Renormalons and power corrections in e+e- annihilation in the three-jet region

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Trento, 7/02/2024

Outline

- Renormalons and linear power corrections in the 2-jet limit, in the 3-jet symmetric limit, and in the full 3-jet region.
- Implications for e^+e^--annihilation shape-variables in the 3-jet region.
- Fits to ALEPH data.

α_s from e^+e^- shape variables

- Historically the framework of choice to measure α_s directly from the $q\bar{q}g$ vertex.
- In practice: very convincing at the 10% level; affected by non-perturbative uncertainties if one wants higher precision
- $\alpha_s(M_Z)$ from NNLO+NLL+Monte Carlo models:
  - 0.1224 ± 0.0039 ALEPH 2009, [arXiv:0906.3436].)
  - 0.1189 ± 0.0043 OPAL 2011, [arXiv:1101.1470])
  - 0.1172 ± 0.0051 JADE 2009, [arXiv:0810.1389]

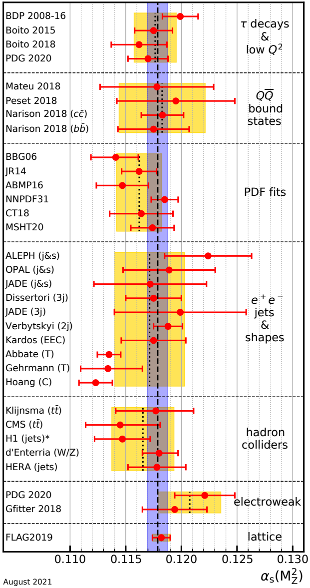
The use of Monte Carlo models to correct for hadronization effects have long been criticized, since the interplay of perturbative and non-perturbative effects in Shower Monte Carlo is not fully clear.

α_s from e^+e^- shape variables

As an alternative, another class of determinations is based upon analytic modeling of non-perturbative effects, using methods like SCET, dispersive models and low scale QCD effective couplings, and using NNLO+N^{3}LL calculations:

- 0.1135 ± 0.0011 R.Abbate *et al*, 2011, [arXiv:0809.3326]
- $0.1134\ ^{+0.0031}_{-0.0025}$ Gehrmann,Luisoni,Monni, 2013,[arXiv:1210.6945]
- 0.1123 ± 0.0015 Hoang *et al*, 2015 [arXiv:1501.04111]

They tend to result in a rather low value, not in good agreement with world data.

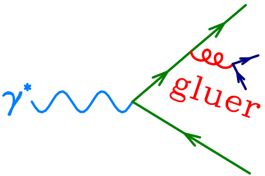


The new developments

- A calculation of linear power corrections was performed in the 3-jet symmetric limit for the C-parameter (Luisoni,Monni,Salam, 2021,[2012.00622]), leading to a result of about $1/2$ of the one in the two-jet limit
- A calculation of linear power corrections in the full 3-jet region was performed in a sequel of papers Caola,Ferrario Ravasio,Limatola,Melnikov, PN 2021,[2108.08897], same authors + Ozcelik 2022[2204.02247], Zanderighi,PN 2023[2301.03607]

The new development; recall why we can do 2-jets

Let us first recall why one can compute NP effects in the 2-jet limit.



NP effects arise from the emission of soft gluons with small off-shellness (so that α_s hits the Landau pole)

$$\langle S \rangle = \int \mathrm{d}\sigma S(q\bar{q} + \text{gluer})$$

and we can write

$$\langle S \rangle = \int \mathrm{d}\sigma \left[S(q\bar{q} + \text{gluer}) - S(q\bar{q}) \right] + \int \mathrm{d}\sigma S(q\bar{q})$$

But, since $S(q\bar{q})$ is constant, the second term is proportional to the inclusive cross section, that does not have linear NP corrections.
The factor in the square bracket suppresses the soft region;
so, the leading NP term must arise from the soft approximation.
Note that the virtual corrections do not contribute to the first term.

The new development; why Luisoni-Salam-Monni works

Now we recall why LSM can compute NP effects in the 3-jet symmetric limit for the C-parameter.

LSM noticed that near the 3-jet symmetric limit the contribution to the C parameter from the hard partons $q, \bar{q}, g$ is

$$C = \frac{3}{4} - \frac{81}{16} \underbrace{(\epsilon_q^2 + \epsilon_q \epsilon_{\bar{q}} + \epsilon_{\bar{q}}^2)}_{\text{quadratic}} + \mathcal{O}(\epsilon^3).$$

where $\epsilon_{q/\bar{q}} = x_{q/\bar{q}} - 2/3$ are the relative deviation of the quark and antiquark energy fraction with respect to the symmetric limit. So

$$C = \int \mathrm{d}\sigma \left[C(q\bar{q}g + \text{gluer}) - S(q\bar{q}g) \right] + \int \mathrm{d}\sigma S(q\bar{q}g)$$

Since $S(q\bar{q}g)$ is almost constant, one can argue that the gluer emission has no linear NP effect in the second term, since it is inclusive in the soft gluer emission. The first term has only real contributions, so we only need to compute a real soft emission, followed by a splitting process.

The new development: the full 3-jet region

Start with $q\bar{q}\gamma$ (large n_f calculation)

- Assume that we have a mapping from the $q\bar{q}\gamma + gluer$ configuration to the $q\bar{q}\gamma$ configuration
  $p_1,\ p_2,\ p_3,\ k \leftrightarrow \tilde{p}_1,\ \tilde{p}_2,\ \tilde{p}_3$
  that for small k is linear in k: $\tilde{p}_i^\mu = p_i^\mu + T^\mu_{i,\nu}(\tilde{p})k^\nu + \mathcal{O}(k_0^2).$
- Then we demonstrated that the integral at fixed $\tilde{p}_i$:

$$\int \mathrm{d}^4 k \frac{\partial^4 \sigma(k, p(\tilde{p}, k))}{\partial k^4} \times G(k)$$

  where G is any function of k that is linear in k for small k, is free from linear NP corrections.

This result is in fact very general, holding not only for e^+e^- annihilation but also for collider processes, like the differential cross section for the production of massive colour neutral object in hadronic collisions.

The new development

We write, for a generic shape variable

$$S = \int \mathrm{d}\sigma(p,k)\,[S(p,k) - S(\tilde{p})] + \int \mathrm{d}\sigma(p,k) S(\tilde{p}),$$

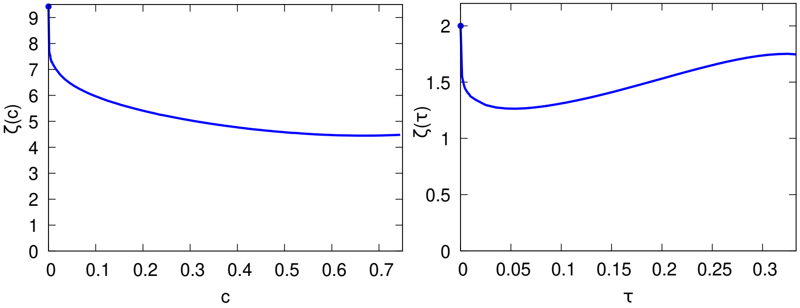
and according to our finding the second term cannot yield linear NP corrections. So, only the first term is left, and it receives contributions only from the real emission of a soft gluer, that, thanks to the suppression of the square bracket, can be computed in the soft limit.

Since the correction depends only upon the soft eikonal approximation for the soft emission, we can put forward the hypothesis that it can be extended to final states involving also gluons.

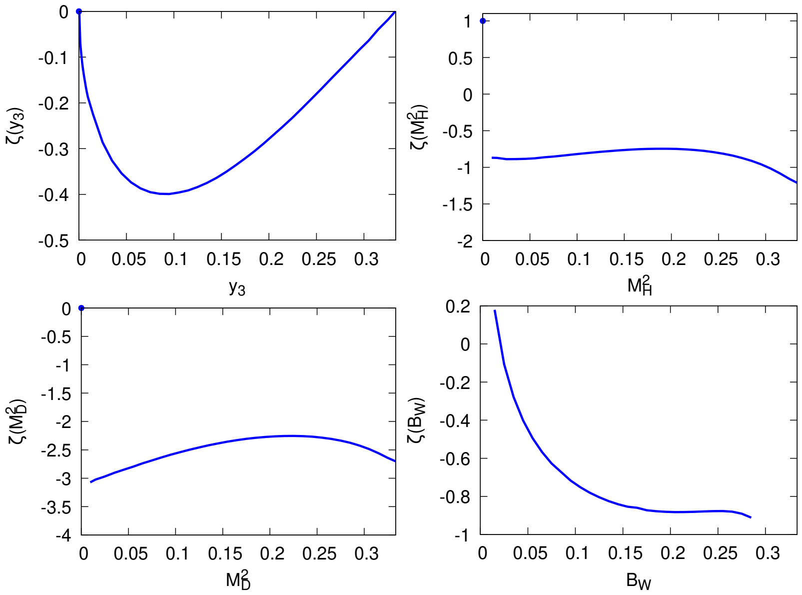
Non-perturbative corrections can be parametrized as a shift in the perturbative cumulant distribution:

$$\Sigma(s) \longrightarrow \Sigma(s + H_{\mathrm{NP}}\zeta(s)), \quad \text{where} \quad \Sigma(s) = \int \mathrm{d}\sigma(\Phi)\theta(s - s(\Phi))$$

and $H_{\mathrm{NP}} \approx \Lambda/Q$ is a non-perturbative parameter that must be fitted to data.



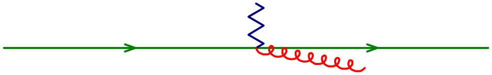
The dot in the plots represents the constant value that was used in earlier studies. The value of $\zeta(c)$ at the symmetric point $c = 3/4$ was also computed by Luisoni,Monni,Salam 2021.



(G.Zanderighi,P.N.2023) In some cases ζ is negative!

Rapid variations near $v = 0$

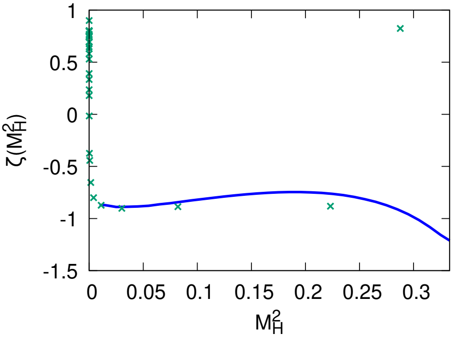
Near $v = 0$, the Born amplitude is dominated by the soft-collinear region.



$$\text{radiation} = \frac{C_A}{2} M_{\bar{q}g} + \frac{C_A}{2} M_{qg} + \left(C_F - \frac{C_A}{2} \right) M_{q\bar{q}}$$

but $M_{qg} \approx 0$, $M_{\bar{q}g} \approx M_{q\bar{q}}$, so the total is $\approx C_F M_{q\bar{q}}$.

Our $\zeta(v)$ functions, for $v \to 0$ MUST approach the 2-jet limit value; but up to single logs!, i.e. terms of relative order $1/|\log(v)|$.



Insist on $v \to 0$ (quadruple precision, log scale histogram). Two-jet limit reached, but subleading terms are extremely important!

Fits to data

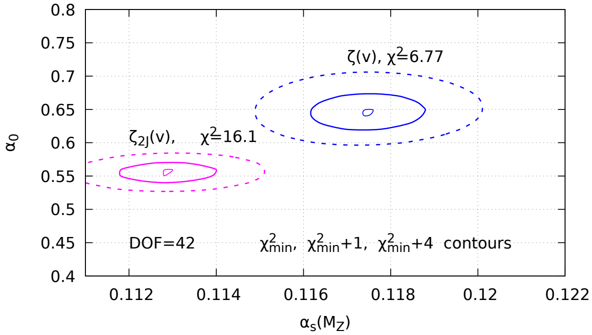
Our calculation indicates that there can be large differences between the power corrections estimated in the two-jet limit with respect to those in the three-jet regime.
So:

- We attempted to fit ALEPH data using our findings.
- The purpose of the fit is to see to what extent the data supports our finding for the non-perturbative corrections.
- Rather than attempting to quote a value for α_s, we have tried to examine all possible sources of uncertainties

RESULTS

Simultaneous fit to C, t and y_3, both for our newly computed $\zeta(v)$, and, for comparison, with $\zeta(v) \to \zeta_{2J}(v) = \zeta(0)$ (traditional method for the computation of power corrections).

(we excluded variables with "bizarre" behaviour near the 2-jet limit)



The central value is at $\alpha_s(M_Z) = 0.1174$, $\alpha_0 = 0.64$. The "traditional" method leads to smaller values of α_s.

Fit details

Take v_i to span all bins of all shape variables considered; we define

$$\chi^2 = \sum_{ij} \Delta_i V_{ij}^{-1} \Delta_j, \quad \Delta_i = \left(\frac{1}{\sigma_{\mathrm{exp}}} \frac{\mathrm{d}\sigma_{\mathrm{exp}}(v_i)}{\mathrm{d}v_i} - \frac{1}{\sigma_{\mathrm{th}}} \frac{\mathrm{d}\sigma_{\mathrm{th}}(v_i)}{\mathrm{d}v_i} \right),$$

$$V_{ij} = \delta_{ij}(R_i^2 + T_i^2) + (1 - \delta_{ij}) C_{ij} R_i R_j + \mathrm{Cov}_{ij}^{(\mathrm{Syst})}$$

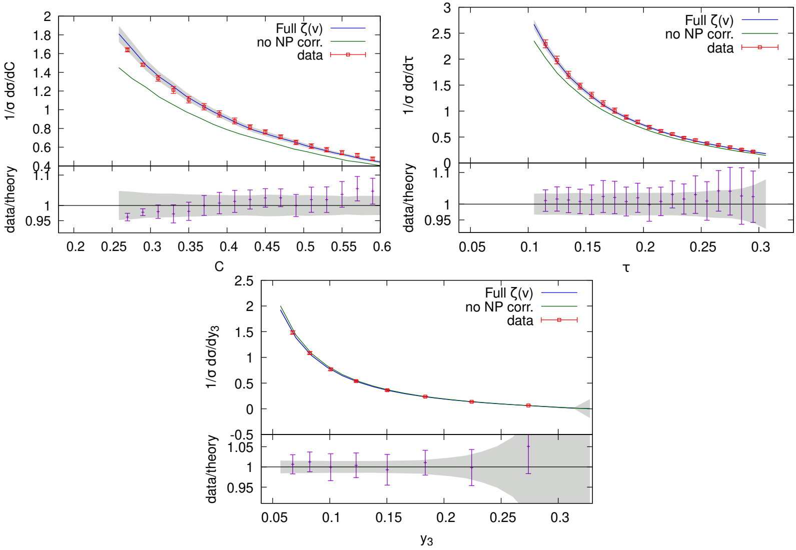
- R_i: statistical error
- T_i: theoretical error (scale variation plus error estimate of non-perturbative shift).
- C_{ij} statistical correlation (from Monte Carlo simulation)
- $\mathrm{Cov}_{ij}^{(\mathrm{Syst})}$: systematics covariance matrix

Variation	$\alpha_s(M_Z)$	α_0	χ^2	$\frac{\chi^2}{N_{\text{deg}}}$
Default setup	**0.1174**	**0.64**	**6.8**	**0.15**
Ren. sc. $Q/4$	0.1180	0.60	6.1	0.14
Ren. sc. Q	0.1182	0.68	7.9	0.18
NP sch. (b)	0.1186	0.79	6.4	0.15
NP sch. (c)	0.1194	0.84	4.7	0.11
NP sch. (d)	0.1184	0.66	5.2	0.12
P-scheme	0.1150	0.63	9.5	0.22
D-scheme	0.1188	0.79	5.1	0.12
Std. scheme	0.1168	0.58	8.1	0.18
No hq corr.	0.1176	0.68	6.2	0.14
`Herwig 6`	0.1174	0.60	14.7	0.33
`Herwig 7`	0.1174	0.60	10.9	0.25
Ranges (2)	0.1166	0.62	12.3	0.22
Ranges (3)	0.1178	0.69	2.4	0.07
Alt. correl.	0.1180	0.62	5.8	0.13
y_3 clustered	0.1166	0.67	7.6	0.17
C	0.1252	0.47	0.9	0.06
τ	0.1188	0.64	0.7	0.03
y_3	0.1196	1.90	0.0	0.00
C, τ	0.1230	0.51	2.0	0.05

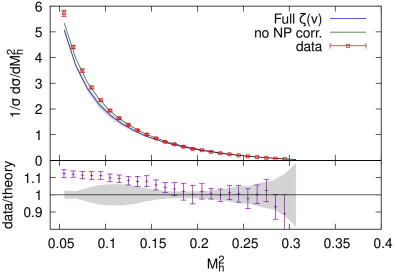
Several variations of setup parameters/methods lead to variations of the central value of order 1%. Among them

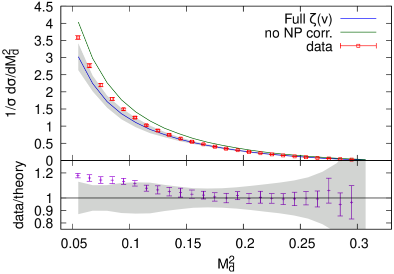
- Central ren. scale
- Ambiguity in implementation of NP corrections
- Treatment of correlation in systematic errors
- Treatment of hadron masses (P, D and std. schemes)

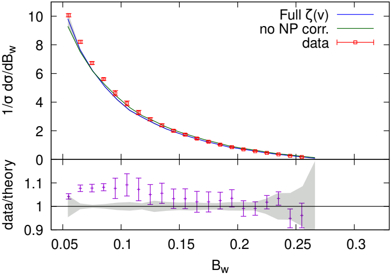
Quality of the fit for C, τ and y_3, using the new calculation of the non-perturbative effect (i.e. the full $\zeta(v)$ dependence.)



Prediction for M_H^2, M_D^2 and B_W using the values of α_s and α_0 obtained by fitting C, τ and y_3.

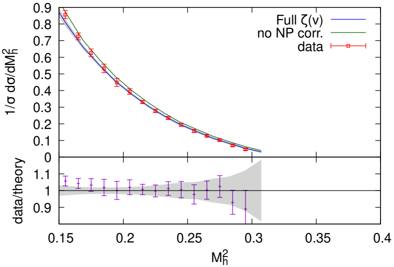


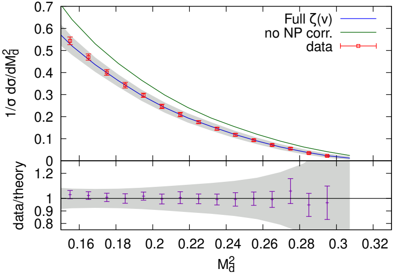


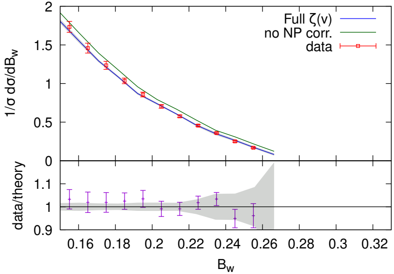


Good fit away from the two jet region.

Prediction for M_H^2, M_D^2 and B_W using the values of α_s and α_0 obtained by fitting C, τ and y_3.

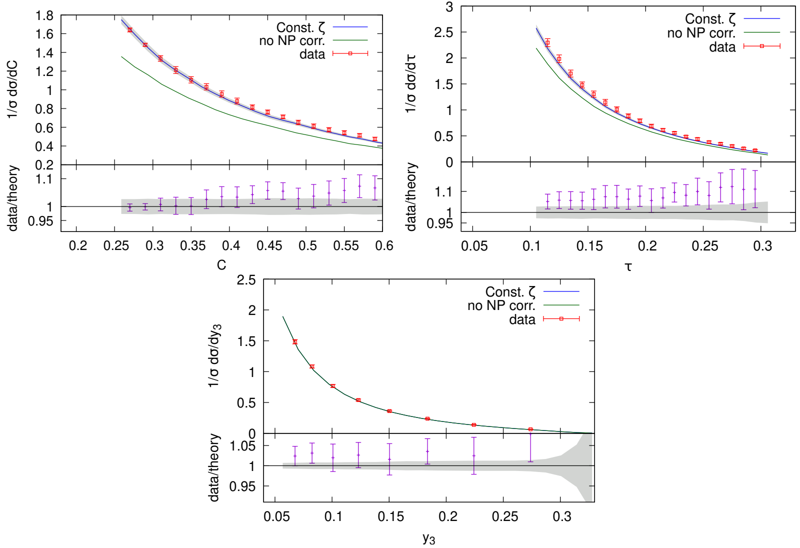




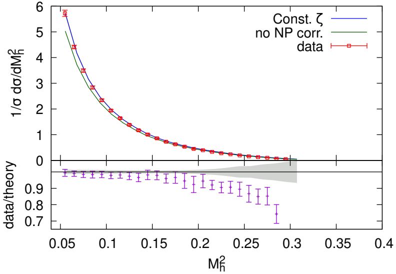


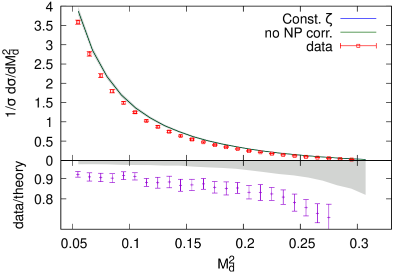
Good fit far away from the two jet region.

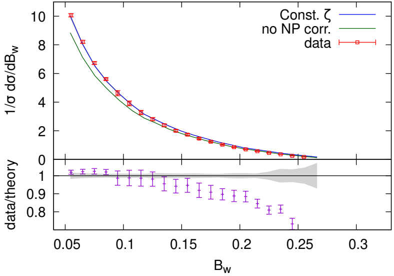
Quality of the fit for C, τ and y_3, obtained setting $\zeta(v) = \zeta(0)$.



Prediction for M_H^2, M_D^2 and B_W using the fitted values of α_s and α_0 obtained by fitting C, τ and y_3.

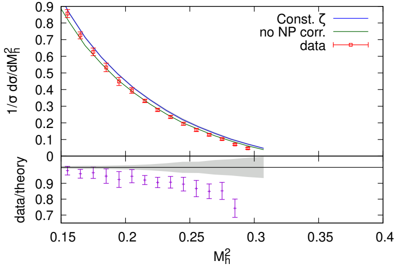


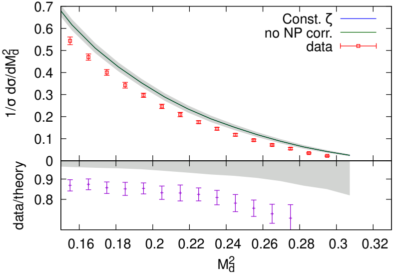


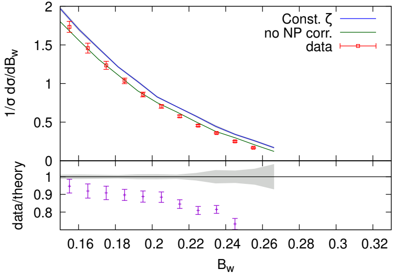


No Good fit in any region.

Prediction for M_H^2, M_D^2 and B_W using the fitted values of α_s and α_0 obtained by fitting C, τ and y_3.







No Good fit in any region.

Conclusions

- Some new, intriguing results regarding linear power corrections in collider observables have been obtained.
- They suggest that power corrections computed in the two-jet limit cannot be safely extrapolated to the three jet region.
- The NP corrections we computed in the three-jet region seem to be supported by data.