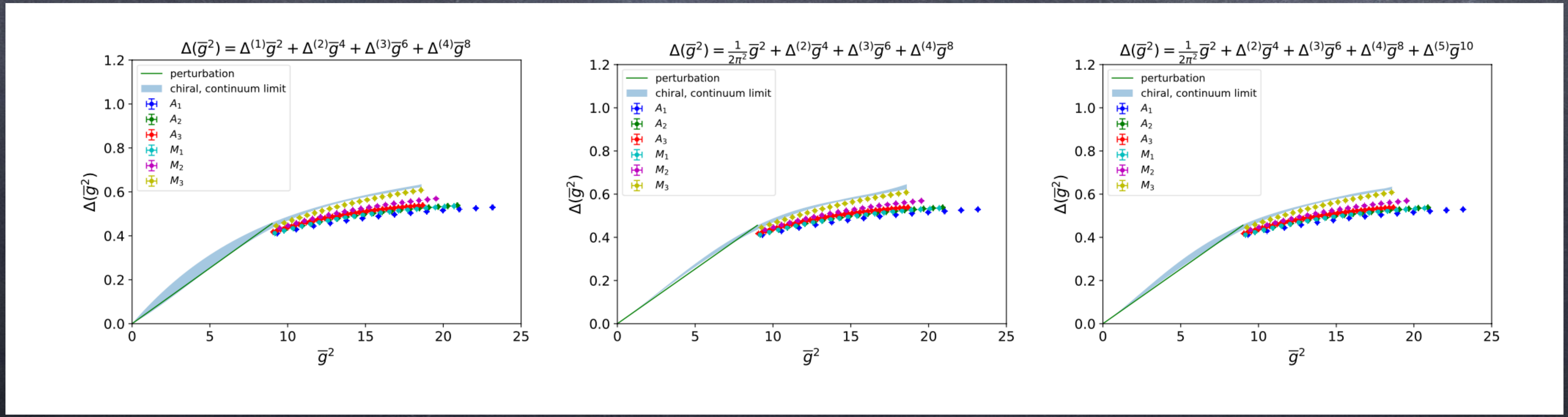
Kim, Luu, Rizik, A.S.:2020

$$\Gamma_{CP}(x_4;t) = a^3 \sum_{\mathbf{x}} \left\langle \mathcal{O}^{ij}_{CE}(x_4,\mathbf{x};t) P^{ji}(0,\mathbf{0};0) \right\rangle$$

$$\Gamma_{PP}(x_4;t) = a^3 \sum_{\mathbf{x}} \langle P^{ij}(x_4,\mathbf{x};t) P^{ji}(0,\mathbf{0}) \rangle$$

$$\overline{R}_{\mathrm{P}}(x_4;t) = t \frac{\Gamma_{\mathrm{CP}}(x_4;t)}{\Gamma_{\mathrm{PP}}(x_4,t)}$$





Quark-Chromo EDM

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

$$[\mathcal{O}_i(t)]_{\mathrm{R}} = \sum_i c_{ij}(t,\mu)\,[\mathcal{O}_i(t=0,\mu)]_{\mathrm{R}} + O(t)$$

$$\mathcal{O}_{CE}(x,t) = \bar{\chi}(x,t)\tilde{\sigma}_{\mu\nu}G_{\mu\nu}(x,t)\chi(x,t)$$

$$\tilde{\sigma}^{\mathrm{HV}}_{\mu\nu} = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\sigma_{\alpha\beta} \qquad \tilde{\sigma}^{\mathrm{NDR}}_{\mu\nu} = \sigma_{\mu\nu}\gamma_5$$

$$\mathcal{O}_P(x) = \bar{\psi}(x)\gamma_5\psi(x)$$

$$\mathcal{O}_{m^2P}(x) = m^2\bar{\psi}(x)\gamma_5\psi(x)$$

$$\mathcal{O}_{m\theta}(x) = m\mathrm{tr}[G_{\mu\nu}\widetilde{G}_{\mu\nu}]$$

$$\mathcal{O}_E(x) = \bar{\psi}(x)\tilde{\sigma}_{\mu\nu}F_{\mu\nu}(x)\psi(x)$$

$$\mathcal{O}^R_{CE}(x;t) = c_P(t,\mu)\mathcal{O}^{\mathrm{MS}}_P(x;\mu) + c_{m\theta}(t,\mu)\mathcal{O}^{\mathrm{MS}}_{m\theta}(x;\mu) + c_E(t,\mu)\mathcal{O}^{\mathrm{MS}}_E(x;\mu) + c_{CE}(t,\mu)\mathcal{O}^{\mathrm{MS}}_{CE}(x;\mu) + c_{mP^2}(t,\mu)\mathcal{O}^{\mathrm{MS}}_{m^2P}(x;\mu) + \cdots$$

$$Z_\chi^{-n/2}\left\langle(\psi)^{n_\psi}\,(\bar{\psi})^{n_{\bar{\psi}}}\,(A_\mu)^{n_A}\,\mathcal{O}_i(t)\right\rangle^{\mathrm{amp}} = c_{ij}(t)\,\left(Z^{\mathrm{MS}}_{jk}\right)^{-1}\left\langle(\psi)^{n_\psi}\,(\bar{\psi})^{n_{\bar{\psi}}}\,(A_\mu)^{n_A}\,\mathcal{O}_k\right\rangle^{\mathrm{amp}}$$

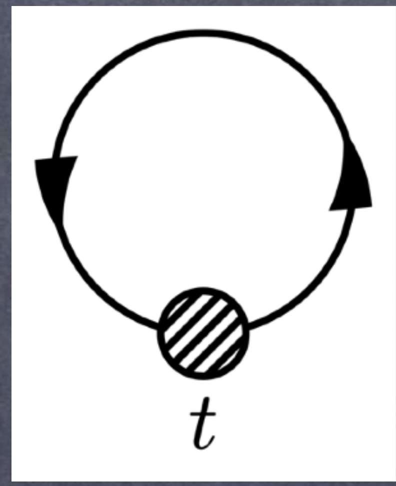
$$c_{ij}(t,\mu) = \delta_{ij} + \frac{\alpha_s(\mu)}{4\pi}c^{(1)}_{ij}(t,\mu) + O(\alpha_s^2)$$

Quark-Chromo EDM

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

$$Z_\chi^{-n/2} \left\langle (\psi)^{n_\psi} \, (\bar{\psi})^{n_{\bar{\psi}}} \, (A_\mu)^{n_A} \, \mathcal{O}_i(t) \right\rangle^{\text{amp}} = c_{ij}(t) \, \left(Z_{jk}^{\text{MS}}\right)^{-1} \left\langle (\psi)^{n_\psi} \, (\bar{\psi})^{n_{\bar{\psi}}} \, (A_\mu)^{n_A} \, \mathcal{O}_k \right\rangle^{\text{amp}}$$

$$\left\langle \mathring{\bar{\chi}}(x;t) \overleftrightarrow{\not{D}} \, \mathring{\chi}(x;t) \right\rangle = -\frac{2N_c N_f}{(4\pi)^2 t^2}$$



Makino, Suzuki: 2014

2-loops

Harlander, Kluth, Lange :2018

Artz, Harlander, Lange, Neumann, Prausa: 2019

$$\chi_R(x;t) = (8\pi t)^{\varepsilon/2} \zeta_\chi^{1/2} \mathring{\chi}(x;t)$$

$$\bar{\chi}_R(x;t) = (8\pi t)^{\varepsilon/2} \zeta_\chi^{1/2} \mathring{\bar{\chi}}(x;t)$$

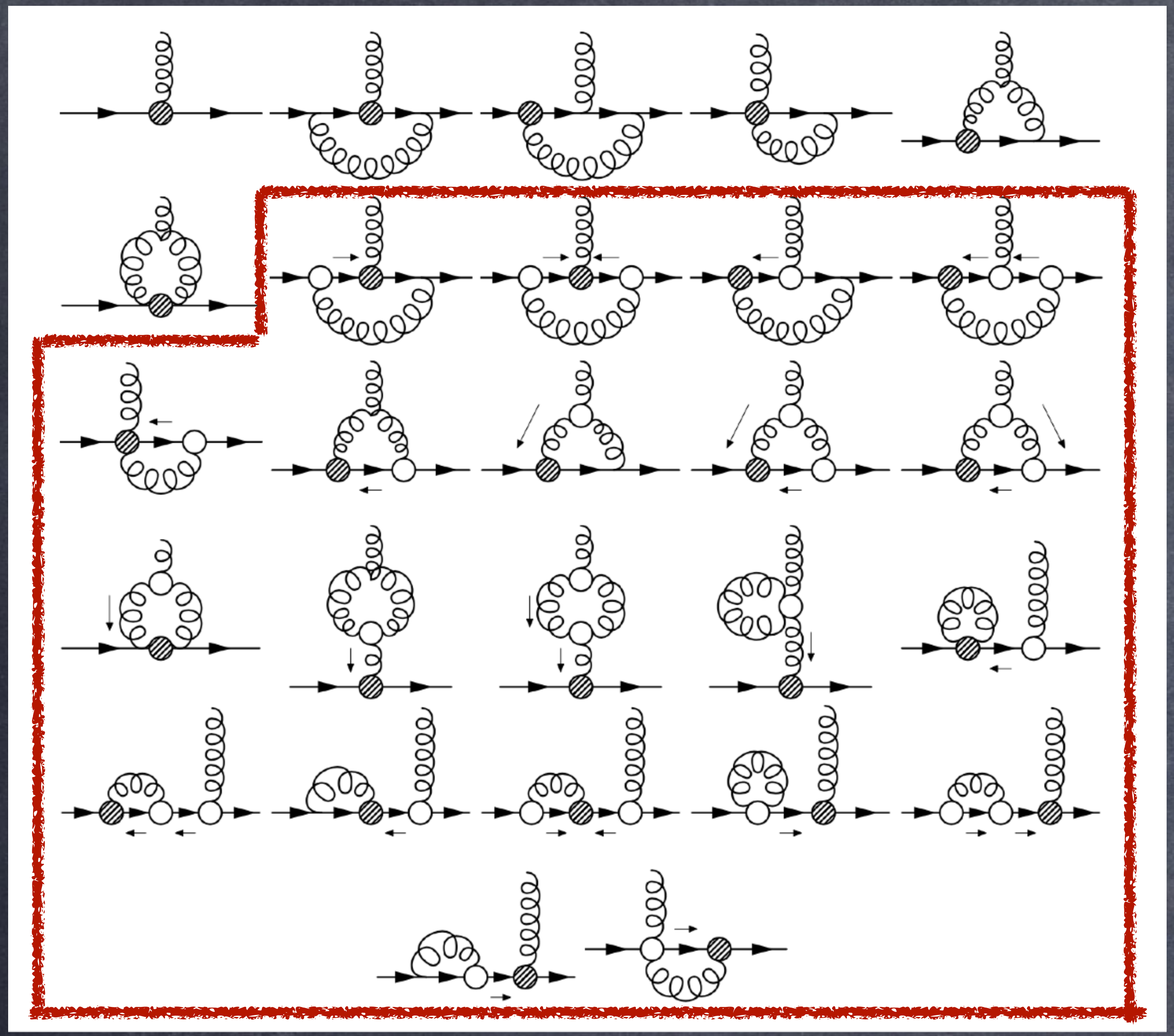
$$\zeta_\chi = 1 - \frac{\alpha_s C_F}{4\pi} \left(3 \log\left(8\pi\mu^2 t\right) - \log(432)\right) + O(\alpha_s^2)$$

$$\mathcal{O}_{CE}^R(x;t) = \mathring{\bar{\chi}}(x;t) \tilde{\sigma}_{\mu\nu} G_{\mu\nu}(x;t) \mathring{\chi}(x;t)$$

Quark-Chromo EDM

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

$c_{CE}(t,\mu)$



- Expand integrands of loop integrals in all scales excluding t
  - Analytic structure altered —> distortion of IR structure
  - in matching equation the IR modification drops out in the difference
  - Expanding loop integrals in the RHS vanish in DR —> UV and IR are identical
  - The LHS is UV-finite, beside the renormalization of the bare parameters and flowed fermion fields
  - The IR singularities on the LHS exactly match the UV MS counterterms

$$c_{CE}(t,\mu) = \zeta_\chi^{-1} + \frac{\alpha_s}{4\pi}\left[2(C_F - C_A)\log(8\pi\mu^2 t) - \frac{1}{2}\Big((4+5\delta_{\rm HV})C_A + (3-4\delta_{\rm HV})C_F\Big)\right]$$

$$= 1 + \frac{\alpha_s}{4\pi}\Big[(5C_F - 2C_A)\log(8\pi\mu^2 t) - \frac{1}{2}\Big((4+5\delta_{\rm HV})C_A + (3-4\delta_{\rm HV})C_F\Big) - \log(432)C_F\Big]$$

SYMLAT Status

- Theta-term nucleon EDM —> first results 1409.2735 1507.02343 1809.03487 1902.03254
  - Renormalization, S/N
- Quark-chromo EDM —> renormalization
  - Power divergences —> PT 1810.05637 2005.04199 2111.1149
    Non-perturbative 1810.10301 2106.07633
  - Logs/mixing —> 2111.1149 2212.09824
- 3 gluon operator —> PT power divergences 2005.04199
  Preliminary studies for renormalization (power divergences) 1711.04730 1810.05637
  —> Logs/mixing 2308.16221

Neutron EDM from Lattice QCD

Quark EDM –>
simplest calculation with Lattice QCD. Precision 3%-5%. No Disc.

Theta-term nucleon EDM –> few calculations: 2 σ effect

–> new result have stronger signal

3 gluon operator –> No Lattice QCD calculation, 1-loop matching

4-fermion operators –> No Lattice QCD calculation, 1-loop matching

Quark-chromo EDM –>
First result with LO renormalization
New promising approach based on gradient flow –>
1-loop matching, NP power divergence,
2-loop in progress

	Renormalization	Continuum limit	Chiral extrapolation	Finite Volume	Excited States
θ - term	🟢	🟢	🟡	🟡	🟡
quark EDM	🟢	🟢	🟢	🟢	🟢
quark-chromo	🟢	🔴	🟡	🔴	🔴
3-gluon	🟡	🔴	🟡	🔴	🔴
4-fermion	🟡	🔴	🔴	🔴	🔴

Scalar content of the nucleon

Composite operators

$$\mathcal{S}(x) = \overline{\psi}(x)\psi(x) \qquad \overline{\psi}(x)\psi(y) \overset{x\to y}{\sim} C_m(x-y)\frac{m}{|x-y|^2} + C_{m^3}(x-y)\, m^3 + C_S(x-y)\mathcal{S}(x) + \cdots$$

$$\mathcal{S}_R(x) = Z_S(a)\left[c_0(a)\frac{1}{a^3} + c_1(a)\frac{m}{a^2} + c_2(a)\frac{m^2}{a} + c_3(a)m^3 + \mathcal{S}(x)\right]$$

Lüscher: 2013

$$S^{rs}(t,x) = \overline{\chi}_r(t,x)\chi_s(t,x)$$

$$S^{rs}(t,x) = c_0(t)M^{rs} + c_1(t)M^{rs}\mathrm{Tr}\left[M^2\right] + c_2(t)\left(M^3\right)^{rs} + \underline{c_3(t)}S^{rs}(0,x) + O(t)$$

$$P^{rs}(t,x) = \underline{c_3(t)}P^{rs}(0,x) + O(t)$$

$$\int d^3x\,\langle P^{ud}(0,x)P^{du}(t,x)\rangle = -\frac{G_\pi G_{\pi,t}}{M_\pi}\mathrm{e}^{-M_\pi x_0} \quad \Rightarrow \quad c_3(t) = \frac{G_{\pi,t}}{G_\pi} + O(t)$$

Strategy

A.S., de Vries, Luu: 2014

$$\mathcal{C}^{\rm sub}(t,x) = \langle \mathcal{N} S^{rs}(t) \mathcal{N}^\dagger \rangle - \langle S^{rs}(t) \rangle \langle \mathcal{N}\mathcal{N}^\dagger \rangle$$

$$\mathcal{C}^{\rm sub}(t,x) = c_3(t)\mathcal{C}^{\rm sub}(0,x) + \mathrm{O}(t)$$

$$\mathcal{C}_{\rm sub}(0,x) = \frac{G_\pi}{G_{\pi,t}} \cdot \left[\langle \mathcal{N} S^{rs}(t) \mathcal{N}^\dagger \rangle - \langle S^{rs}(t) \rangle \langle \mathcal{N}\mathcal{N}^\dagger \rangle \right] + O(t)$$

$$R_f(y_4; x_4, t)|_{\rm disc} = \frac{a^6 \sum_{\underline{xy}} \langle N(y_4) \left[\overline{\psi}_f(x_4,t) \psi_f(x_4,t) \right] \overline{N}(0) \rangle}{a^3 \sum_{\underline{y}} \langle N(y_4) \overline{N}(0) \rangle}$$

$$g_f^{\rm S}(t)|_{\rm disc} = \langle N | \overline{\psi}_f(t) \psi_f(t) | N \rangle$$

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Sigma terms

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Ens	β	κ_l	κ_s	V	m_π [MeV]	m_N [GeV]	N_{conf}	a [fm]
M_1	1.90	0.13700	0.1364	$32^3 \times 64$	699.0(3)	1.585(2)	399	0.0907(13)

Ens	β	κ_l	κ_s	V	m_π [MeV]	m_N [GeV]	N_{conf}	a [fm]
M_2	1.90	0.13727	0.1364	$32^3 \times 64$	567.6(3)	1.415(3)	400	0.0907(13)
M_3	1.90	0.13754	0.1364	$32^3 \times 64$	409.7(7)	1.219(4)	450	0.0907(13)

$$\sigma_f = Z_A m_f \frac{G_\pi}{G_{\pi,t}} g_f^S(t) + O(t)$$

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