Discussion session: systematic effects in heavy-light observables setting

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Theory institute, Cern, July 2024

HELMHOLTZ ASSOCIATION

NIC

DESY

Systematic effects

Systematic effects

- we don't want any
- we don't like to discuss them (it's no fun)
- but it is necessary, especially since Nature $\approx$ SM
- discussion usually starts when something happens, e.g.

  WI is violated in nucleon matrix elements

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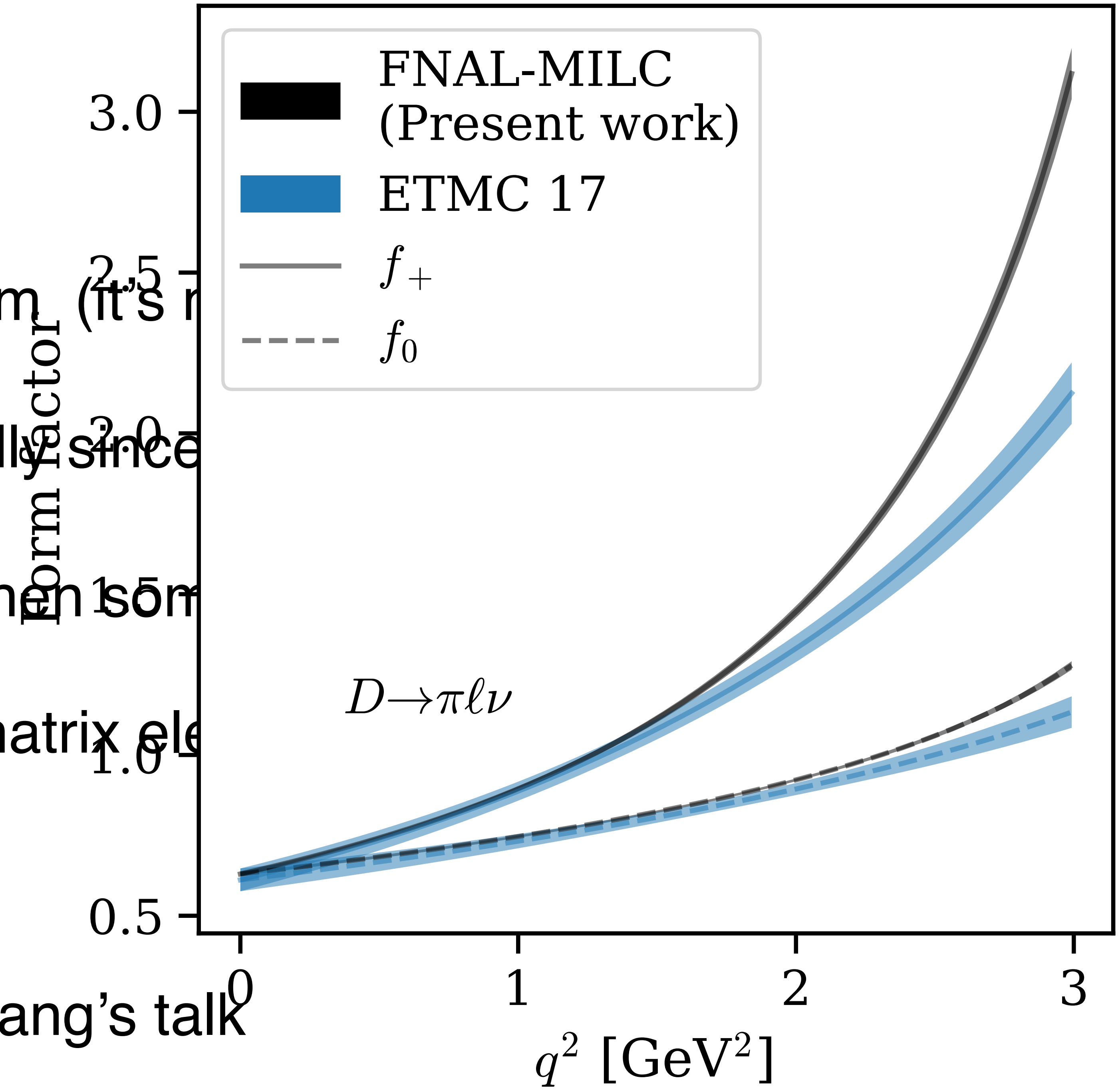
- or
  (arXiv:2212.12648) or T. Tsang’s talk



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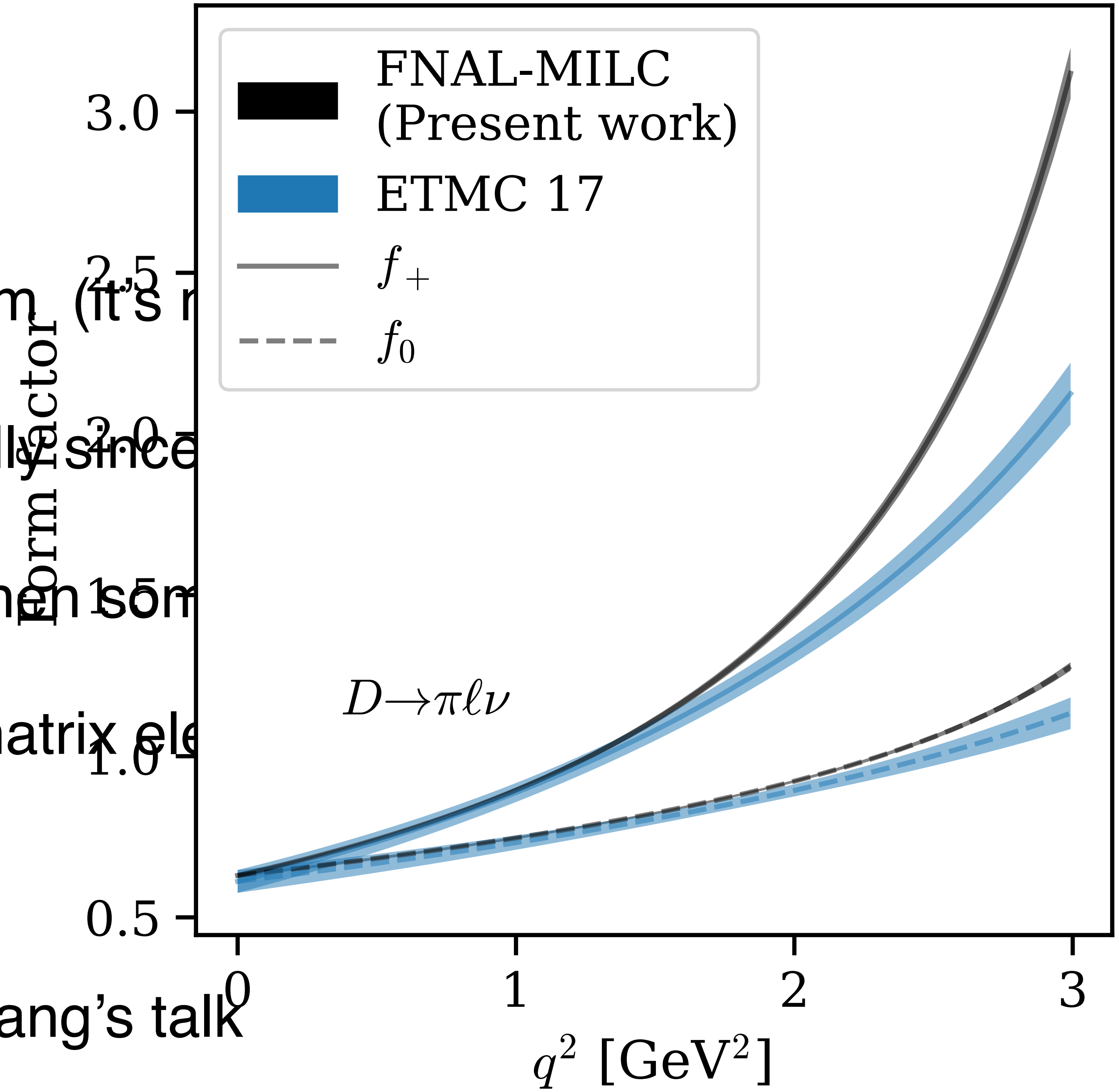
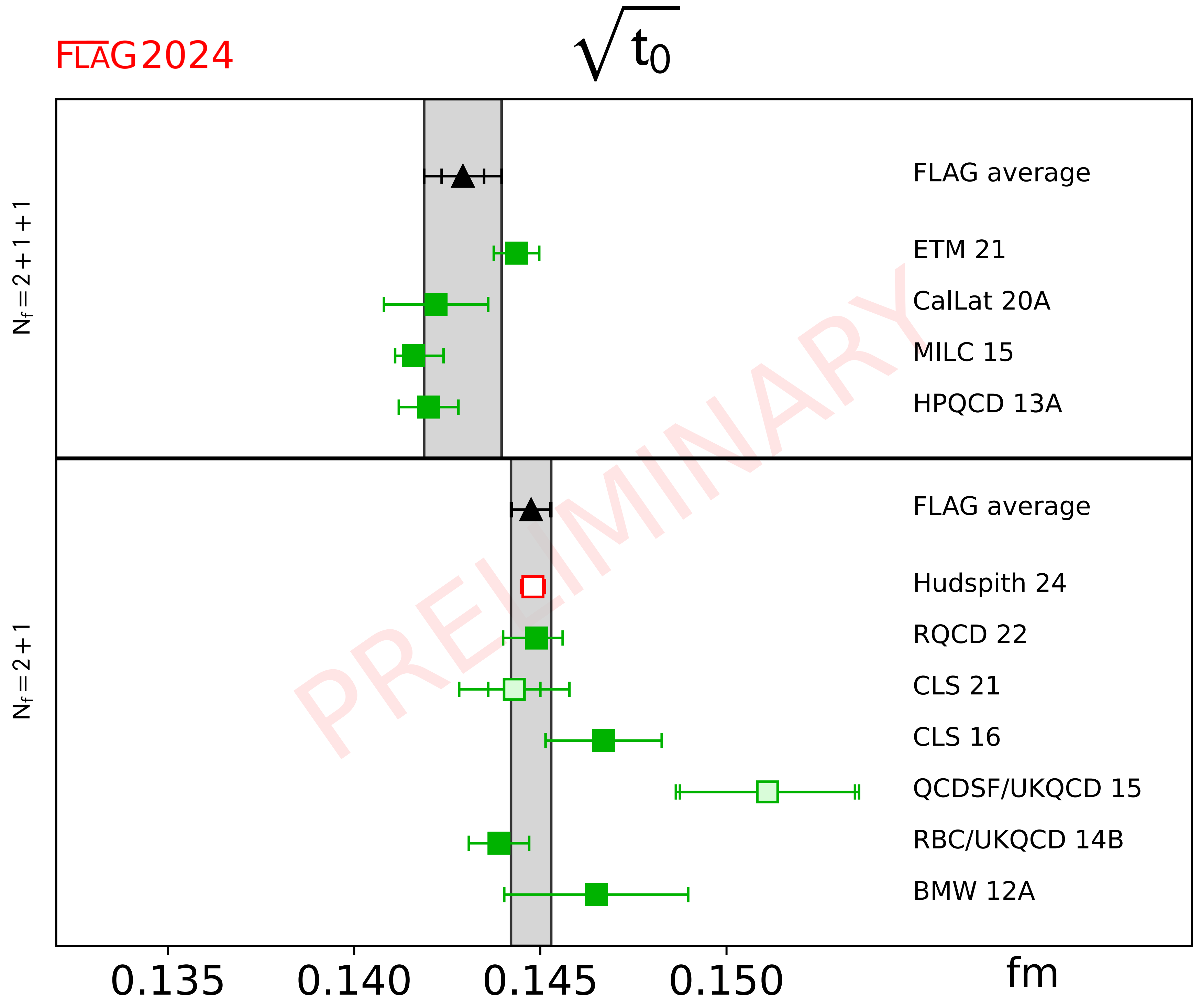
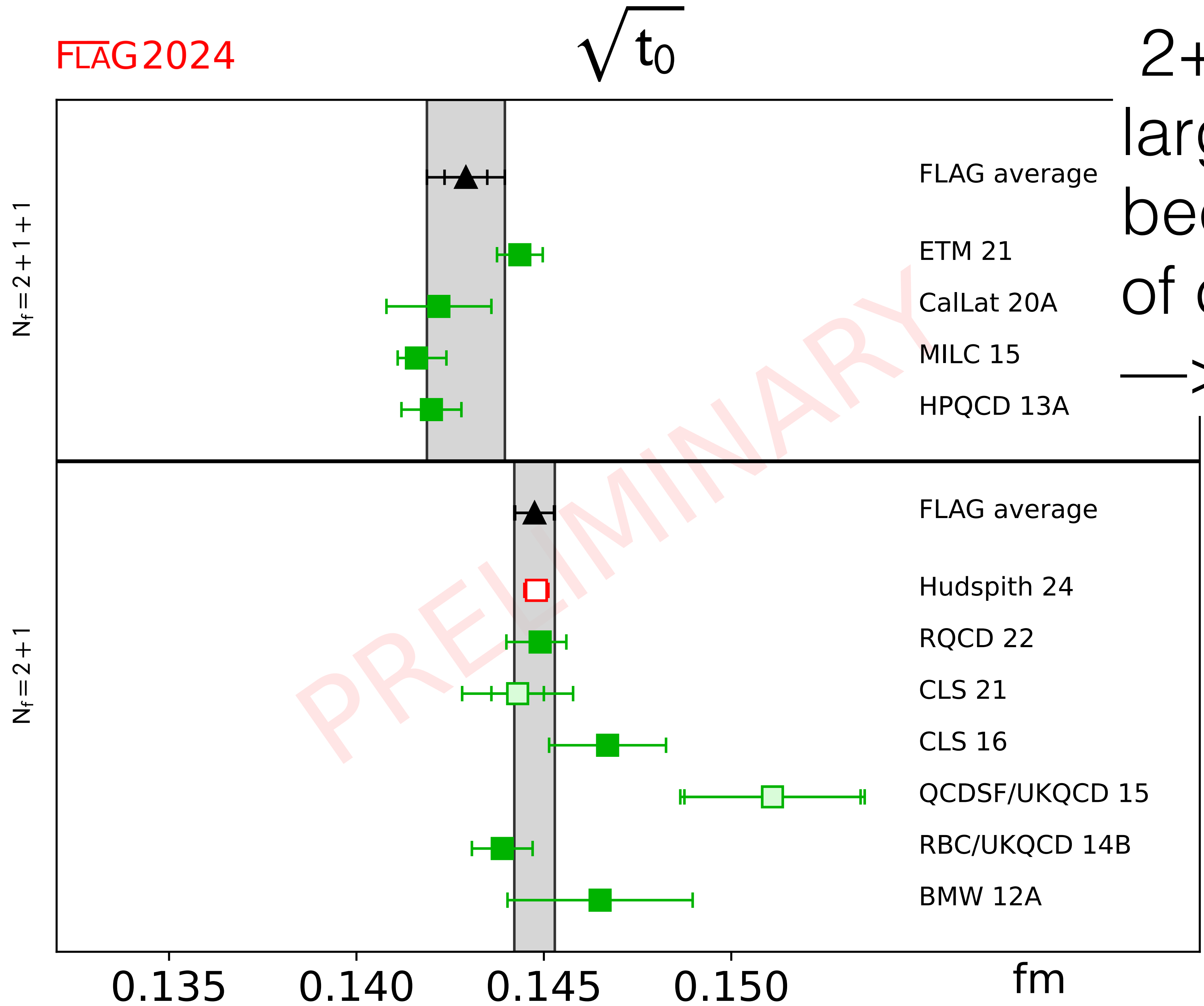


- imo this is late and common errors may go unnoticed
  let's start now

Systematics in most simple (?) quantity: t_0



t_0, possible interpretations

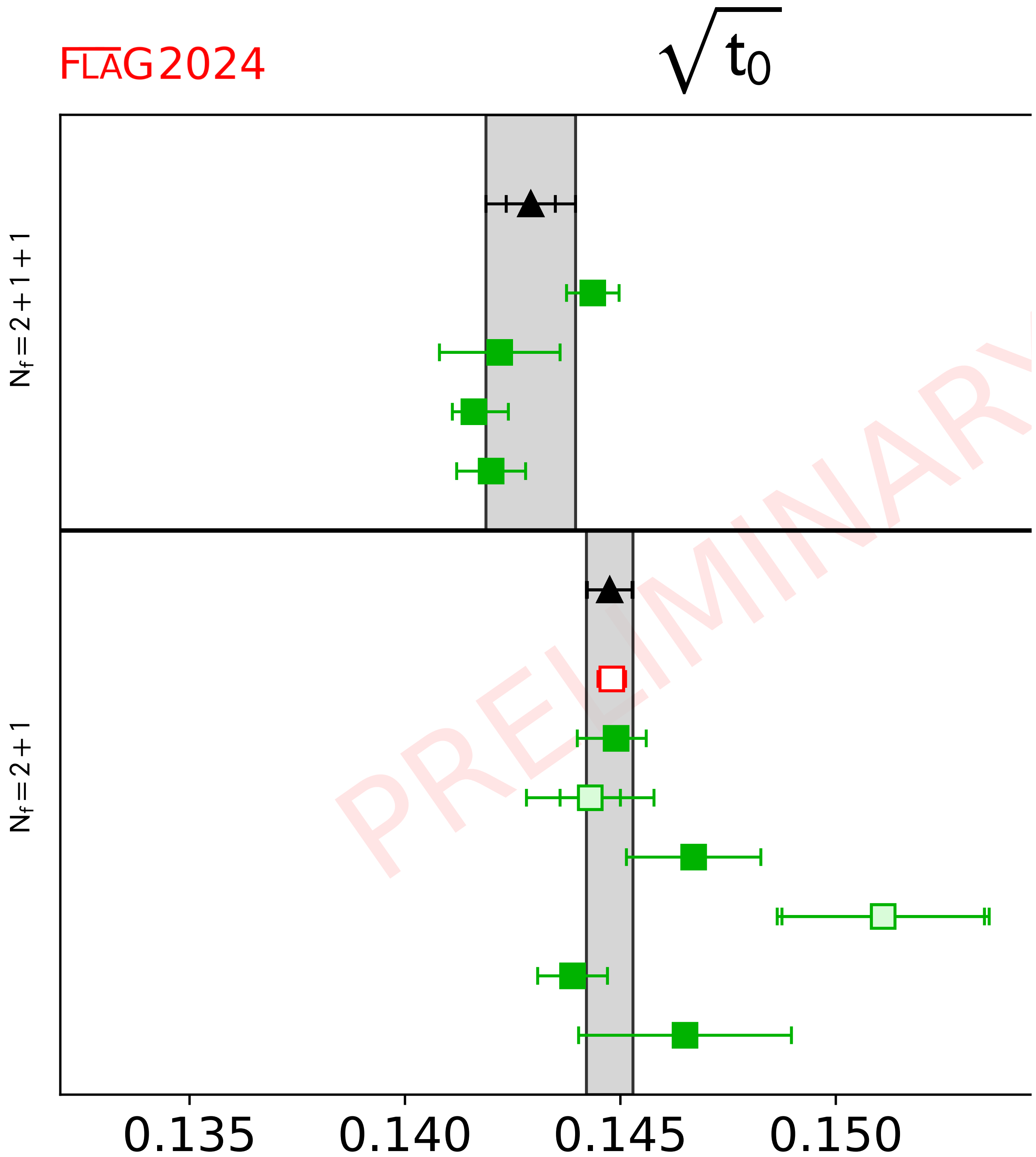


2+1+1:

large error of average because of bad chisq of constant fit

—> stretching factor

t_0, possible interpretations

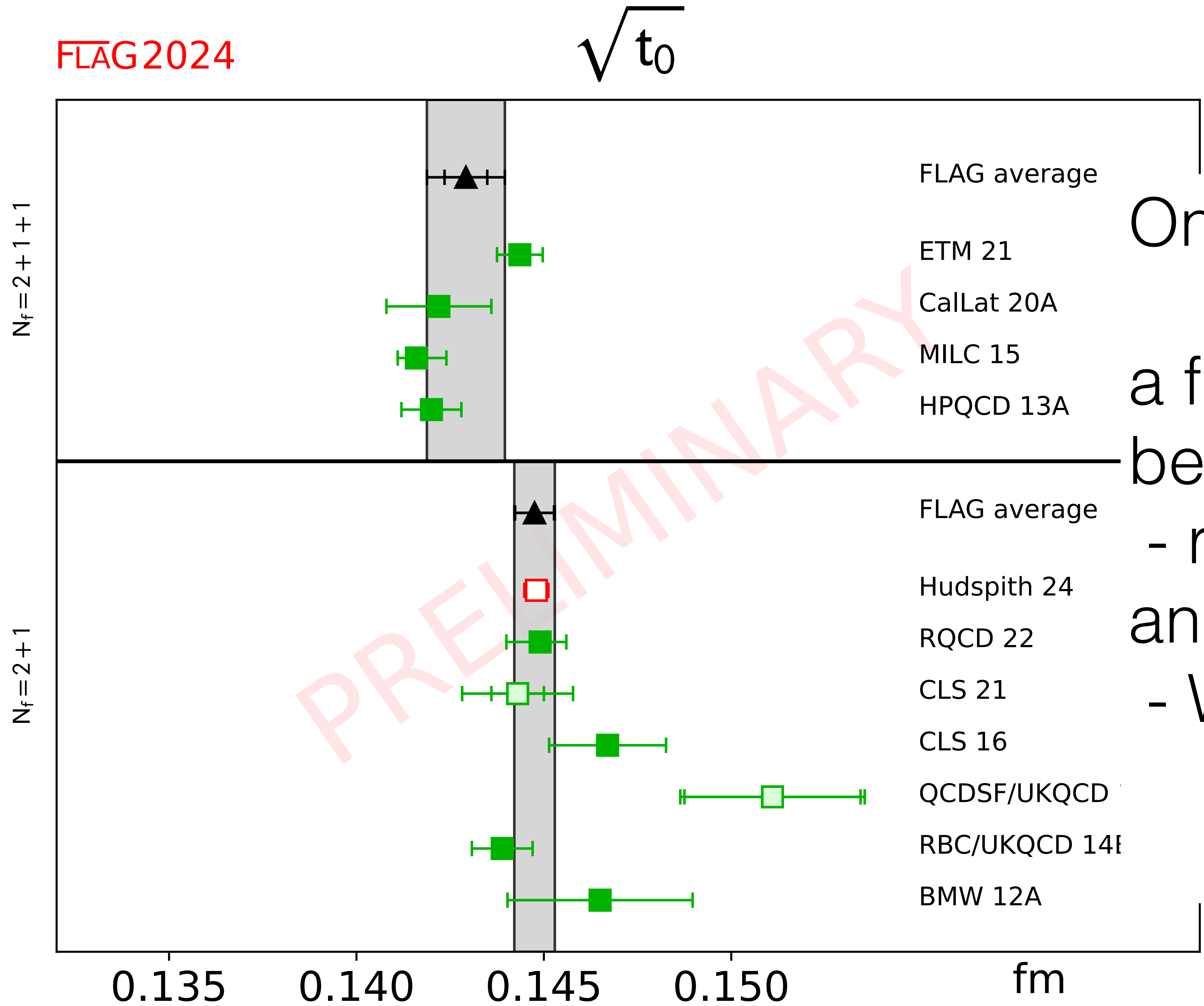


effects of charm quark (loop effects) expected to be very small (ALPHA)

data: a few sigma diff. between 2+1 and 2+1+1

why?
- expectation really wrong?
- cited errors too small? (plateaux, continuum limit, etc: **systematics**))

t_0, possible interpretations



One may also say:

a few sigma differences between
- rooted staggered

and
- Wilson-like

t_0

- how do the differences affect g-2?

- let's switch to B-physics

Systematics for B-observables

- we are mostly “just” dealing with extrapolations
  - 1) excited state effects: $x_0 \to \infty$
  - 2) continuum limit: $a \to 0$
  - 3 … not discussed here.
- there is a big difference between 1) and 2)

  1) exponential convergence

  2) polynomial with logs
- start with the “easy” one

Excited state effects in B-meson observables

- dominant effects: states with additional pions

  —> ChPT (HMChPT)
  —> this is a (good, I think) approximation and definitely gives a good general picture for what is relevant

  follow what has been done by O. Bär for the nucleon, apart from construction of Lagrangian, renormalisation, …

- References for what I discuss here:
  - $B\pi$ excited-state contamination in lattice calculations of B-meson correlation functions, O. Bär, A. Broll, RS Eur.Phys.J.C 83 (2023) 8, 757 , 2306.02703 [hep-lat]
  - Lattice 2022 talk by A. Broll
  - Thesis A. Broll https://edoc.hu-berlin.de/handle/18452/28796

Bπ and B*π states[1]

slide stolen from Oliver B.

Consider static B and B*-mesons:

$$\Delta E_n \approx E_\pi(\vec{p}_n) \qquad \vec{p}_n = \frac{2\pi}{L}\vec{n}$$

$M_\pi = 140\,\mathrm{MeV}$

$M_\pi L = 4$

$M_\pi L = 5$

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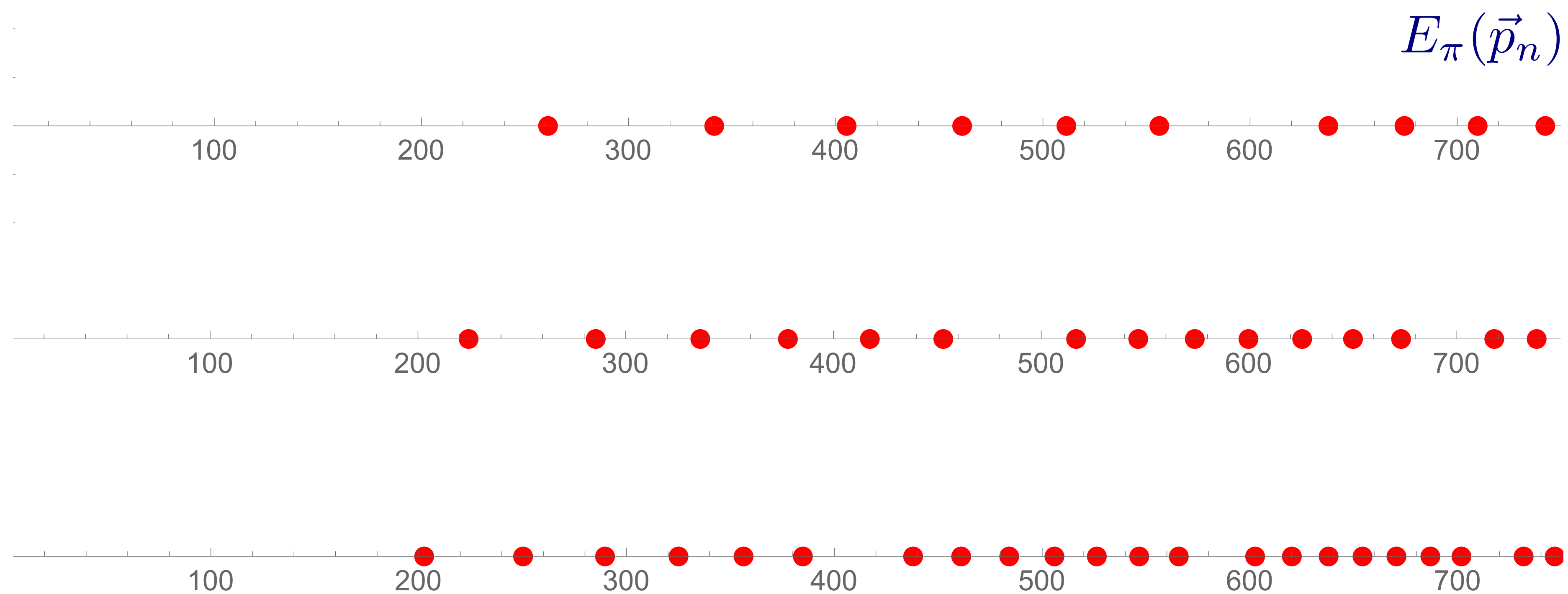

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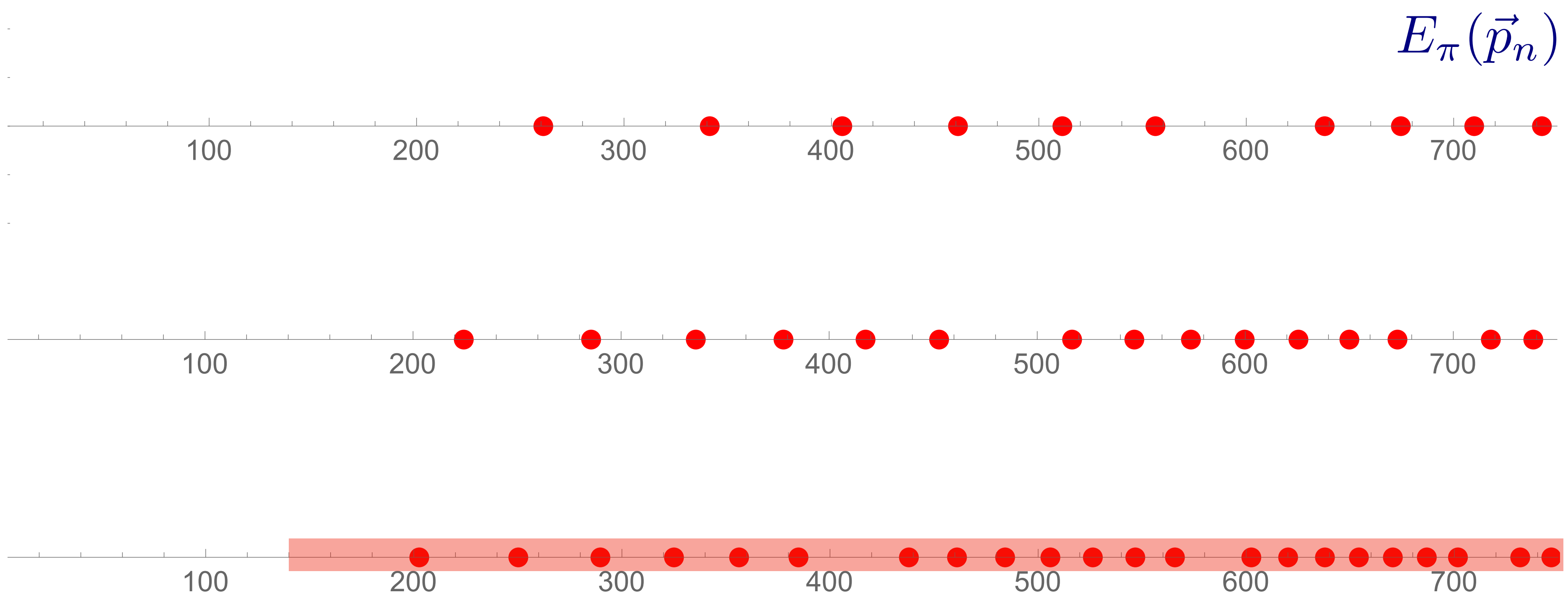


Infinite volume: continuous 2-particle spectrum, threshold = $M_B + M_\pi$

[1] I often won't distinguish between B and B* in the following …

A little on HMChPT

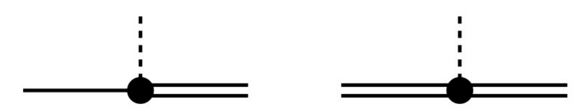
slide stolen from Oliver B.; modified

- Heavy quark spin symmetry → multiplet $H = P_+ \left(iB_k^* \gamma_k + iB\gamma^5\right)$ $\quad P_+ = (1+\gamma_4)/2$

  $\bar{H} = \gamma_4 H^\dagger \gamma_4$

- Relevant interaction given by

$$\mathcal{L}_{\rm int} = i\frac{g}{f}\,{\rm tr}\left(H\gamma_5\gamma_\mu\partial_\mu\pi\bar{H}\right) + \ldots \quad \rightarrow$$



$BB^*\pi$ $\quad$ $B^*B^*\pi$

Note:

- one pion derivative
- two LO LECs f: pion decay constant $\quad f \approx f_\pi \approx 93\,\mathrm{MeV}$

  g: $BB^*\pi$ coupling $\quad g \approx 0.49$ $\quad$ Lattice: A. Gerardin *et al.* (2022)

  (in the chiral limit)

Interpolating B-meson fields

QCD: $\quad \overline{q}_r(x)\Gamma Q(x)$

HMChPT: $\quad \frac{\alpha}{2}[B_k^\star + i\beta_1 B\partial_k\pi + \ldots],\ \beta_1 = 0.14(4)\,\mathrm{GeV}^{-1}$ (from JLQCD form factor)

smeared interpolating field with $r_{sm} \ll 1/m_\pi$: $\quad \beta_1 \rightarrow \tilde{\beta}_1(r_{\rm sm})$. $\quad$ Can it be tuned?

Example 1: 2-pt function, (infinite volume, but this is not important)

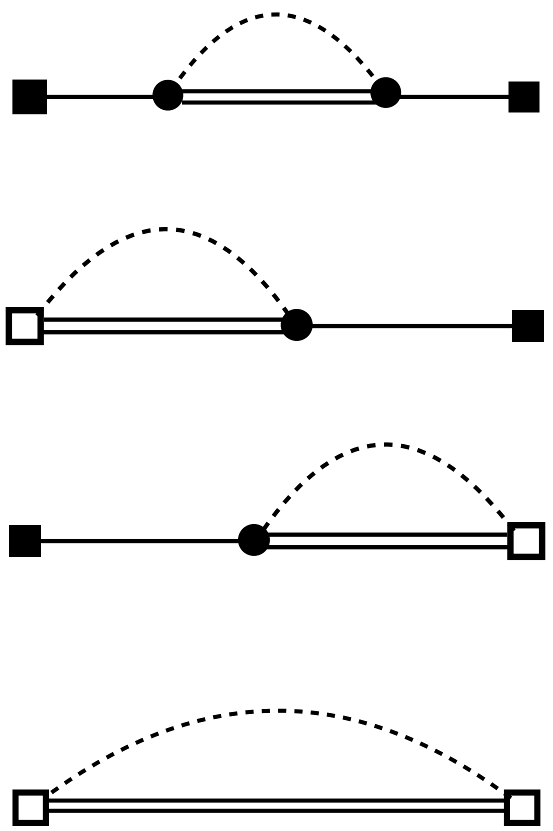
Example $\langle 0|A_4^{\rm RGI}(0)|B(\vec{p}=0)\rangle \equiv \hat{f}$ heavy light decay constant

$\hat{f} = f_B\sqrt{M_B}\,/\,C_{\rm PS}$

Estimator
not unique

$$\hat{f}_{\rm eff}(t) = \sqrt{2}\,\sqrt{C_2(t)}\,e^{\frac{1}{2}M_B^{\rm eff}(t)\,t}$$

$$= \hat{f}\left(1+\Delta\hat{f}^{B\pi}(t)\right) \xrightarrow{t\,\to\,\infty} \hat{f}$$



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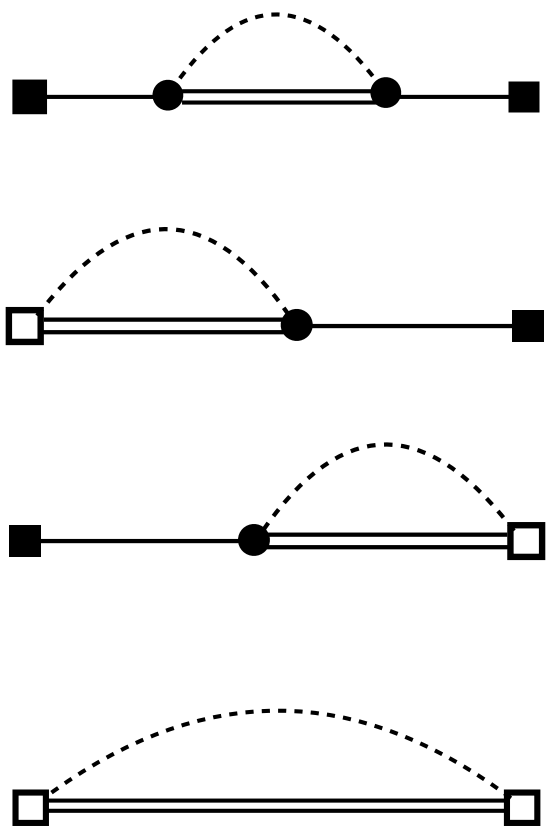
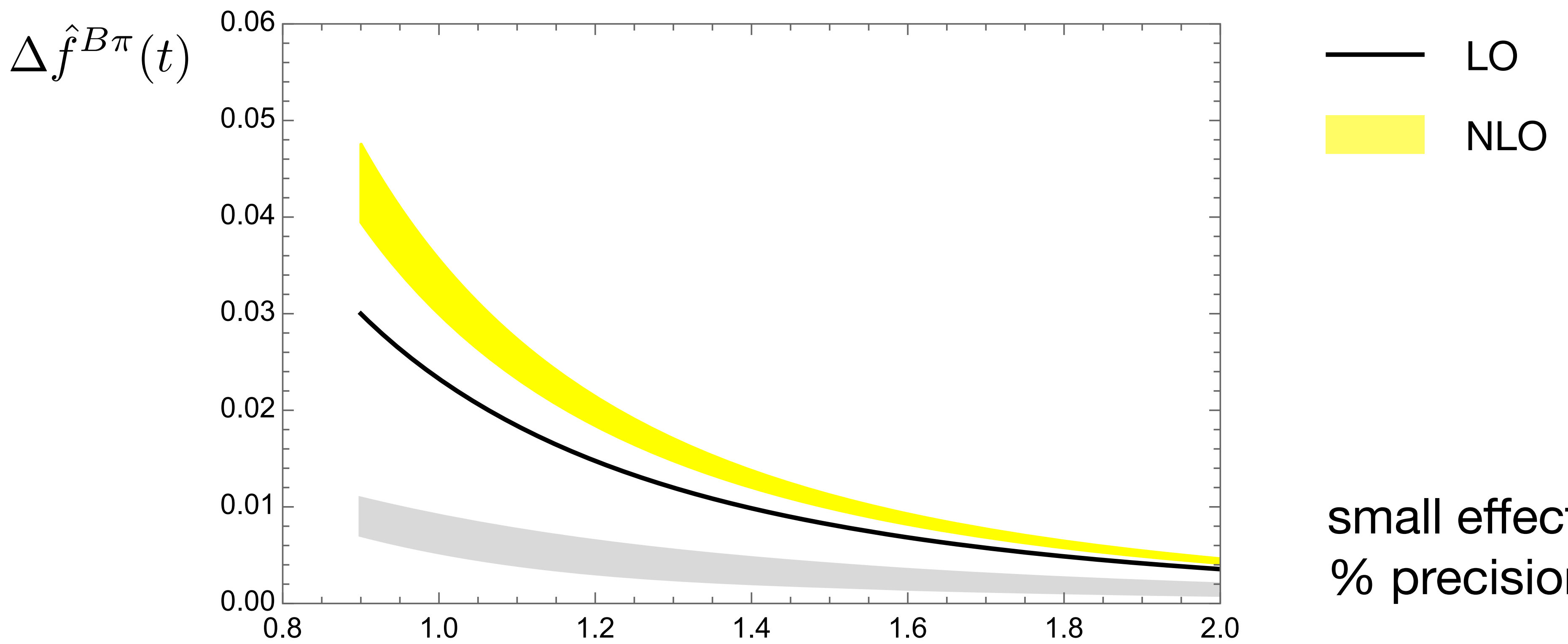




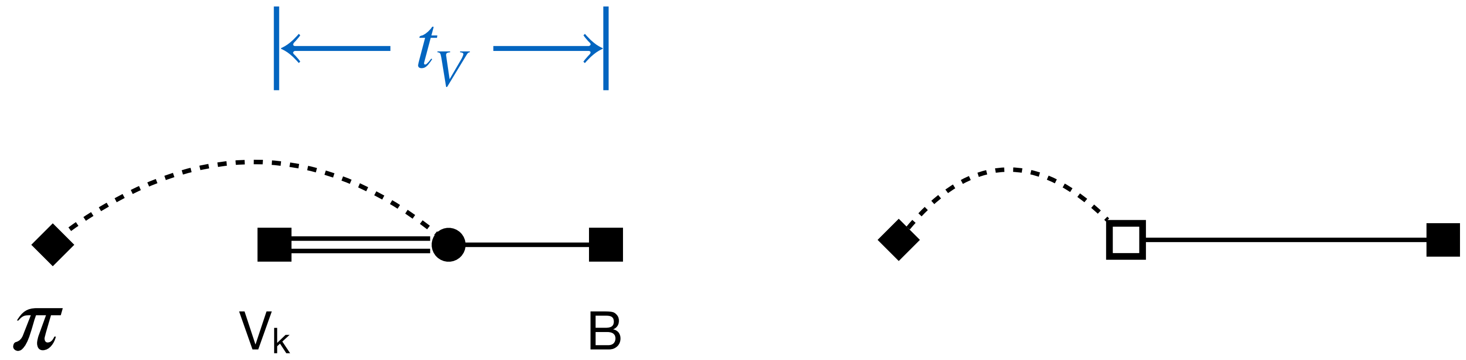
small effect but relevant for
% precision

slide stolen from Oliver B.; modified

Example 2: Bπ contamination in $h_\perp$ (dominant FF in $B \to \pi$)

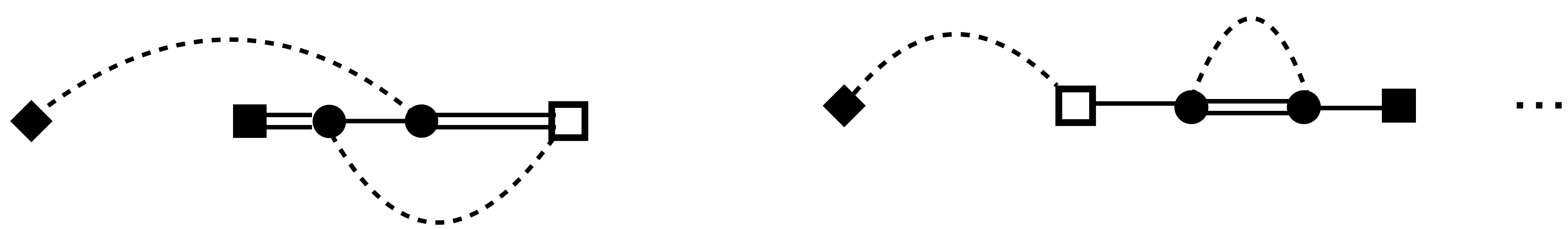
Feynman diagrams for

$C_3^B(t, t_V)$



$C_3^{B\pi}(t, t_V)$

loop diagrams

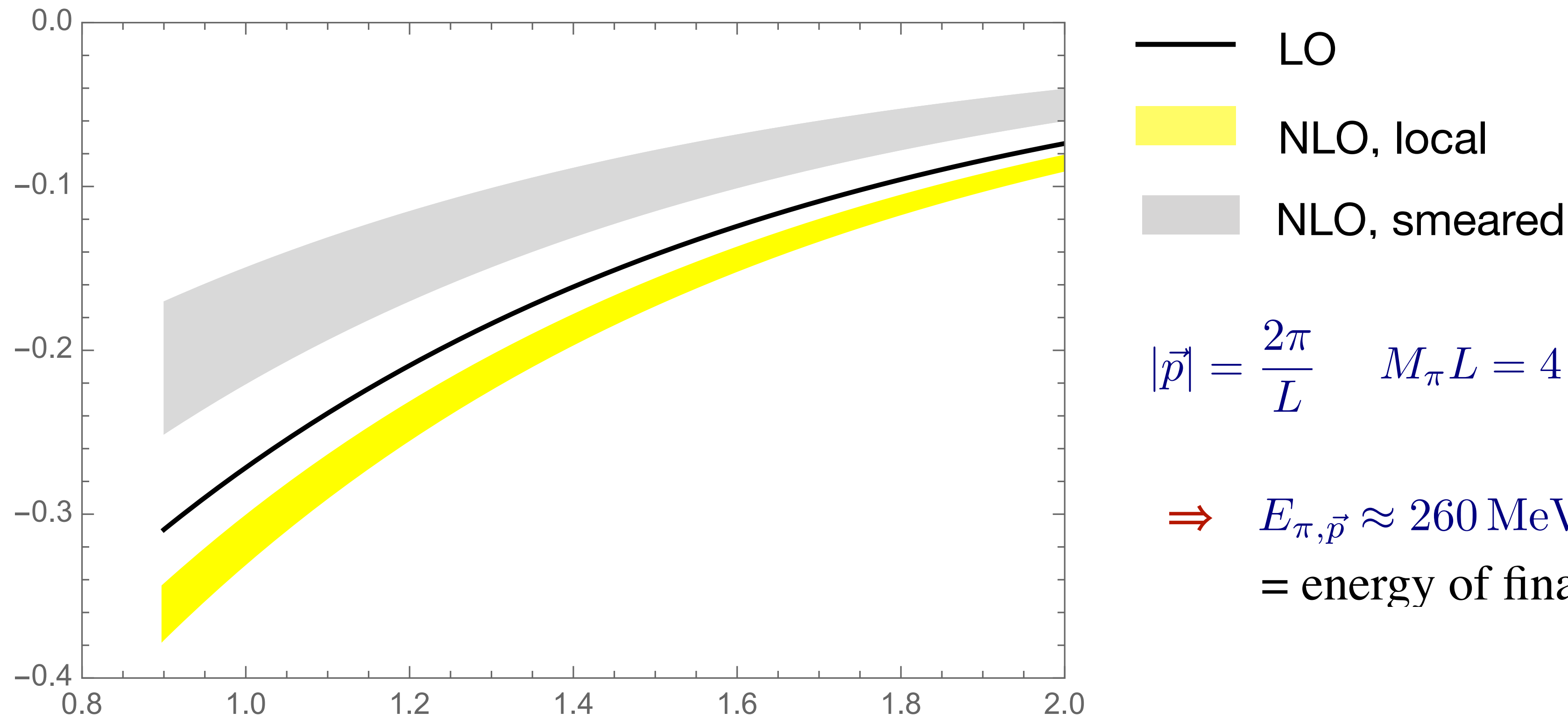


tree diagrams

$$\Delta h_\perp^{B\pi} = \Delta h_\perp^{B\pi,\text{tree}} + \Delta h_\perp^{B\pi,\text{loop}} \approx \Delta h_\perp^{B\pi,\text{tree}}$$

Bπ contamination $\Delta h_\perp$

slide stolen from Oliver B.; modified



$|\vec{p}| = \frac{2\pi}{L} \qquad M_\pi L = 4$

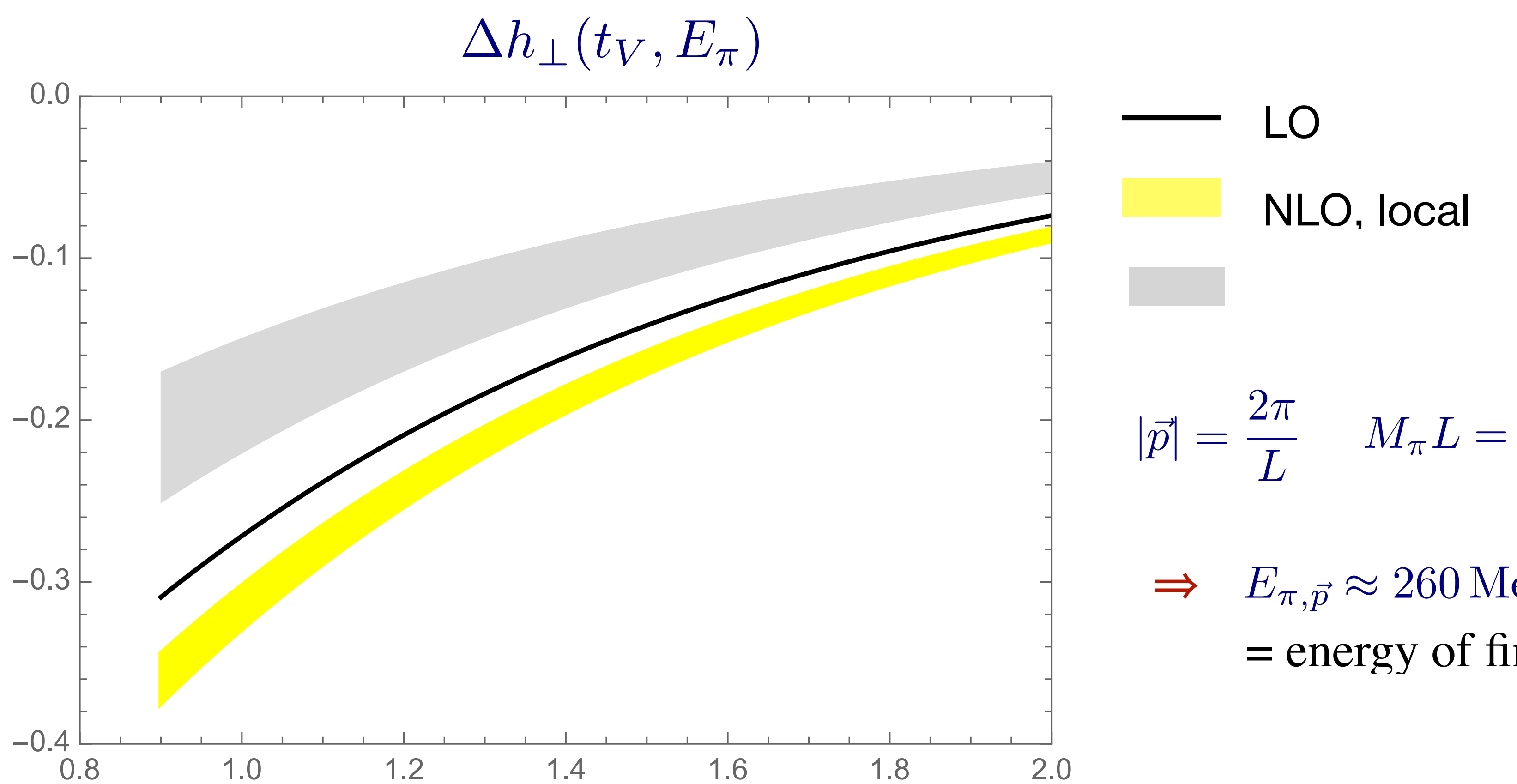
$\Rightarrow \; E_{\pi,\vec{p}} \approx 260\,\mathrm{MeV}$

= energy of final state π

$$\Delta h_\perp^{B\pi}(t_V, E_{\pi,\vec{p}}) \approx \Delta h_\perp^{B\pi,\mathrm{tree}}(t_V, E_{\pi,\vec{p}}) = -\frac{g + \tilde{\beta}_1 E_{\pi,\vec{p}}}{g - \beta_1 E_{\pi,\vec{p}}} e^{-E_{\pi,\vec{p}}\, t_V} \quad = -1 \times \mathrm{e}^{-E_\pi t_V} + \mathrm{NLO}$$

- no factor $1/L^3$ $\Rightarrow$ sometimes called "*volume enhanced*" contribution
- no sum over pion momenta (i.e. no tower of states), *<u>one fixed pion momentum</u>*
- $t_V = 1.2$ fm: underestimate of ~ 20% !
- is it taken into account by standard excited state fitting ?

Bπ contamination $\Delta h_\perp$



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- in principle this can be suppressed by negative $\tilde{\beta}_1$
- since it is a LEC there are various handles to determine it (see Bär,Broll, S)
- Lattice 2024 talk: Antoine Gérardin "$B^* - \pi$ excited-state contamination in B-physics observables"

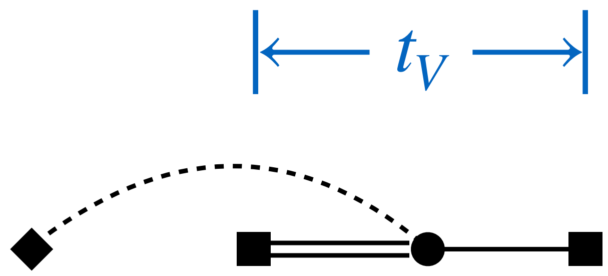
  abstract: … "The LECs can be determined well and it turns out
  that the investigated smearings do not suppress excited states significantly." …

btw: subtraction of tree-level Bpi contamination in the determination of LEC works very well (too well?)

Relation to standard fitting procedure of 3-point + 2-point

- $B \to \pi$

- assume final state π has t large enough
  for ground state dominance
  just for simplicity



- 2-pt correlator

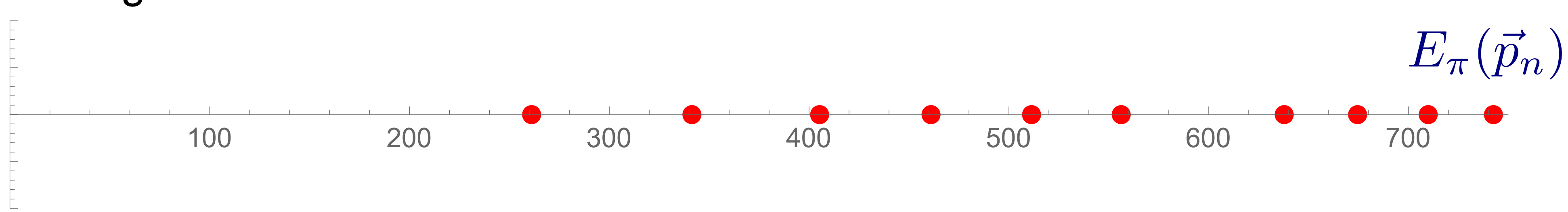
$$C_B^{(2)}(t) = \langle O(t)\bar{O}(0)\rangle = \sum_{n=1}^{\infty} \mathrm{e}^{-E_n(B)t}\psi_n\psi_n^* \,,$$

3-pt correlator

$$C^{(3)}(t, t_v) = \langle \pi^+(t)\ V_\mu(t_v)\ \bar{O}_j(0)\rangle = \sum_{n,m=1}^{\infty} \phi_m\, \mathrm{e}^{-E_m(\pi)(t-t_v)}\, \mathscr{M}^\mu_{mn}\, \mathrm{e}^{-E_n(B)t_v}\, \psi_n^*$$

$$C^{(3)}(t, t_v) \approx \sum_{n=1}^{\infty} \phi_1\, \mathrm{e}^{-E_1(\pi)(t-t_v)}\, \mathscr{M}^\mu_{1n}\, \mathrm{e}^{-E_n(B)t_v}\, \psi_n^*$$

- spectrum relative to ground state



Relation to standard fitting procedure of 3-point + 2-point

- 2-pt correlator

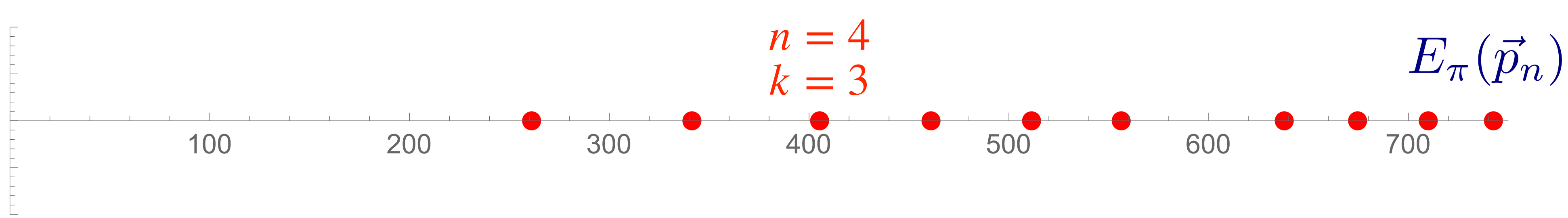
$$\langle O(t)\bar{O}(0)\rangle = \sum_{n=1}^{\infty} \mathrm{e}^{-E_n t}\psi_n\psi_n^* \,, \qquad \left|\frac{\psi_{n>1}}{\psi_1}\right| = \left|\frac{\langle 0|B|k+1\rangle}{\langle 0|B|1\rangle}\right| = \left|\frac{\langle 0|B|B^*\pi,k)\rangle}{\langle 0|B|B\rangle}\right|$$

$$= \Big(N_{\rm deg}(k)\Big)^{1/2}\frac{\sqrt{3}}{4}(g+\beta_1 E_{\pi,k})\frac{2\pi k}{fL}\frac{1}{(LE_{\pi,k})^{3/2}}$$

3-pt correlator $\langle \pi^+(t)\; V_\mu(t_v)\; \bar{O}_j(0)\rangle \approx \sum_{n=1}^{\infty} \phi_1\, \mathrm{e}^{-E_1(\pi)(t-t_v)}\, \mathcal{M}^{\mu}_{1n}\, \mathrm{e}^{-E_n t_v}\, \psi_n^*$

$$\frac{\mathcal{M}^j_{1n}(p)}{\mathcal{M}^j_{11}(p)} = \delta_{n,k+1}\frac{1+\beta_1 E_\pi/g}{1-\beta_1 E_\pi/g} + \mathrm{O}((LE_\pi)^{-3})\,, \qquad p = |\vec{p}\,| = \sqrt{k}2\pi/L\,, \quad k>0$$

- spectrum relative to ground state



Relation to standard fitting procedure of 3-point + 2-point

part of spectrum which contributes in 2-pt function
is very suppressed in 3-pt function

effectively: different states in 3-pt fct than in 2-pt fct
message: allow for this in fits .

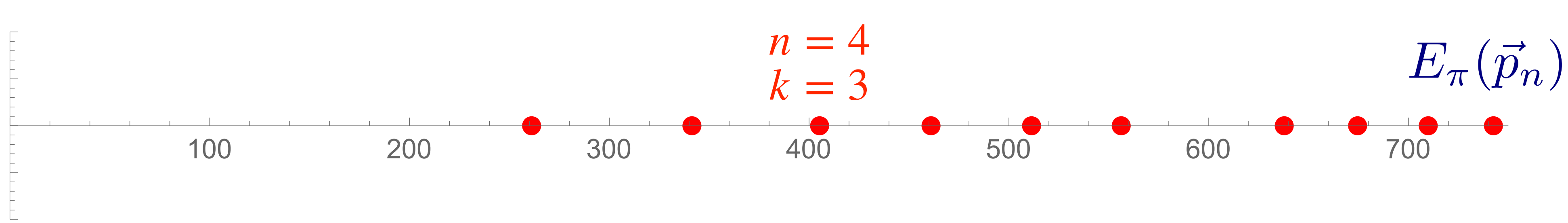
furthermore: energy of dominant correction in 3-pt function is known

structure expected to remain beyond HMChPT

This is all rather simple
But: signal/noise problem limits accessible t

> 0

▸



Excited states: discussion

- let’s do it after the next topic

Discretization errors, continuum extrapolation

- two issues
  - 1) a -expansion is not a Taylor expansion

    it is non-analytic, given by Symanzik EFT
    (according to standard wisdom)

    non-integer powers of $\alpha_s(1/a)$

  - 2) Symanzik EFT is for energies below a^{-1}

    but $m_b > a^{-1}$

    RHQ actions are a separate discussion.
    CERN cafeteria (still with leather chairs) AK, MdM, RS
    " should not extrapolate $a \to 0$ "

    NRQCD is yet another discussion —> Lepage and B. Thacker

- 1) is worrying to me (and to Peter Weisz and to ?)
  The combination with 2) makes it scary .

  The "interpolation method" (S. Kuberski) avoids 2)

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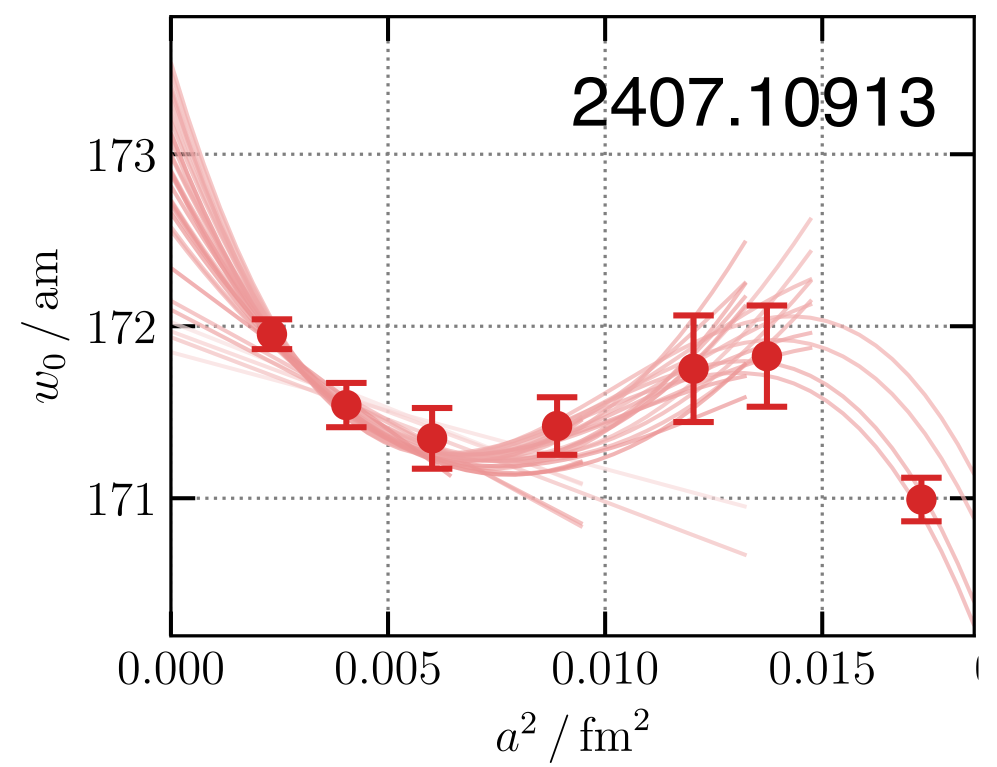

maybe authors of 2407.10913

Briefly on Symanzik EFT

- $$\mathscr{L}_a = \mathscr{L}_0 + a^2 \mathscr{L}_2 + \ldots$$

  $$\mathscr{L}_2(x) = \sum_i c_i \, \mathscr{O}_i(x)$$ different local fields depending on symmetry of lattice action

  e.g. $$\mathscr{O}_2 = \frac{1}{g_0^2} \sum_{\mu,\nu} D_\mu F_{\mu\nu} \, D_\mu F_{\mu\nu}$$ H(4) invariant but not O(4)

  but e.o.m. can be used

- enough for energies
  more is needed for ME's of local fields, susceptibilities …
  (see N. Husung)

  and more for Gradient Flow

  $\mathscr{O}_i(x)$ as insertions into correlation functions

  $$\delta C^{\mathscr{O}}(x-y) = -\, a^2 \int \mathrm{d}^4 z \, \langle \, \Phi(x)\Phi(y) \, \mathscr{O}(z) \, \rangle^{\mathrm{con}}_{\mathrm{cont}}$$

Non-integer powers of $\alpha_s(1/a)$: the usual EFT story

SymEFT = continuum EFT, regularize, renormalize: $\alpha_{\overline{\rm MS}}(\mu)$

Match to Lattice PT (expanded in a): gives matching coefficients $\bar{c}_i(\alpha_{\overline{\rm MS}}(\mu), a\mu)$

Do RG improvement $\mu = a^{-1}$ —> $\bar{c}_i(\alpha_{\overline{\rm MS}}(a^{-1}),1)$

Introduce RGI operators to express result in terms of (non-perturbative) constants.
Needs anomalous dimension matrix! (N. Husung)

$$\Delta_{\mathscr{P}}(a) = -\,a^2 \sum_i c_i^{(0)} \left[\alpha(a^{-1})\right]^{\hat{\gamma}_i} \mathscr{M}^{\rm RGI}_{\mathscr{P},i}\,[1 + {\rm O}(\alpha(a^{-1})] + {\rm O}(a^3)$$

$\hat{\gamma}_i = \gamma_i^{(0)}/(2b_0)$ eigenvalues of 1-loop AD matrix of $\left\{ \mathscr{O}_j(x) \right\}$

known for theory with flavor symmetry

not for staggered fermions (or minimally doubled) or …

Examples what is done: 1

$$f_X^{B_s\to K}(M_\pi, E_K, a^2) = \frac{\Lambda}{E_K + \Delta_X}\left[c_{X,0}\left(1 + \frac{\delta f(M_\pi^s) - \delta f(M_\pi^p)}{(4\pi f_\pi)^2}\right) + c_{X,1}\frac{\Delta M_\pi^2}{\Lambda^2} + c_{X,2}\frac{E_K}{\Lambda} + c_{X,3}\frac{E_K^2}{\Lambda^2} + c_{X,4}(a\Lambda)^2\right],$$

"We include a term proportional to a^2 to account for the dominant lattice-spacing dependence.

The domain-wall fermion and Iwasaki gluon actions are expected to have discretization errors $O((a\Lambda_{QCD})^2)$,

about 3% (5%) on the F (M) ensemble(s) for ΛQCD = 500 MeV, while power-counting estimates of errors in the RHQ action and heavy-light current are smaller, below 2%. "

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Yes, $a^2\mathscr{L}_2 = a^2\mathcal{O}_2 + \ldots = a^2\frac{1}{g_0^2}\sum_{\mu,\nu} D_\mu F_{\mu\nu}\, D_\mu F_{\mu\nu} + \ldots$ but SymEFT says

$$\delta C^{\mathscr{L}}(x-y) = -\int \mathrm{d}^4 z\,\langle\,\Phi(x)\Phi(y)\,\mathscr{L}_2(z)\,\rangle^{\mathrm{con}}_{\mathrm{cont}} \;\to\; a^2\mathscr{M}^{\mathrm{RGI}}_{\mathscr{P},2} = a^2\Lambda_{\mathrm{QCD}}\times\varphi(E_K/\Lambda, m_b/\Lambda, m_\pi/\Lambda)$$

with some function φ not $(a\Lambda_{\mathrm{QCD}})^2$,

(here I ignored mixing, AD, …)

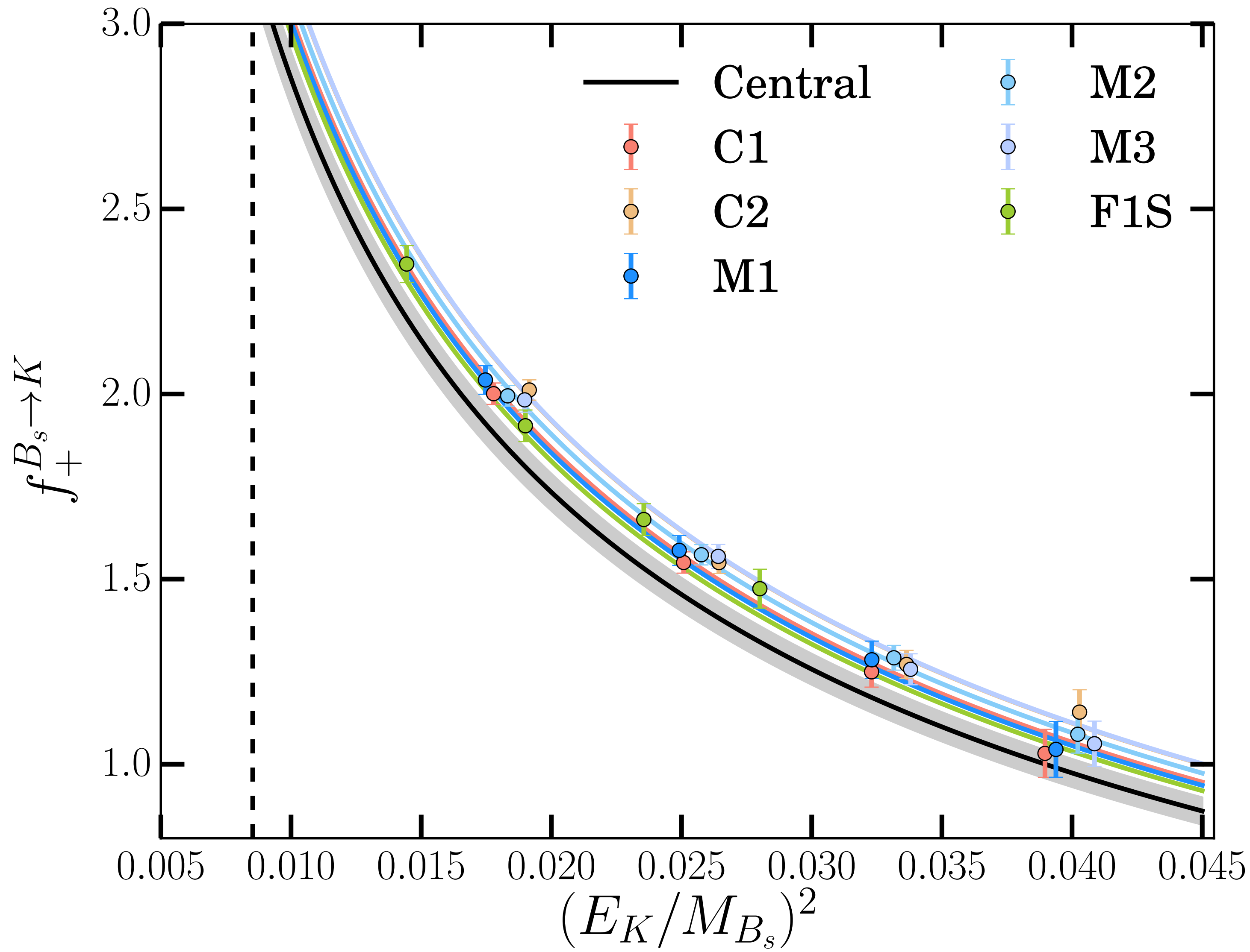
and question: how do these power-counting estimates go? Are they independent of E_K ?

Examples what is done: 1

$$+ c_{X,4}(a\Lambda)^2$$

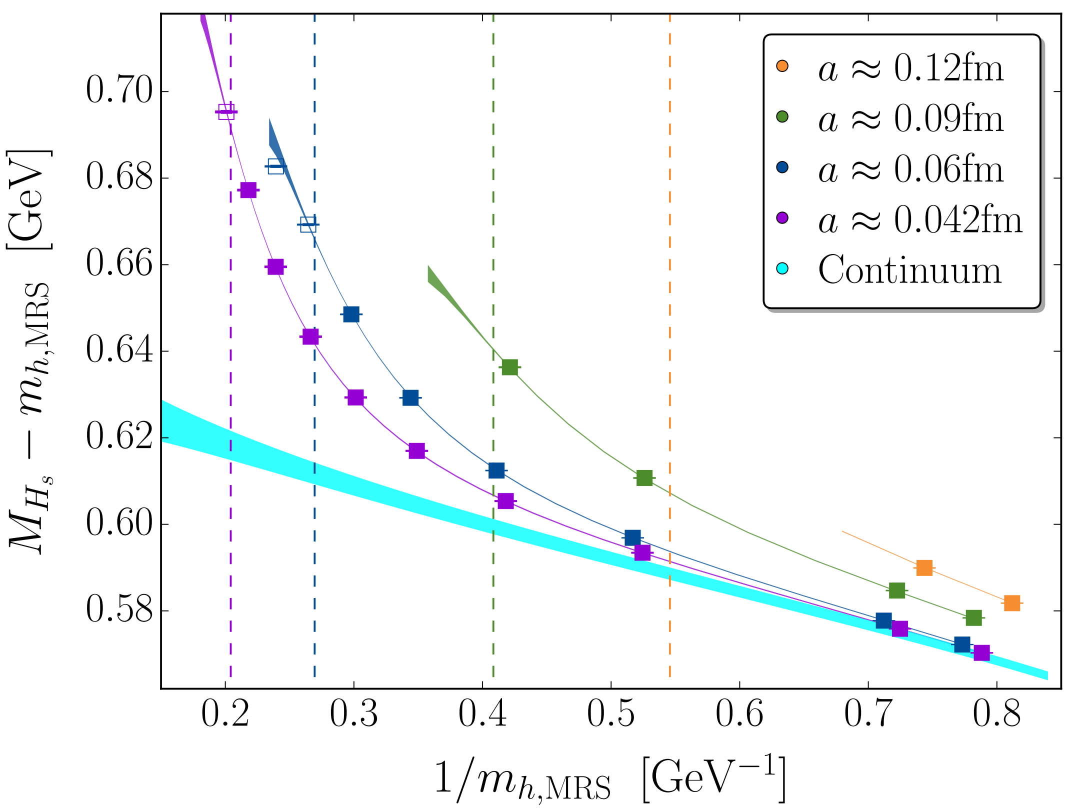
effect of continuum limit simultanously determined by all data points

error in CL becomes small even at large E where data have large errors



footnote: discussion of systematics involves an $(aE)^2$ term
but it does not affect the statistical error of the CL (as far as I understand)

Example 2: most precise determination of b-quark mass



$$
\begin{aligned}
\mathcal{F} = {} & \breve{m}_{h,\mathrm{MRS}} + \breve{\bar{\Lambda}}_{\mathrm{MRS}} + \frac{\breve{\mu}_\pi^2}{2m_{h,\mathrm{MRS}}} - \frac{\breve{\mu}_G^2(m_b)}{2m_{h,\mathrm{MRS}}}\frac{C_{\mathrm{cm}}(m_h)}{C_{\mathrm{cm}}(m_b)} \\
& + 2\breve{\lambda}_1 B_0(m_x - m_l) + 2\breve{\lambda}_1' B_0(2m_l' + m_s' - 2m_l - m_s) \\
& + \delta M_{H_x}(m_x; \{m_l', m_l', m_s'\}; a) - \delta M_{H_l}(m_l; \{m_l, m_l, m_s\}; 0) \\
& + \Lambda_{\mathrm{HQET}}\left[\rho_1 w_h^2 + \rho_2 w_h^3 + \rho_3 w_h^4\right] \\
& + f_\pi \left[\sum_{i=1}^{4} q_i \left(1 + q_i' w_h + \tilde{q}_i \alpha_s y^2\right) x_i^2 + \sum_{j=5}^{11} q_j x_j^3\right],
\end{aligned}
$$

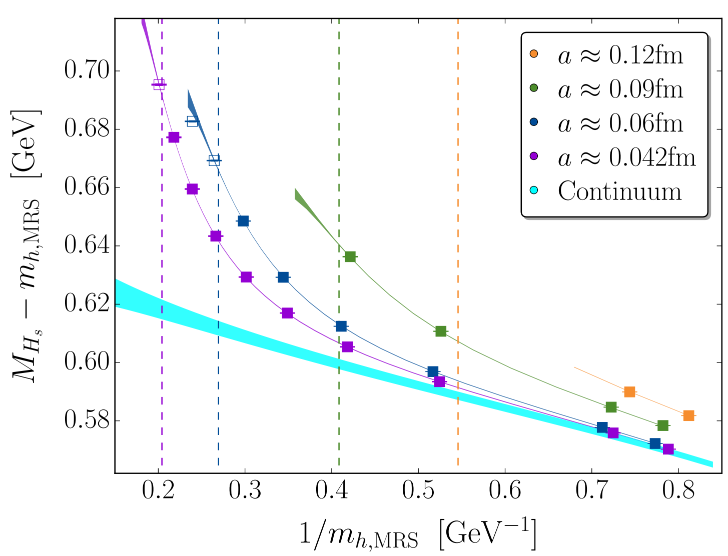
$$
\breve{\bar{\Lambda}}_{\mathrm{MRS}} = \bar{\Lambda}_{\mathrm{MRS}}\left(1 + \bar{c}_1 \alpha_s y + \bar{c}_2 y^2\right)\left(\frac{m_c'}{m_c}\right)^{2/27}\left(1 + k_1' \frac{\delta m_c'}{m_c}\right),
$$

$$
\begin{aligned}
\breve{\lambda}_1 &= \lambda_1\left(1 + c_1\alpha_s y + c_2 y^2 + c_3 \bar{w}_h \alpha_s y + c_4 \bar{w}_h + c_5 \bar{w}_h^2 + c_6 \bar{w}_h^3\right), \\
\breve{\lambda}_1' &= \lambda_1'\left(1 + c_1'\alpha_s y + c_2' y^2 + c_3' \bar{w}_h \alpha_s y + c_4' \bar{w}_h + c_5' \bar{w}_h^2 + c_6' \bar{w}_h^3\right), \\
\breve{g}_\pi &= g_\pi\left(1 + g_1\alpha_s y + g_2 y^2 + g_3 \bar{w}_h \alpha_s y + g_4 \bar{w}_h + g_5 \bar{w}_h^2 + g_5 \bar{w}_h^3\right), \\
\breve{\mu}_\pi^2 &= \mu_\pi^2\left(1 + p_\pi \alpha_s y + r_\pi \alpha_s x_h^2\right), \\
\breve{\mu}_G^2(m_b) &= \mu_G^2(m_b)\left(1 + p_G \alpha_s y + r_G \alpha_s x_h^2\right),
\end{aligned}
$$

$$y \sim a^2$$

$$
\begin{aligned}
\breve{m}_{h,\mathrm{MRS}} = {} & m_{p4s,\overline{\mathrm{MS}}}(2\ \mathrm{GeV})\left[\frac{C(\alpha_{\overline{\mathrm{MS}}}(\overline{m}_h))}{C(\alpha_{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}))}\right]_{\mathrm{Eq.}\ (3.23)}\left[\frac{m_{h,\mathrm{MRS}}}{\overline{m}_h}\right]_{\mathrm{Ref.}\ [1]}\left[\frac{am_{0h}}{am_{0,p4s}}\right]_{\mathrm{sim}} \times \\
& \left(1 + \alpha_{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})\sum_{n=1}^{4} k_n x_h^n\right) \times \left(1 + \tilde{c}_1 \alpha_s y + \tilde{c}_2 y^2 + \tilde{c}_3 y^3\right).
\end{aligned}
$$

Example 2: most precise determination of b-quark mass



$$\mathcal{F} = \breve{m}_{h,\mathrm{MRS}} + \breve{\bar{\Lambda}}_{\mathrm{MRS}} + \frac{\breve{\mu}_\pi^2}{2m_{h,\mathrm{MRS}}} - \frac{\breve{\mu}_G^2(m_b)}{2m_{h,\mathrm{MRS}}}\frac{C_{\mathrm{cm}}(m_h)}{C_{\mathrm{cm}}(m_b)} + 2\breve{\lambda}_1 B_0(m_x - m_l) + 2\breve{\lambda}_1' B_0(2m_l' + m_s' - 2m_l - m_s) + \delta M_{H_x}(m_x; \{m_l', m_l', m_s'\}; a) - \delta M_{H_l}(m_l; \{m_l, m_l, m_s\}; 0) + \Lambda_{\mathrm{HQET}}\left[\rho_1 w_h^2 + \rho_2 w_h^3 + \rho_3 w_h^4\right] + f_\pi\left[\sum_{i=1}^{4} q_i\left(1 + q_i' w_h + \tilde{q}_i \alpha_s y^2\right) x_i^2 + \sum_{j=5}^{11} q_j x_j^3\right],$$

$$\breve{\bar{\Lambda}}_{\mathrm{MRS}} = \overline{\Lambda}_{\mathrm{MRS}}\left(1 + \bar{c}_1\alpha_s y + \bar{c}_2 y^2\right)\left(\frac{m_c'}{m_c}\right)^{2/27}\left(1 + k_1'\frac{\delta m_c'}{m_c}\right),$$

$$\breve{\lambda}_1 = \lambda_1\left(1 + c_1\alpha_s y + c_2 y^2 + c_3\bar{w}_h\alpha_s y + c_4\bar{w}_h + c_5\bar{w}_h^2 + c_6\bar{w}_h^3\right),$$

$$\breve{\lambda}_1' = \lambda_1'\left(1 + c_1'\alpha_s y + c_2' y^2 + c_3'\bar{w}_h\alpha_s y + c_4'\bar{w}_h + c_5'\bar{w}_h^2 + c_6'\bar{w}_h^3\right),$$

$$\breve{g}_\pi = g_\pi\left(1 + g_1\alpha_s y + g_2 y^2 + g_3\bar{w}_h\alpha_s y + g_4\bar{w}_h + g_5\bar{w}_h^2 + g_5\bar{w}_h^3\right),$$

$$\breve{\mu}_\pi^2 = \mu_\pi^2\left(1 + p_\pi\alpha_s y + r_\pi\alpha_s x_h^2\right),$$

$$\breve{\mu}_G^2(m_b) = \mu_G^2(m_b)\left(1 + p_G\alpha_s y + r_G\alpha_s x_h^2\right),$$

$y \sim a^2$

- various places: $Q \times (1 + c_1\alpha_s a^2 + c_2 a^4)$
- Symanzik: $Q \times (1 + \{\, c_{11}[\alpha_s(1/a)]^{\hat{\Gamma}_1} + c_{12}[\alpha_s(1/a)]^{\hat{\Gamma}_2} + \ldots \,\}a^2 + \ldots)$
- yes, tree-level improvement buys a $\hat{\Gamma}_i = \hat{\gamma}_i + 1$ but $\hat{\gamma}_i > 0$ is unknown (Nikolai?)

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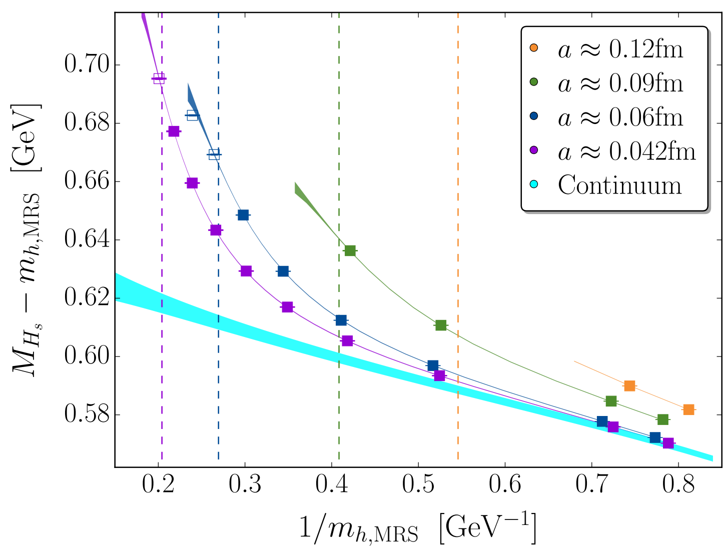
Example 2: most precise determination of b-quark mass



$$\begin{aligned}\mathcal{F} = \breve{m}_{h,\mathrm{MRS}} + \breve{\overline{\Lambda}}_{\mathrm{MRS}} + \frac{\breve{\mu}_\pi^2}{2m_{h,\mathrm{MRS}}} - \frac{\breve{\mu}_G^2(m_b)}{2m_{h,\mathrm{MRS}}}\frac{C_{\mathrm{cm}}(m_h)}{C_{\mathrm{cm}}(m_b)} \\ + 2\breve{\lambda}_1 B_0(m_x - m_l) + 2\breve{\lambda}_1' B_0(2m_l' + m_s' - 2m_l - m_s) \\ + \delta M_{H_x}(m_x; \{m_l', m_l', m_s'\}; a) - \delta M_{H_l}(m_l; \{m_l, m_l, m_s\}; 0) \\ + \Lambda_{\mathrm{HQET}}\left[\rho_1 w_h^2 + \rho_2 w_h^3 + \rho_3 w_h^4\right] \\ + f_\pi \left[\sum_{i=1}^{4} q_i \left(1 + q_i' w_h + \tilde{q}_i \alpha_s y^2\right) x_i^2 + \sum_{j=5}^{11} q_j x_j^3\right],\end{aligned}$$

$$\begin{aligned}\breve{\overline{\Lambda}}_{\mathrm{MRS}} &= \overline{\Lambda}_{\mathrm{MRS}}\left(1 + \bar{c}_1\alpha_s y + \bar{c}_2 y^2\right)\left(\frac{m_c'}{m_c}\right)^{2/27}\left(1 + k_1'\frac{\delta m_c'}{m_c}\right), \\ \breve{\lambda}_1 &= \lambda_1\left(1 + c_1\alpha_s y + c_2 y^2 + c_3\bar{w}_h\alpha_s y + c_4\bar{w}_h + c_5\bar{w}_h^2 + c_6\bar{w}_h^3\right), \\ \breve{\lambda}_1' &= \lambda_1'\left(1 + c_1'\alpha_s y + c_2' y^2 + c_3'\bar{w}_h\alpha_s y + c_4'\bar{w}_h + c_5'\bar{w}_h^2 + c_6'\bar{w}_h^3\right), \\ \breve{g}_\pi &= g_\pi\left(1 + g_1\alpha_s y + g_2 y^2 + g_3\bar{w}_h\alpha_s y + g_4\bar{w}_h + g_5\bar{w}_h^2 + g_5\bar{w}_h^3\right), \\ \breve{\mu}_\pi^2 &= \mu_\pi^2\left(1 + p_\pi\alpha_s y + r_\pi\alpha_s x_h^2\right), \\ \breve{\mu}_G^2(m_b) &= \mu_G^2(m_b)\left(1 + p_G\alpha_s y + r_G\alpha_s x_h^2\right),\end{aligned}$$

$y \sim a^2$

- complicated extrapolation to physical quark mass and continuum limit
- non-trivial theory (MRS scheme, HMrASχPT)
- How do we get convinced that the combined extrapolation is correct? Symanzik-like expansion with $am_b \approx 1$! Why can we truncate?
- Does the behavior at $am_b \ll 1$ really tell us much about $am_b \approx 1$

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Continuum extrapolations

- General form allows for very general functions
- Assumptions needed, e.g. just explore one power at a time
- A good strategy seems:

  — computations with different discretisations
  — compare
  — perform combined continuum extrapolations
  better to disentangle continuum extrapolations
  and other extrapolations
  — even better: cancel renormalisation
  (S. Kuberski: $h_\perp(E_\pi)\,/\,h_\perp(E_\pi^{\rm ref})$ or $h_\perp(E_\pi)\,/\,f_V$)
  simple heavy quark mass scaling!
  — or develop some new ideas
  (means work! credited?)

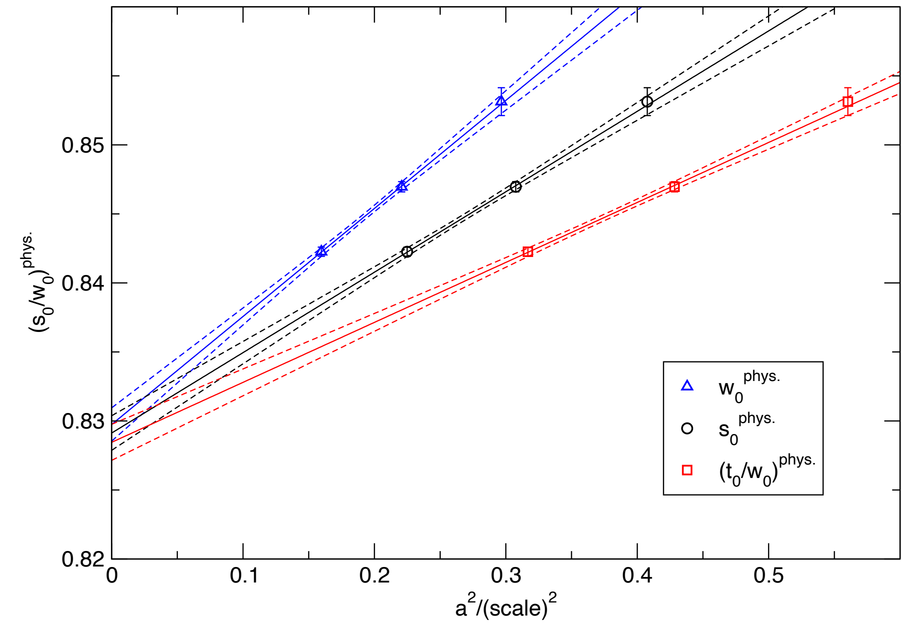
New continuum extrapolation criterion in FLAG6

- Issue: there are quantities with a strong dependence on a

  Given the discussed uncertainties in the functional form of the a effects, extrapolating too far is dangerous.
  "Far" should be measured in the (total) error cited for the result, $\sigma_Q^{\rm cont}$

- distance measure:

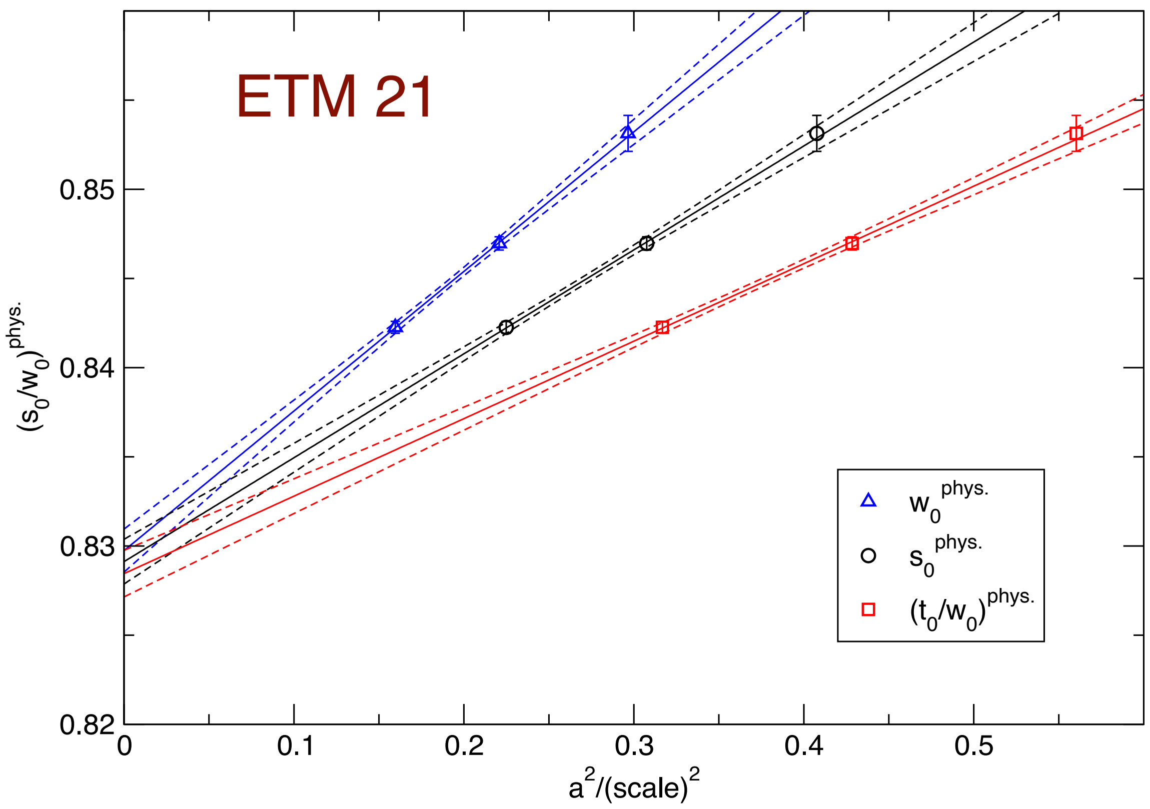
$$\delta(a) = \frac{|Q(a) - Q(0)|}{\sigma_Q^{\rm cont}}$$



- $\delta(a_{\rm min}) \leq 3$ considered fine: extrapolation by $3 \times \sigma_Q^{\rm cont}$

- $\delta(a_{\rm min}) > 3$ some stretching of the uncertainty before averaging

FLAG5 scale setting: update



Fits: $a^2\,[\alpha_s(1/a)]^{\hat\Gamma}\,,\quad \hat\Gamma = 0$

indeed (Husung)

$\hat\Gamma_{\rm lead} = 0\,,$ for $N_f = 0$ (also for $N_f > 0$?)

Still: described entirely by the leading term?

Summary for discussion

- imo. B-physics on the lattice is not in good shape

- I find it good that some discrepancies/tensions are there because they provide motivation for doing better
  — The real worry is wrong results in agreement with others

- Some news on excited state effects
  $B^{\star}\pi$ states dominate at large t and can be estimated
  by HMChPT
  — mostly small (loop effects; L^{-3} in finite volume, but …)
  — but known, large, tree-level effect (only $L^{-3/2}$ in finite vol.)
  in $f_{\perp}$ of $B \to \pi$

- Continuum extrapolations remain scary
  — interpolation to B (—> Simon Kuberski) helps
  — perform computations of benchmark computations with
  different discretisations, compare, constrain continuum limit
  — avoid fits with $am_b \approx 1$

- I plead for more work (compare to g-2; that is only one number!)
  It is worth it, after all: QCD $\equiv \lim_{a\to 0}$ (Lattice QCD)