QCD-Collapsed Domain Walls

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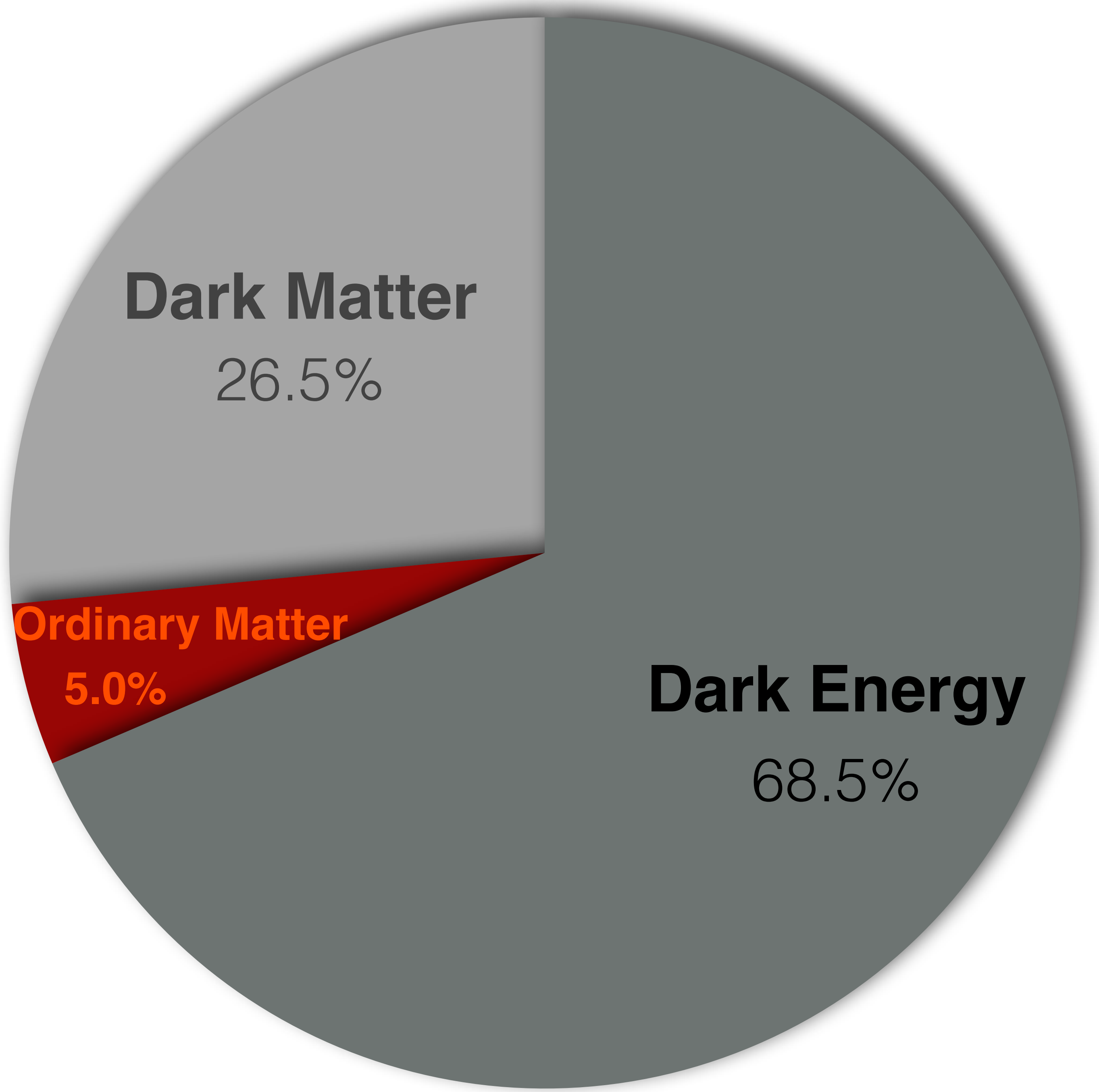
University of Wisconsin-Madison

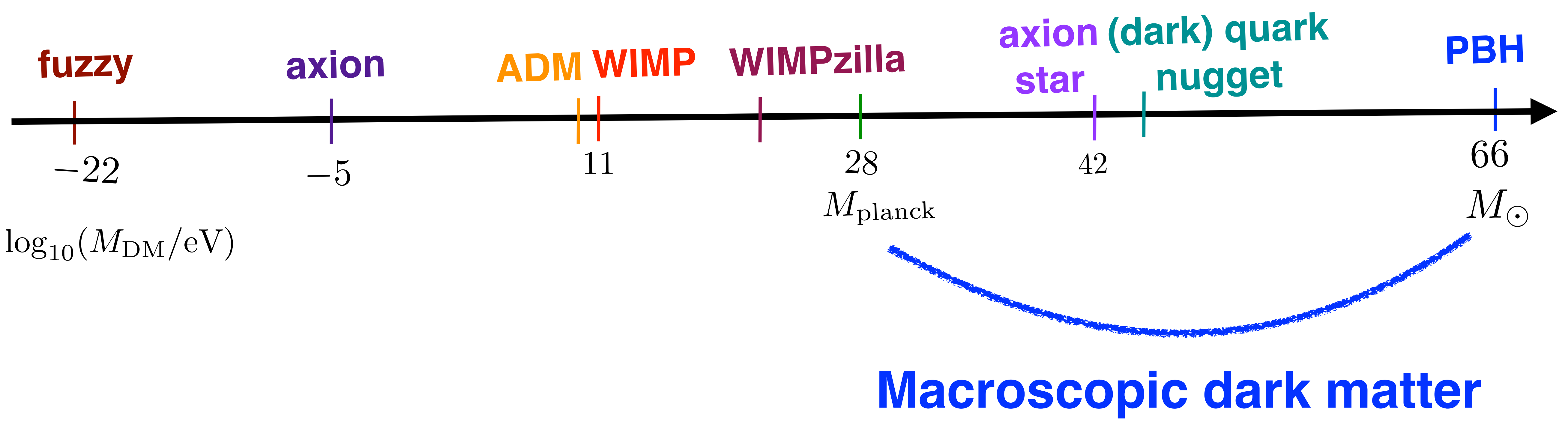
Unravelling the Universe with PTAs, Dec. 1, 2023

Dark world



from PLANCK, 1807.06209

(Incomplete list) Dark Matter Models



$$\frac{26.5\,\%}{5.0\,\%} \approx \mathbf{5}$$

Dark matter is coincident with ordinary matter! Why?

Baryon-anti-baryon asymmetry

$$\frac{n_B - n_{\overline{B}}}{n_B + n_{\overline{B}}} \sim 10^{-10}$$

- **Sakharov's three "general" conditions:**

  I. **Baryon number violation**

  II. **C and CP violation**

  III. **Departure from thermal equilibrium**

- **Models: electroweak baryogenesis, leptogenesis, spontaneous baryogenesis, ……**

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- Sakharov's three "general" conditions:

    I. Baryon number violation

    II. C and CP violation

    III. Departure from thermal equilibrium

- Models: electroweak baryogenesis, leptogenesis, spontaneous baryogenesis, ……

- Why is dark matter comparable to the leftover of baryon asymmetry?

Asymmetric dark matter

$$\frac{m_{\rm DM}\, n_{\rm DM}}{m_p\, n_p} = \frac{26.5\,\%}{5.0\,\%} \approx \mathbf{5}$$

- **Two conditions:**
  - **I.** $n_{\rm DM} \sim n_p$
  - **II.** $m_{\rm DM} \sim m_p$

I. $n_{\mathrm{DM}} \sim n_p$

The first condition can be satisfied by introducing some non-trivial number density history

Barr, Chivukula, Farhi, PLB, 241, 387 (1990)

David B. Kaplan, PRL, 68, 741 (1992)

Dodelson, Greene, Widrow, NPB, 372, 467 (1992)

Fujii, Yanagida, PLB, 542, 80 (2002)

Kitano, Low, PRD, 71, 023510 (2005)

Farrar, Zaharijas, PRL, 96, 041302 (2006)

Gudnason, Kouvaris, Sannino, PRD, 73, 115003 (2006)

Kaplan, Luty, Zurek, PRL, 79, 115016 (2009)

Shelton, Zurek, PRD, 82, 123512 (2010)

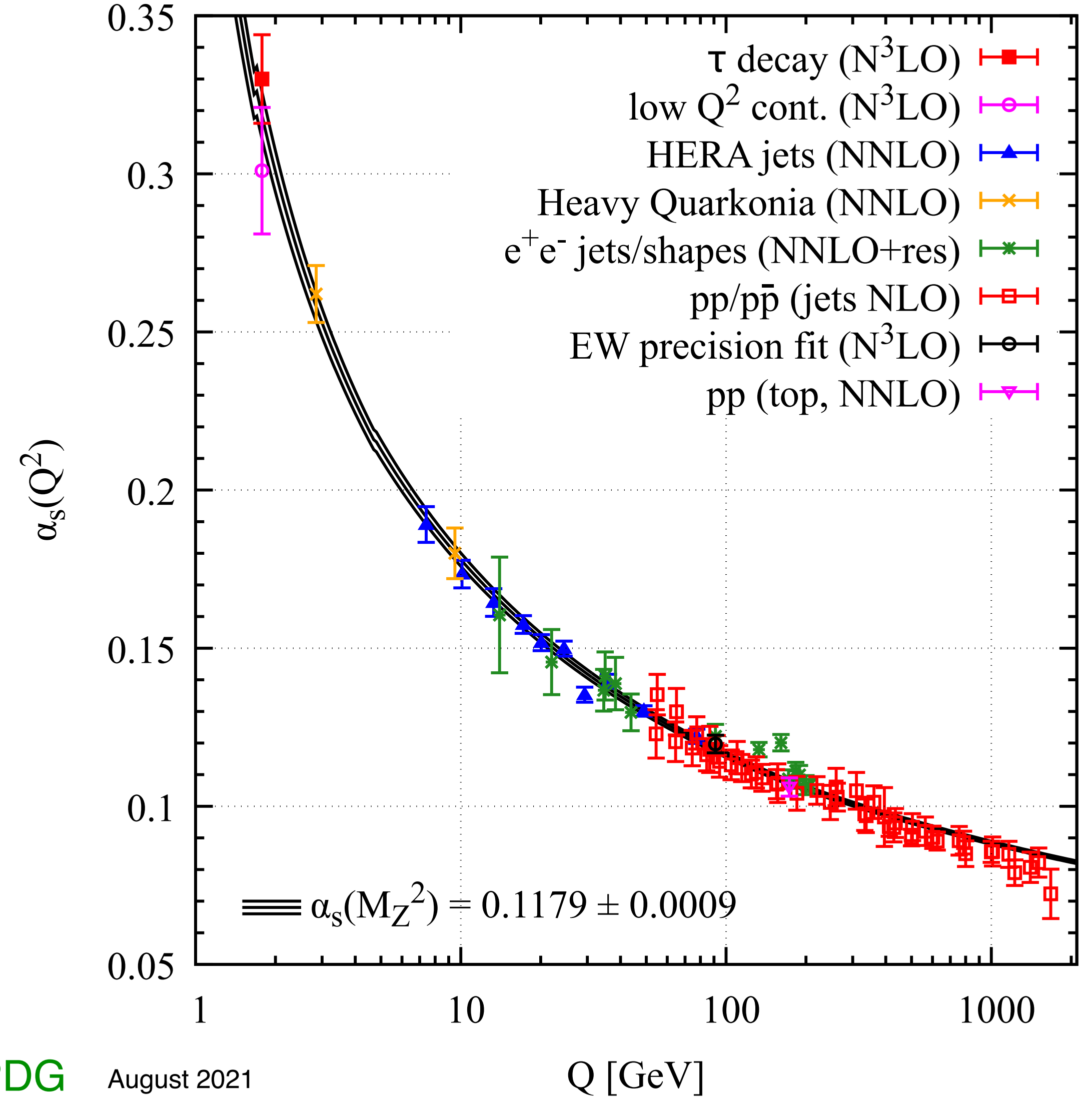
Davoudiasl, Morrissey, Sigurdson, Tulin, PRL, 105, 211304 (2010)

Buckley, Randall, JHEP, 1109, 009 (2011)

……

II. $m_{\mathrm{DM}} \sim m_p$

90% of the mass of ordinary matter emerges from QCD



PDG August 2021

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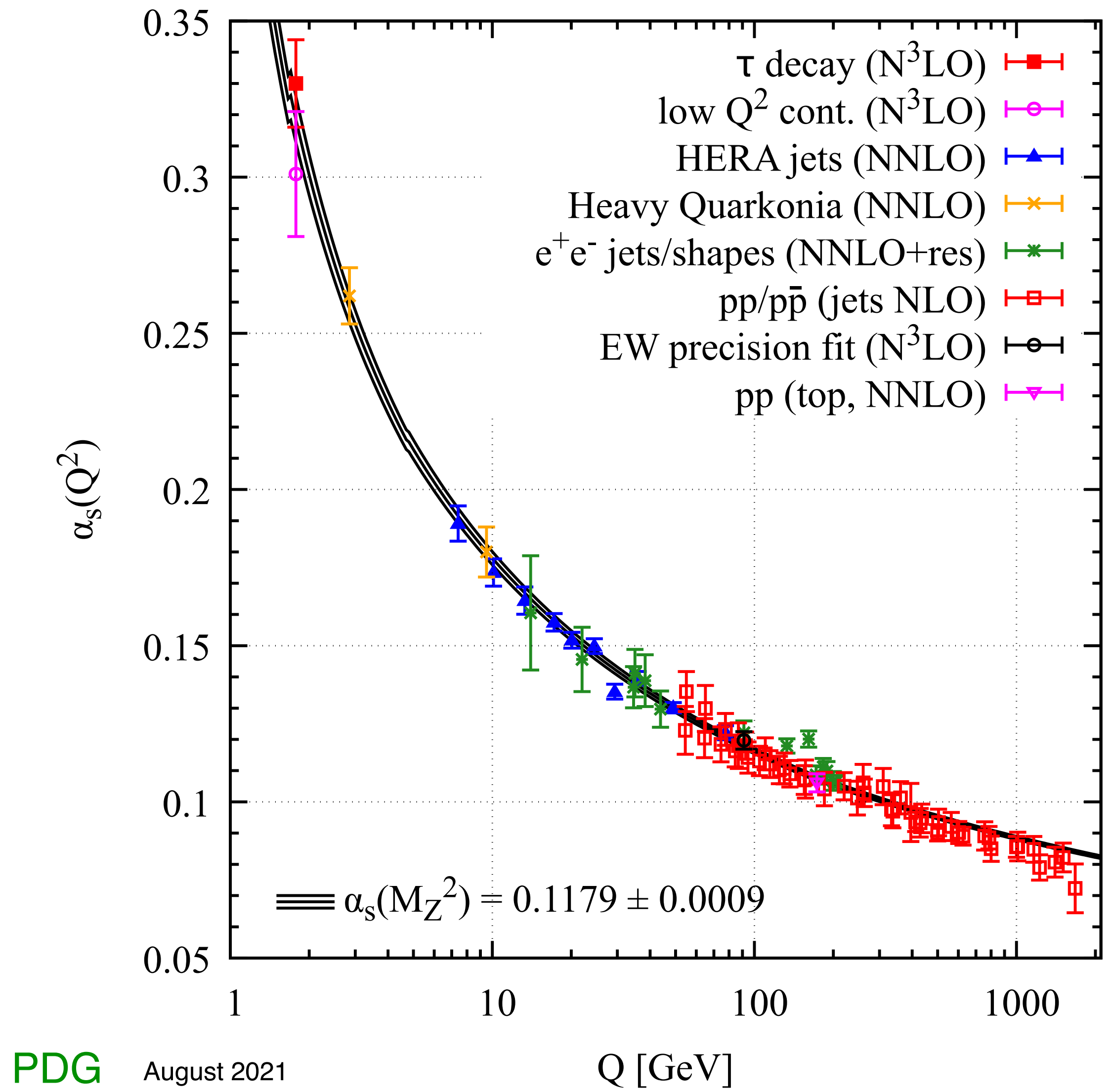


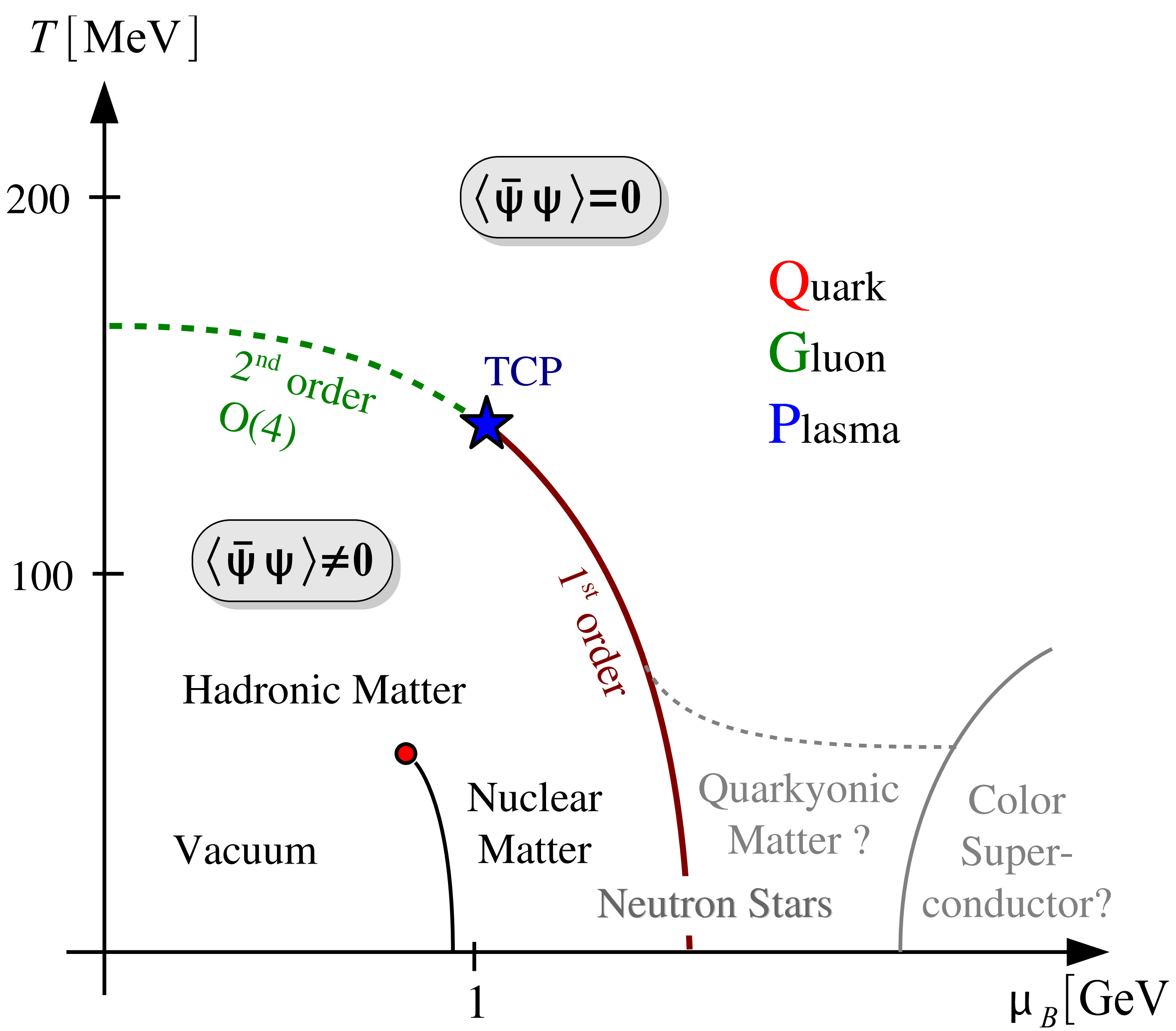
Does it mean that dark matter knows the QCD scale?

Dark matter from our QCD

- **Other states in QCD?**

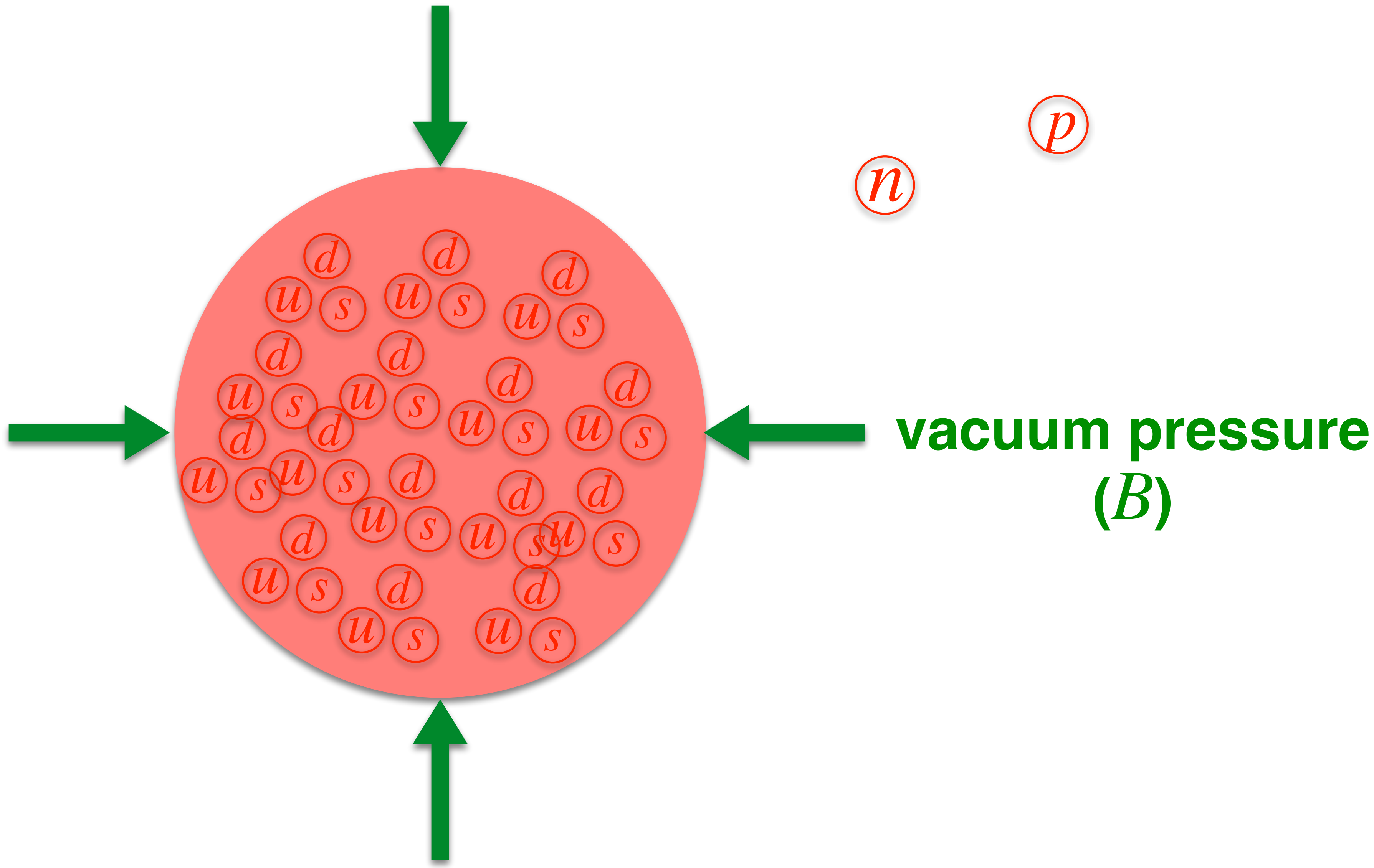
Dark matter from our QCD

- **Other states in QCD?**



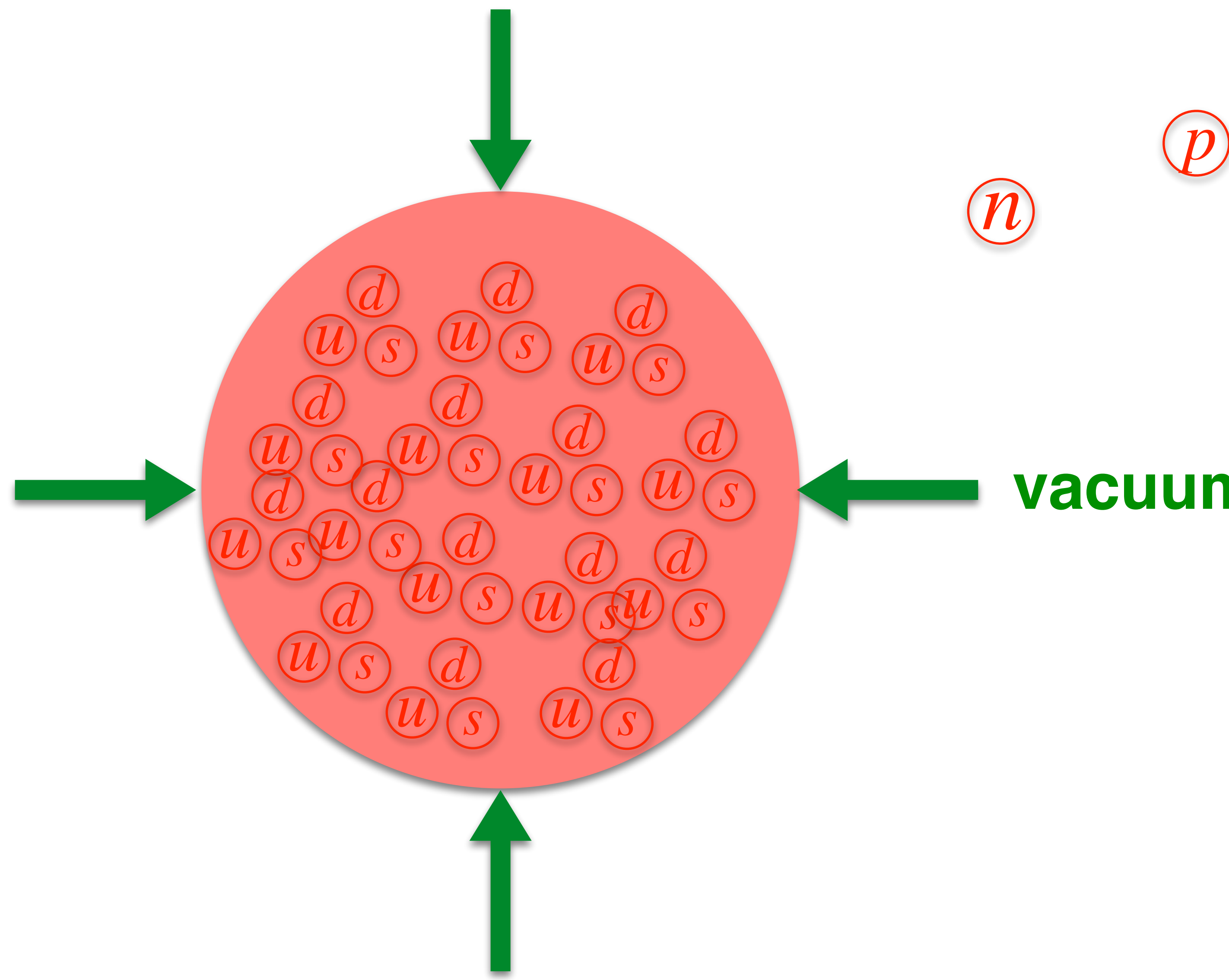
Forcrand, et. al., arXiv:1503.08140

Quark nuggets



- Balanced between vacuum and degenerate Fermi pressures

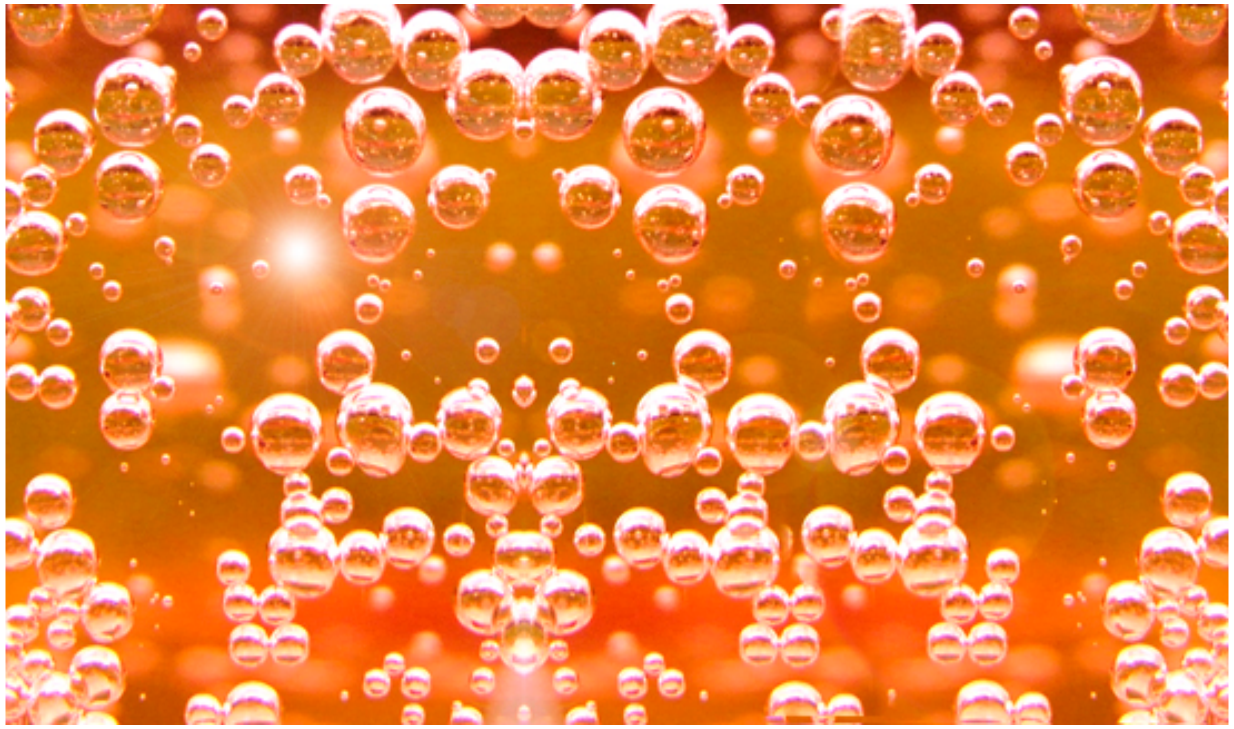
Quark nuggets



- **Balanced between vacuum and degenerate Fermi pressures**

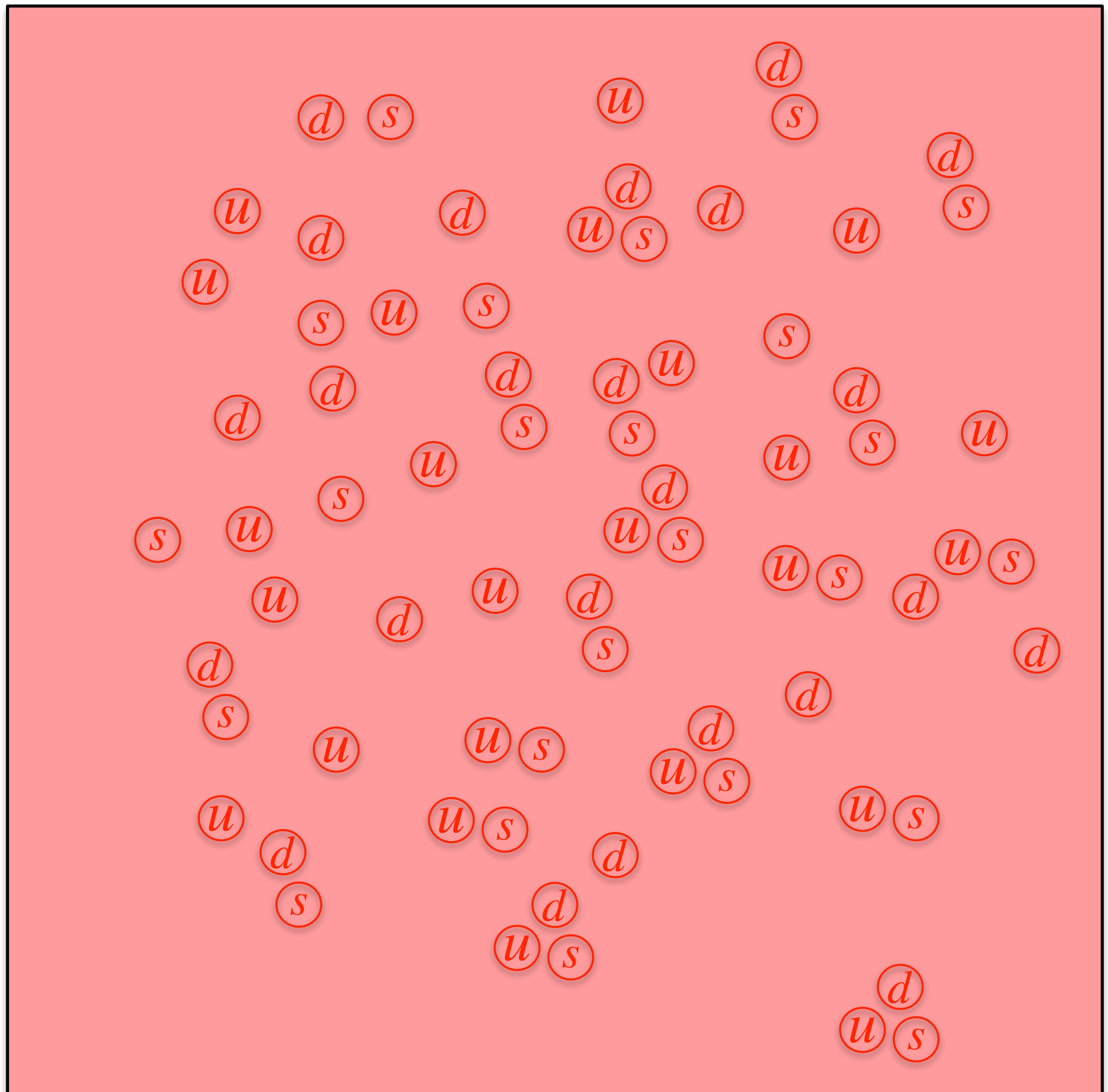
$$\frac{M}{A} \approx 5.7\, B^{1/4} = 912\,\mathrm{MeV} \times \frac{B^{1/4}}{160\,\mathrm{MeV}} < \frac{M_{\mathrm{Fe}}}{A_{\mathrm{Fe}}} \approx 930\,\mathrm{MeV}$$

Formation from 1'st order phase transition



Witten, '1984

T > Tc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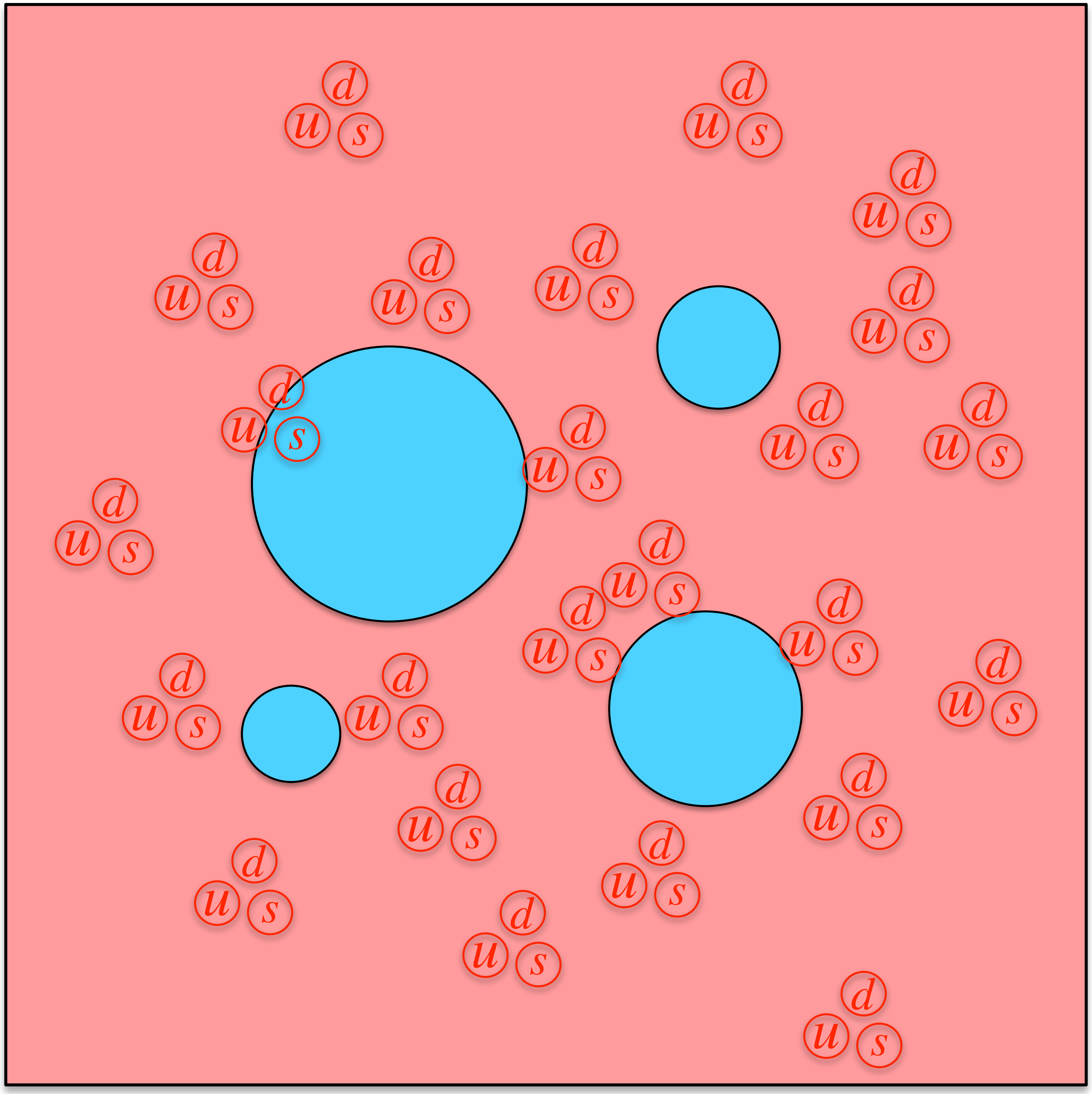
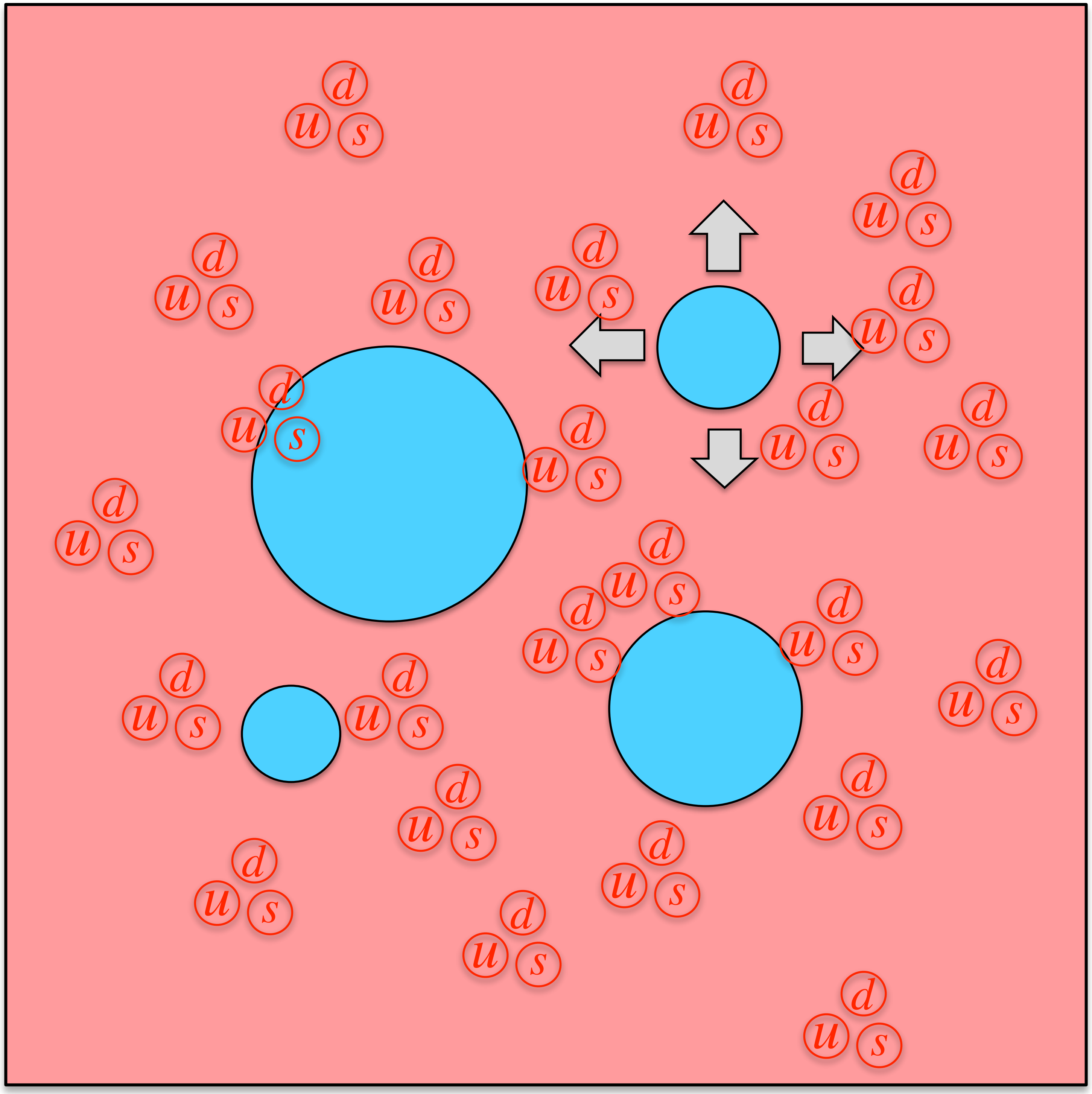
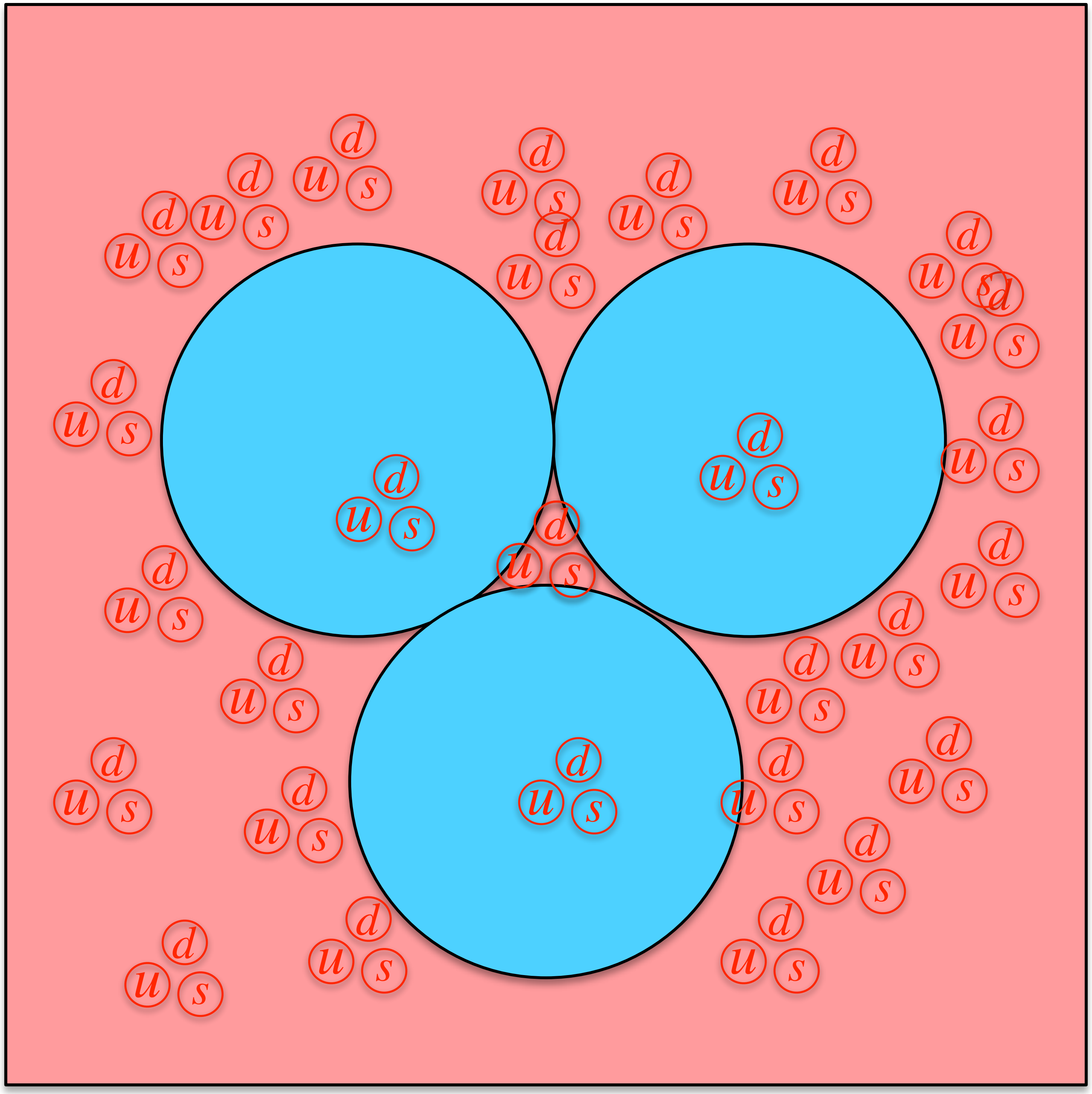


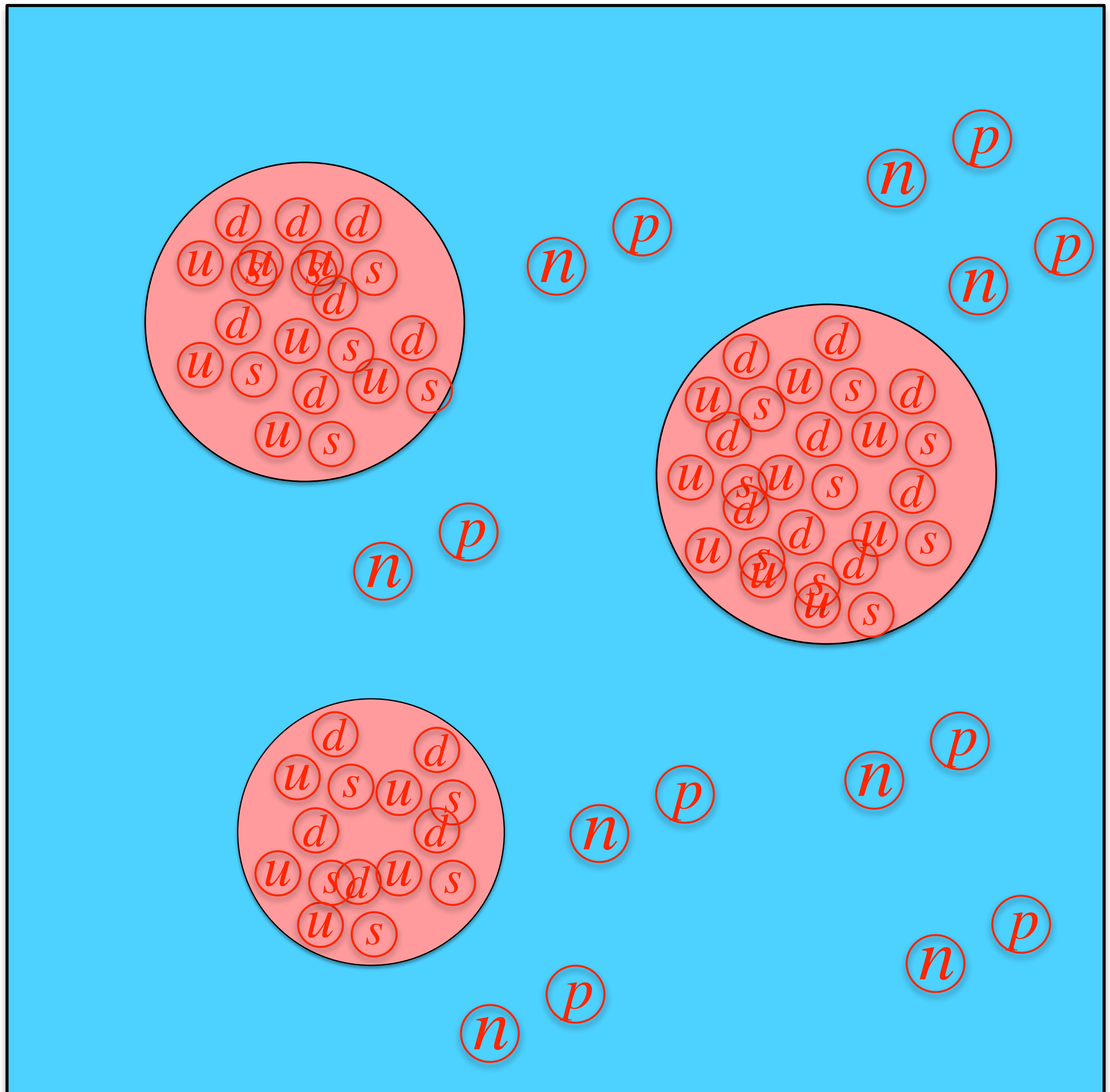
T ~ Tc



Hadron bubbles grow



Isolated quark nuggets



Properties of quark nuggets

- **The mass of the quark nugget is**

$$M_{\rm QN} \sim 10^{14}\,{\rm g}$$

- **The radius of the quark nugget is**

$$R_{\rm QN} \sim 1\,{\rm cm}$$

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- **The energy density of the QM is similar to a Neutron Star, except with a much smaller radius**

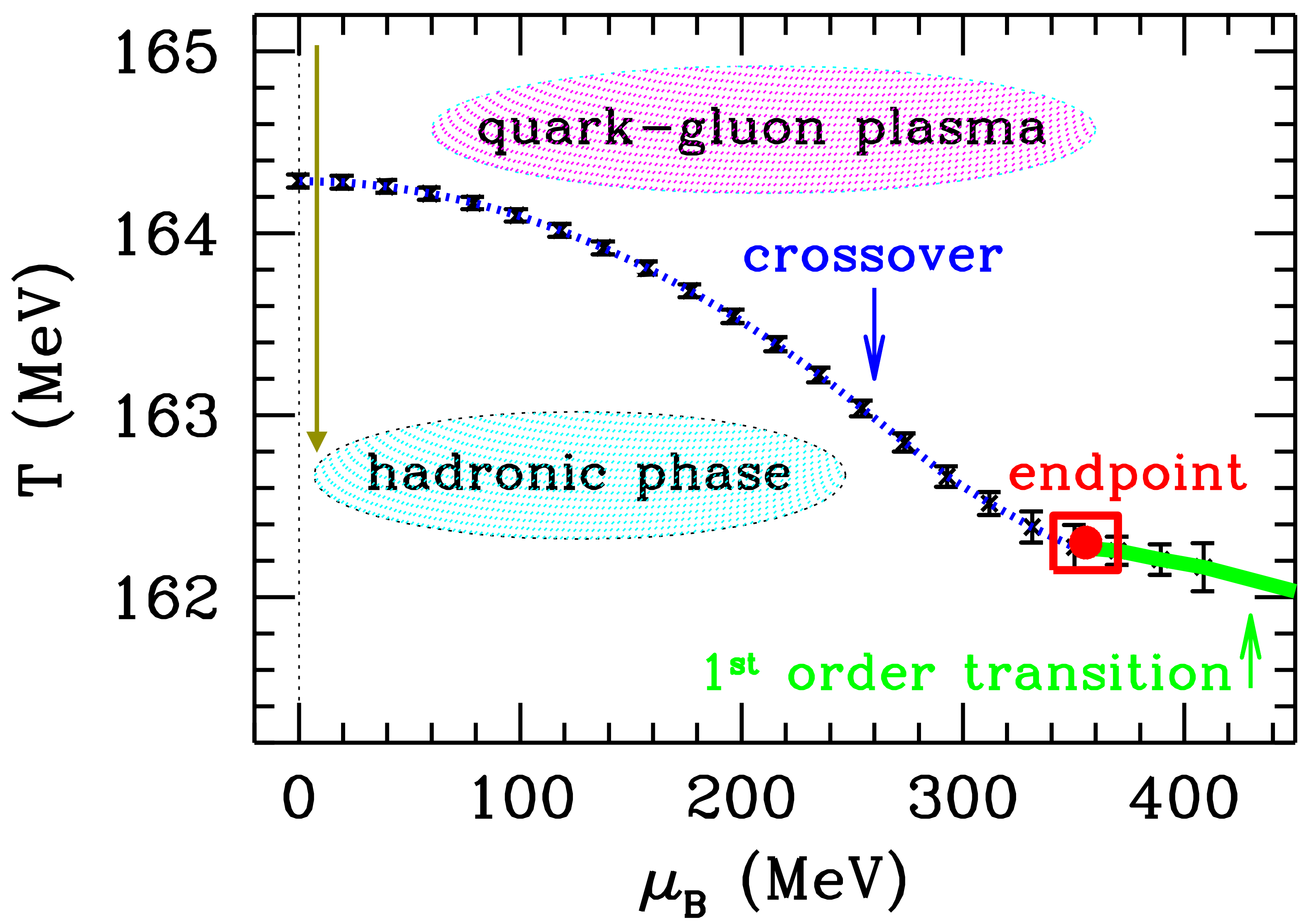
"micro Neutron Star"

- **One example of Macroscopic Dark Matter**

QCD phase transition

- **Crossover in the minimal Standard Model of Particle Physics with the normal early universe history**

Fodor and Katz, '2004



How to have QCD 1’st-order PT?

- **Making the strange quark lighter during the transition time (FOPT for 3 massless quarks)**

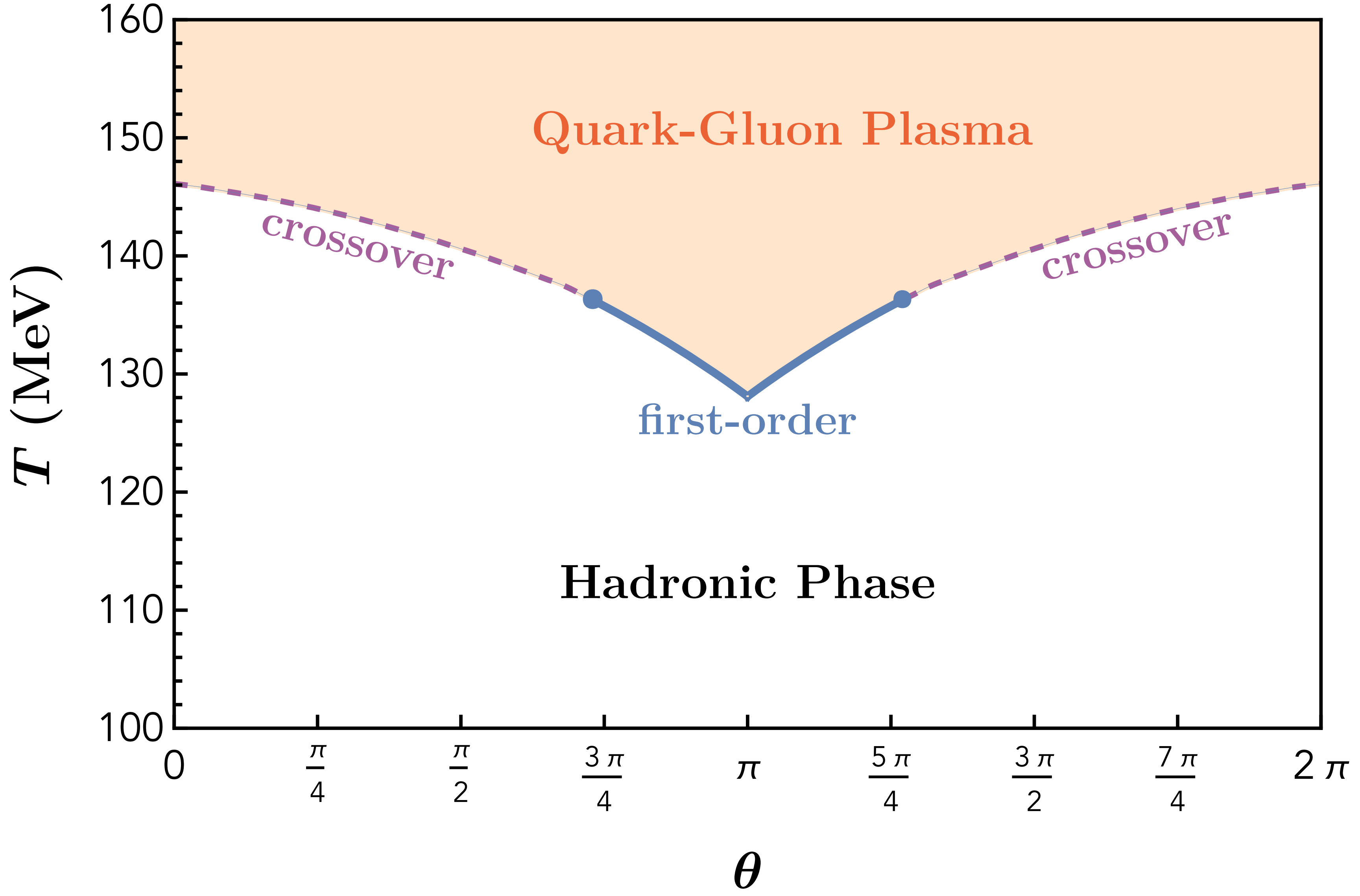
  For instance, using Froggatt-Nielsen fields to dynamically control quark masses (suffers fine-tuning and flavor constraints)

- **Supercool the electroweak phase transition to be below the QCD scale (requires a non-trivial flat potential)**

- **Existing a large lepton number chemical potential (suffers from BBN and CMB constraints)**

QCD phase transition with $\theta \neq 0$

$$\mathscr{L} \supset -\frac{1}{32\pi^2} G^{\mu\nu} \widetilde{G}_{\mu\nu} \theta$$

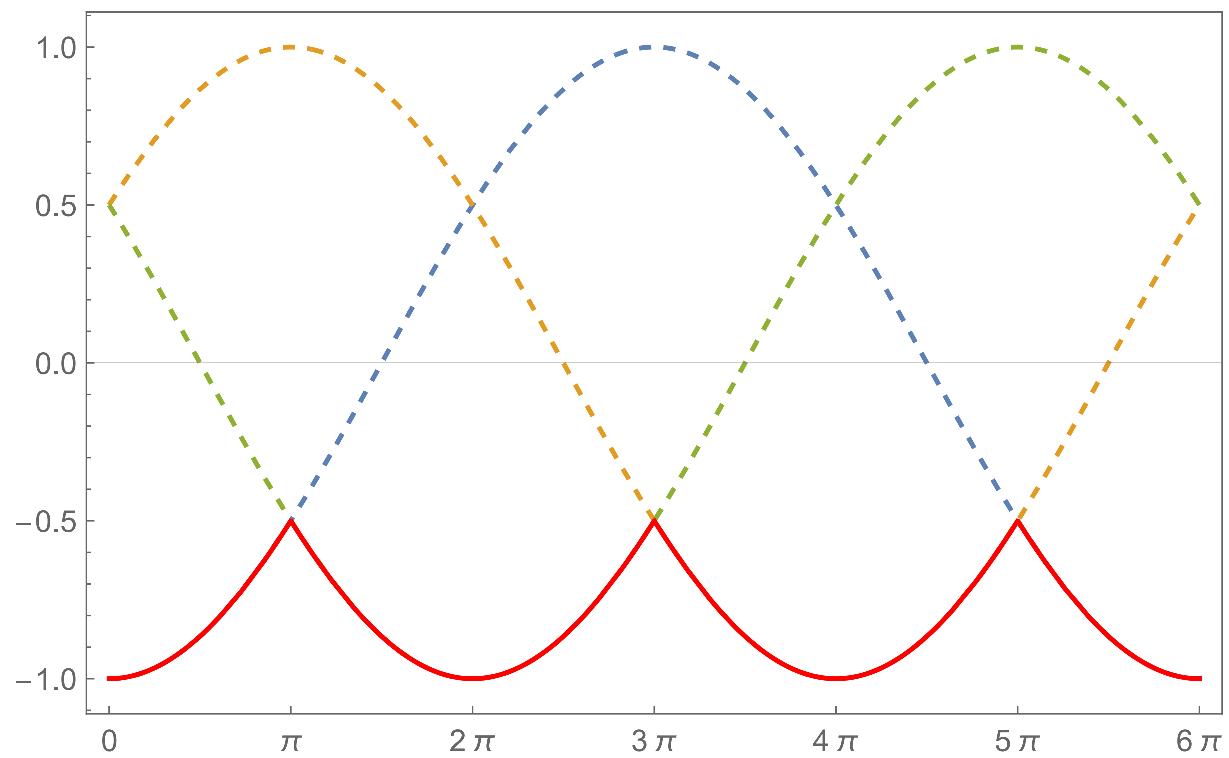


YB, Chen, Korwar, 2306.17160

QCD with $\theta = \pi$

- **In large N_c, the periodicity in θ and continuity of the vacuum energy function suggests a multibranched function**

$$V(\theta) = -N_c^2 \Lambda^4 \min_k \left[\cos\left(\frac{\theta + 2\pi k}{N_c} \right) \right], \quad k = 0, \cdots, N_c - 1$$



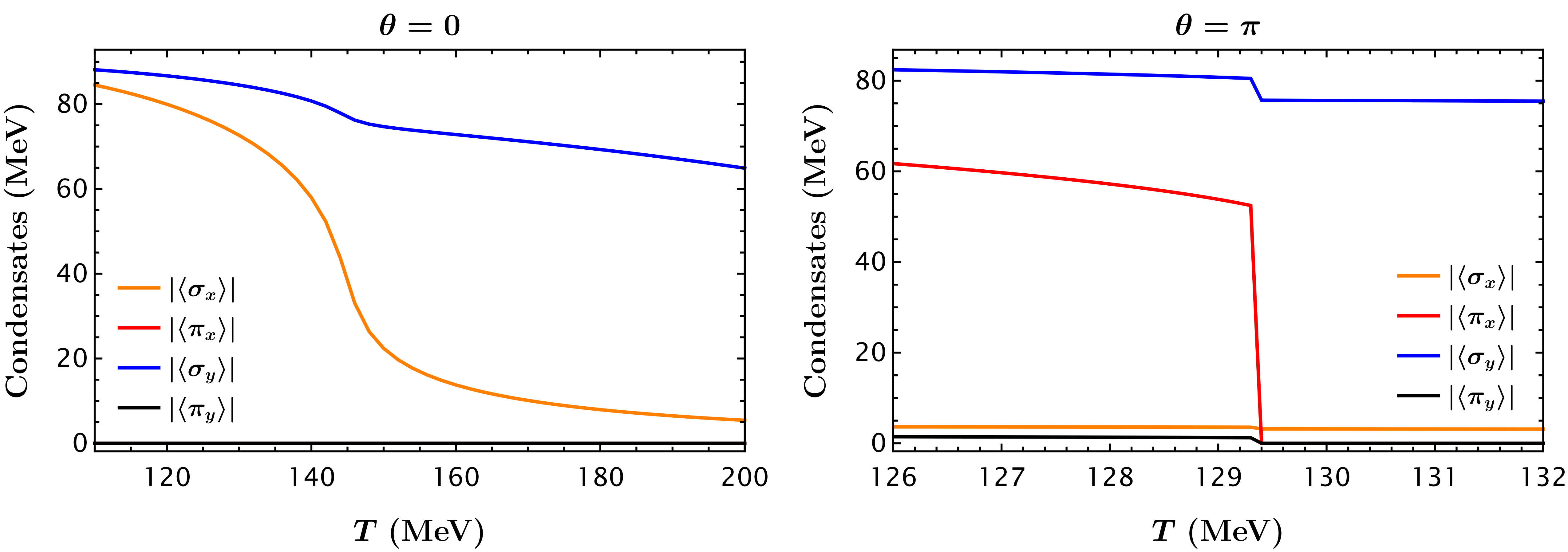
Dashen '1971; Witten '1980;
Gaiotto, Kapustin, Komargodski, Seiberg, '2017

Phenomenological LSMq

$$V(\Phi) = \mu^2 \mathrm{Tr}\left(\Phi^\dagger \Phi\right) + \lambda_1 \left[\mathrm{Tr}\left(\Phi^\dagger\Phi\right)\right]^2 + \lambda_2 \mathrm{Tr}\left[\left(\Phi^\dagger\Phi\right)^2\right]$$
$$-\frac{\kappa}{2}\left[e^{-i\theta}\det\left(\Phi\right) + e^{i\theta}\det\left(\Phi^\dagger\right)\right] - \mathrm{Tr}\left[H\left(\Phi+\Phi^\dagger\right)\right]$$

Pisarski, hep-ph/9601316

$$\Phi = T_a\left(\sigma_a + i\pi_a\right) \qquad H = T_a h_a \qquad \mathscr{L}_{\mathrm{Yukawa}} \supset \bar{q}\left[-gT_a\left(\sigma_a + i\gamma^5\pi_a\right)\right] q$$



QCD PT inside domains

- **The early universe could have different domains with different effective θ angle**

0	π	0	π	0
π	0	π	0	π
0	π	0	π	0
π	0	π	0	π

- **One half of the domains could have FOPT for QCD**

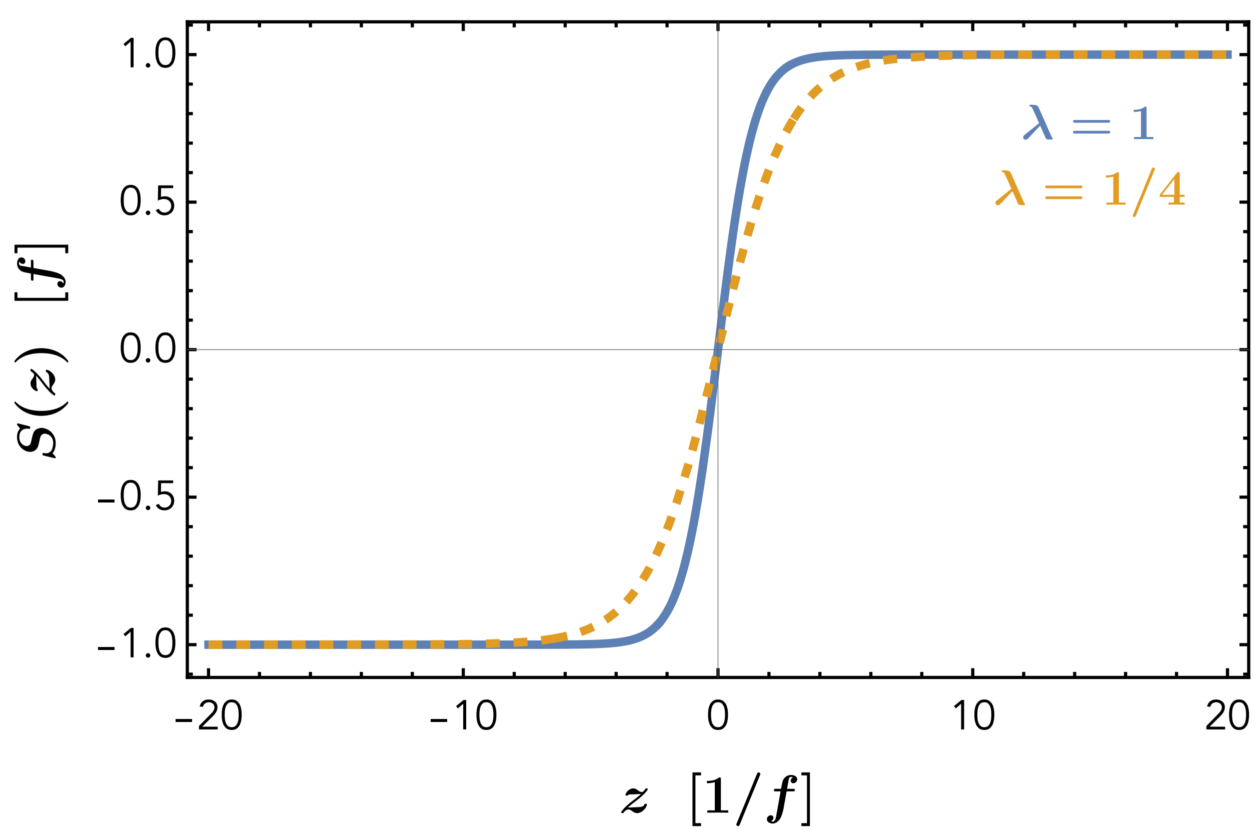
Discrete symmetry and domain wall

- **Spontaneous breaking of discrete symmetries generate domain walls in the early universe** Zeldovich, Kobzarev, Okun, '1974

$$V(S) = \frac{\lambda}{4}(S^2 - f^2)^2 \qquad \mathbb{Z}_2$$

- **A non-perturbative solution to transit from one vacuum $\langle S \rangle = -f$ to the other vacuum $\langle S \rangle = +f$**

$$S(z) = f\ \tanh\left[\sqrt{\frac{\lambda}{2}}\, f\, z\right]$$



- **Wall tension**

$$\sigma \equiv \int_{-\infty}^{\infty} T_{00} dz = \frac{2\sqrt{2}}{3}\sqrt{\lambda} f^3$$

QCD-anomalous discrete symmetry

- **For QCD to care different domains, one need to have the discrete symmetry ($\mathbb{Z}_N$) anomalous under QCD**

Preskill, Trivedi, Wilczek, Wise, '1991

$$V(S) \Rightarrow \langle S \rangle_j = f e^{i\, 2\pi j/N}, \quad \text{with} \quad j = 0, 1, \cdots, N-1$$

$$\mathscr{L} \supset -\frac{1}{32\pi^2} G^{\mu\nu} \widetilde{G}_{\mu\nu} \left[\theta_0 + \sum_{\psi} 2\, q_\psi\, C(r_\psi)\, \arg(S) \right] \qquad C(\mathbf{3}) = 1/2$$

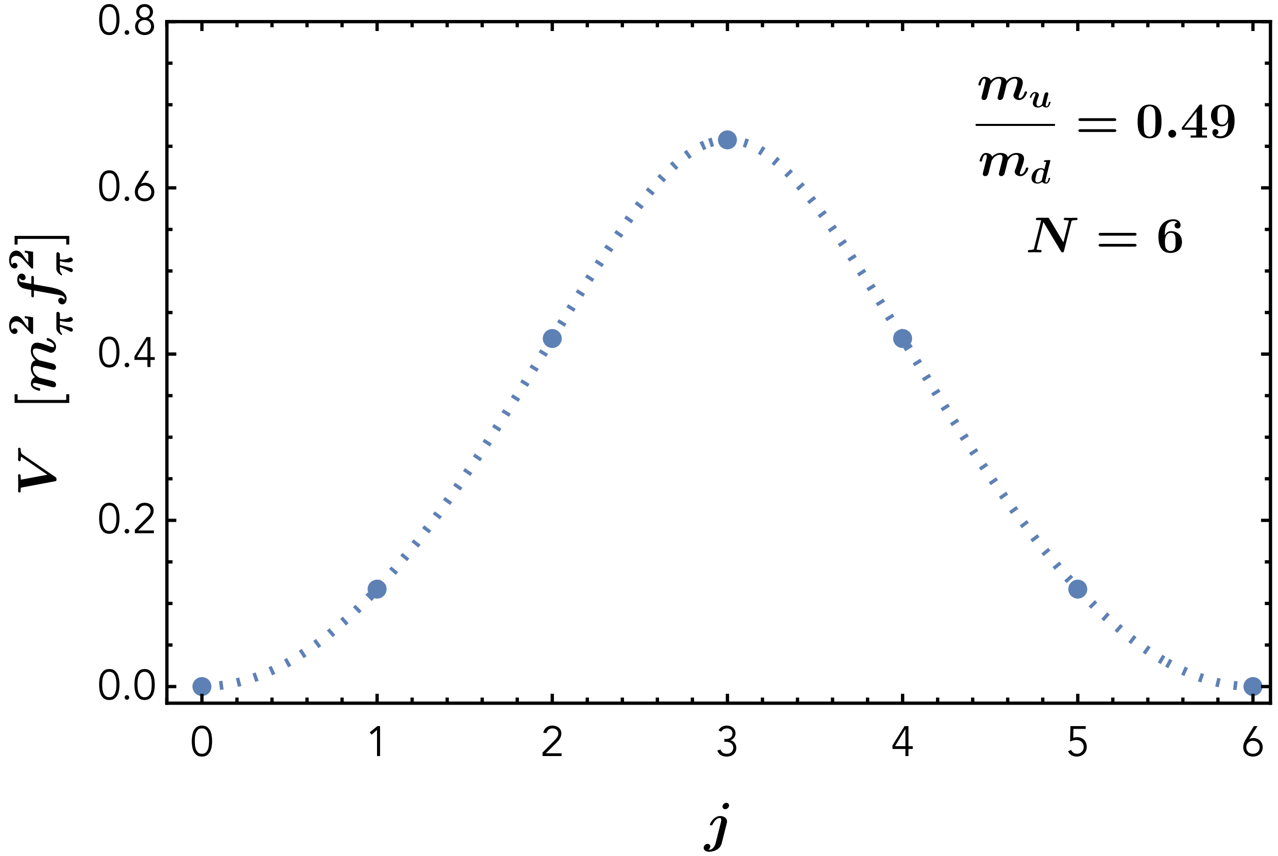
- **Assume $\theta_0 = 0$, for instance from the Nelson-Barr mechanism**

For $q_\psi = 1,\ \theta_j = 2\pi\, j\, n_f / N$. For $\gcd(n_f, N) = 1$, the QCD instanton effects break all $\mathbb{Z}_N$ symmetry

QCD-anomalous discrete symmetry

- **Based on the chiral Lagrangian, different domains have different effective potential**

$$V\left(\langle S\rangle_j\right) = -\,m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4\,m_u\,m_d}{(m_u+m_d)^2}\,\sin^2\left(\frac{\pi\,j}{N}\right)}$$



$$V_{\rm bias}^{\rm max} = 0.66\,m_\pi^2\,f_\pi^2 \approx (100.4\,\text{MeV})^4 \qquad \text{for} \quad N = \text{even}$$

- **A smaller $N-$dependent value when $N = \text{odd}$**

A Nelson-Barr model

- **CP is a good symmetry at the UV and is spontaneously broken in the IR**

Nelson, '1983; Barr, '1984

$$\mathscr{L} \supset Y^u_{ij}\, \widetilde{H}_u\, \overline{Q}_{iL}\, u_{jR} + (\eta_j\, \phi + \kappa_j\, \phi^*)\, \bar{\psi}_L\, u_{jR} + \mu\, \bar{\psi}_L\, \psi_R \; + \; Y^d_{ij}\, H_d\, \overline{Q}_{iL}\, d_{jR} \; - \; V(H_u, H_d, \phi)$$

$$\mathbb{Z}_2^{\rm NB} : H_u \to -H_u\,, \quad H_d \to H_d\,, \quad \phi \to -\phi$$

$$u_R \to -u_R\,, \quad \psi_{L,R} \to \psi_{L,R}\,, \quad Q_L \to Q_L\,, \quad d_R \to d_R$$

- **4 domains:**

$$(\langle H_d \rangle, \langle H_u \rangle, \langle \phi \rangle) = \quad (v\, \cos\beta, v\, \sin\beta, f e^{i\alpha})\;, \qquad (v\, \cos\beta, v\, \sin\beta, f e^{-i\alpha})\;,$$

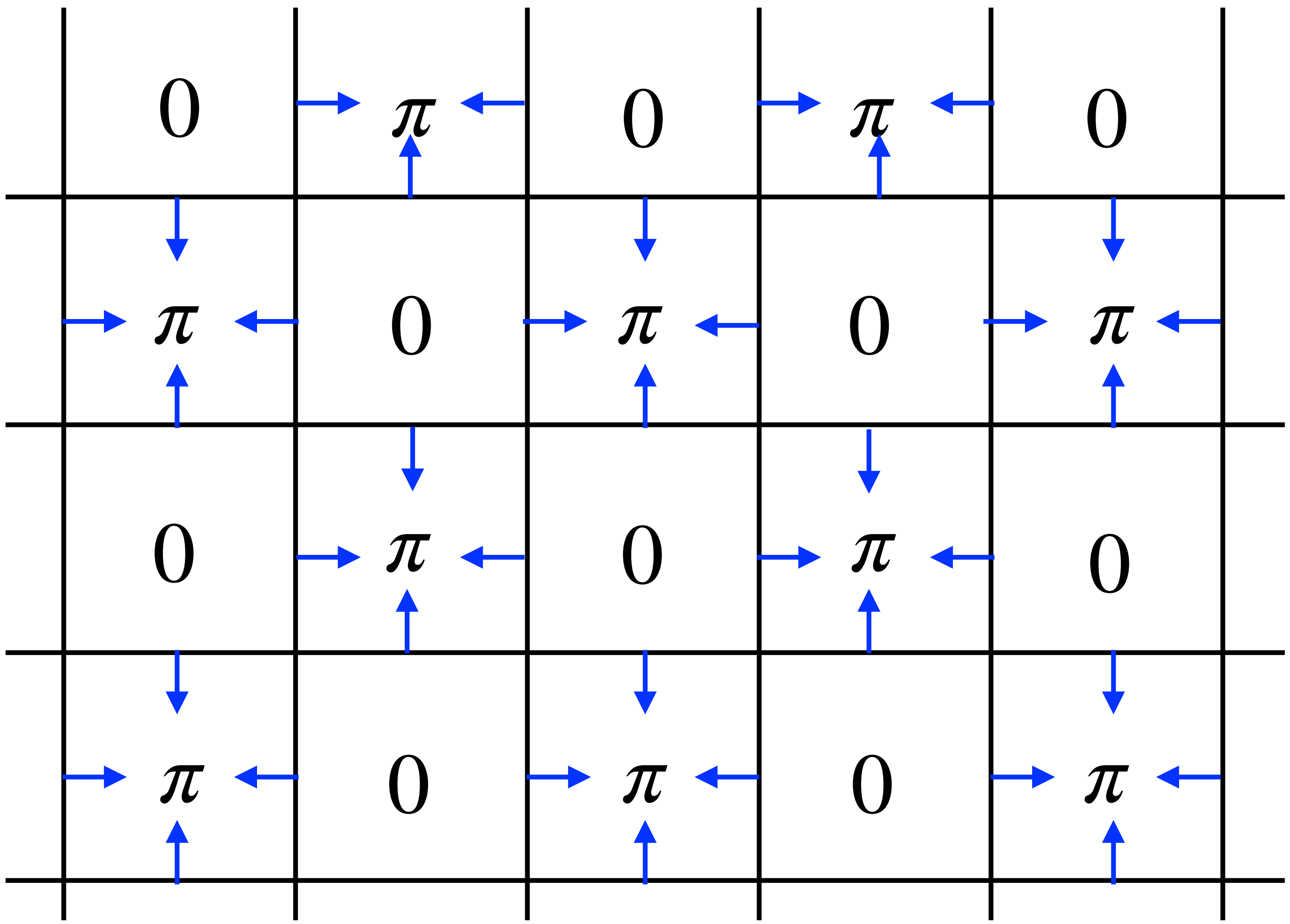
$$(v\, \cos\beta, -v\, \sin\beta, -f e^{i\alpha})\;, \quad (v\, \cos\beta, -v\, \sin\beta, -f e^{-i\alpha})$$

$$\mathscr{M}_u = \begin{pmatrix} Y^u \langle \widetilde{H}_u \rangle & 0 \\ \eta_j \langle \phi \rangle + \kappa_j \langle \phi \rangle^* & \mu \end{pmatrix}$$

arg[det($\mathscr{M}_u$)] = 0 or π

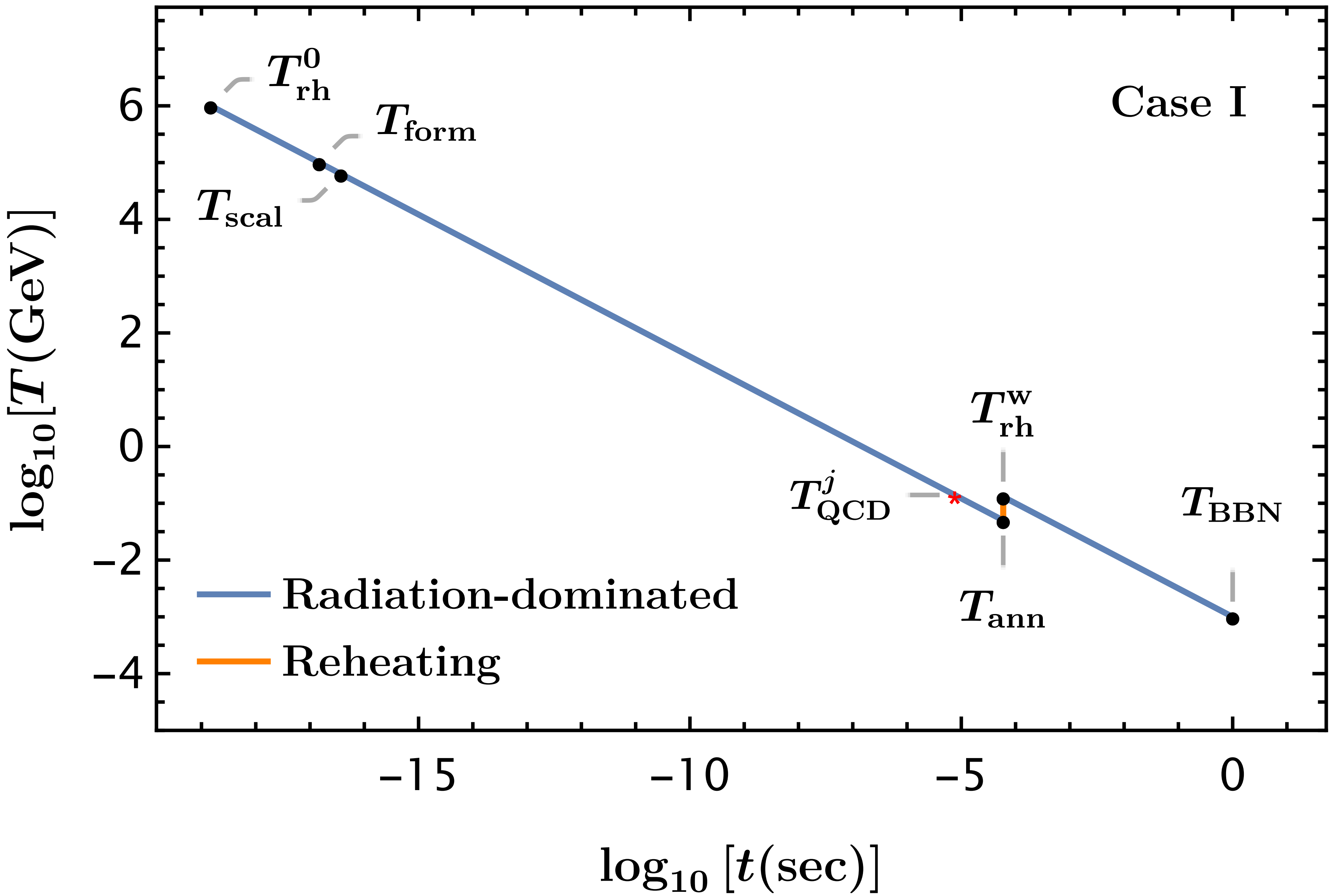
QCD-collapsed domain walls

- Around the QCD phase transition temperature, the domains with a lower potential expand and push walls to collapse



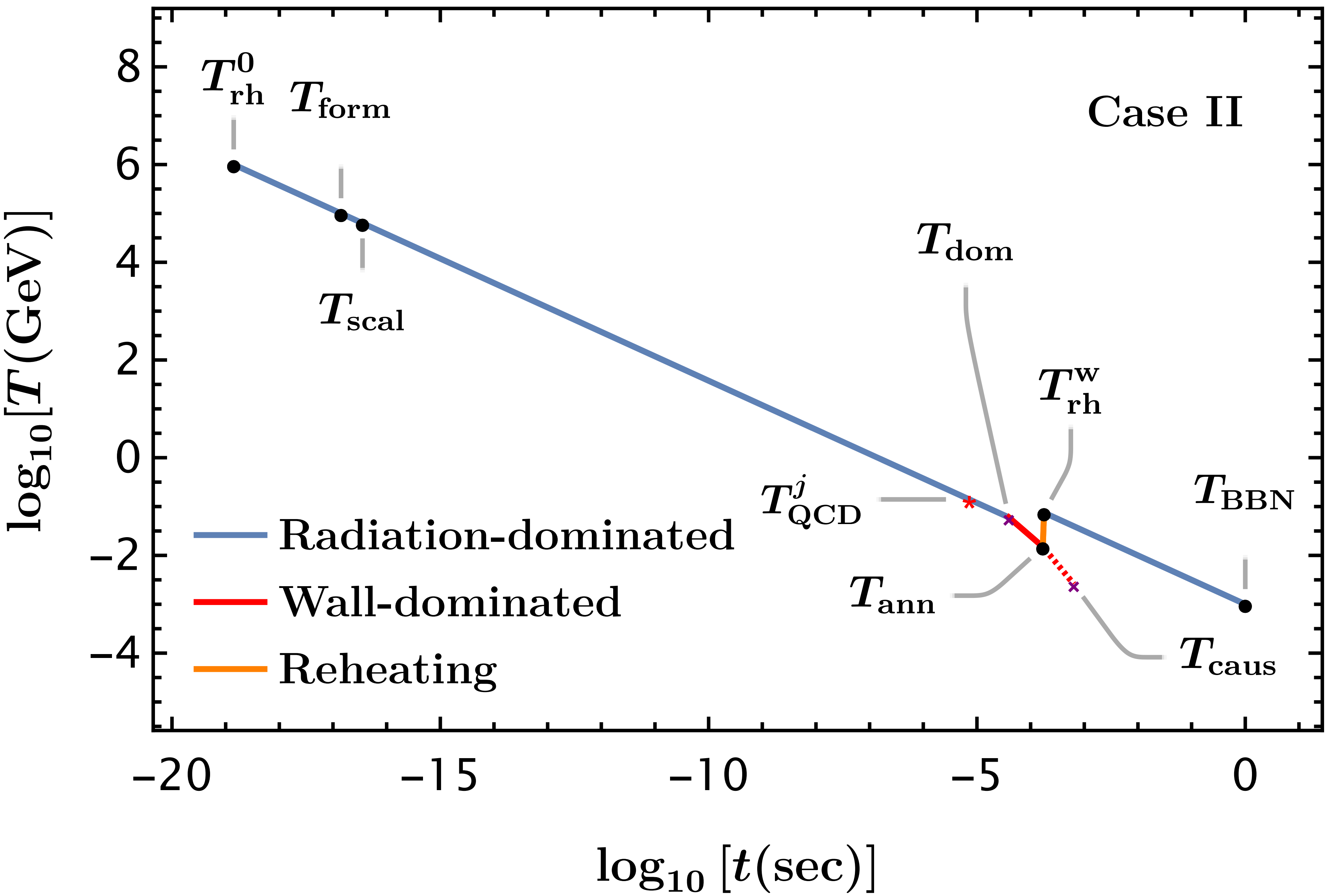
Domain wall evolution

❖ **Case I: domain walls never dominate the universe**



Domain wall evolution

- Case II: domain walls could dominate the universe



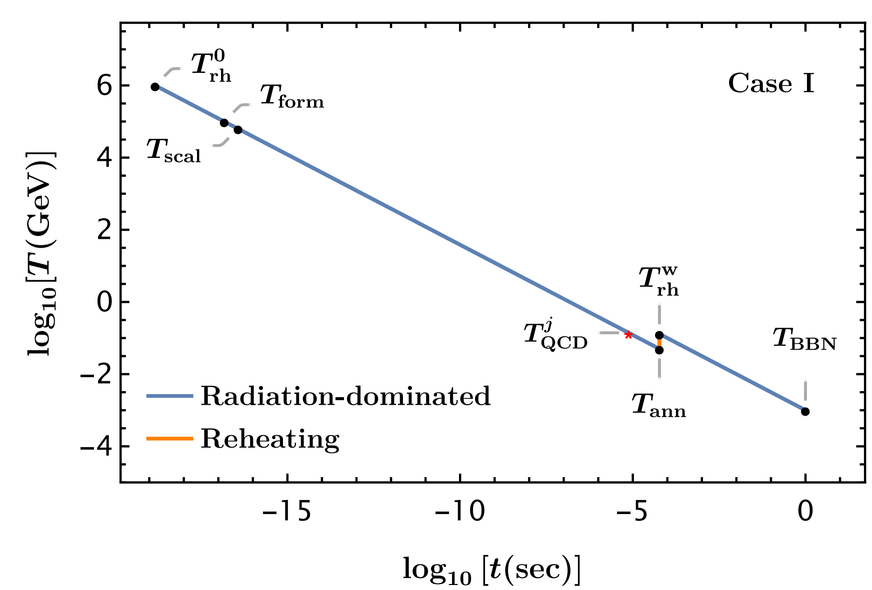
Domain wall evolution

- **After formation, the domain walls reach the so-called scaling region with an order-one number of walls per Hubble patch**
- **The wall energy density is $\rho_w \approx \sigma/L$, with $L \simeq t$, so $\rho_w \propto t^{-1}$, which drops slower than radiation energy density $\rho_R \propto t^{-2}$**
- **The walls will dominate the universe if they exist till**

$$T_{\rm dom} \approx 45\,{\rm MeV}\left(\frac{\sigma}{10^{16}\,{\rm GeV}^3}\right)^{1/2}\left(\frac{g_*}{10}\right)^{-1/4}$$

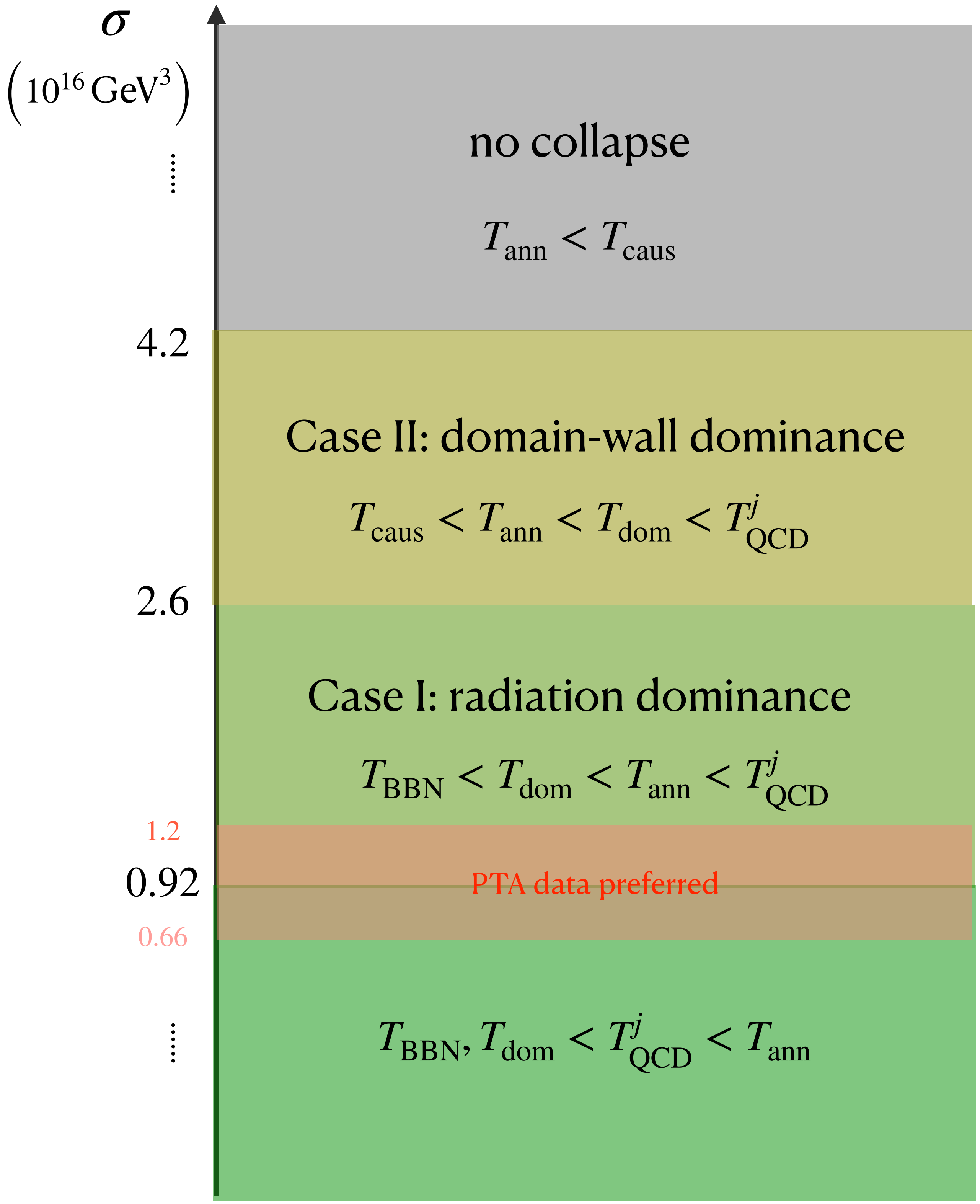
- **The walls annihilate at ($p_T \approx p_V$)**

$$T_{\rm ann} \approx 120\,{\rm MeV}\left(\frac{V_{\rm bias}}{(100\,{\rm MeV})^4}\right)^{1/2}\left(\frac{\sigma}{10^{16}\,{\rm GeV}^3}\right)^{-1/2}\left(\frac{g_*}{10}\right)^{-1/4}$$



Simulations: Martins et. al, 1602.01322, M. Kawasaki et. al, 1412.0789

Different scenarios



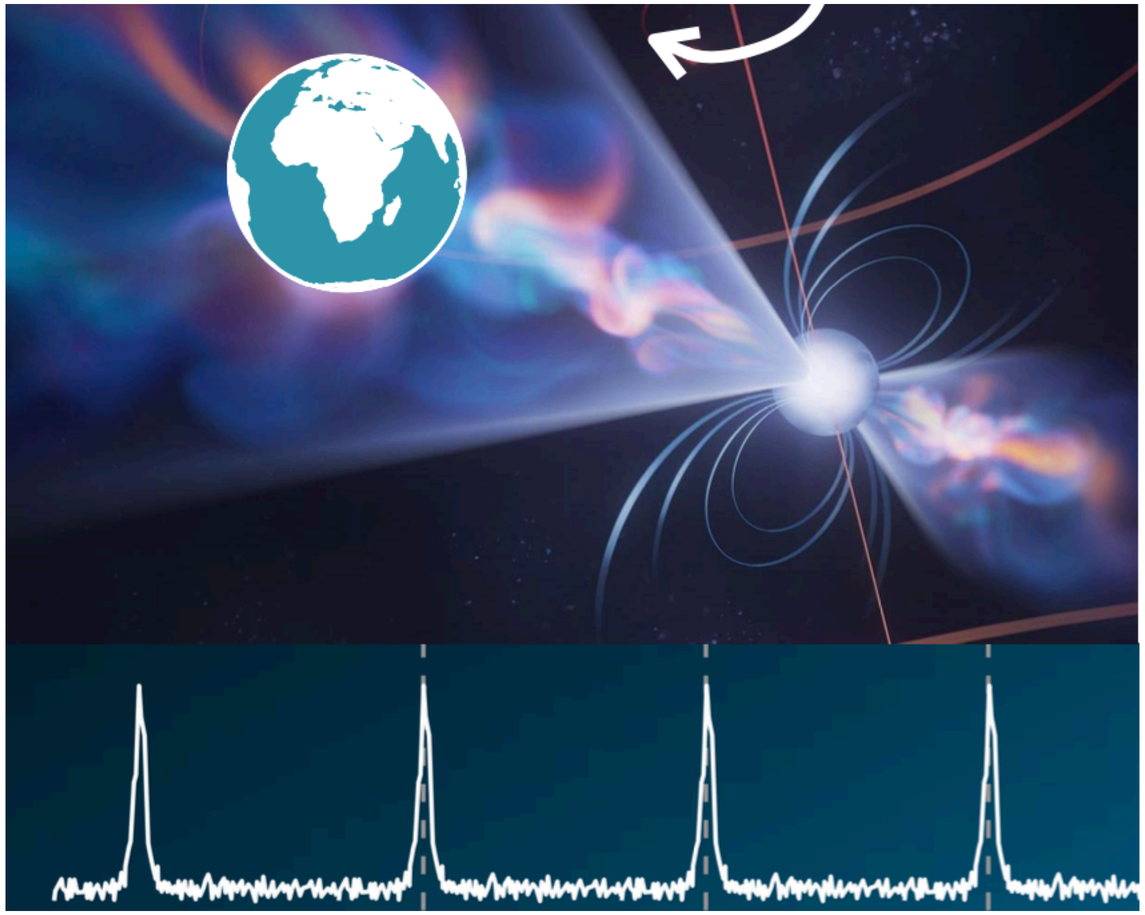
GW for pulsar timing array (PTA)

- **Millisecond pulsar has a very stable rotation frequency**

$$\nu(t) = \nu_0 + \dot{\nu}_0 t$$

$$\dot{\nu}_0/\nu_0 \sim 10^{-23} - 10^{-20}\,\text{Hz}$$

- **Using pulsar timing to find the common stochastic red process**



Credit: Kurzgesagt

Opportunities for detecting ultralong gravitational waves

M. V. Sazhin

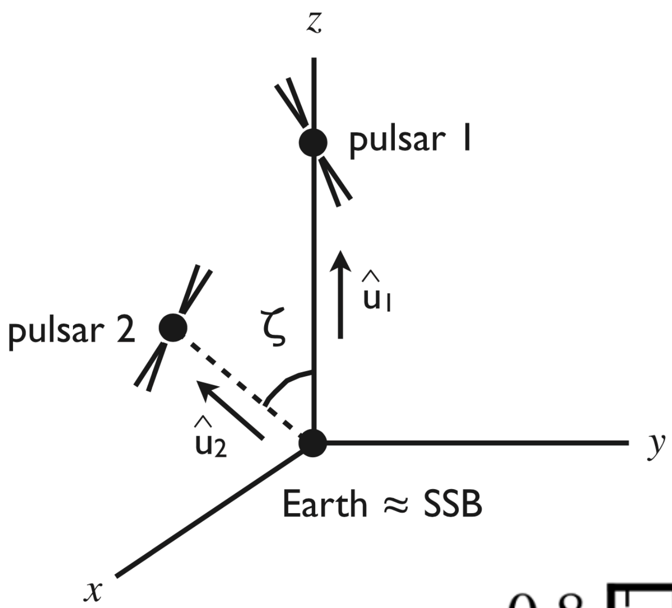
Shternberg Astronomical Institute, Moscow

(Submitted June 14, 1977)

Astron. Zh. **55**, 65–68 (January–February 1978)

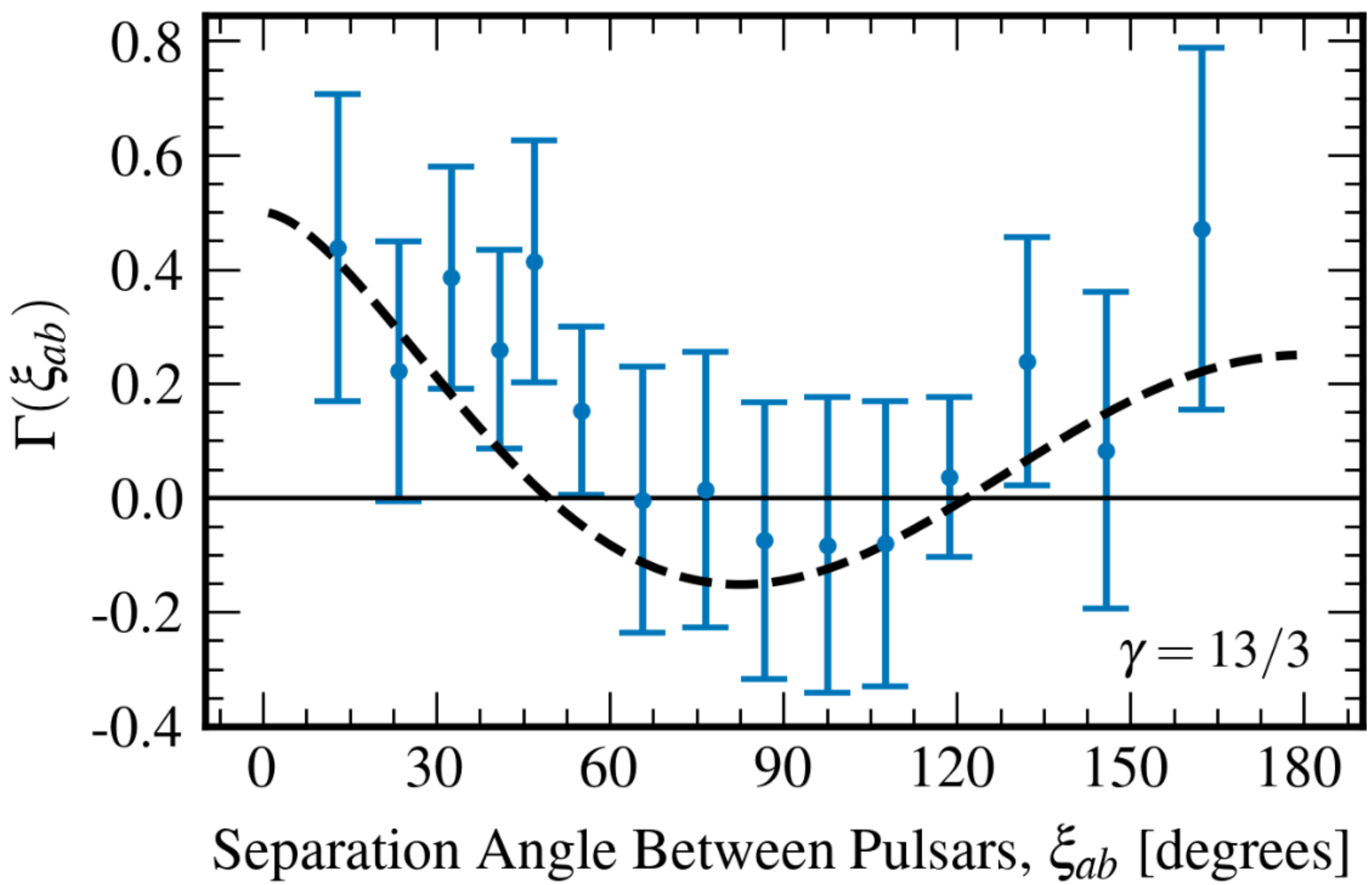
The influence of ultralong gravitational waves on the propagation of electromagnetic pulses is examined. Conditions are set forth whereby it might be possible to detect gravitational waves arriving from binary stars. There are some prospects for detecting gravitational radiation from double superstars with masses $\mathfrak{M}_1 \approx \mathfrak{M}_2 \approx 10^{10}\,\mathfrak{M}_\odot$.

Hellings-Downs curve



$$\chi(\zeta) = \frac{1}{2} - \frac{1}{4}\left(\frac{1-\cos\zeta}{2}\right) + \frac{3}{2}\left(\frac{1-\cos\zeta}{2}\right)\ln\left(\frac{1-\cos\zeta}{2}\right)$$

Hellings, Downs, '1983



NANOGrav-15, 2306.16213,
see also CPTA,EPTA, PPTA

GW from domain wall collapse

- **From Einstein's quadrupole formula $P_{\rm GW} \sim G\,(\dddot{Q})^2$, with Q as the transverse-traceless part of the quadrupole moment of matter.**
- **For domain walls, $Q \sim M_{\rm wall} L(t)^2$. So, $\rho_{\rm GW} \sim P_{\rm GW} H^{-1}/L^3$**
- **The peak frequency, $f(t_{\rm ann}) \sim H(t_{\rm ann})$**
- **Red-shift to today's universe**

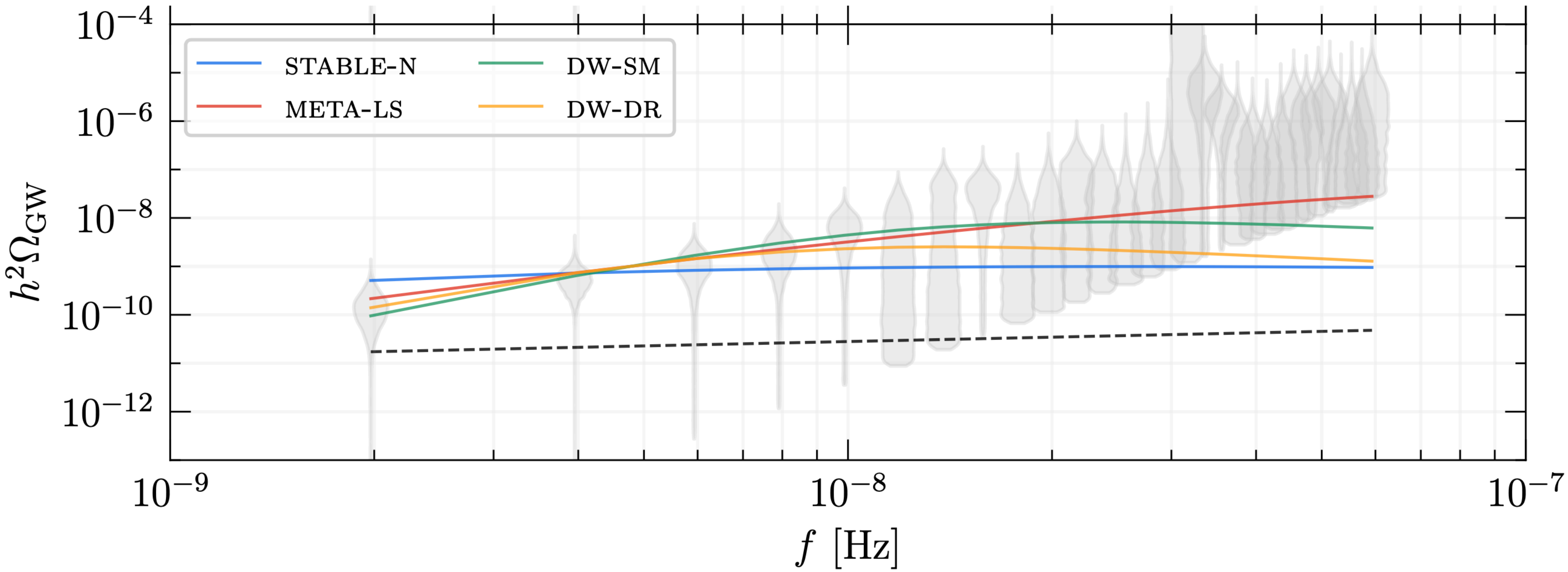
$$\Omega_{\rm GW} h^2(t_0)\Big|_{\rm peak} = 3\times 10^{-8}\left(\frac{\sigma}{10^{16}\,{\rm GeV}^3}\right)^2\left(\frac{T_{\rm ann}}{100\,{\rm MeV}}\right)^{-4}\left(\frac{g_{*s}}{10}\right)^{-4/3}$$

$$f_{\rm peak} = 1.1\times 10^{-8}\,{\rm Hz}\left(\frac{g_*(T_{\rm ann})}{10}\right)^{1/2}\left(\frac{g_{*s}(T_{\rm ann})}{10}\right)^{-1/3}\left(\frac{T_{\rm ann}}{100\,{\rm MeV}}\right)$$

- **It scales like f^3, for $f < f_{\rm peak}$ and f^{-1} for $f > f_{\rm peak}$. A harder spectrum for $N > 2$, from simulations**

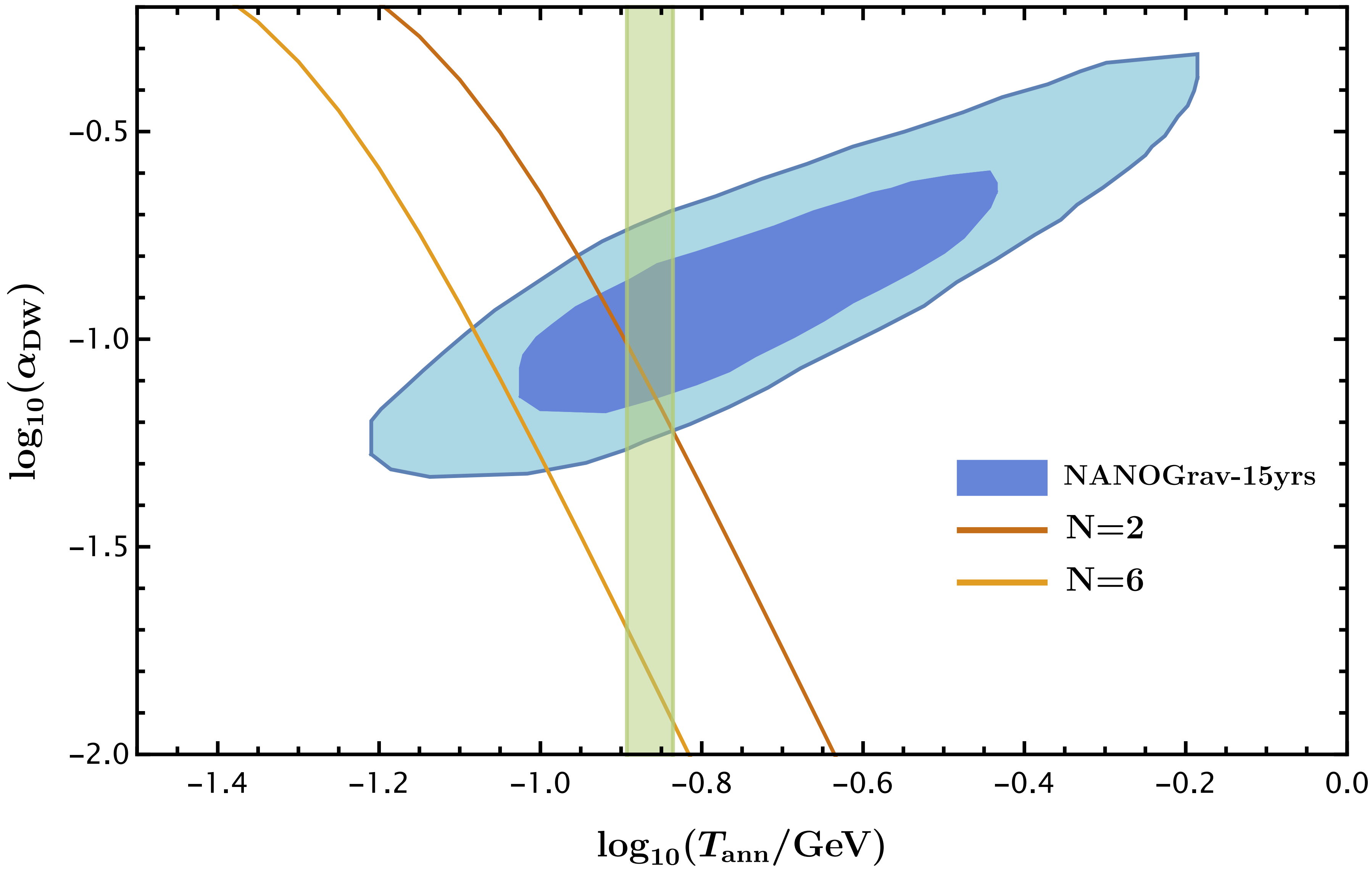
M. Kawasaki et. al, 1207.3166

Domain-wall interpretation for PTA



NANOGrav-15, 2306.16219

Domain-wall interpretation for PTA



green band indicate QCD phase transition T for different θ

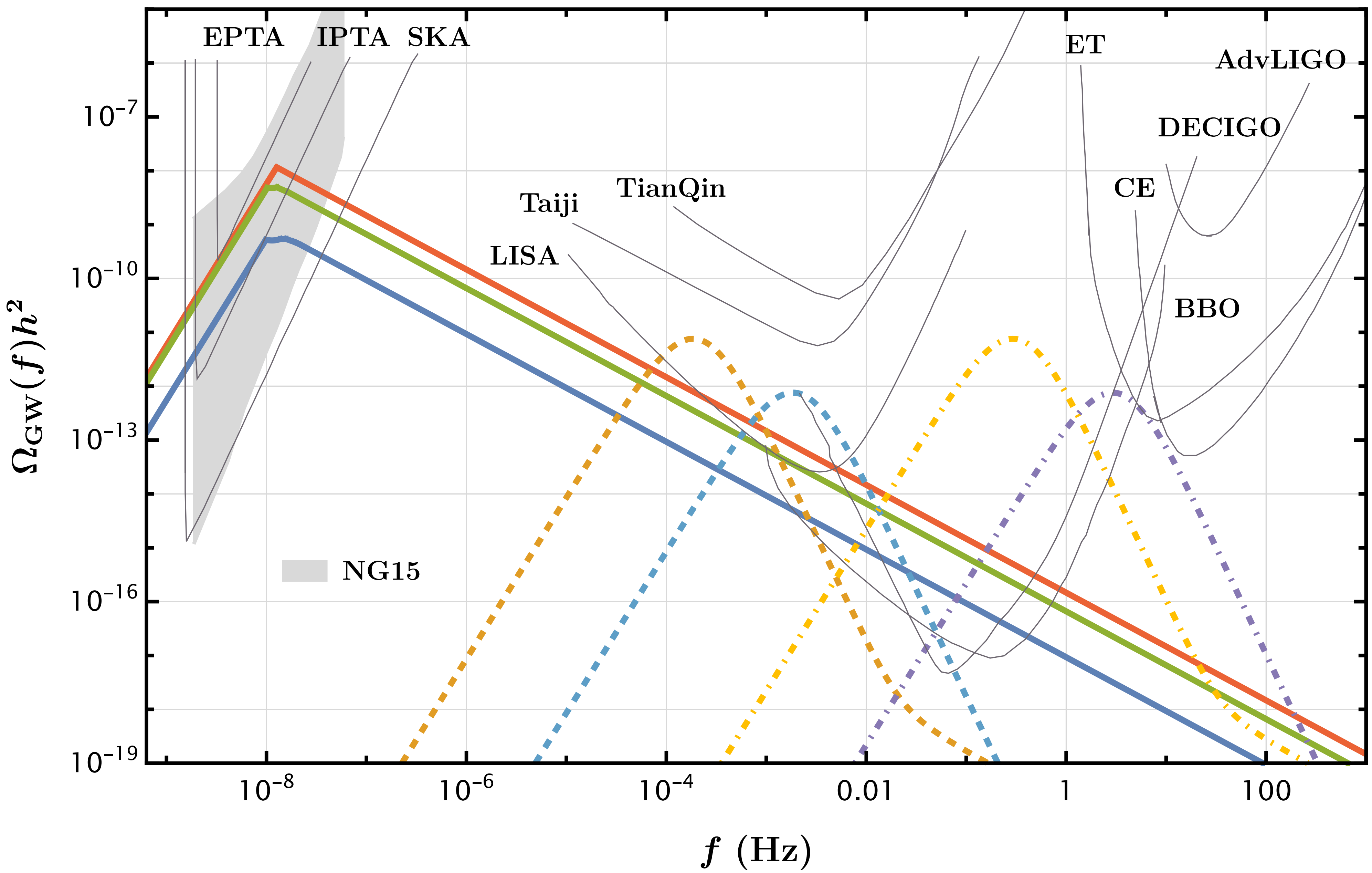
$$\alpha_{\rm DW} \equiv \frac{\rho_w(T_{\rm ann})}{\rho_w(T_{\rm ann}) + \rho_R(T_{\rm ann})}$$

$$N = 2 : \sigma \in (0.66, 1.2) \times 10^{16}\,{\rm GeV}^3$$

$$N = 6 : \sigma \in (0.9, 1.3) \times 10^{16}\,{\rm GeV}^3$$

Discrete symmetry breaking scale: $f \simeq 100\,{\rm TeV}$

Gravitational wave spectroscopy



QCD 1'st PT

$(\alpha, \beta/H) = (0.5, 10^4)$

$(\alpha, \beta/H) = (0.5, 10^5)$

$\mathbb{Z}_N$ 1'st PT

$(\alpha, \beta/H) = (0.5, 10^4)$

$(\alpha, \beta/H) = (0.5, 10^5)$

Conclusions

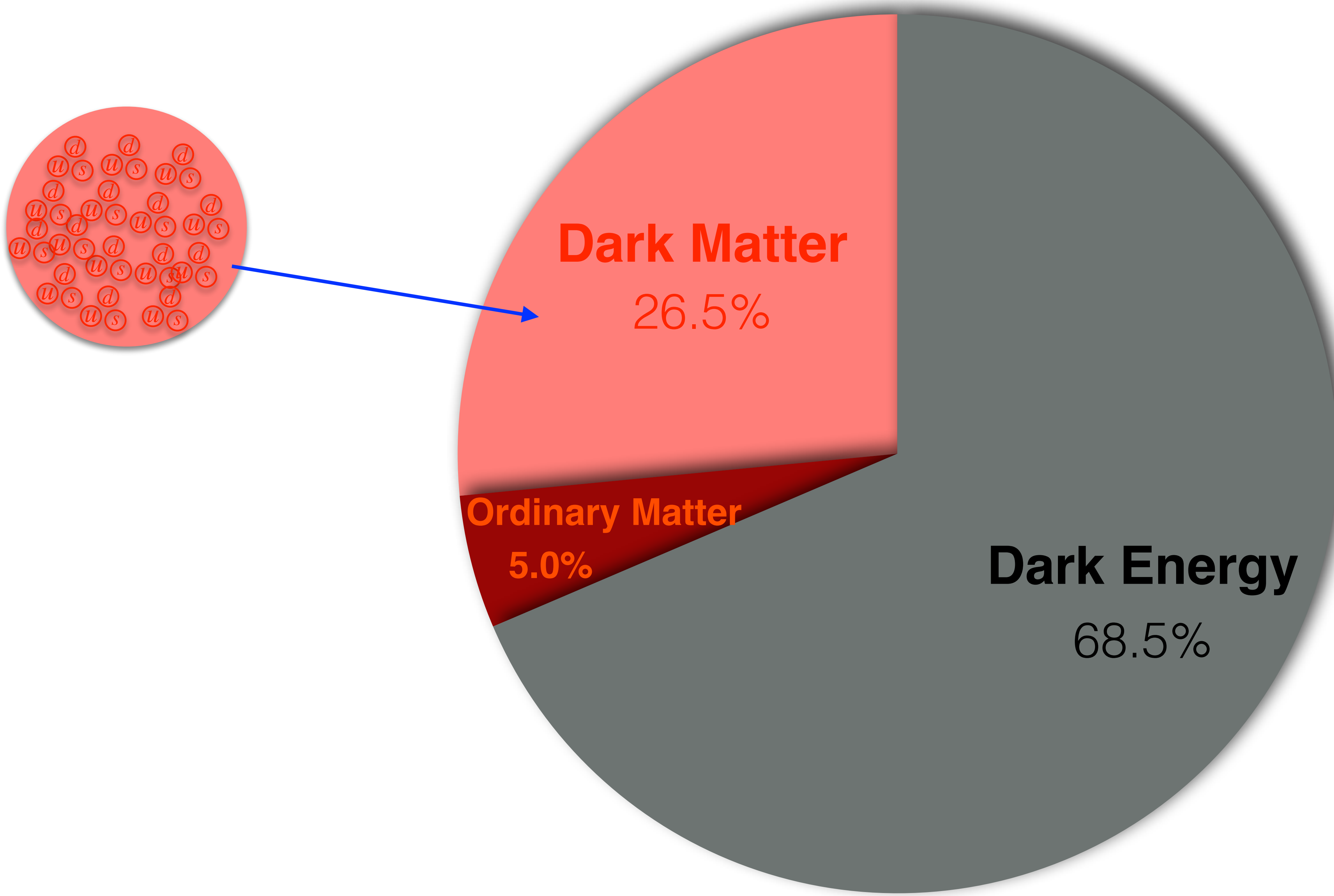
- The dark matter coincidence problem suggests a non-trivial relationship between dark matter and QCD

- The quark nugget is a compelling dark matter candidate, with additional BSM physics that can modify the QCD phase transition

- Domain walls from a QCD-anomalous discrete symmetry could play such a role

- The stochastic GW from domain collapses with the discrete symmetry breaking scale ~100 TeV has both frequency and amplitude match the PTA observed one

Dark matter is not dark, but rare!



Thanks!