Computation of the HVP contribution to the muon anomaly to 4.6 per mil

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BMW-DMZ

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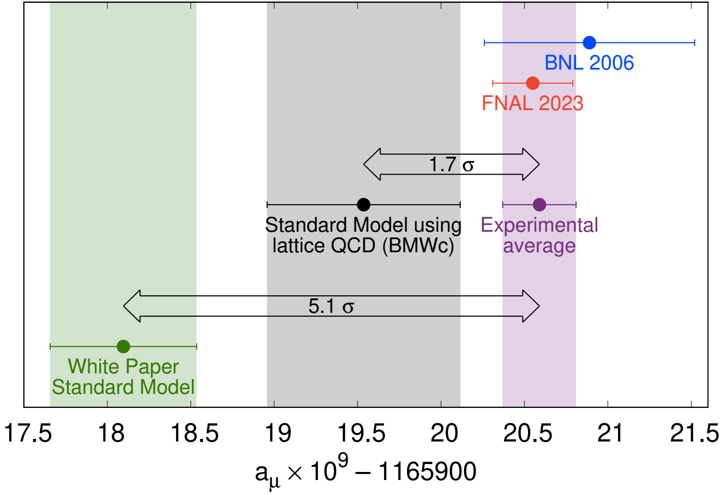
arXiv:2407.10913

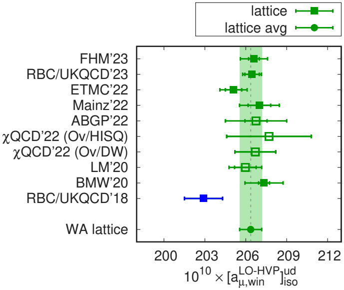
Thanks to Max Hansen, Luchang Jin, Agostino Patella

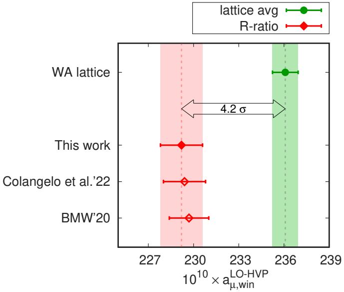
Situation on Aug. 10, 2023 (1)



New physics ?

Situation on Aug. 10, 2023 (2)





- Intermediate window $a_{\mu,\text{win}}^{\text{LO-HVP}}$: contribution to $a_\mu^{\text{LO-HVP}}$ from $t \in 0.4-1.0\,\text{fm}$ [RBC/UKQCD '18]

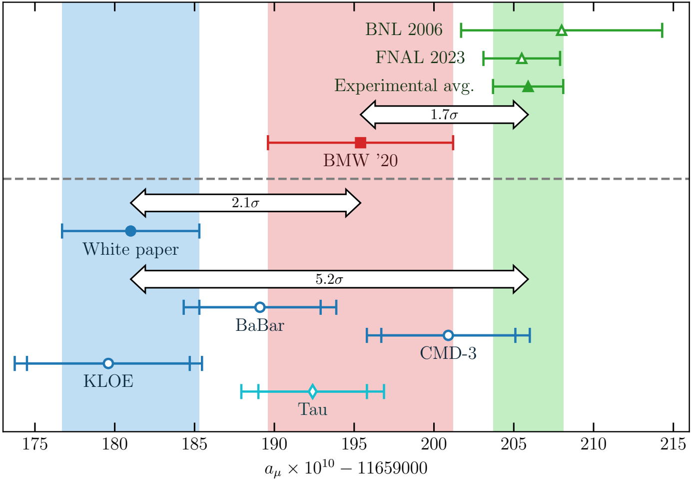
- 9 lattice calculations agree on $\left[a_{\mu,\text{win}}^{\text{LO-HVP}}\right]_{\text{iso}}^{\text{ud}}$

- Lattice $a_{\mu,\text{win}}^{\text{LO-HVP}}$ is 4.2σ larger than data-driven

  $\Rightarrow$ lattice and data-driven result of $a_\mu^{\text{LO-HVP}}$ cannot both be right!

(Plots from BMW-DMZ '23)

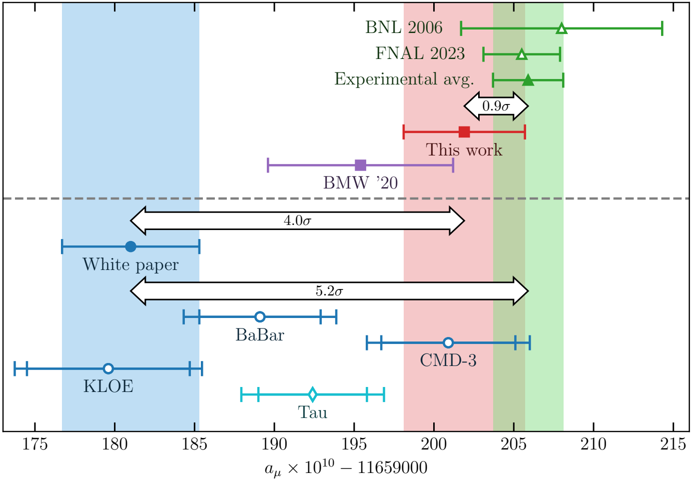
Situation on Monday night (for you!)



Discrepancies between between data-driven results probably due to issues w/ radiative corrections [DHLMZ '23]

New physics ??

Situation on Tuesday morning



Standard model confirmed to 0.37 ppm !!

(Fine print: result should be confirmed by others. . .)

Challenges

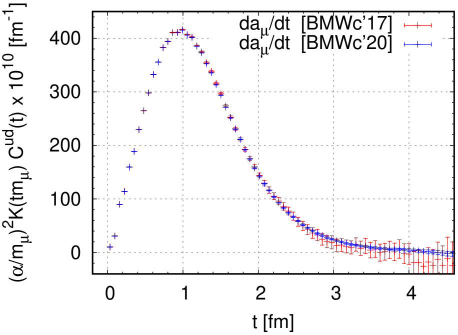
(a) Statistical uncertainties of light and disconnected contributions

(b) Finite T and V corrections on $I = 1$ contribution

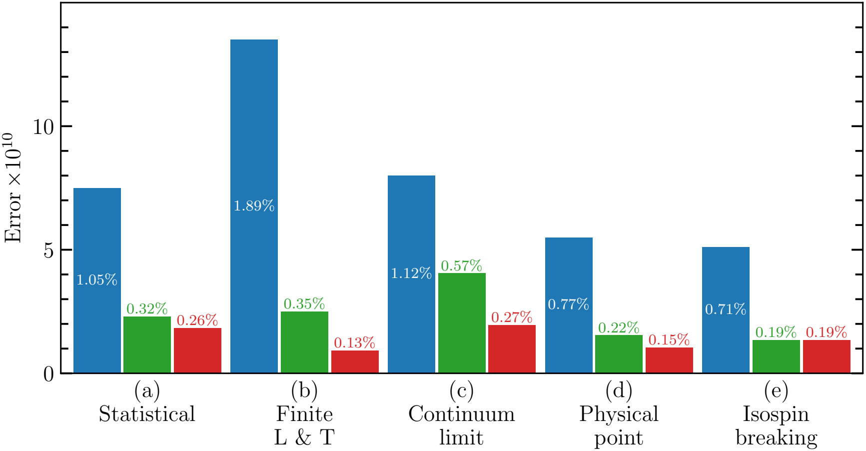
(c) Continuum limits

(e) Tuning of physical point $\rightarrow$ very precise determination QCD parameters: lattice scale and m_u, m_d, m_s, m_c masses

(f) QED + SIB corrections



Uncertainty reduction



$\Rightarrow$ uncertainty reduced by 40%: 5.5 $\rightarrow$ 3.3

Strategy for improvement

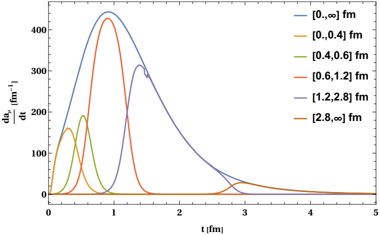
- New simulations on finer ("Monster") lattice spacing: $128^3 \times 192$ w/ $a = 0.048\,\mathrm{fm}$
- Completely revamped analysis vs BMW '20
- Break up analysis into optimized set of windows: $0-0.4$, $0.4-0.6$, $0.6-1.2$, $1.2-2.8\,\mathrm{fm}$
- Combined fit to $a_{\mu,\mathrm{win},04\text{-}06}^{\mathrm{LO\text{-}HVP}}$, $a_{\mu,\mathrm{win},06\text{-}12}^{\mathrm{LO\text{-}HVP}}$, $a_{\mu,\mathrm{win},12\text{-}28}^{\mathrm{LO\text{-}HVP}}$
- Continuum extrapolate $I = 0$ instead of disconnected

$\rightarrow$ reduces statistical uncertainty
$\rightarrow$ reduces $a \rightarrow 0$ error

- Data-driven evaluation of tail: $a_{\mu,28\text{-}\infty}^{\mathrm{LO\text{-}HVP}}$ (proposed and used w/ $1\,\mathrm{fm} \rightarrow \infty$ [RBC/UKQCD '18])

$\rightarrow$ reduces FV effect $18.5(2.5) \rightarrow 9.3(9)$, i.e. cv $\div 2$ & err $\div 3$
$\rightarrow$ reduces LD noise
$\rightarrow$ reduces LD taste breaking and $a \rightarrow 0$ error

- Analysis unblided last Friday



(plot made w/ KNT '18 data set)

Landscape (not swampland)

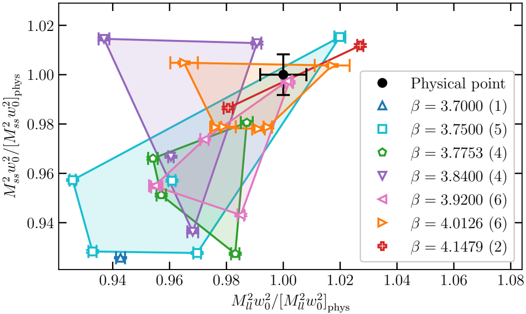
28, $N_f = 2+1+1$ smeared staggered large-scale simulations bracketing physical m_{ud}, m_s, m_c

- New lattice spacing $a = 0.048\,\mathrm{fm}$ (same cost as all of BMW'20)
  Reduced $a = 0.064\,\mathrm{fm}$ from $\rightarrow$ divides a^2 effect by 2
- 7 a's $a = 0.048 \rightarrow 0.132\,\mathrm{fm}$, $L = 6.1 \rightarrow 6.6\,\mathrm{fm}$, $T = 8.6 \rightarrow 11.3\,\mathrm{fm}$
- 4 dedicated $N_f = 2+1$, 4-HEX simulations w/ $a = 0.112\,\mathrm{fm}$ and $L = 6.3$ and $10.7\,\mathrm{fm}$ bracketing physical to compute FV corrections
- 4 dedicated, $N_f = 2+1$ $L \simeq 3.1 \rightarrow 6.3\,\mathrm{fm}$ simulations for sea-quark QED effects

- State-of-the-art techniques:
  - EigCG [Strathopoulos et al '08]
  - Low mode averaging [Neff et al '01, Giusti et al '04, ...]
  - All mode averaging [Blum et al '13]
  - Solver truncation [Bali et al '09]

$\rightarrow$ Over 30,000 gauge configurations

$\rightarrow$ 10's of millions of measurements



Fixing the physical isospin point

- Scale setting w/ w_0 and M_{Ω^-}

  $[w_0]_{\rm ph} = 0.17245(22)(46)[51]\,\mathrm{fm}$

- m_u, m_d, m_s fixed w/ [BMW'20, PDG'24]:

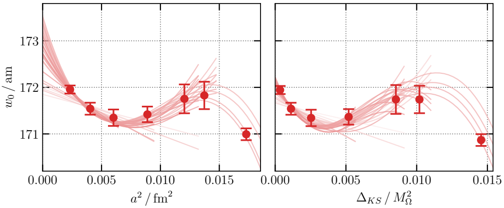
  $[\hat{M}]_{\rm ph} = 134.9768(5)\,\mathrm{MeV}$

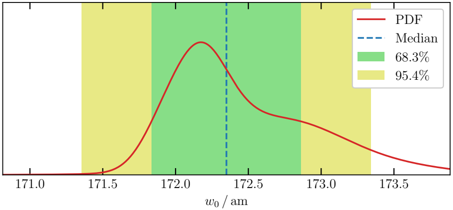
  $[\Delta M^2]_{\rm ph} = 13170(320)(270)[420]\,\mathrm{MeV}^2$

  $[M_{ss}]_{\rm ph} = 689.89(28)(40)[49]\,\mathrm{MeV}$

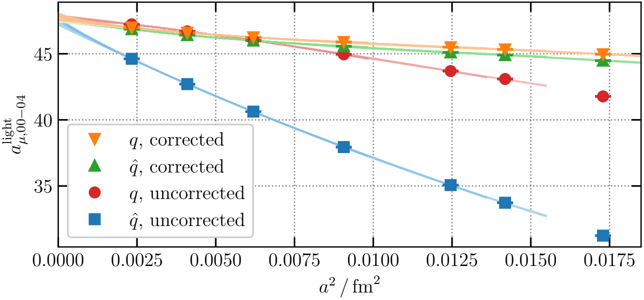
- $m_s/m_c = 11.85$ [HPQCD'10]

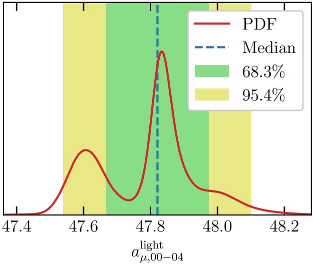
- $a \to 0$ ugly
  - Includes $a^2[\alpha_s(1/a)]^\Gamma$ from pion-mass splitting
  - Satisfies Rainer's criterion: $|Q(a_{\min}) - Q(0)| \leq 3 \times \sigma_Q^{\rm cont}$
  - Generous errors

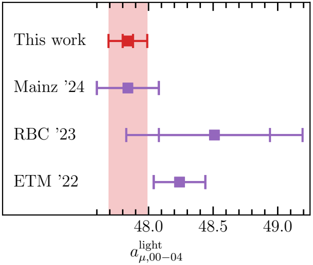




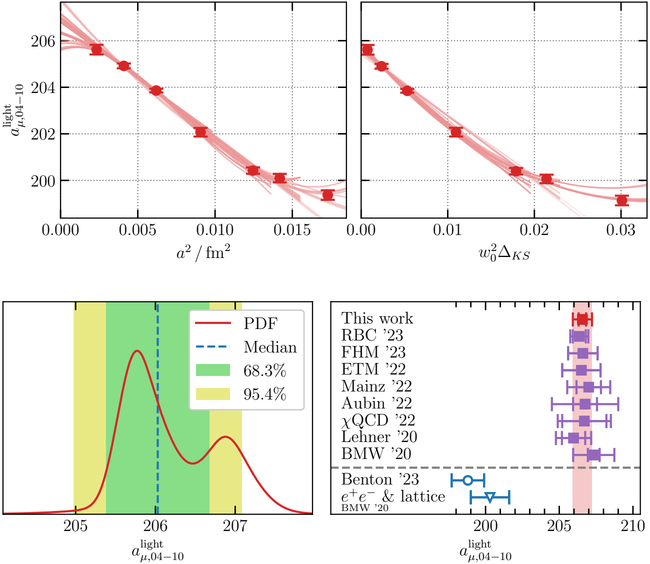
0−0.4 fm (SD) window



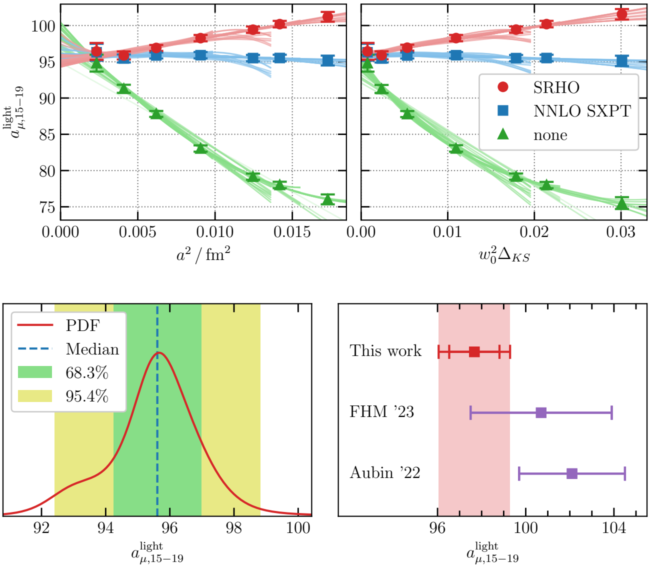




0.4−1.0 fm (ID) window

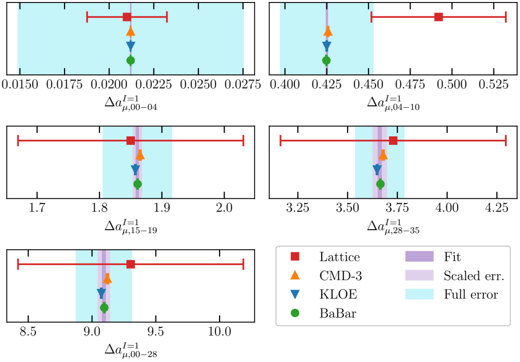


1.5−1.9 fm (Aubin et al '22) window

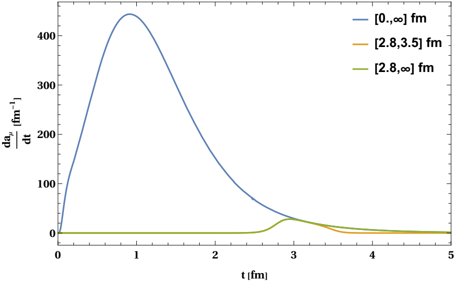


Finite-volume corrections

- Dedicated simulation in volumes up to $11\,\mathrm{fm}^4$
- Combination of Hansen-Patella ['19, '20] for $t < t_\star$ and Meyer-Lellouch-Lüscher ['11, '01, '91] $t \geq t_\star$ w/ $t_\star = (M_\pi L/4)^2/M_\pi = 1.682\,\mathrm{fm}$
- Checked independence on $t_\star \in\sim [1.2, 2.2]\,\mathrm{fm}$
- Use latest $e^+e^- \to$ hadrons data [DHLMZ '23]



Of window and tail



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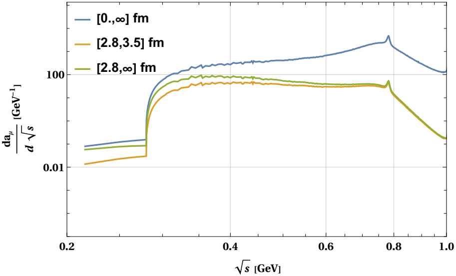
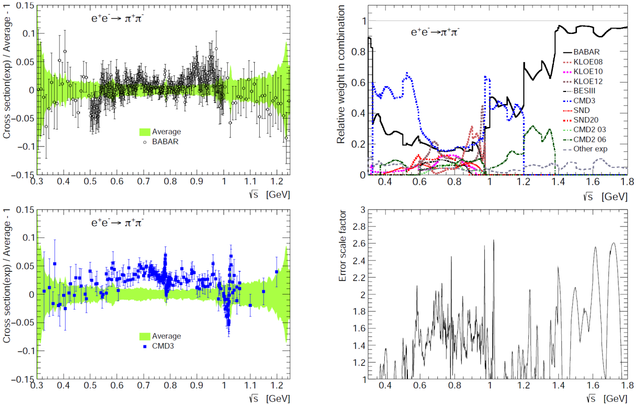


- Tail $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ contributes $\lesssim 5\%$ to final result for a_μ
- Tail dominated by cross section below ρ peak: $\sim 75\%$ for $\sqrt{s} \leq 0.63\,\mathrm{GeV}$
- Partial tail $a_{\mu,28\text{-}35}^{\text{LO-HVP}}$ for comparison with lattice dominated by cross section below ρ peak: $\sim 70\%$ for $\sqrt{s} \leq 0.63\,\mathrm{GeV}$
- For small $\sqrt{s}$ radiative-correction issues are less pronounced [DHLMZ '23]
- Region well controlled by theory (χPT, analyticity, unitarity, ...) and other experimental constraints (e.g. $\langle r_\pi^2 \rangle$)

(plots made w/ KNT '18 data set)

Cross section and the tail

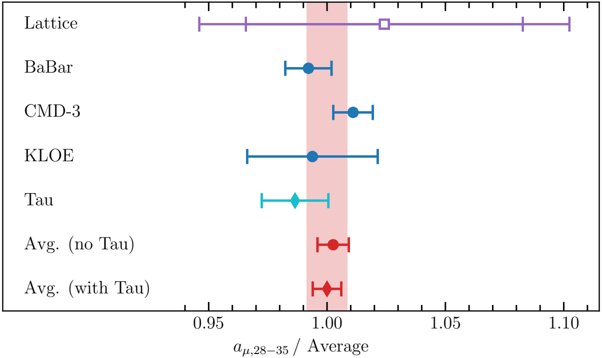
Tail $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ dominated cross section below ρ peak: $\sim 75\%$ for $\sqrt{s} \leq 0.63\,\text{GeV}$



All measurements agree to within 1.4σ for $\sqrt{s} \lesssim 0.55\,\text{GeV}$

$\Rightarrow$ tensions that plague $a_\mu^{\text{LO-HVP}}$ & $a_{\mu,\text{win}}^{\text{LO-HVP}}$ not present here

Data-driven partial-tail comparison with lattice

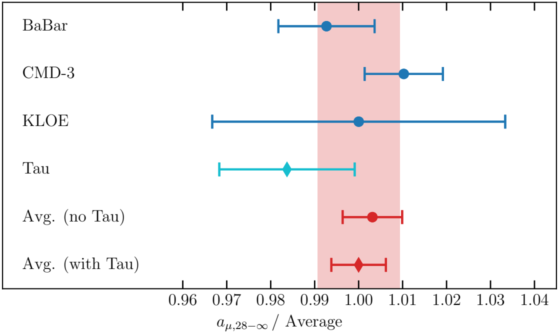


- All data-driven result agree very well
- Weighted average taken w/ and w/out τ: $\chi^2\text{dof} = 1.1$ for both
- Final number: average w/ τ, PDG factor, and systematic = full difference τ/no-τ added linearly

$$a_{\mu,28\text{-}35}^{\text{LO-HVP}} = 18.12(11)(5)[16]$$

- Excellent agreement w/ lattice, but uncertainty reduced by factor ~ 15

Data-driven tail



- All data-driven result agree very well
- Weighted average taken w/ and w/out τ: χ^2dof $= 1.0$ and 0.8
- Final number: average w/ τ, and systematic = full difference τ/no-τ added linearly

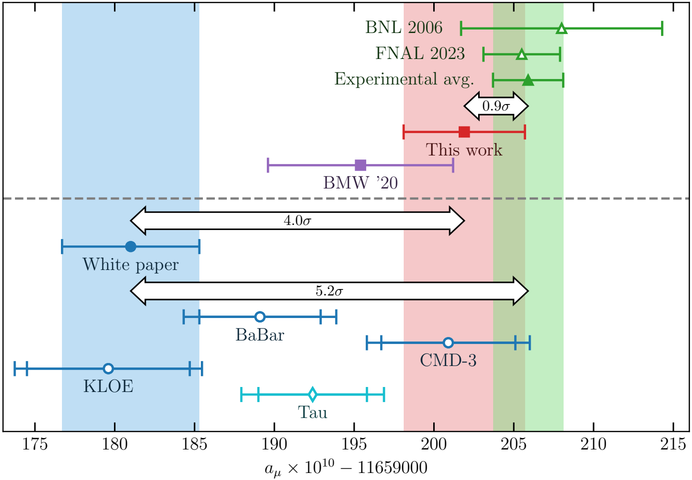
$$a_{\mu,28\text{-}\infty}^{\text{LO-HVP}} = 27.59(17)(9)[26]$$

- Only $\lesssim 5\%$ of final result for a_μ
- Contributes $\sim 65\%$ to total squared uncertainty uncertainty improvement: $5.5 \to 3.3$

All contribution to a_μ

light and disconnected $00 - 28$	618.6(1.9)(2.3)[3.0]	this work
strange $00 - 28$	53.19(13)(16)[21]	this work
charm $00 - 28$	14.64(24)(28)[37]	this work
light qed	$-1.57(42)(35)$	BMW'20 Table 15 corrected
light sib	6.60(63)(53)	BMW'20, Table 15
disconnected qed	$-0.58(14)(10)$	BMW'20, Table 15
disconnected sib	$-4.67(54)(69)$	BMW'20, Table 15
disconnected charm	0.0(1)	, BMW'20, Section 4 in Supp. Mat.
strange qed	$-0.0136(86)(76)$	BMW'20, Table 15
charm qed	0.0182(36)	ETM'19
bottom	0.271(37)	HPQCD'14
tail from data-driven $28 - \infty$	27.59(17)(9)[26]	this work
total	714.1(2.2)(2.5)[3.3]	

Situation on Tuesday morning



Standard model confirmed to 0.37 ppm !!

(Fine print: result should be confirmed by others. . .)

Conclusions

- New calculation of $a_\mu^{\text{LO-HVP}}$ to 0.47%
- Analysis blinded until last Friday
- Lattice calculation of $0-2.8\,\mathrm{fm}$ window $>$ 95% of total
- Data-driven evaluation of $2.8-\infty\,\mathrm{fm}$ window $\leq$ 5% of total
- Error reduction
  - $\sim$ 35% on error square from lattice
  - $\sim$ 65% on error square from data-driver
- SM confirmed to 0.37 ppm
- Need confirmation from other groups
- Eagerly await
  - Fermilab 0.1 ppm measurement of a_μ in 2025
  - J-PARC entirely new method for a_μ measurement in $\geq$ 2025
  - Entirely new BaBar $e^+e^- \rightarrow$ hadrons analysis in 2025
  - MuONe for spacelike HVP