Event Generator(s) for Specific Applications

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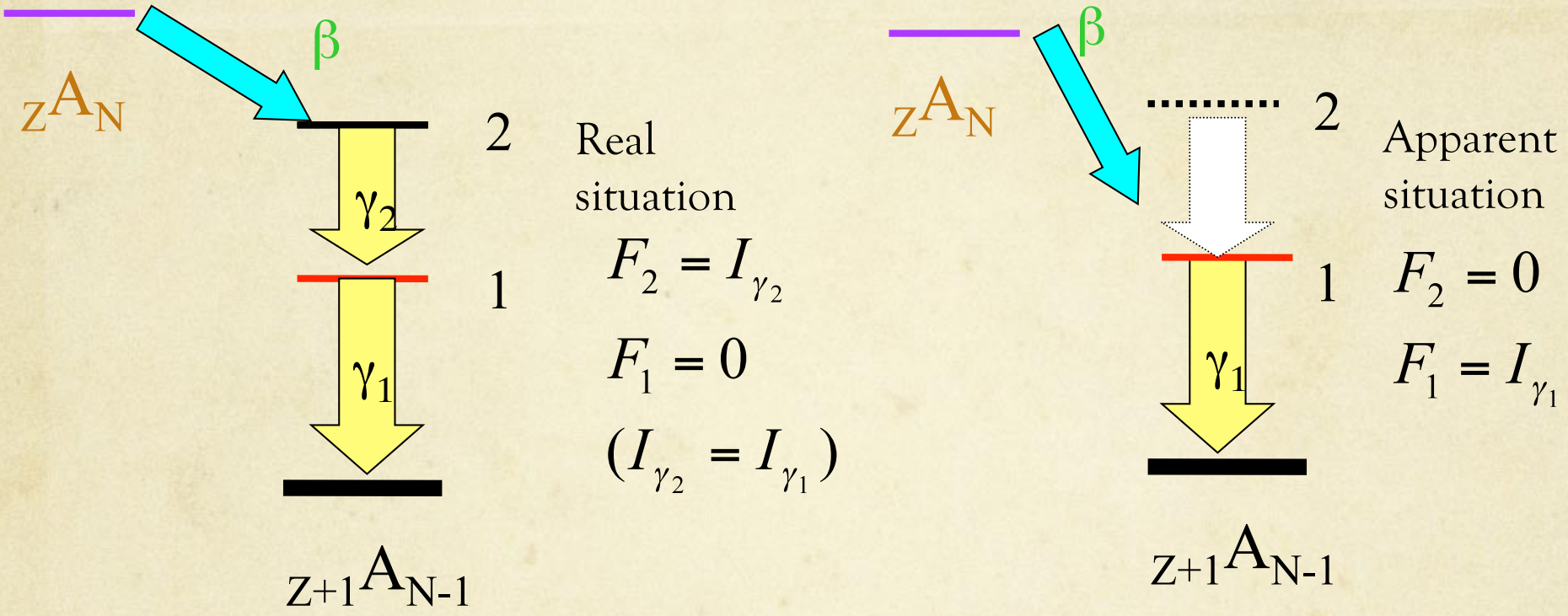
Why Geant4 ?

- Our group started to use G4 in 2000 related to the application of the total absorption technique. We were motivated by the development of the low-energy EM physics extension
- We developed our own beta-decay event generators in the early 2000. These early generators can be considered equivalent to the existing Radioactive Decay Module and were used for testing the response of our detectors to standard radioactive sources

Total Absorption Spectroscopy very briefly

The problem of measuring the beta feeding (no delayed particle emission case)



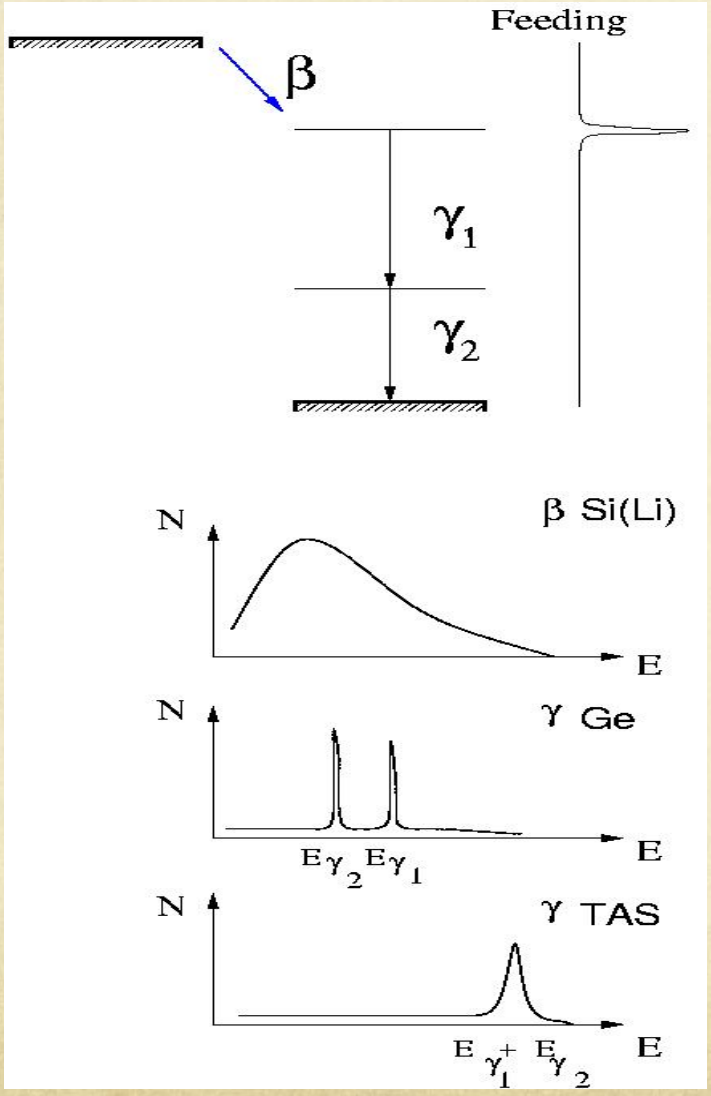
- We use Ge detectors to construct the level scheme populated in the decay
- From the gamma intensity balance we deduce the beta feeding
- What happens if we miss some gamma intensity ?

Pandemonium (The Capital of Hell)

introduced by John Milton (XVII) in his epic poem Paradise Lost

John Martin (~ 1825), Louvre

TAS measurements



Since the gamma detection is the only reasonable way to solve the problem, we need a highly efficient device:

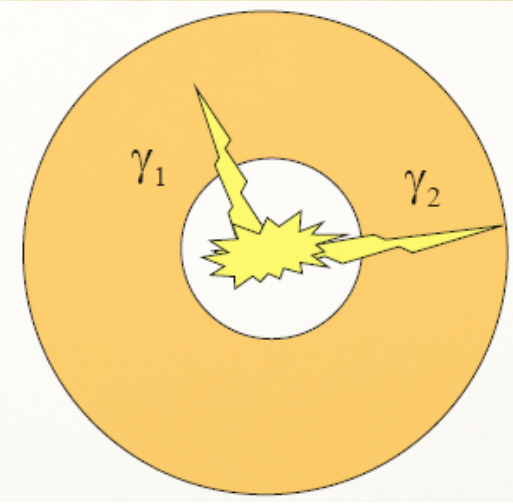
A TOTAL ABSORTION SPECTROMETER

But we need a change in philosophy. Instead of detecting the individual gamma rays we sum the energy deposited by the gamma cascades in the detector.

A TAS is like a calorimeter!

Big crystal, 4π

$$d_i = \sum_j R_{ij} f_j \quad or \quad \mathbf{d} = \mathbf{R} \cdot \mathbf{f}$$

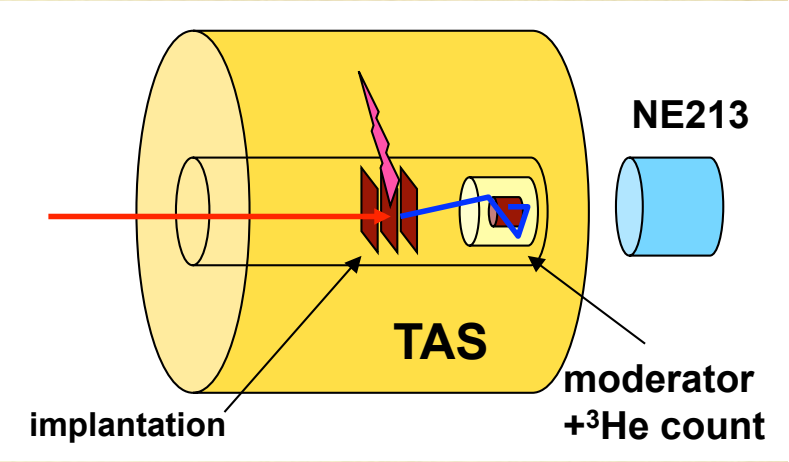


A Decay Event Generator for DESPEC (FAIR)

Objective:

- **Develop a Monte Carlo event generator for the design, optimization and performance characterization of DESPEC experimental set-ups**

One example:

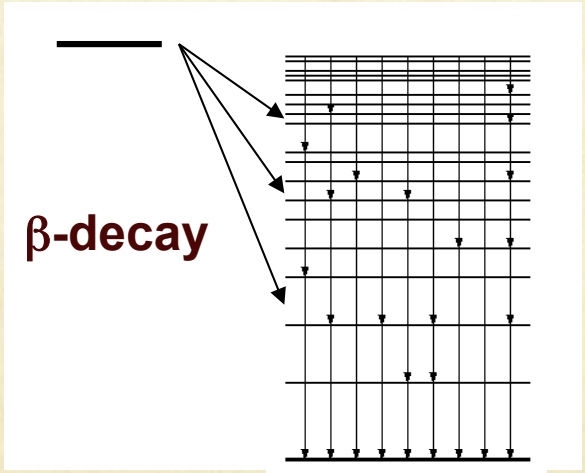


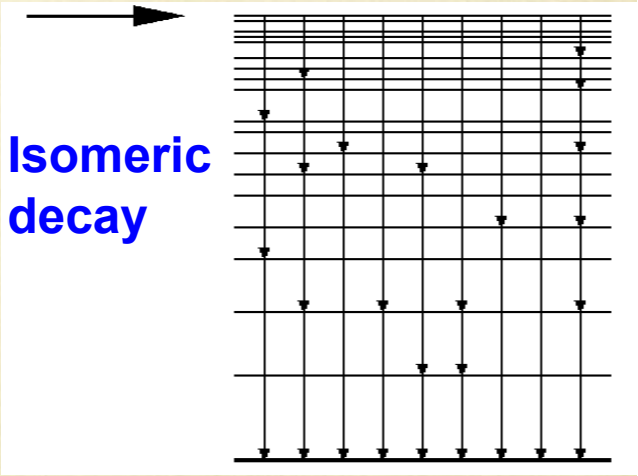
Primary processes:

- β^- **-decay**
- β^+**/EC –decay**
- **Isomeric decay**

Also:

- β**n -decay**



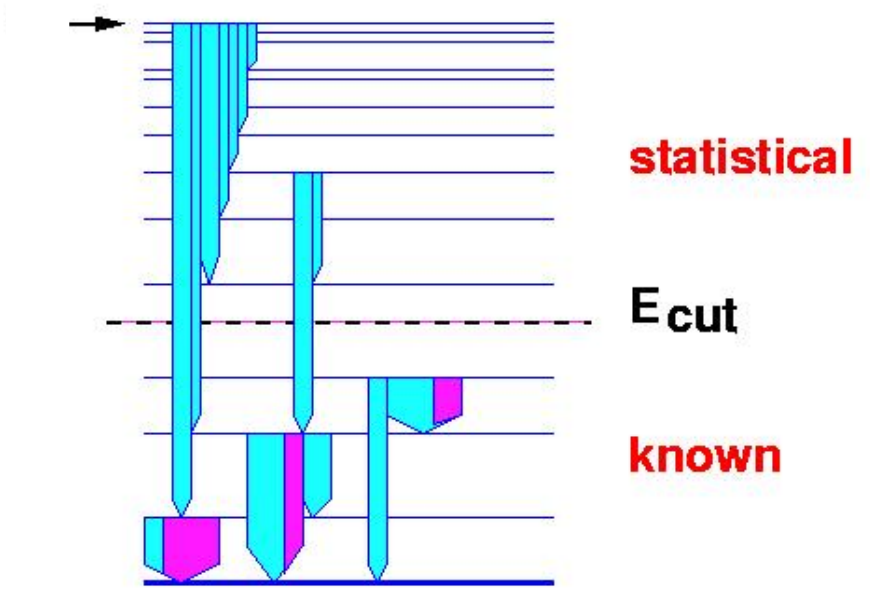


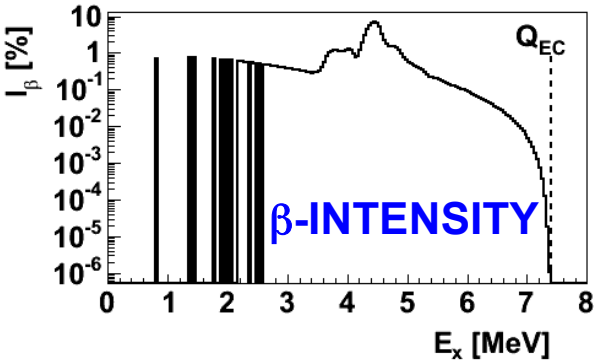
Itt goes beyond what is available in ENSDF !

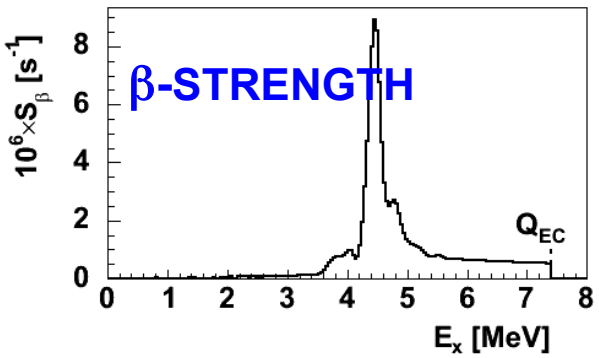
Basis for generator:

- **Statistical model for population and decay of the nucleus**
- **Known level scheme**

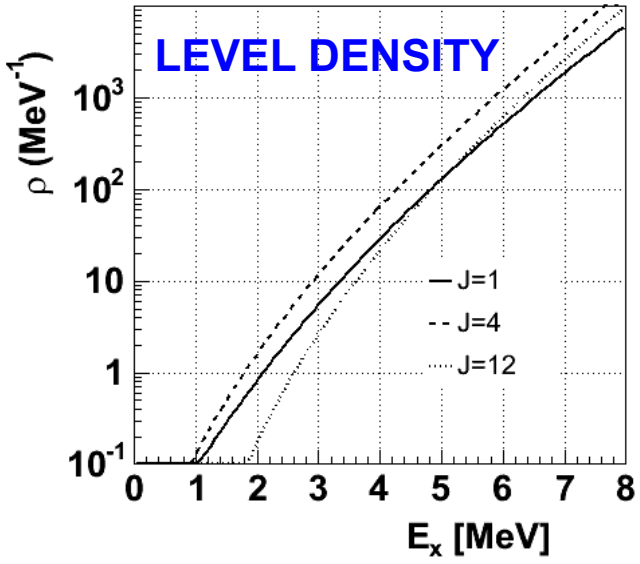
Electromagnetic cascades

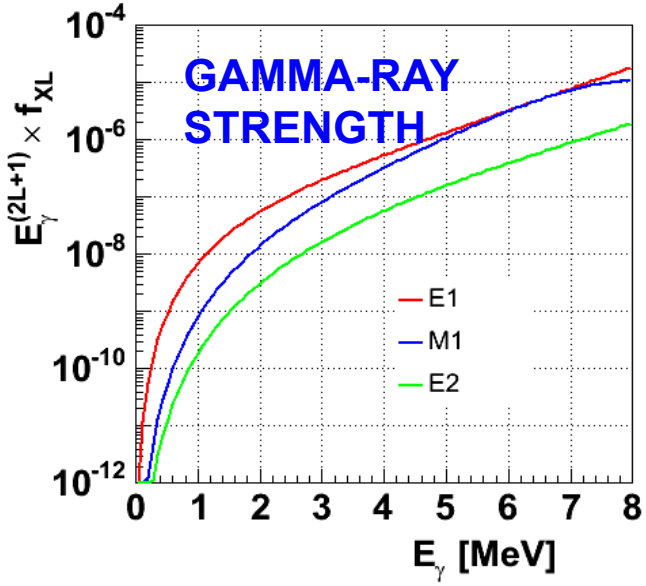




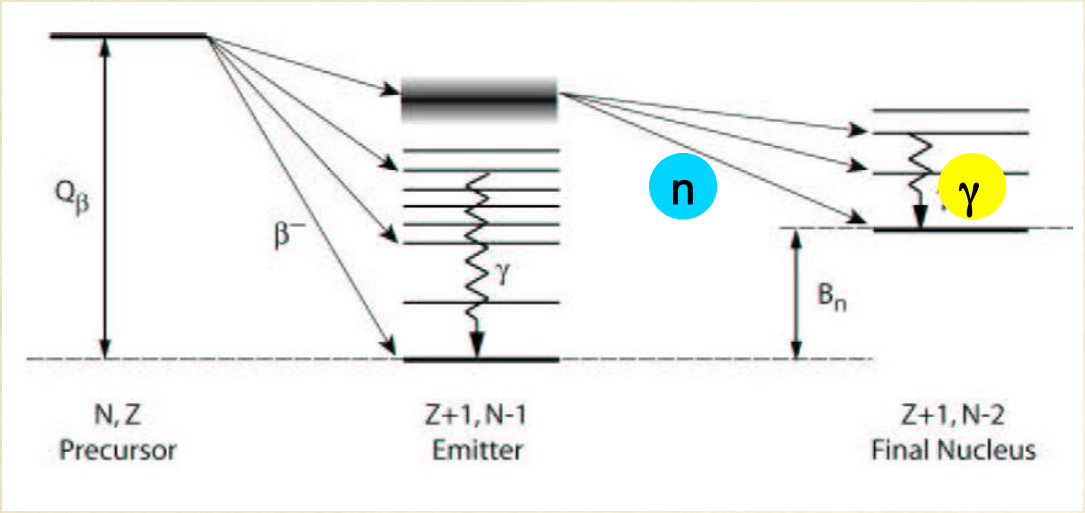


$$S_\beta(E_x) = \frac{I_\beta(E_x)}{f(Q_\beta - E_x)T_{1/2}}$$

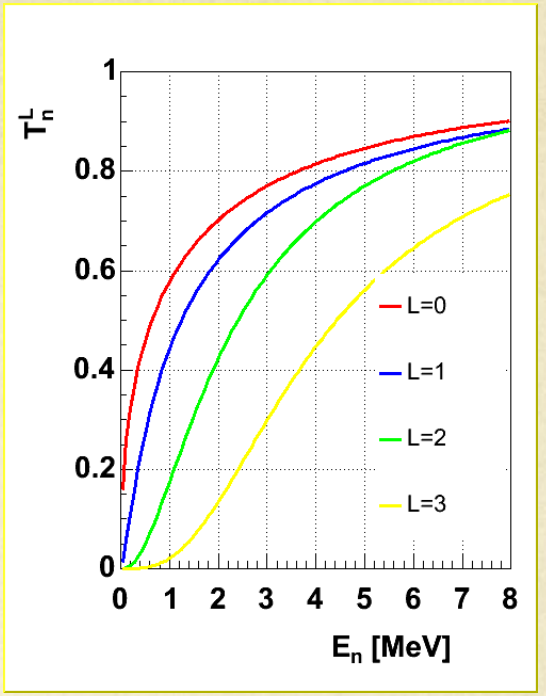




Secondary processes:
• β-delayed neutron emission



Neutron transmission calculated from square well potential



It will include **also**:
• Generator of the decaying nuclei **implantation distribution**
• Generator of **background**:
– implantation **bremsstrahlung**
– **beam-line** related background
– **ambient** background

Parameters (simplified):

- Parent nucleus/level variables (A,Z,$T_{1/2}$, J^{π})
- Type of decay (β^-, β^+/EC, ID)
- Q-value, beta-intensity distribution, neutron separation energy
- Known level scheme (Elev, J^{π}, $E\gamma$, $I\gamma$, α)
- Fermi functions: f_0^- , $f_0^{\varepsilon} + f_0^+$, $f_0^{\varepsilon} / f_0^+$ (internal)
- Unknown level scheme: level density and gamma-strength parameters (internal)
- Neutron penetrability (internal)

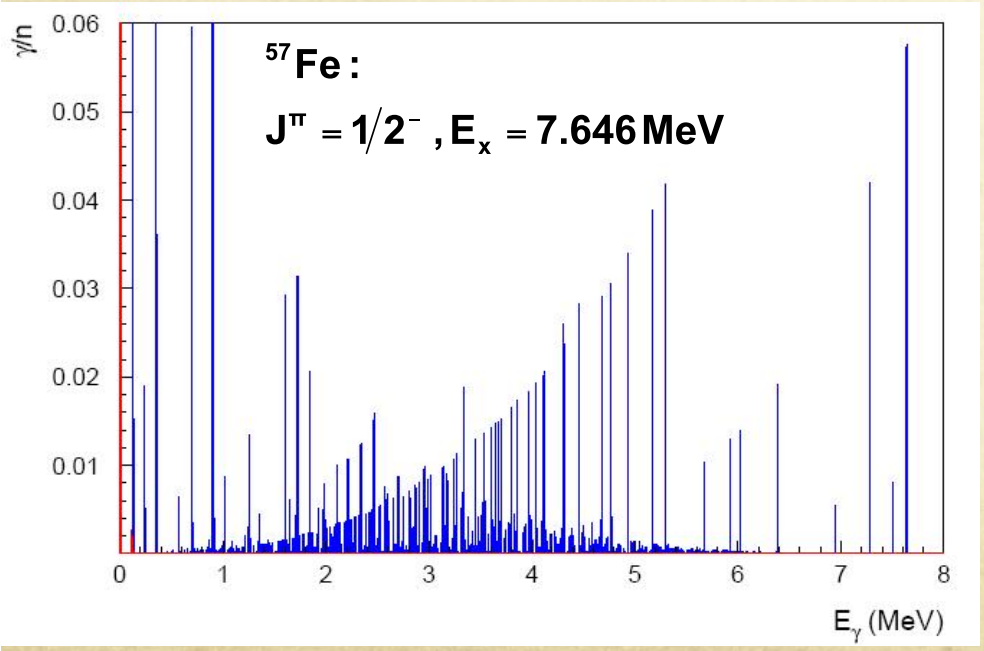
Approximations/simplifications:

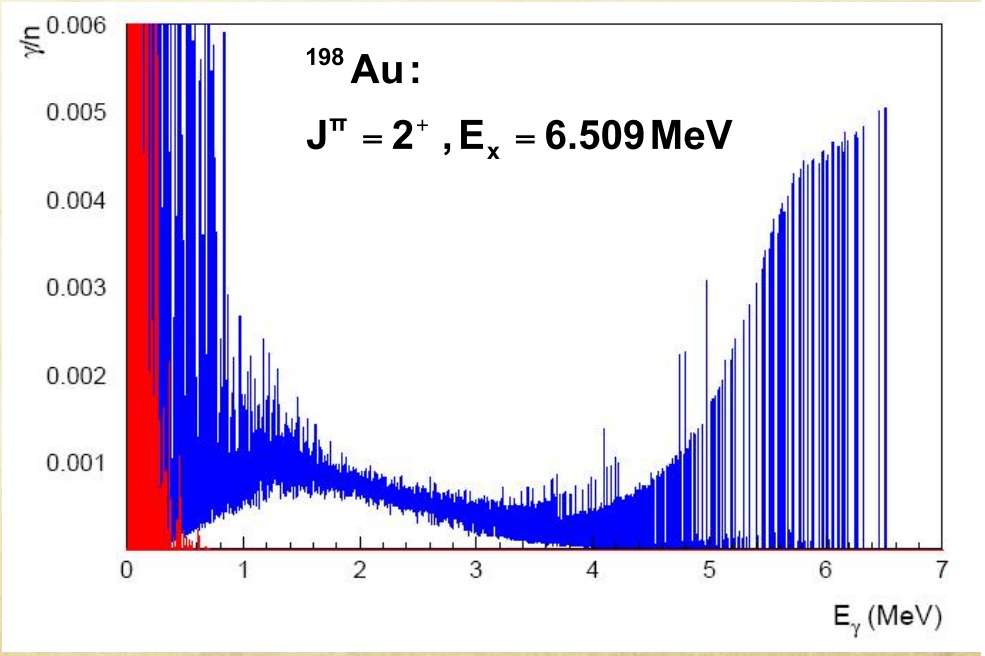
- β-spectral shape: allowed
- electrons (EC or CE) only from K-shell
- no γ/n competition (n emitted)
- fluctuations in the statistical model are omitted

Integrated within Geant4 Simulation Toolkit framework

- DecayGeneratorAction.cc
- DecayGeneratorInput.dat
- KnownLevelScheme.dat
- BetaIntensity.dat
- DelayedNeutronLevelScheme.dat

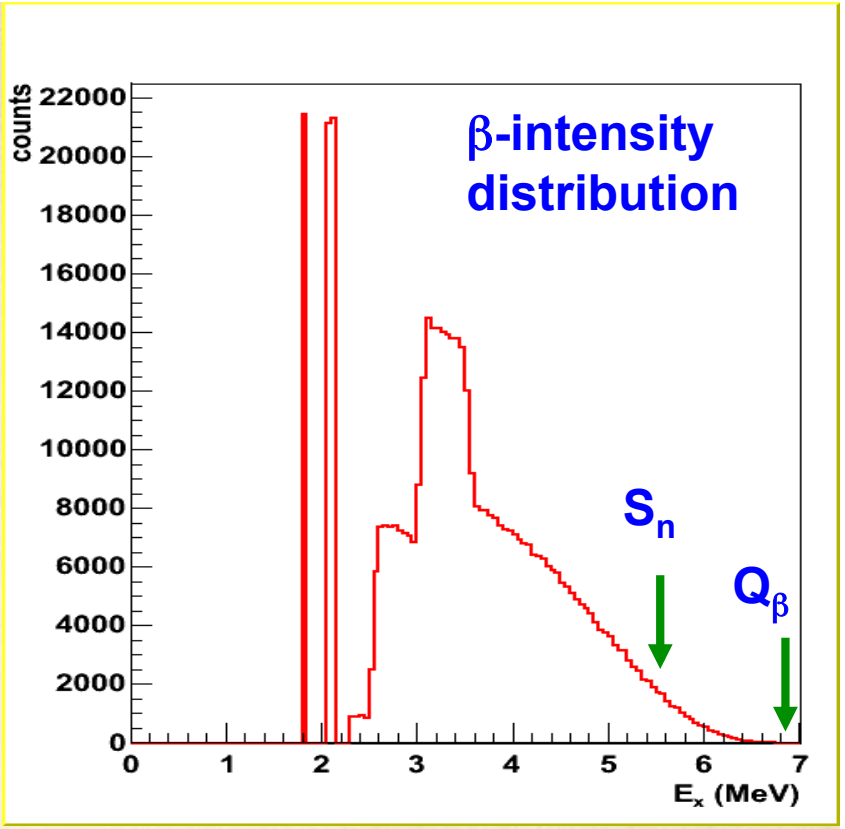
Example: gamma-ray and conversion electron spectra after neutron capture (another possible appl.)

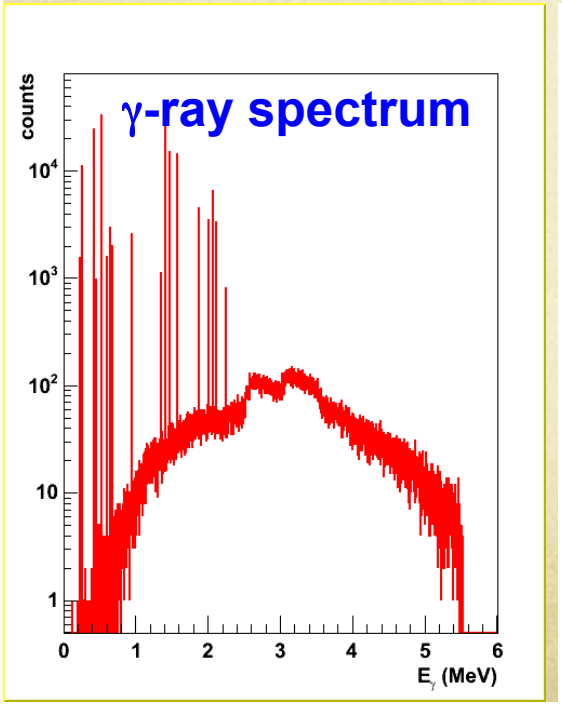


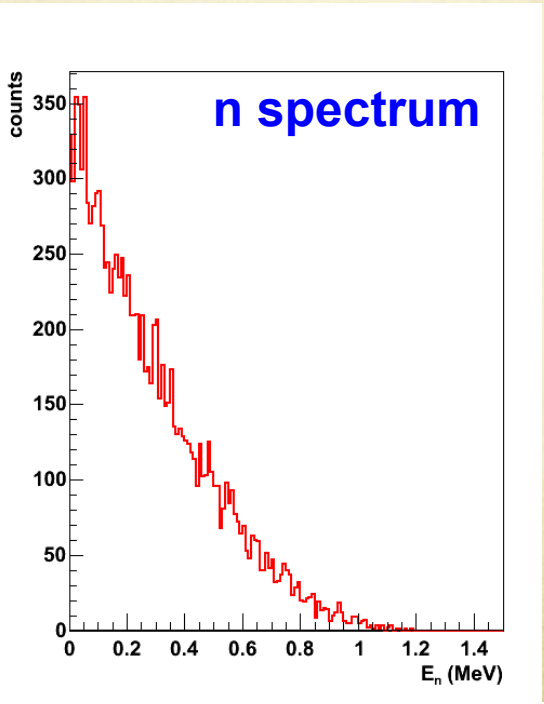


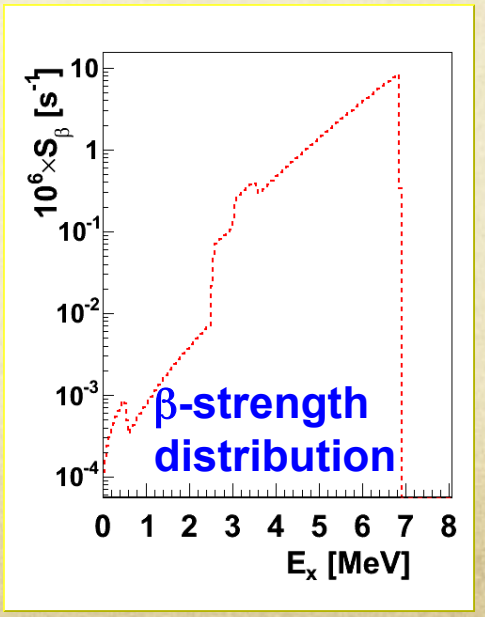
Test cases: ^{87}Br β-decay

$T_{1/2}$= 55.65 s ; J^{π}= 3/2$^-$; Q_{β} = 6.852 MeV
^{87}Kr S_n= 5.515 MeV ; P_n= 2.7 %
^{87}Kr known levels: 14 (E_x < 2.3 MeV)
(total: 6×10^3)
^{86}Kr levels: only g.s.



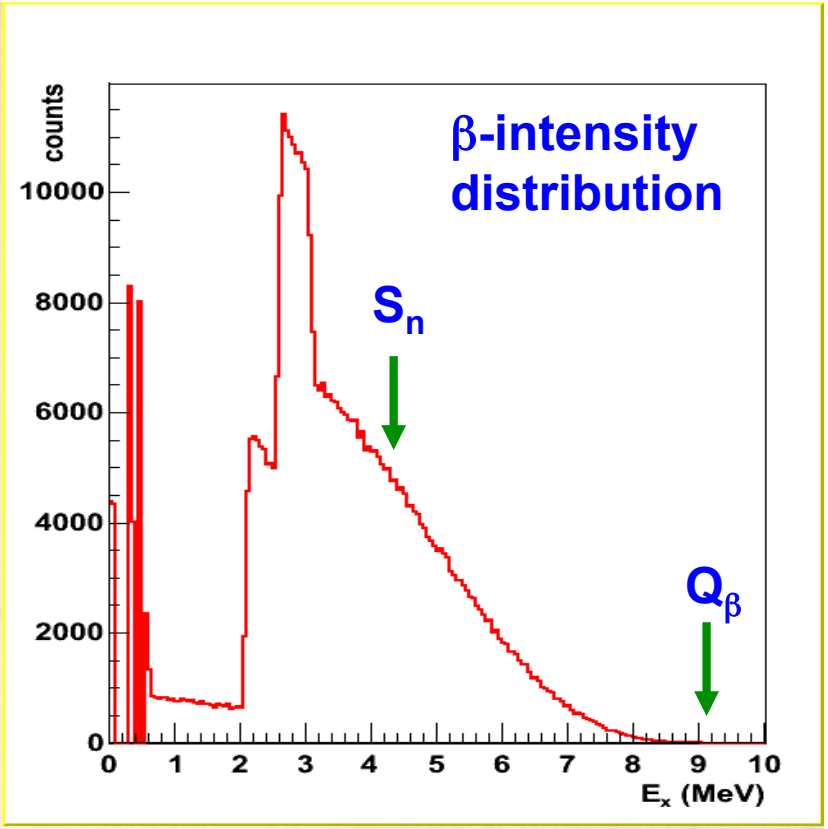


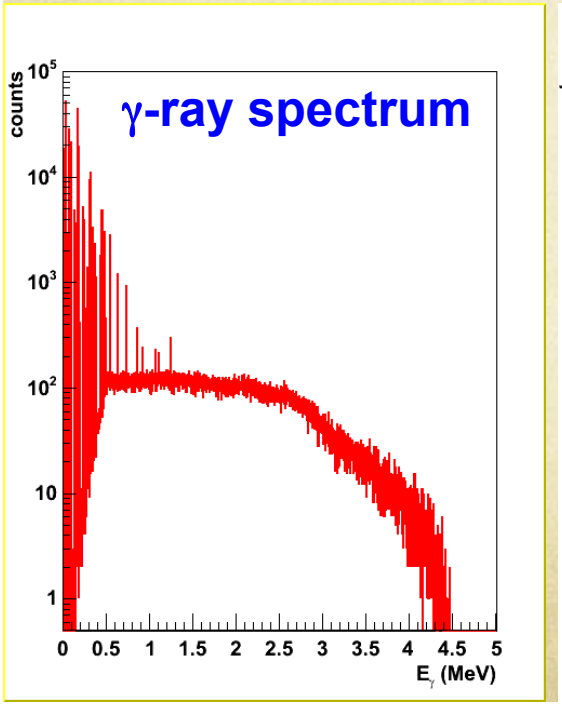


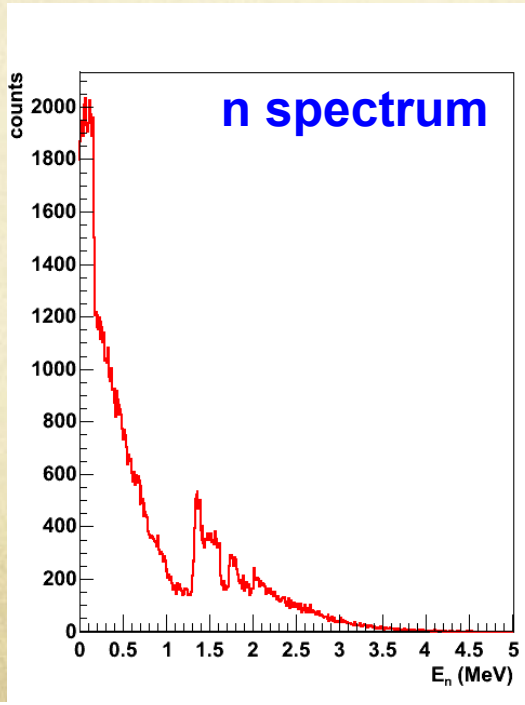


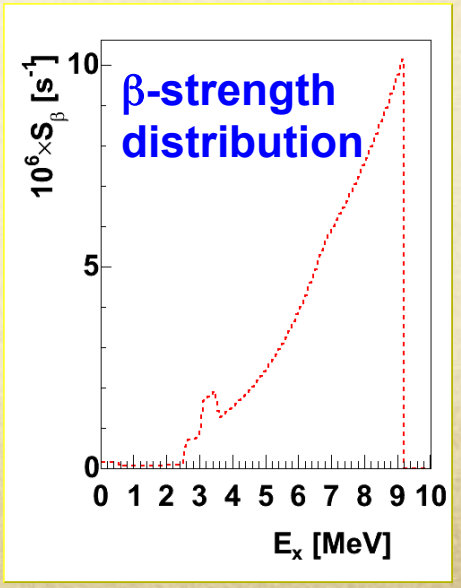
Test cases: ^{147}Cs β-decay

$T_{1/2}$= 0.235 s ; J^{π}= 3/2$^+$; Q_{β} = 9.2 MeV
^{147}Ba S_n= 4.45 MeV ; P_n= 27.5 %
^{147}Ba known levels: 20 (E_x < 0.5 MeV)
(total: 6×10^7)
^{146}Ba levels: up to 4.071 MeV









Statistical model for population and decay of the nucleus

- average quantities to describe level spacing and transition strengths

+

- probability distribution laws to describe its fluctuations

1) Nuclear level density:

Semi-empirical parameterizations: Back Shifted Fermi Gas, Constant Temperature, Gilbert-Cameron

BSFG formula:

(a, Δ)

$$\rho(J,\pi,E_x) = \rho(J,\pi)\rho(E_x)$$

$$\rho(J,\pi) = \frac{2J+1}{4\sigma^2}\exp\left(-\frac{J(J+1)}{2\sigma^2}\right)$$

$$\rho(E_x) = \frac{1}{12\sqrt{2}}\frac{\exp\left(2\sqrt{a(E_x-\Delta)}\right)}{\sigma a^{0.25}(E_x-\Delta-t)^{1.25}}$$

$$\sigma^2 = 0.015A^{1.67}t \quad , \quad t = \frac{1+\sqrt{1+4a(E_x-\Delta)}}{2a}$$

• Average inter-level spacing: $\langle d \rangle \approx \rho^{-1}$

• Actual inter-level spacing for states of **same J^{π}** follow a Wigner distribution, or Poisson distribution or something intermediate

Wigner distribution: $$P_W(x) = \frac{1}{2}\pi \exp\left(-\frac{1}{4}\pi x^2\right) \quad , \quad x = \frac{d}{\langle d \rangle}$$

$J_i^{\pi_i} \rightarrow J_f^{\pi_f}$

2) Average radiation width: for XL transitions from individual levels to levels in an energy interval:

$$\langle \Gamma_\gamma \rangle = \frac{1}{\rho(J_i, \pi_i, E_x)} \sum_{XL} \sum_{J_f^{\pi_f}} \int_{E}^{E+\Delta E} E_\gamma^{2L+1} f_{XL}(E_\gamma) \rho(J_f, \pi_f, E_x - E_\gamma) dE_\gamma$$

X : E or M character, L : multipolarity

Strength functions f_{XL} from Single Particle estimates or Giant Resonances (Axel-Brink hypothesis):

GR functions:
$(E_0, \Gamma_0, \sigma_0)$

$$f_{E1}\left(E_\gamma\right) = 8.68\times10^{-8}\sigma_0\Gamma_0\left[\frac{E_\gamma\Gamma\left(E_\gamma\right)}{\left(E_\gamma^2 - E_0^2\right)^2 - E_\gamma^2\Gamma^2\left(E_\gamma\right)} + \frac{2.8\pi^2T^2\Gamma_0}{E_0^5}\right]$$

$$\Gamma\left(E_\gamma\right) = \Gamma_0\frac{E_\gamma^2 + 4\pi^2T^2}{E_0^2} \quad , \quad T = \sqrt{\frac{B_n - E_\gamma}{a}}$$

$$f_{M1}\left(E_\gamma\right) = 8.68\times10^{-8}\frac{\sigma_0E_\gamma\Gamma_0^2}{\left(E_\gamma^2 - E_0^2\right)^2 - E_\gamma^2\Gamma_0^2}$$

$$f_{E2}\left(E_\gamma\right) = 5.22\times10^{-8}\frac{\sigma_0E_\gamma^{-1}\Gamma_0^2}{\left(E_\gamma^2 - E_0^2\right)^2 - E_\gamma^2\Gamma_0^2}$$

- average branching ratios: $b_{if} \propto \left\langle\Gamma_\gamma\right\rangle_{if}$

- actual branching ratios follow a Porter-Thomas distribution:

$$P_{PT}(y) = \frac{1}{\sqrt{2\pi y}}\exp\left(-\frac{1}{2}y\right) \quad , \quad y = \frac{\Gamma_\gamma}{\left\langle\Gamma_\gamma\right\rangle}$$

3) Beta strength: for decay into permitted J^{π} levels

$$S_{\beta}(E_x) = \frac{1}{D}\sum_{J^{\pi}} \left\langle B^{J^{\pi}} \right\rangle \rho(J,\pi,E_x)$$

$$\left\langle B^{J^{\pi}} \right\rangle = \frac{2J+1}{\sum_{J}(2J+1)} \langle B \rangle$$

- average beta-decay intensity: $I_{\beta}(E_x) = f(Q_{\beta} - E_x) T_{1/2} S_{\beta}(E_x)$

- actual beta-decay intensity follow the Porter-Tomas distribution

4) Neutron width: for emission of L-orbital neutron $J_i^{\pi_i} \xrightarrow{L} J_f^{\pi_f}$

$$\left\langle \Gamma_n^L \right\rangle = \frac{1}{2\pi\rho(J_i,\pi_i,E_x)} T_n^L(E_n)$$

$T_n^L(E_n)$: neutron transmission from optical model or square well potential

$$T_n^L(E_n) = \frac{4xXv_L(x)}{X^2 + \left(2xX + x^2 v_L'(x)\right)v_L(x)}$$

$$x = \frac{1}{\hbar} R\sqrt{2\mu_n E_n} \quad ; \quad X = \frac{1}{\hbar} R\sqrt{2\mu_n (E_n + V_0)}$$

Square-well transmission

L	$v_L(x)$	$v_L'(x)$
0	1	1
1	$\frac{x^2}{1+x^2}$	$\frac{1}{x^2} + \left(1 - \frac{1}{x^2}\right)^2$
2	$\frac{x^2}{9+3x^2+x^4}$	$\left(1 - \frac{6}{x^2}\right)^2 + \left(\frac{6}{x^3} - \frac{3}{x^2}\right)^2$
...		

- actual neutron intensity follow the Porter-Tomas distribution

In summary

We developed an complex beta decay/isomer event generator that goes beyond what is available from databases (ENSDF). Certain applications may require it (main reason: data in main databases may suffer from the Pandemonium effect)

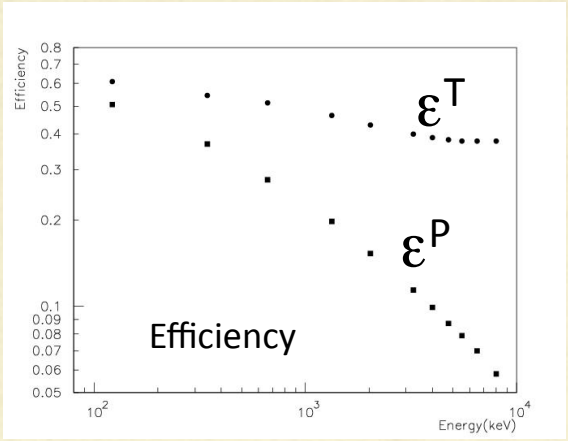
It uses real decay information at low excitation energies, and applies the Statistical Model for the high energy part of the populated level scheme.

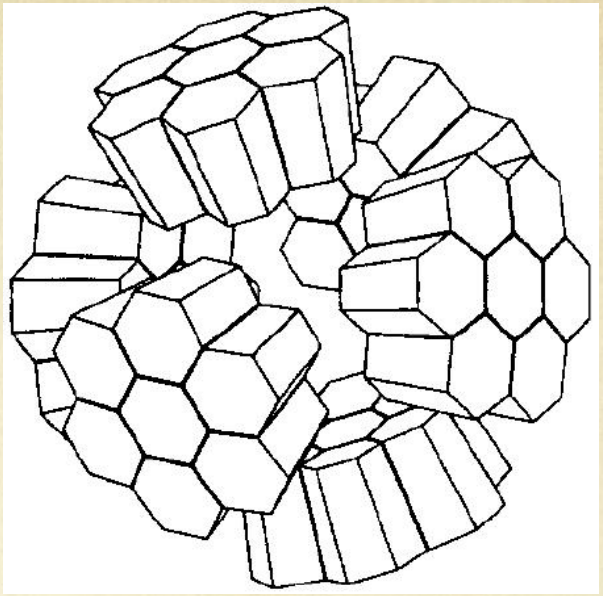
THANK YOU

The Pandemonium effect in ^{150}Ho decay:

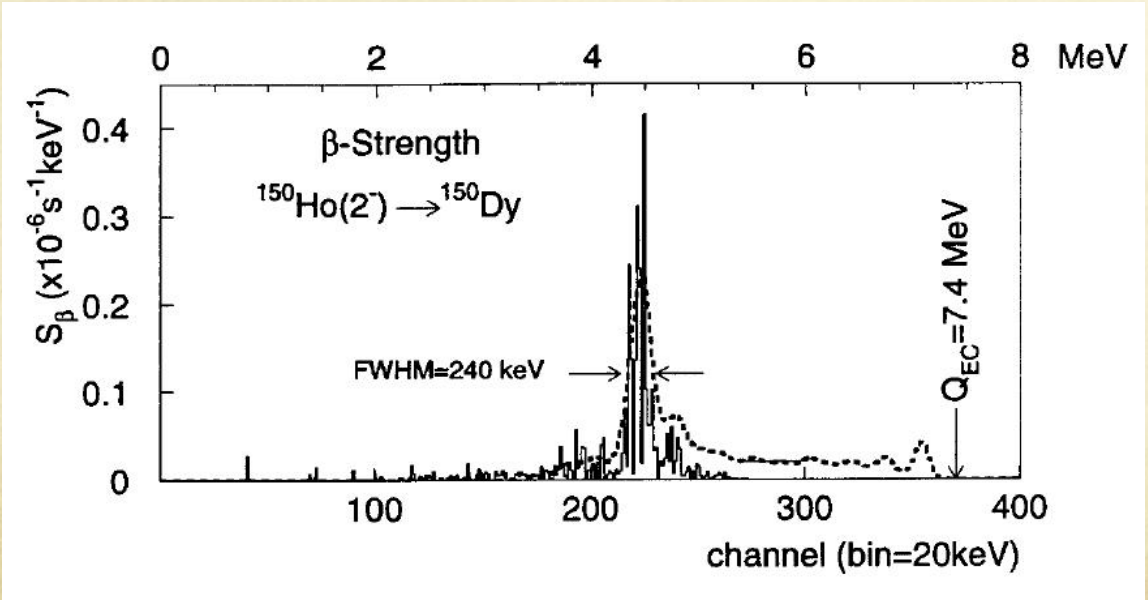
CLUSTER-CUBE: 6 EUROBALL Clusters in cubic geometry

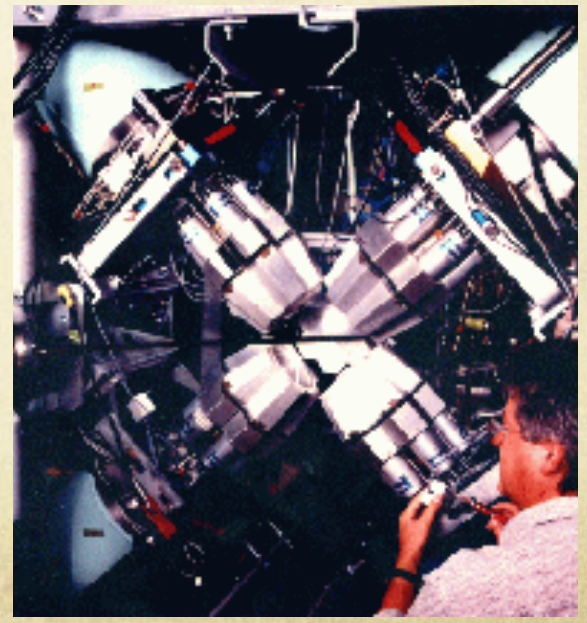
CLUSTER: 7 Ge detectors, 60% each





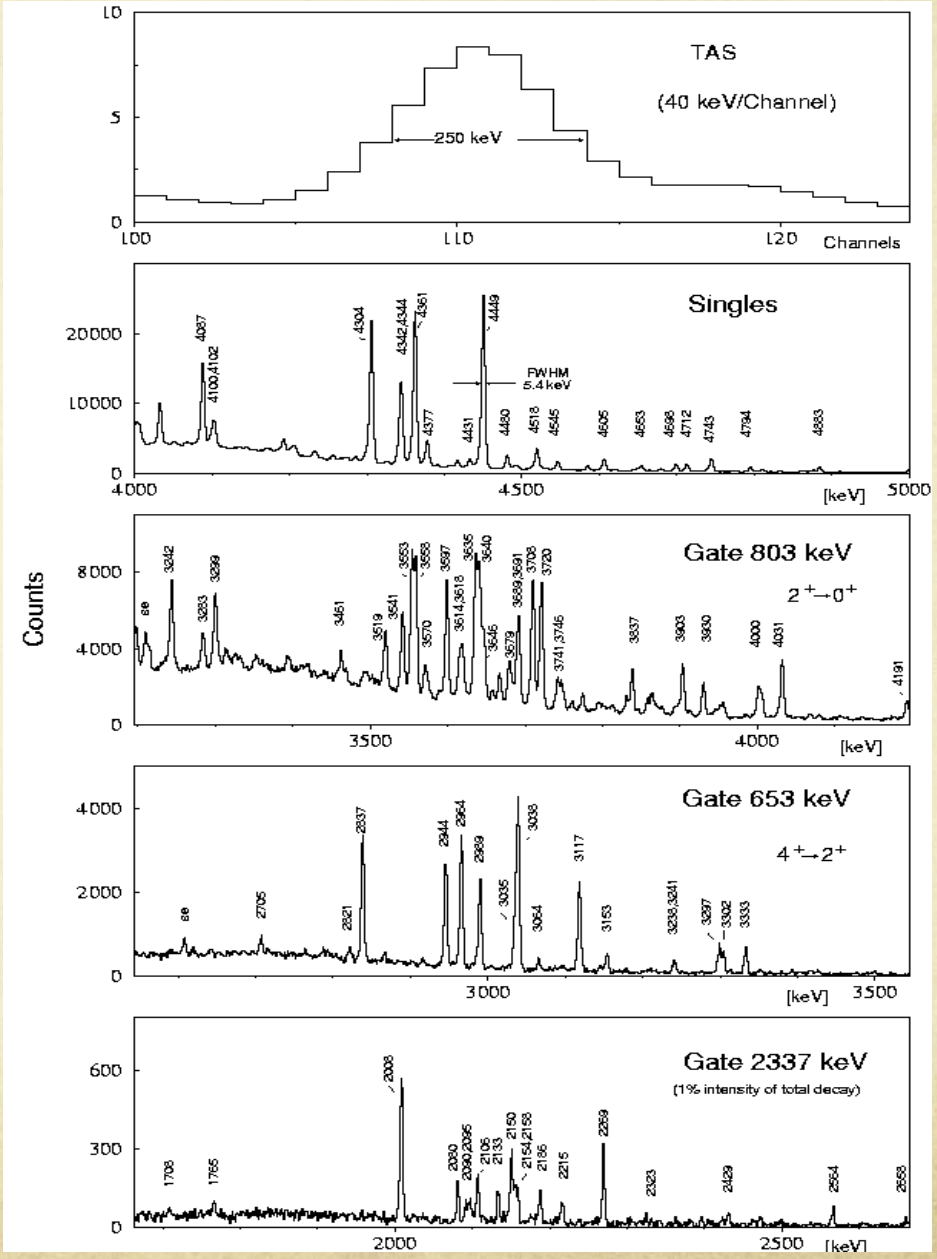
CLUSTER-CUBE at GSI





The decay of ^{150}Ho 2^- isomer

High resolution results

- No. total of γ: ~ 1064
- No. total of levels:~295
- Sharp resonance ~ 4.4 MeV
- B(GT) is approx. 47 % of the TAS result.

Algora et al. PRC 68 (2003) 034301