Exploring high-purity multi-parton scattering at the LHC

Alba Soto Ontoso
QCD seminar
CERN, 4th December, 2023
Exploring high-purity multi-parton scattering at hadron colliders

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arXiv:2307.05693

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$\langle #\text{MPI} \rangle$ in Drell-Yan production at hadron colliders

Mean # of MPI

- Tevatron
- LHC Run1
- HL-LHC
- FCC

Pythia 8.307
$Z/\gamma \rightarrow \mu \mu$, $66 < m_{\mu\mu} < 116$ GeV

Primary hard-scattering

Secondary scatterings

$\mathcal{O}(10)$ additional parton collisions per Drell-Yan event
Era of precision physics for the primary hard scattering. What’s the status on MPI modelling?
Option A: MPI as part of the MC toolkit

Figure 1: Schematic of the structure of a pp → t̅t̅ event, as modelled by PYTHIA. To keep the layout relatively clean, a few minor simplifications have been made: 1) shower branchings and final-state hadrons are slightly less numerous than in real PYTHIA events, 2) recoil effects are not depicted accurately, 3) weak decays of light-flavour hadrons are not included (thus, e.g. a K_0_S meson would be depicted as stable in this figure), and 4) incoming momenta are depicted as crossed (p→−p). The latter means that the beam remnants and the pre- and post-branching incoming lines for ISR branchings should be interpreted with “reversed” momentum, directed outwards towards the periphery of the figure; this avoids beam remnants and outgoing ISR emissions having to criss-cross the central part of the diagram.

[Bierlich et al SciPost Phys. Codebases 8 (2022)]
MPI modelling in general purpose MCs in a nutshell

Standard 2 → 2 cross section

\[
\frac{d\sigma}{dp^2_\perp} = \sum_{i,j,k} \int \int f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\hat{\sigma}^k_{ij}}{d\hat{t}} \delta \left( p^2_\perp - \frac{\hat{t}}{\hat{s}} \right) dx_1 dx_2 d\hat{t}
\]

\[
\frac{d\hat{\sigma}}{d\hat{t}} \propto \frac{\alpha_s^2(Q^2)}{\hat{t}^2} \Rightarrow \frac{d\hat{\sigma}}{dp^2_\perp} \propto \frac{\alpha_s^2(p^2_\perp)}{p^4_\perp}
\]

divergent when \( p_\perp \rightarrow 0 \)

\[
\frac{d\hat{\sigma}}{dp^2_\perp} \sim \frac{\alpha_s^2(p^2_\perp)}{p^4_\perp} \rightarrow \frac{\alpha_s^2(p^2_{\perp0} + p^2_\perp)}{(p^2_{\perp0} + p^2_\perp)^2}
\]

with \( p_{\perp0}(\sqrt{s}) \) a free parameter
MPI modelling in general purpose MCs in a nutshell

\[
\frac{d\sigma}{dp_{\perp}^2} = \sum_{i,j,k} \int \int f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\hat{\sigma}_{ij}^k}{d\hat{t}} \delta \left( p_{\perp}^2 - \frac{\hat{t}\hat{u}}{\hat{s}} \right) dx_1 dx_2 d\hat{t}
\]

Interactions occur independently (w/ momentum, flavour sum rules): Poisson stat

\[
\frac{d\mathcal{P}}{dp_{\perp i}} = \frac{1}{\sigma_{nd}} \frac{d\sigma}{dp_{\perp i}} \exp \left( - \int_{p_{\perp i}}^{p_{\perp i-1}} \frac{1}{\sigma_{nd}} \frac{d\sigma}{dp_{\perp}'} dp_{\perp}' \right)
\]

\( \sigma_{nd} = 50 \text{ mb} \)

+ some impact-parameter distribution
Multi-parton interactions are interleaved with the rest of the showering.

\[
\frac{d\mathcal{P}}{dp_{\perp}} = \left( \frac{d\mathcal{P}_{\text{MPI}}}{dp_{\perp}} + \sum \frac{d\mathcal{P}_{\text{ISR}}}{dp_{\perp}} + \sum \frac{d\mathcal{P}_{\text{FSR}}}{dp_{\perp}} \right) \times \exp\left( -\int_{p_{\perp}}^{p_{\perp,max}} \left( \frac{d\mathcal{P}_{\text{MPI}}}{dp'_{\perp}} + \sum \frac{d\mathcal{P}_{\text{ISR}}}{dp'_{\perp}} + \sum \frac{d\mathcal{P}_{\text{FSR}}}{dp'_{\perp}} \right) dp'_{\perp} \right)
\]

Tuning parameters: \( p_{\perp 0}(\sqrt{s}) \) and transverse geometry profile.
MPI/underlying event tuning

Figure 3: CDF data at \( p_T^{\text{max}} > 0.5 \text{ GeV and } p_T^{\text{max}} > 900 \text{ GeV} \) on particle (top) and charged-particle density (bottom) in Tevatron and LHC-Ron1. The data are compared to CMS data and tunes of PYTHIA8 Tune 4C, CUETP8S1-CTEQ6L1, CUETP8S1-HERAPDF1.5LO, and CUETP8M1. The ratios of MC events to data are given below each panel. The green bands in the ratios represent the total experimental uncertainties.

Tunes performed in events with at least one hard scatter
MPI/underlying event tuning

Is it possible to describe MPI with QFT tools?

Tunes performed in events with at least one hard scatter
Option B: double-parton scattering as QFT playground

The double-parton scattering (DPS) cross section can be written as

\[
\frac{1}{\lambda^2_{\text{QCD}}} F_{ik}(x_i, x_k, b) \otimes \hat{\sigma}_{ij \rightarrow Z} \hat{\sigma}_{kl \rightarrow \text{jets}} \otimes F_{jl}(x_j, x_l, b) = \mathcal{O}\left(\frac{\lambda^2_{\text{QCD}}}{Q^4}\right)
\]

where we have introduced the double-parton density \( F^{ab}(x_a, x_b, b) \)

Double parton scattering can be used for proton tomography, i.e. extract partonic correlations
Option B: double-parton scattering as QFT playground

To **first approximation**, double-parton density is given by

\[ F^{ab}(x_a, x_b, b) \approx f(x_a)f(x_b) \int_s G(s)G(b + s) \]

This leads to the so-called **pocket-formula**

\[ \sigma^A_{DPS} \sigma^B_{DPS} = \frac{\sigma^A_{SPS} \sigma^B_{SPS}}{\sigma_{eff}} \]

\[ \sim \pi R^2 \sim 50 \text{ mb} \]
Option B: double-parton scattering as QFT playground

To **first approximation**, double-parton density is given by

\[ F^{ab}(x_a, x_b, b) \approx f(x_a)f(x_b) \int G(s)G(b + s) \]

This leads to the so-called **pocket-formula**

\[ \sigma_{DPS}^{A,B} = \frac{\sigma_{SPS}^A \sigma_{SPS}^B}{\sigma_{\text{eff}}} \]

i.e.

\[ \approx \pi R^2 \sim 50 \text{ mb} \]
Experimental extractions of $\sigma_{\text{eff}}$


$$\sigma_{\text{eff}} = \frac{\sigma_{\text{DPS}}^A \sigma_{\text{DPS}}^B}{\sigma_{\text{SPS}}^A \sigma_{\text{SPS}}^B}$$

- CMS, $\sqrt{s}=13$ TeV, J/$\psi$+J/$\psi$+J/$\psi$
- CMS*, $\sqrt{s}=7$ TeV, J/$\psi$+J/$\psi$
- ATLAS, $\sqrt{s}=8$ TeV, J/$\psi$+J/$\psi$
- D0, $\sqrt{s}=1.96$ TeV, J/$\psi$+J/$\psi$
- D0*, $\sqrt{s}=1.96$ TeV, J/$\psi$+Y
- ATLAS*, $\sqrt{s}=7$ TeV, W+J/$\psi$
- ATLAS*, $\sqrt{s}=8$ TeV, Z+J/$\psi$
- ATLAS*, $\sqrt{s}=8$ TeV, Z+b$\rightarrow$J/$\psi$
- D0, $\sqrt{s}=1.96$ TeV, $\gamma$+b/c+2-jet
- D0, $\sqrt{s}=1.96$ TeV, $\gamma$+3-jet
- D0, $\sqrt{s}=1.96$ TeV, 2-$\gamma$+2-jet
- D0, $\sqrt{s}=1.96$ TeV, $\gamma$+3-jet
- CDF, $\sqrt{s}=1.8$ TeV, $\gamma$+3-jet
- UA2, $\sqrt{s}=640$ GeV, 4-jet
- CDF, $\sqrt{s}=1.8$ TeV, 4-jet
- ATLAS, $\sqrt{s}=7$ TeV, 4-jet
- CMS, $\sqrt{s}=7$ TeV, 4-jet
- CMS, $\sqrt{s}=13$ TeV, 4-jet
- CMS, $\sqrt{s}=7$ TeV, W+2-jet
- ATLAS, $\sqrt{s}=7$ TeV, W+2-jet
- CMS, $\sqrt{s}=13$ TeV, WW
Classic experimental challenge in DPS

Same experimental signature: Z boson (2 leptons) + jets
Illustration: first LHC DPS measurement with $W(\rightarrow \ell \nu) + jj$

[Introduce a metric to characterise MPI-likelihood: $\Delta = |\vec{p}_{T}^{J1} + \vec{p}_{T}^{J2}|$]

Signal (2HS)

Background (1HS)
Illustration: first LHC DPS measurement with W+2-jets


Low 2HS purities require very good understanding of 1HS
Avoid QCD radiation issue: same-sign $W^\pm W^\pm$

Traditional gold-plated observable for MPI:

$\mathcal{O}(\alpha^2)$

$\mathcal{O}\left(\alpha^4, \alpha_s^2 \alpha^2\right)$

Experimental signature: $e^\pm \mu^\pm, \mu^\pm \mu^\pm + p_{T,\text{miss}}$
Avoid QCD radiation issue: same-sign $W^\pm W^\pm$

Traditional gold-plated observable for MPI suffers from background:

![Graph showing the BDT discriminant distribution](image)

**Just 6.2σ statistical significance with full Run 2 dataset**
Theory challenges in DPS: beyond pocket-formula

Perturbative interconnection, i.e. $1 \rightarrow 2$

$\lim_{b \to 0} F^{q\bar{q}}(x_q, x_{\bar{q}}, b) \sim \alpha_s \frac{f(x_q + x_{\bar{q}})}{x_q + x_{\bar{q}}} P_{g\to q\bar{q}} \left( \frac{x_q}{x_q + x_{\bar{q}}} \right) \frac{1}{b^2}$

Delicate interplay with loop corrections to 1HS: need to avoid double counting

Extend 1HS theory to 2HS: double PDFs, colour flow, sum rules, DGLAP

Substantial progress in describing 2HS with MC tools: dShower
Rest of this talk: present an experimental strategy to optimally disentangle 1HS from MPI
We explore Drell-Yan events and study the $p_{tZ} \to 0$ limit. Two concurring mechanisms:

1. Exponential suppression of the spectrum (Sudakov peak)

$$p_{tZ}^2 \sim k_{t,i}^2 \ll M_Z$$
Idea: exploit Parisi-Petronzio lesson from 1979


We explore Drell-Yan events and study the $p_{tZ} \rightarrow 0$ limit. Two concurring mechanisms:

\[ \mathcal{O}(p_t) \] suppression of the spectrum (dominant for $p_{tZ} \rightarrow 0$)

\[ \sum k_{t,i}^2 \approx 0, \quad p_{tZ}^2 \ll k_{t,i}^2 \ll M_Z \]
Idea: exploit Parisi-Petronzio lesson from 1979


Leading jet pt vs ptZ

Intercept calculable

\[ \langle p_{\ell}^t \rangle_{p_{tZ}\to 0} \sim \Lambda \left( \frac{M}{\Lambda} \right) \kappa \ln \left( \frac{2+\kappa}{1+\kappa} \right) \sim 2-3 \text{ GeV} \]

\[ \kappa = \frac{2C_F}{\pi \beta_0} \]
Key observation to suppress 1HS contribution

By constraining $p_{tZ}$ we can forbid QCD radiation from 1HS above 2-3 GeV

[RadISH: Monni, Re, Torrielli PRL 116, 242001, Monni, Rottoli, Torrielli PRL 124 (2020) 25, 252001] [MINNLO: Monni et al JHEP 05 (2020) 143]
What happens when switching on MPI?

MPI off: $\langle p_{ij}^\ell \rangle_{p_{iZ}\to 0} \sim 2.5$ GeV

MPI on: $\langle p_{ij}^\ell \rangle_{p_{iZ}\to 0} \sim 10$ GeV

Suggests we should study MPI with help of a tight cut on $p_{iZ}$

[RadISH: Monni, Re, Torrielli PRL 116, 242001, Monni, Rottoli, Torrielli PRL 124 (2020) 25, 252001] [MINNLO: Monni et al JHEP 05 (2020) 143]
Did nobody think about this before?

There has been some past study of MPI with $p_{tZ}$ cuts

Underlying event study

Enhanced MPI with $p_{tZ} < 10$ GeV
This study: establish what cut to use, explore new opportunities

We want balance between:

- maximising stats (loose $p_{tZ}^{\text{cut}}$)
- minimise 1HS (tight $p_{tZ}^{\text{cut}}$)

Optimum at $p_{tZ}^{\text{cut}} = 2$ GeV

Experimental feasibility:

- $p_{tZ}^{\text{cut}} = 2$ GeV: 4-5% $\sigma_{\text{DY}}$

Corresponds to 12 million events in Run 3 at LHC
New observables: cumulative jet spectrum with $p_{tZ} < C_Z$

Average number of jets above $p_{tj,\text{min}}$ for a given cut $C_Z$ on $p_{tZ}$:

$$\langle n(p_{tj,\text{min}}) \rangle_{C_Z} = \frac{1}{\sigma(p_{tZ} < C_Z)} \int_{p_{tj,\text{min}}} dp_{tj} \frac{d\sigma_{\text{jet}}(p_{tZ} < C_Z)}{dp_{tj}}$$

For small jet radii, $R$, the total spectrum is a linear sum, i.e.:

$$\langle n(p_{tj,\text{min}}) \rangle_{C_Z} \simeq \sum_{i}^{n-\text{HS}} \langle n(p_{tj,\text{min}}) \rangle_{C_Z}^{i} = \langle n(p_{tj,\text{min}}) \rangle_{C_Z}^{\text{MPI-off}} + \langle n(p_{tj,\text{min}}) \rangle_{C_Z}^{\text{MPI-on}}$$
New observables: cumulative jet spectrum with $p_{tZ} < 2$ GeV

Less than 1 jet/event from the primary hard scattering
New observables: cumulative jet spectrum with $p_{tZ} < 2$ GeV

Around 10 jets/event from multi-parton interactions
New observables: cumulative jet spectrum with $p_{tZ} < 2$ GeV

Around 10 jets/event from multi-parton interactions

Tight cut on $p_{tZ}$ yields high-purity MPI samples. How can we exploit them?

Around 10 jets/event from multi-parton interactions
We introduce the pure MPI contribution to the inclusive jet spectrum

\[
\langle n(p_{tj, \text{min}}) \rangle_{C_Z}^{\text{pure-MPI}} \equiv \langle n(p_{tj, \text{min}}) \rangle_{C_Z} - \langle n(p_{tj, \text{min}}) \rangle_{C_Z}^{\text{no-MPI}}
\]

In the pocket-formula approach this reduces to

\[
\sigma_{DPS}^{A,B} = \frac{\sigma_{A}^{SPS} \sigma_{B}^{SPS}}{\sigma_{\text{eff}}}
\]

\[Z+jj@NLO \text{ theory prediction}\]
Pure MPI cumulative jet spectrum with $p_t Z < C_Z$

We introduce the pure MPI contribution to the inclusive jet spectrum

$$\langle n(p_{tj, \text{min}}) \rangle_{C_Z}^{\text{pure-MPI}} \equiv \langle n(p_{tj, \text{min}}) \rangle_{C_Z} - \langle n(p_{tj, \text{min}}) \rangle_{C_Z}^{\text{no-MPI}}$$

In the pocket-formula approach this reduces to

$$\langle n(p_{tj, \text{min}}) \rangle_{C_Z}^{\text{pure-MPI}} \simeq \frac{1}{\sigma_{\text{eff}}} \int_{p_{tj, \text{min}}} dp_{tj} \frac{d\sigma_{\text{jet}}}{dp_{tj}}$$

Pocket formula predicts $\langle n(p_{tj, \text{min}}) \rangle_{C_Z}^{\text{pure-MPI}}$ to be independent of $C_Z$
New observable: ratio of $\langle n(p_{tj, \text{min}}) \rangle_{\text{pure-MPI}}^{C_Z}$ with different $p_{tZ} < C_Z$

We propose to measure

$$r_{15/2} = \frac{\langle n(p_{tj, \text{min}}) \rangle_{\text{pure-MPI}}^{15}}{\langle n(p_{tj, \text{min}}) \rangle_{\text{pure-MPI}}^{2}}$$

- Pocket formula: $r_{15/2} = 1$
- Pythia: $r_{15/2} \approx 1$ (colour reconnection)
New observable: ratio of $\langle n(p_{tj, \min}) \rangle^\text{pure-MPI}_{C_Z}$ with different $p_{tZ} < C_Z$

We propose to measure

$$r_{15/2} = \frac{\langle n(p_{tj, \min}) \rangle_{15}^\text{pure-MPI}}{\langle n(p_{tj, \min}) \rangle_{2}^\text{pure-MPI}}$$

- Pocket formula: $r_{15/2} = 1$
- Pythia: $r_{15/2} \approx 1$ (colour reconnection)
- Perturbative interconnection:

These splittings result in higher $p_{tZ} \Rightarrow r_{15/2} > 1$
Probing deviations from the pocket-formula

Can one see effect of perturbative interconnection in data?
Assessing statistical significance of perturbative interconnection

Assume dShower size for signal. Evaluate few assumptions for:

- theory uncertainty on 1HS subtraction
  \[
  \langle n(p_{tj, \text{min}}) \rangle_{C_Z}^{\text{pure-MPI}} - \langle n(p_{tj, \text{min}}) \rangle_{C_Z}^{\text{no-MPI}}
  \]

- + their correlation for different $C_Z$

![Plots showing significance of perturbative interconnection in simulation](image)
Assessing statistical significance of perturbative interconnection

Assume dShower size for signal. Evaluate few assumptions for:

- theory uncertainty on 1HS subtraction

\[ \langle n(p_{tj, \text{min}}) \rangle_{C_Z}^{\text{pure-MPI}} \equiv \langle n(p_{tj, \text{min}}) \rangle_{C_Z} - \langle n(p_{tj, \text{min}}) \rangle_{C_Z}^{\text{no-MPI}} \]

- + their correlation for different \( C_Z \)

Just barely feasible. Motivation for NNLO (matched) Z+2j calculations to reduce theory uncertainty
Assessing statistical significance of perturbative interconnection

Assume dShower size for signal. Evaluate few assumptions for:

- theory uncertainty on 1HS subtraction

\[
\langle n(p_{tj,\text{min}}) \rangle_{\text{pure-MPI}}^C Z \equiv \langle n(p_{tj,\text{min}}) \rangle_C Z - \langle n(p_{tj,\text{min}}) \rangle_{\text{no-MPI}} C Z
\]

- + their correlation for different 1HS

Just barely feasible. Motivation for NNLO (matched) Z+2j calculations

Can we go beyond 2HS?
Final topic: seeing 3HS via azimuthal correlations

Measure $\Delta \phi$ between leading jets using a tight cut on $p_{TZ}$

Signal (3HS)

Background (2HS)
Final topic: seeing 3HS via azimuthal correlations

Clear signal of 3HS in terms of a plateau for all values of $\Delta \phi$
Conclusions

Study of Drell-Yan events with tight cut on $p_{tZ}$ opens door to new MPI studies

High-purity 2HS samples

Perturbative interconnection

3HS (and potentially 4HS)

Potential for significant impact on conceptual and quantitative understanding of multi-parton interactions