# Determining $\alpha_S$ from Thrust with Power Corrections


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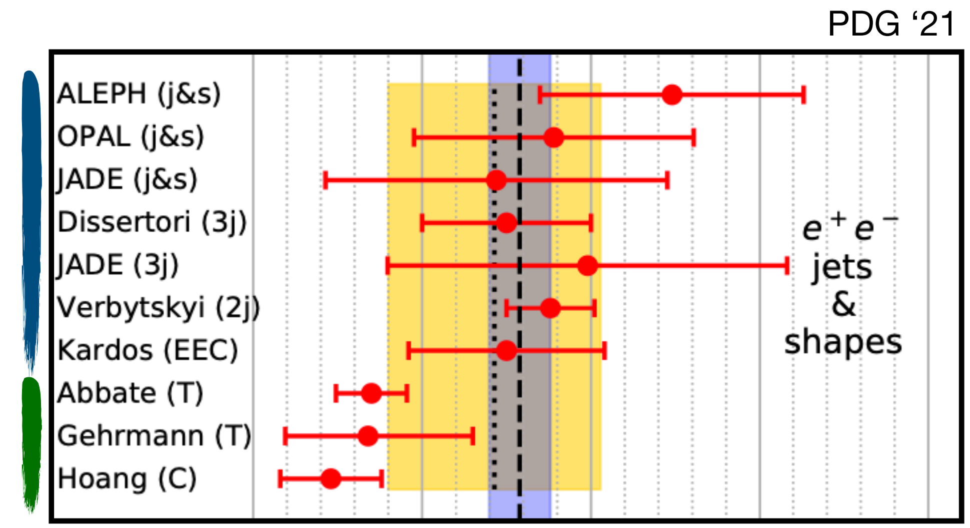
MBR, A. Hoang, V. Mateu, I. Stewart, G. Vita - work in progress

# Overview

Discrepancy between $\alpha_s$ determinations using non-perturbative corrections based on **Monte Carlos**, and those based on **analytic approaches**




Discrepancy between $\alpha_s$ determinations from $e^+e^-$ event shape observables based on analytic treatment of power corrections and world average

Triggered new analyses on treatment of power corrections in 3-jet region
**Luisoni, Monni, Salam '20  Caola et al '21  Caola et al '22**

New $\alpha_s$ determination (used new model for treatment of PCs based on 3-jet kinematics, no resummation, only $m_Z$ data, many observables used together)
**Nason, Zanderighi '23**

Analysis suggests that uncertainties on $\alpha_s$ should increase

**<u>Topics addressed in this talk:</u>**

Improvements compared to the analysis carried out in 2010

Address questions raised in the recent literature/criticism directed towards the 2010 analysis

How to carry out a determination of the strong coupling from thrust in 2024

# Theory: Dijet Factorization Formula for $e^+e^-$ Thrust distribution

Perform tail region fits of Thrust Distribution in dijet region

$$\tau = 1 - \max_{\vec{n}_T}\left(\frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}\right)$$

Based on factorization formula from SCET Fleming, Hoang, Mantry, Stewart ‘07 Bauer, Lee, Fleming, Sterman ‘08 Abbate, Fickinger, Hoang, Mateu, Stewart ‘10

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}_s}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{ns}}{\mathrm{d}\tau}\right)\left(\tau - \frac{k}{Q}\right)\delta\left(k - \frac{2\Omega_1}{Q}\right)$$

**Includes Resummation**

$$\frac{\mathrm{d}\hat{\sigma}_s^{\mathrm{QCD}}}{\mathrm{d}\tau} = Q\sum_I \sigma_o^I H_Q^I(Q,\mu_H)U_H(Q,\mu_H,\mu)$$
$$\times \int \mathrm{d}s\mathrm{d}s'\; J_\tau(s',\mu_J)U_J^\tau(s-s',\mu,\mu_J)$$
$$\times \int \mathrm{d}k' U_S^\tau(k',\mu,\mu_s)e^{\frac{-2\delta(R,\mu_s)}{Q}\frac{\partial}{\partial\tau}} S_\tau^{\mathrm{part}}\left(Q\tau - \frac{s}{Q} - k',\mu_s\right)$$
$$\approx \sum_n \alpha_s^n \delta(\tau) + \sum_{n,l}\alpha_s^n\left[\frac{\ln^l(\tau)}{\tau}\right]_+ + \text{subtractions}$$

Singular partonic for massless quarks QCD final states

$$\frac{\mathrm{d}\hat{\sigma}_{ns}}{\mathrm{d}\tau} = \sum_n \alpha_s^n f_n(\tau) + \text{subtractions}$$

$$f_n(\tau) = \sum_{i,j=0}^{\infty} a_{ij}^n \tau^i \ln^j(\tau)$$

$\mathcal{O}(\alpha_s)$ contribution known analytically

$\mathcal{O}(\alpha_s^2)$ from EVENT2 (incl. more statistics)

$\mathcal{O}(\alpha_s^3)$ from Colorful (formerly EERAD3)

Nonsingular partonic

→ Vicent’s talk

Nonperturbative soft function

# Theory: Improvements

2015 Profile functions: achieve correct partonic description → updated compared to 2010
Hoang, Kolodrubetz, Mateu, Stewart ‘15

Updated Nonsingular at $\mathcal{O}(\alpha_s^3)$

**In addition we now have information on:**

- $s_2 \longrightarrow$ 2-loop non-log coefficient of renormalized soft function Gehrmann, Luisoni, Monni ‘11 Kelley et al ‘11
- $j_3 \longrightarrow$ 3-loop coefficient of partonic jet function Brüser, Ze Long Liu, Stahlhofen ‘18 Banerjee et al ‘18
- $\Gamma_4^{\rm cusp} \longrightarrow$ 4-loop cusp anomalous dimension Moch et al ‘18 Henn, Korchemsky, Mistlberger ‘19

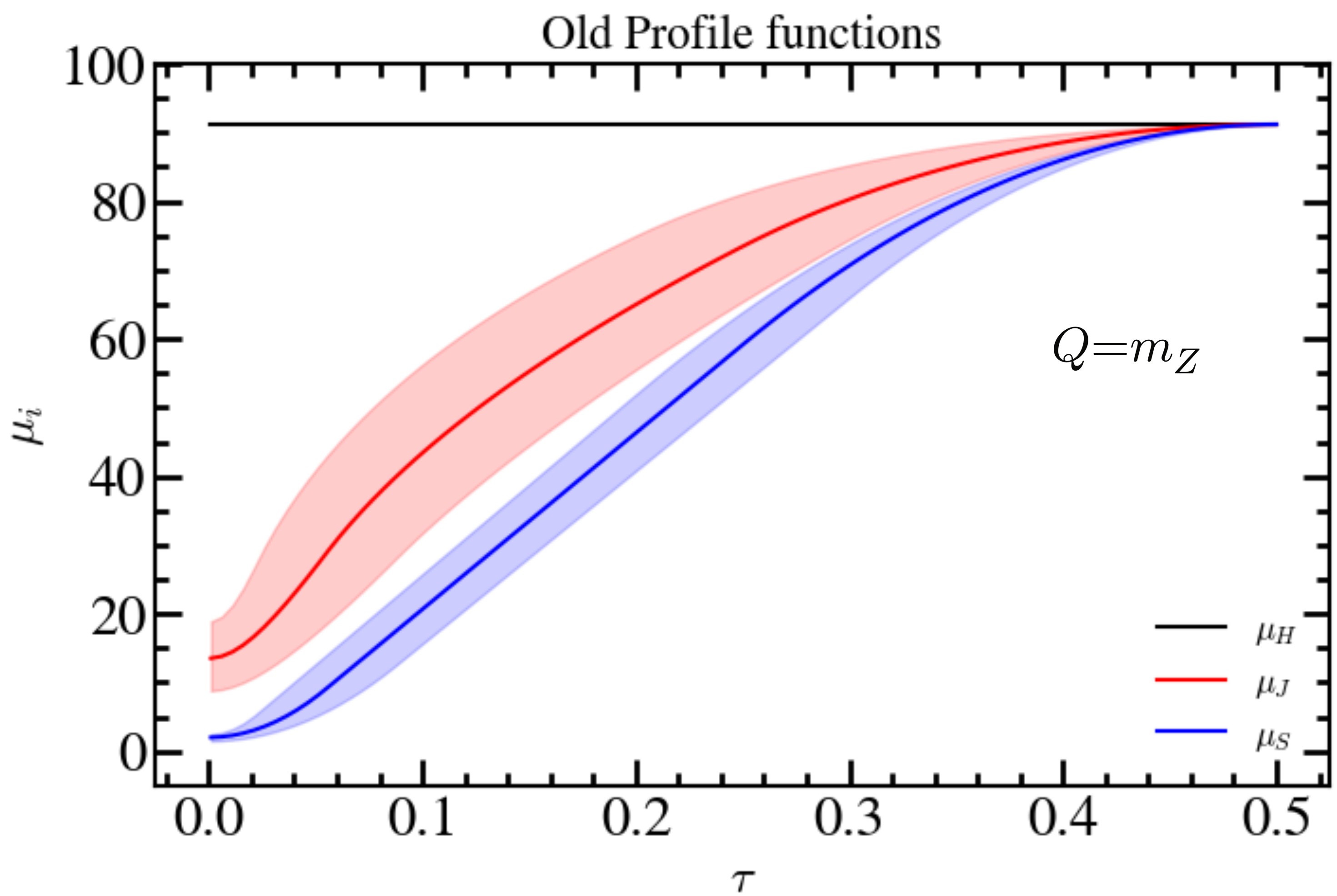
# Updated Profile functions

Factorization formula for the singular partonic thrust distribution governed by three renormalization scales

Hard scale: $\mu_H$ Jet scale: $\mu_J$ Soft scale: $\mu_S$

To avoid large logs while not spoiling singular vs non-singular cancellation in the far tail, the above scales must satisfy
Abbate, Fickinger, Hoang, Mateu, Stewart '10

$$\begin{aligned}
&\text{peak:} && \mu_H \sim Q\,, \quad \mu_J \sim \sqrt{\Lambda_{\mathrm{QCD}} Q}\,, && \mu_S \gtrsim \Lambda_{\mathrm{QCD}}\\
&\text{tail:} && \mu_H \sim Q\,, \quad \mu_J \sim Q\sqrt{\tau}\,, && \mu_S \sim Q\tau\\
&\text{far-tail:} && \mu_H = \mu_J = \mu_S \sim Q
\end{aligned}$$



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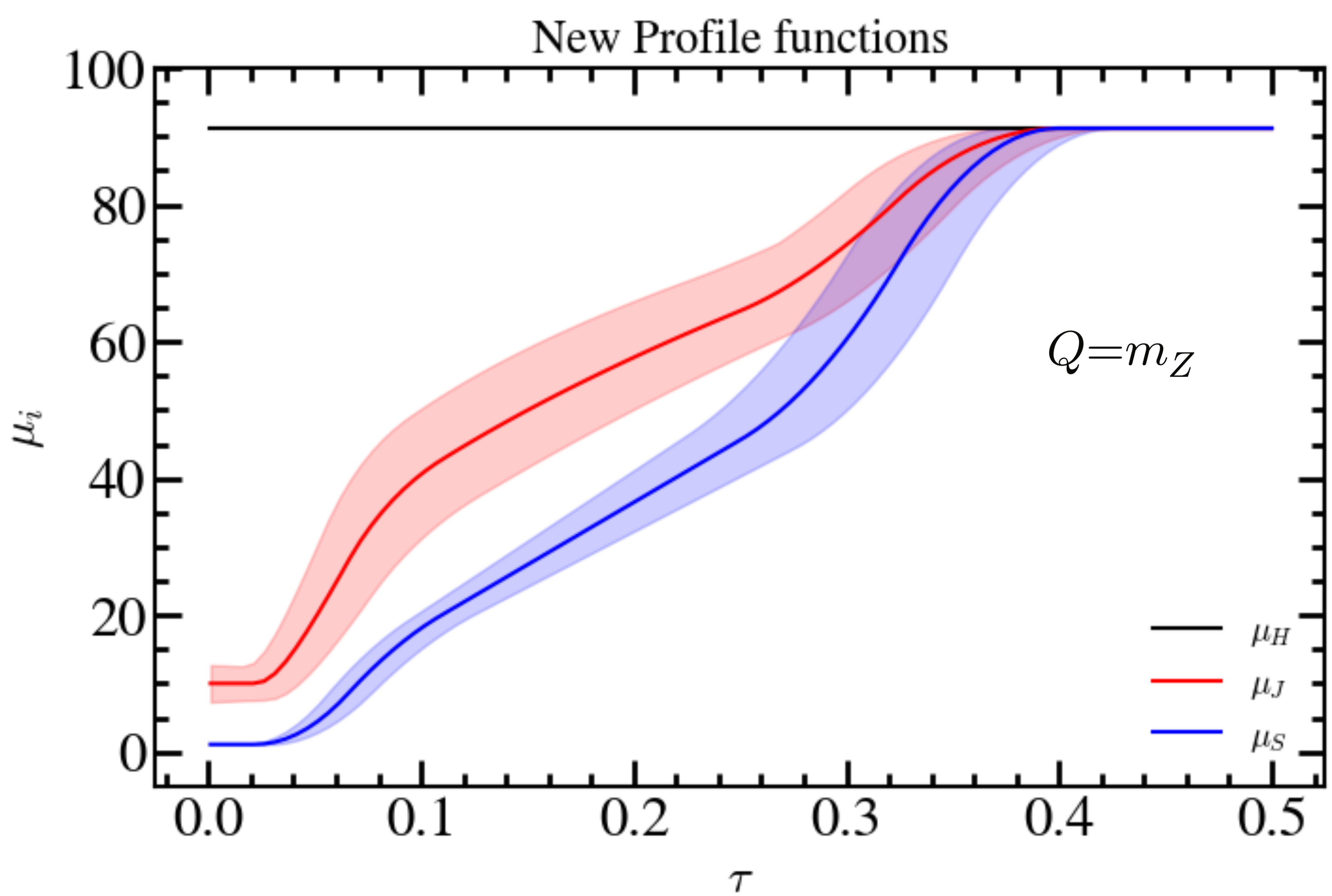
$$\begin{aligned} &\text{peak:} && \mu_H \sim Q\,, \quad \mu_J \sim \sqrt{\Lambda_{\rm QCD} Q}\,, && \mu_S \gtrsim \Lambda_{\rm QCD} \\ &\text{tail:} && \mu_H \sim Q\,, \quad \mu_J \sim Q\sqrt{\tau}\,, && \mu_S \sim Q\tau \\ &\text{far-tail:} && \mu_H = \mu_J = \mu_S \sim Q \end{aligned}$$



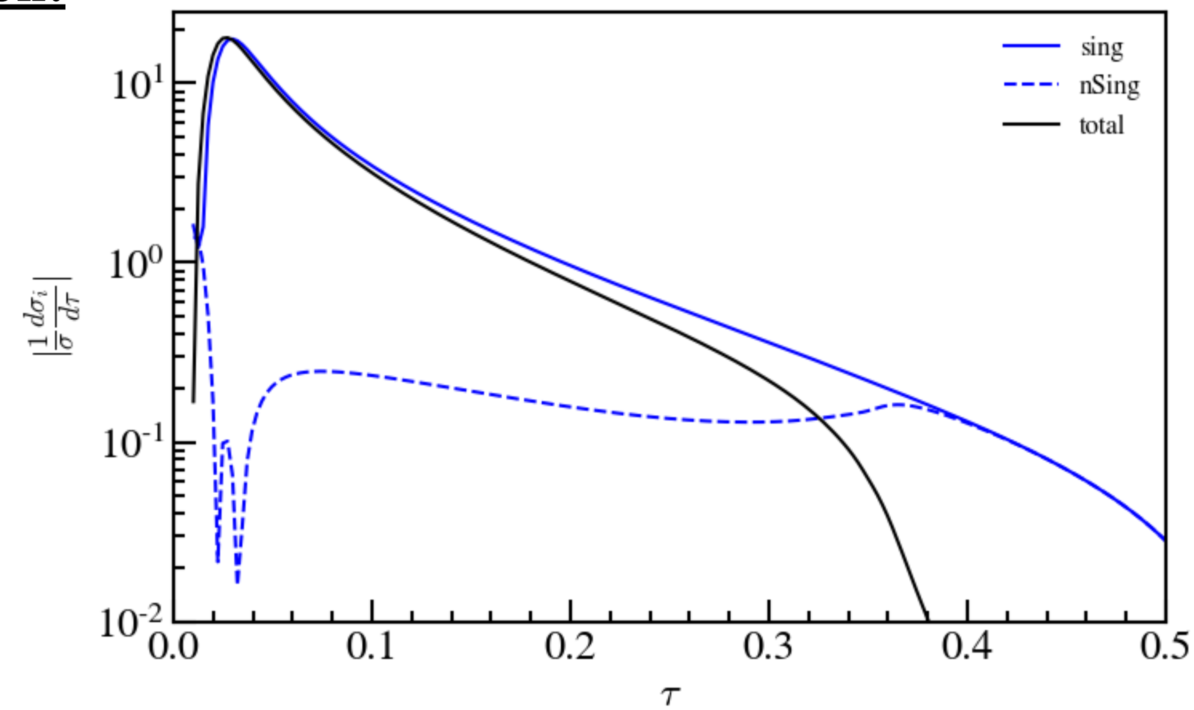

- implements correctly canonical scale setting in tail region
- variable slope
- parameters for different regions more independent
- scales merge earlier

→ fit stability

# Theory: Final comments

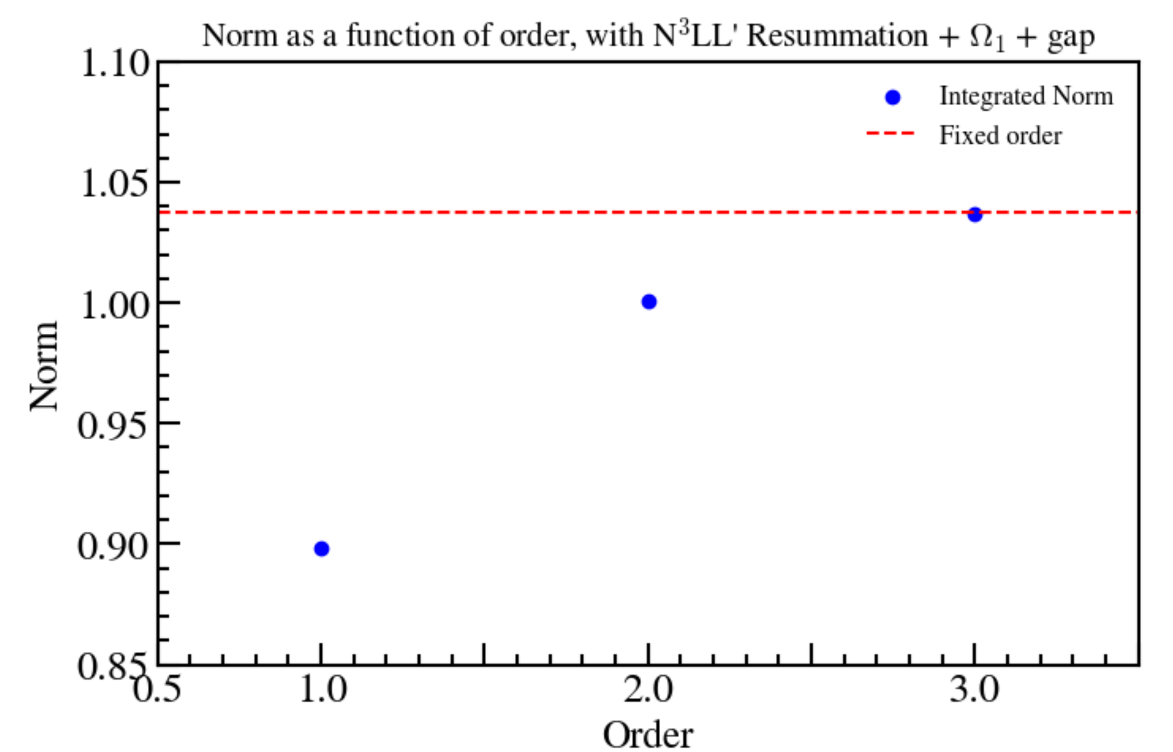
**Singular/Nonsingular contributions to total Cross Section:**

- In both peak and tail region, singular cross section significantly dominates over nonsingular
- As one approaches threshold to far-tail region ($\tau \to 0.33$), singular and nonsingular appear with opposite signs and largely cancel




**Self normalization of Cross Section:**

- Experimental data normalized to total number of events
- Need to normalize distribution to the total Cross section
- Can use fixed-order result for total hadronic Cross Section or integral of $\mathrm{d}\sigma/\mathrm{d}\tau$ distribution
- For gap scheme, integrated norm and fixed-order norm give compatible results



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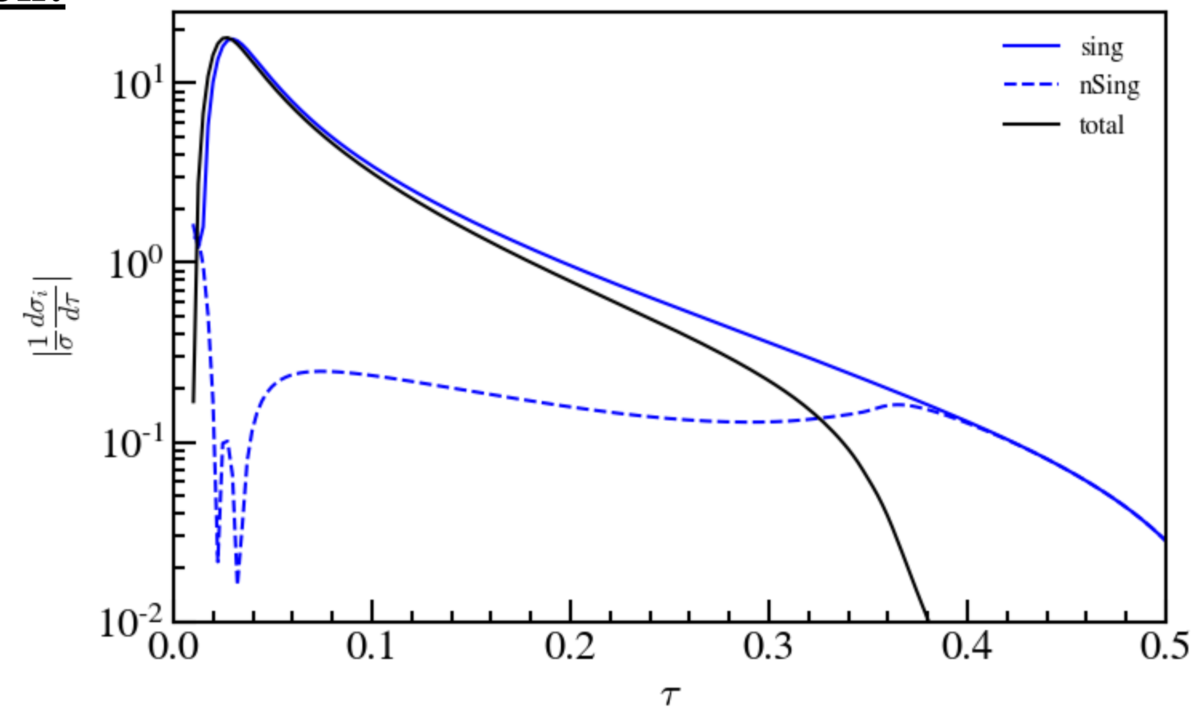
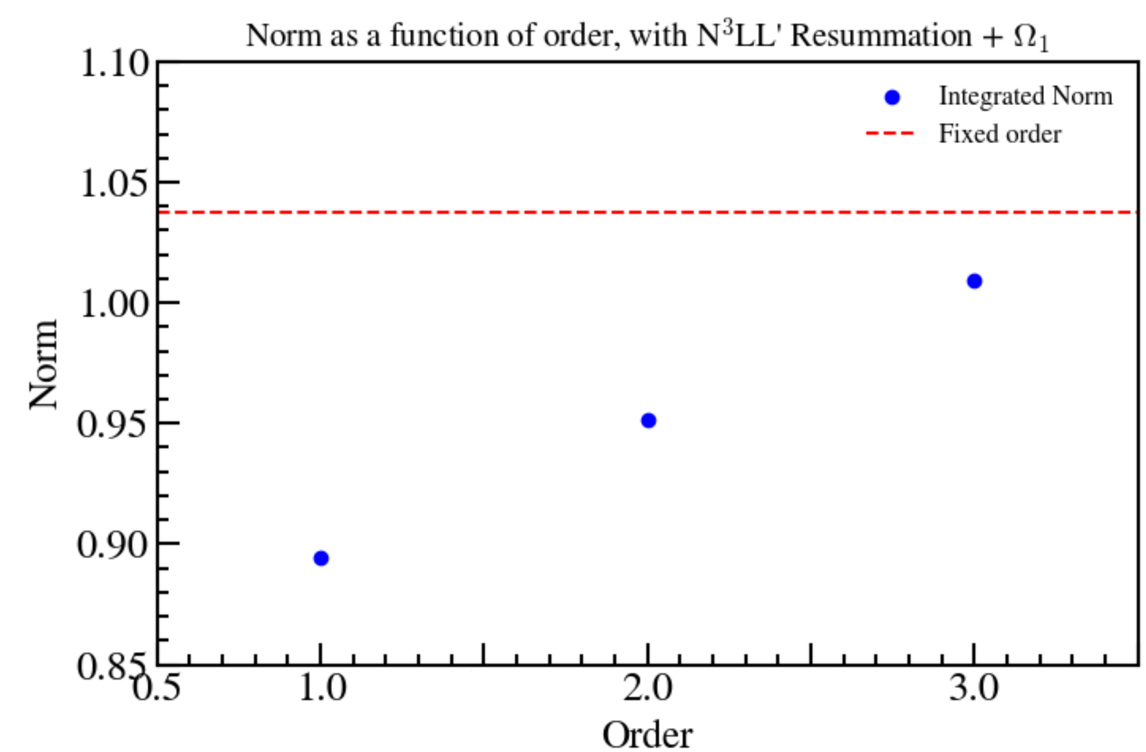
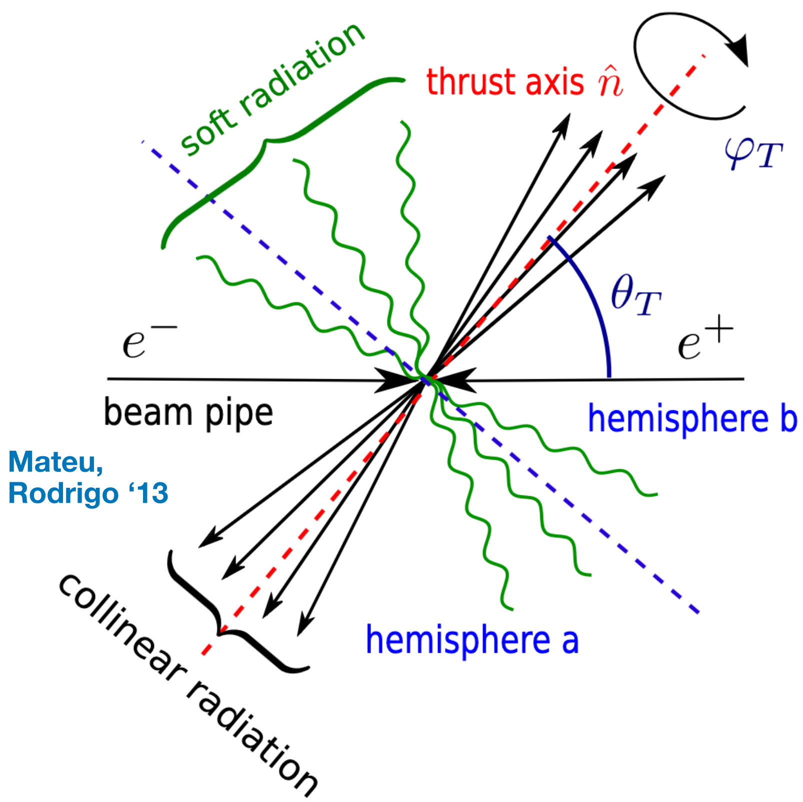



**Self normalization of Cross Section:**

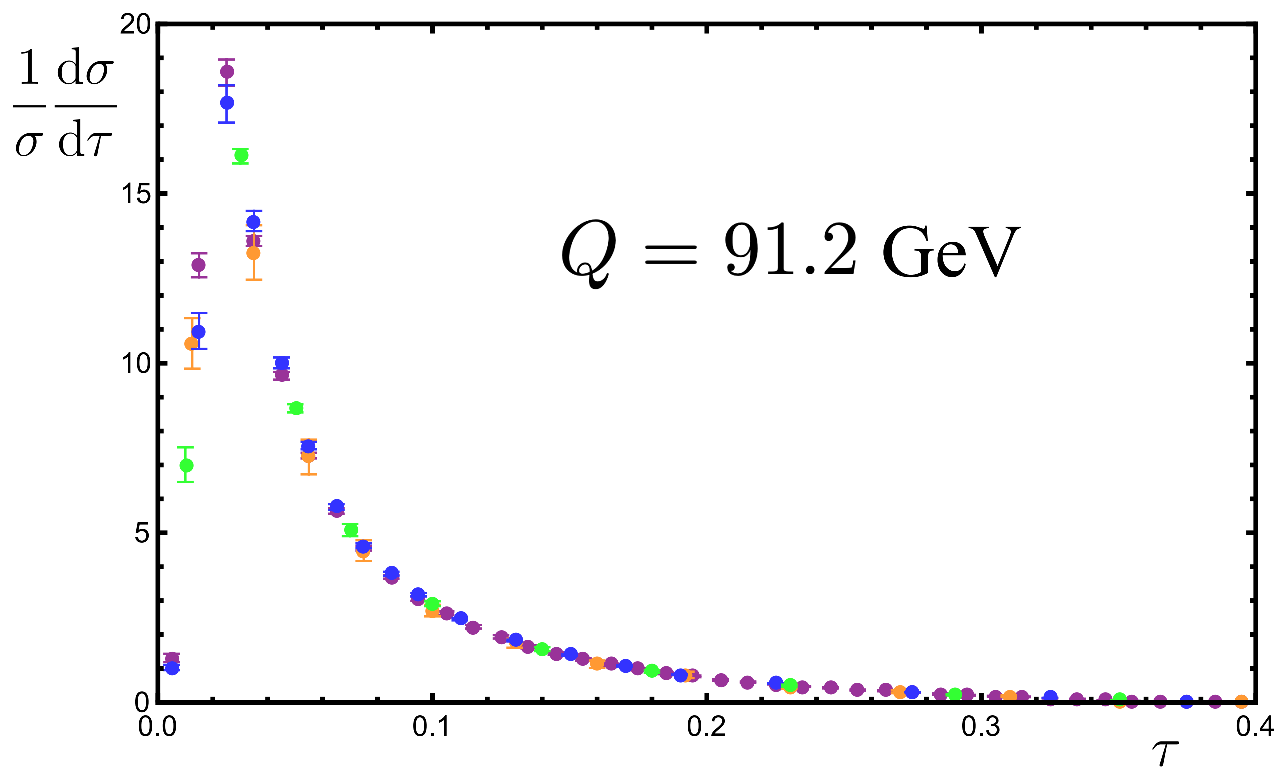
- Experimental data normalized to total number of events
- Need to normalize distribution to the total Cross section
- Can use fixed-order result for total hadronic Cross Section or integral of $\mathrm{d}\sigma/\mathrm{d}\tau$ distribution
- With uncanceled renormalon in Power Correction, one needs to use integrated norm



# Experimental data




| | Experiment | Values of the center of mass energy Q |
|---|---|---|
| LEP | ALEPH | {91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0} |
| | DELPHI | {45.0, 66.0, 76.0, 89.5, 91.2, 93.0, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0} |
| | OPAL | {91.0, 133.0, 177.0, 197.0} |
| | L3 | {41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2} |
| SLAC | SLD | {91.2} |
| DESY | TASSO | {14.0, 22.0, 35.0, 44.0} |
| | JADE | {35.0, 44.0} |
| KEK | AMY | {55.2} |



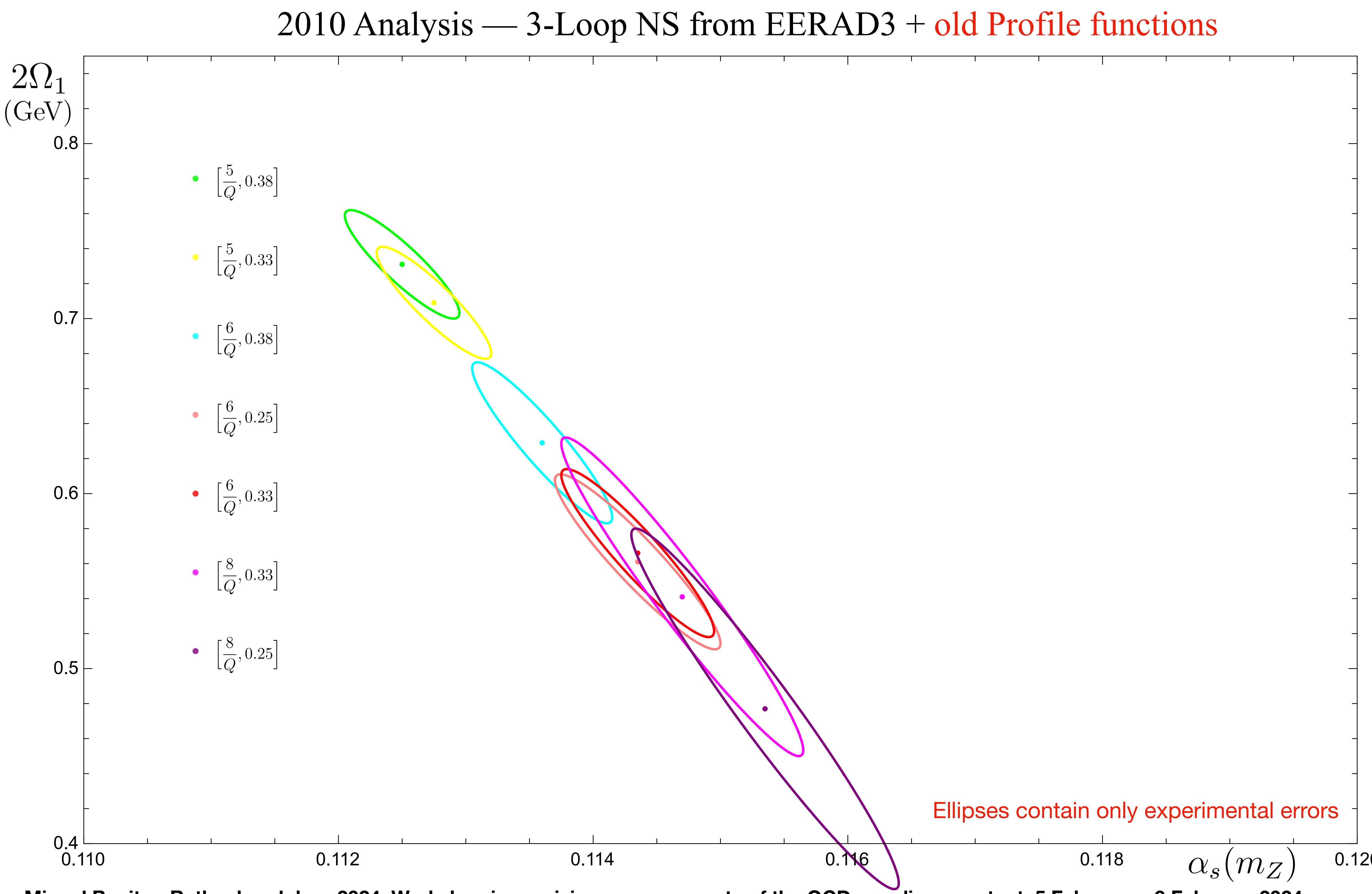

Standard dataset: 35 GeV $\leq$ Q

$$\frac{6\ \text{GeV}}{Q} \leq \tau \leq 0.33$$

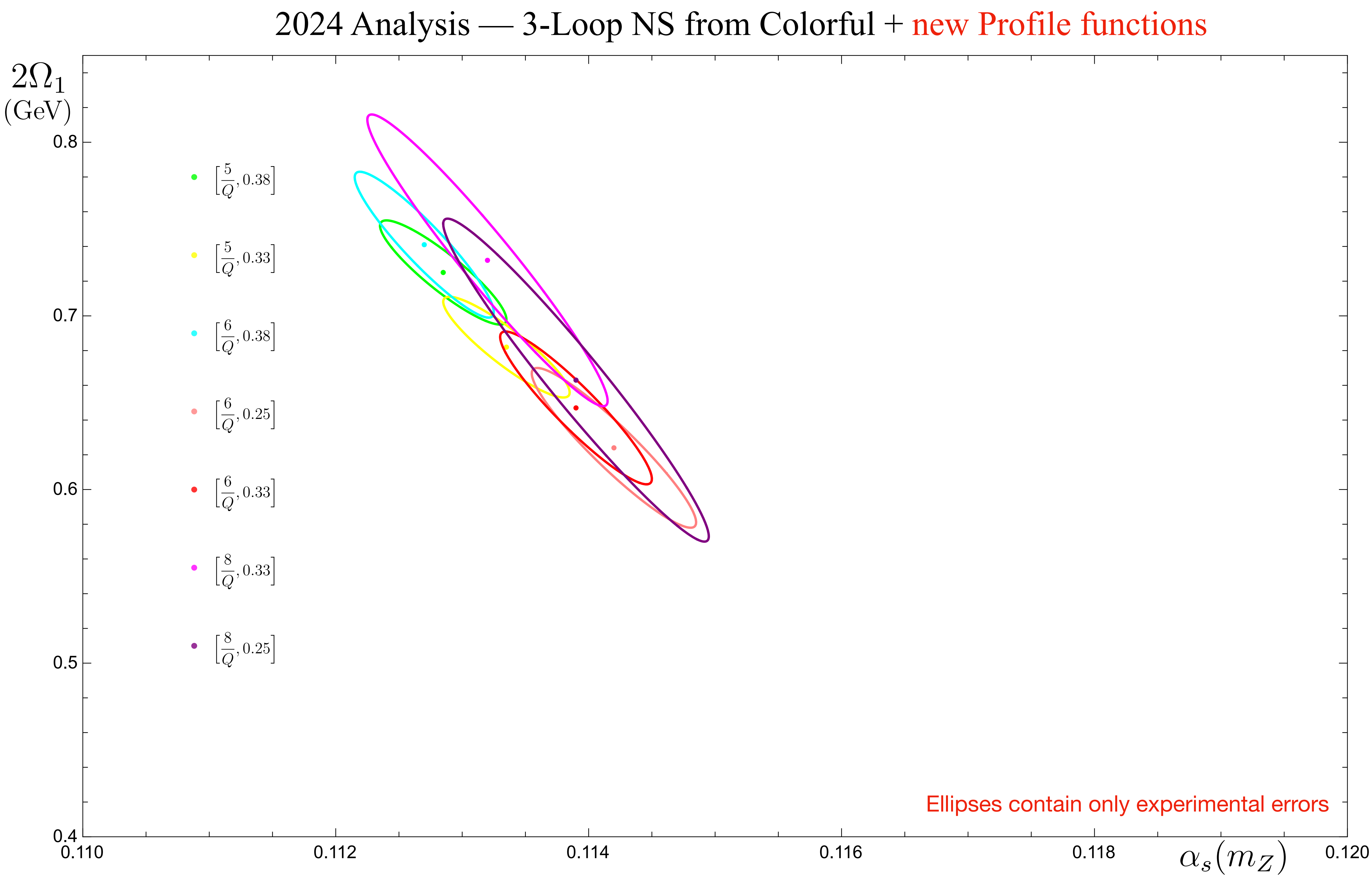
487 bins

Correlations treated in minimal overlap model

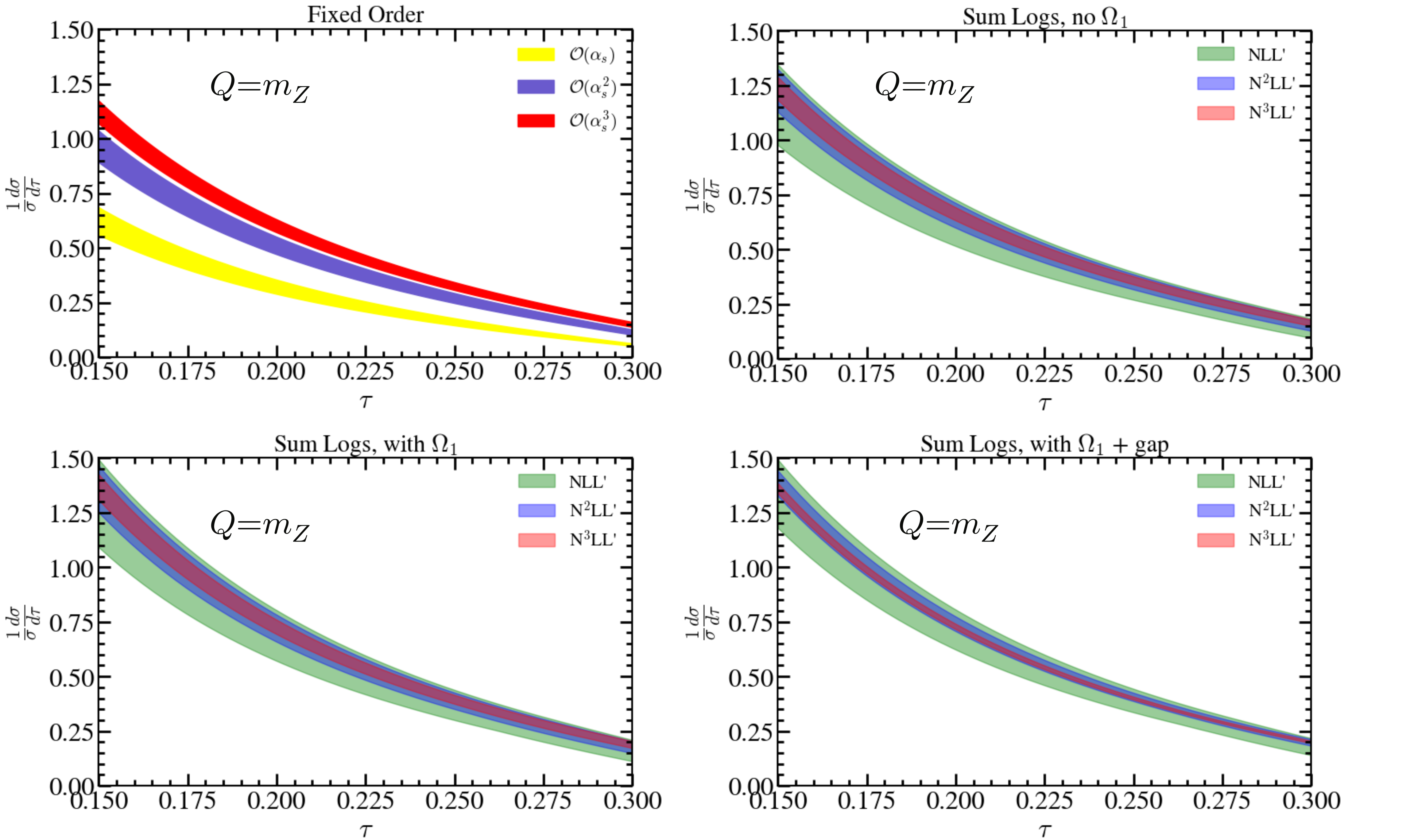
# Stability of Results at $N^3LL'$



# Stability of Results at $\mathrm{N^3LL'}$



# Impact of Resummation



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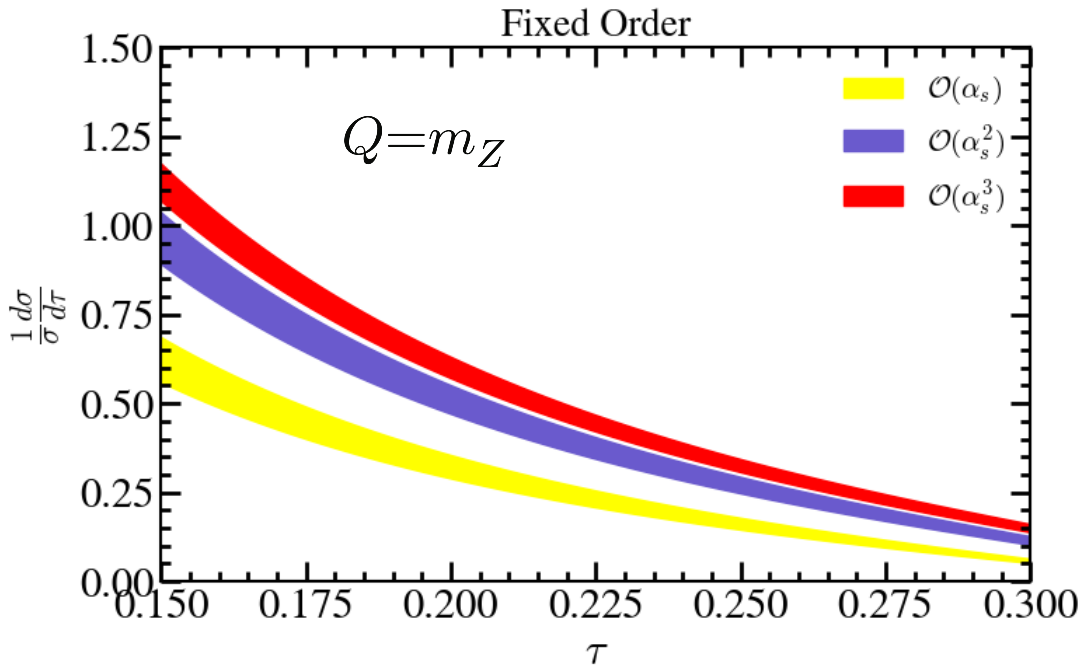


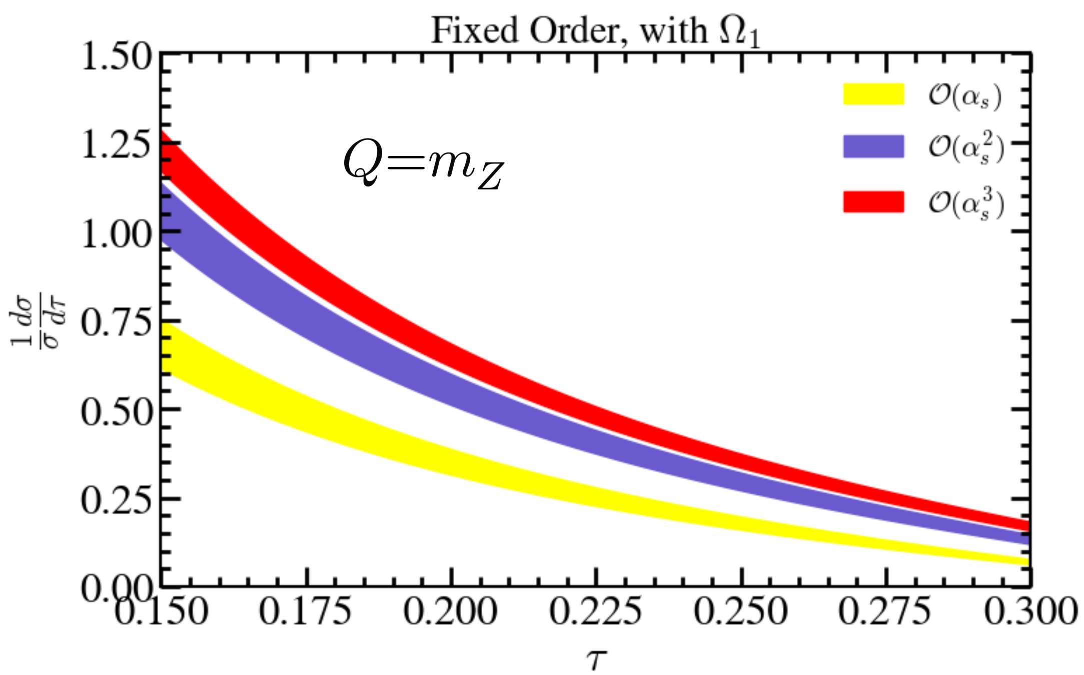


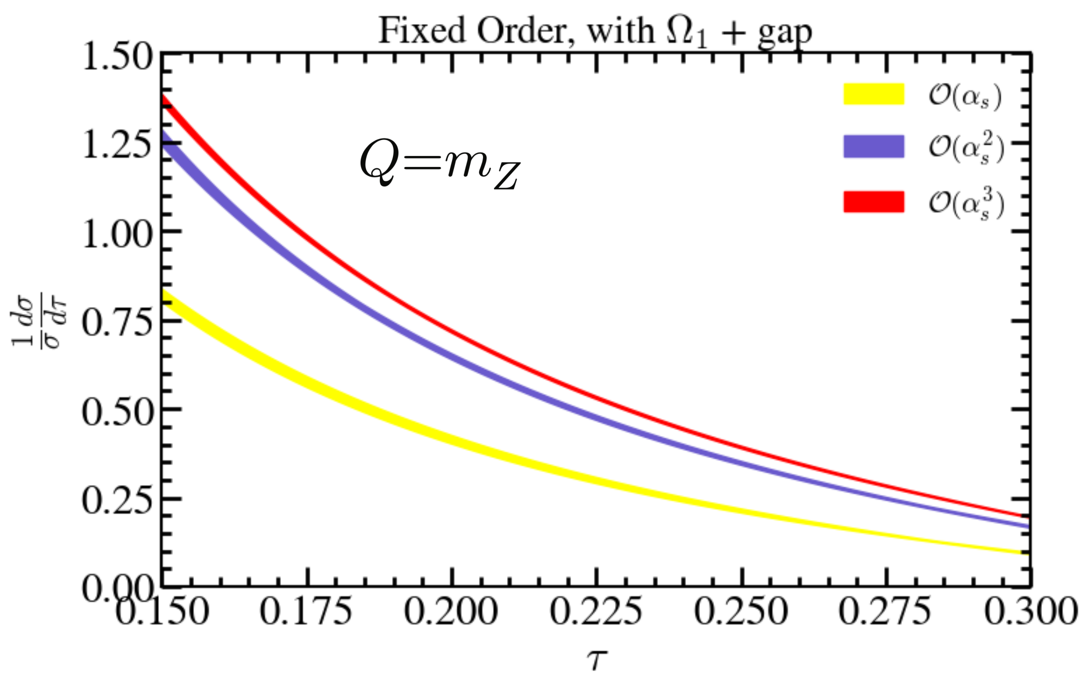

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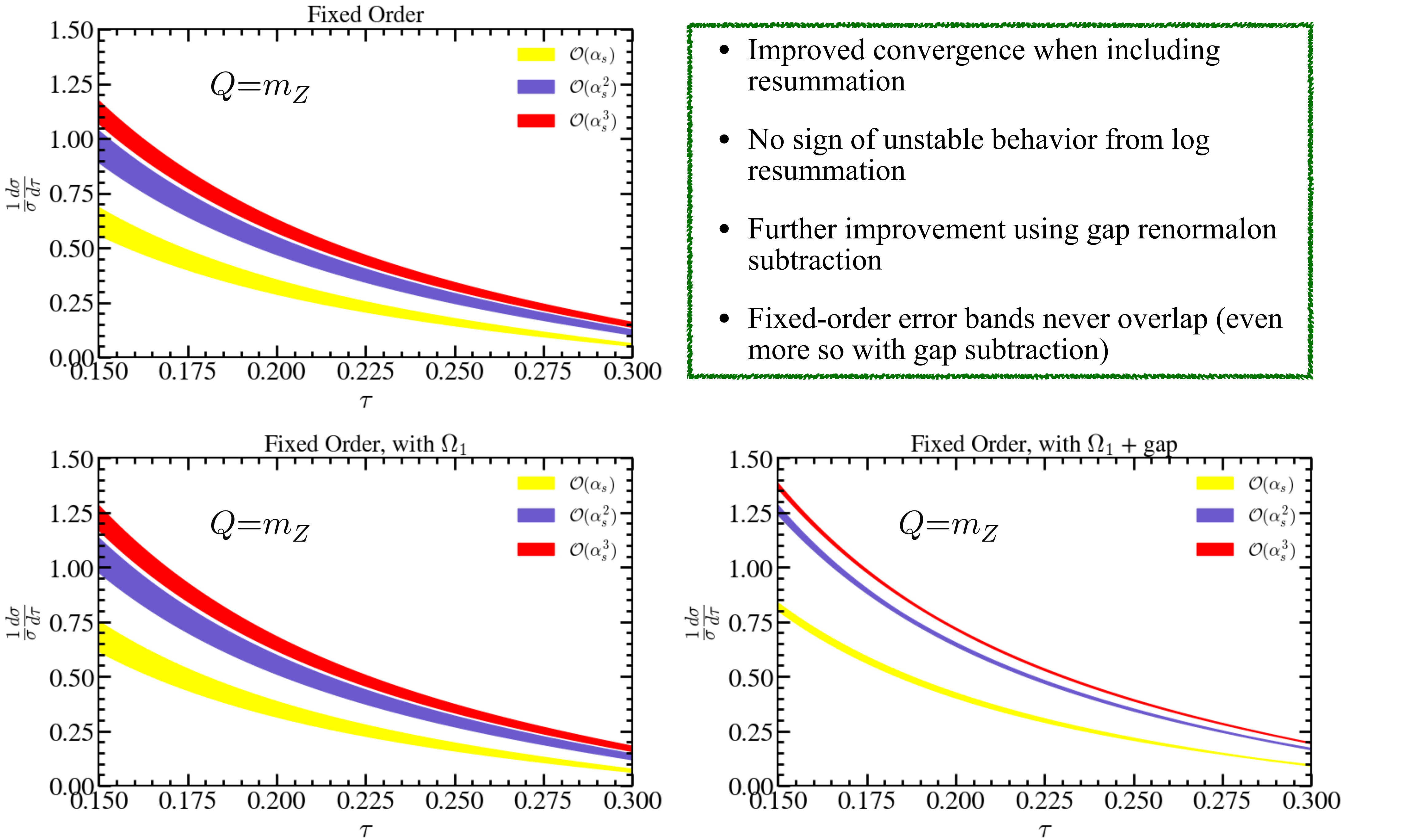
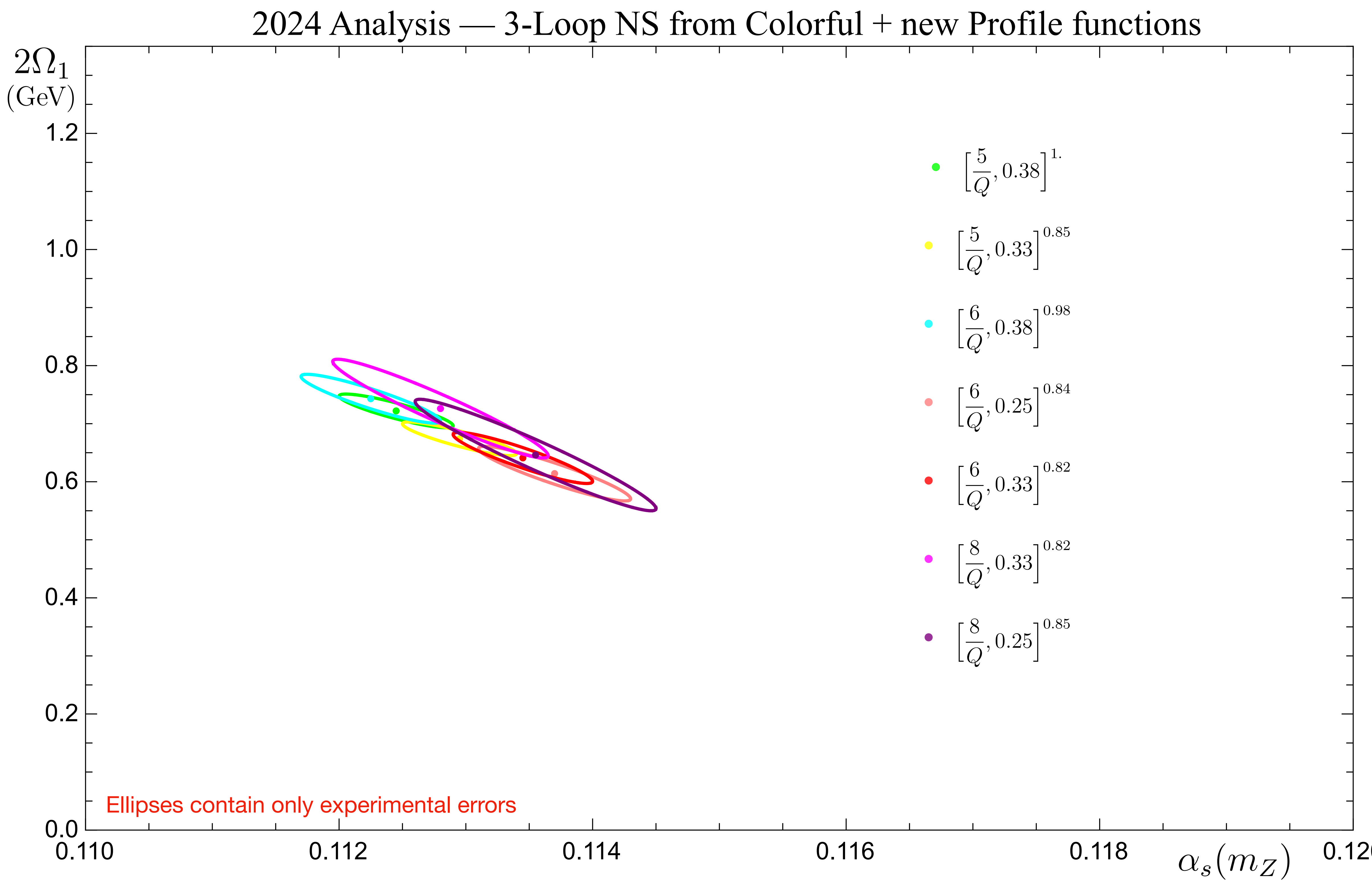
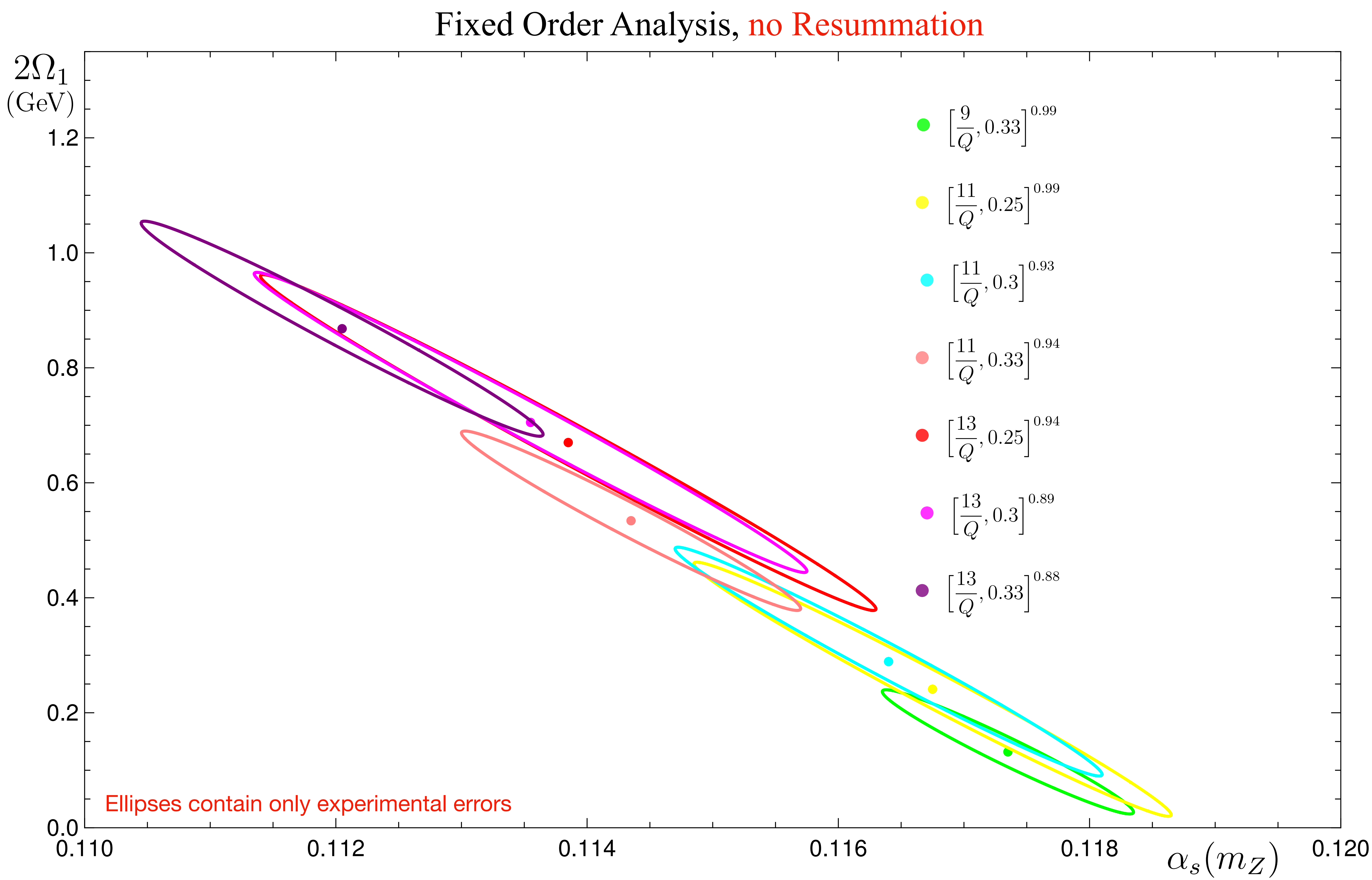



- Improved convergence when including resummation
- No sign of unstable behavior from log resummation
- Further improvement using gap renormalon subtraction
- Fixed-order error bands never overlap (even more so with gap subtraction)

# Reminder: Stability of Results at $N^3LL'$



# Impact of Resummation on Stability of Results at $N^3LL'$



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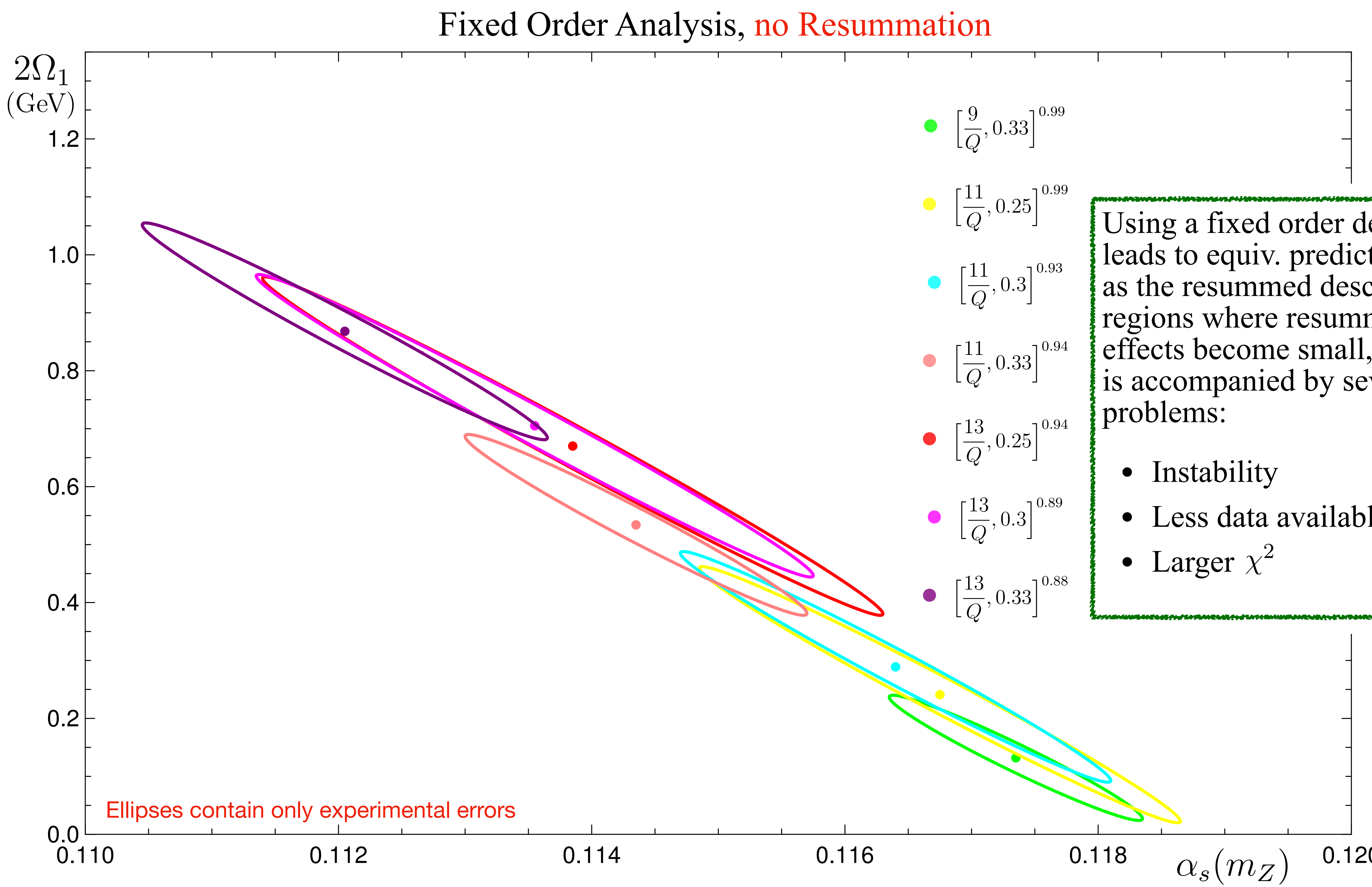


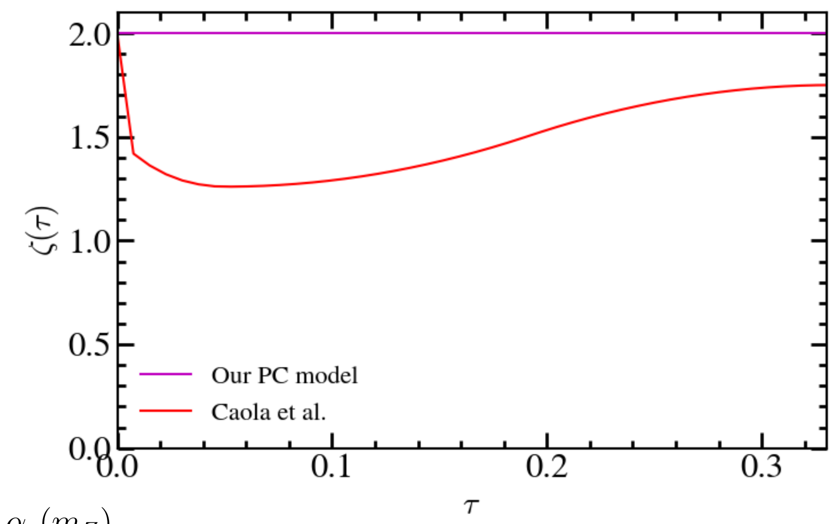

Using a fixed order descr. leads to equiv. predictions as the resummed descr. in regions where resummation effects become small, but is accompanied by severe problems:

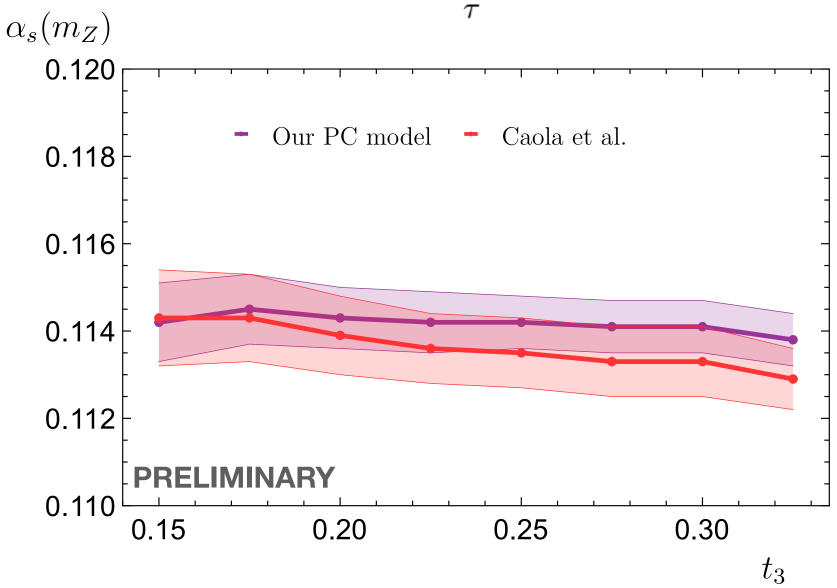
- Instability
- Less data available
- Larger $\chi^2$

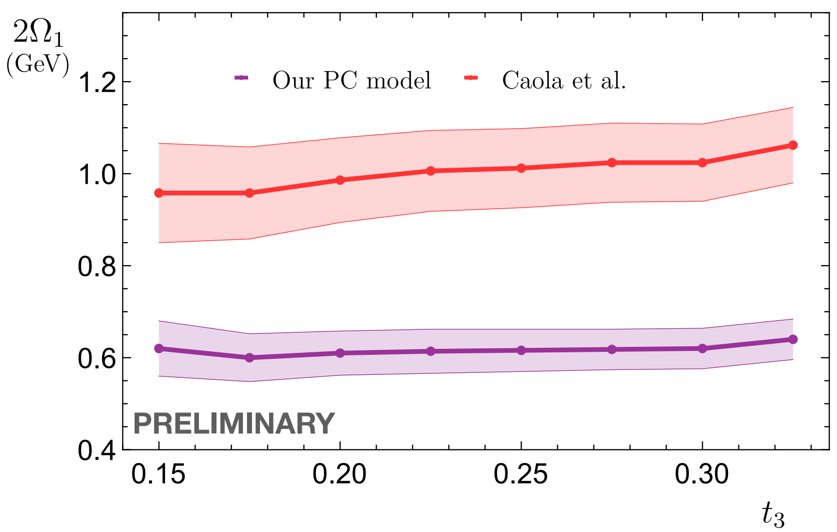
# Treatment of Power Corrections

We compare fit results for $\alpha_s$ and $\Omega_1$ for two different treatments of PC

- Our previous treatment of PC, i.e. flat behavior across thrust spectrum
- Model predictions for 3-jet region




What we observe: (Error bands contain only experimental errors)

- Outcome for strong coupling gives compatible results for the fit range $\tau \in [6/Q, t_3]$
- Nonperturbative matrix element is shifted towards larger values
- Expected, since model predictions are flat in the region where we perform the fit → this only acts as a shift in the distribution but will not influence $\alpha_s$

# Treatment of Power Corrections

We know: Dijet region is located safely within $\tau < 0.11$ (→ talk by Vicent)

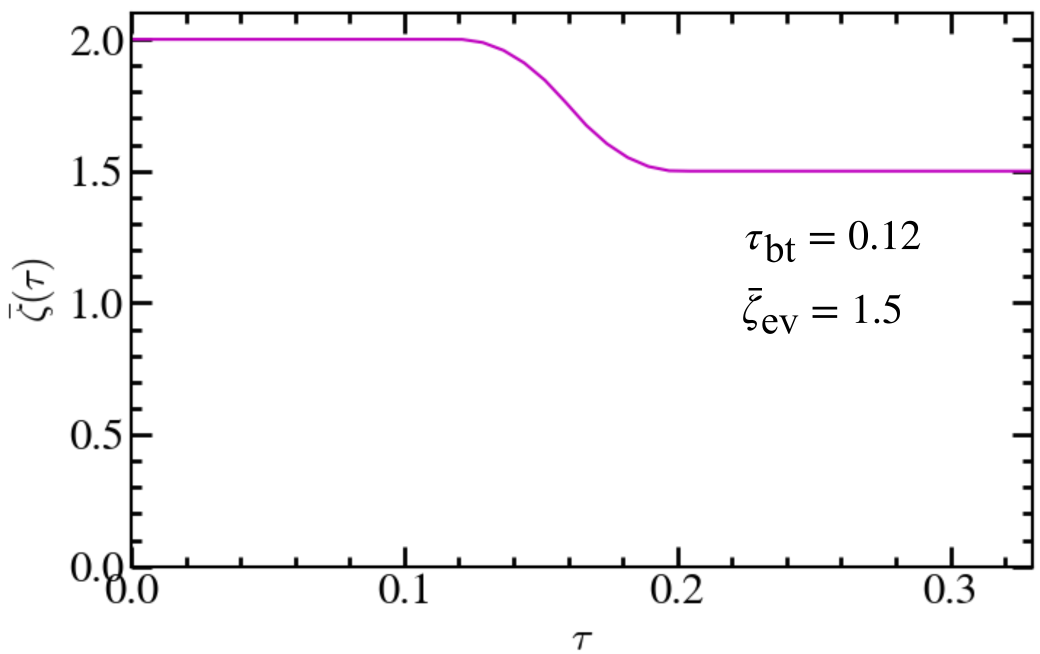
Question: How to obtain an error estimate for deviations from the dijet treatment of the power correction for $\tau > 0.11$?

**From now on:** restrict fit range to [6/Q,0.15] → possible due to new profiles which provide stability of fit

Proceed as follows:

- Use model function for PC

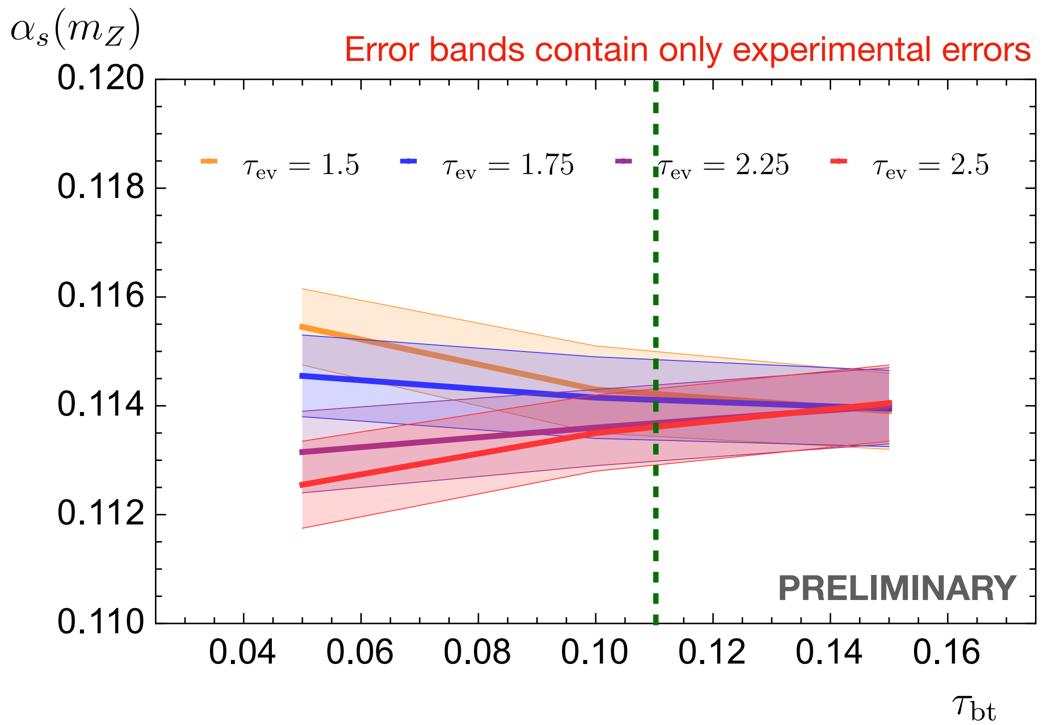
$$\bar{\zeta}(\tau) = \begin{cases} \bar{\zeta}_{\text{iv}} & 0 \leq \tau < \tau_{\text{bt}} \\ \zeta_C(\tau_{\text{iv}}, 0, 0, \tau_{\text{ev}}, \tau_{\text{bt}}, \tau_{\text{et}}, \tau) & \tau_{\text{bt}} \leq \tau < \tau_{\text{et}} \\ \bar{\zeta}_{\text{ev}} & \tau_{\text{et}} \leq \tau < 0.5 \end{cases}$$

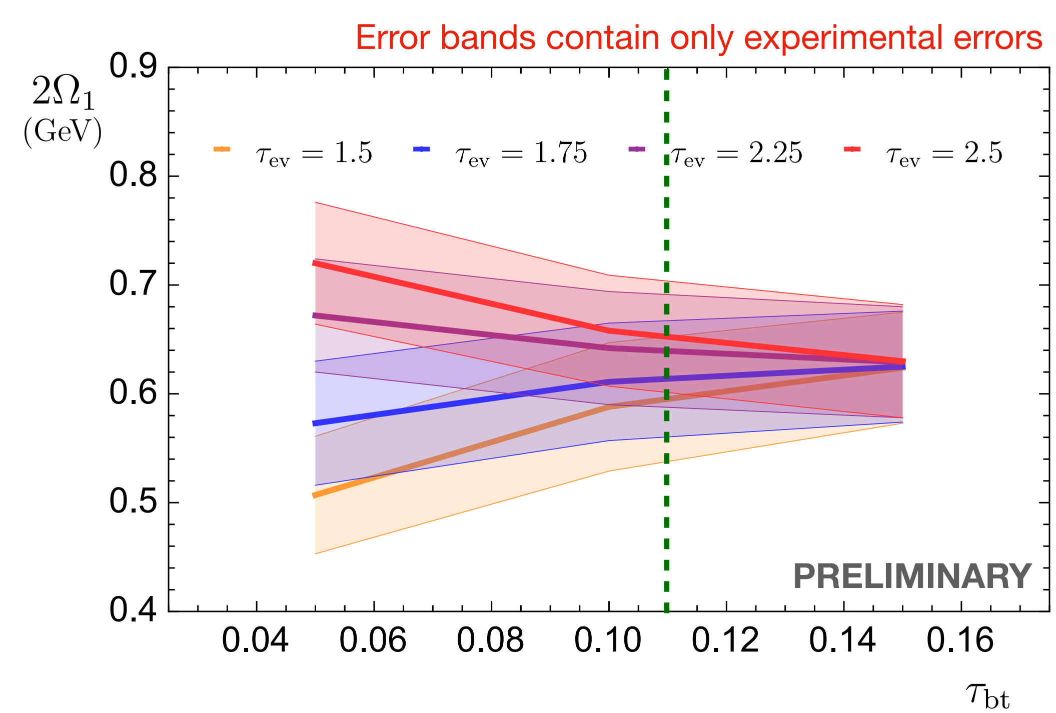



- Vary parameter responsible for deviation away from dijet treatment of PC's, as well as parameter defining the value the model function attains at $\tau = 0.5$

- Look at impact on $\alpha_s$ and $\Omega_1$

# Treatment of Power Corrections




Consider upper part of $\tau_{\rm bt}$ range to make the error estimate

**Uncertainty estimate:** Half of the difference between the maximum and minimum best fit value of $\alpha_s$ obtained when varying $\bar{\zeta}_{\rm ev}$ in the range [1.5, 2.5] for constant $\tau_{\rm bt} = 0.11$

# New default setup

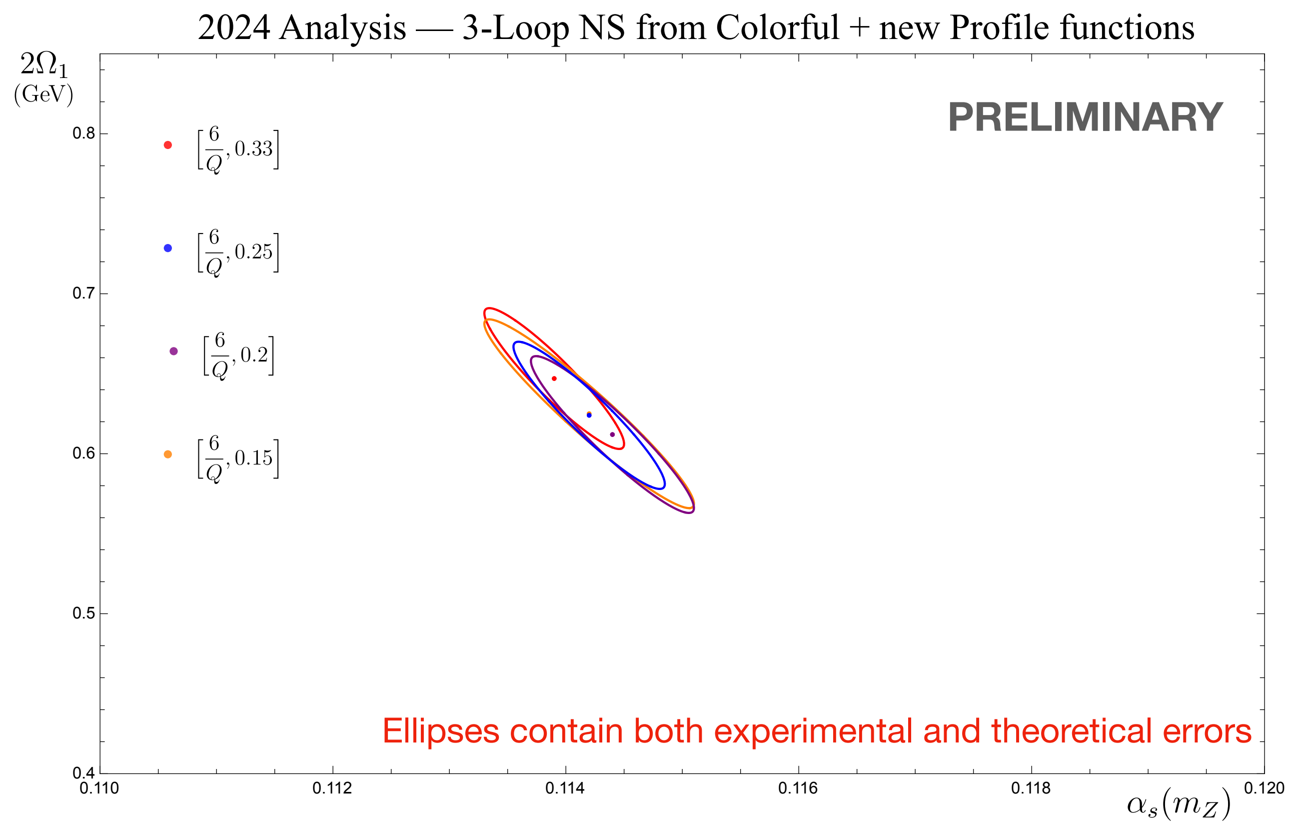
Treatment of power corrections in 3-jet region from first principles so far unknown

Adapt default setup in the following way:

- Include improvements in theory description (updated profiles, improved nonsingular, new perturbative information)
- Since 2015 profiles yield improved stability of the fits → restrict fit region to [6/Q,0.15], which is mostly dijet
- Assume deviations from dijet treatment of power correction can be parametrized by model function

$$\bar{\zeta}(\tau) = \begin{cases} \bar{\zeta}_{\mathrm{iv}} & 0 \leq \tau < \tau_{\mathrm{bt}} \\ \zeta_C(\tau_{\mathrm{iv}}, 0, 0, \tau_{\mathrm{ev}}, \tau_{\mathrm{bt}}, \tau_{\mathrm{et}}, \tau) & \tau_{\mathrm{bt}} \leq \tau < \tau_{\mathrm{et}} \\ \bar{\zeta}_{\mathrm{ev}} & \tau_{\mathrm{et}} \leq \tau < 0.5 \end{cases}$$

# New default setup




From choosing the fit range [6/Q,0.15] we get

$$\alpha_s = 0.1142 \pm 0.0006_{\text{pert}} \pm 0.0009_{\text{exp}} \pm 0.0004_{\text{had}} = 0.1142 \pm 0.0012_{\text{tot}}$$

$$\Omega_1 = 0.313 \pm 0.033_{\text{pert}} \pm 0.030_{\text{exp}} \pm 0.018_{\text{had}} = 0.313 \pm 0.048_{\text{tot}}$$

$$\langle \chi^2 \rangle / \text{dof} = 0.86$$

In the 2010 analysis, the result (ignoring QED effects and considering a massless bottom quark) for the fit range [6/Q,0.33] gave **Abbate, Fickinger, Hoang, Mateu, Stewart '10**

$$\alpha_s = 0.1140 \pm 0.0008, \qquad \Omega_1 = 0.332 \pm 0.045$$

# Historical Outline & Conclusions

- 2010 analysis used profile functions, which gave rise to unstable behavior of fits and furthermore used a wider default fit range, i.e. $\tau \in [6/Q, 0.33]$
- 2015 analysis used profile functions that significantly improve stability of the fit w.r.t. a variation of the fit range for Thrust but took the same default fit range, $\tau \in [6/Q, 0.33]$
- 2024 assessment shows that resummation is important when carrying out fits in this context, improves order by order convergence of the Cross Section and significantly increases the stability of the fit
- 3-jet model of Caola et al. only has marginal effect on outcome of strong coupling
- Due to improved stability of the the fit → able to restrict fit region to [6/Q, 0.15], which is mostly dijet
- Even in new improved default setup, which is mostly dijet and contains error estimate of deviations from dijet treatment of power correction, strong coupling is still small compared to the world average → this is in agreement with Bell et al.