PULSAR TIMING PROBES OF SMALL SCALE STRUCTURE

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ARXIV:1901.04490
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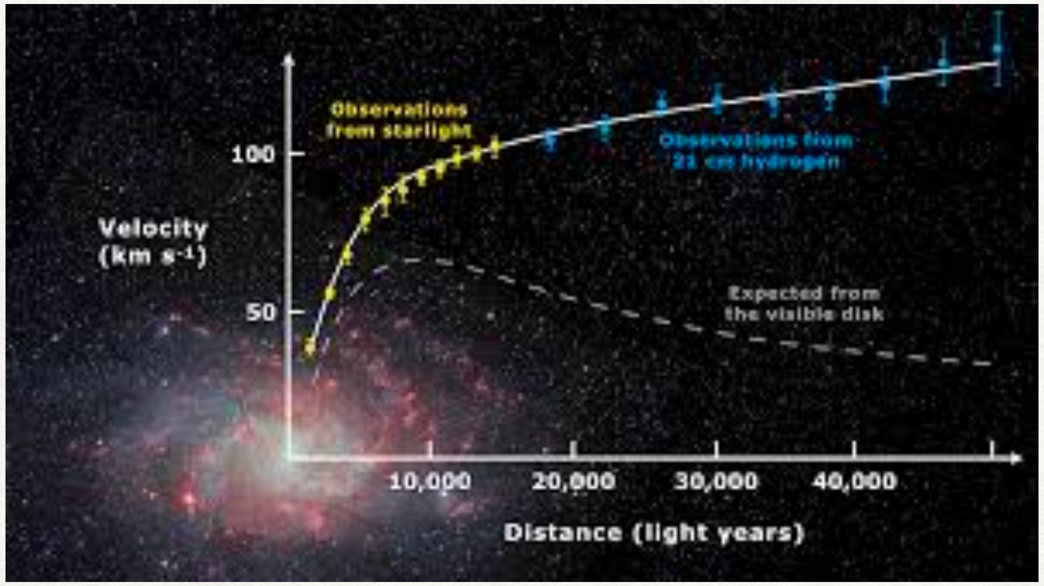
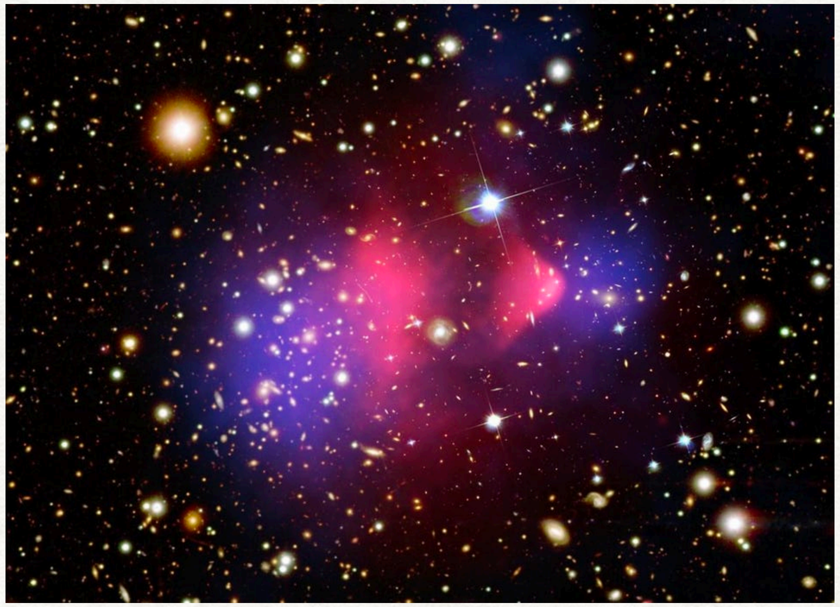
DARK MATTER





- Overwhelming evidence for Dark Matter (DM)
- Only through Gravitational Interactions
- Direct/Indirect/Collider efforts set stringent limits
- Nightmare Scenario: Only Gravitational Interactions

PROBING THE NIGHTMARE SCENARIO

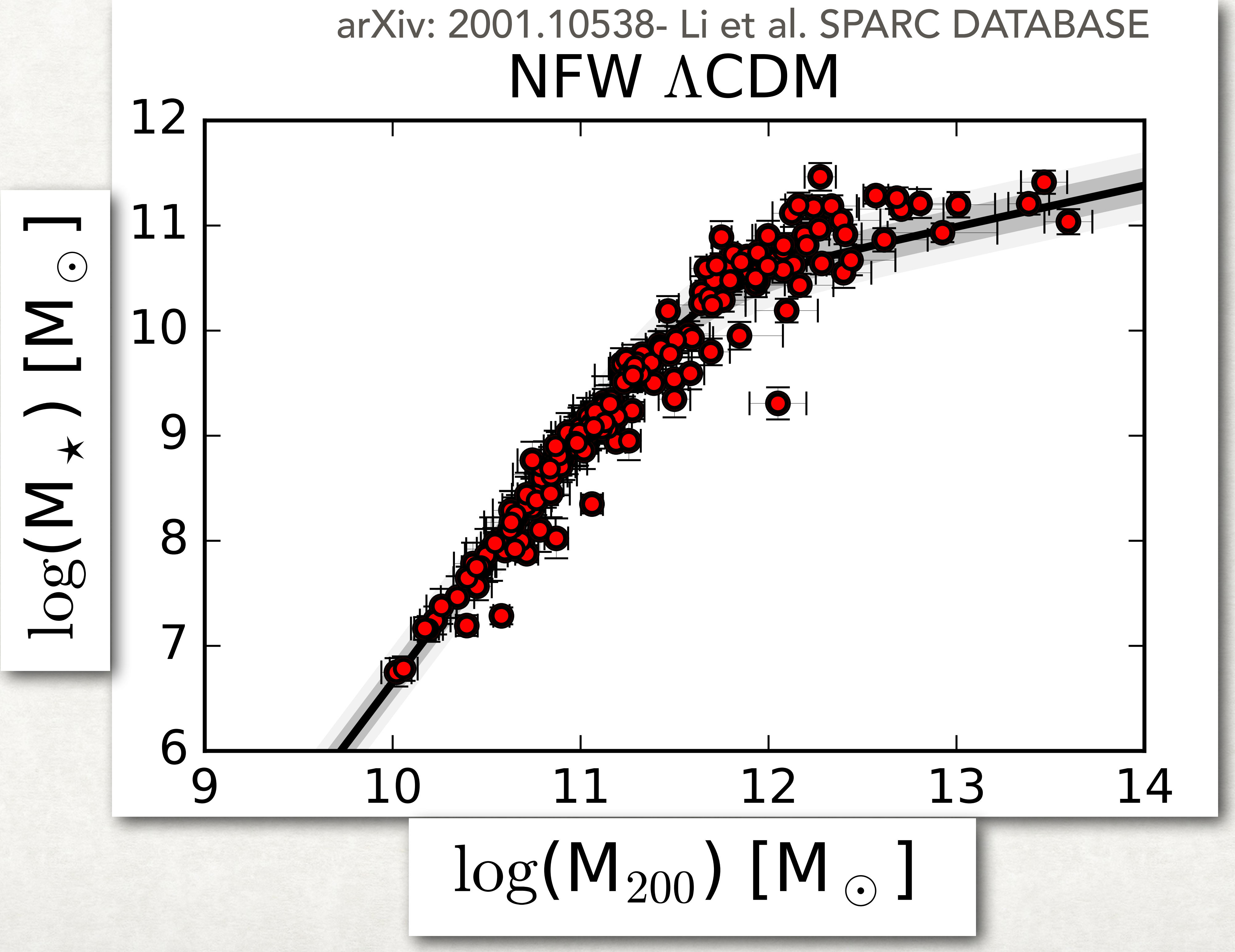
- Extremely difficult
- Could learn about underlying particle physics purely from gravity?
- Look for gravitationally collapsed structure, substructure etc

HIERARCHICAL STRUCTURE FORMATION

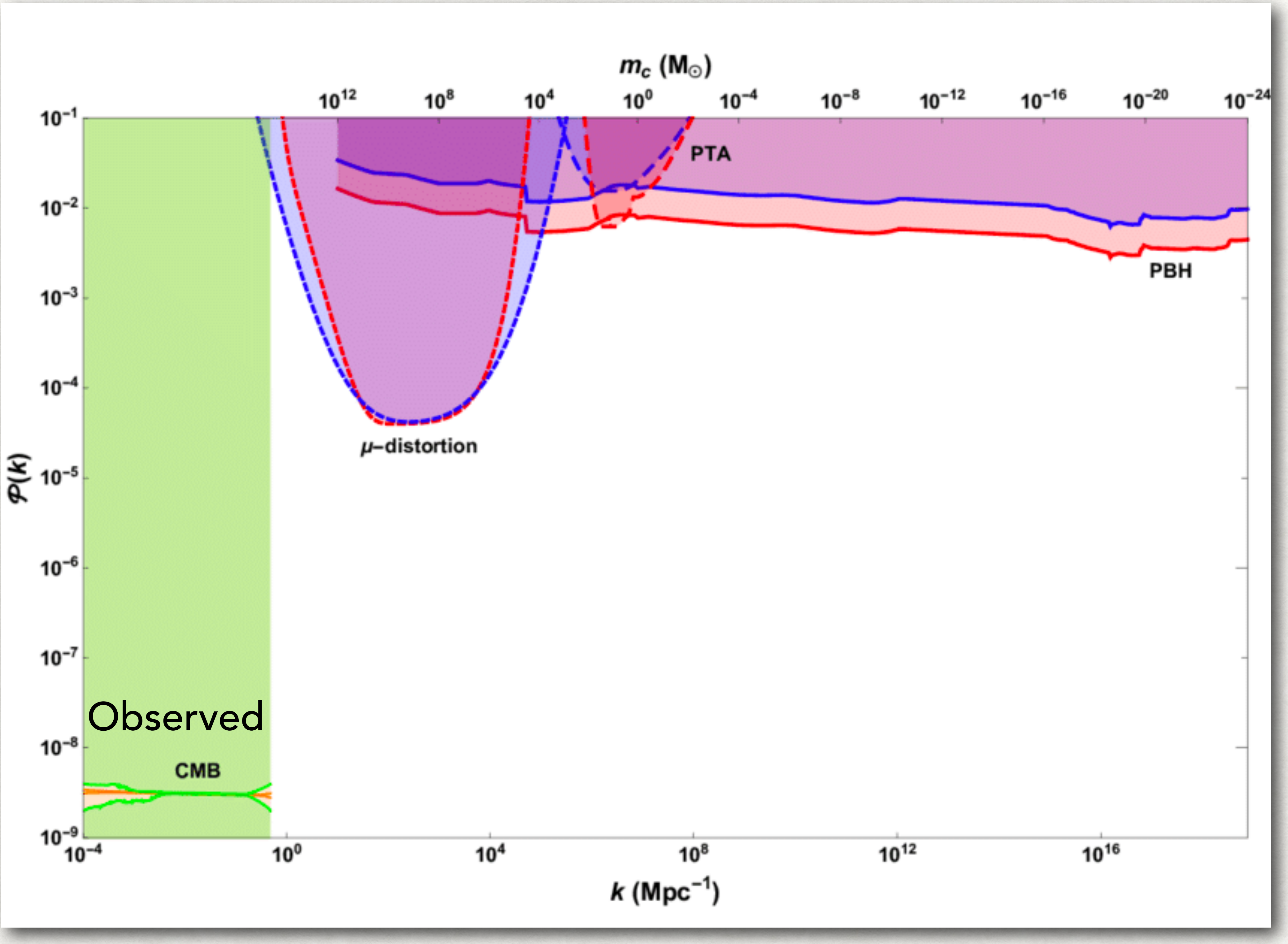
- Over-densities predicted by primordial power spectrum P(k)
- Collapse when reaching criticality
- Comoving wavenumber k determines mass of collapsed structure
- Smaller structure collapse first, act as seeds for larger structure
- Smallest structures (clumps/substructure/minihalos) can make up intra-galactic structure

WHAT DO WE KNOW TODAY?
WHAT CAN BE GLEANED IN THE FUTURE?

LOW MASS HALOS: DIFFICULT



ΛCDM+SCALE INVARIANT POWER



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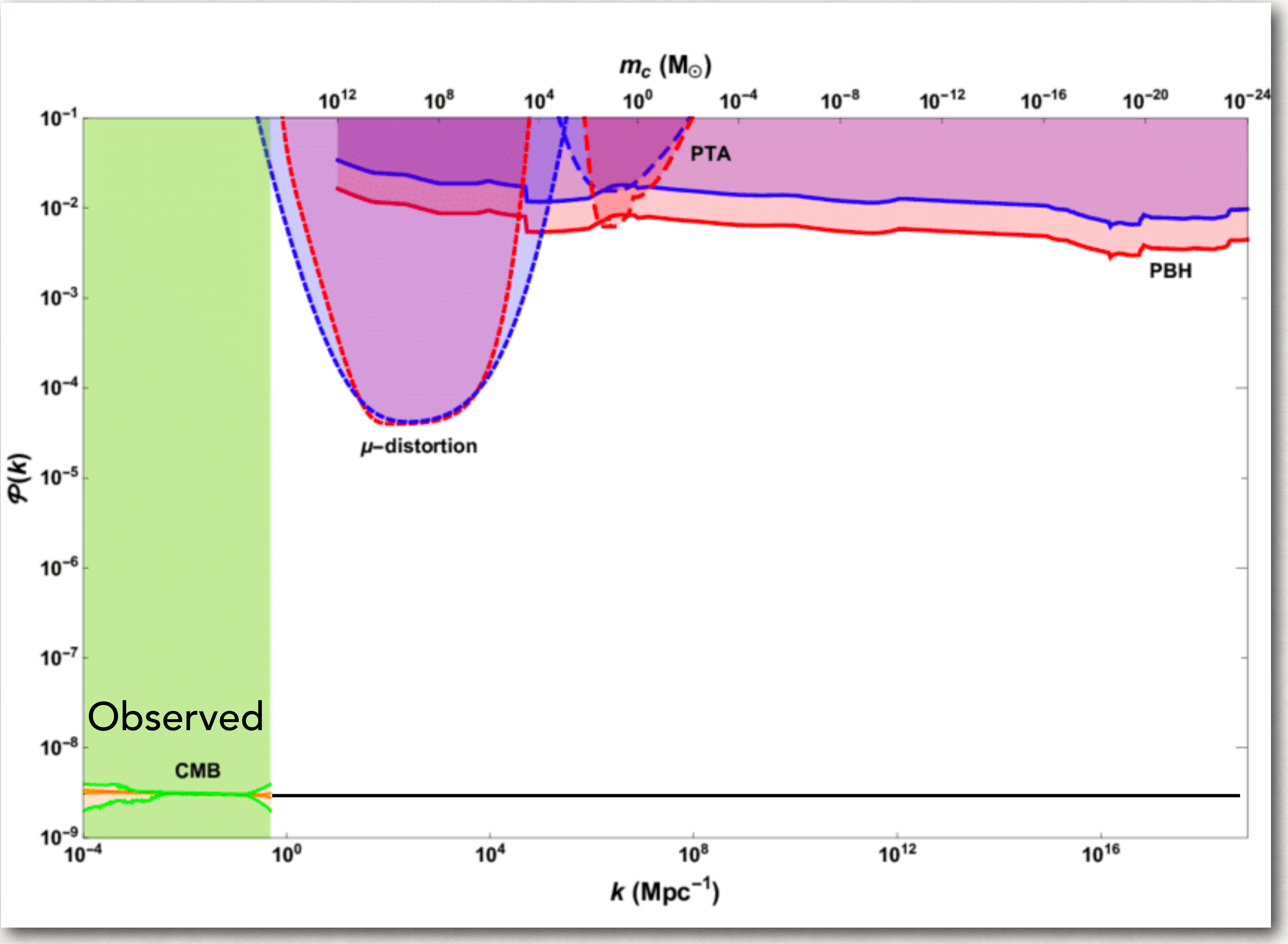
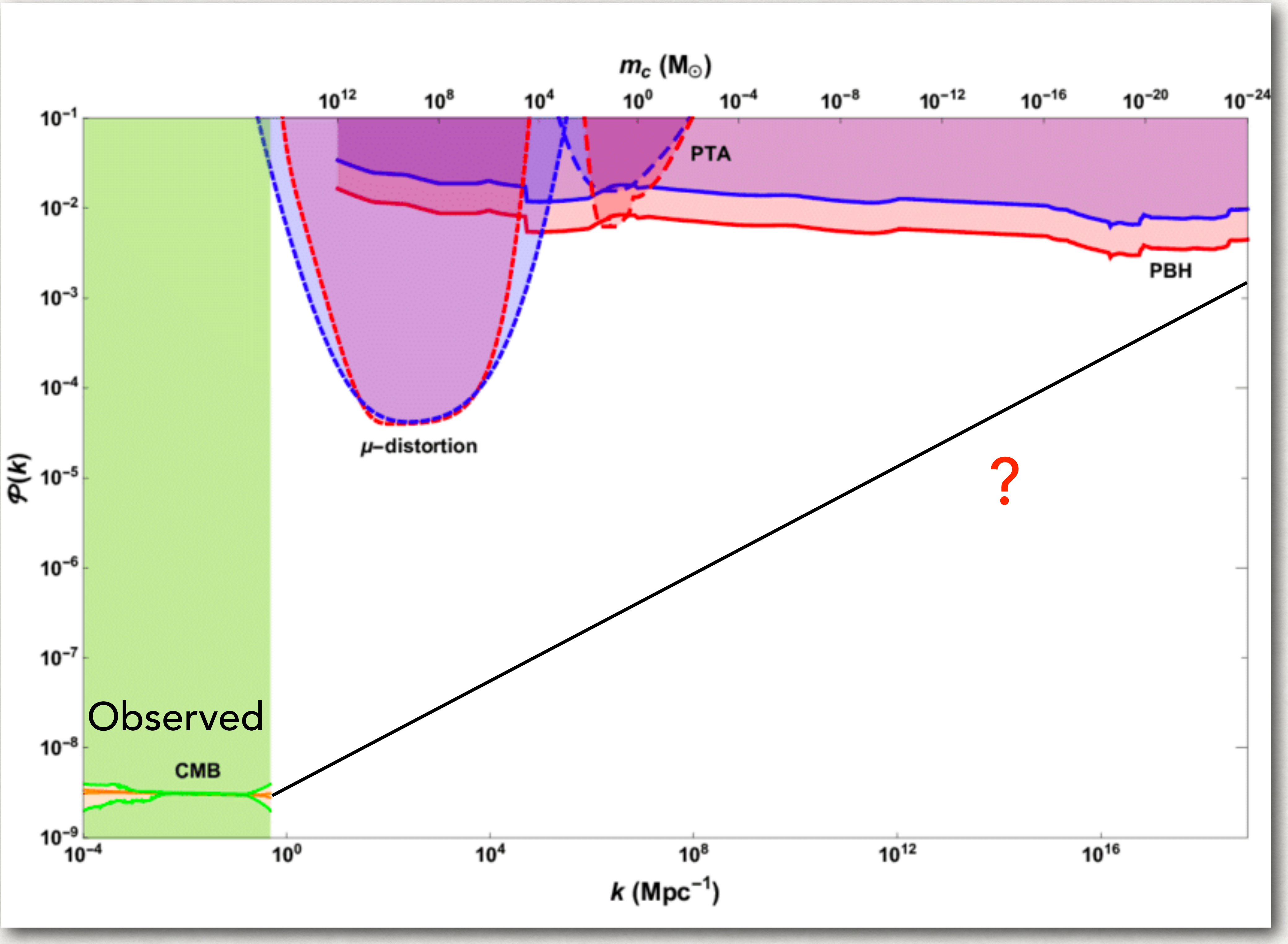
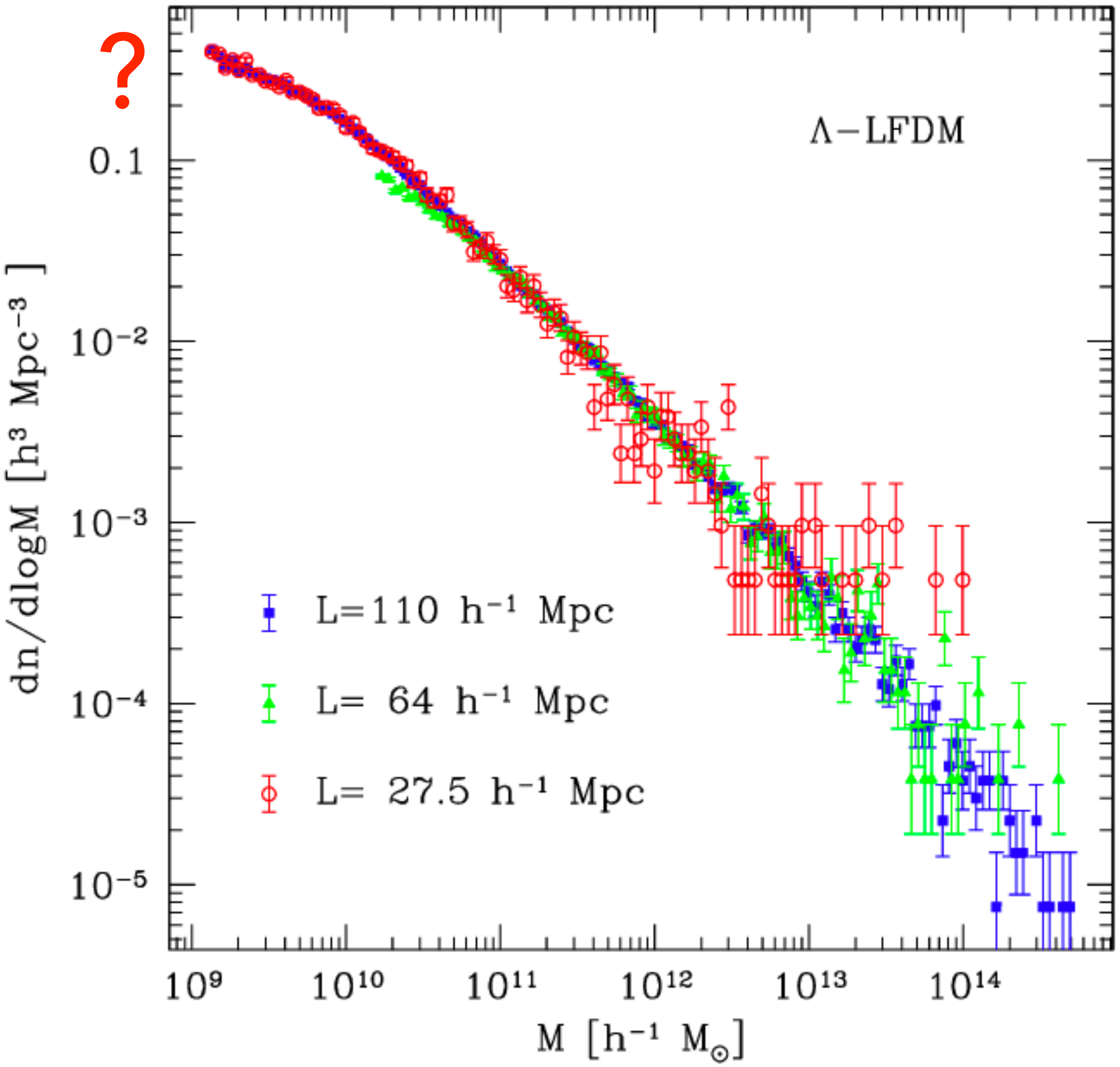


ΛCDM+EXCESS POWER



HALO MASS FUNCTION



LARGE SUBSTRUCTURE AT SMALL SCALES?

- Enhanced Primordial Power Spectrum
- Long Range Self-Interactions - Earlier collapse
- Early Matter Domination
- Strong short-range interactions (SIDM) + Gravothermal Collapse
- Dissipative Dark Matter
- Dark Nucleosynthesis

HOW TO LOOK FOR?

- Usual Answer: Lensing.
- Lensing only sensitive to extremely dense halos
- PBHs/MACHOS/Stars
- More diffuse objects?

PULSAR TIMING ARRAYS

PULSAR TIMING

- Phase: $\phi(t) = \phi_0 + \nu\, t + \frac{1}{2}\dot{\nu}\, t^2 + \frac{1}{6}\ddot{\nu}\, t^3 + ...$
- ν ~ kHz
- $\dot{\nu}/\nu$ ~ 10^{-23} to 10^{-20} Hz
- $\ddot{\nu}/\nu$ < 10^{-31} Hz^2, not included in fits
- After fitting away the period and derivative, residuals are remarkably small* (and stable).

$$t_{\rm RMS} \equiv \sqrt{\frac{1}{N}\sum_n (t_n^{\rm data} - t_n^{\rm fit})^2} \sim 50 \text{ ns}$$

*in reality, some other delays, shall describe a relevant few later

CONSIDER MONOCHROMATIC PBH DISTRIBUTION WITH MASS M_{PBH}

That make up a fraction f_{DM} of the local DM density

DOPPLER DELAY

- Recognize the ratio $\dfrac{\delta\nu}{\nu}$ is v_{rel}/c
- Thus sensitive to tiny accelerations

$$\left(\frac{\delta\nu}{\nu}\right)_D = \hat{\mathbf{d}} \cdot \int \nabla\Phi \, dt,$$

- velocity shape for a point object transit looks like:

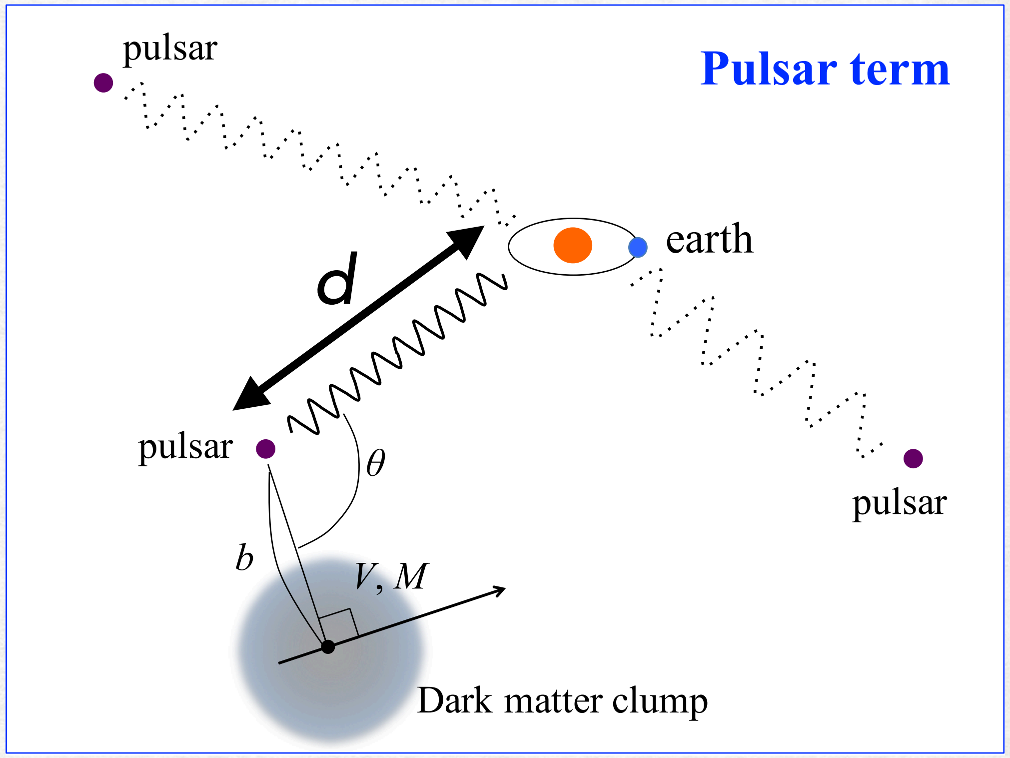
$$\left(\frac{\delta\nu}{\nu}\right)_D = \frac{GM}{v^2\tau_D}\frac{1}{\sqrt{1+x_D^2}}\left(x_D\hat{\mathbf{b}} - \hat{\mathbf{v}}\right)\cdot\hat{\mathbf{d}}$$

Signal period

Dimensionless time variable

Impact parameter

$$\boxed{|\mathbf{b}| = \tau v}$$



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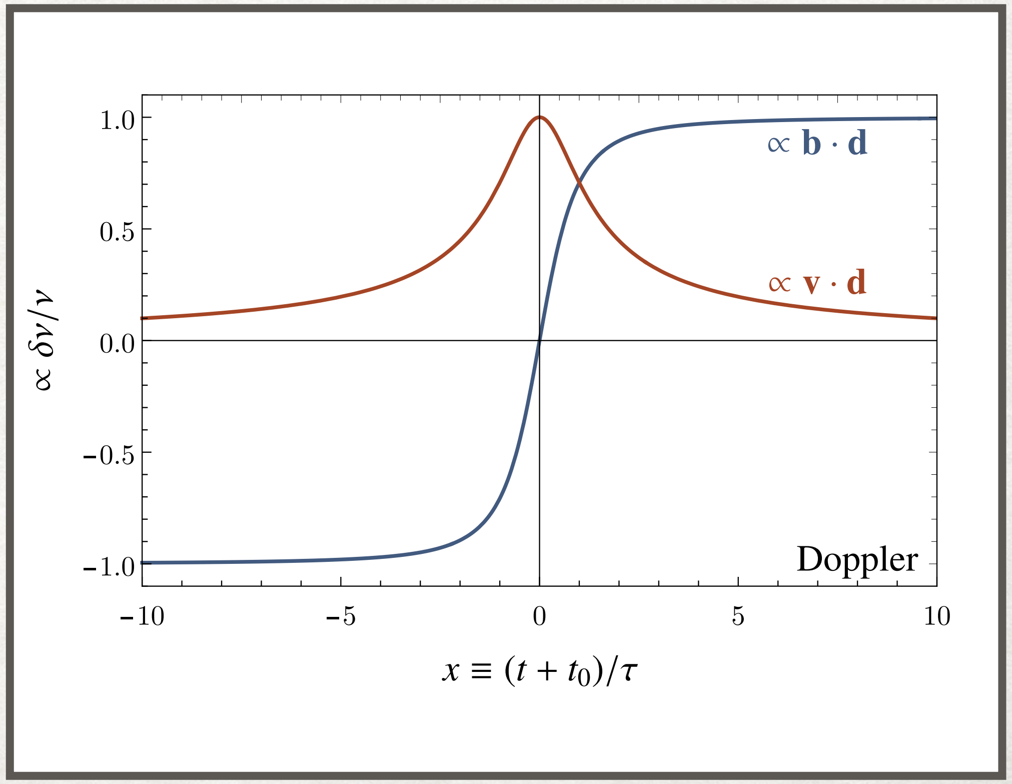


DYNAMIC VS STATIC

Dynamic if

$$\underbrace{\Delta t}_{\text{Cadence}} \ll \overset{\text{Characteristic signal period}}{\tau} \ll \underbrace{T}_{\text{Total Time of observation}}$$

Static otherwise

$$\tau \gtrsim T$$

DETECTING DYNAMIC SIGNALS

- Similar to a bump hunt / LIGO signal / Microlensing signal
- Doppler - leaves a permanent imprint
- Shapiro - Blip (As we will see)
- SNR is a solved problem in signal processing.

Fourier transform of Signal

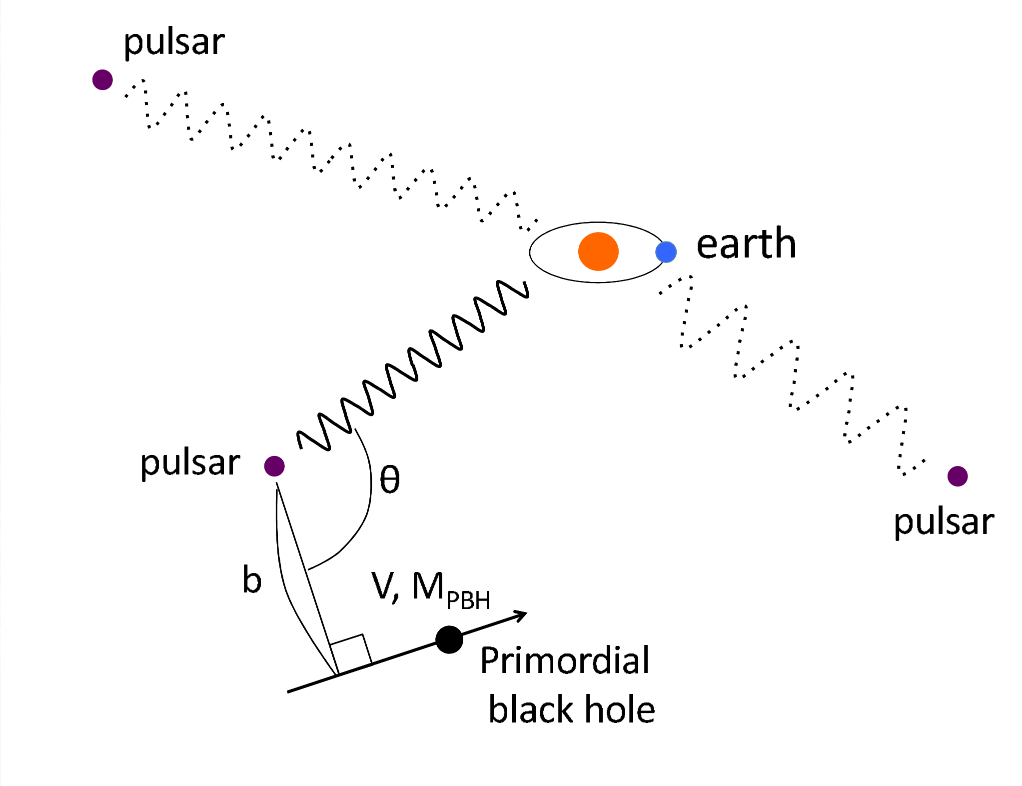
$$\mathrm{SNR}^2 = 4\int_0^\infty df\,\frac{|\widetilde{h}(f)|^2}{S_{\dot{\delta t}}(f)}$$

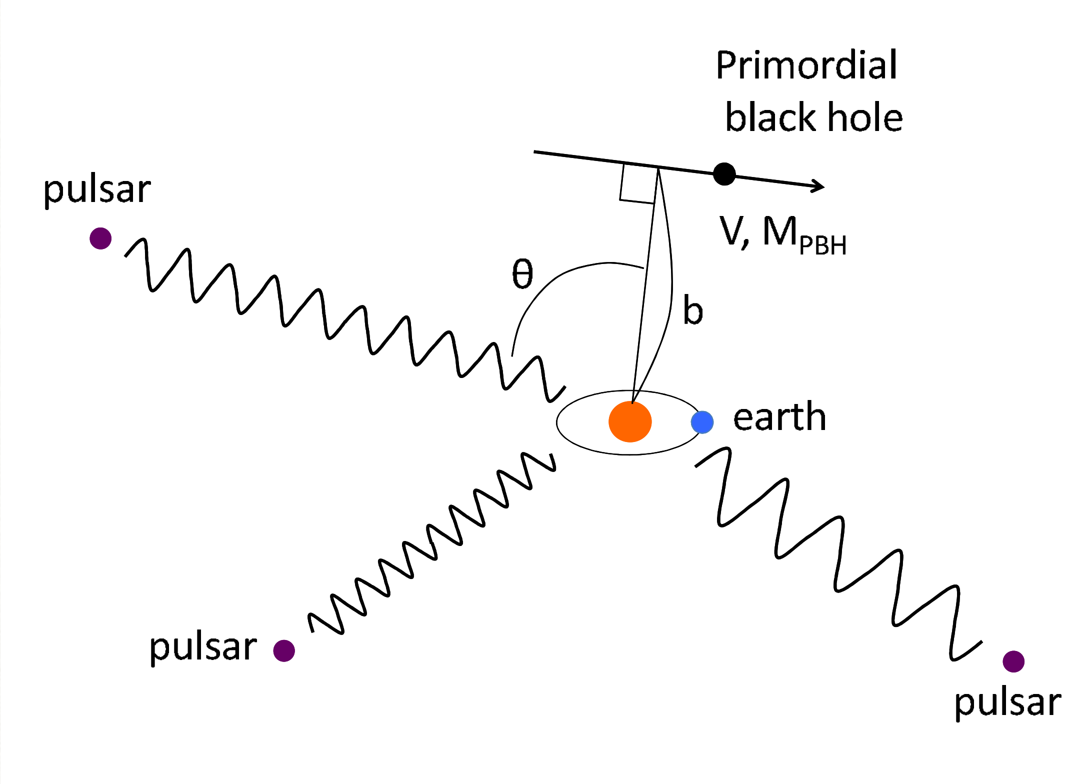
Cadence ~ 2 weeks

$$S_{\dot{\delta t}}(f) \equiv 8\pi^2 t_{\mathrm{RMS}}^2 \Delta t\, f^2$$

PULSAR TERM VS EARTH TERM FOR DOPPLER

- Many more pulsars → impact parameter smallest for one lucky pulsar.
- Angular correlations → sensitivity far higher for earth term
- Earth term - more robust to foregrounds



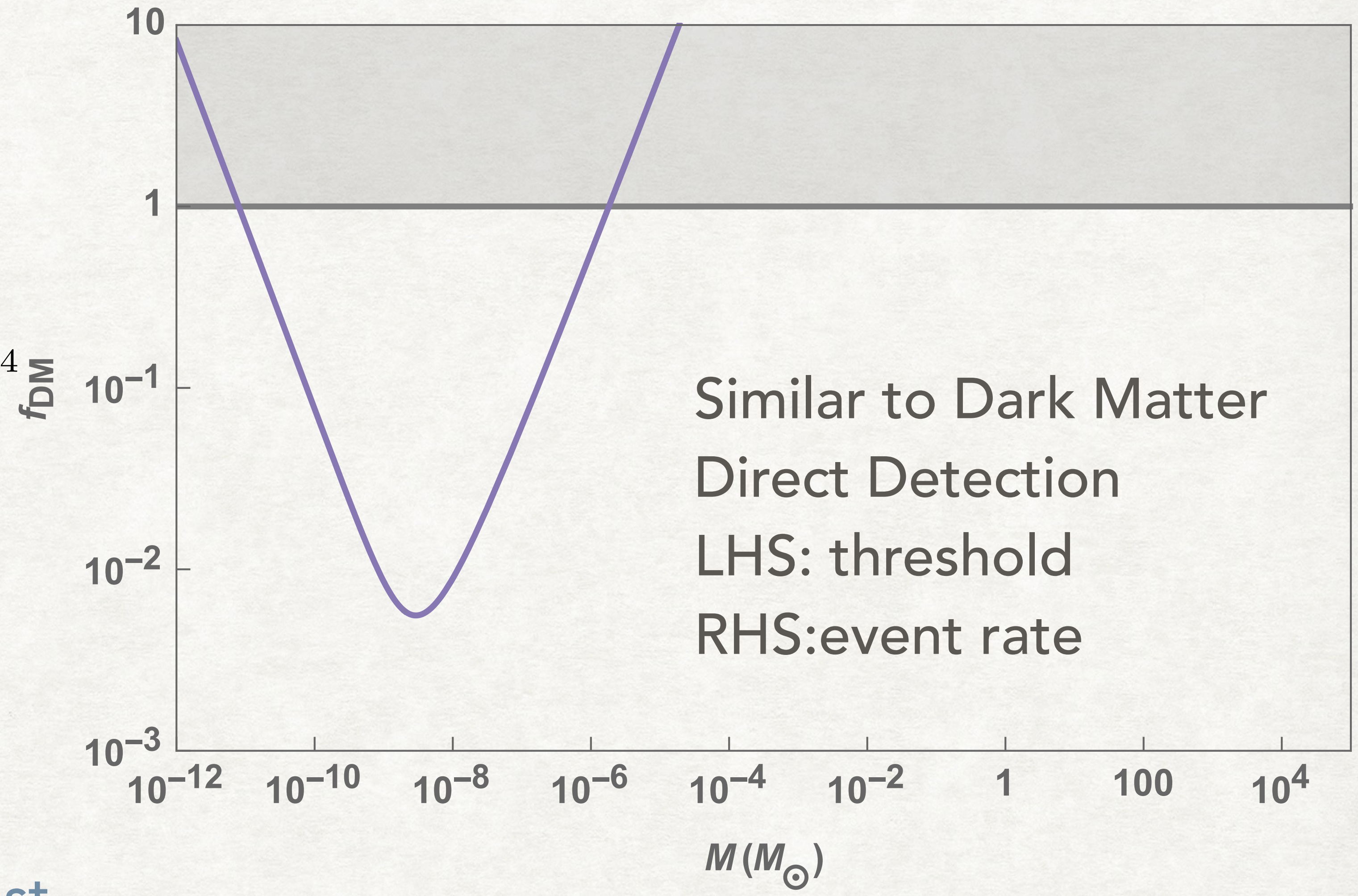


BOUNDS FROM DYNAMIC SIGNALS (DOPPLER)

- For Doppler Pulsar Term,
- LHS:

$$f^{L}_{\mathrm{D,\ dyn}} \lesssim 0.01 \left(\frac{10^{-9}\,M_\odot}{M}\right)\left(\frac{200}{N_P}\right)\left(\frac{20\ \mathrm{yr}}{T}\right)^4$$

Earth term scales the same way



Similar to Dark Matter Direct Detection

LHS: threshold

RHS:event rate

- At some Mass M, even the nearest PBH starts failing dynamic constraint.

$$f^{R}_{\mathrm{D,\ dyn}} \lesssim 3 \left(\frac{M}{10^{-7}\,M_\odot}\right)\left(\frac{200}{N_P}\right)\left(\frac{20\ \mathrm{yr}}{T}\right)^3$$

Earth term has Np=1

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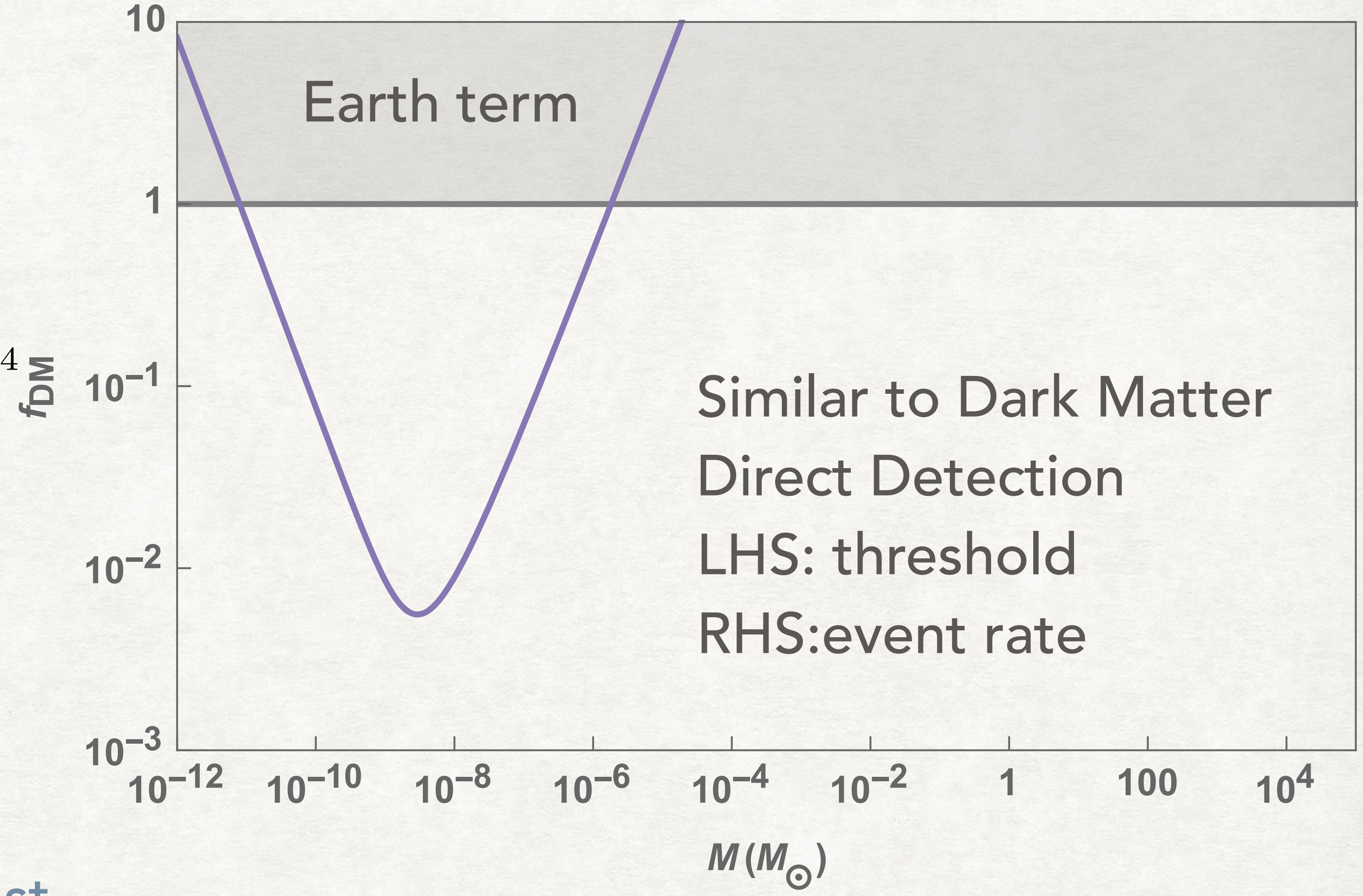

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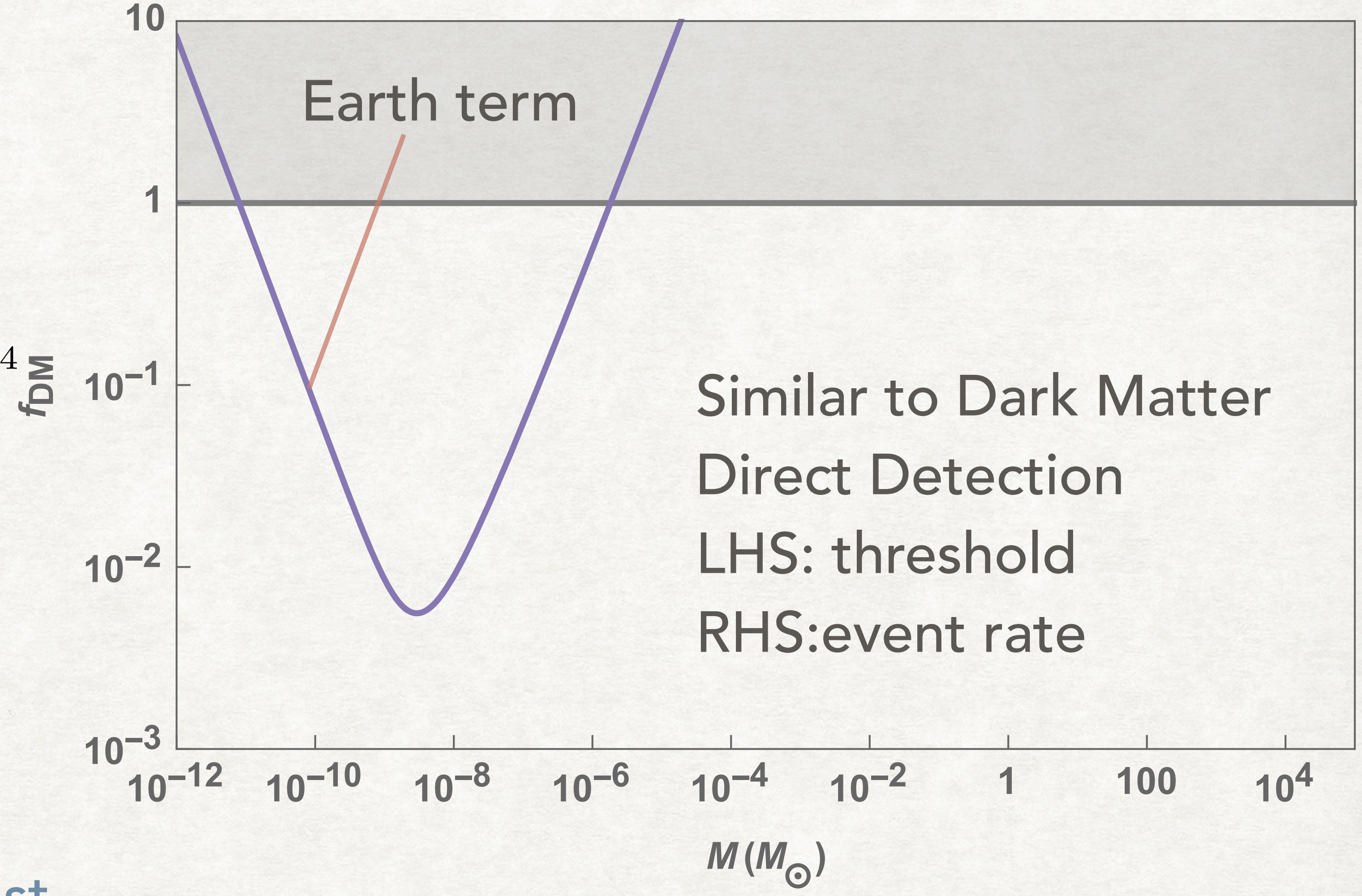

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Earth term has Np=1

SHAPIRO DELAY

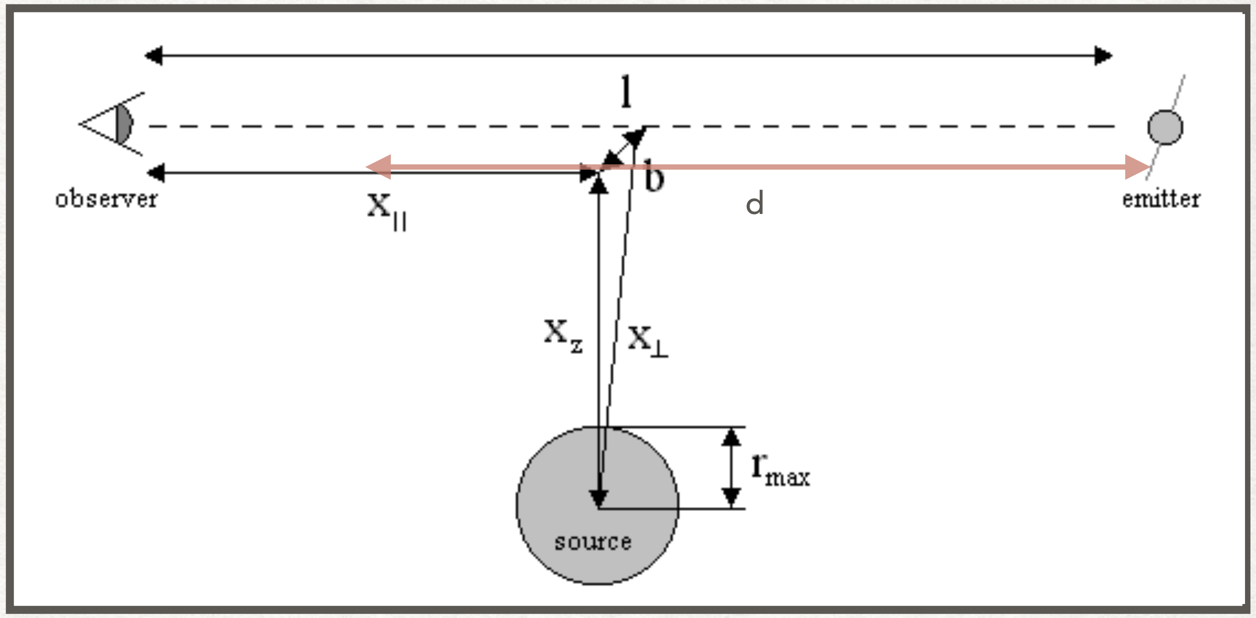
- Similar to Sachs-Wolfe effect
- In frequency domain given by,

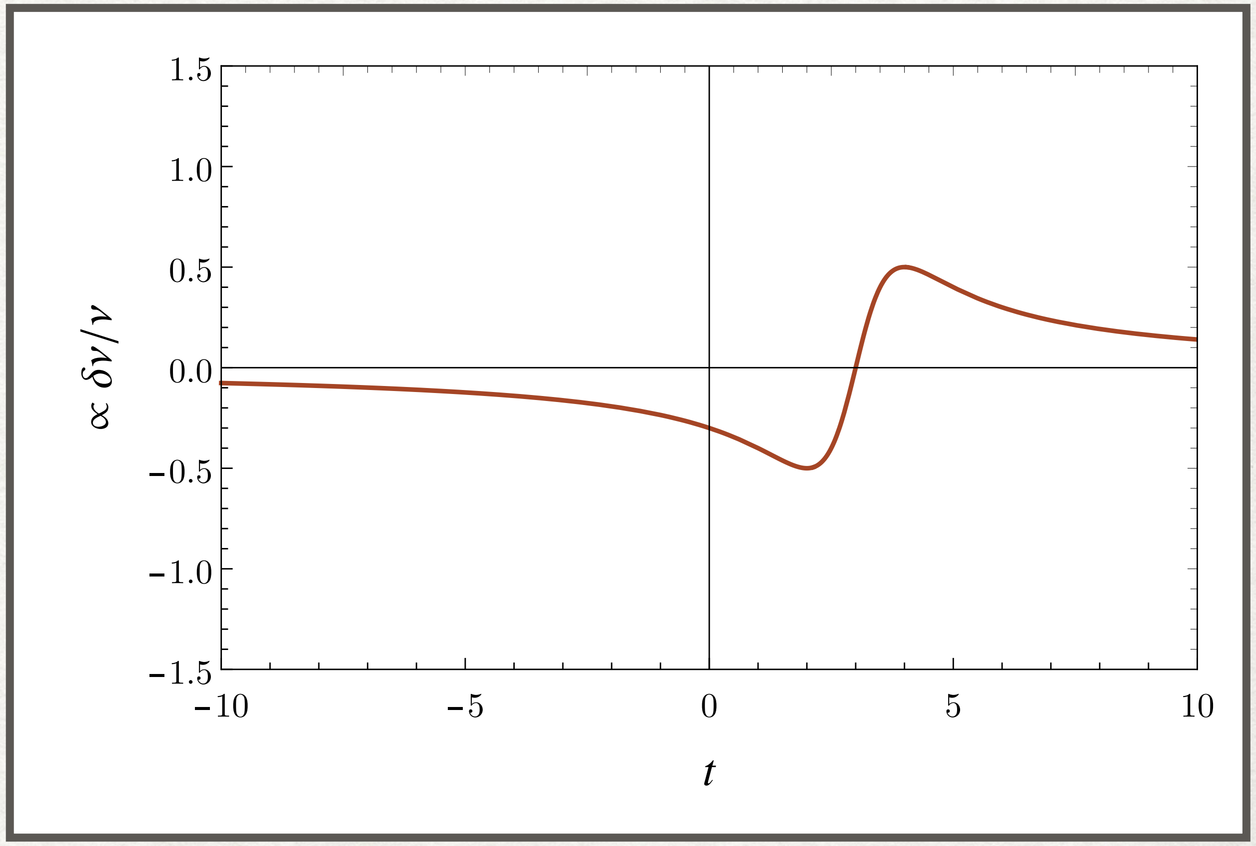
$$\left(\frac{\delta\nu}{\nu}\right)_S = -2\int \mathbf{v}\cdot\nabla\Phi\,dz$$

- For a point object,
- $\left(\frac{\delta\nu}{\nu}\right)_S = \frac{4GM}{\tau_S}\frac{x_S}{1+x_S^2}$

Duration of signal

Dimensionless time variable





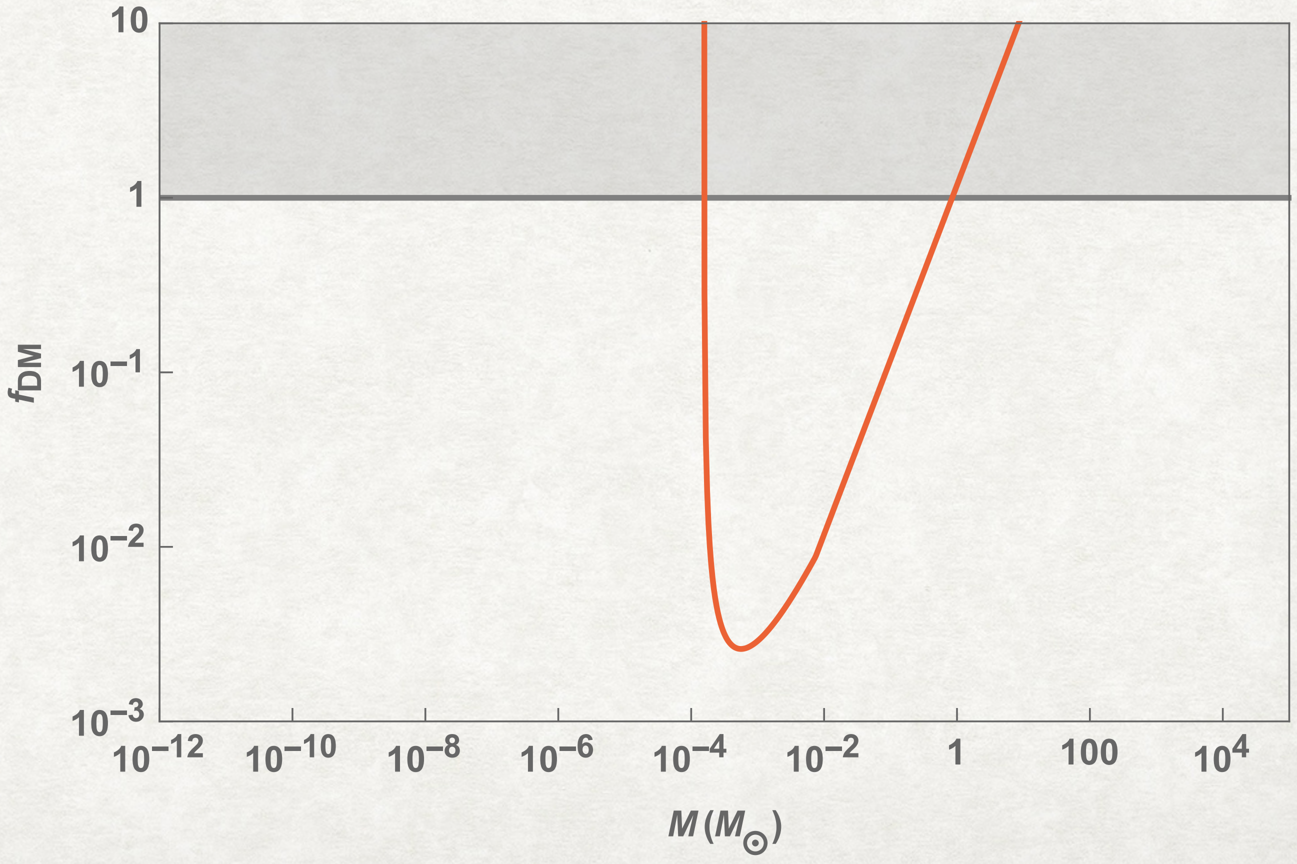
BOUNDS FROM DYNAMIC SIGNALS (SHAPIRO)

- For small enough $\tau_{\min}$,

$$\mathrm{SNR} = 4\frac{GM}{c^6 t_{\mathrm{rms}}}\sqrt{\frac{T}{\Delta t}}$$

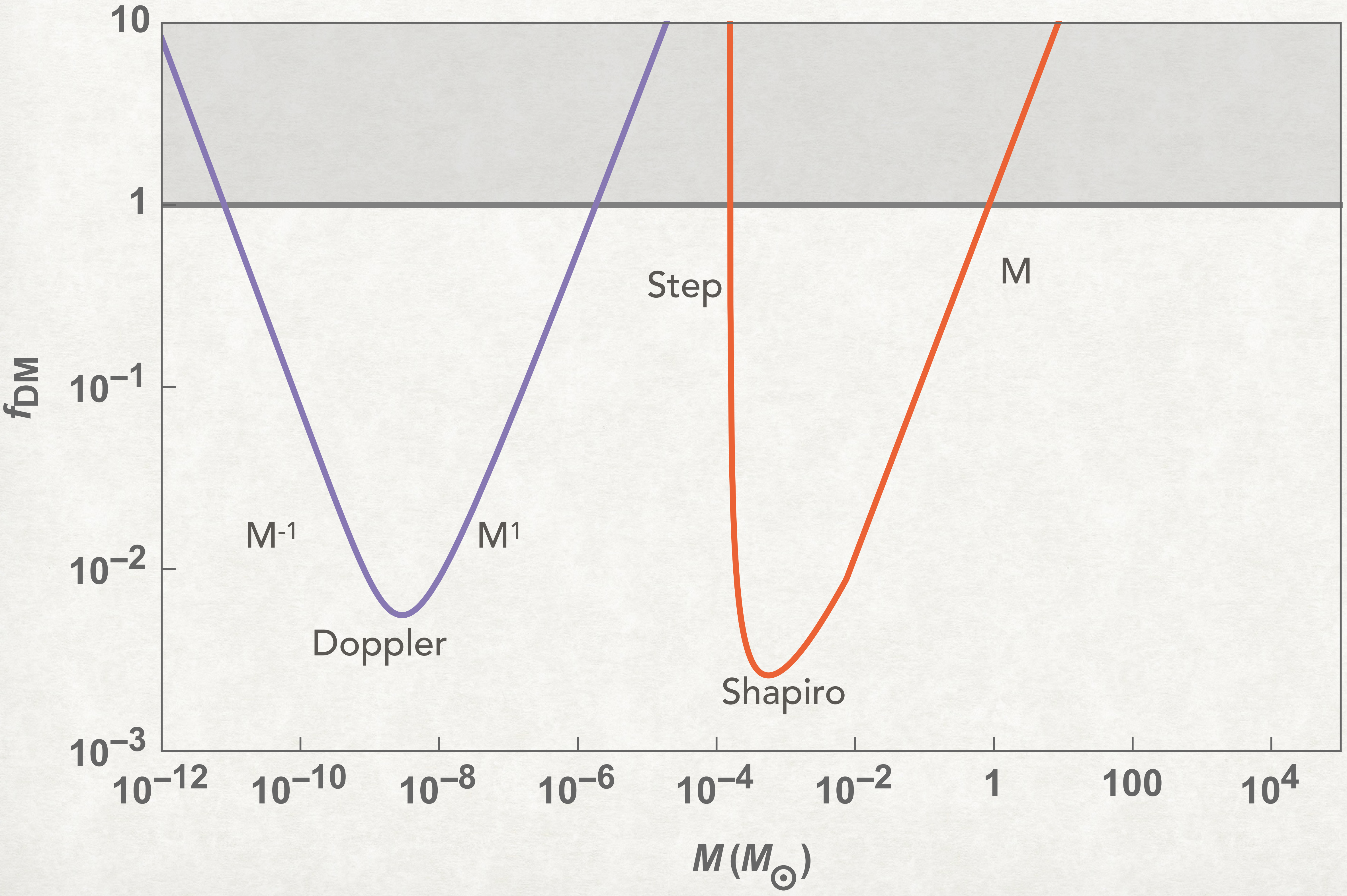
- Low enough masses are simply incapable of producing signal
- RHS just like before, f ~ M,

$$f^{R}_{\mathrm{S,\ dyn}} \lesssim 0.8\left(\frac{M}{10^{-2}\,M_\odot}\right)\left(\frac{200}{N_P}\right)\left(\frac{20\ \mathrm{yr}}{T}\right)^2$$



SHAPIRO SIGNAL SELDOM HAVE AN EARTH TERM:
SAMPLING VOLUMES DO NOT OVERLAP

FRACTION VS M SCALING - DYNAMIC LIMIT

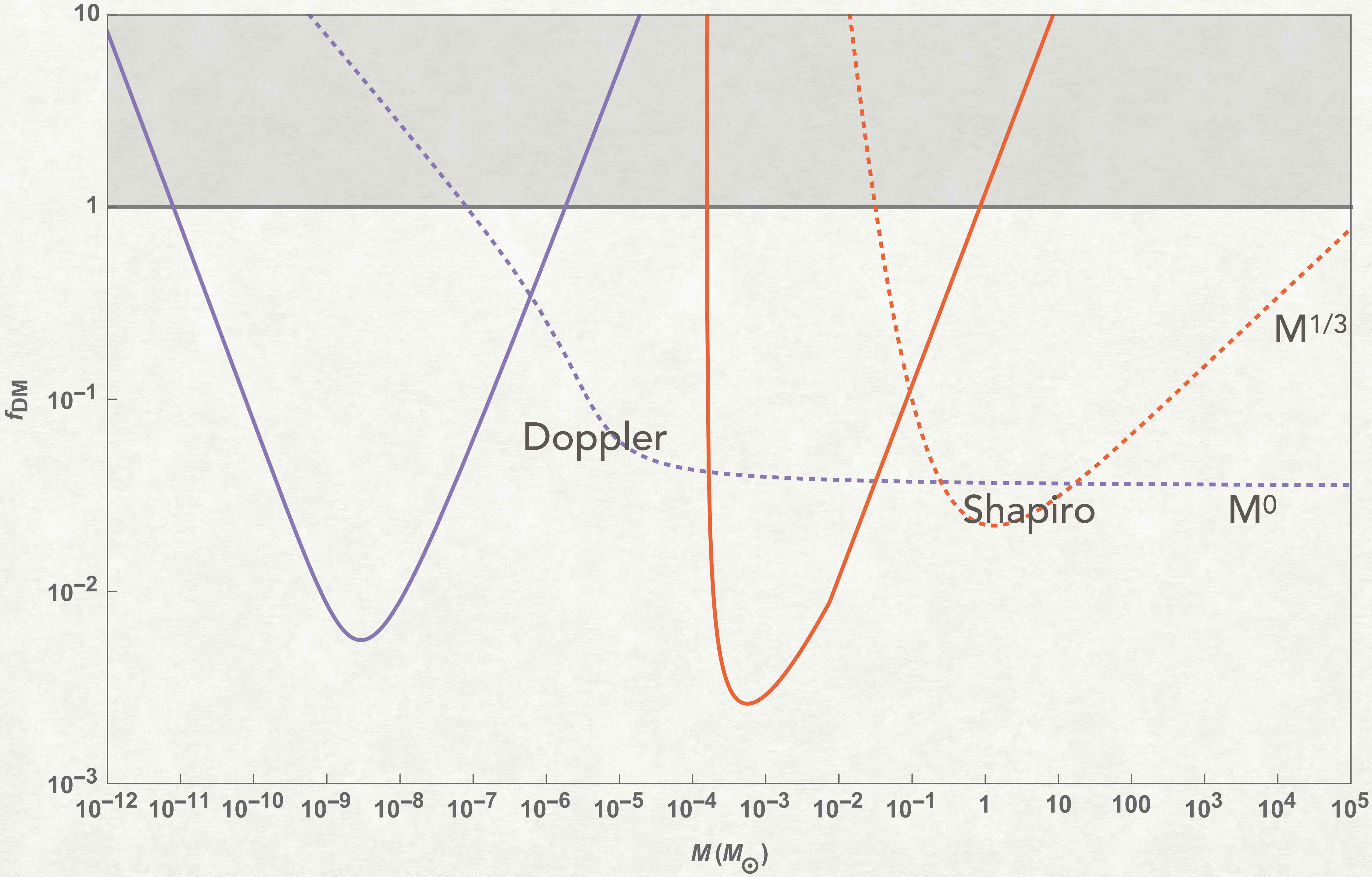


SIGNAL TO NOISE RATIO (STATIC SIGNALS)

Signal Duration $\gg$ Time of observation

- Taylor expand signal
- A constant first derivative i.e. spin-down already observed (incalculable from first principles).
- Subtracted as part of the fitting procedure.
- Second derivative much less common.
- Non-observation of second derivative can be used to set constraints.

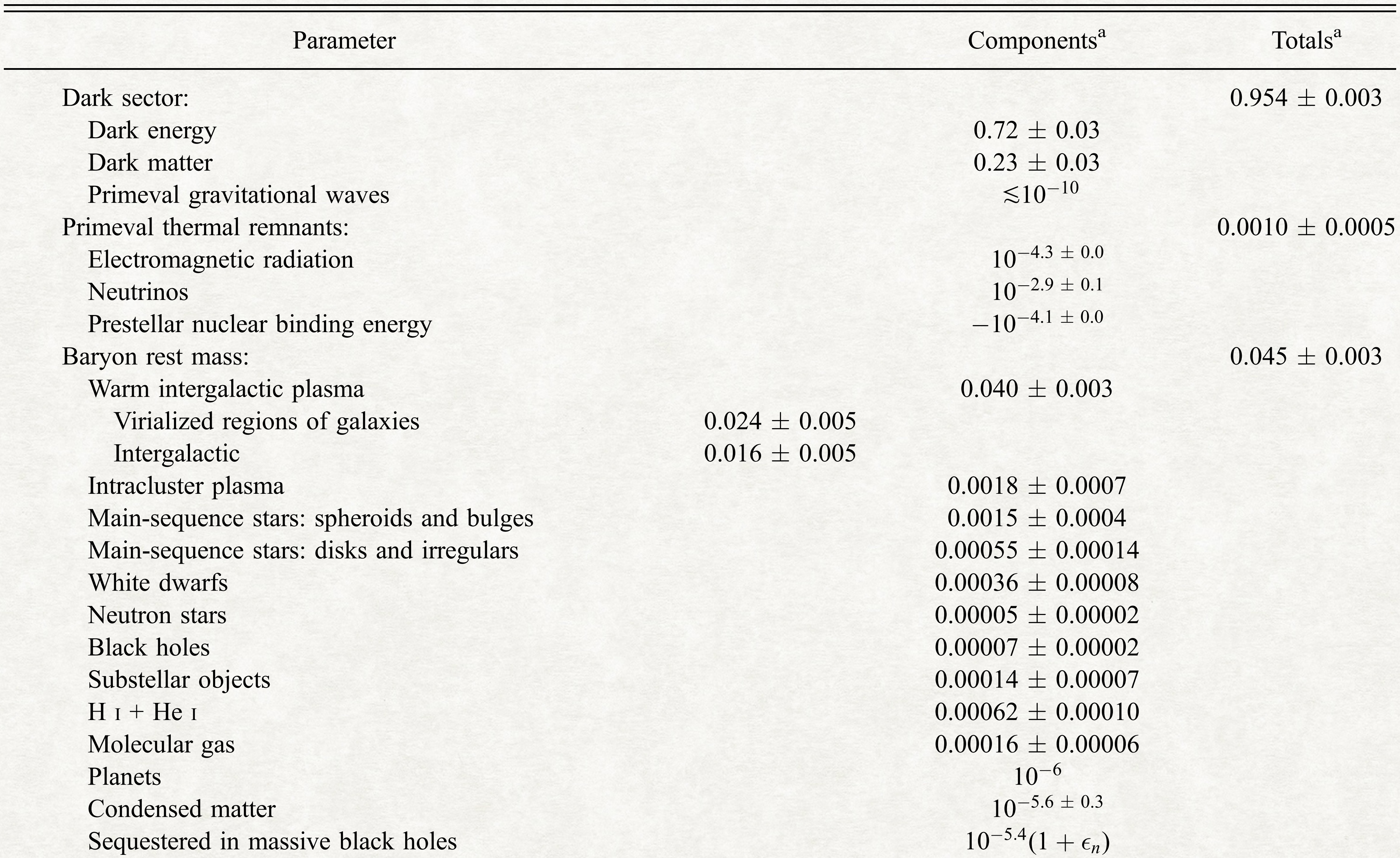
FRACTION VS M SCALING -STATIC



MAJOR BACKGROUNDS

BARYONS

THE COSMIC ENERGY INVENTORY Masataka Fukugita, P. J. E. Peebles, 0406095



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Dark energy		0.72 ± 0.03	
Dark matter		0.23 ± 0.03	
Primeval gravitational waves		$\lesssim 10^{-10}$	
Primeval thermal remnants:			0.0010 ± 0.0005
Electromagnetic radiation		$10^{-4.3 \pm 0.0}$	
Neutrinos		$10^{-2.9 \pm 0.1}$	
Prestellar nuclear binding energy		$-10^{-4.1 \pm 0.0}$	
Baryon rest mass:			0.045 ± 0.003
Warm intergalactic plasma		0.040 ± 0.003	
Virialized regions of galaxies	0.024 ± 0.005		
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Main-sequence stars: spheroids and bulges		0.0015 ± 0.0004	
Main-sequence stars: disks and irregulars		0.00055 ± 0.00014	
White dwarfs		0.00036 ± 0.00008	
Neutron stars		0.00005 ± 0.00002	
Black holes		0.00007 ± 0.00002	
Substellar objects		0.00014 ± 0.00007	
H I + He I		0.00062 ± 0.00010	
Molecular gas		0.00016 ± 0.00006	
Planets		10^{-6}	
Condensed matter		$10^{-5.6 \pm 0.3}$	
Sequestered in massive black holes		$10^{-5.4}(1 + \epsilon_n)$	

Most of the baryonic component will also be co-rotating with pulsar or earth

MAJOR BACKGROUNDS

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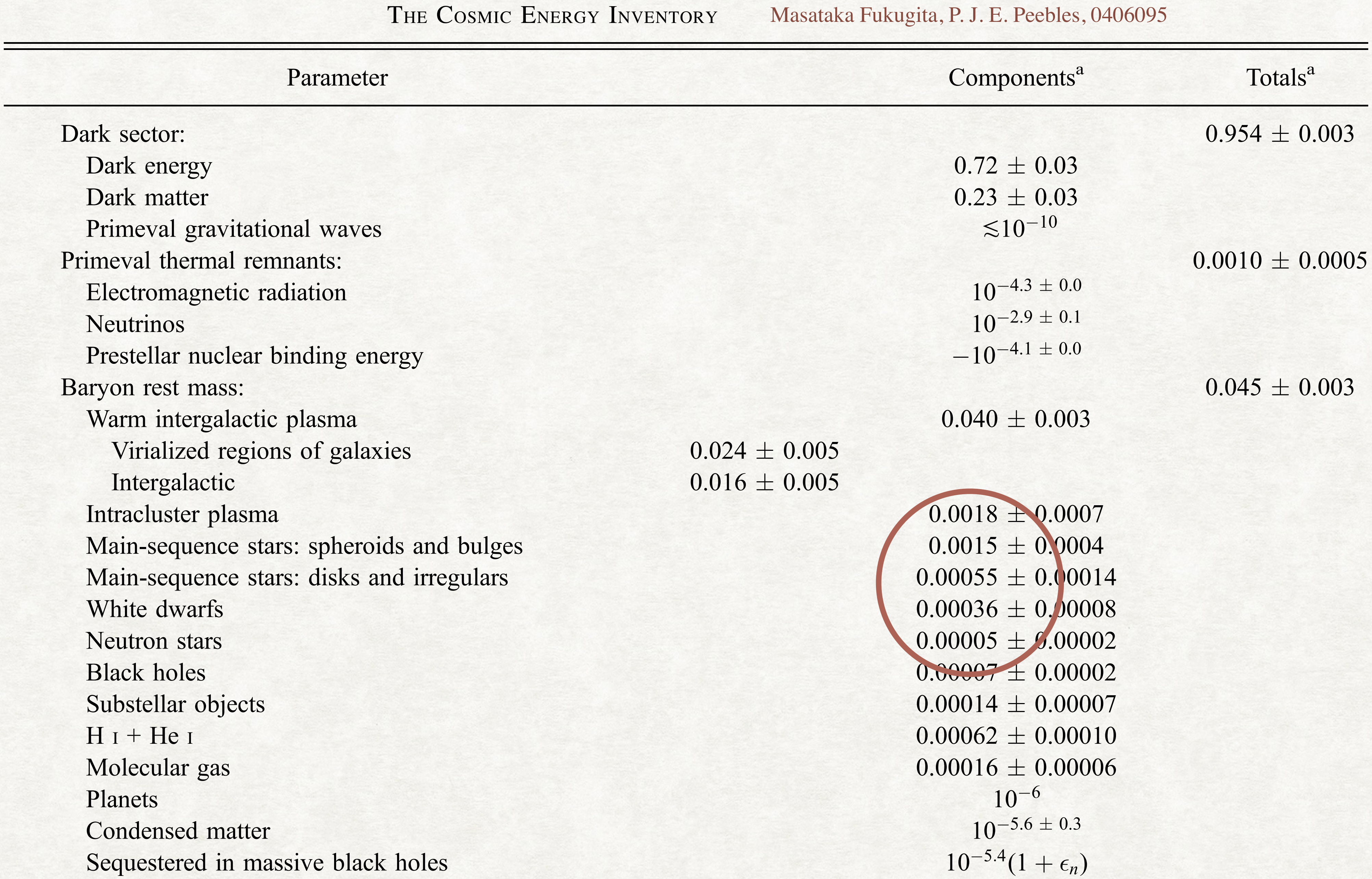

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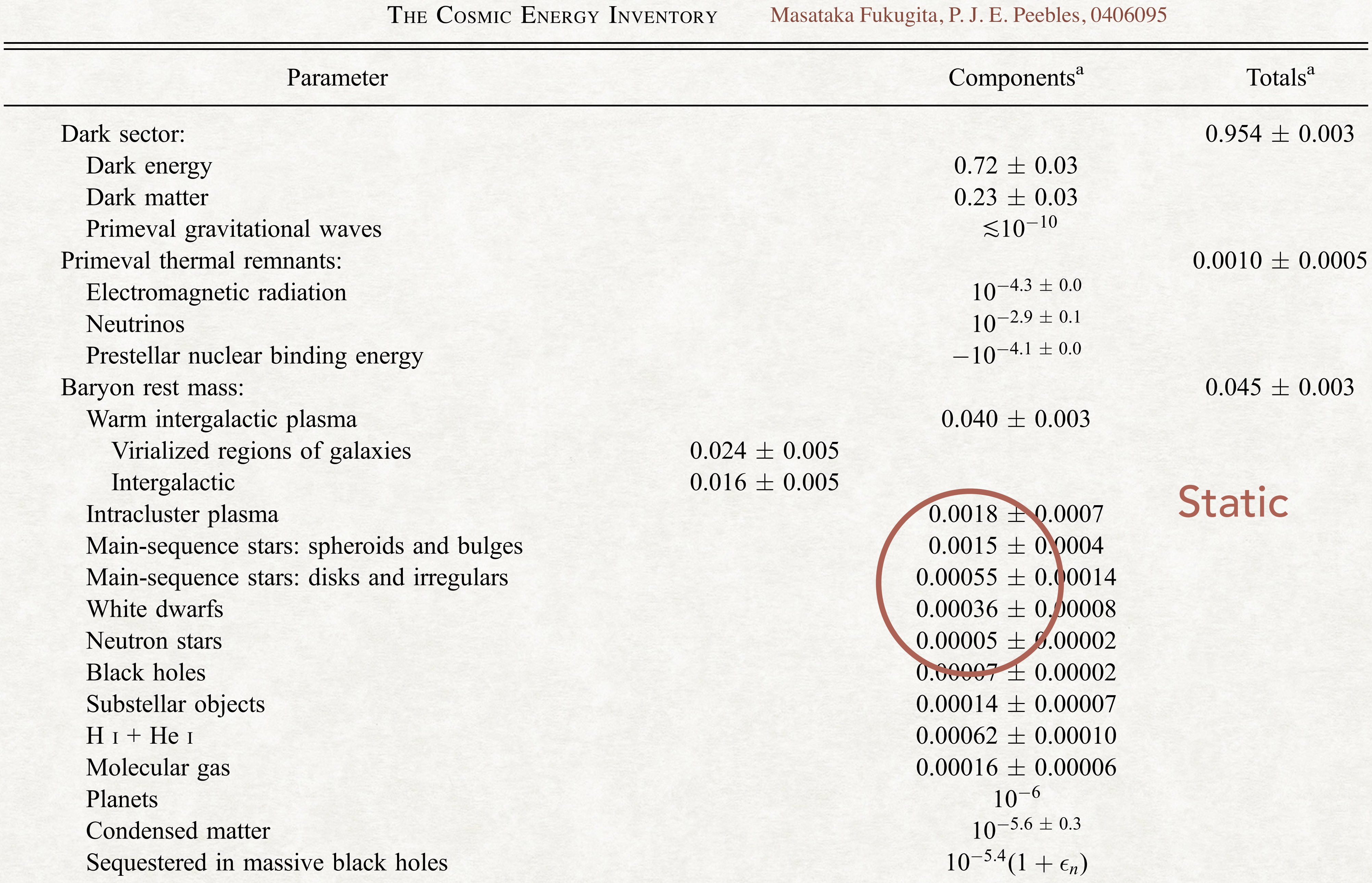

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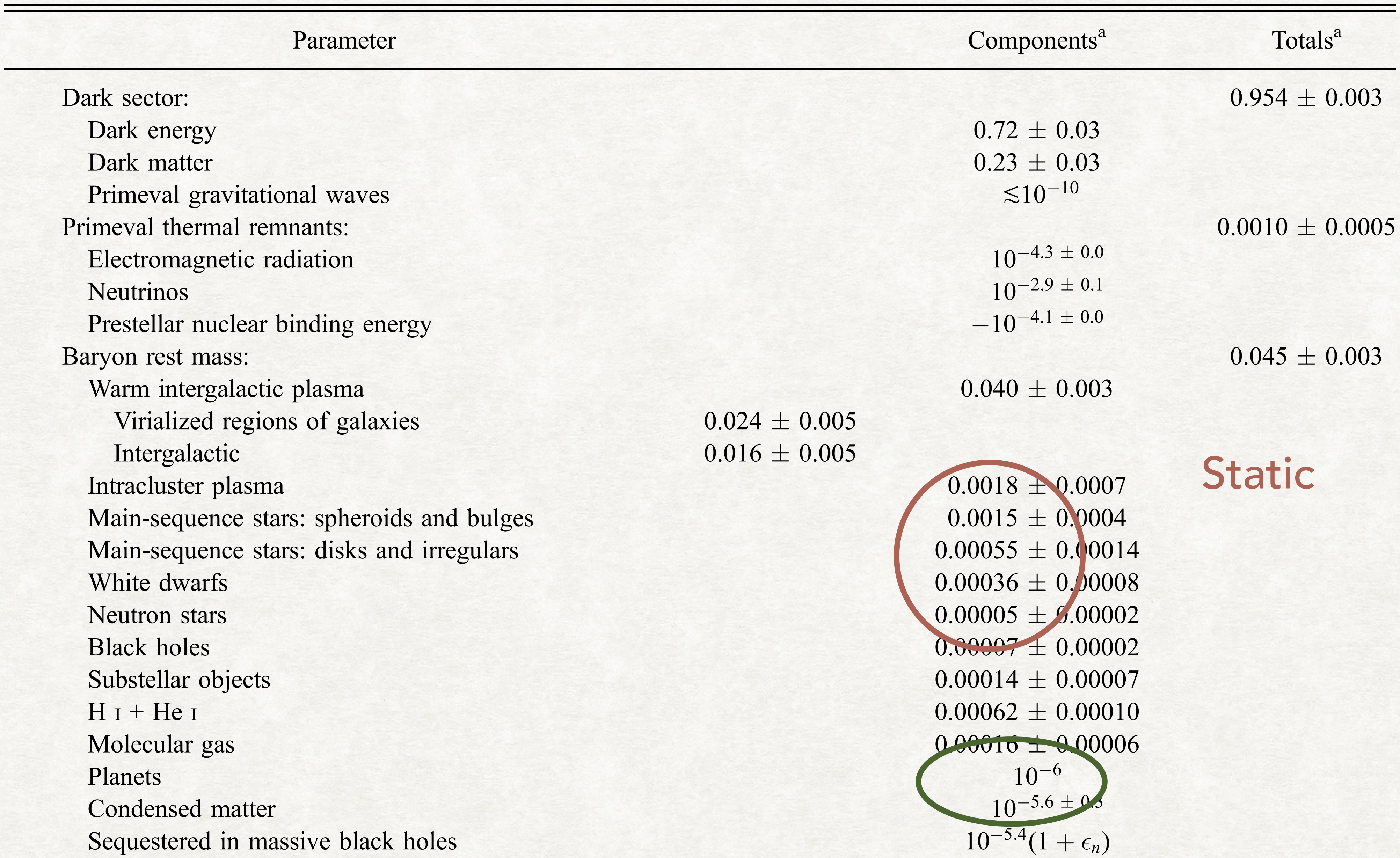

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Dynamic

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SOURCES OF BACKGROUND

- Glitches: Sudden increase in frequency, followed by a slow relaxation (days-year). Reduced significantly for Earth Term
- We considered a simplistic white noise
- In reality:
- Dispersion through interstellar medium - frequency dependent and red
- Some pulsars also suffer from intrinsic red noise
- Next step: use collaboration code to check signal survival

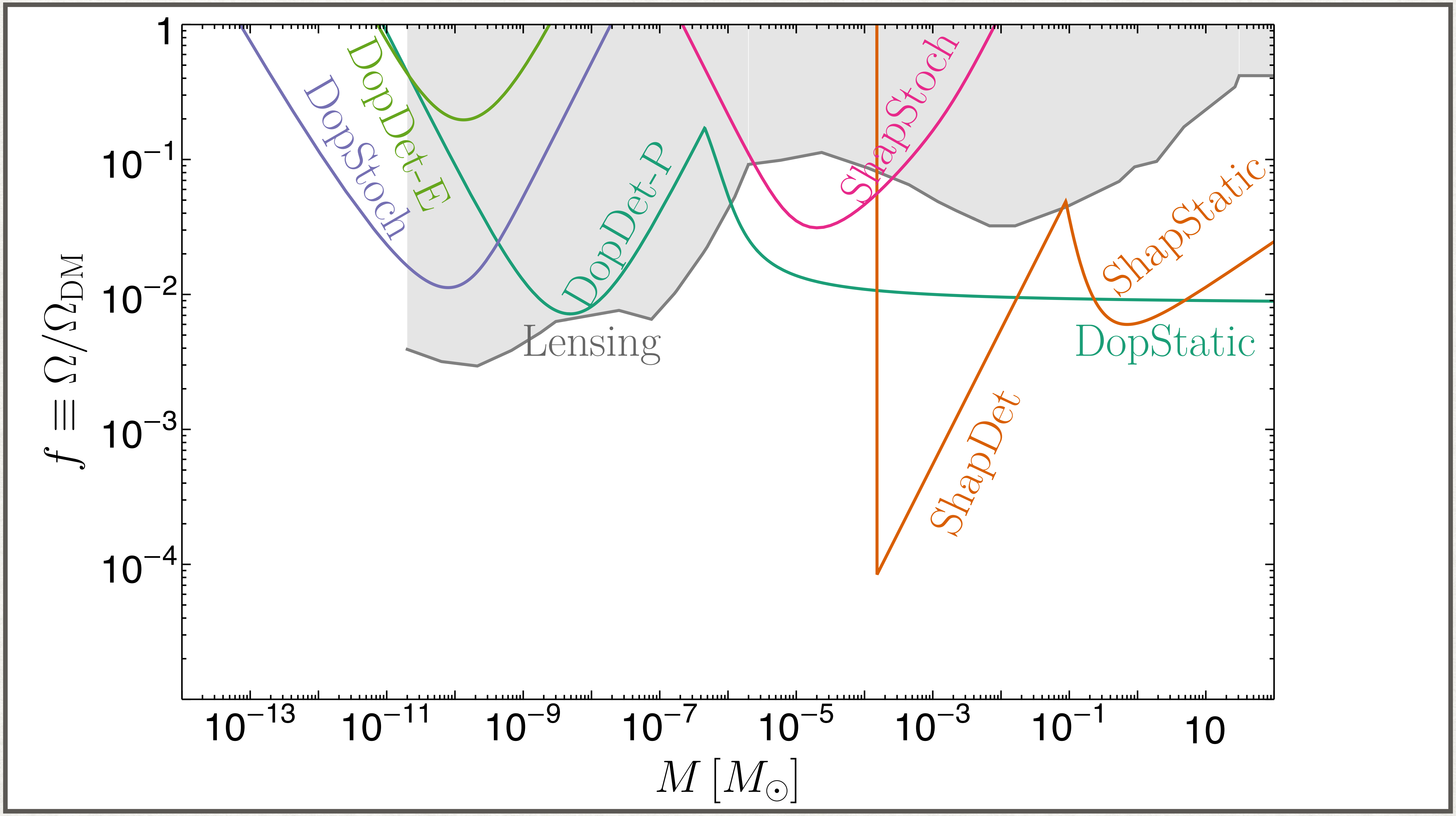
MONTECARLO SIMULATION

- Assume PBHs randomly distributed
- Isotropic Maxwell distribution with velocity truncated at v_{esc}.
- Simulate N_P randomly distributed pulsars at appropriate distances.
- Simulate order O(10^5) universes and require more than 95% universes pass SNR cut.

OPTIMISTIC ASSUMPTIONS

- $N_P = 1000$
- $T = 30$ yr
- $t_{rms} = 10$ ns
- $\Delta t = 1$ week
- $z_0 = 10$ kpc

RESULTS FOR PBH : OPTIMISTIC



Lensing constraint from Subaru, Machos, Eros, Ogle (MEO) and SN lensing

MORE DIFFUSE OBJECTS

- We have seen point-like objects till now.
- If size of the object < impact parameter,
  Newton’s shell theorem: treat object as point like
- Signal loss if object size > impact parameter.
- Can get conservative estimate with $M_{enc}(b)$.

EXTENDED OBJECTS

- Parametrize the profile as NFW.

$$\rho(r, M_{\rm vir}) = \frac{\rho_s}{(r/r_s)^{\alpha}\,(1 + r/r_s)^{\beta}}$$

$$\alpha = 1,\ \beta = 2$$

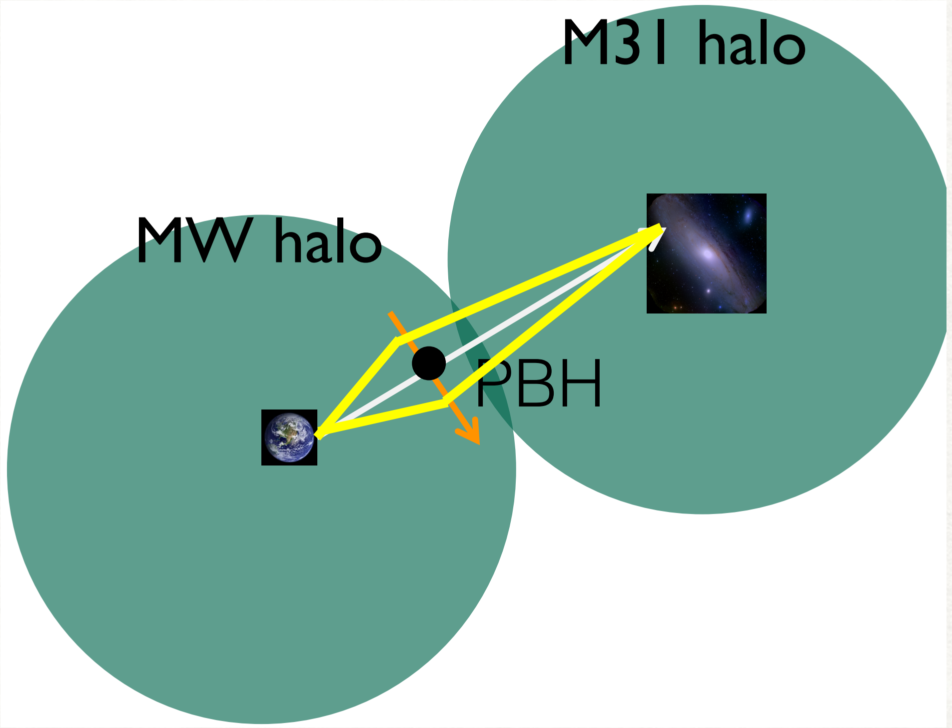
$$r_{\rm vir} \equiv (3M_{\rm vir}/800\pi\rho_c)^{1/3}$$

$$c \equiv r_{\rm vir}/r_s$$

Retrieve PBH in the large c limit

COMPARISON TO MICROLENSING

- Microlensing constraints from looking at M31/LMC



Source:Subaru

- Einstein radius

$$r_E \simeq \left(4GM\frac{(D_S - D_L)D_L}{D_S}\right)^{1/2}$$

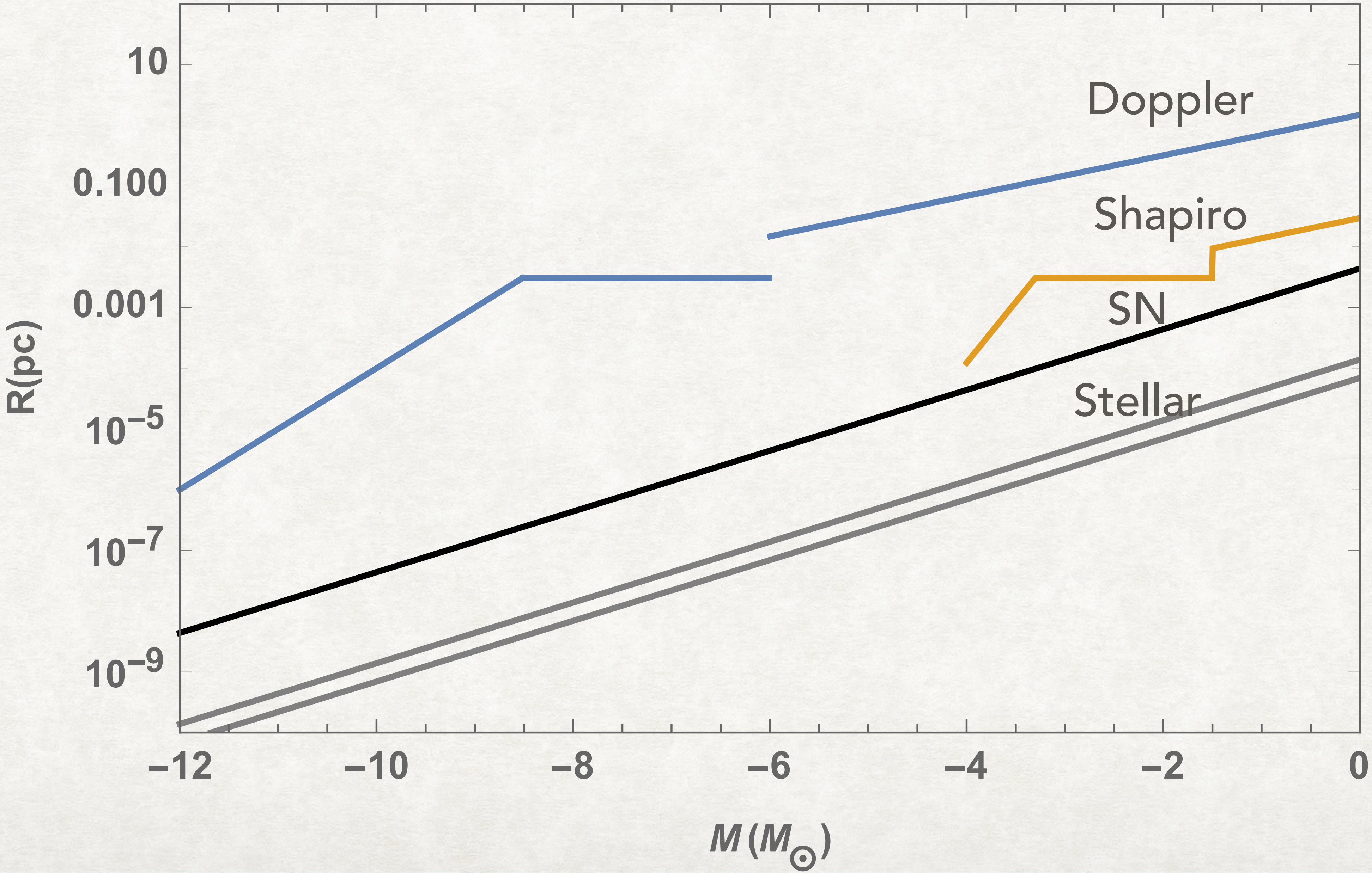
D_S= Earth Star distance
D_L= Earth Lens distance

IMPACT PARAMETER: PTA VS LENSING

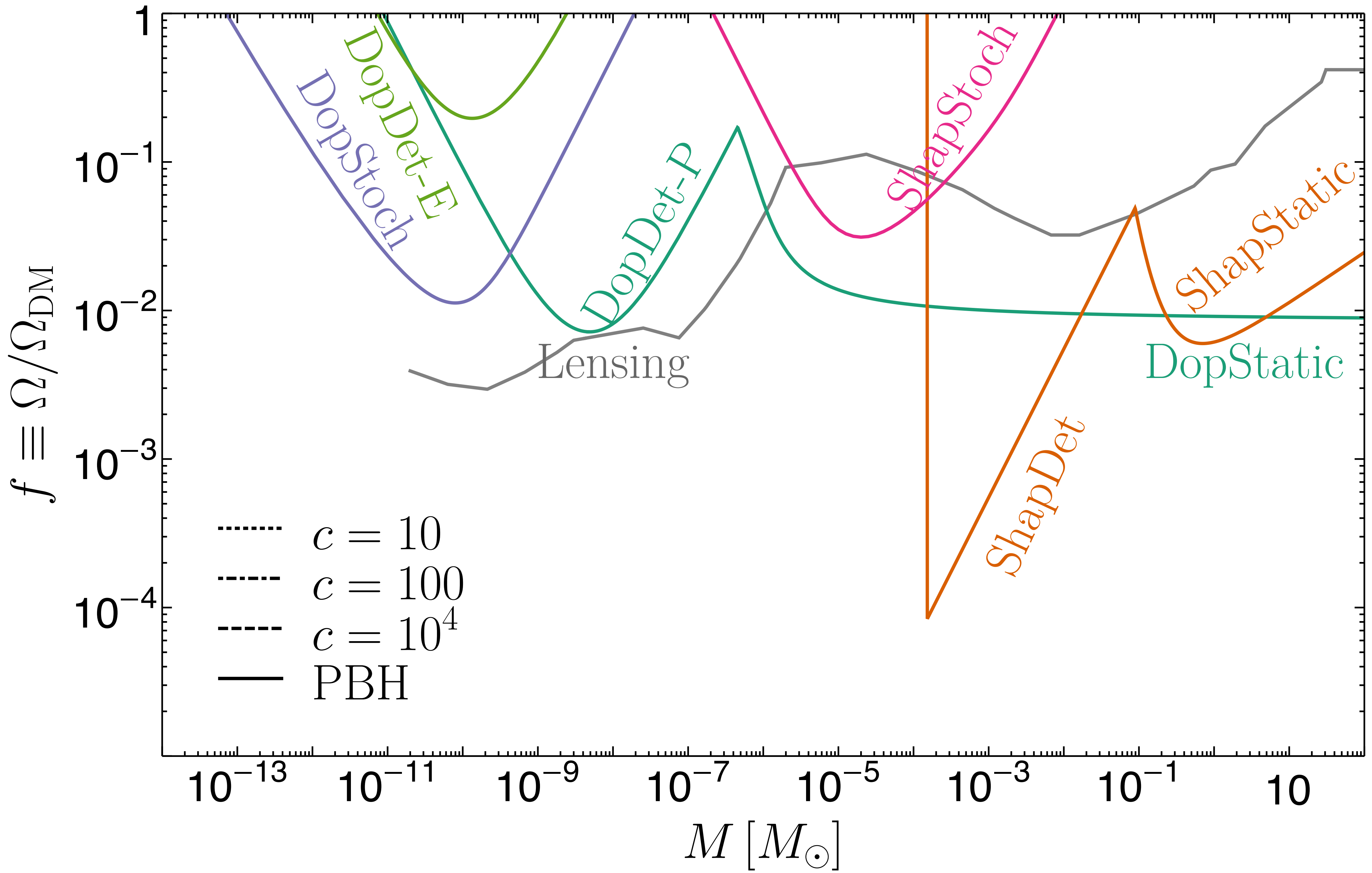
- Lensing: (Billion Stars x few hours) small impact parameter
- PTA: (100-1000 pulsars x few years) enormous impact parameter

$$r_{\rm PTA} \sim 10^{-3}\ \text{pc} \times \begin{cases} \frac{M}{10^{-9}\,M_\odot} & \text{(Doppler Dynamic)} \\ \left(\frac{M}{10^{-3}\,M_\odot}\right)^2 & \text{(Shapiro Dynamic)} \end{cases}$$

$$r_E \sim \begin{cases} 10^{-6}\ \text{pc}\left(\frac{M}{10^{-4}\,M_\odot}\right)^{\frac{1}{2}} & \text{(Stellar Lensing)} \\ 10^{-2}\ \text{pc}\left(\frac{M}{10\,M_\odot}\right)^{\frac{1}{2}} & \text{(Supernovae Lensing)} \end{cases}$$



LIMITS FOR DIFFUSE OBJECTS



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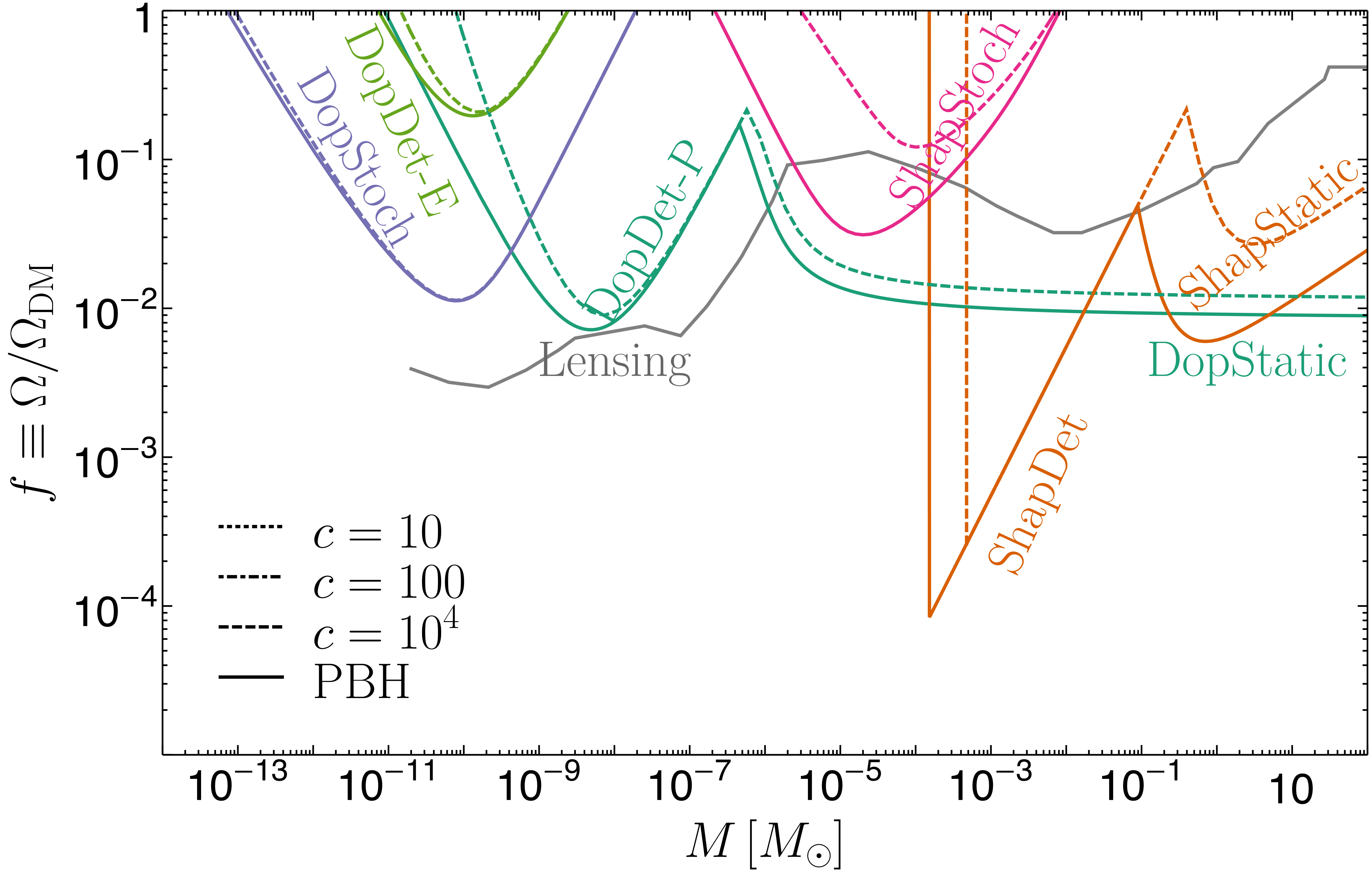

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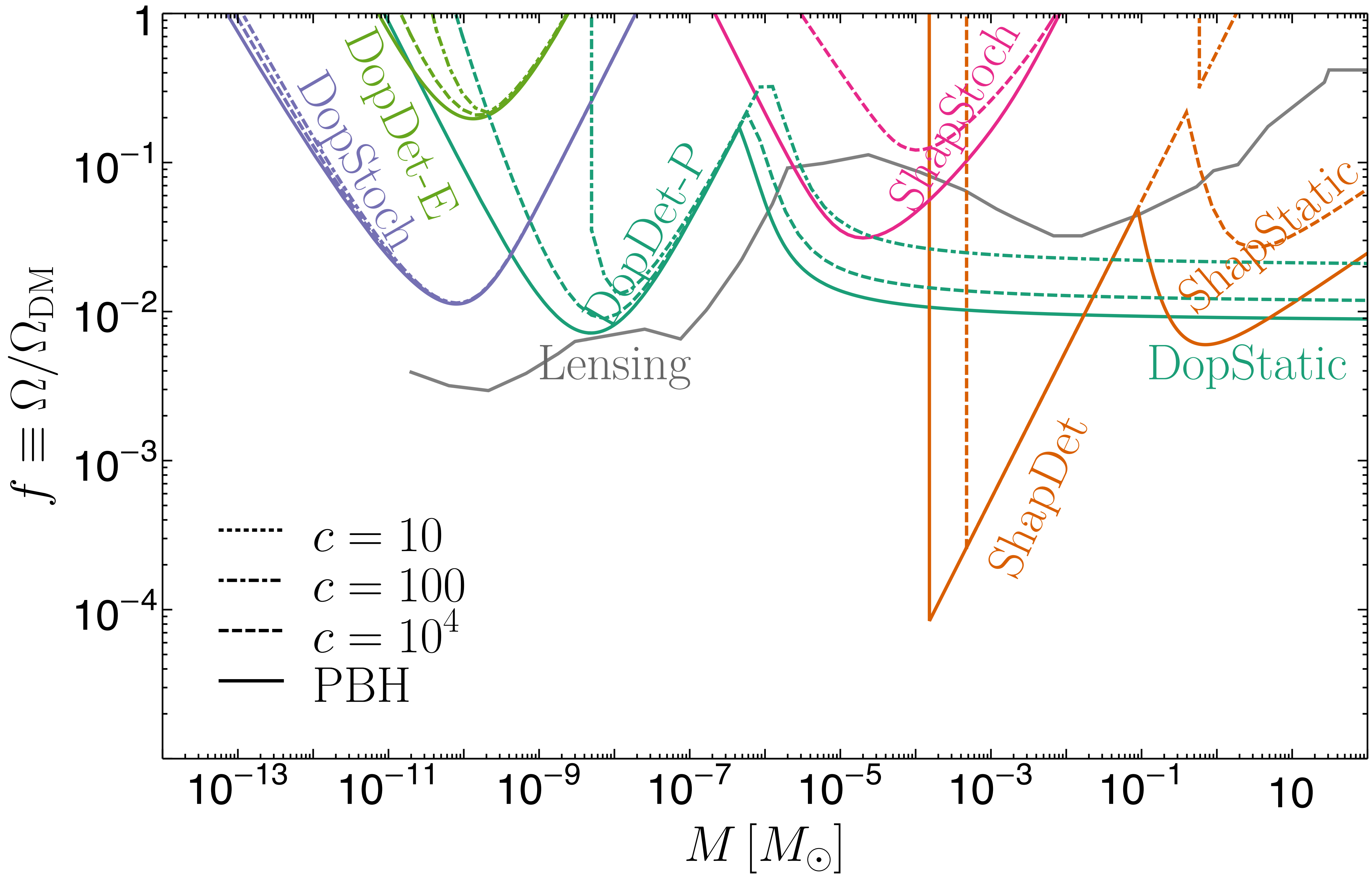
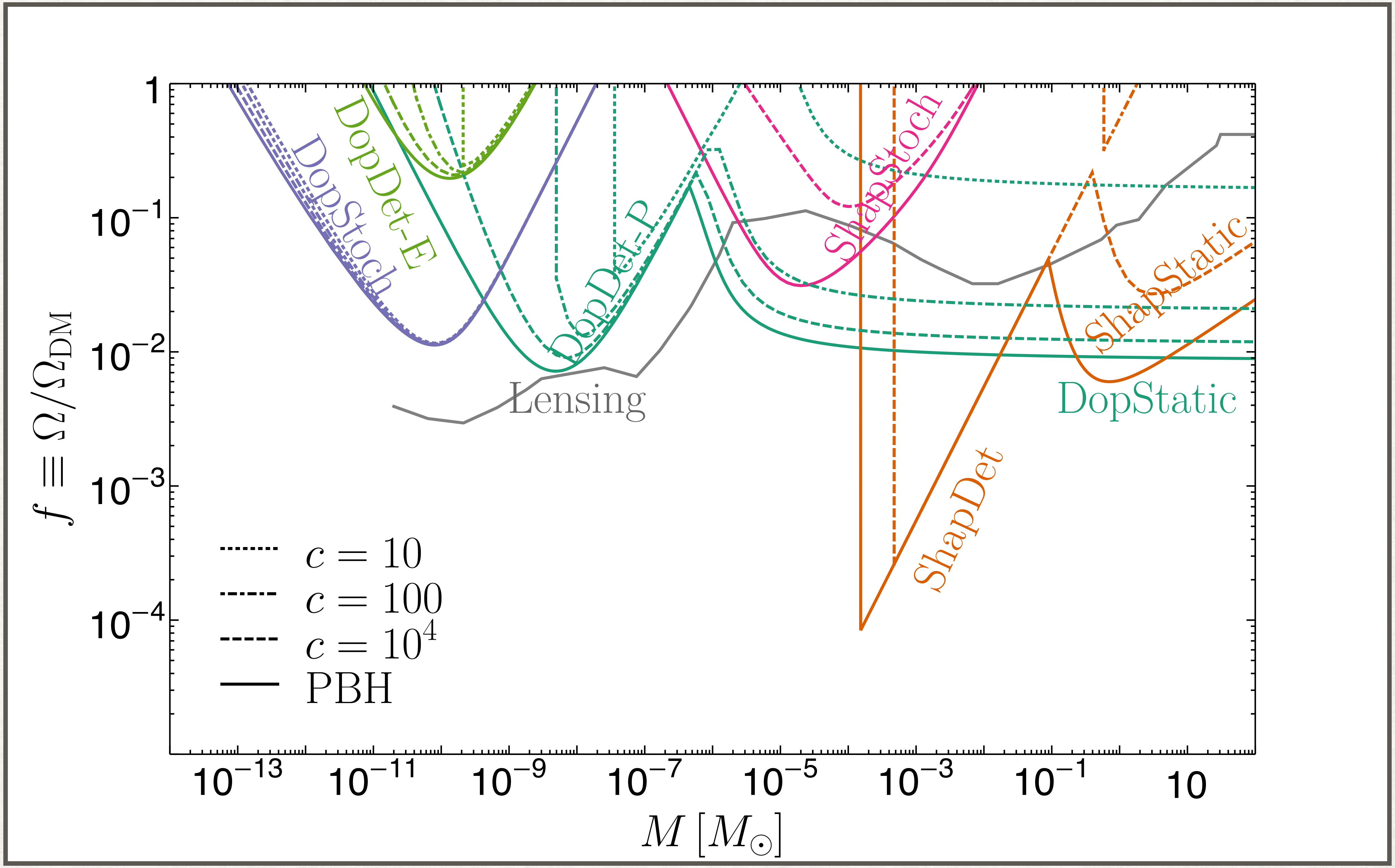


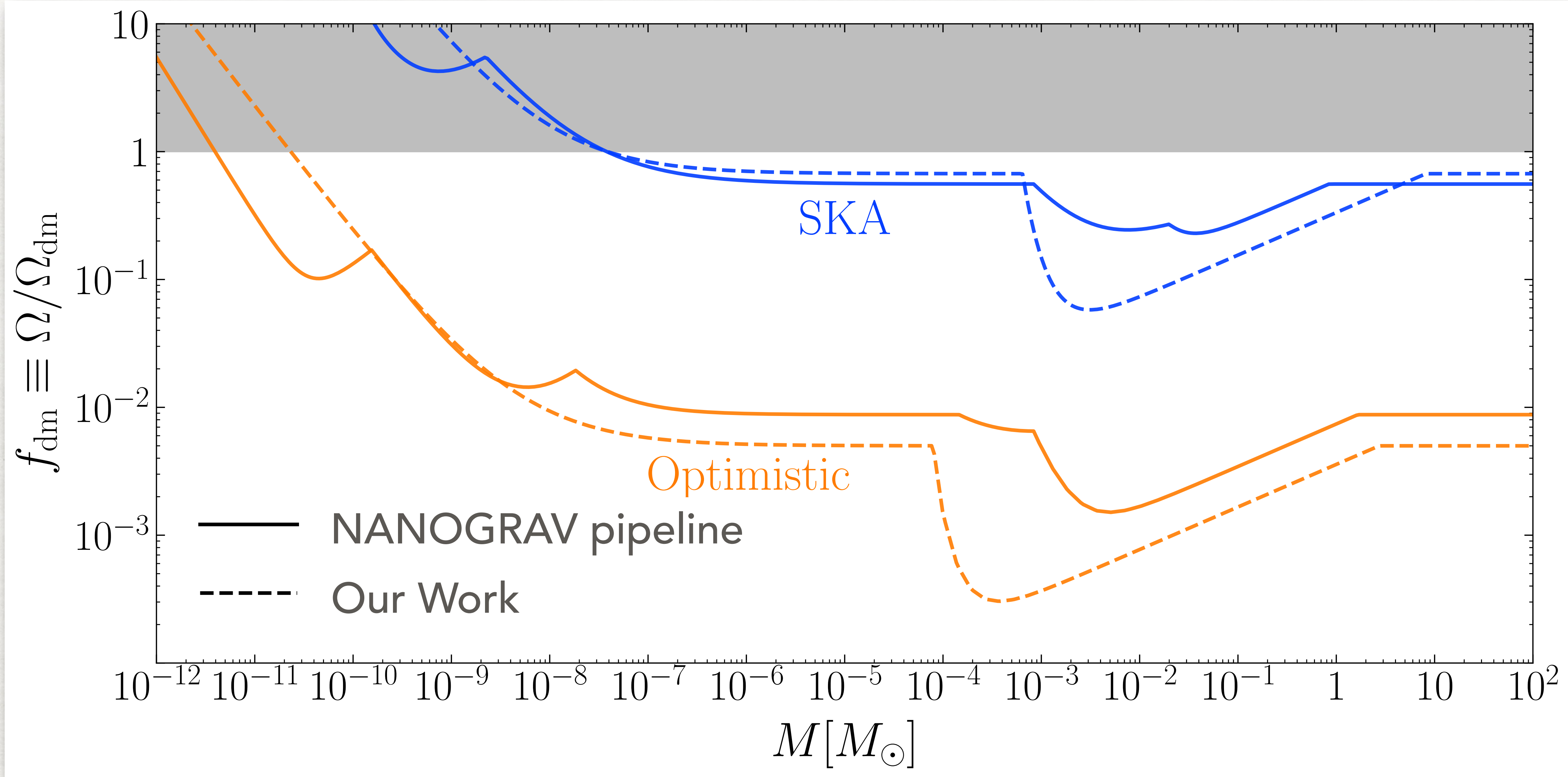
LIMITS FOR DIFFUSE OBJECTS



NANOGRAV ANALYSIS

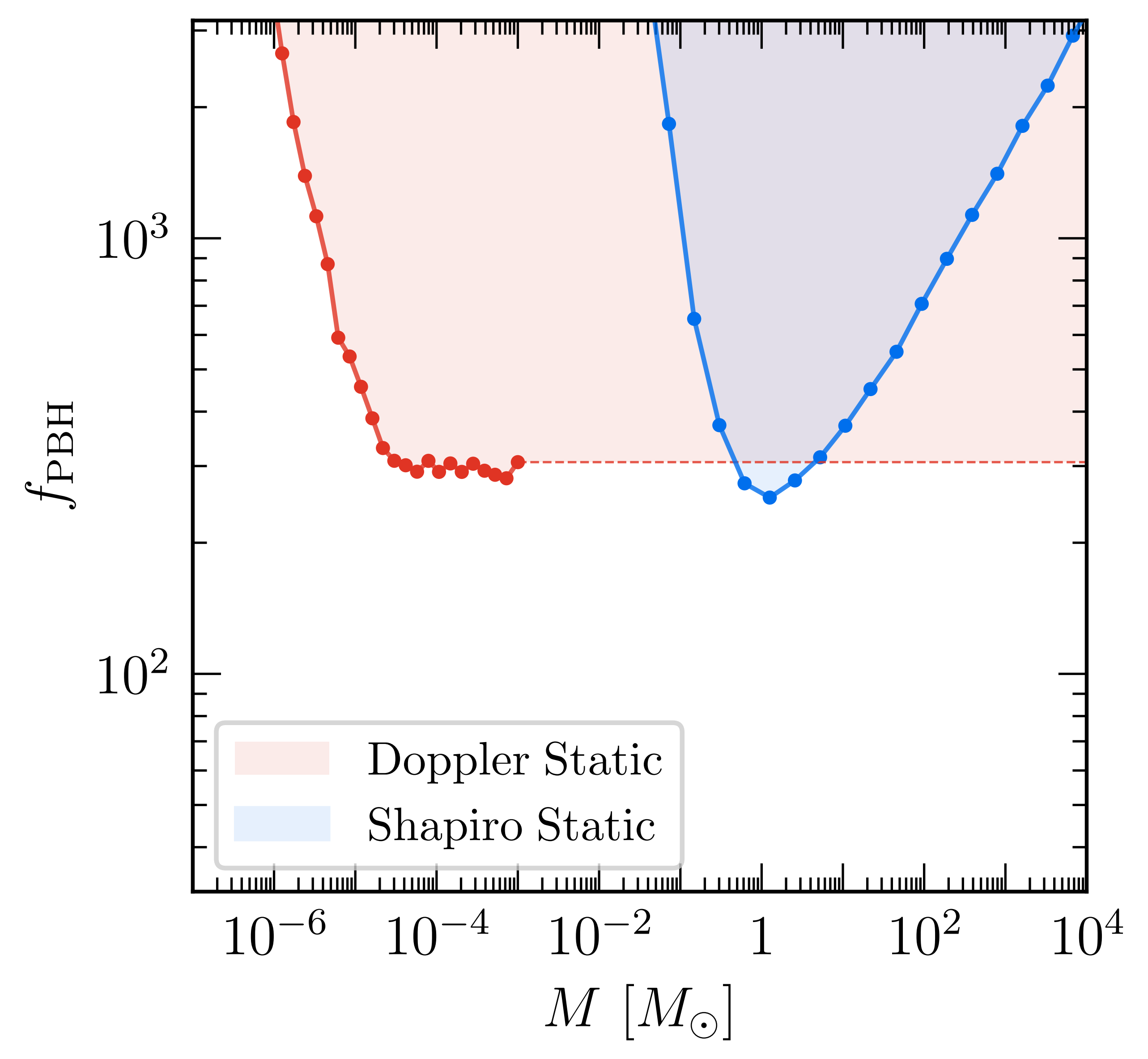
NANOGRAV PIPELINE

- Mock Data analysis. arXiv: 2104.06717 (Lee, Taylor, Tricke, Zurek)
- Includes subtractions of known background sources



CURRENT DATA RELEASE

- arXiv: 2306.16219 [The NANOGrav 15-year Data Set: Search for Signals from New Physics]



OUTLOOK

- MSPs across the GC?
- in DM rich environments?
- Extra galactic MSPs?
- Non-gravitational long range forces?

- Better understanding of subhalos today given an initial Power Spectrum
- Limits on subhalos today into limits on primordial power spectrum?
- Understanding better the map between substructure or the lack thereof today and particle physics models.

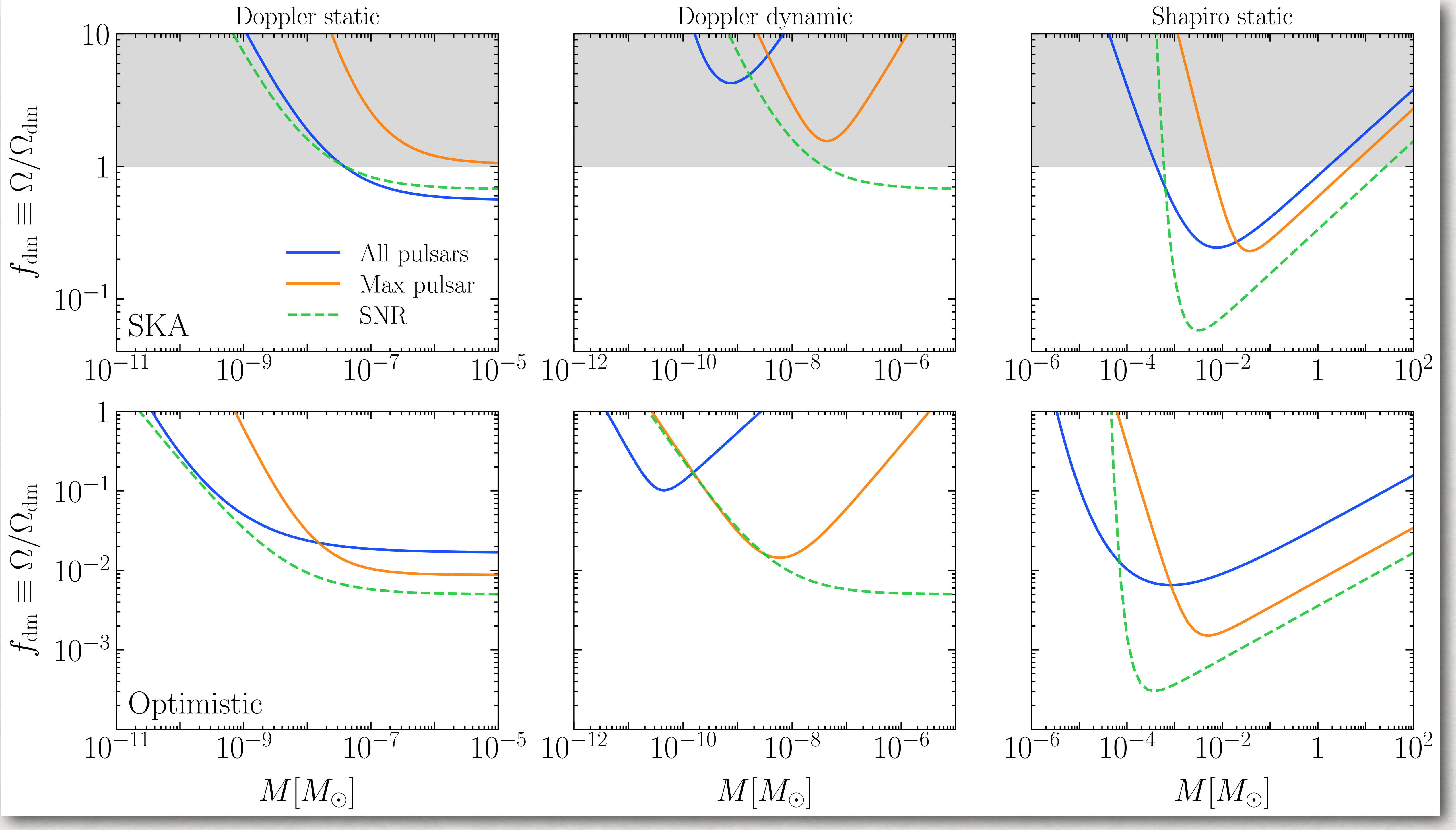
CONCLUSIONS

- Pulsar timing can probe structure at a wide range of small scales.
- Doppler and Shapiro delays, especially in the dynamic regime, can provide a compelling discovery signal for DM subhalos.
- Probing CDM subhalos could be viable.

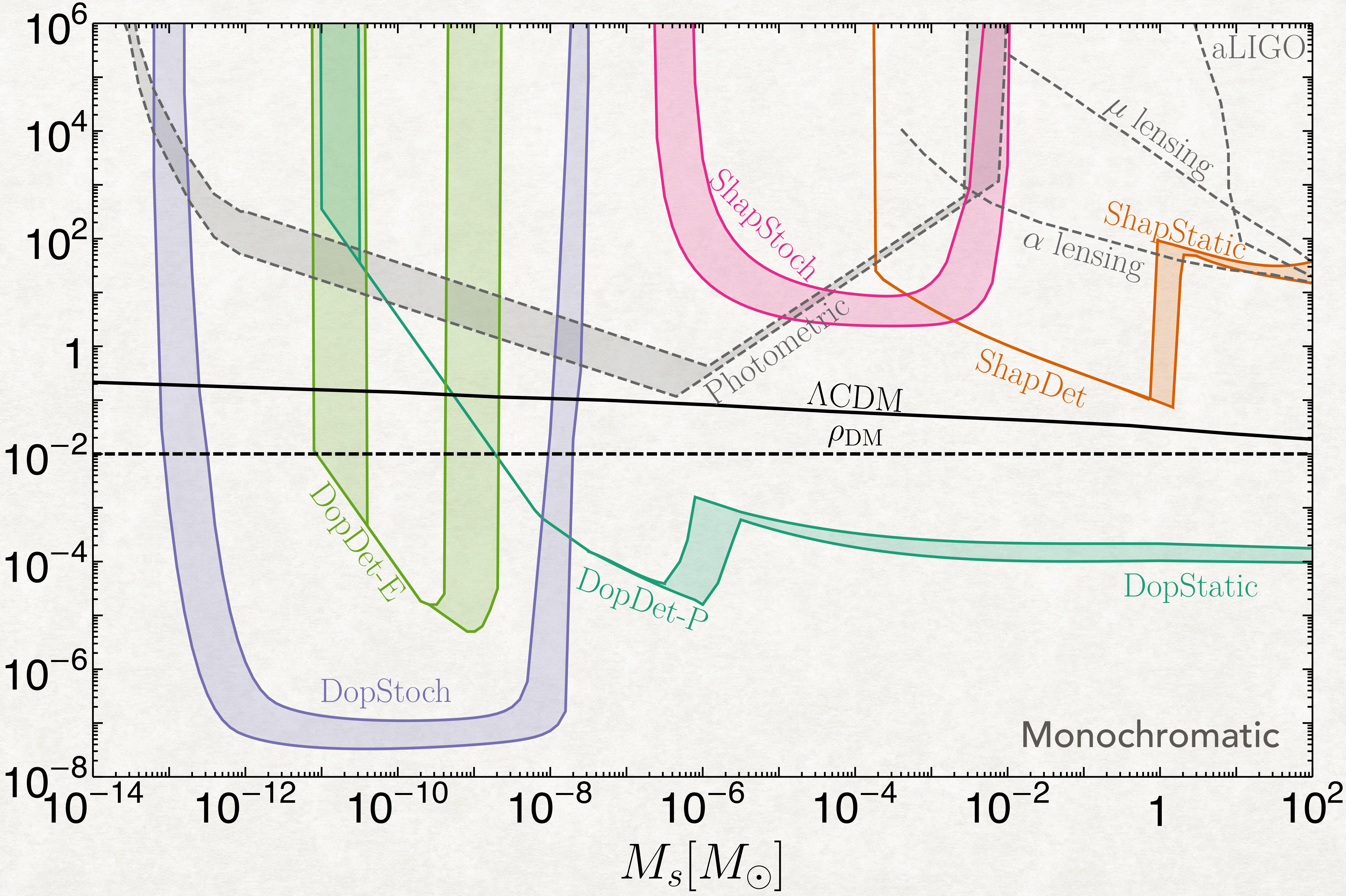
BACKUP

COMPARISON TO MOCK DATA

BAYESIAN ANALYSIS



TIDALLY STRIPPED CORES - OPTIMISTIC



Error bands correspond to f=1 and f=0.3

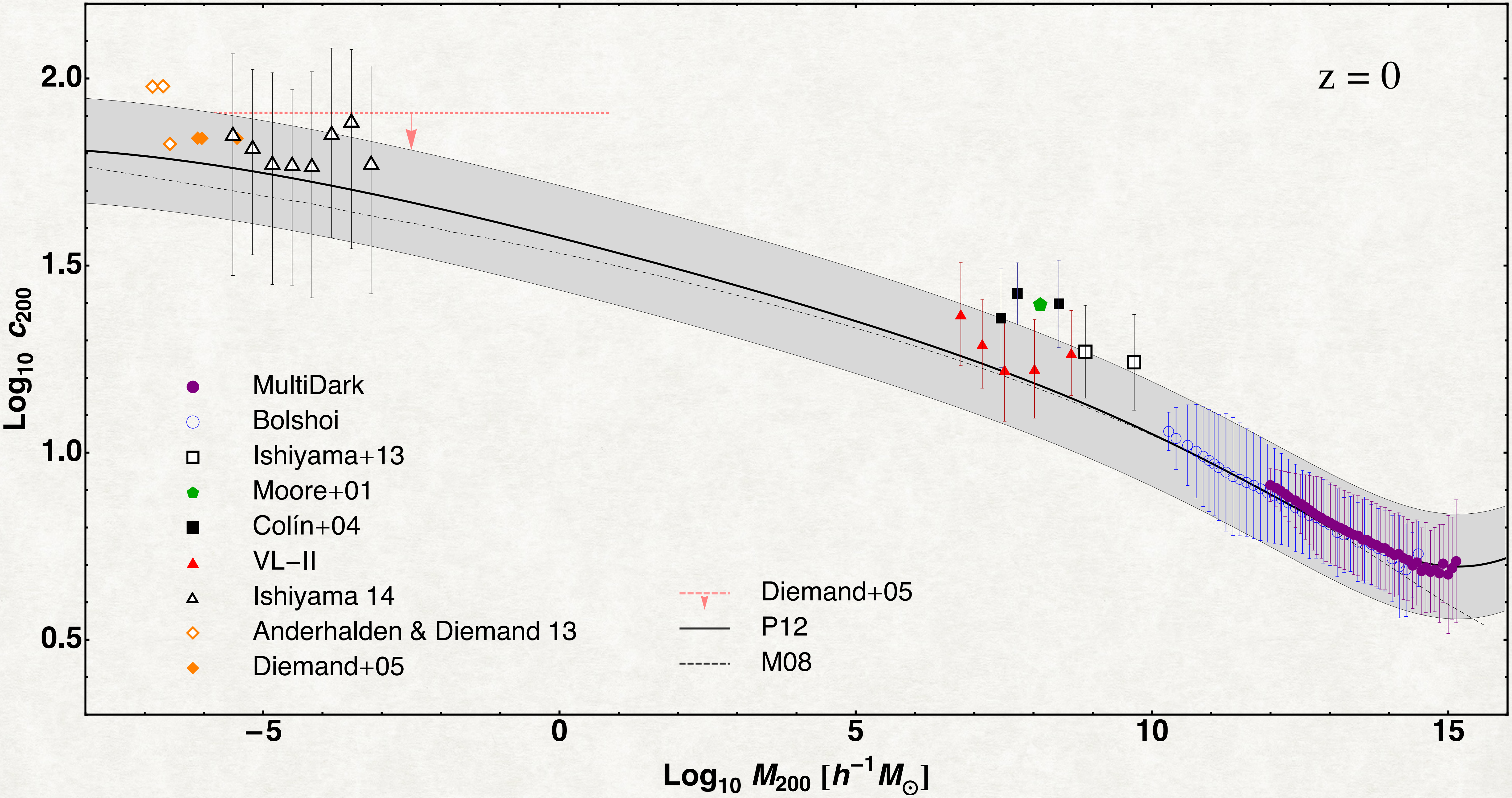
EXTENDED HALO MASS FUNCTION

- Assume typical scale-free Halo mass function from Press-Schechter.
- $dn/dM \sim M^{-2}$
- Abrupt cutoffs:M_{min}, and M_{max}
- Equal amount of DM in every decade of masses,
- Even large M_{max}/M_{min} can be probed using sensitivity solely in a small subset window.

LIMIT SETTING PARAMETERS

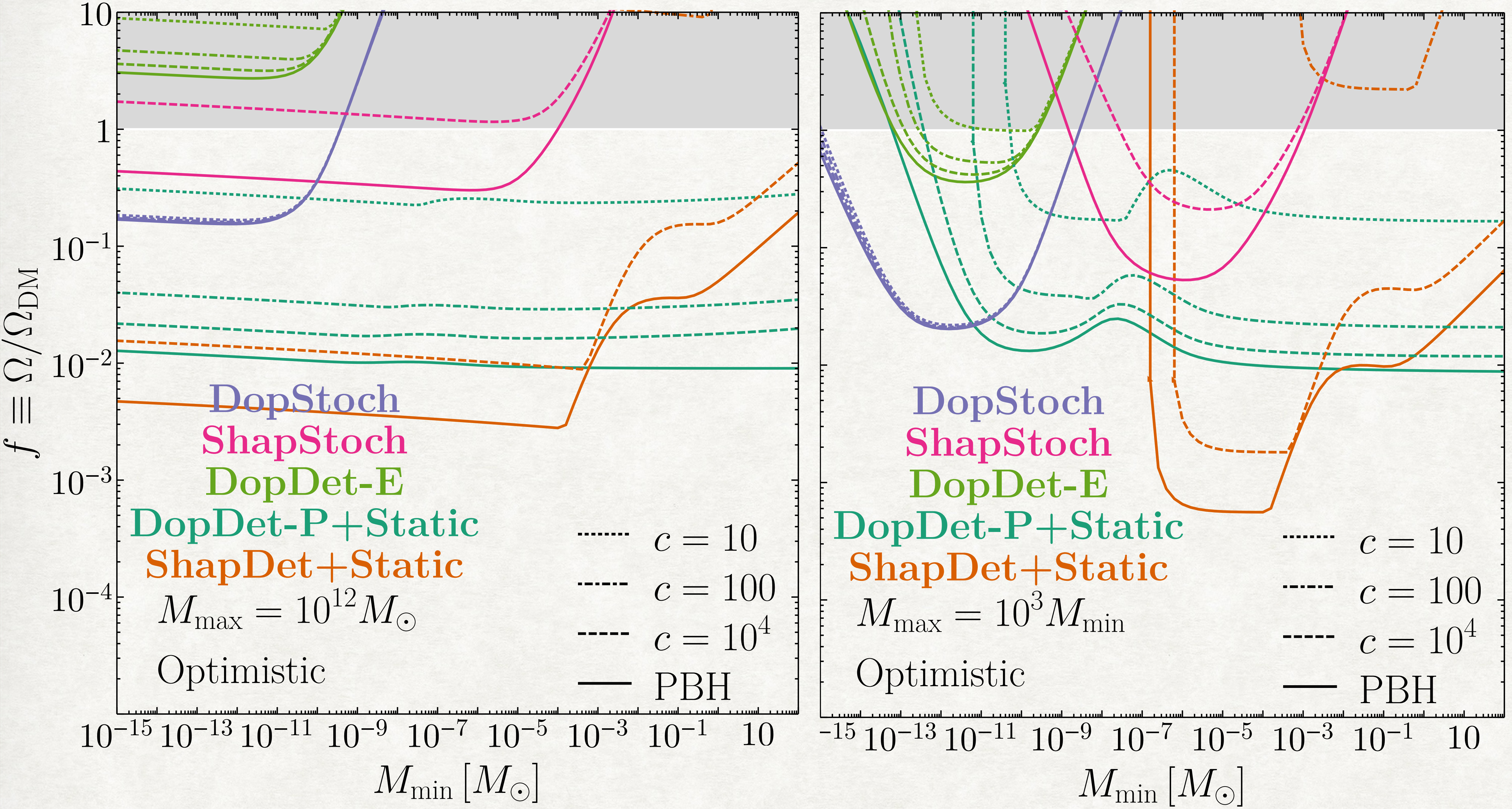
- Set Limits for
- c, the concentration parameter
- f the fraction of dark matter that has not disrupted
- Ignoring tidal disruption and sweeping it into c and f

GOAL: C=100



M usually cut off at 10^{-6} because WIMPs wash out small-scale structure…

EXTENDED HMF



BOUNDS FROM DYNAMIC SIGNALS (DOPPLER)

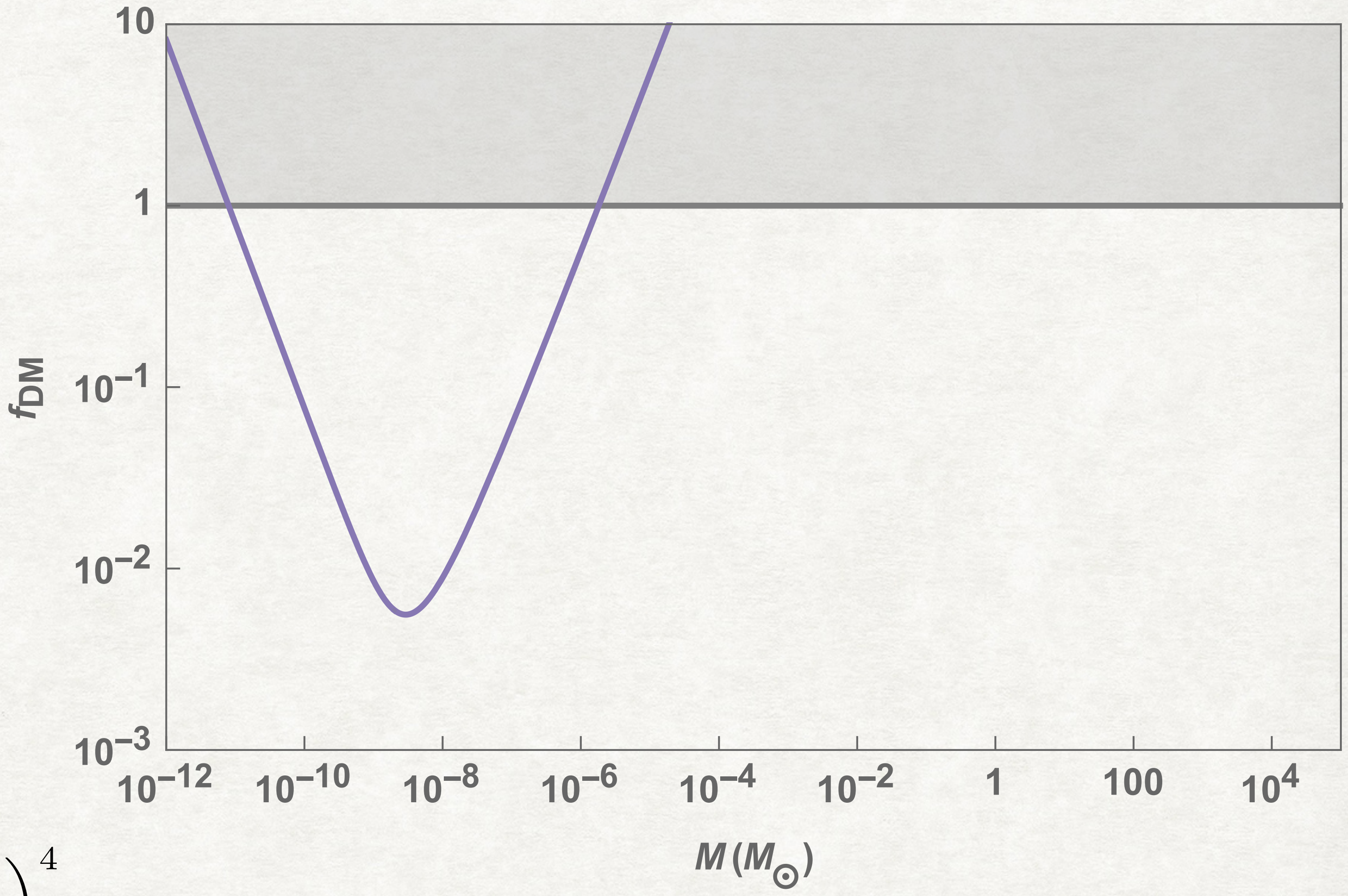
- For Doppler

$$\mathrm{SNR}_D = \frac{1}{2\sqrt{3}}\frac{GMT^{\frac{3}{2}}}{c\,t_{\mathrm{rms}}v^2\sqrt{\Delta t}\,\tau}$$

$$\boxed{\tau_{\mathrm{min}} \propto \sqrt{\frac{M}{f}}}$$

- Requiring the closest approaching PBH to have SNR>4.
- f scales as 1/M

$$f^{L}_{\mathrm{D,\ dyn}} \lesssim 0.01\left(\frac{10^{-9}\,M_\odot}{M}\right)\left(\frac{200}{N_P}\right)\left(\frac{20\ \mathrm{yr}}{T}\right)^4$$



Earth term scales the same way

- At some Mass M, even the nearest PBH starts failing dynamic constraint.
- This condition on f scales as M

$$f^{R}_{\mathrm{D,\ dyn}} \lesssim 3\left(\frac{M}{10^{-7}\,M_\odot}\right)\left(\frac{200}{N_P}\right)\left(\frac{20\ \mathrm{yr}}{T}\right)^3$$

Earth term has Np=1

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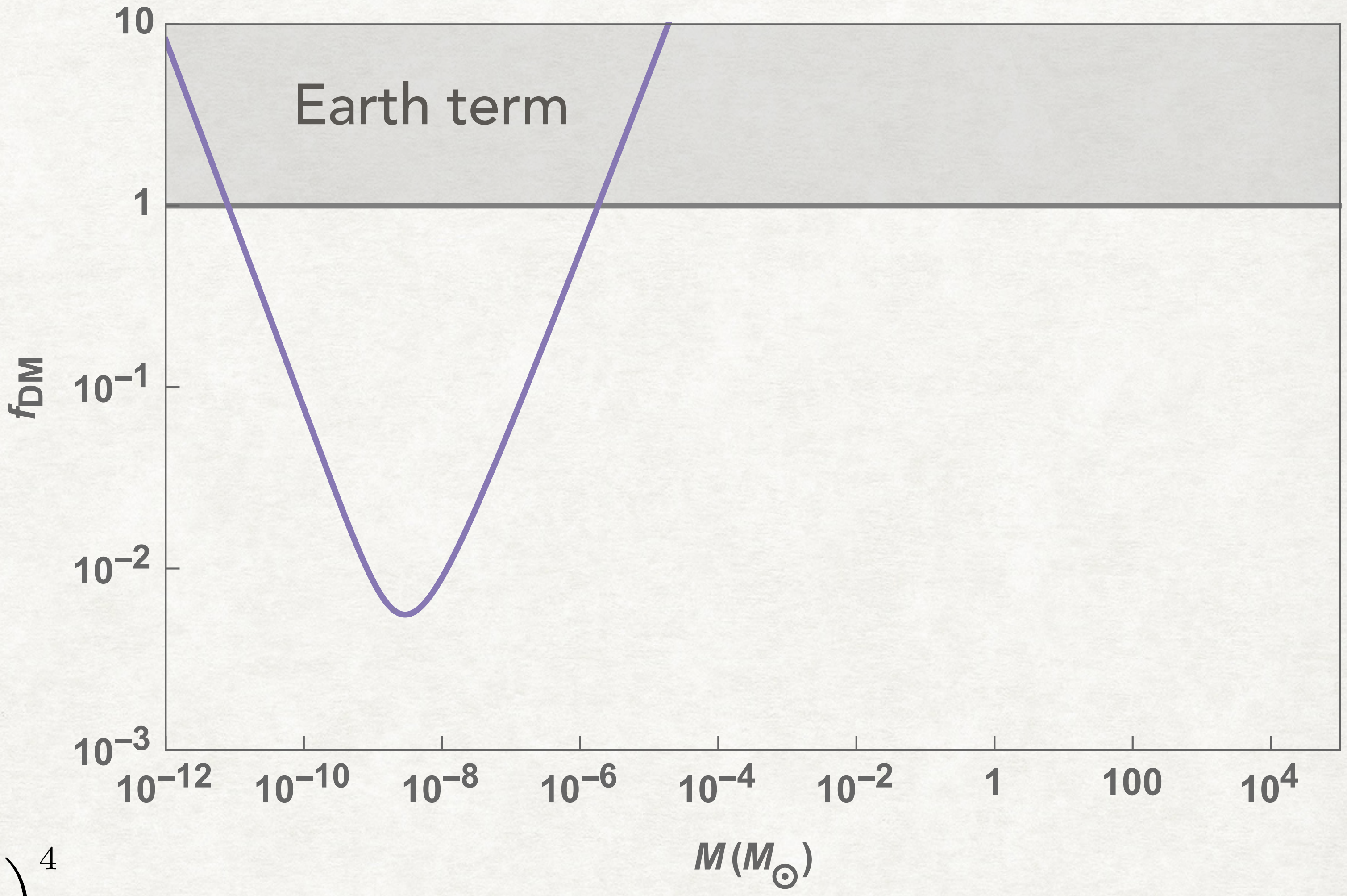

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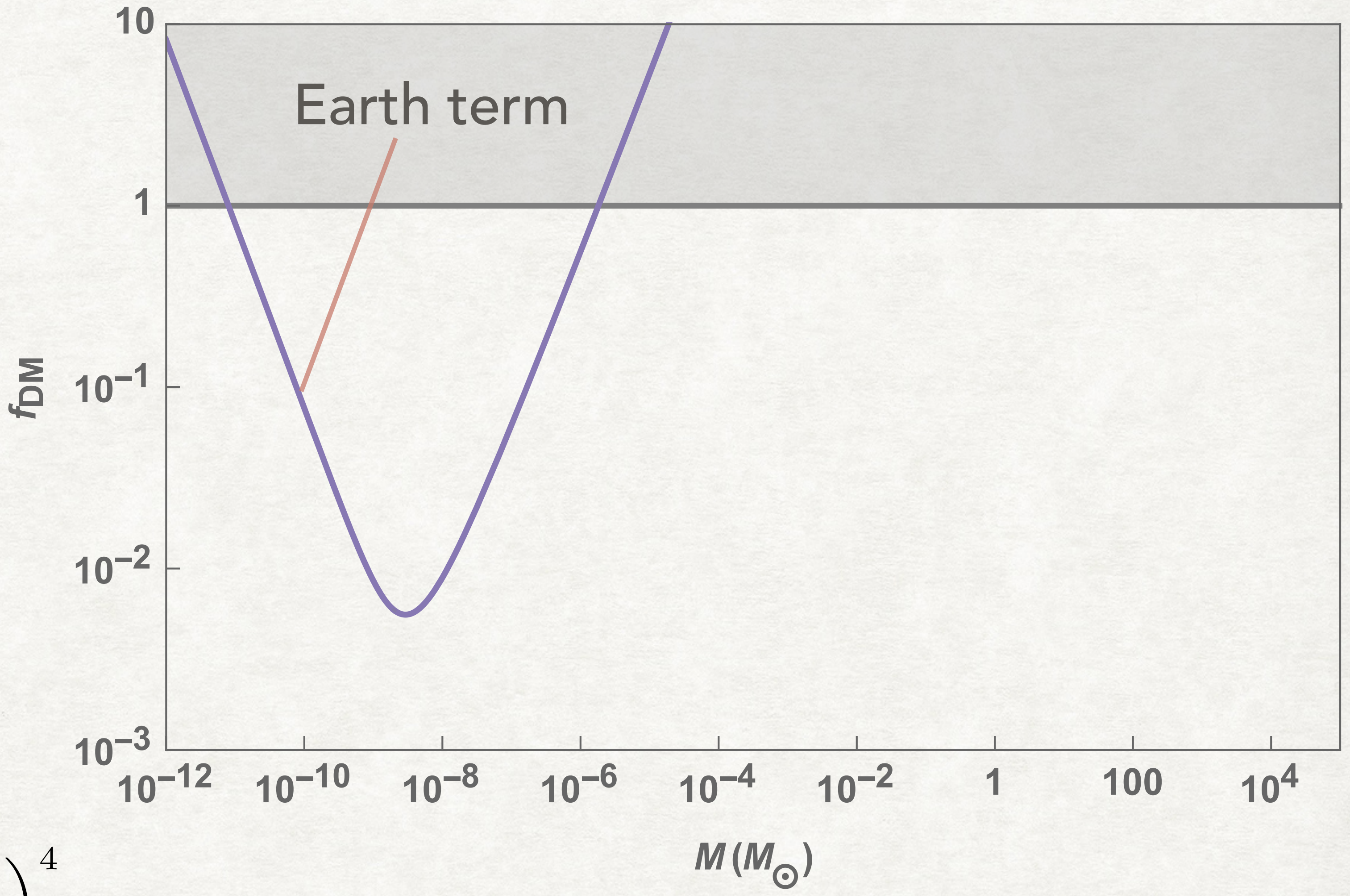

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Earth term has Np=1

STATIC SIGNAL SENSITIVITY

Doppler

$$\frac{\ddot{\nu}}{\nu} \simeq \frac{2GMv}{r_{\mathrm{min}}^3} \sim 3 \times 10^{-32} \left(\frac{N_P f}{200}\right) \ \mathrm{Hz}^2$$

Shapiro

$$\frac{\ddot{\nu}}{\nu} \simeq \frac{16GMv^3}{r_{\times,\mathrm{min}}^3}$$

$$\sim 8 \times 10^{-33} \left(\frac{N_P f}{200}\right)^{\frac{3}{2}} \left(\frac{M_\odot}{M}\right)^{\frac{1}{2}} \left(\frac{d}{\mathrm{kpc}}\right)^{\frac{3}{2}} \ \mathrm{Hz}^2$$

Uncertainty in second derivative purely from rms fluctuations

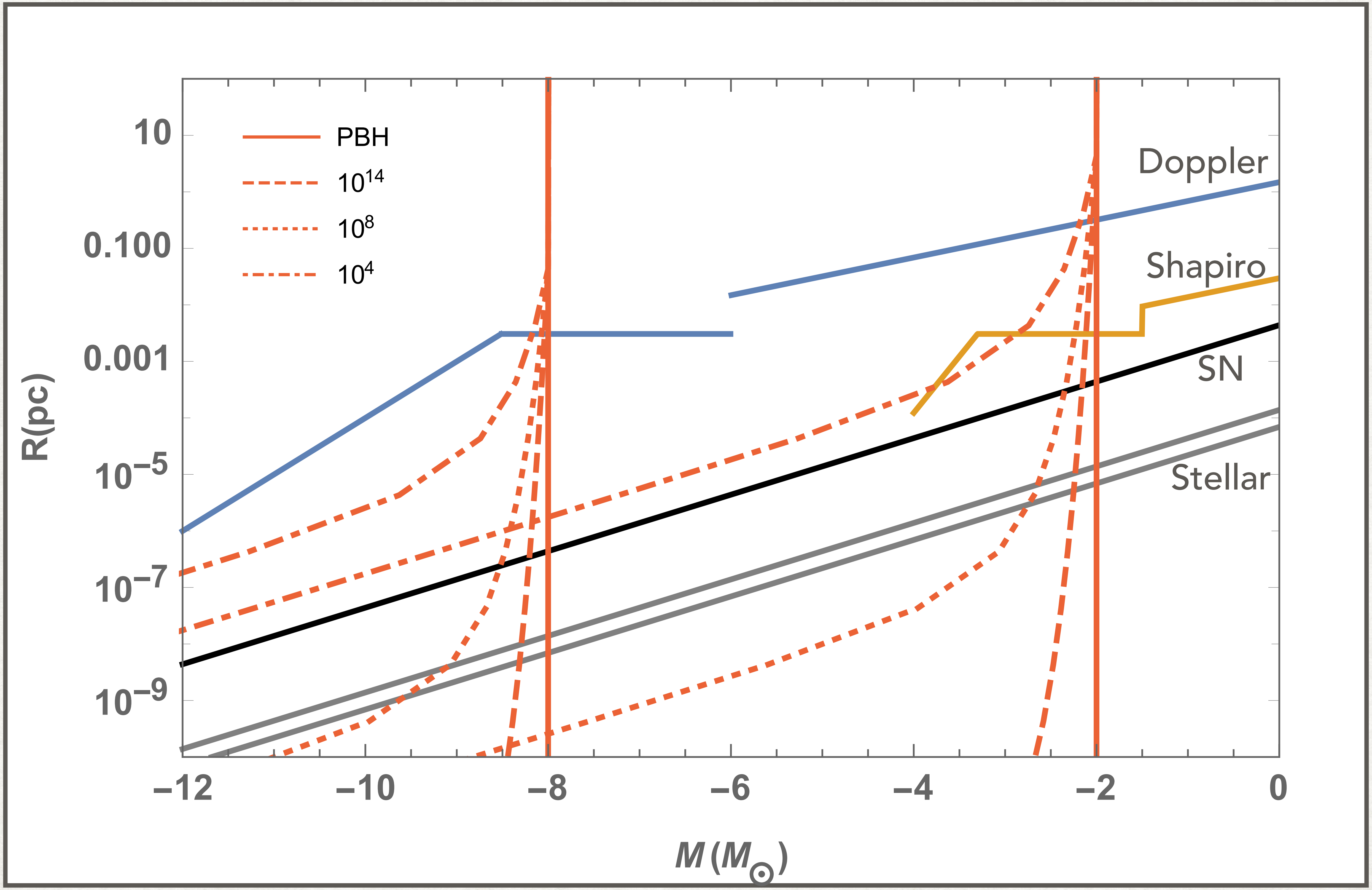
$$\sigma_{\ddot{\nu}/\nu} = 6\sqrt{\frac{2800\Delta t}{T}\frac{t_{\mathrm{RMS}}}{T^3}}$$

$$f_{\mathrm{D,\ stat}} \lesssim 0.4 \left(\frac{200}{N_P}\right) \left(\frac{20\ \mathrm{yr}}{T}\right)^{\frac{7}{2}}$$

$$f_{\mathrm{S,\ stat}} \lesssim \left(\frac{200}{N_P}\right) \left(\frac{M}{M_\odot}\right)^{\frac{1}{3}} \left(\frac{20\ \mathrm{yr}}{T}\right)^{\frac{7}{3}} \left(\frac{\mathrm{kpc}}{d}\right)$$

Notice no M dependence here

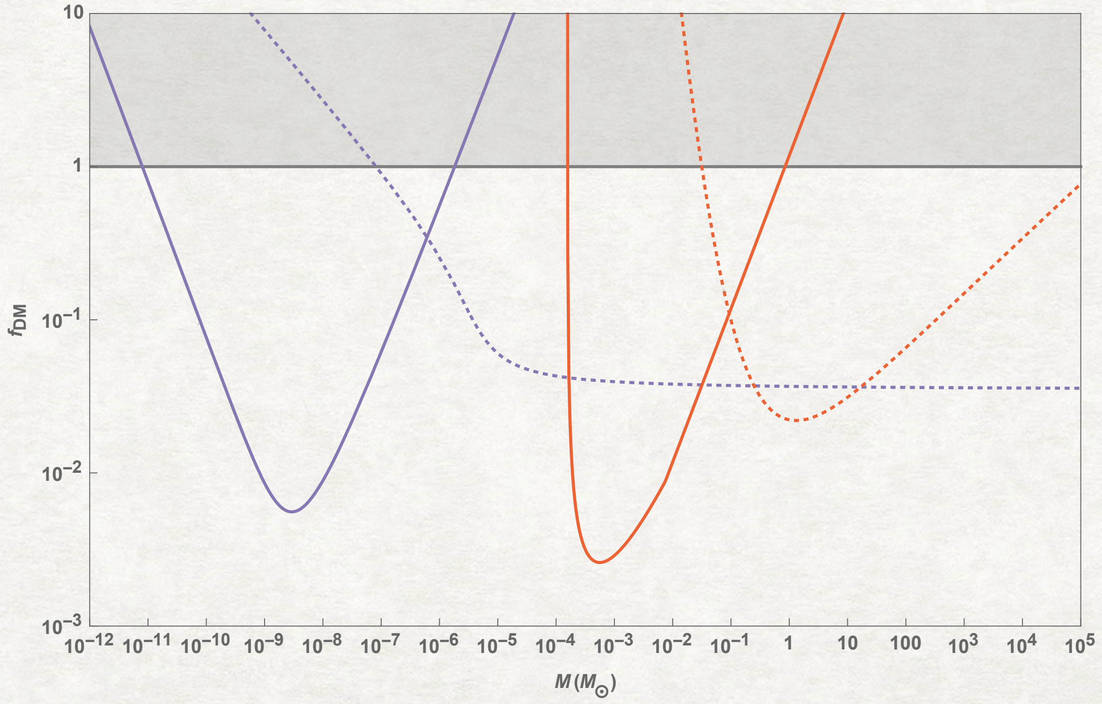
SENSITIVITY TO DIFFUSE HALOS



Limits iff red line intersects a probe radius

STOCHASTIC SIGNAL

- Multiple events at lower masses which do not pass the threshold SNR individually
- Lose ability to fit for deterministic signal shape



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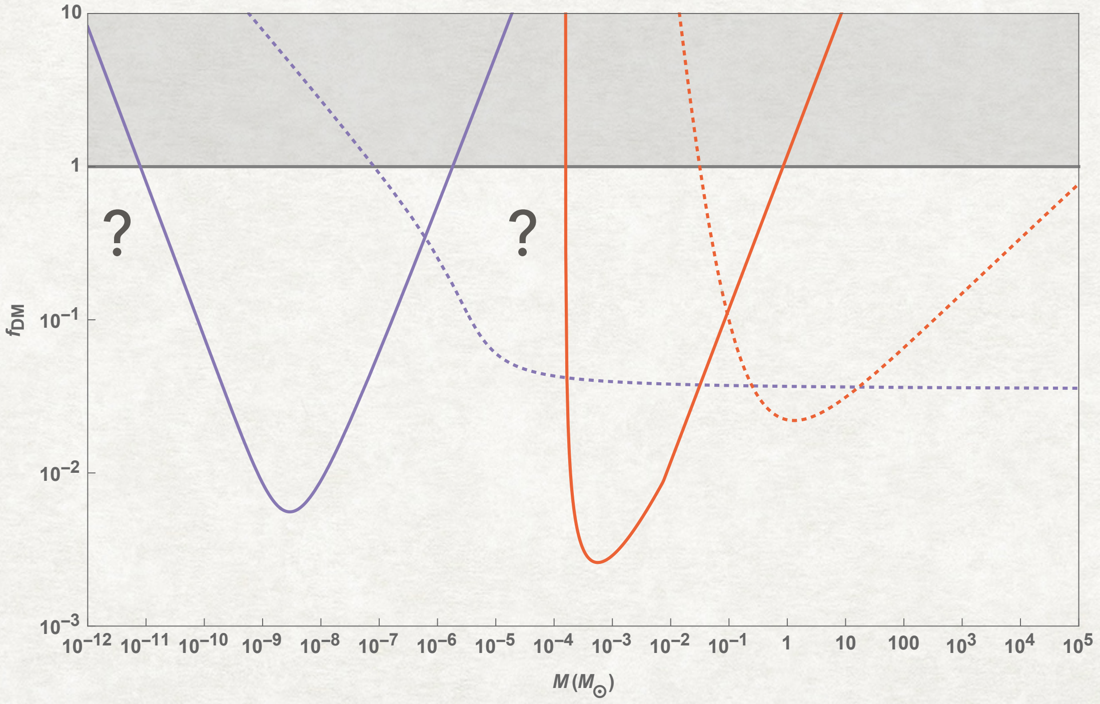


STOCHASTIC SNR

$$\langle\delta\phi(t)\delta\phi(t')\rangle = \sum_{i=1}^{N}\langle\delta\phi_i(t)\delta\phi_i(t')\rangle + \sum_{i\neq j}^{N(N-1)}\langle\delta\phi_i(t)\delta\phi_j(t')\rangle \equiv R_1(t,t') + R_2(t,t')$$

1-halo

2-halo

$$\mathrm{SNR}_P^2 = \frac{N_P}{2\widetilde{N}^2}\int dt dt'\langle R_I^{\mathrm{sub}}(t,t')^2\rangle_{\mathcal{P}}$$

$$\mathrm{SNR}_E^2 = \frac{N_P(N_P-1)}{2\widetilde{N}^2}\int dt dt'\left\langle R_{IJ}^{\mathrm{sub}}(t,t')^2\right\rangle_{\mathcal{P}}$$

Sum over all events i,
Average over ensemble

Deterministic Signal: care about the single closest event. A random “Best pulsar” exists

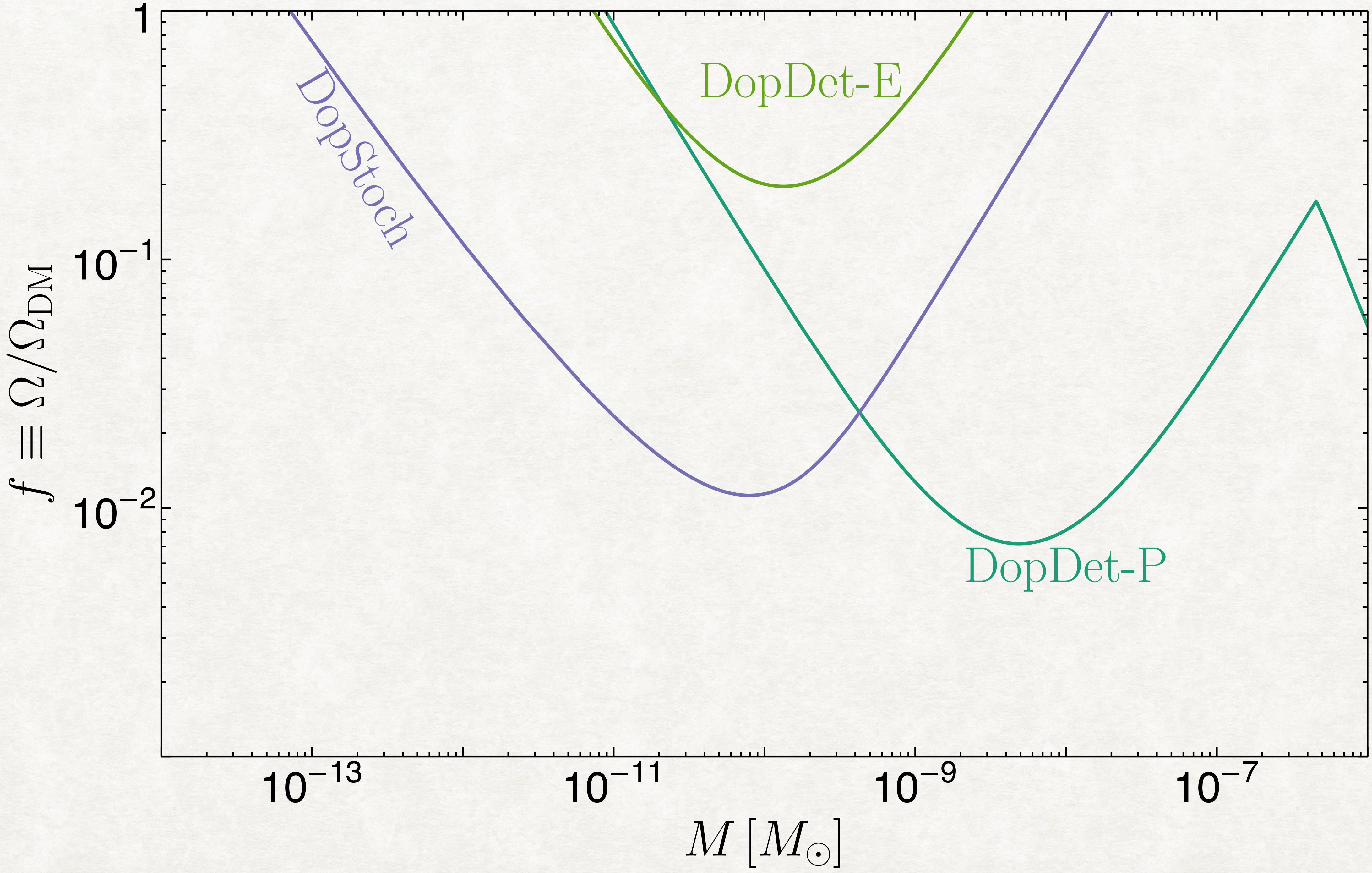
Stochastic signal:

Pulsar Term - N_P pulsars accumulating more statistics
Earth Term - can cross-correlate across pulsars with angular correlations.

For the highest single die roll, helps to roll die several times,
For sum of 100 die rolls, no point repeating the 100 roll.

DOPPLER SUMMARY

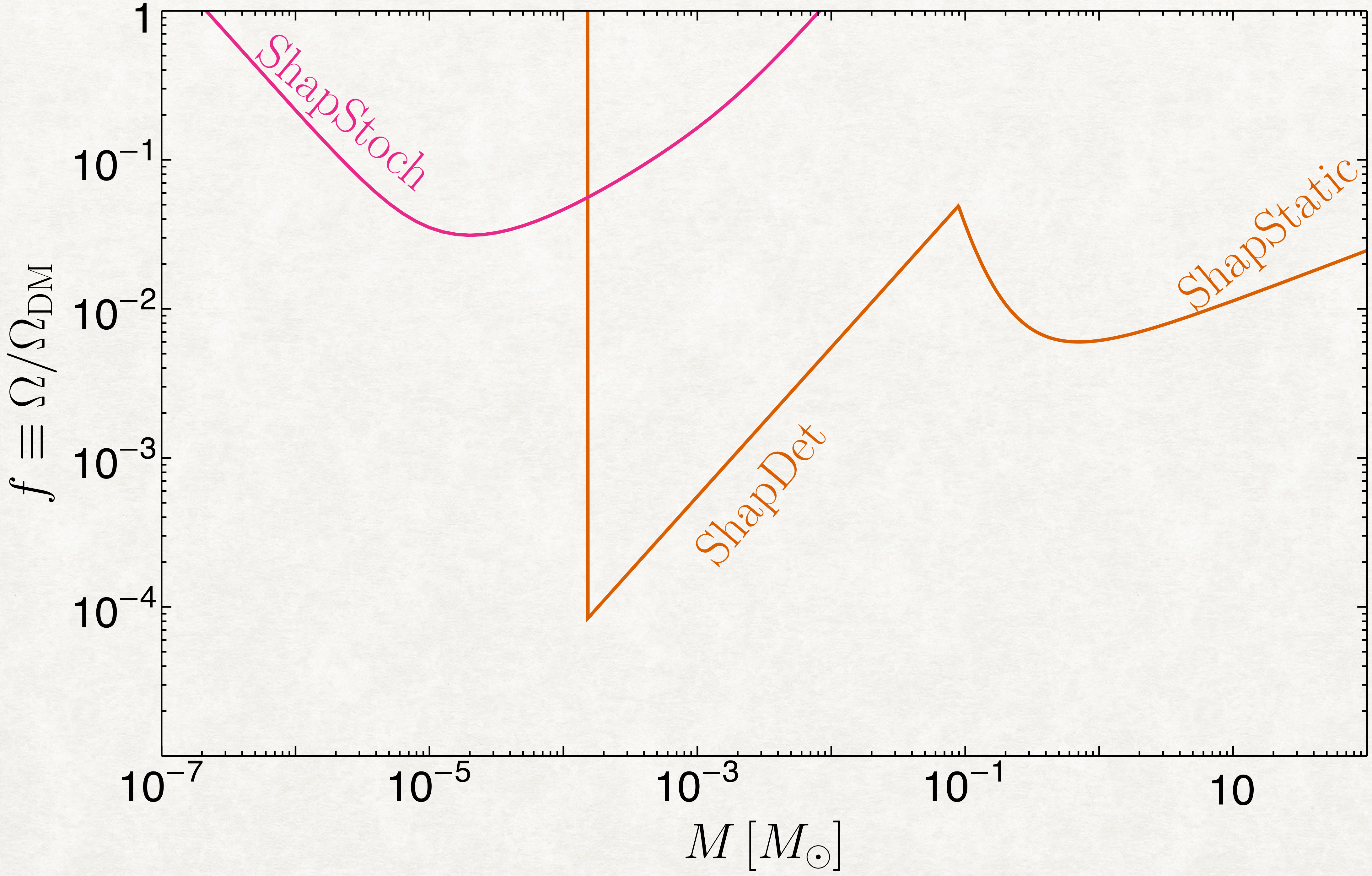
Stochastic Signal: Random walk in velocity



OPTIMISTIC

SHAPIRO SUMMARY

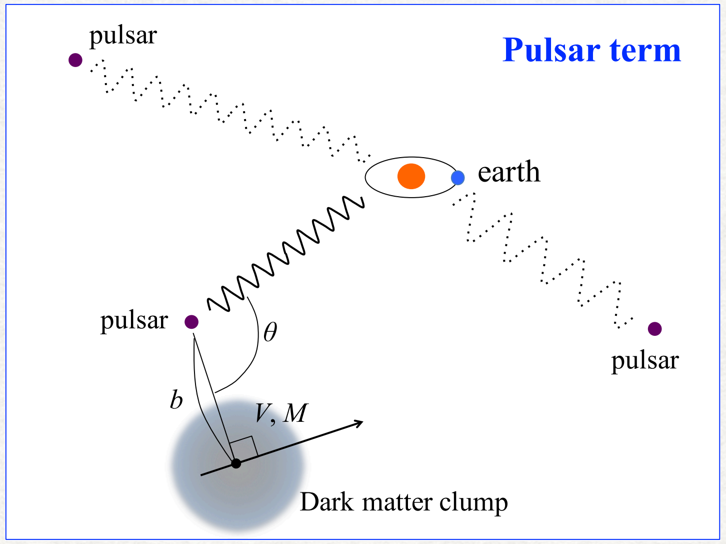
Stochastic Signal: Random addition of blips



OPTIMISTIC

DOPPLER GEOMETRY

- To determine typical timescale, let us determine object of closest approach
- Cross-section for Doppler, is a circle.
- Remembering $|\mathbf{b}| = \tau v$



N$_P$ pulsars
N$_P$ x cross-section

$$\tau_{\rm min} \simeq \frac{1}{v}\sqrt{\frac{M}{N_P f \rho_{\rm DM} v T}}$$

$$\sim \frac{20\ \mathrm{yr}}{\sqrt{N_P f}} \left(\frac{M}{10^{-9}\, M_\odot}\right)^{\frac{1}{2}} \left(\frac{20\ \mathrm{yr}}{T}\right)^{\frac{1}{2}}$$

Number of pulsars

Fraction of DM in M mass PBH

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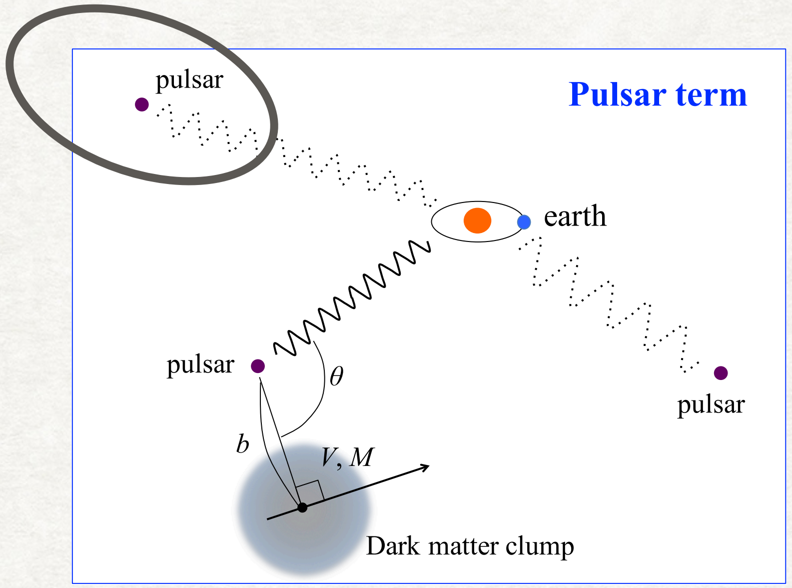

N

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N_P pulsars

N_P x cross-section

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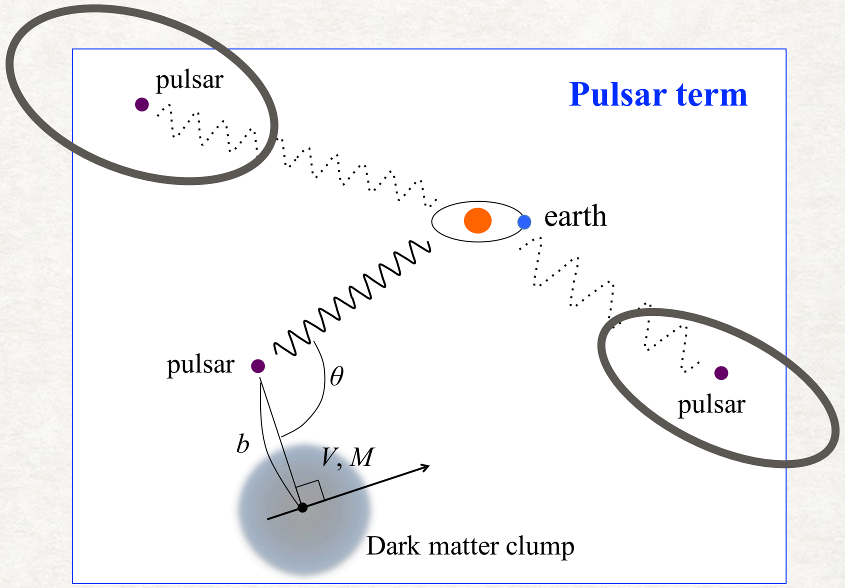

N_P pulsars

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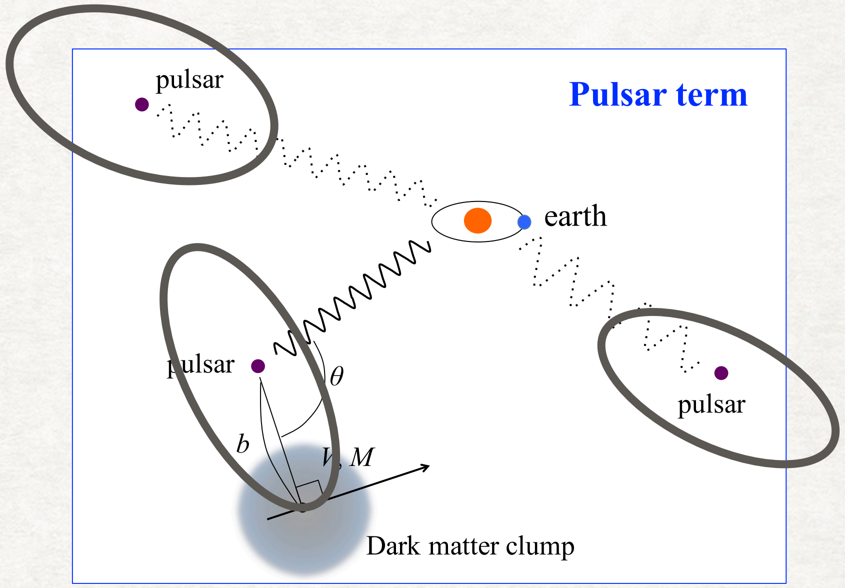

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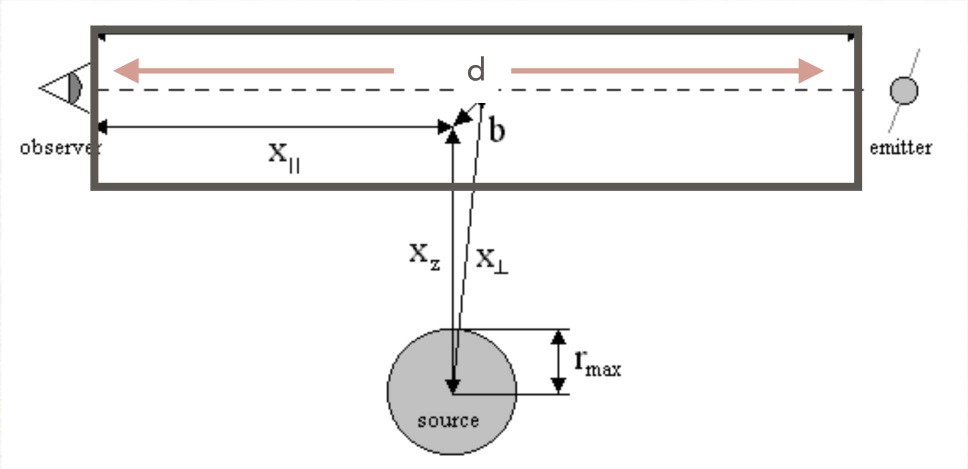
Number of pulsars

Fraction of DM in M mass PBH

SHAPIRO CROSS-SECTION

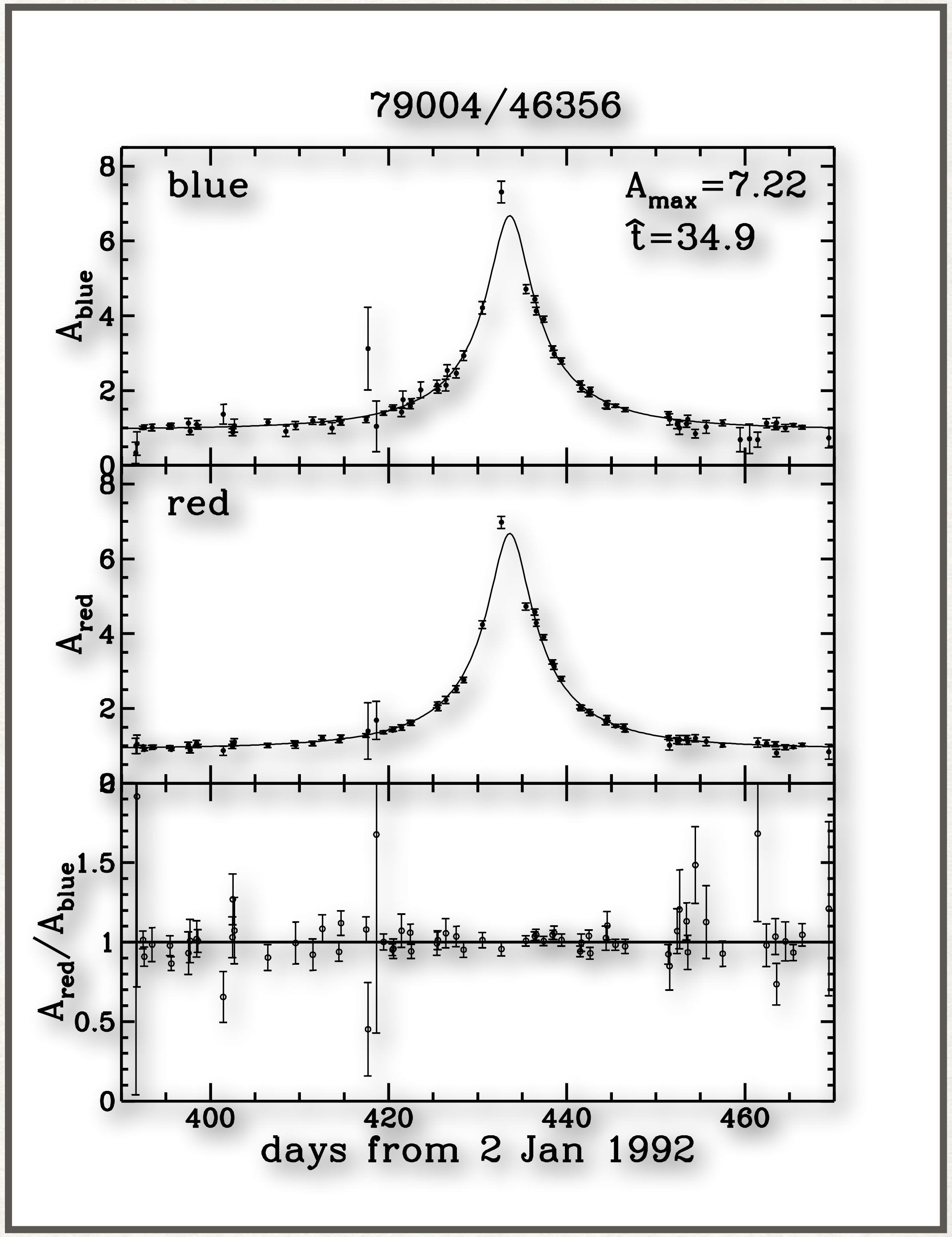
- Cross-section for Shapiro is a rectangle



$$
\begin{aligned}
\tau_{\min} &\simeq \frac{2}{v}\frac{M}{N_P f \rho_{\mathrm{DM}} v T d}\,, \\
&\sim \frac{20\ \mathrm{yr}}{N_P f}\left(\frac{M}{10^{-4}\,M_\odot}\right)\left(\frac{20\ \mathrm{yr}}{T}\right)\left(\frac{\mathrm{kpc}}{d}\right)
\end{aligned}
$$

DYNAMIC BACKGROUNDS

- Dynamic signal more spectacular than static signal.
- Shape differences could help differentiate from glitches etc.
- DM signals are non-dispersive
- Baryonic structure too few at these masses



Dispersion used in Microlensing to differentiate lensing blip from a dispersive blip

STATIC BACKGROUNDS

- A few pulsars already display non-zero second derivative.
- Will need to supplement with E&M observations to subtract known nearby objects.