Could the Higgs Boson be the Inflaton?

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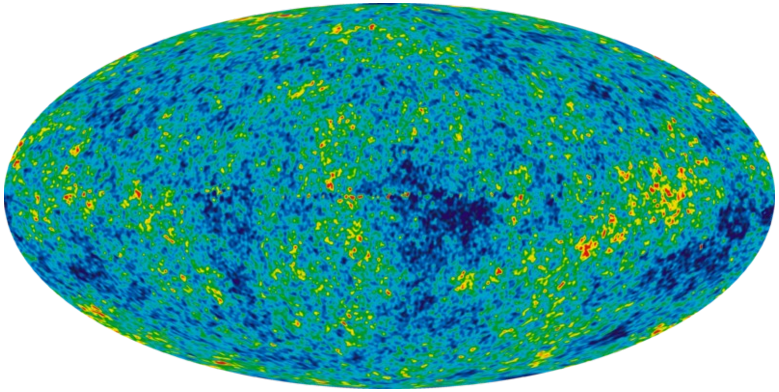
Outline

- Why inflation?
- The Higgs as the inflaton
- Perturbative unitarity as a tool in particle physics
- Unitarity and Higgs inflation

Why Inflation?

•Why does the universe appear flat, homogeneous and isotropic?



The CMB temperature fluctuations
from the 7-year WMAP data

Temperature of CMB is 2.725K ± 0.0002K – extremely uniform!

•Can be explained if the universe went through a very early period of exponential expansion - inflation.

•Inflation also explains the origin of the large-scale structure of the cosmos.

Slow Roll Inflation

•Inflation is driven by a negative-pressure vacuum energy density.

•Example: slowly rolling scalar field

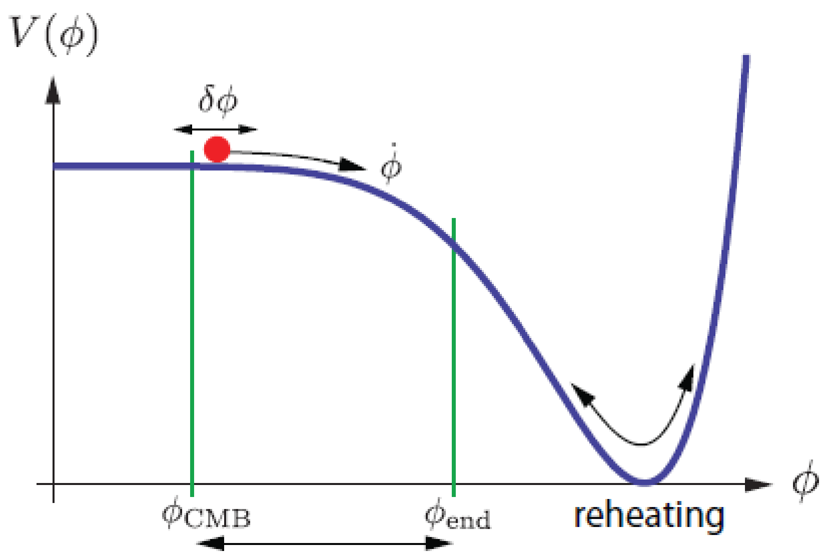
$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

$$\Rightarrow \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\begin{cases} \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases} \Rightarrow \quad \rho + 3p = 2(\dot{\phi}^2 - V(\phi))$$

$$\text{if} \quad \dot{\phi}^2 < V(\phi) \quad \Longrightarrow \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

accelerated expansion



[Source - arXiv:0907.5424]

CMB fluctuations are created by quantum fluctuations $\delta\varphi$ about 60 *e*-folds before the end of inflation.

Higgs Inflation

The standard model Higgs potential is not flat $$V = \lambda_H \left(\mathcal{H}^\dagger\mathcal{H} - \frac{v_H^2}{2}\right)^2$$

However, scalar fields can (should?) be non-minimally coupled to gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{\bar{M}_P^2}{2}R - \xi\mathcal{H}^\dagger\mathcal{H}R + \mathcal{L}_{SM}\right]$$

Can transform to the Einstein frame

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\ , \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2} \qquad \frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2h^2/M_P^2}{\Omega^4}}$$

$$S_E = \int d^4x\sqrt{-\hat{g}}\left\{-\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial^\mu\chi}{2} - U(\chi)\right\}$$

where the potential is $$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 + \exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)\right)^{-2}$$

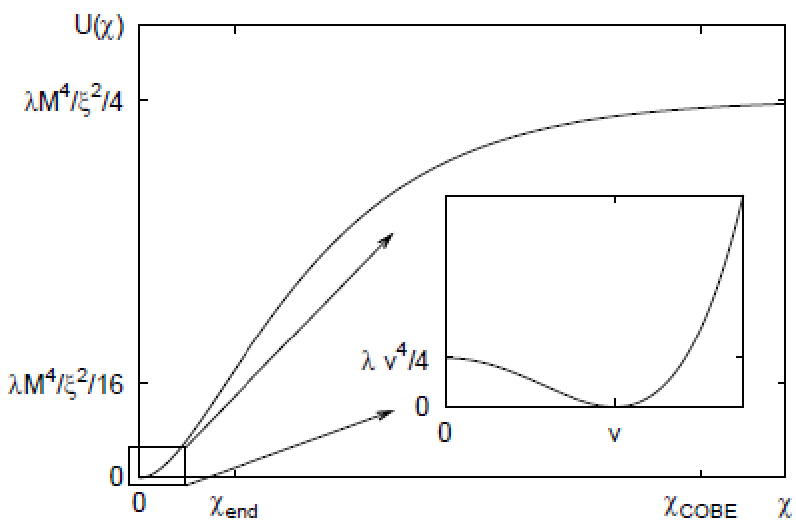
[F.L. Bezrukov & Mikhail Shaposhnikov]

Higgs Inflation

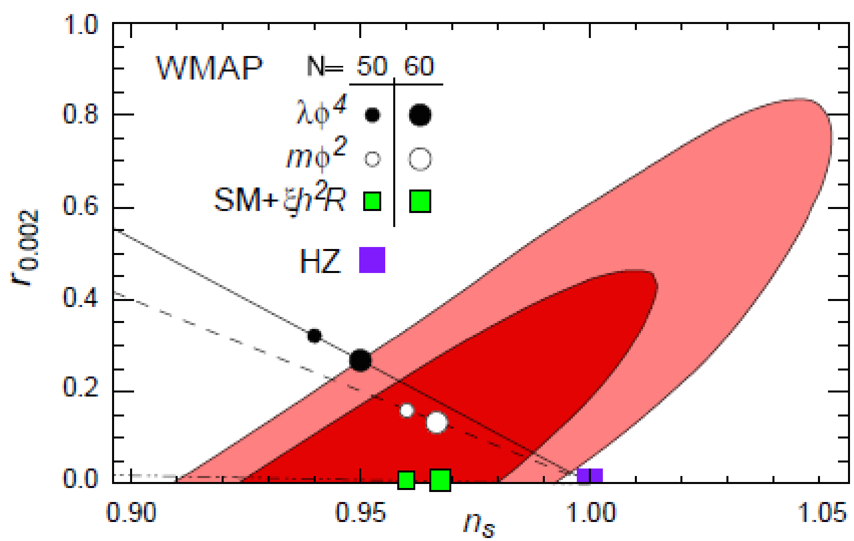
$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 + \exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)\right)^{-2}$$

When $\chi \gg M_P \; (h \gg M_P/\sqrt{\xi})$, the potential is flat and slow roll inflation can occur.

However it is found that we need $\xi \sim 10^4$ to obtain the correct amplitude of density fluctuations.



Potential in the Einstein frame.



The allowed WMAP region for inflationary parameters spectral index *n*, and the tensor to scalar ratio *r*.

[Source - arXiv:0710.3715]

Unitarity in Quantum Field Theory

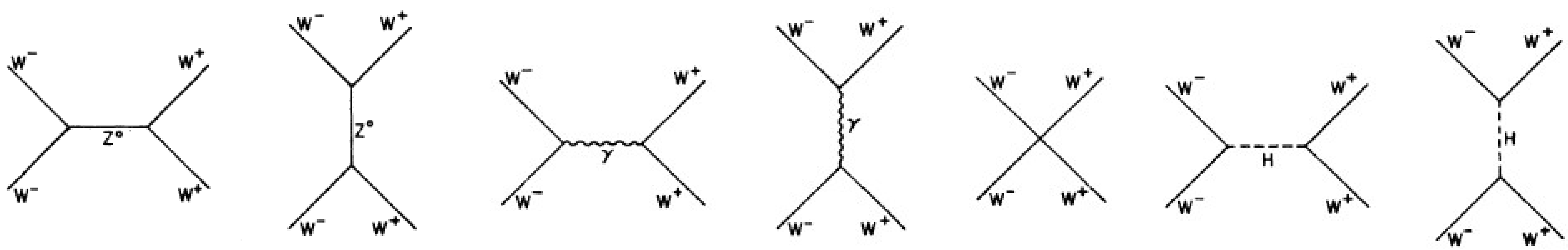
•Follows from the conservation of probability, i.e. unitarity of the S-matrix: $S^\dagger S = 1$

• Implies that amplitudes do not grow too fast with energy.

• Can derive a bound on the size of the partial wave amplitudes:

$$\mathcal{A} = 16\pi \sum_j (2j+1) P_j(\cos\theta)\, a_j \qquad |\mathrm{Re}\, a_j| \leq \frac{1}{2}$$

• Well known example is the bound on the Higgs boson mass in the Standard Model (m<790 GeV).



Unitarity in WW Scattering

With no Higgs we find the *j=0* partial wave grows with energy as

$$a_0 = \frac{g^2 s}{128\pi M_W^2}$$

and the maximum energy for perturbative unitarity is

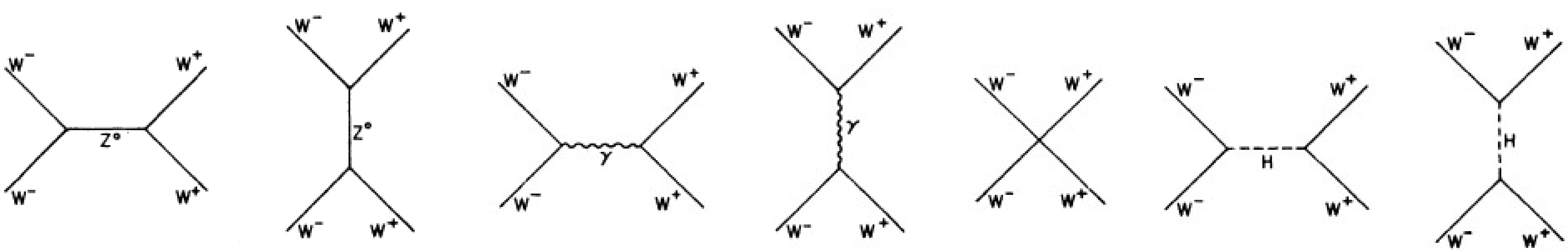
$$E \lesssim 1.7 \text{ TeV}$$

Including the Higgs we find the *j=0* partial wave is given by

$$a_0 = -\frac{G_F m_h^2}{4\sqrt{2}\pi}$$

and the maximum Higgs mass to maintain perturbative unitarity is

$$m_H \lesssim 790 \text{ GeV}$$



Unitarity in Higgs Inflation

The large value of $\xi \sim 10^4$ might make one concerned from a particle physics perspective.

Let us consider gravitational scattering of Higgs bosons (we impose different in and out states – *s*-channel only) in the Jordan frame

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_P}$$

$$a_0 = \frac{\pi}{3}\frac{s}{M_P^2}(1+12\xi)^2 \sim \frac{\xi^2}{M_P^2}s \leq \frac{1}{2} \qquad \text{for } \xi \gg 1$$

$$\Rightarrow \Lambda \lesssim \frac{M_P}{\xi}$$

[MA & X. Calmet] [Burgess, Lee & Trott] [Barbon & Epinosa]

But remember inflation takes place for $h \gg M_P/\sqrt{\xi}$ which is therefore above the regime of validity for the effective theory!

Einstein vs. Jordan Frame [Hertzberg] [Burgess, Lee, Trott]

The cut off (Λ) should be the same in both frames. But if we look at the Einstein frame action again

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - U(\chi) \right\}$$

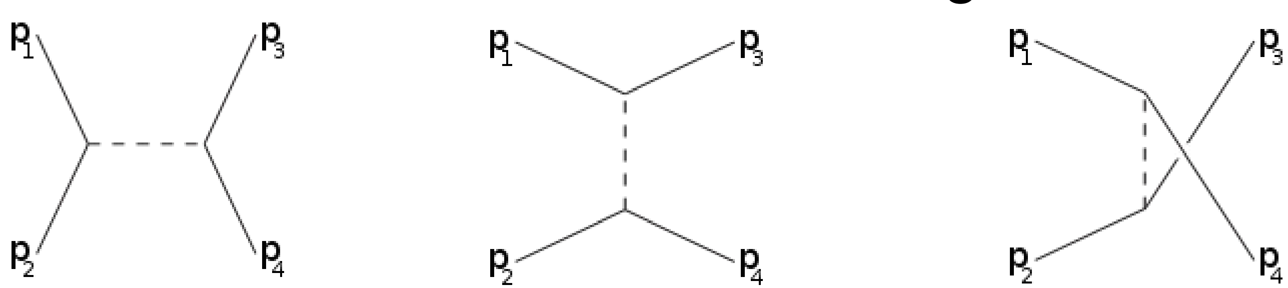
we see that the cut off is just the usual gravitational cut off ($\Lambda = M_P$).

Einstein Frame

Cannot canonically normalise all the fields of the Higgs doublet so cannot actually get Einstein frame potential with multiple scalars.

Jordan Frame

If only have a single field need to include *s, t* and *u* channels. Then we find a cancellation between the three diagrams leaving ($\Lambda = M_P$).



New Model of Higgs Inflation

To get around the unitarity problems a new model of Higgs inflation was proposed [Germani & Kehagias]

$$S = \int d^4x \sqrt{-g} \left[\frac{\bar{M}_P^2}{2} R - \frac{1}{2} (g^{\mu\nu} - w^2 G^{\mu\nu}) \partial_\mu \Phi \partial_\nu \Phi - \frac{\lambda}{4} \Phi^4 \right]$$

where $G^{\mu\nu} = R^{\mu\nu} - \frac{R}{2} g^{\mu\nu}$ is the Einstein tensor.

Expanding around the inflating background we find an interaction

$$I \simeq \frac{1}{2H^2 \bar{M}_P} \partial^2 h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi.$$

Which gives a cut-off

$$\Lambda \simeq (2H^2 \bar{M}_P)^{1/3}. \simeq 2 \times 10^{-3} \bar{M}_P.$$

But during inflation we have $2.1 \times 10^{-2} \bar{M}_P < \Phi_0 < 2.7 \times 10^{-2} \bar{M}_P$

and so again the inflationary scale exceeds the realm of validity of the effective theory. [MA & X Calmet]

Background Dependence

[Bezrukov, Magnin, Shaposhnikov & Sibiryakov]

For the original Higgs inflation model we expanded around *φ=0*. We could expand around inflating background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \ , \qquad \phi = \bar{\phi} + \delta\phi \ .$$

Then we find an interaction term

$$\frac{\xi\sqrt{M_P^2 + \xi\bar{\phi}^2}}{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}(\delta\hat{\phi})^2\Box\hat{h}$$

Leading to a $\bar{\phi}$ dependent cut-off

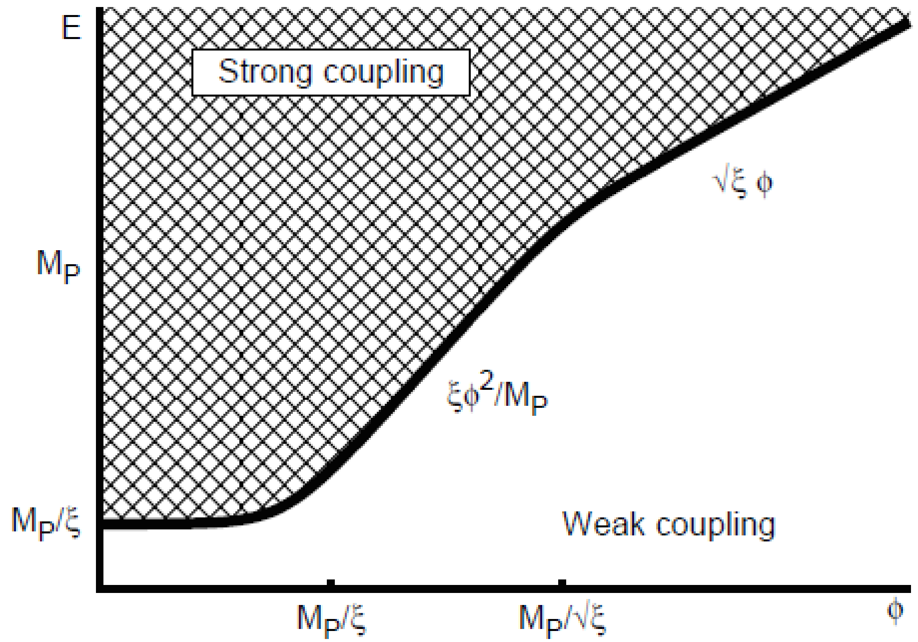
$$\Lambda^J(\bar{\phi}) = \frac{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}{\xi\sqrt{M_P^2 + \xi\bar{\phi}^2}}$$

Small field: $\bar{\phi} \ll M_P/\xi \qquad \Lambda^J \simeq \frac{M_P}{\xi}$

Re-heating: $M_P/\xi \ll \bar{\phi} \ll M_P/\sqrt{\xi} \qquad \Lambda^J \simeq \frac{\xi\bar{\phi}^2}{M_P}$

Inflation: $\bar{\phi} \gg M_P/\sqrt{\xi} \qquad \Lambda^J \simeq \sqrt{\xi}\bar{\phi}$

New Physics or Strong Coupling?

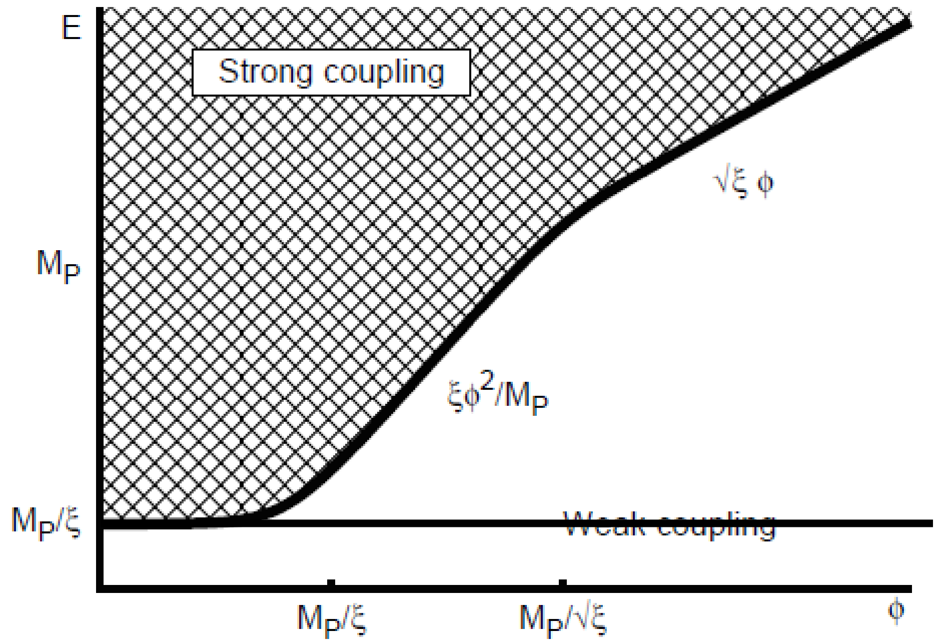


Cut-off as a function of the background value of Higgs field.

[Source - arXiv:1008.5157]

During inflation still in perturbative regime.

New Physics or Strong Coupling?



Cut-off as a function of the background value of Higgs field.

[Source - arXiv:1008.5157]

During inflation still in perturbative regime.

However if new physics is required to unitarise the theory at small background field values, potential must include the operators

$$\frac{(H^\dagger H)^n}{\Lambda_0^{2n-4}}$$

Appearing at $\Lambda_0 = \frac{M_P}{\xi}$. and spoiling the flat potential.

[MA & X Calmet]

Unitarising Higgs Inflation [Giudice & Lee]

Can unitarise Higgs inflation by using an analogy with the non-linear sigma model. Consider the kinetic term in Einstein frame

$$\mathcal{L}_{\rm kin} = -\frac{1}{2(1+\xi_0\vec{\phi}^2/M_P^2)}\left(\delta_{ij} + \frac{6\xi_0^2\phi_i\phi_j/M_P^2}{1+\xi_0\vec{\phi}^2/M_P^2}\right)\partial_\mu\phi_i\partial^\mu\phi_j$$

So we can complete in the UV by introducing a σ field with $\sigma^2 \doteq \Lambda^2 + \vec{\phi}^2$ with $\Lambda^2 \equiv M_P^2/\xi_0$

$$\begin{aligned}\frac{\mathcal{L}_J}{\sqrt{-g_J}} &= \frac{1}{2}\left(\bar{M}^2 + \xi\bar{\sigma}^2 + 2\zeta\mathcal{H}^\dagger\mathcal{H}\right)R - \frac{1}{2}(\partial_\mu\bar{\sigma})^2 - |D_\mu\mathcal{H}|^2 \\ &\quad -\frac{1}{4}\kappa\left(\bar{\sigma}^2 - \bar{\Lambda}^2 - 2\alpha\mathcal{H}^\dagger\mathcal{H}\right)^2 - \lambda\left(\mathcal{H}^\dagger\mathcal{H} - \frac{v^2}{2}\right)^2.\end{aligned}$$

with $\xi \sim \mathcal{O}(10^4)\ ,\ \zeta \sim \mathcal{O}(1)$

Low energy theory is the usual Higgs inflation Lagrangian, but at high energies the sigma field propagates and the cut-off scales with the background to allow control over the potential.

Two More Scenarios

1. Asymptotic Safety [MA & X Calmet]

The theory is non-perturbatively renormalisable and approaches a non trivial fixed point in the UV. Planck mass will grow in the UV and ξ will decrease, so growth of amplitudes with energy as $\xi^2 s/\bar{M}_P^2$ could be compensated by this running. Or at least no new physics is required.

2. Composite Inflation [Channuie, Jørgensen, Sannino]

The inflaton emerges as a composite field of a four dimensional, strongly interacting gauge theory.

$$\frac{\xi}{2}\frac{(QQ)^\dagger QQ}{\Lambda_{ECI}^4}R$$

the scale of inflation is the grand unified one, the composite inflaton cannot be identified with the composite Higgs state.

Conclusions

- Inflation explains why the universe appears flat, homogeneous and isotropic
- With a large non-minimal coupling the Higgs boson could drive inflation which agrees with CMB data.
- The Higgs inflation models (old and new) suffer from unitarity problems.
- This may be bypassed if one can show that no new physics required to fix unitarity scales with the size of the Higgs background.
- We saw three ways around this problem: sigma field, asymptotic safety and composite inflation.