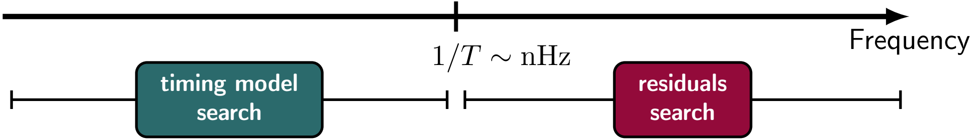
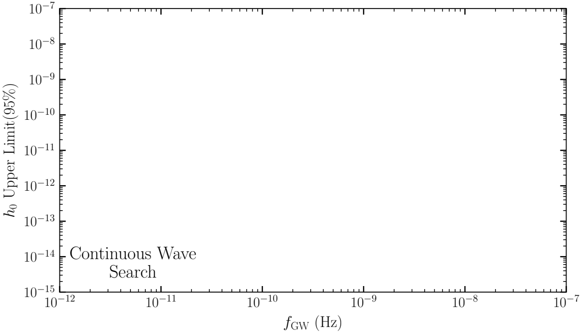
Sub-nHz Gravitational Waves Detection with Pulsars

PITT PACC Workshop
Jeff Dror

PRL (TBD) w/ DeRocco ['23]
PRD 108, 103011 w/ DeRocco ['23]
...



Continuous Source Search



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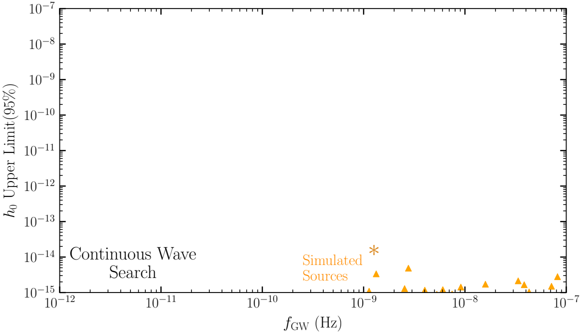
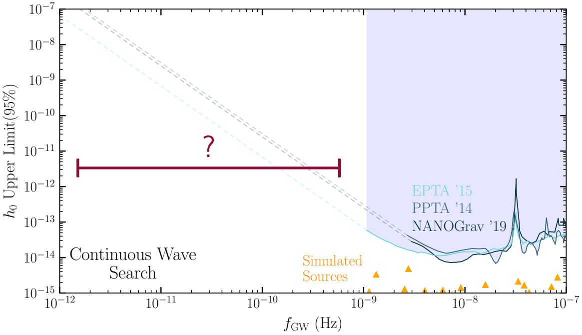


*[Kocsis ,Sesana '10]

Continuous Source Search



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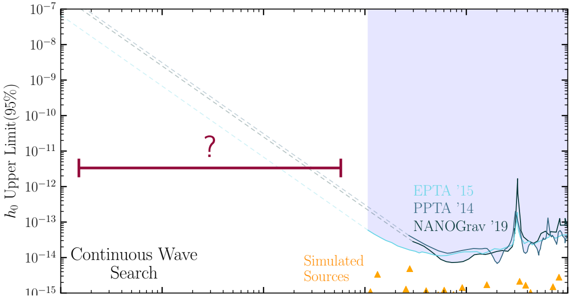
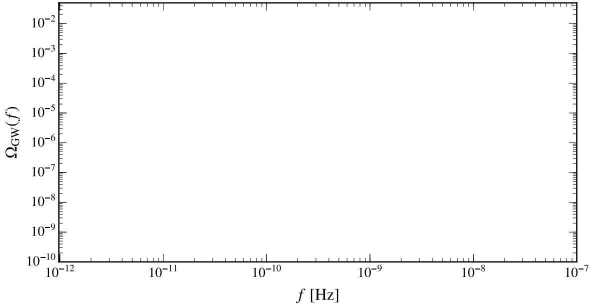
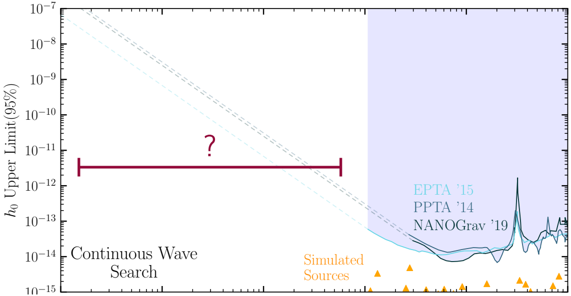


Stochastic Search



Continuous Source Search



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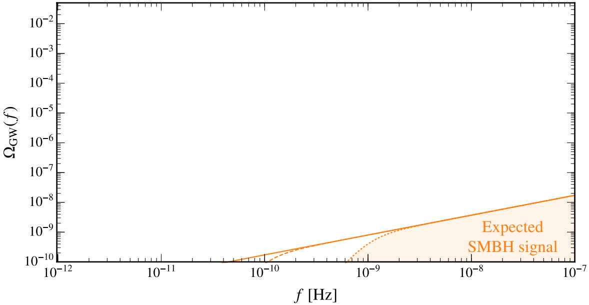
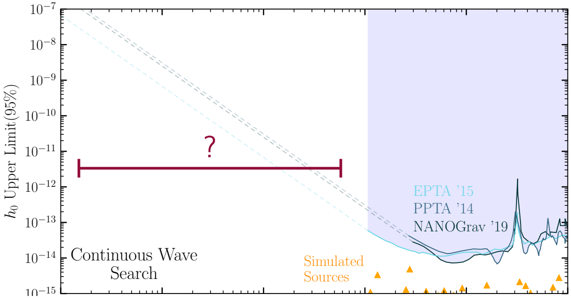
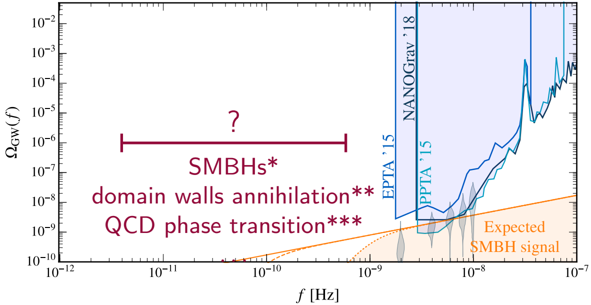


[amplitude set to $A_\star \sim 10^{-15}$]

Continuous Source Search



Stochastic Search



*Luke Kelly's talk
**Yang Bai's talk
***Arthur Kosowsky's talk
***Tina Kahniashvili's talk

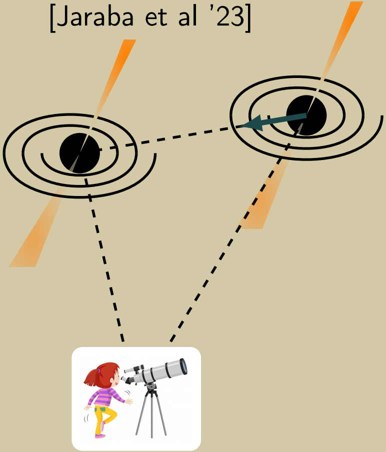
Existing sub-nHz Frequency Efforts

the ultralow frequency challenge → requires measuring "secular drifts"

Quasar Astrometry

[Gwinn et al '96]
[Book, Flanagan '10]
[Darling et al '18]
[Jaraba et al '23]

Existing sub-nHz Frequency Efforts



Existing sub-nHz Frequency Efforts

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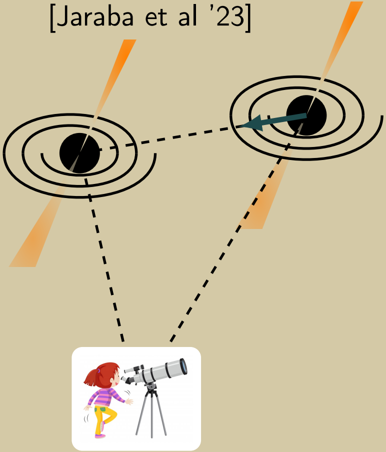

Pulsar Timing Model

[Bertotti, Carr, Rees, '83]
[Kopeikin '97] [Kopeikin '99]
[Kopeikin, Potapov '04]
[Pshirkov '09] [Yonemaru et al '18]
[Kumamoto et al '19] [Kumamoto et al '21]
[Kikunaga et al '21]

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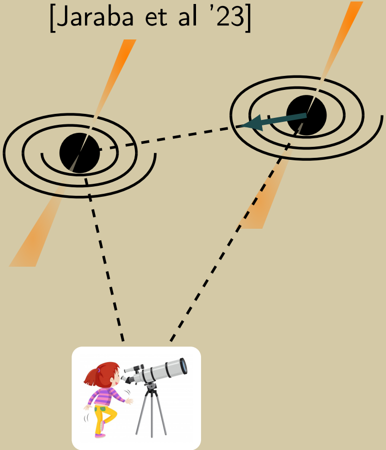
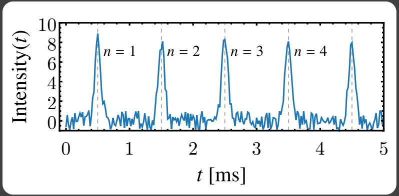
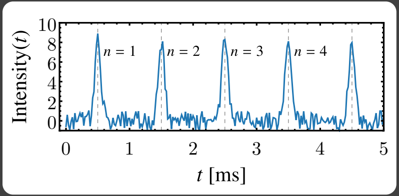


Our Work ([JD, DeRocco '23] [JD, DeRocco '23])

1. show backgrounds under control through correlated signal
2. reach realistic continuous and stochastic signal strengths


Intensity(t)
10
8
6
4
2
0
n = 1
n = 2
n = 3
n = 4
0
1
2
3
4
5
t [ms]


P


Intensity(t)
10
8
6
4
2
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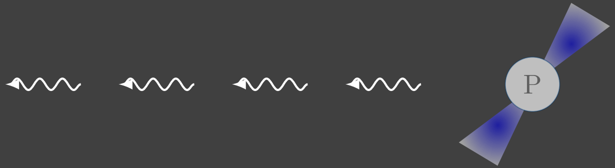
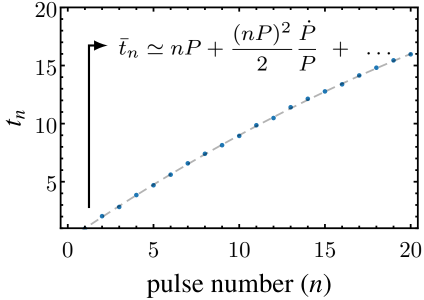
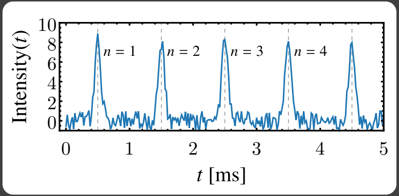
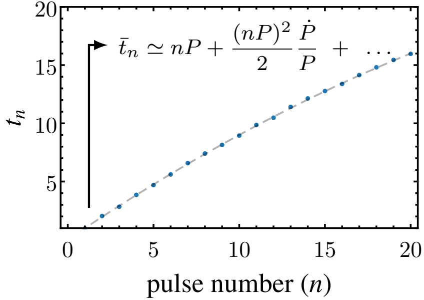
$n = 1$
$n = 2$
$n = 3$
$n = 4$
0
1
2
3
4
5
t [ms]


P

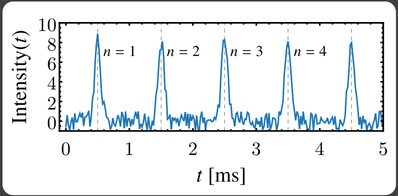

$\bar{t}_n \simeq nP + \frac{(nP)^2}{2}\frac{\dot{P}}{P} + \ldots$
t_n
20
15
10
5
0
5
10
15
20
pulse number (n)

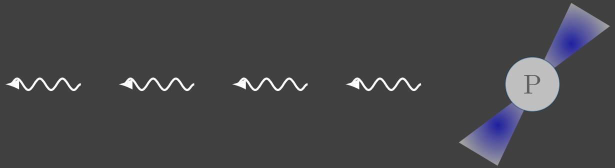


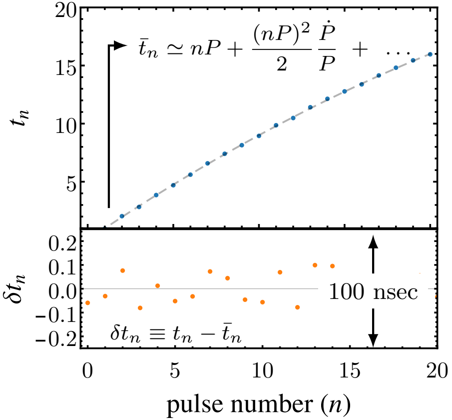


Timing Model

- $\mathcal{O}(10)$ parameters
- includes $P, \dot{P}, \ddot{P}, \ldots$
- if binary: includes $P_b, \dot{P}_b, \ldots$







usual analysis

Timing Model

- $\mathcal{O}(10)$ parameters
- includes $P, \dot{P}, \underline{\ddot{P}}, \ldots$
- if binary: includes $P_b, \underline{\dot{P}_b}, \ldots$

focus of this talk

First Derivative of Period

$$\frac{\dot{P}_{\rm obs}}{P} = \frac{\dot{P}_{\rm int}}{P} - \frac{v_\perp^2}{d_a} - a_{\rm MW}$$

First Derivative of Period

Observed value
$\mathcal{O}(10^{-18} \pm 10^{-24})\ \mathrm{sec}^{-1}$

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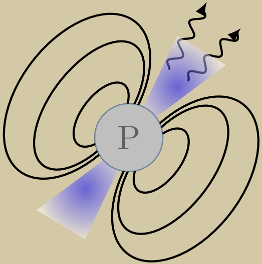
“Intrinsic” spin-down

$\sim 10^{-18}\mathrm{sec}^{-1}$



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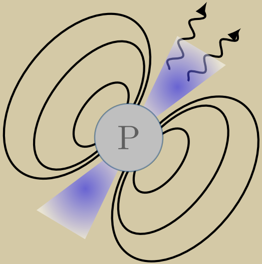
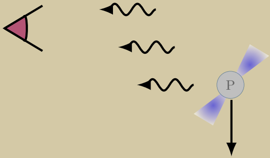


Kinematic

$\simeq 10^{-18}\ \text{sec}^{-1} \frac{v_\perp^2}{v_{\text{gal}}^2} \frac{\text{kpc}}{d_a}$



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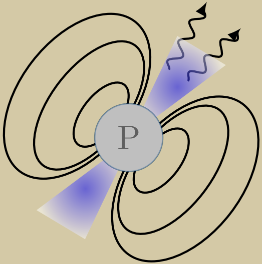
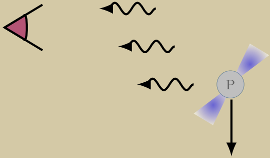


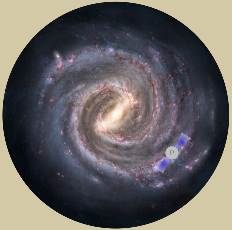
Kinematic

$\simeq 10^{-18}\ \text{sec}^{-1}\,\frac{v_\perp^2}{v_{\text{gal}}^2}\,\frac{\text{kpc}}{d_a}$



Galactic

$\sim 10^{-21}\text{sec}^{-1}$



First Derivative of Period

$$\frac{\dot{P}_{\rm obs}}{P} = \frac{\dot{P}_{\rm int}}{P} - \frac{v_\perp^2}{d_a} - a_{\rm MW} - a_{\rm GW}$$

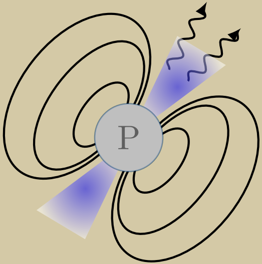
Observed value

$\mathcal{O}(10^{-18} \pm 10^{-24})\ \mathrm{sec}^{-1}$

Gravitational waves

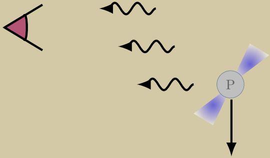
"Intrinsic" spin-down

$\sim 10^{-18}\mathrm{sec}^{-1}$



Kinematic

$\simeq 10^{-18}\ \mathrm{sec}^{-1}\, \frac{v_\perp^2}{v_{\rm gal}^2} \frac{\rm kpc}{d_a}$



Galactic

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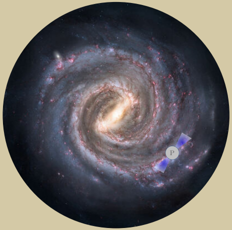

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Gravitational waves

"Intrinsic" spin-down

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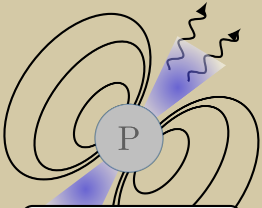
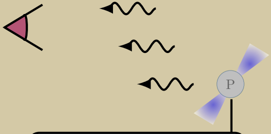


unpredictable, unmeasurable

Kinematic

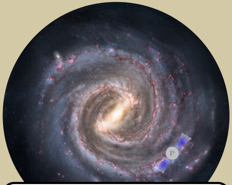
$\simeq 10^{-18}\ \mathrm{sec}^{-1}\,\frac{v_\perp^2}{v_{\rm gal}^2}\,\frac{\mathrm{kpc}}{d_a}$



independently measurable

Galactic

$\sim 10^{-21}\mathrm{sec}^{-1}$



estimate with galactic model

“Observed” *vs* “True” Parameters

"Observed" vs "True" Parameters

binary period derivative → $\dot{P}_b$ follows similarly to $\dot{P}$

$$\frac{\dot{P}_{b,\mathrm{obs}}}{P_b} = \frac{\dot{P}_{b,\mathrm{int}}}{P_b} - \frac{v_\perp^2}{d_a} - a_{\mathrm{MW}} - a_{\mathrm{GW}}$$

↓

$$\sim -10^{-19}\mathrm{sec}^{-1}\left(\frac{M_i}{M_\odot}\right)^{5/3}\left(\frac{\mathrm{day}}{P_b}\right)^{8/3}$$

“Observed” vs “True” Parameters

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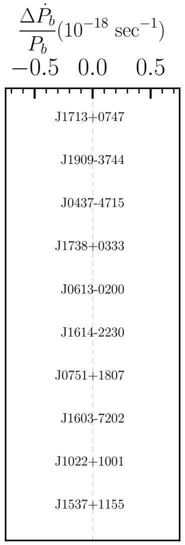
compiled by [Chakrabarti et al '20]



dominant uncertainty : obs $v_\perp^2/d_a$ int

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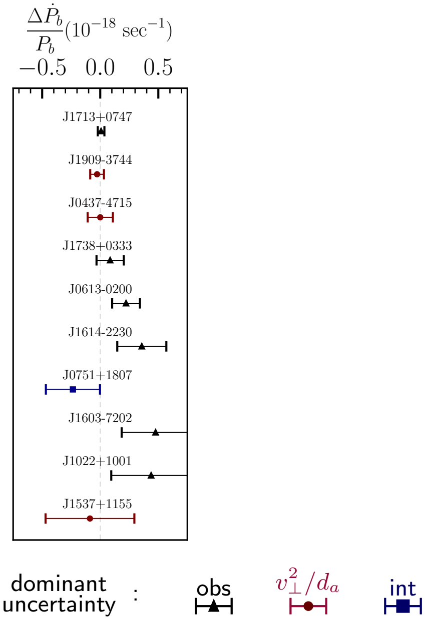

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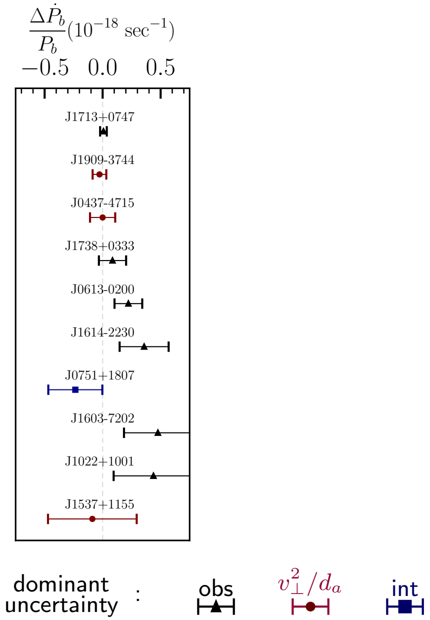
compiled by [Chakrabarti et al '20]

pulsar period second derivative → negligible kinematic, galactic terms

$$\frac{\ddot{P}_{\mathrm{int}}}{P} \sim \left(\frac{\dot{P}_{\mathrm{int}}}{P}\right)^2$$

$$\frac{\ddot{P}_{\mathrm{obs}}}{P} = j_{\mathrm{GW}}$$



“Observed” *vs* “True” Parameters

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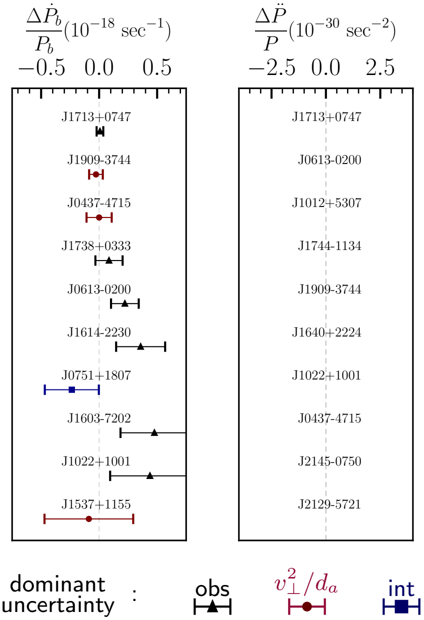
$\ddot{P}$s from [Liu et al '19] using...

[Desvignes et al (EPTA) '16]
[Reardon et al (PPTA) '15]



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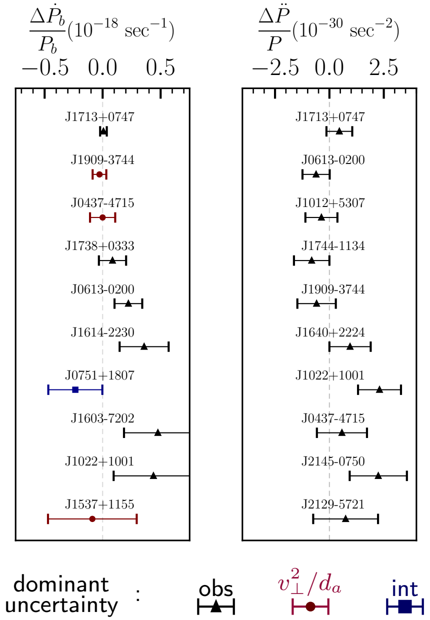
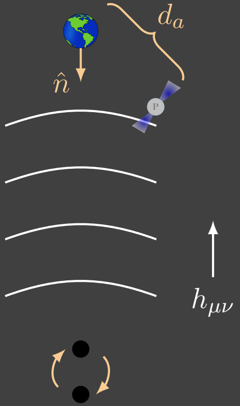


Continuous Gravity Waves on Timing Model [JD, DeRocco '23]



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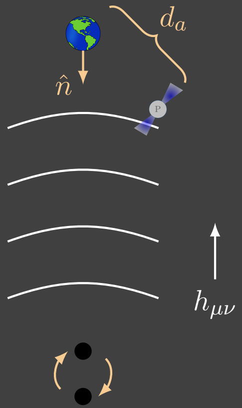
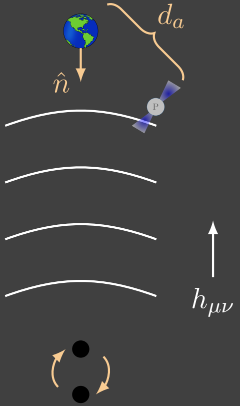


$$v_{\rm GW}(t) = \sum_{A=+,\times} F_A(\hat{n}) \left[h_A(t,0) - h_A(t - d_a, \vec{\mathbf{d}}_a) \right]$$

Pattern functions ↑ Earth term ↑ Pulsar term ↑

Continuous Gravity Waves on Timing Model [JD, DeRocco '23]



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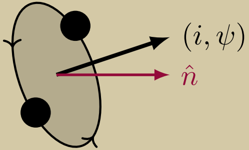
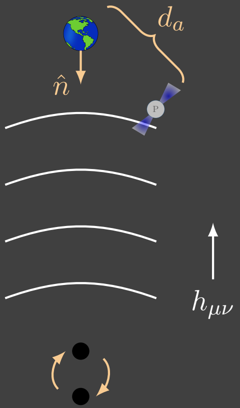


$$h_A \sim h_0 \sin(\pi f_{\rm GW} t + \Phi_0)$$

Continuous Gravity Waves on Timing Model [JD, DeRocco '23]



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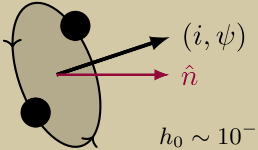
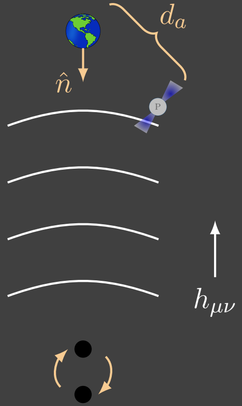


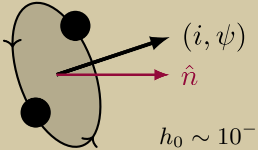
$$h_A \sim h_0 \sin(\pi f_{\rm GW} t + \Phi_0)$$

$$h_0 \sim 10^{-14} \left[\frac{M_i}{10^8 M_\odot} \right]^{5/3} \left[\frac{f_{\rm GW}}{1\ {\rm nHz}} \right]^{2/3} \left[\frac{100\ {\rm kpc}}{d_L} \right]$$

Continuous Gravity Waves on Timing Model [JD, DeRocco '23]



$$v_{\mathrm{GW}}(t) = \sum_{A=+,\times} F_A(\hat{n}) \left[h_A(t,0) - h_A(t - d_a, \vec{\mathbf{d}}_a) \right]$$



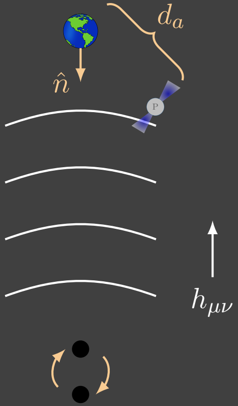
$$h_A \sim h_0 \sin(\pi f_{\mathrm{GW}} t + \Phi_0)$$

$$h_0 \sim 10^{-14} \left[\frac{M_i}{10^8 M_\odot} \right]^{5/3} \left[\frac{f_{\mathrm{GW}}}{1\ \mathrm{nHz}} \right]^{2/3} \left[\frac{100\ \mathrm{kpc}}{d_L} \right]$$

Parameters:

4 angles	1 phase	frequency	amplitude
$(\psi, i, \hat{n})$	Φ_0	f_{GW}	h_0

Continuous Gravity Waves on Timing Model [JD, DeRocco '23]



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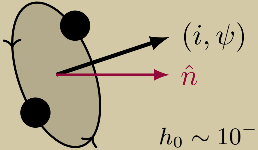

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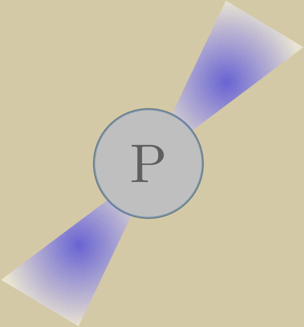
Parameters:

4 angles	1 phase	frequency	amplitude
$(\psi, i, \hat{n})$	Φ_0	$f_{\rm GW}$	h_0

$$a_{\rm GW} \sim f_{\rm GW} h_0 \sim 10^{-20} {\rm sec}^{-1} \left(\frac{f_{\rm GW}}{\rm nHz} \right) \frac{h_0}{10^{-12}}$$

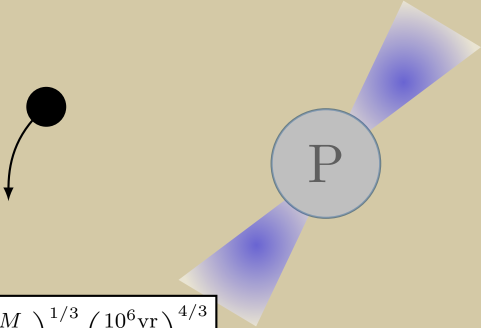
$$j_{\rm GW} \sim f_{\rm GW}^2 h_0 \sim 10^{-31} {\rm sec}^{-2} \left(\frac{f_{\rm GW}}{\rm nHz} \right)^2 \frac{h_0}{10^{-14}}$$

Backgrounds



unidentified wide binary ← **Backgrounds**

cannot model companion if $P_b \gg T$



$$\frac{\Delta \dot{P}_b}{P_b} \sim 10^{-19}\text{sec}^{-1} \left(\frac{M}{M_\odot}\right)^{1/3} \left(\frac{10^6\text{yr}}{P_b}\right)^{4/3}$$

$$\frac{\Delta \ddot{P}}{P} \sim 10^{-29}\text{sec}^{-2} \left(\frac{10^6\text{yr}}{P_b}\right)^{2} \left(\frac{v}{100\,\frac{\text{km}}{\text{sec}}}\right)$$

→ remove outliers (1 in dataset)

unidentified wide binary ← **Backgrounds** → poorly-modeled red noise

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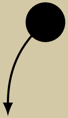

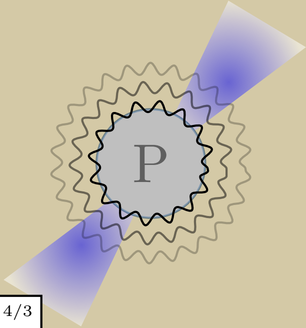

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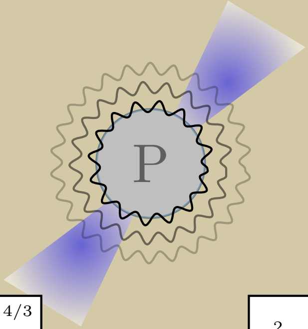

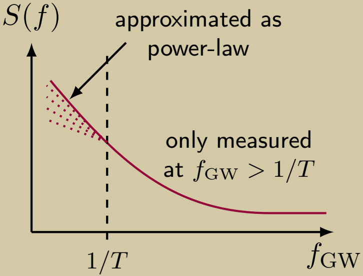

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$$\sigma^2_{\rm RN} \equiv \left\langle \left(\frac{\Delta \dot{P}_b}{P_b}\right)^2 \right\rangle \simeq \int_0^{1/4T} df\; S_{\rm RN}(f)(2\pi f)^2$$

$$\sigma^2_{\rm RN} \equiv \left\langle \left(\frac{\Delta \ddot{P}}{P}\right)^2 \right\rangle \simeq \int_0^{1/4T} df\; S_{\rm RN}(f)(2\pi f)^4$$

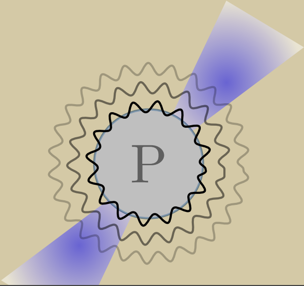
→ repeat w/ $10\times$ red noise

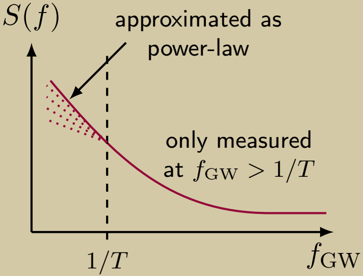
Backgrounds

unidentified wide binary

poorly-modeled red noise

cannot model companion if $P_b \gg T$





backgrounds uncorrelated among pulsars

$$\mathcal{L}(h_0, f_{\mathsf{GW}}, \boldsymbol{\theta} | \{y_a\}) = \prod_{a=1}^{N_{\mathsf{p}}} \frac{1}{\sqrt{2\pi}\sigma_a} \exp\left[-\frac{(y_a - \bar{y}_a(h_0, f_{\mathsf{GW}}, \boldsymbol{\theta}))^2}{2\sigma_a^2}\right]$$

$y = \{\Delta \dot{P}_b / P_b\}$ or $\{\Delta \ddot{P}/P\}$; $\sigma_a = \sqrt{\sigma_{0,a}^2 + \sigma_{\mathrm{RN},a}^2}$

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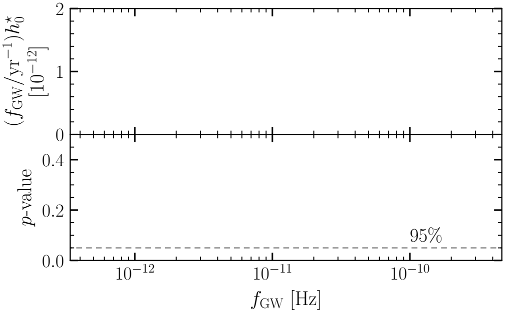
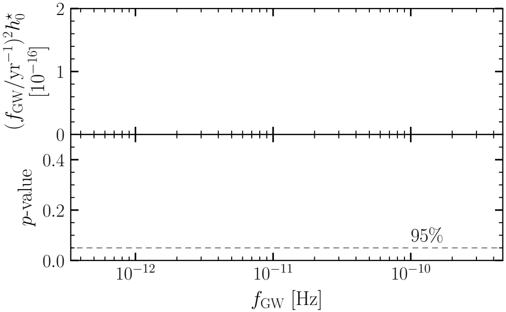
$\dot{P}_b$ search



$\ddot{P}$ search



$$\mathcal{L}(h_0, f_{\mathsf{GW}}, \boldsymbol{\theta} | \{y_a\}) = \prod_{a=1}^{N_{\mathsf{p}}} \frac{1}{\sqrt{2\pi}\sigma_a} \exp\left[-\frac{(y_a - \bar{y}_a(h_0, f_{\mathsf{GW}}, \boldsymbol{\theta}))^2}{2\sigma_a^2}\right]$$

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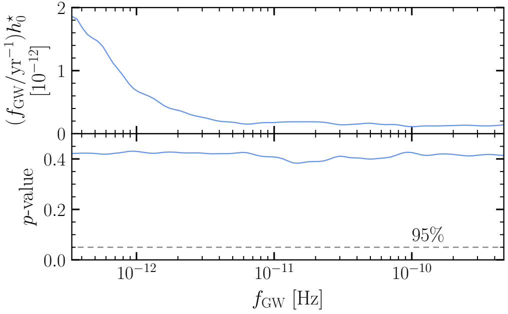
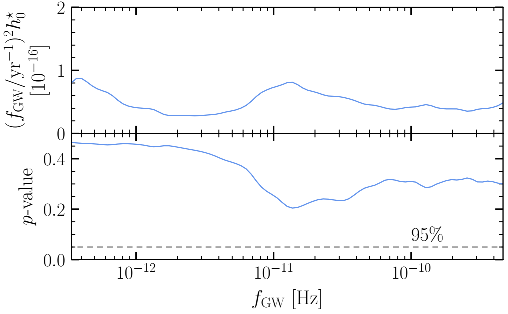
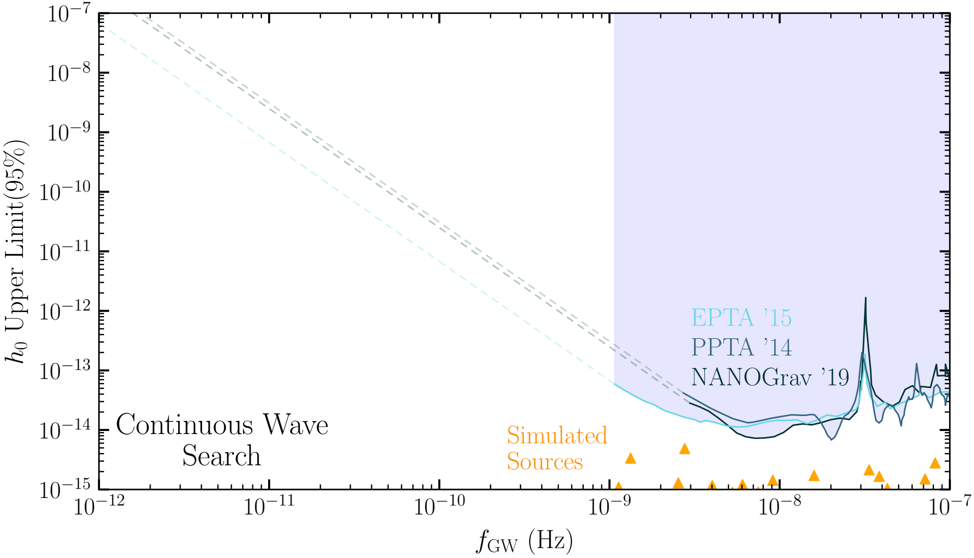
$\dot{P}_b$ search

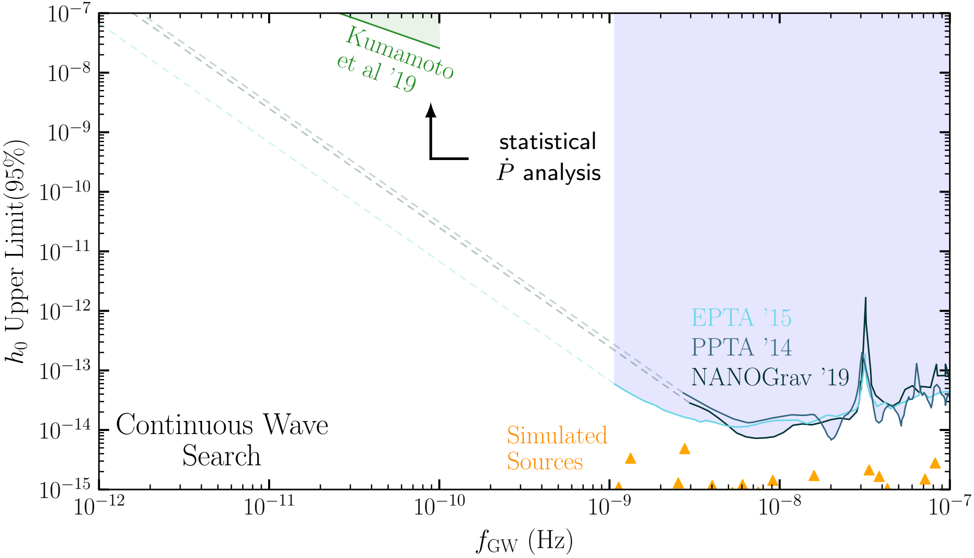


$\ddot{P}$ search





10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
h_0 Upper Limit(95%)
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}
10^{-7}
f_{GW} (Hz)
EPTA '15
PPTA '14
NANOGrav '19
Continuous Wave Search
Simulated Sources



Kumamoto et al '19
statistical $\dot{P}$ analysis
EPTA '15
PPTA '14
NANOGrav '19
Continuous Wave Search
Simulated Sources
h_0 Upper Limit(95%)
f_{GW} (Hz)
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-12}
10^{-11}
10^{-10}
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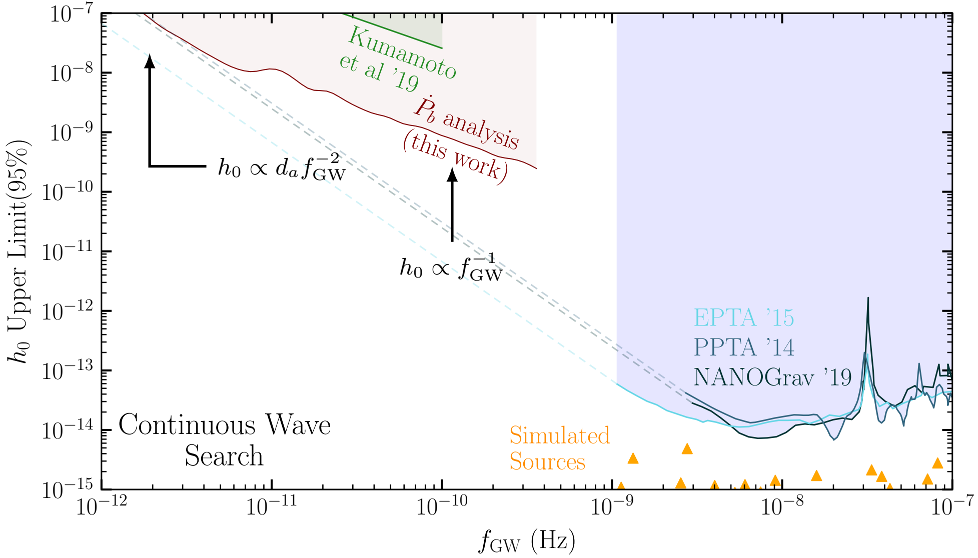
10^{-8}
10^{-7}



Kumamoto et al '19
$\dot{P}_b$ analysis (this work)
$h_0 \propto d_a f_{\rm GW}^{-2}$
$h_0 \propto f_{\rm GW}^{-1}$
EPTA '15
PPTA '14
NANOGrav '19
Continuous Wave Search
Simulated Sources
h_0 Upper Limit(95%)
$f_{\rm GW}$ (Hz)
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
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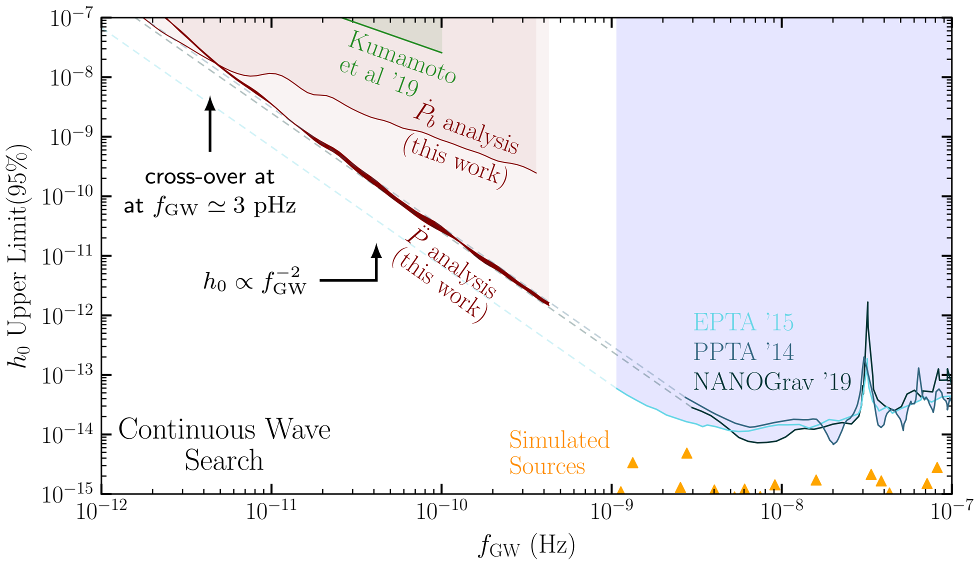
10^{-8}
10^{-7}



10^{-7}
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h_0 Upper Limit(95%)
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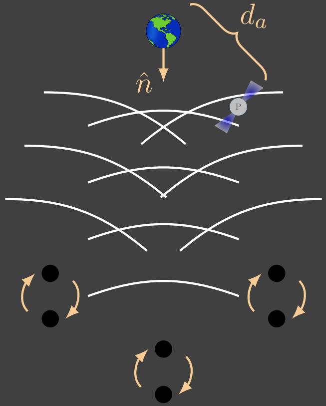
10^{-9}
10^{-8}
10^{-7}
f_{GW} (Hz)
Kumamoto et al '19
$\dot{P}_b$ analysis (this work)
$\ddot{P}$ analysis (this work)
cross-over at
at $f_{\mathrm{GW}} \simeq 3$ pHz
$h_0 \propto f_{\mathrm{GW}}^{-2}$
EPTA '15
PPTA '14
NANOGrav '19
Simulated Sources
Continuous Wave Search

Stochastic Background on Timing Model [JD, DeRocco in prep]



$$\Omega_{\rm GW}(f) \equiv \frac{4\pi^2 f^3}{3H_0^2} S_h(f)$$

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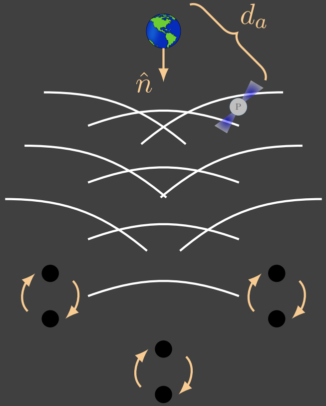

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$a^{(a)}_{\rm GW}$, $j^{(a)}_{\rm GW}$ are **random** variables

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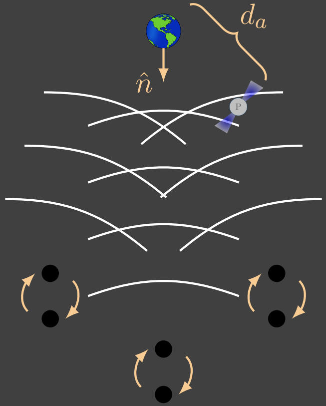

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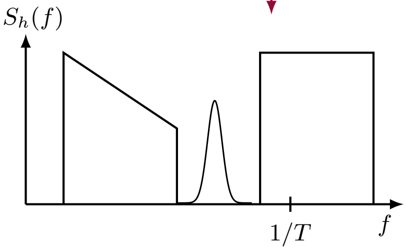

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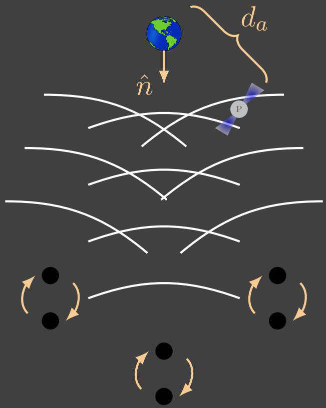

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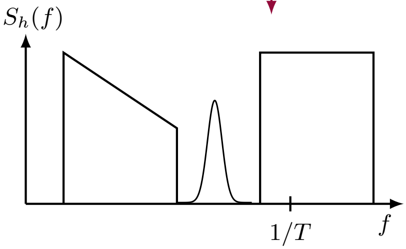

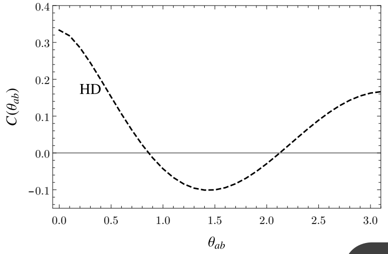

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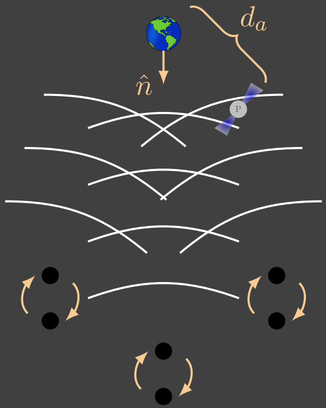

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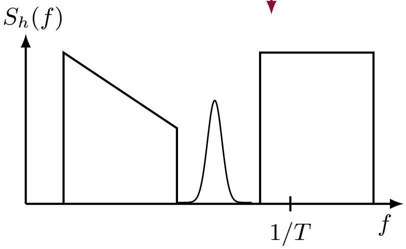

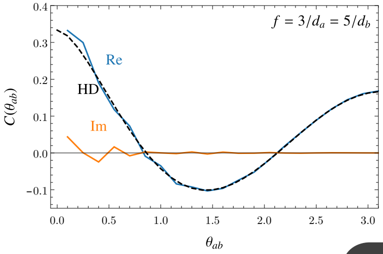

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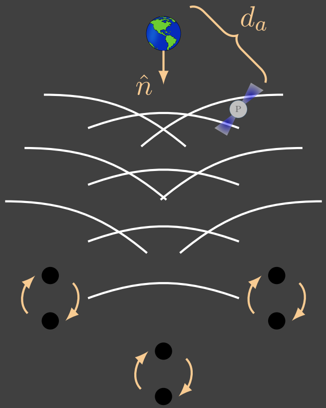

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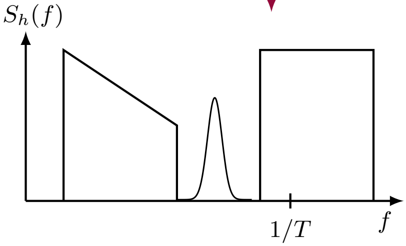

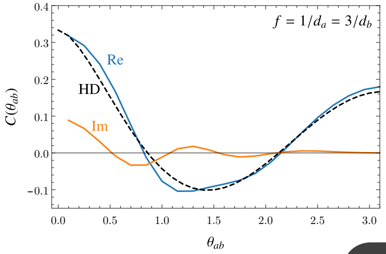

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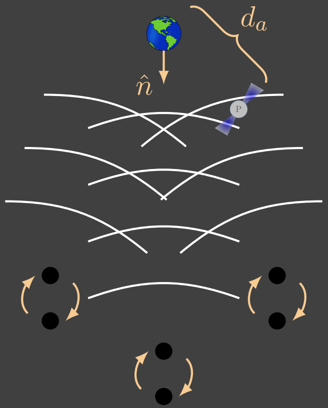

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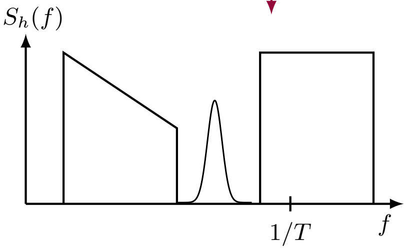
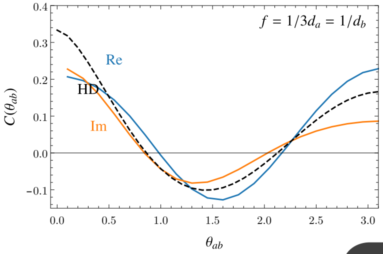
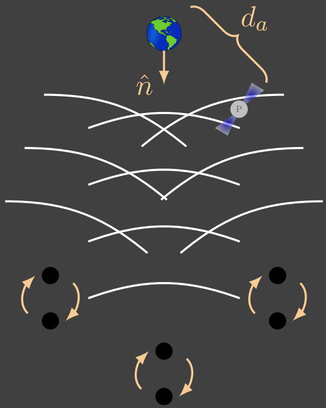




Stochastic Background on Timing Model [JD, DeRocco in prep]



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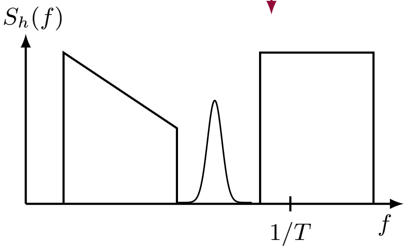
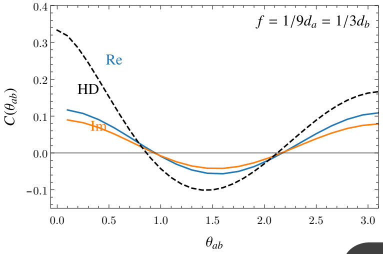
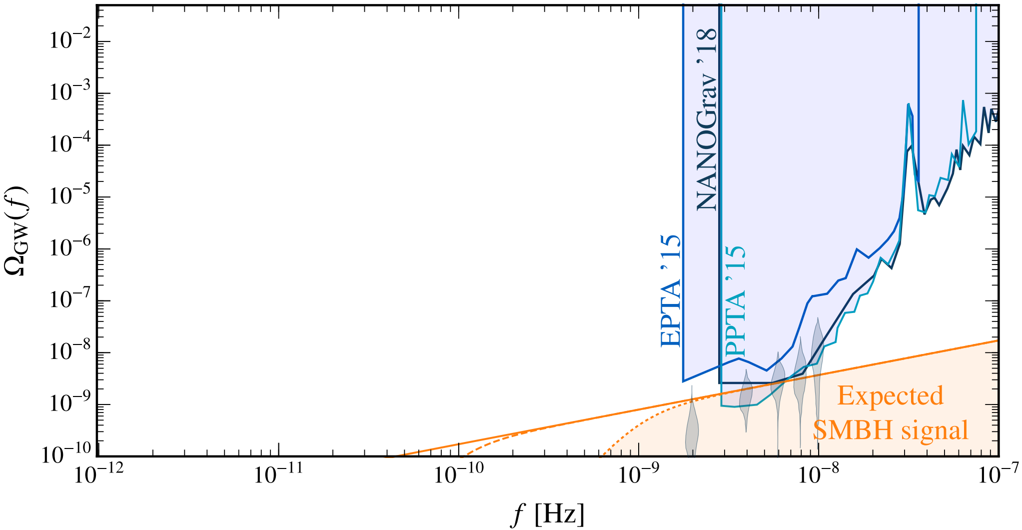




[JD, DeRocco '23]



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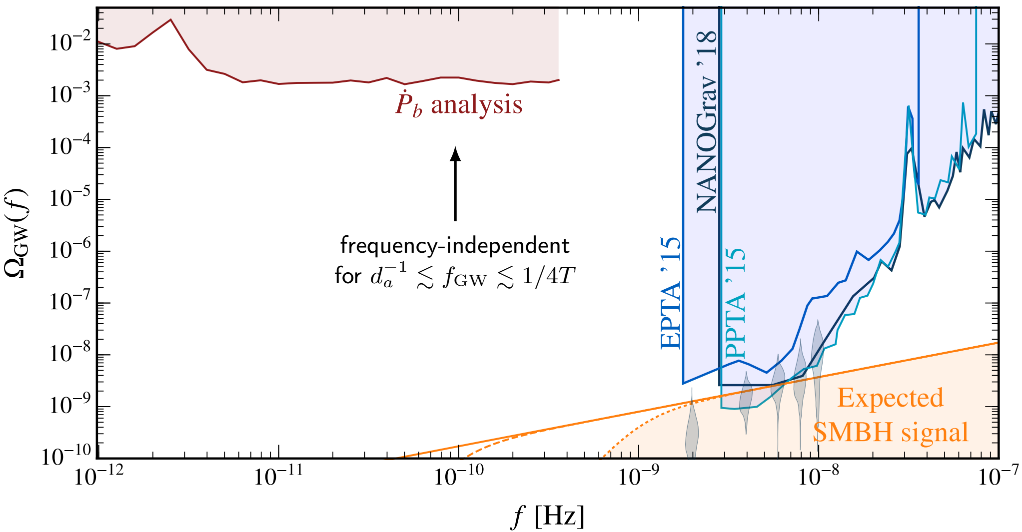

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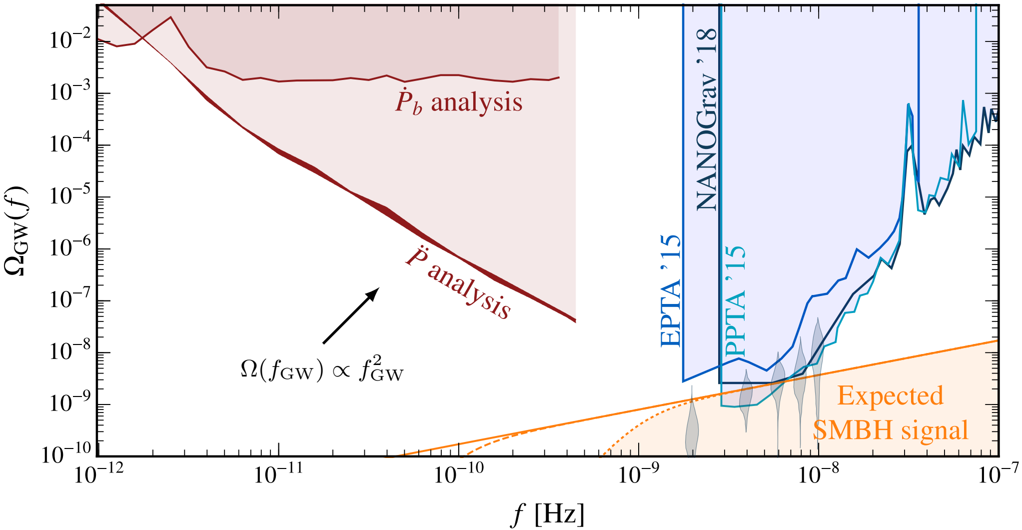
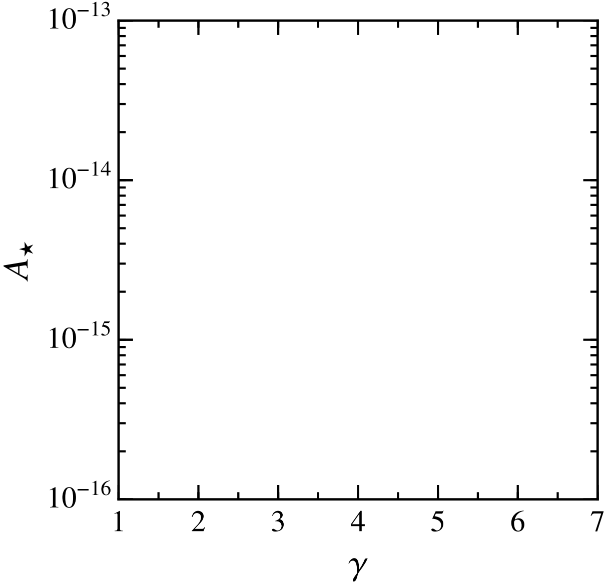
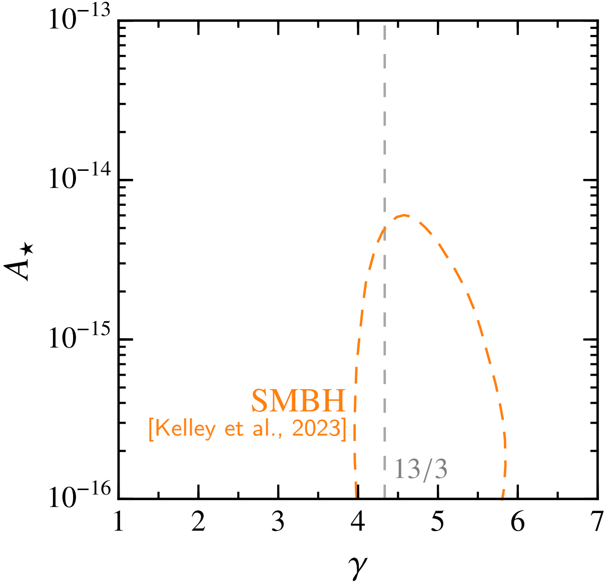




$$S_h(f) = \frac{A_\star^2}{2f_\star}\left(\frac{f}{f_\star}\right)^{2-\gamma}$$



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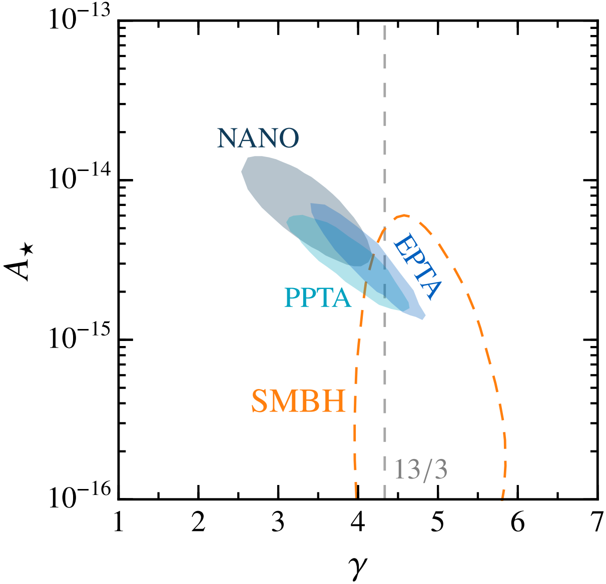

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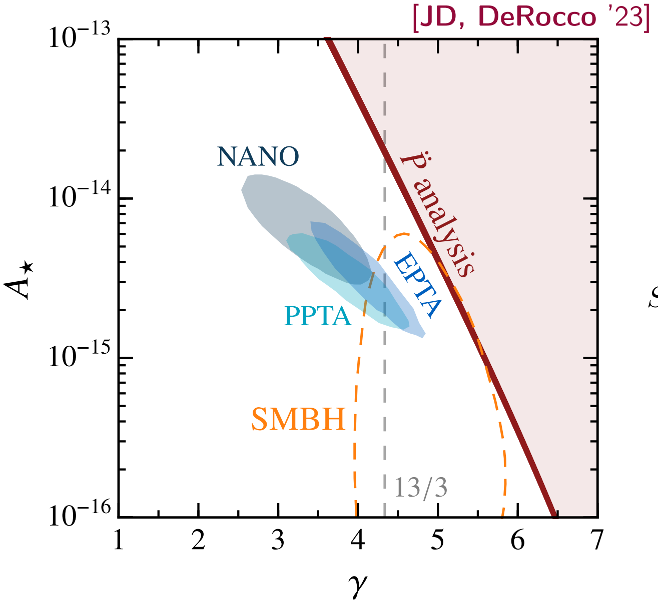

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- build joint analyses
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NANOGrav, PPTA, IPTA (International Pulsar Timing Array), EPTA, InPTA (Indian Pulsar Timing Array), CPTA

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detection of ultralow-frequency signal