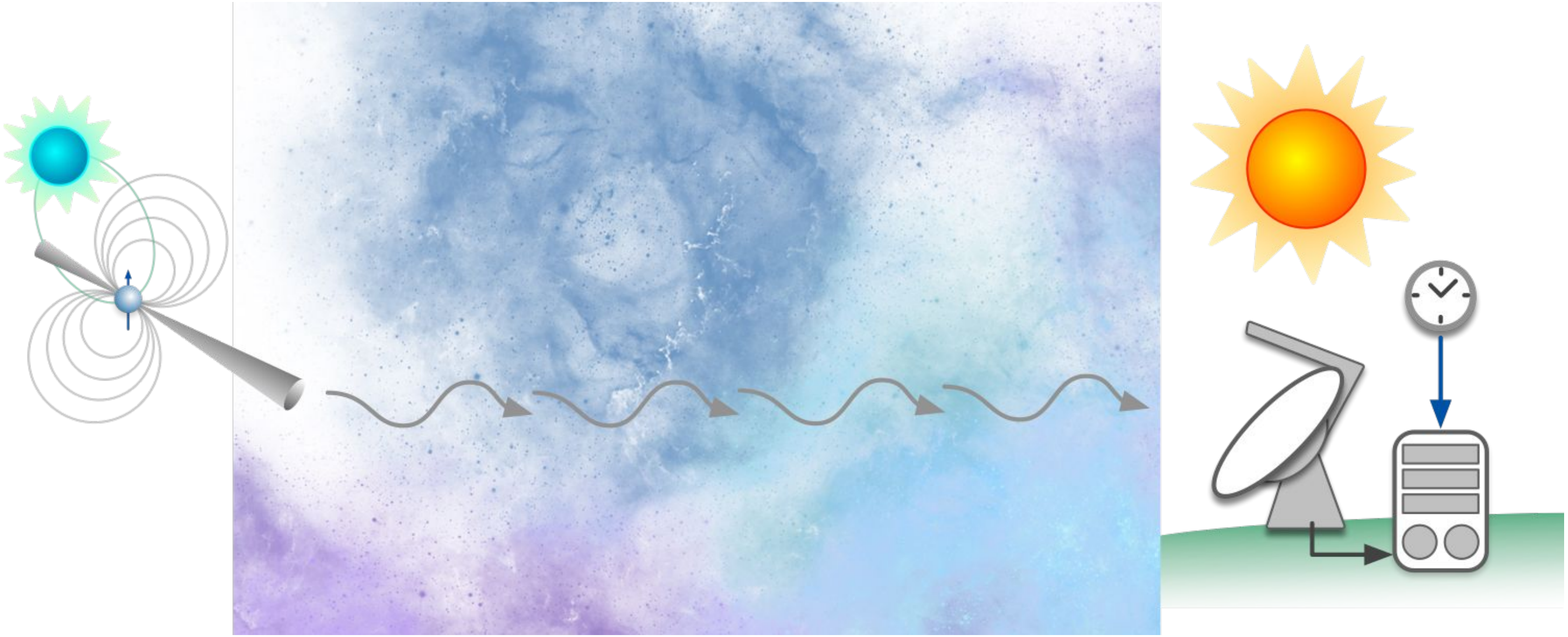
A tour of pulsar timing and PTA noise modeling



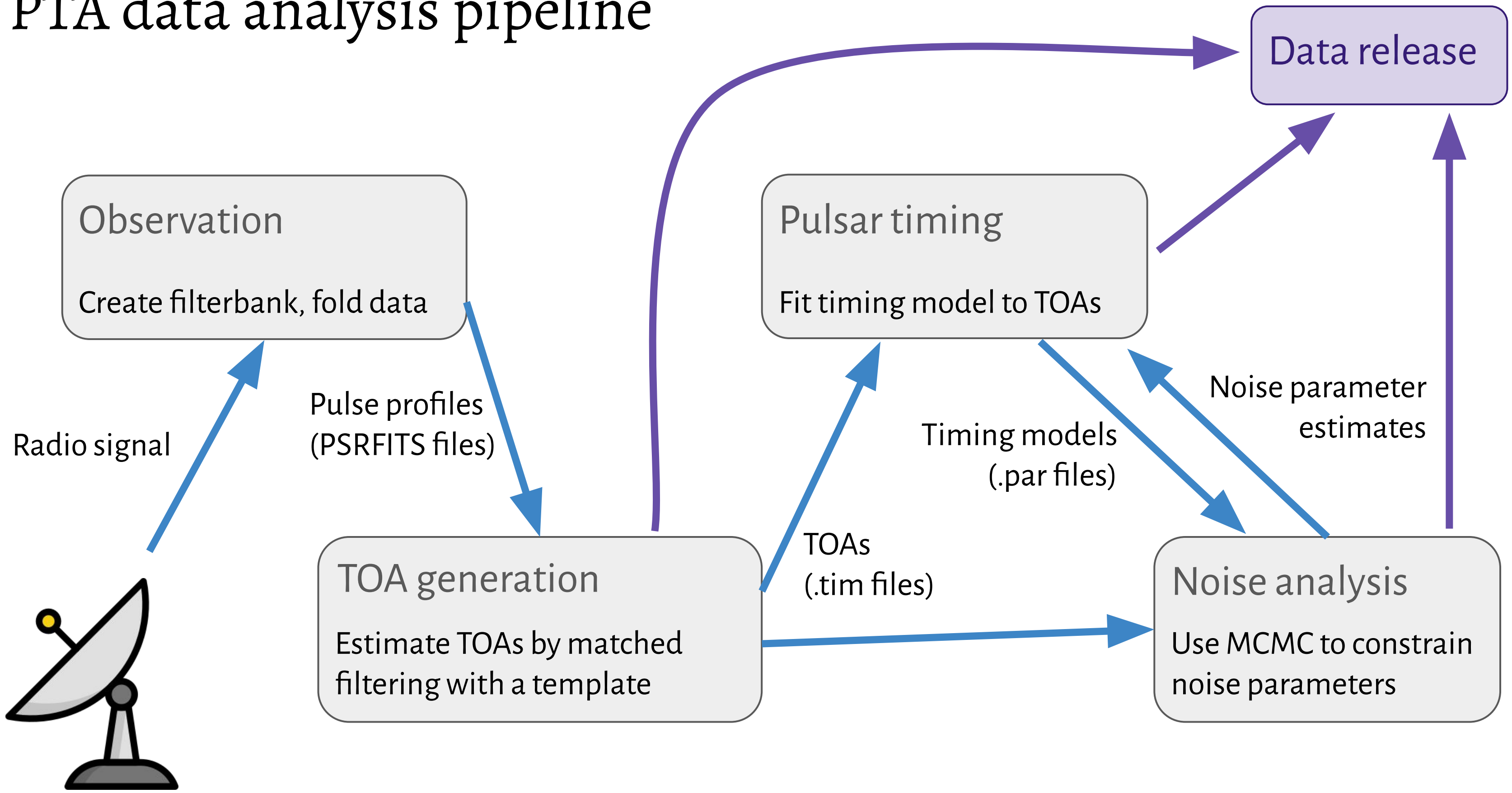
West Virginia University

WVU CENTER FOR GRAVITATIONAL WAVES AND COSMOLOGY

Ross Jennings · U. of Pittsburgh PTA workshop · Nov. 30, 2023

NANOGrav Physics Frontiers Center

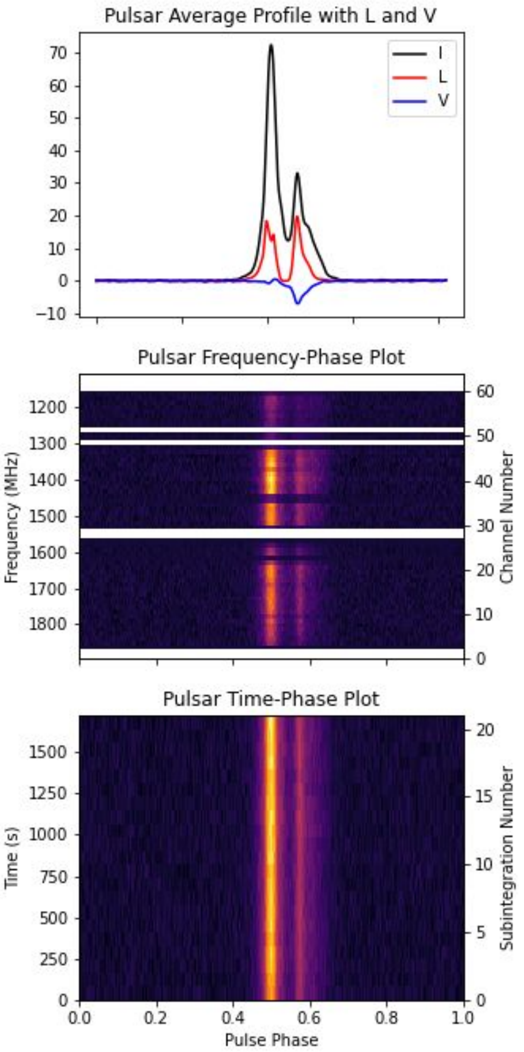
A PTA data analysis pipeline



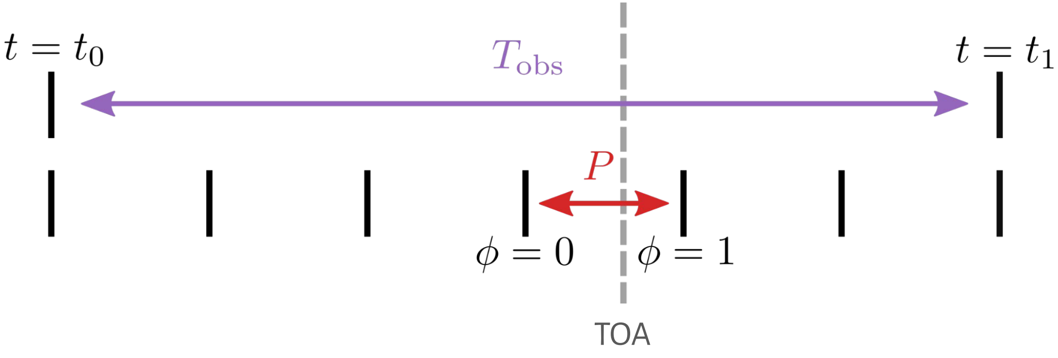
Part 1:

Observation and TOA Generation

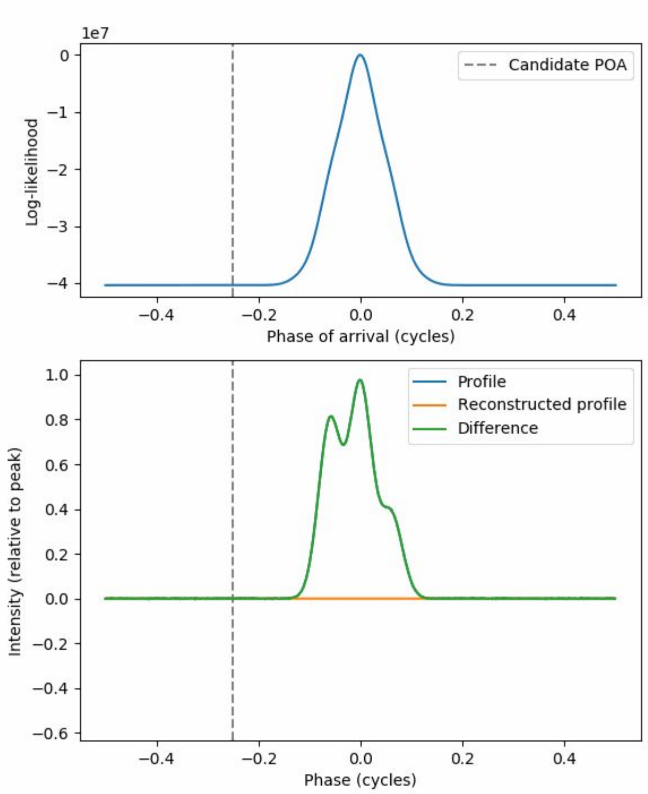
Pulsar observations



Measuring phase and time of arrival



$$\sigma_{\mathrm{TOA}} = \frac{W_{\mathrm{eff}}}{S\sqrt{N_\phi}}$$



Dispersion in the interstellar medium

Dispersion relation in cold plasma:

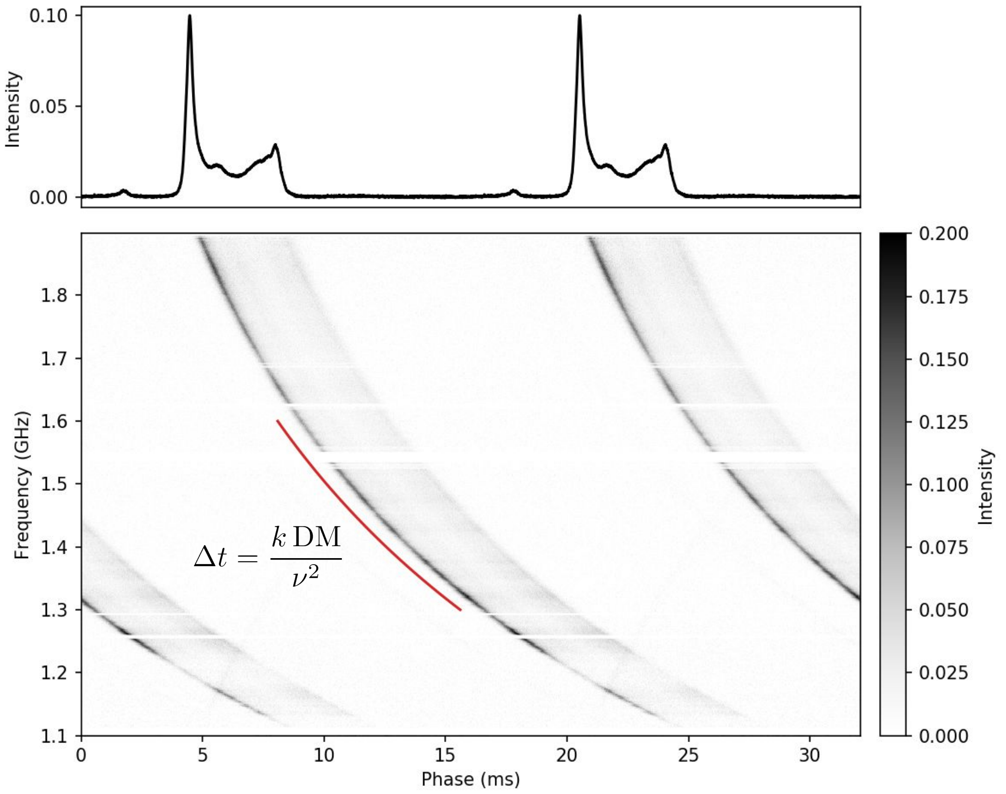
$$\nu^2 = \nu_p^2 + c^2 k^2$$

Plasma frequency:

$$\nu_p = \frac{1}{2\pi}\sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

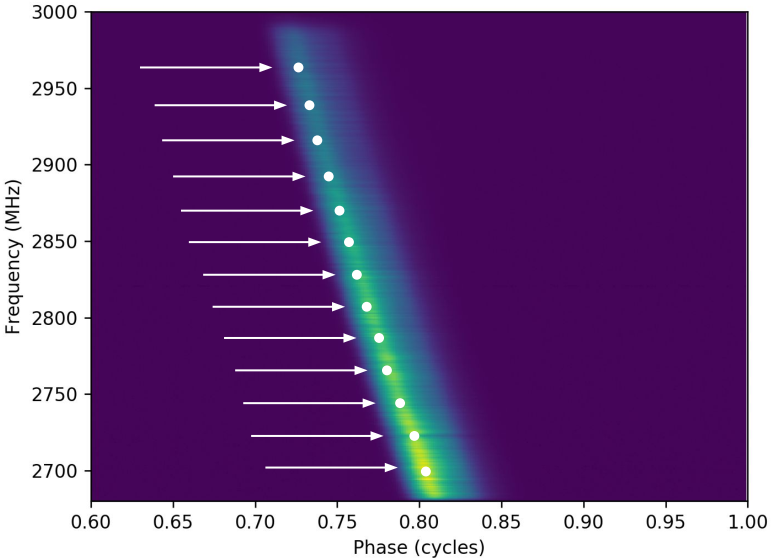
$$\Delta t = \frac{k\,\mathrm{DM}}{\nu^2} \qquad \mathrm{DM} = \int_L n_e\, d\ell$$

Dispersion measure

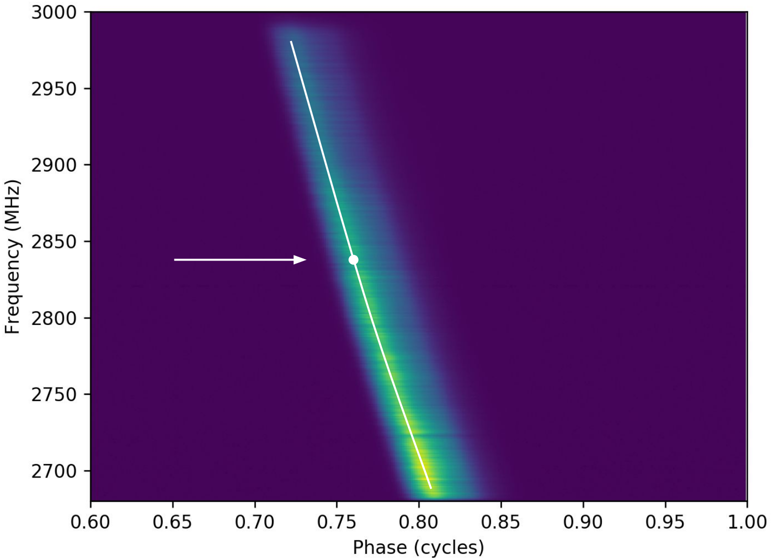


Two approaches to TOA generation

Narrowband



Wideband



`PulsePortraiture` (Pennucci et al. 2014)

Part 2:

The Timing Model

From TOAs to timing residuals

1. Compute phase @ time of arrival:

Delay function: convert observatory time to "pulsar time"

$$\phi = \phi(t - \tau(t))$$

Phase function: account for rotation of the pulsar

Timing model =
delay function + phase function

2. Compute phase residual:

Phase @ time of arrival should be a (specific) integer

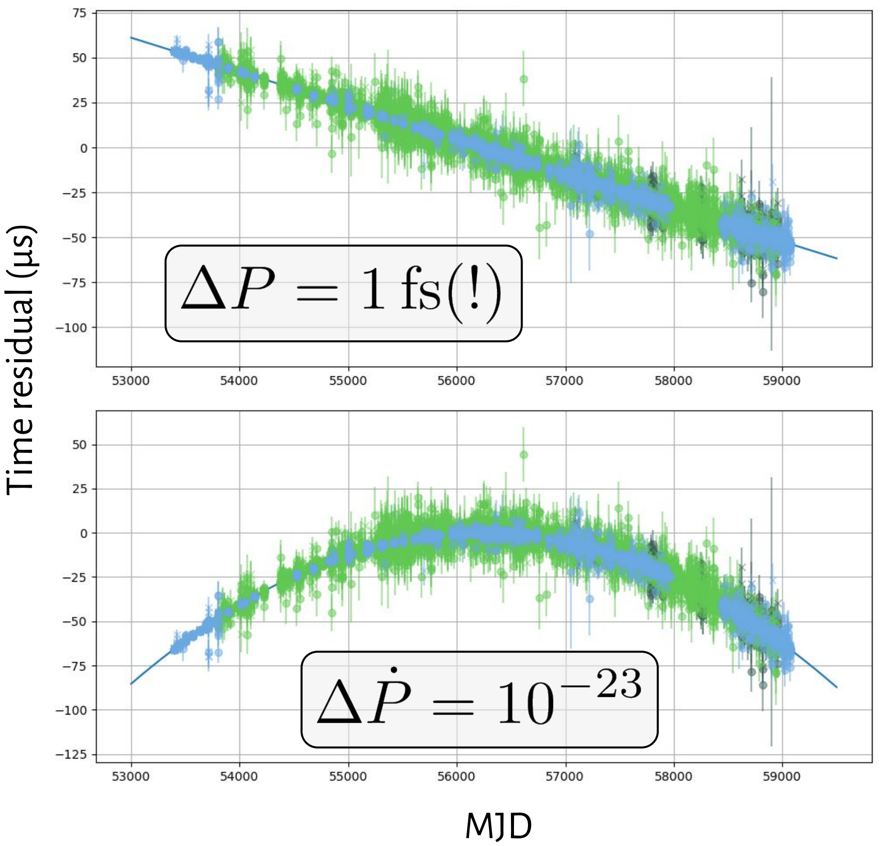
$$\Delta\phi = \phi - n$$

Once the pulsar has been "phase connected", we can just use the nearest integer for *n*.

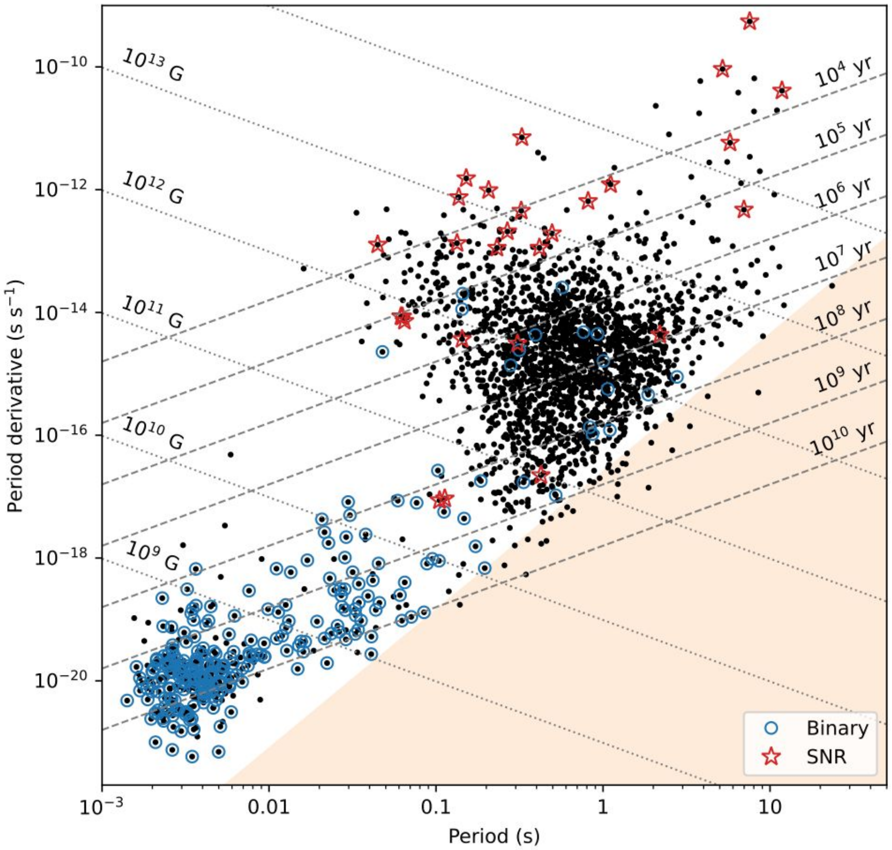
3. Convert phase residual to time residual:

$$\Delta t = \frac{\Delta\phi}{P}$$

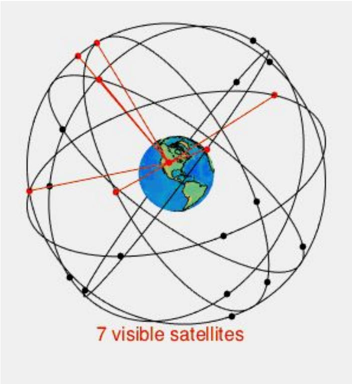
The phase function: spindown



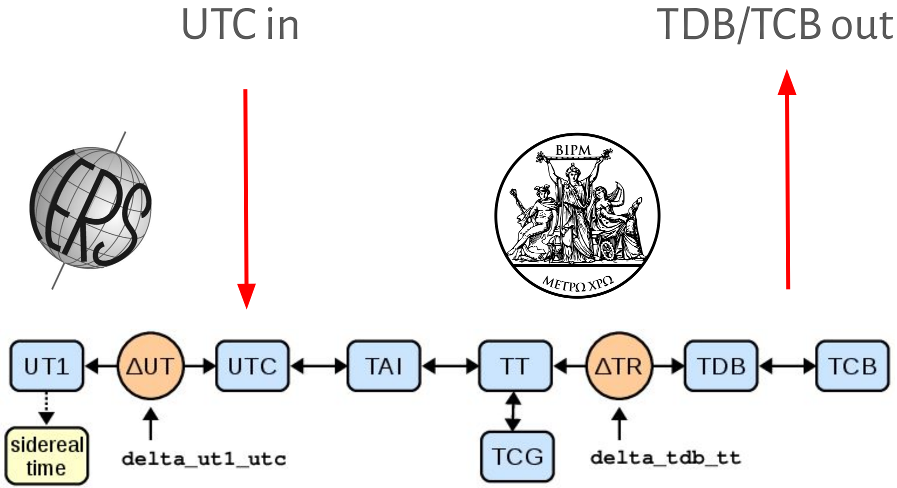
`psrqpy` (Pitkin 2018)



The delay function: clock corrections

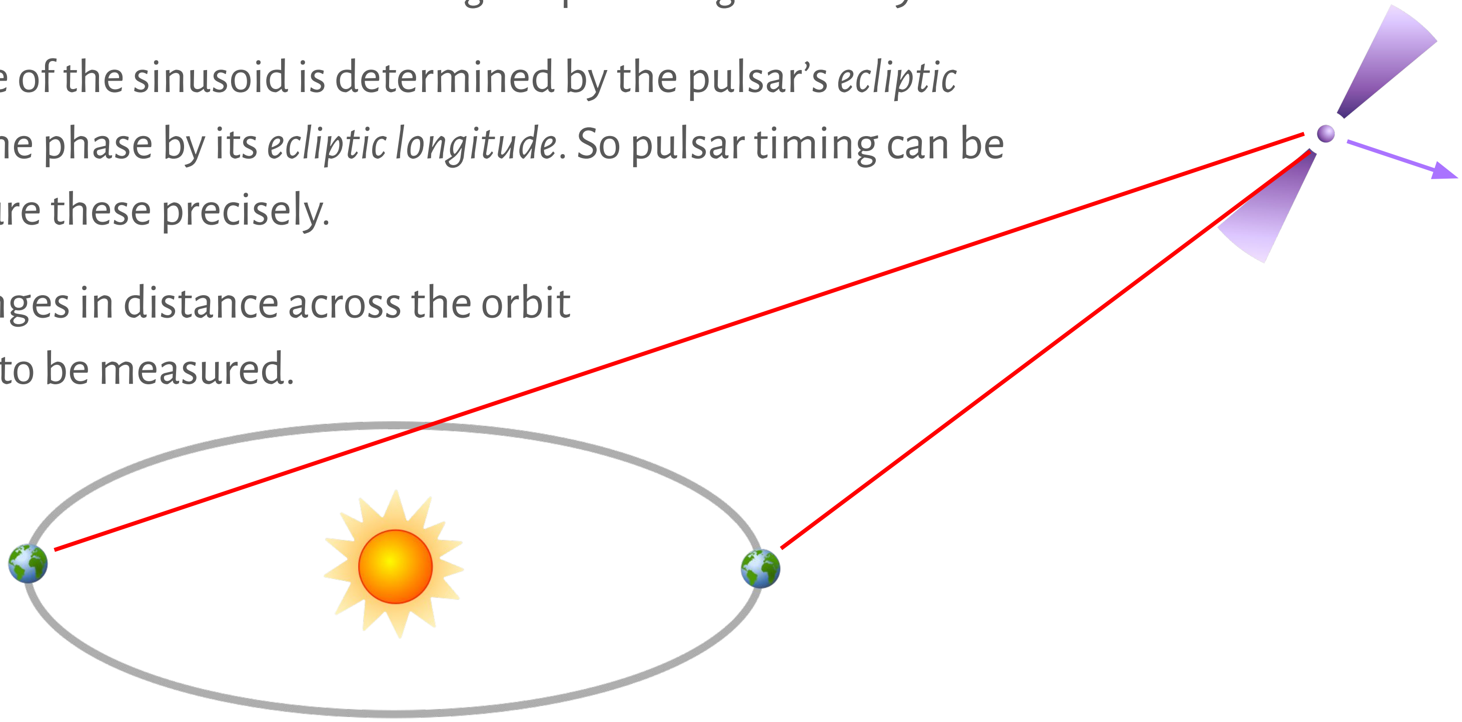


Observatories rely on masers and GPS satellites to keep nanosecond-accurate time.

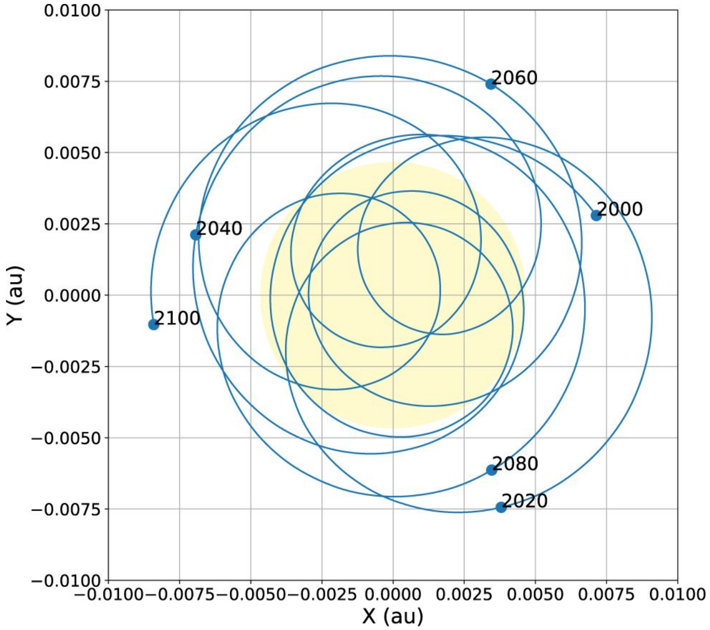


The delay function: position, parallax and proper motion

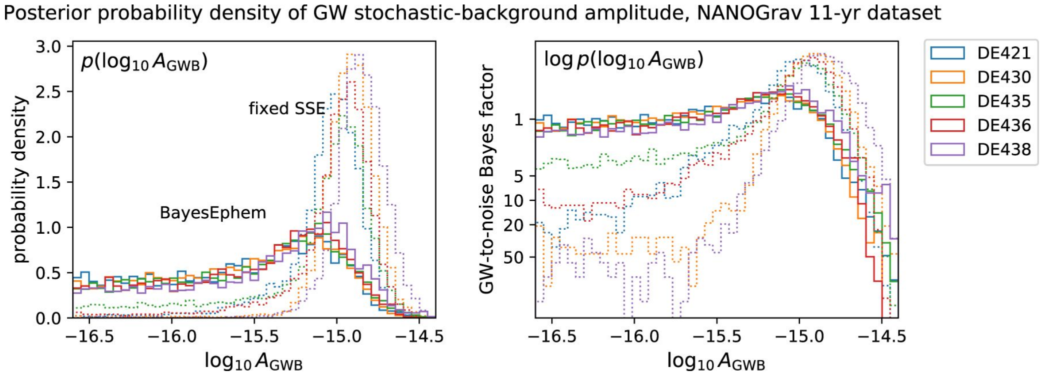
- Earth's orbit produces a sinusoidal change in path length = delay.
- The amplitude of the sinusoid is determined by the pulsar's *ecliptic latitude*, and the phase by its *ecliptic longitude*. So pulsar timing can be used to measure these precisely.
- *Quadratic* changes in distance across the orbit allow *parallax* to be measured.



The delay function: the Solar System ephemeris

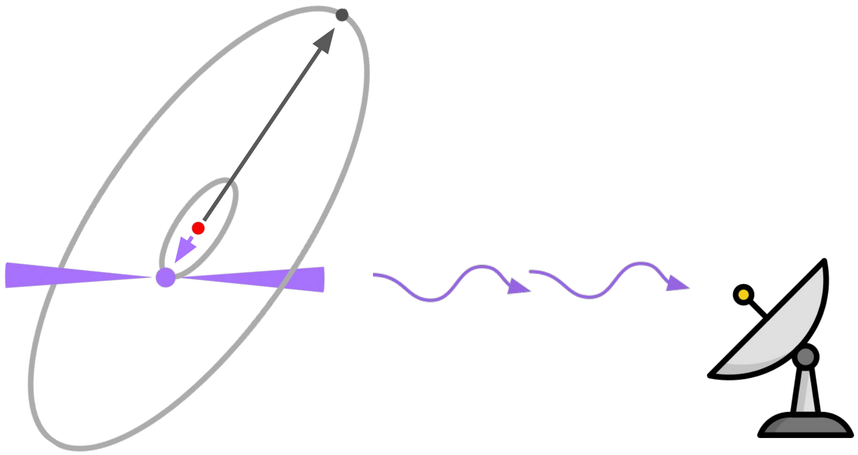


`BayesEphem` (Vallisneri et al. 2020)



The delay function: binary orbit modeling

- Many MSPs are in binaries, so we have to model their orbits too!
- Several different binary models are used, depending on orbital eccentricity (often *very* small, but still measurable) and inclusion of relativistic effects.
- In most precise pulsars, necessary to include "Kopeikin terms" (Kopeikin 1995, 1996) such as annual-orbital parallax.



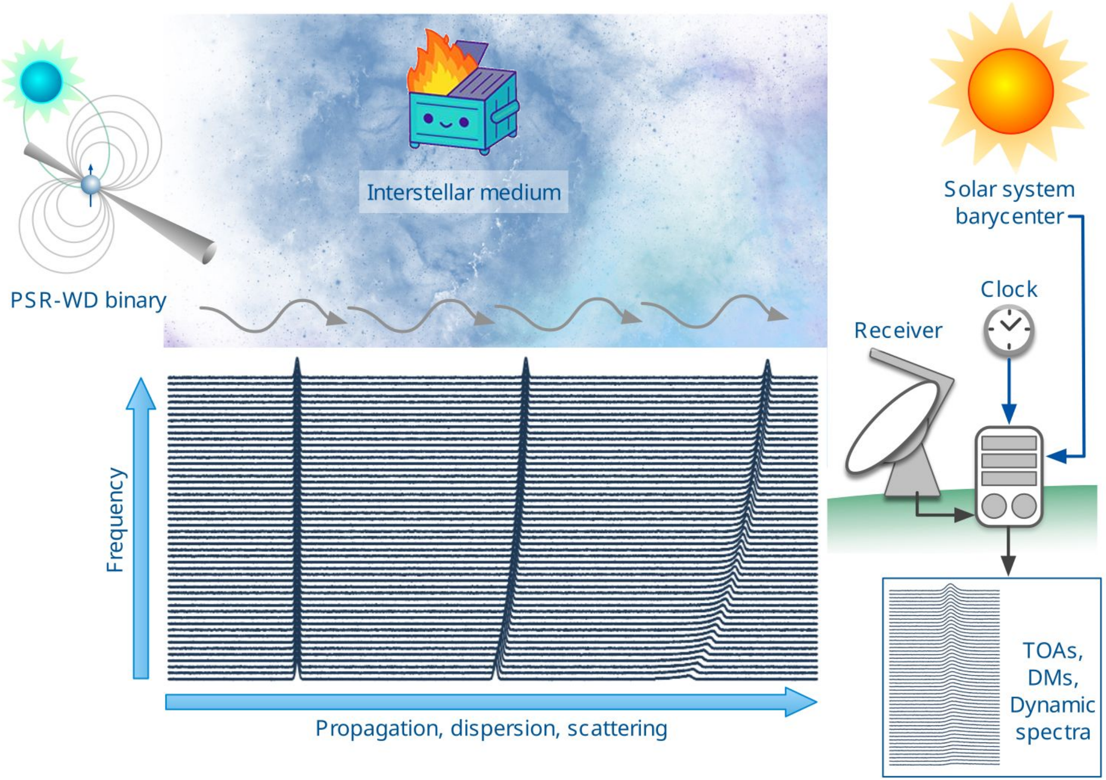
Part 3:

The Noise Model

The PTA "noise budget"

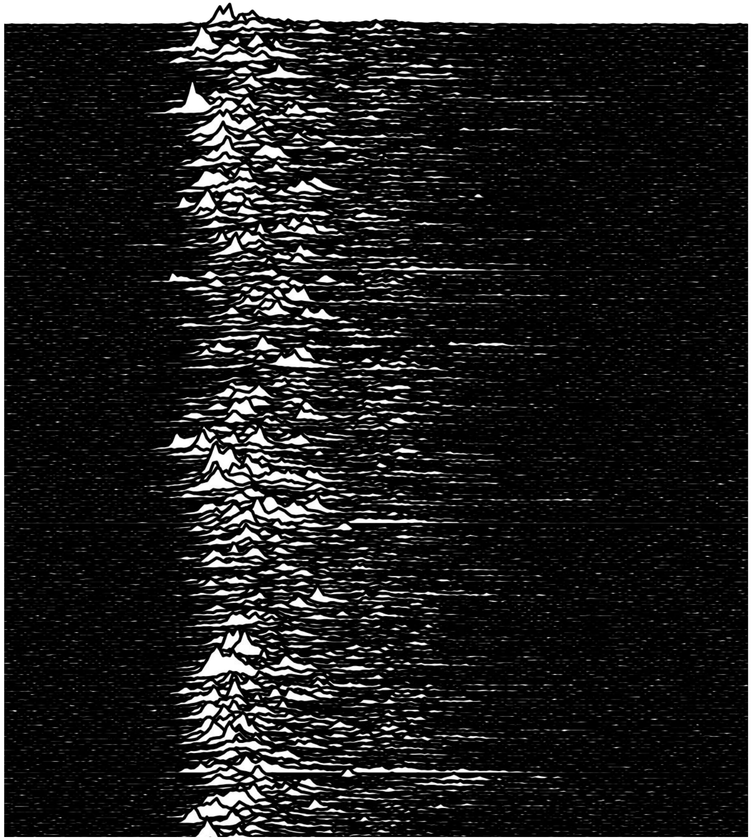
Noise sources:

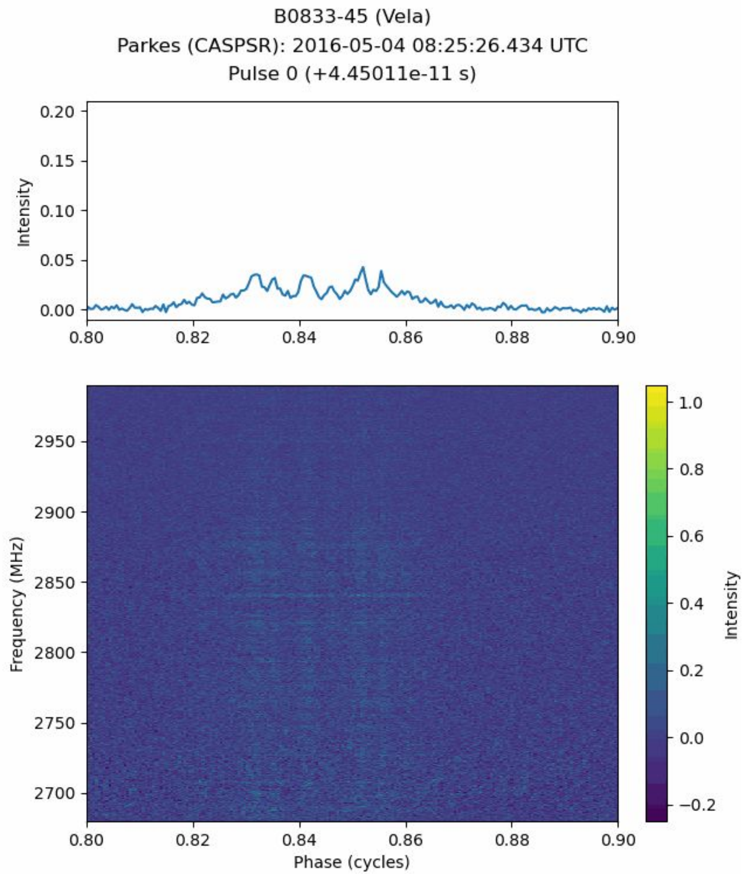
- Radiometer noise
- Pulse jitter
- Spin noise
- Orbital variations
- **Dispersion measure variations**
- **Interstellar scintillation**
- Solar wind
- Solar system ephemeris errors
- Polarization miscalibration
- Clock corrections
- RFI



Noise source	Origin	Time correlations	Frequency dependence or correlations	Spatial correlations
Pulse jitter	Pulsar	No (white)	Yes (correlated)	No
Spin noise	Pulsar	Yes (red)	No (achromatic)	No
Orbital variations	Pulsar system	Yes (red)	No (achromatic)	No
DM variations	ISM	Yes (red)	Yes (ν^{-2})	No
Diffractive scintillation	ISM	No (white)	Yes (ν^{-x}, $x \approx 4$)	No
Solar wind	Solar system	Yes (red)	Yes (ν^{-2})	Yes (solar elongation)
Solar system ephemeris	Solar system	**Yes (red)**	**No (achromatic)**	**Yes (dipolar)**
Polarization calibration	Telescope	Either	Yes (correlated)	No
Clock corrections	Telescope	**Yes (red)**	**No (achromatic)**	**Yes (monopolar)**
RFI	Telescope	No (white)	No (uncorrelated)	No
Gravitational waves	**GW sources**	**Yes (red)**	**No (achromatic)**	**Yes (quadrupolar)**

Pulse jitter

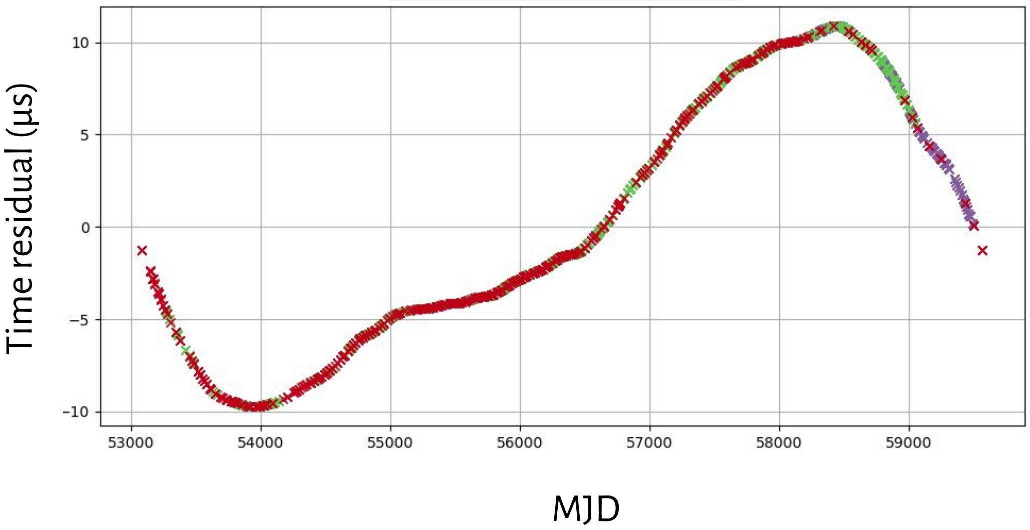




Spin noise

- Some (if not all) pulsars have intrinsic red noise due to rotational irregularities (changes in coupling between the crust and interior, or magnetic torque fluctuations).
- This is more significant in canonical pulsars, but does show up at a lower level in MSPs.
- Importantly, it is not correlated between pulsars.

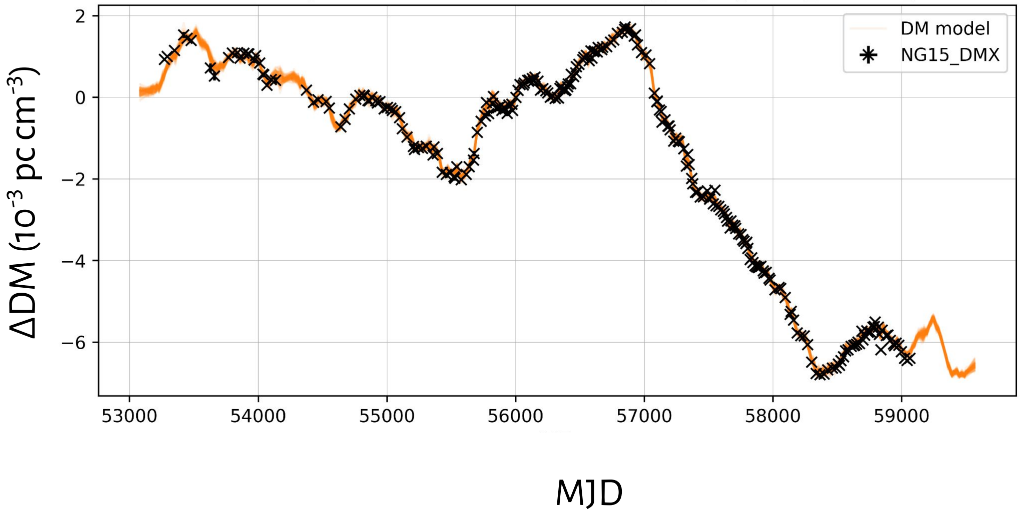
PSR B1937+21



DM variations

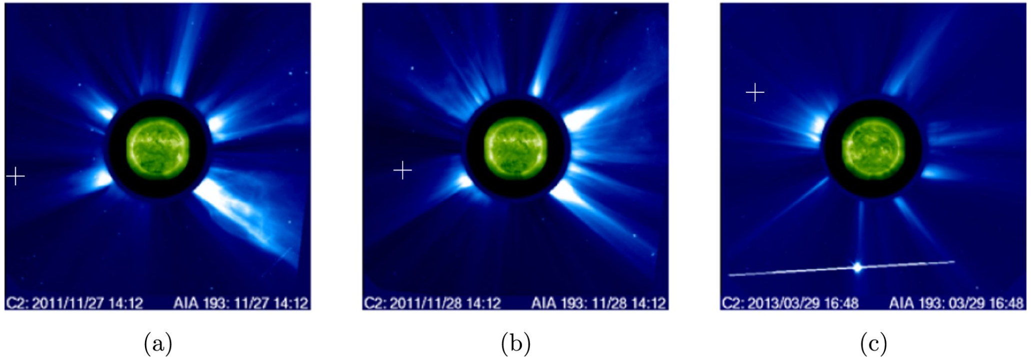
- Dispersion measure (DM) changes with time as a result of changes to the electron density along the line of sight.
- To achieve high timing precision, this effect must be removed from the TOAs.
- NANOGrav's approach to this so far has been to fit a piecewise constant model ("DMX").

PSR B1937+21

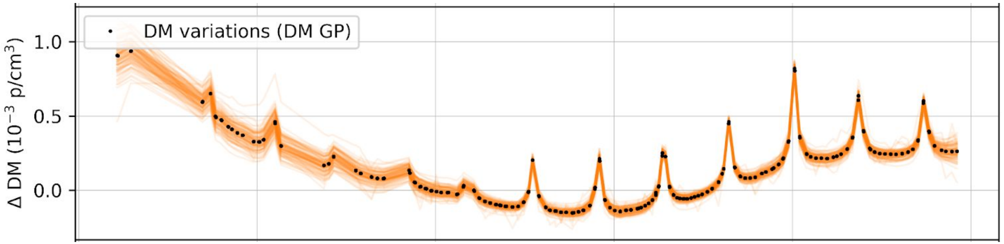


The solar wind

- One notable source of DM variations comes from close to Earth: the solar wind.
- The effect of the solar wind on DM is greatest when the line of sight passes close to the Sun.
- The DM change can be predicted, assuming a spherically-symmetric, static wind. But reality is more complicated.

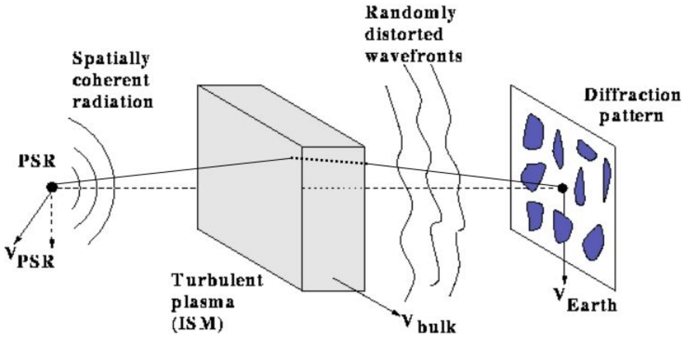


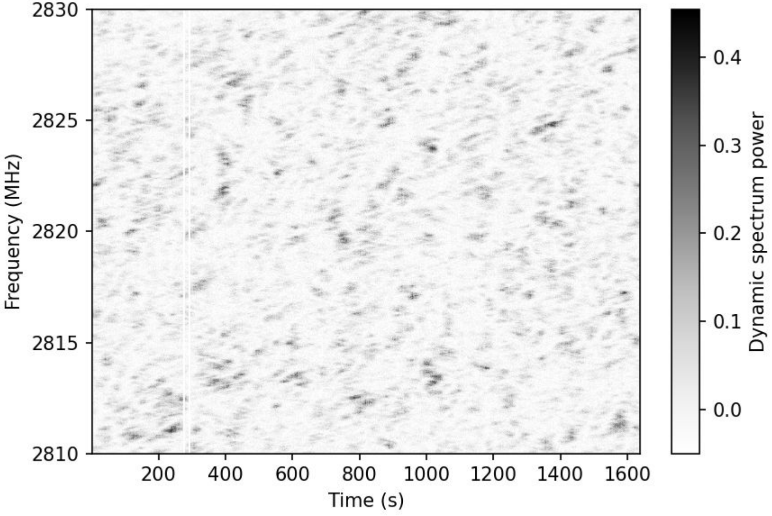
PSR J1744-1134



Interstellar scintillation

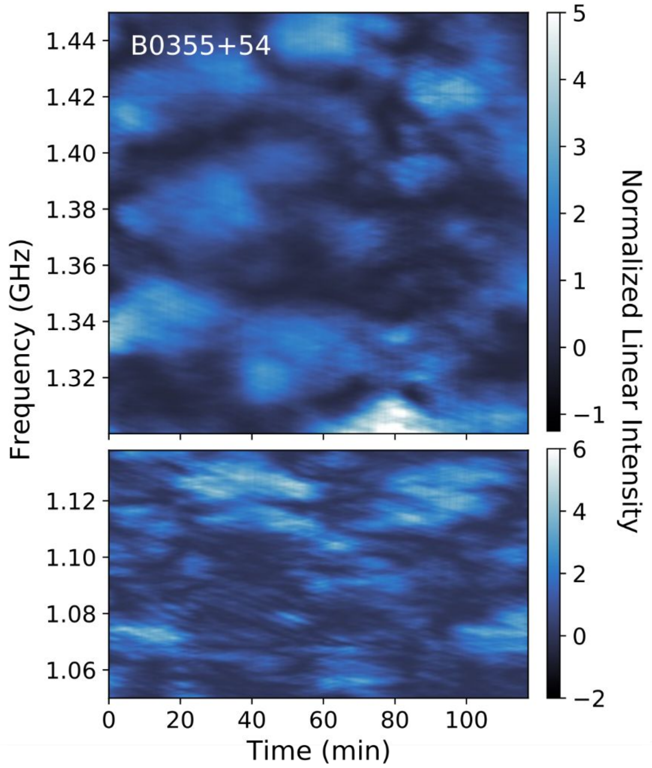
- In addition to being dispersed, pulsar emission is also scattered by density fluctuations in the ISM.
- This leads to a characteristic pattern of brighter and dimmer patches (“scintles”) as a function of frequency and time.
- Additionally, it leads to pulse broadening, which is more pronounced at lower frequencies.

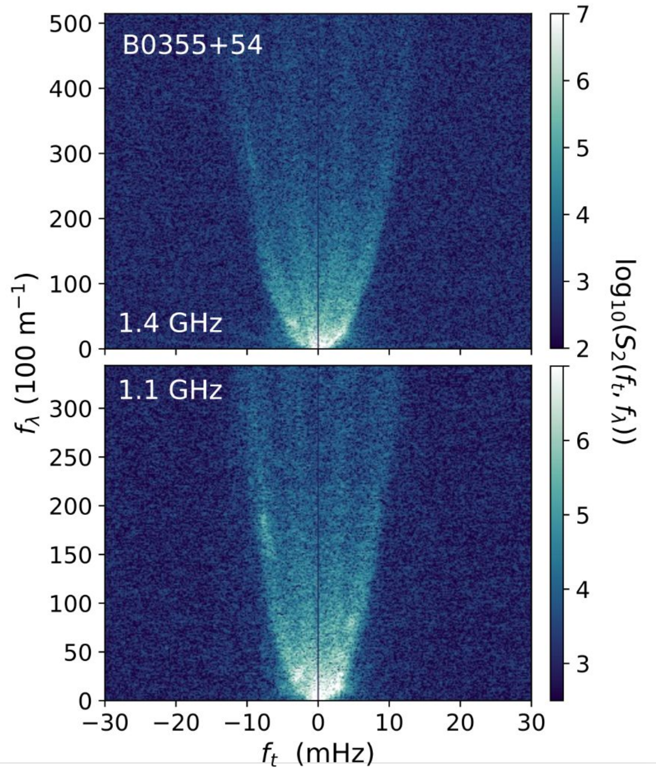




The secondary spectrum and scintillation arcs

dynamic spectrum ⟶ 2D Fourier Transform ⟶ secondary spectrum





Bonus part:

Noise Modeling in Practice

The phenomenological white noise model

Covariance matrix of TOA residuals

$$C = \begin{bmatrix} C^{\text{epoch}}(t_1) & 0 & \cdots & 0 \\ 0 & C^{\text{epoch}}(t_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C^{\text{epoch}}(t_N) \end{bmatrix}$$

EFAC (F) is an overall scaling factor.

$$C^{\text{epoch}} = \begin{bmatrix} \mathcal{F}^2[\sigma^2_{\text{S/N}}(\nu_1) + \mathcal{Q}^2] + \mathcal{J}^2 & \mathcal{J}^2 & \cdots & \mathcal{J}^2 \\ \mathcal{J}^2 & \mathcal{F}^2[\sigma^2_{\text{S/N}}(\nu_2) + \mathcal{Q}^2] + \mathcal{J}^2 & \cdots & \mathcal{J}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{J}^2 & \mathcal{J}^2 & \cdots & \mathcal{F}^2[\sigma^2_{\text{S/N}}(\nu_N) + \mathcal{Q}^2] + \mathcal{J}^2 \end{bmatrix}$$

EQUAD (Q) represents systematic uncertainty not accounted for by the TOA estimation likelihood.

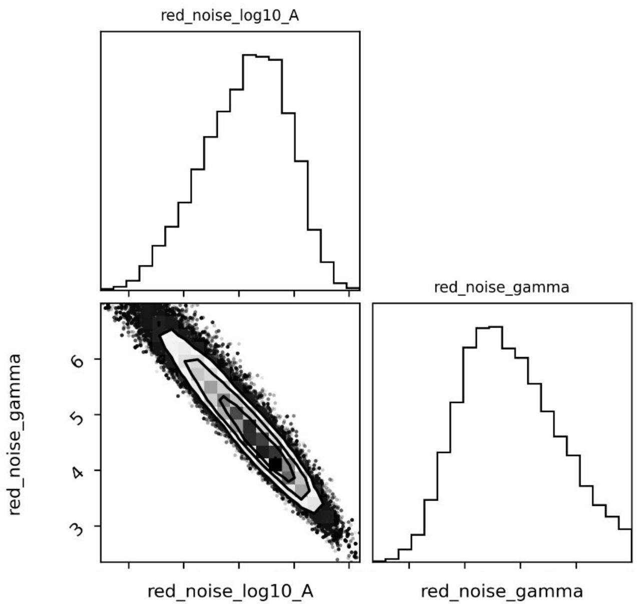
ECORR (J) introduces correlations within an epoch.

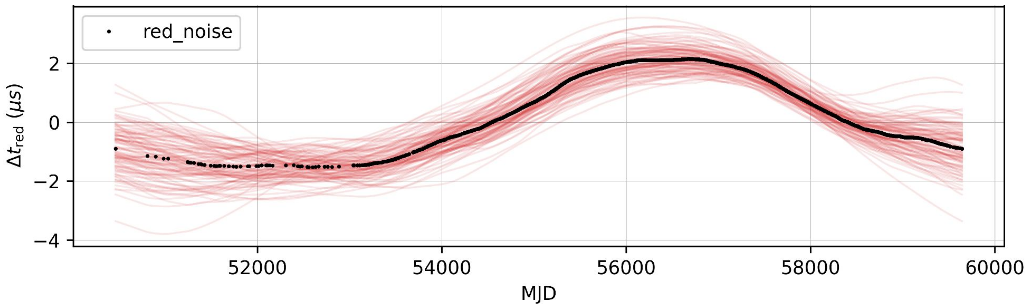
Red noise modeling

- We typically treat red noise as having a power-law spectrum:

$$S(f) = A\left(\frac{f}{f_0}\right)^{-\gamma}$$

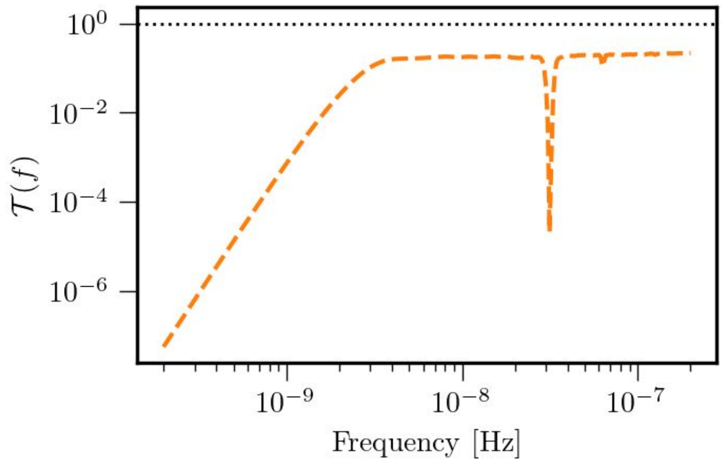
- Really, A and γ are *hyperparameters* that define a prior on the coefficients of the basis functions (typically, Fourier coefficients) that actually make up the signal.

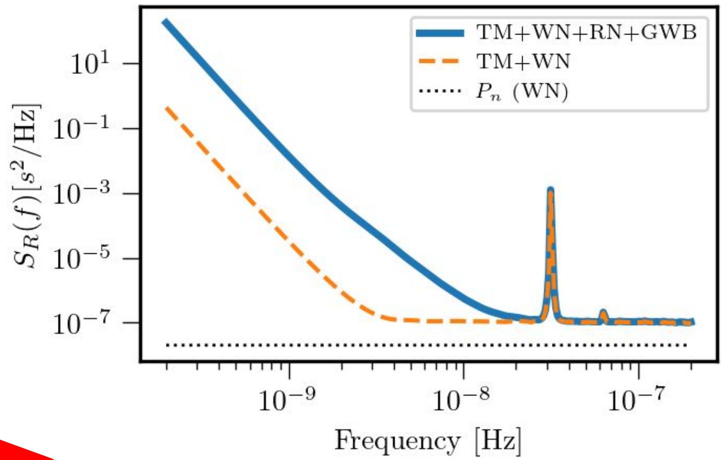


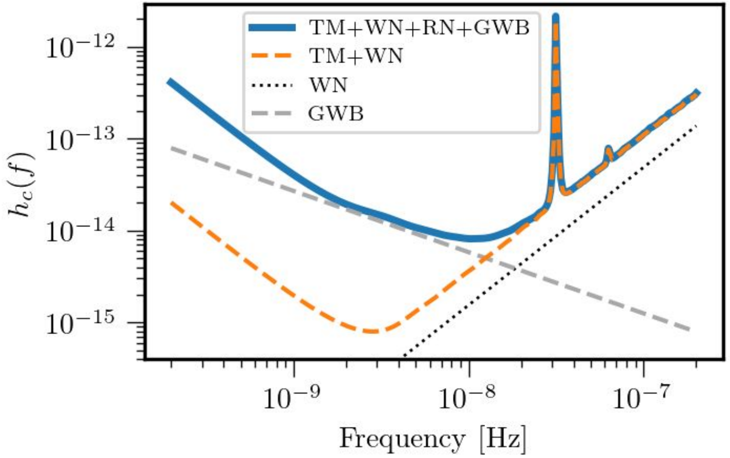


Sensitivity curves

Transmission function



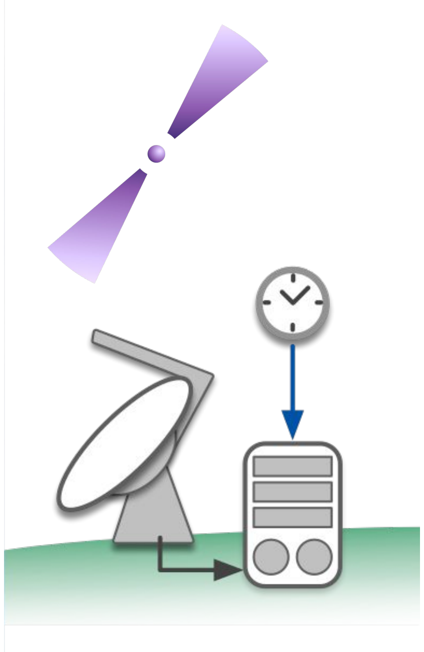




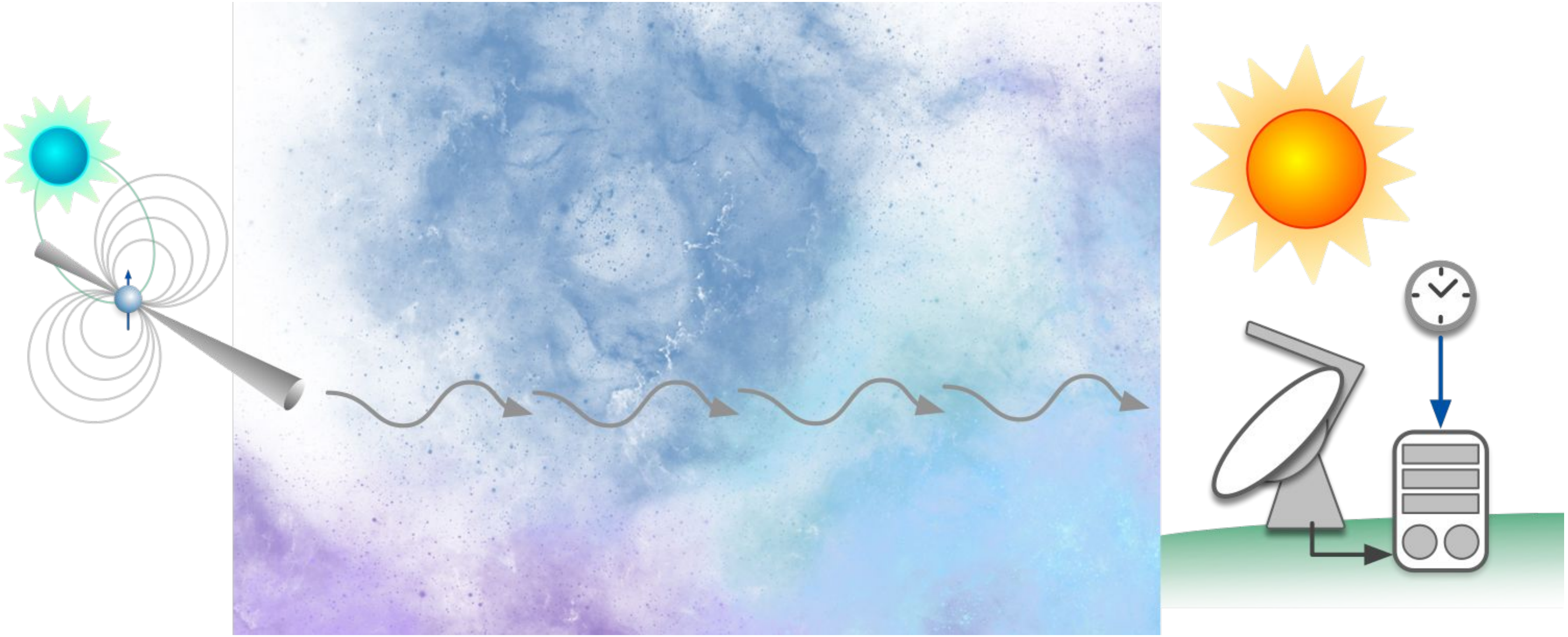
hasasia (Hazboun et al. 2019)

Conclusions: Getting better timing precision

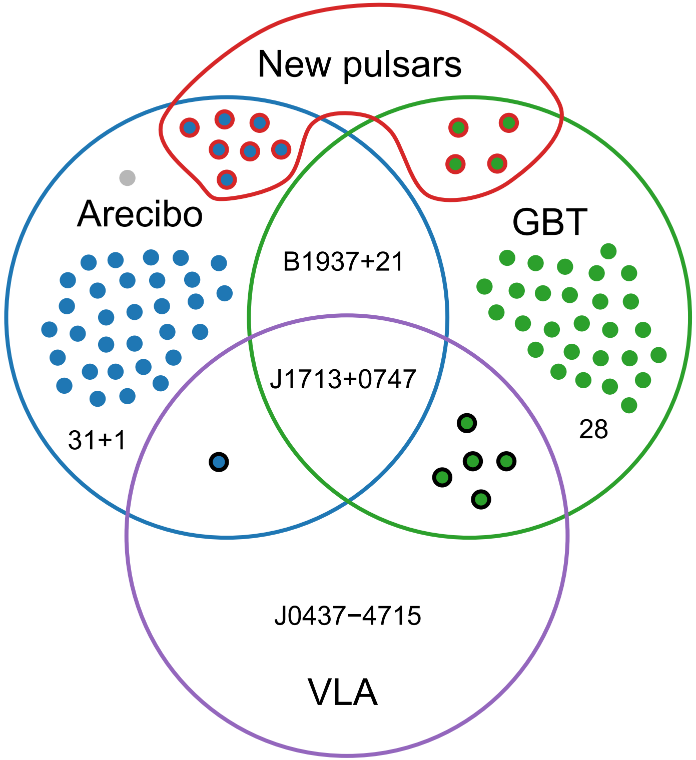
- PTAs don't build our GW detectors, we find them in nature.
- That's great for our budgets, but makes understanding our detectors harder.
- The upside is that our "detector characterization" is also astrophysics!
- To achieve high timing precision, we have to understand all the effects that compete with our signal, and remove them to the best of our ability.



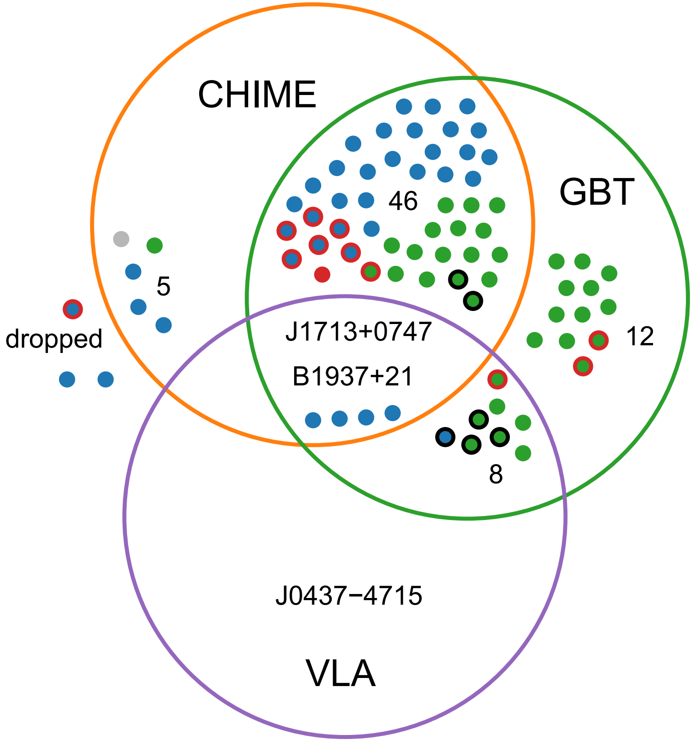
Thank you!



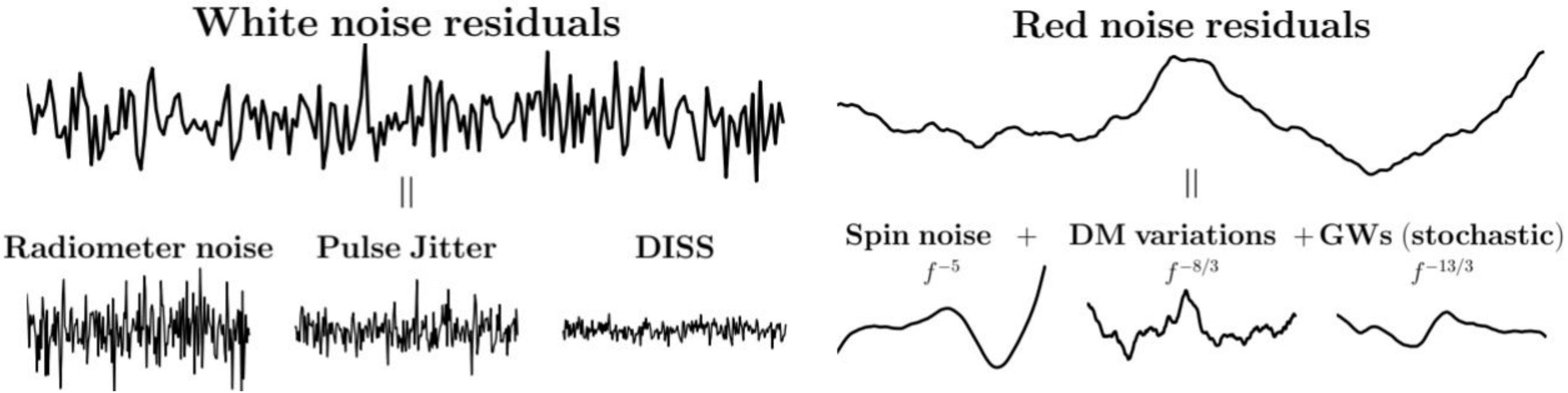
Then (2020)



Now (2023)



Current baseline is 2004-07-30 – 2023-03-08:
18.6 years!


White noise residuals
=
Radiometer noise
Pulse Jitter
DISS
Red noise residuals
=
Spin noise + DM variations + GWs (stochastic)
f^{-5}
$f^{-8/3}$
$f^{-13/3}$