The QCD Coupling at all Scales from Light Front Holography

A. Deur
Jefferson Lab

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Outline:

- α_s as an effective charge.
- HLFQCD (~AdS/QCD in Light-Front form)
- $\alpha_s(M_{z^0})$ from HLFQCD

Work done in collaboration with:

- S. J. Brodsky (SLAC, Stanford U.),
- G. de Téramond (UCR)

Presented on behalf of S. J. Brodsky.

α_s as an effective charge

Prescription: Define effective couplings from an observable's perturbative series truncated to first order in α_s. G. Grunberg, PLB B95 70 (1980); PRD 29 2315 (1984); PRD 40 680(1989).

Proposed for pQCD. Can be extended to non-perturbative QCD.

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Ex: Bjorken sum rule:

$$\int(g^p{}_1-g^n{}_1)dx \triangleq \Gamma_1^{p-n} = \frac{1}{6} g_A\left(1-\frac{\alpha_s}{\pi}-3.58\left(\frac{\alpha_s}{\pi}\right)^2-\ldots\right) + \frac{M^2}{9Q^2}\left[a_2(\alpha_s)+4d_2(\alpha_s)+4f_2(\alpha_s)\right]+\ldots$$

Nucleon axial charge.

pQCD corrections (Leading twist)

Higher Twists: $1/Q^{2n}$ corrections.

Non-perturbative quantities. Express correlations between parton distributions and confinement forces.

$$\Rightarrow \boxed{\Gamma_1^{p-n} \triangleq \frac{1}{6} g_A\left(1-\frac{\alpha_{g1}}{\pi}\right)}$$

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This means that additional short distance effects, and long distance confinement force and parton distribution correlations are now folded into the definition of α_s.

Analogy with the original coupling constant becoming an effective running coupling when short distance quantum loops are folded into its definition.

α_s as an effective charge

The effective charge is then:

- Extractable at any Q^2;
- Free of divergence;
- Renormalization scheme independent.

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However these α_s can be related (Commensurate Scale Relations).

S. J. Brodsky & H. J. Lu, PRD 51 3652 (1995)

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$\Rightarrow$ pQCD retains it predictive power.

Such definition of α_s using a particular process is equivalent to a particular choice of renormalization scheme.

(process dependence) $\Leftrightarrow$ (scheme dependence)

$\alpha_{g1} = \alpha_s$ in the "g_1 scheme".
Relations between g_1 scheme and other schemes are known in pQCD domain, e.g.,$\Lambda_{g_1} = 2.70\Lambda_{\overline{MS}} = 1.48\Lambda_{MOM} = 1.92\Lambda_V$.

α_s as an effective charge

Advantages of extracting α_s from the Bjorken Sum Rule:

- Bjorken sum rule: simple perturbative series.
- Data exist at low, intermediate, and high Q^2.
- Rigorous Sum Rules dictate the behavior of α_{g1} in the unmeasured $Q^2 \rightarrow 0$ and $Q^2 \rightarrow \infty$ regions.

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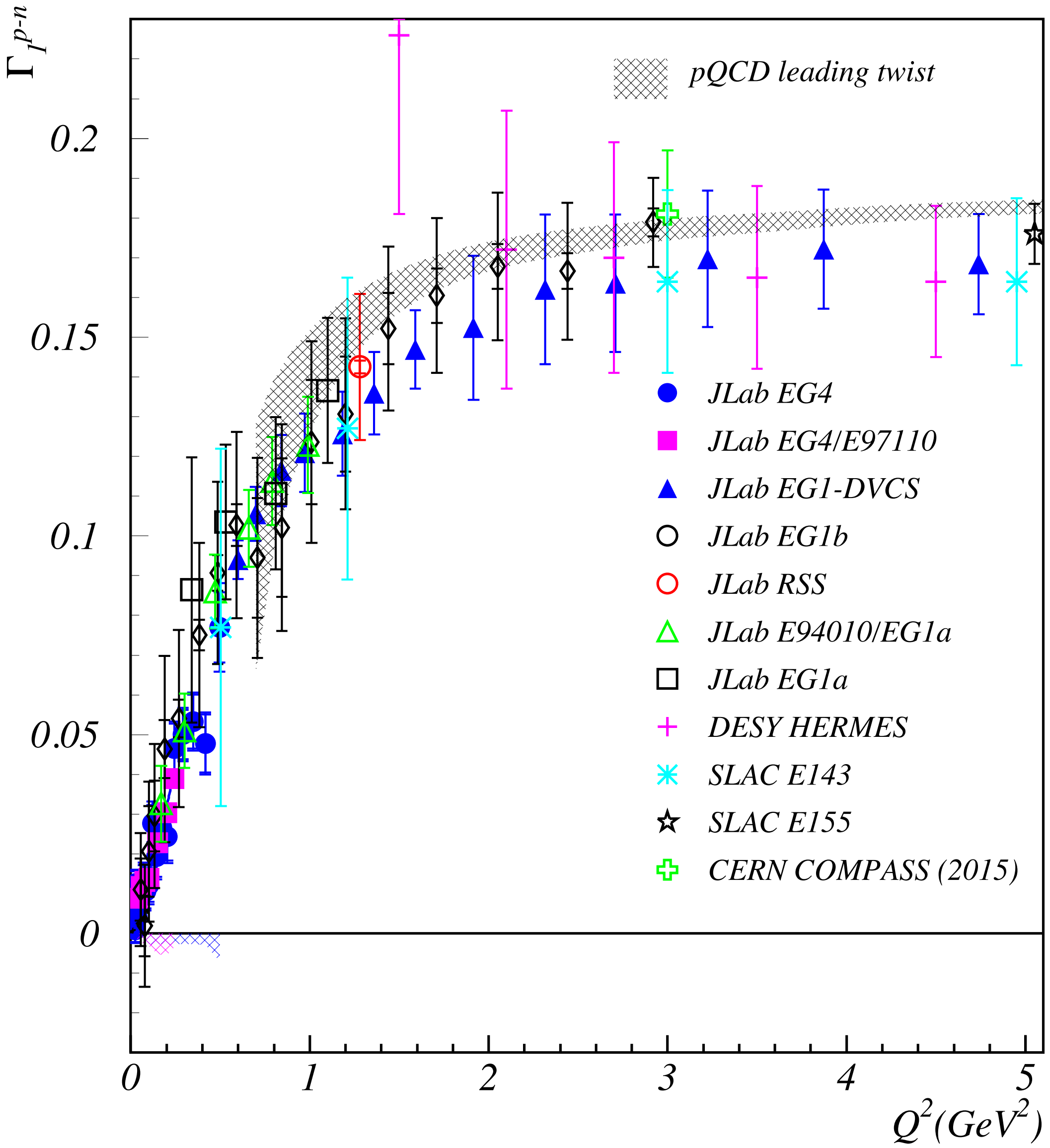
The reason why this is relevant to this workshop is because the value of $\alpha_s(M_{z^0})$ depends on Λ_{QCD}, a non-perturbative quantity,
$\Rightarrow$ To predict it, we need a non-perturbative approach.

α_{g1} from the Bjorken Sum data

Bjorken sum Γ_1^{p-n} measurements



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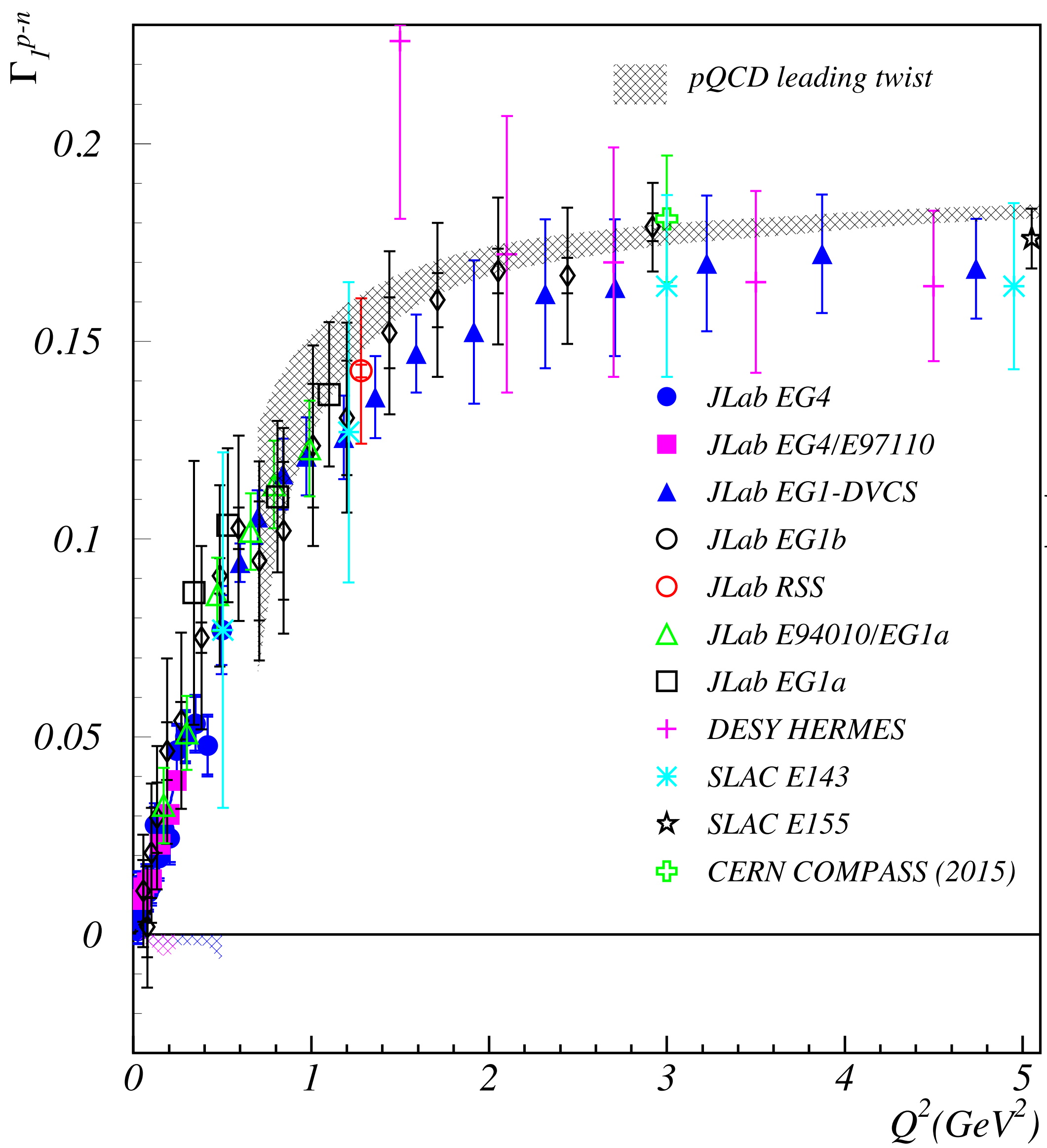
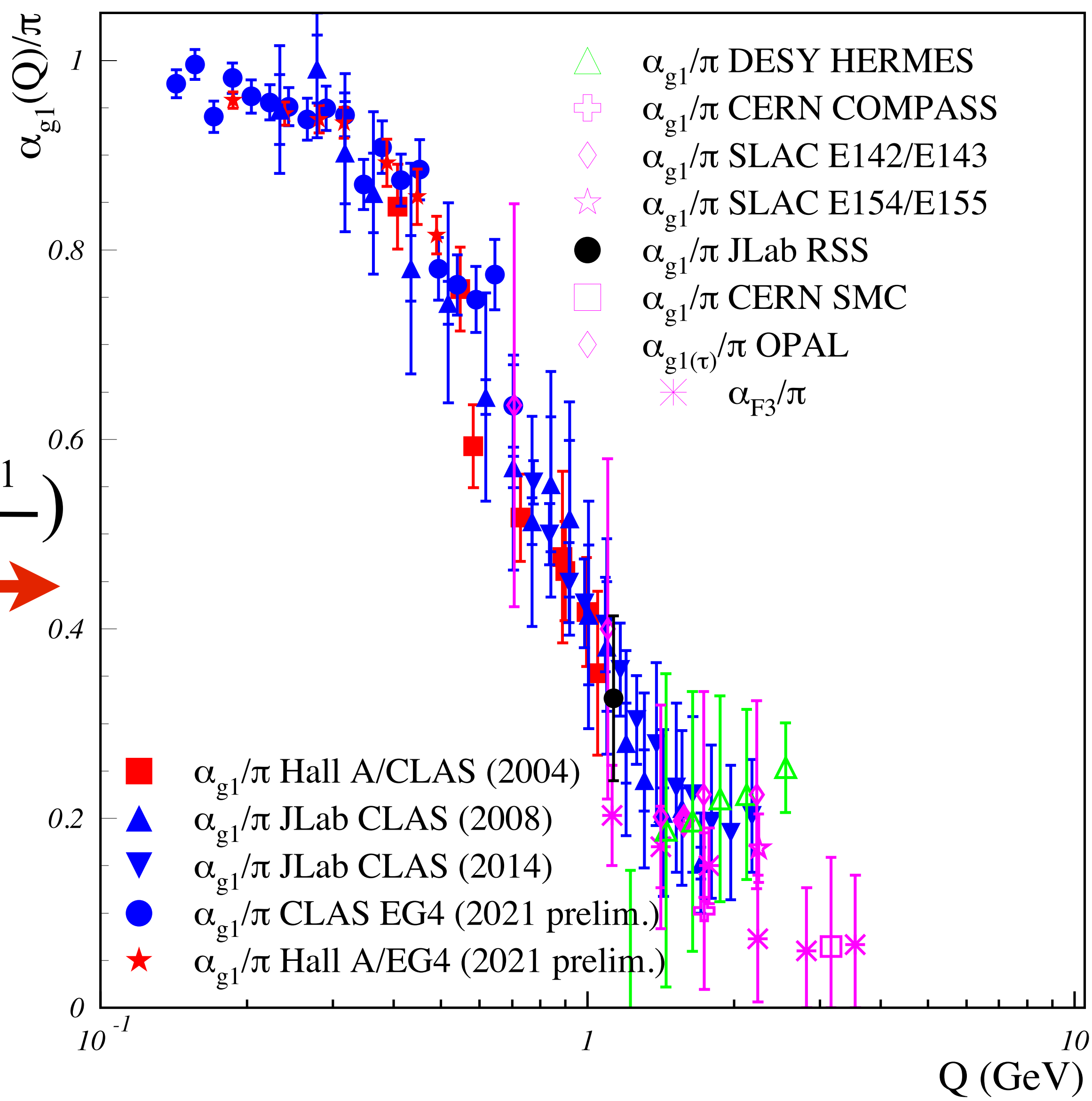
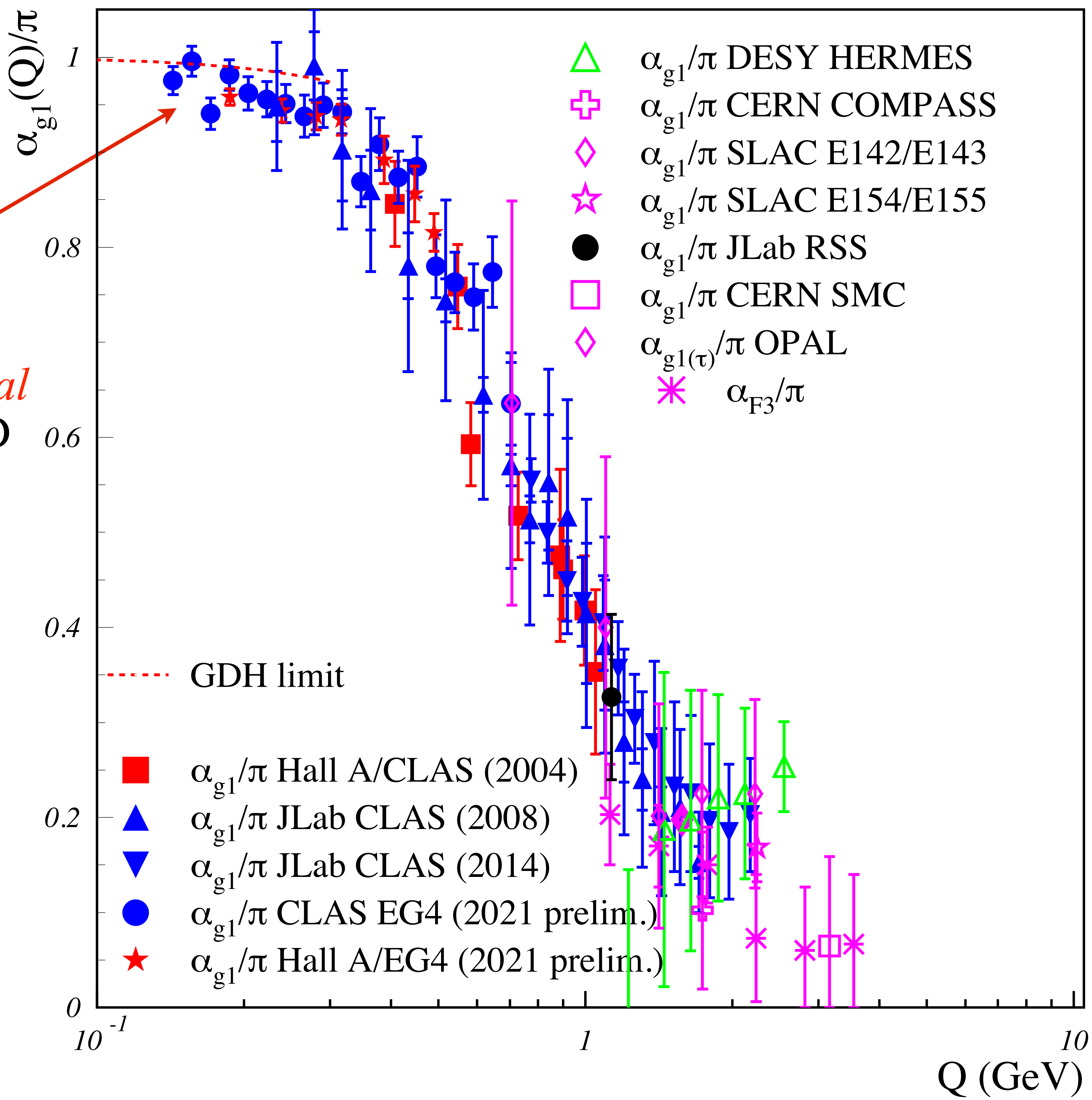


$$\Gamma_1^{p-n} \triangleq \frac{1}{6} g_A \left(1 - \frac{\alpha_{g1}}{\pi}\right)$$



Low Q^2 limit

Experimental evidence of nearly *conformal behavior* (i.e., no Q^2-dependence) of QCD at low Q^2.



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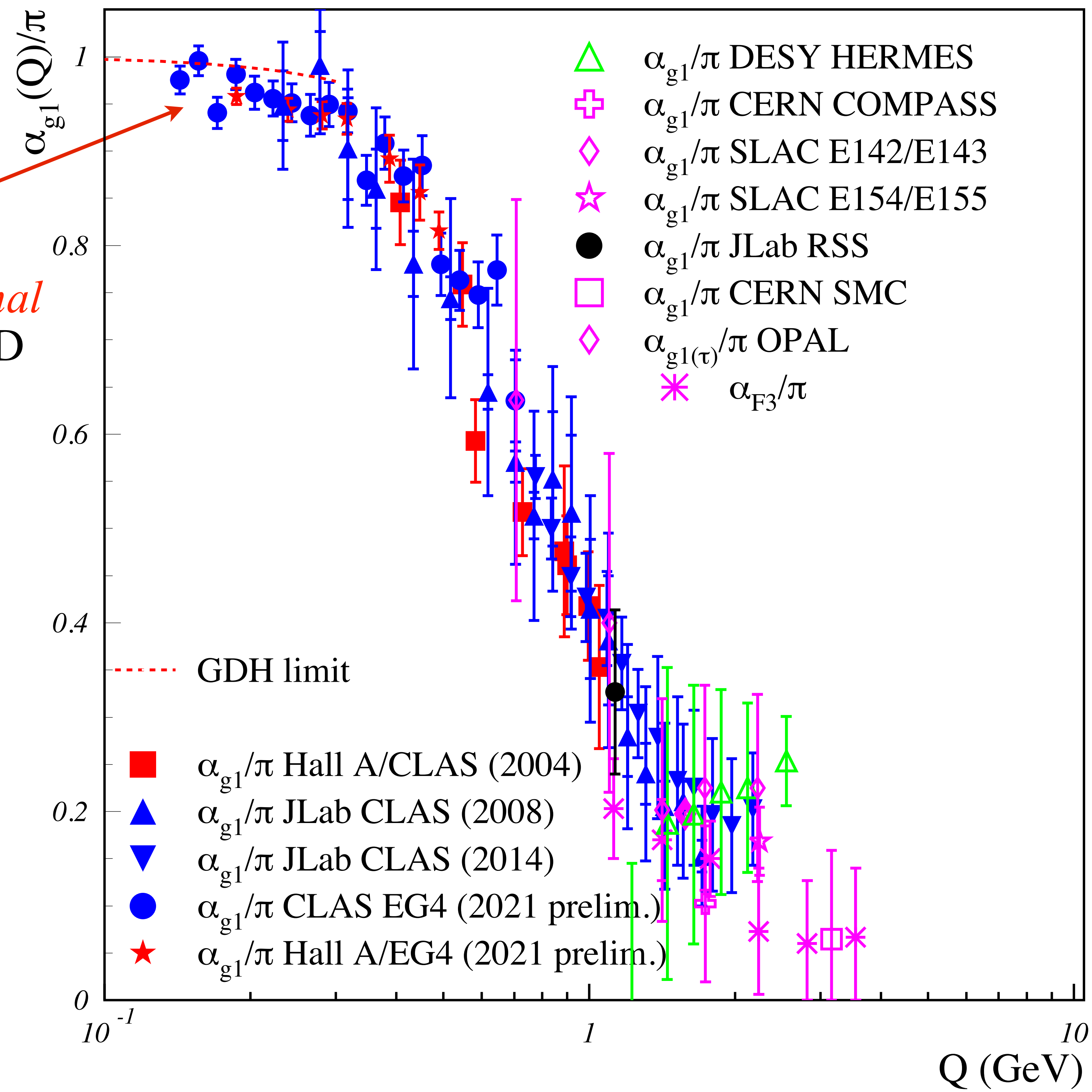

⇒ One can use tools associated with conformal theories to study non-perturbative QCD.

In particular, we can use the AdS/CFT correspondence, with QCD being the Conformal Field Theory.

One incarnation of AdS/QCD is HLFQCD, which we use here.

The Holographic Light-Front QCD approximation (HLFQCD)

• Light-front QCD: Rigorous and exact formulation of non-perturbative QCD. Yields a relativistic Schrödinger-like equation for hadrons. Confining potential calculable in principle but not tractable in 3+1 dimensions.

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Harmonic oscillator on light front $\Rightarrow$ in AdS space, $ds^2 \rightarrow \exp(\kappa^2 z^2) ds^2$
z is the 5th dimension of AdS space. z^2 is the scale at which the hadron is probed, i.e. $1/Q^2$.
κ is the universal scale factor of HLFQCD.

α_s from HLFQCD

Perturbative QCD:

pQCD effective coupling $\alpha_s(Q^2)$: small distance QCD effect are folded into the definition of the coupling constant α_s.

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Non-perturbative QCD:

Likewise for $\alpha_{g1}(Q^2)$ at long distance, confinement forces and parton correlations are folded into the definition of $\alpha_s(Q^2)$: *effective charge.*

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AdS Action: $S \propto \int d^5x \sqrt{g} \frac{1}{g^2_5} F^2$, with F the gauge field and g_5 the coupling

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Confinement potential

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Transforming to momentum space: $\alpha_s^{HLF}(Q^2)=\alpha_s^{HLF}(Q^2=0)e^{(-Q^2/4\kappa^2)}$

Brodsky, de Téramond, Deur.
Phys. Rev. D 81, 096010 (2010)

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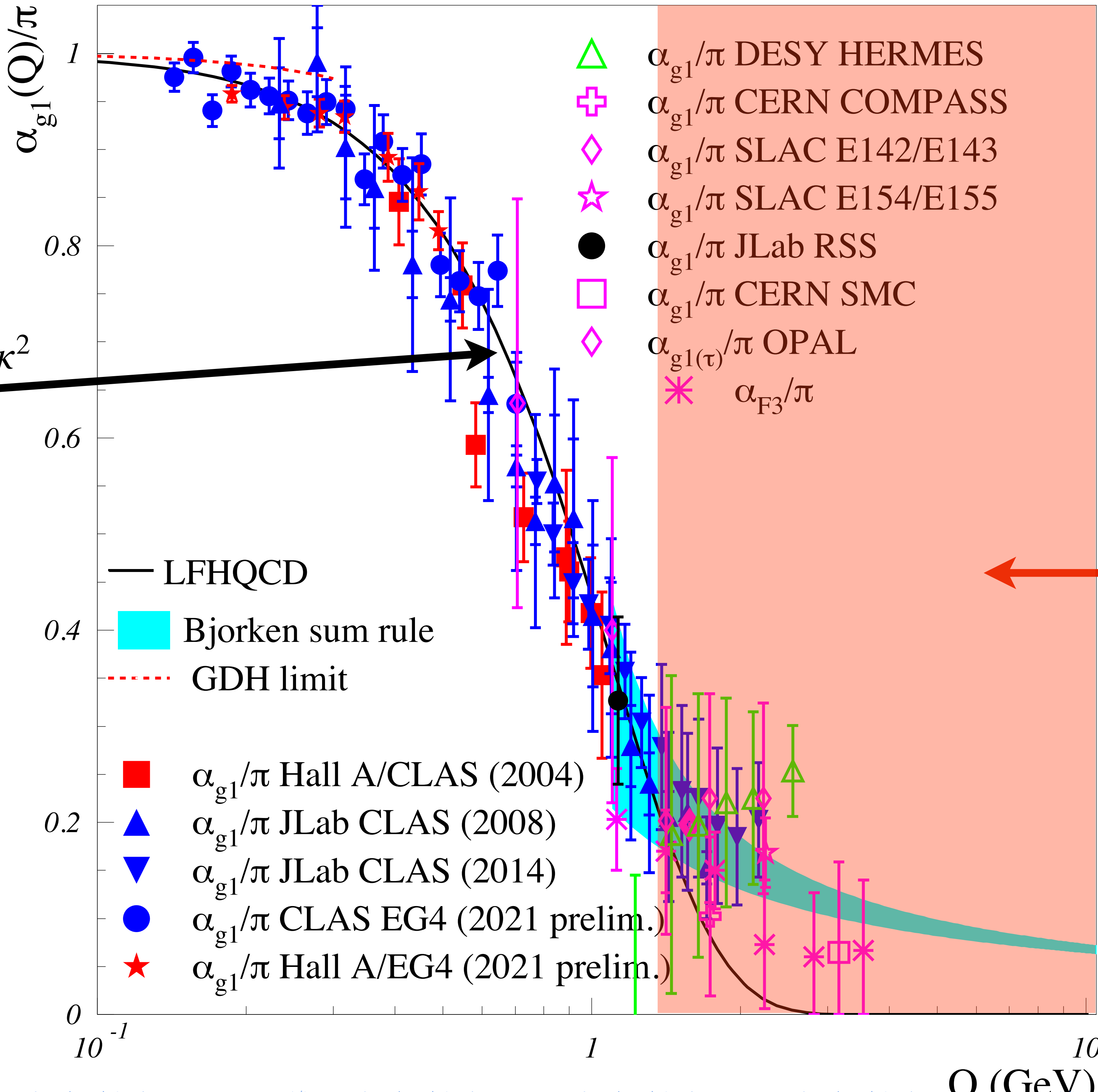
$\alpha_s^{HLF}(0)\equiv\pi$: $\alpha_s^{HLF}(Q^2)$ in the g_1 scheme.

α_s and HLFQCD: Comparison with data

$$\frac{\alpha_s^{LFH}(Q^2)}{\pi} = e^{-Q^2/4\kappa^2}$$

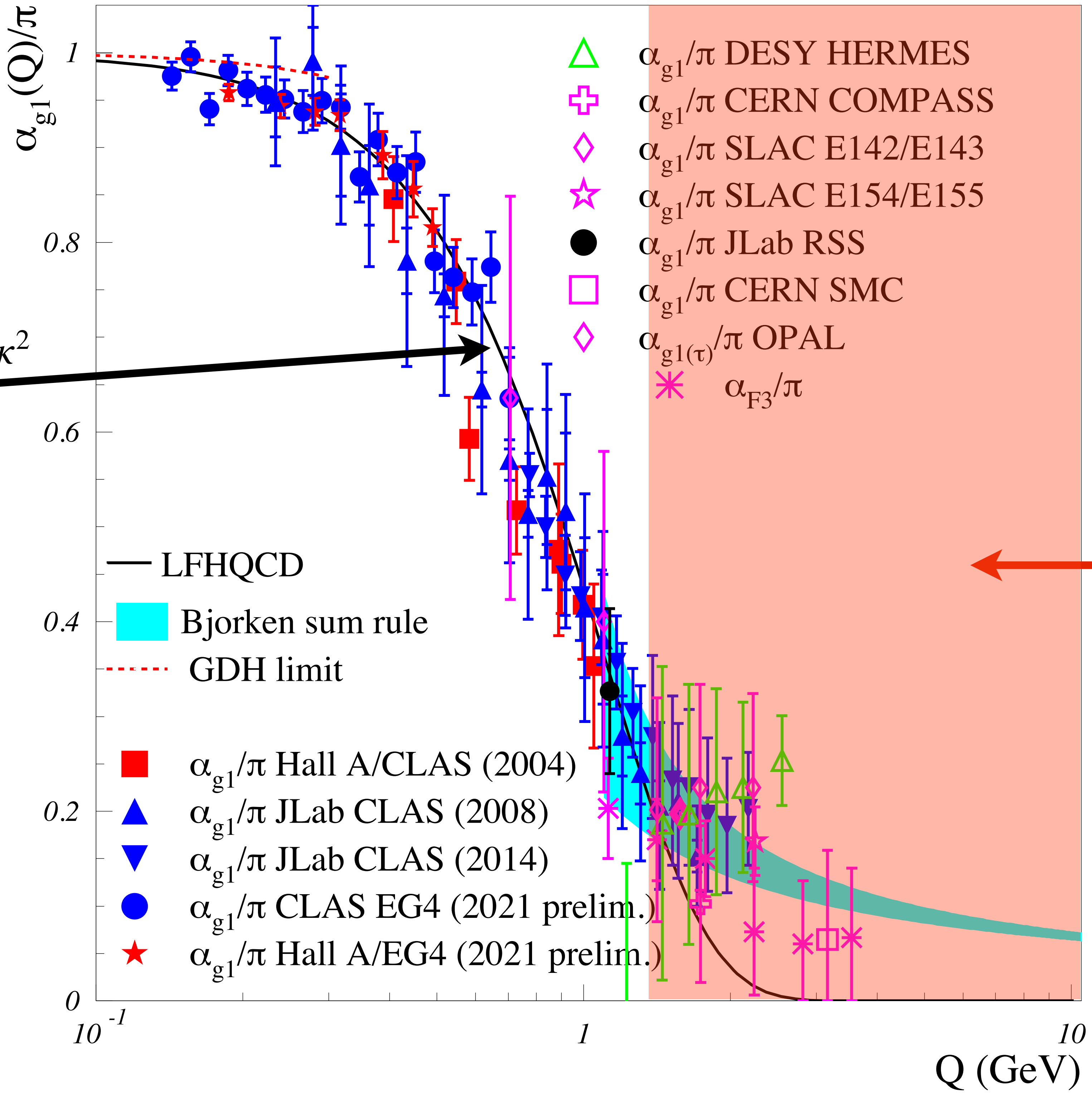


HLFQCD not valid

⇒ **Prediction for α_s at long distances. No free parameters ($\kappa = M_\rho/\sqrt{2}$).**
Agrees very well with the α_s extracted from JLab's Bjorken sum data.

Prediction of Λ_{QCD}

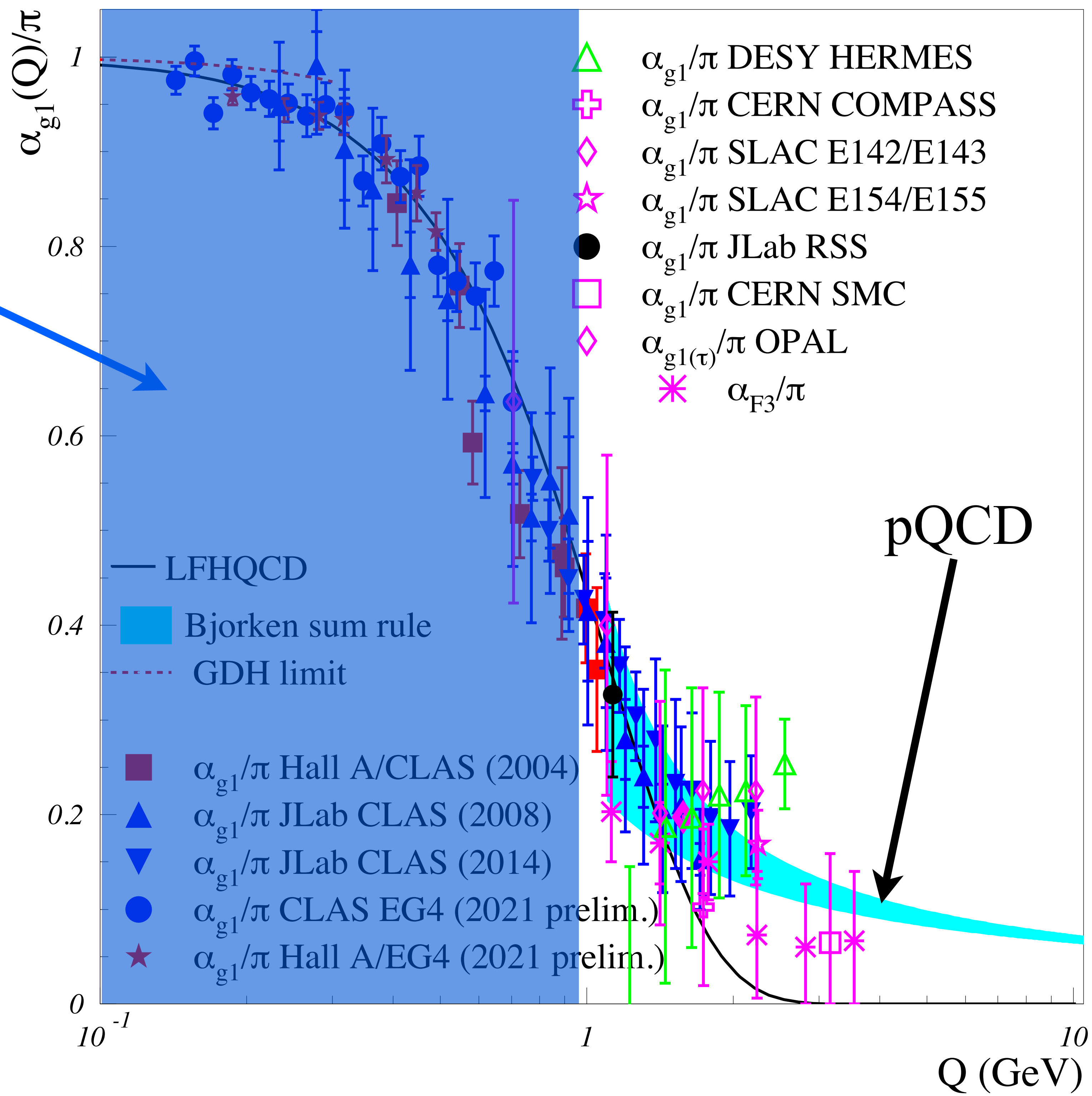
$$\frac{\alpha_s^{LFH}(Q^2)}{\pi} = e^{-Q^2/4\kappa^2}$$



HLFQCD
not valid

Prediction of Λ_{QCD}

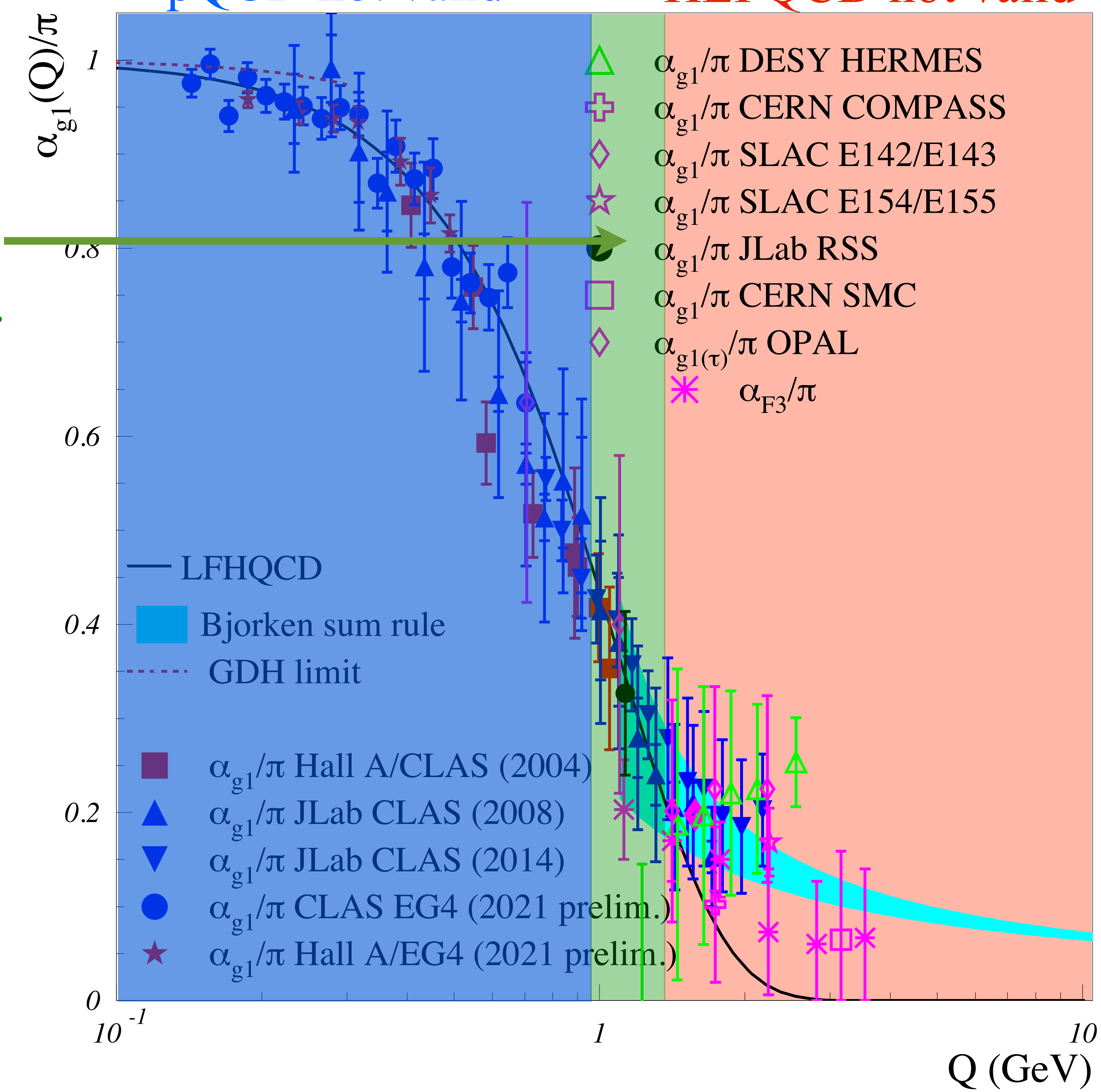
pQCD not valid



Jefferson Lab
Thomas Jefferson National Accelerator Facility
Exploring the Nature of Matter

Prediction of Λ_{QCD}

pQCD and HLFQCD both provide a good description of α_{g1} (i.e. the Bjorken sum)



Match HLFQCD and pQCD expressions of α_{g1} and its β-function:

⇒ Relate fundamental HLFQCD parameter κ to fundamental QCD parameter Λ_{QCD}.

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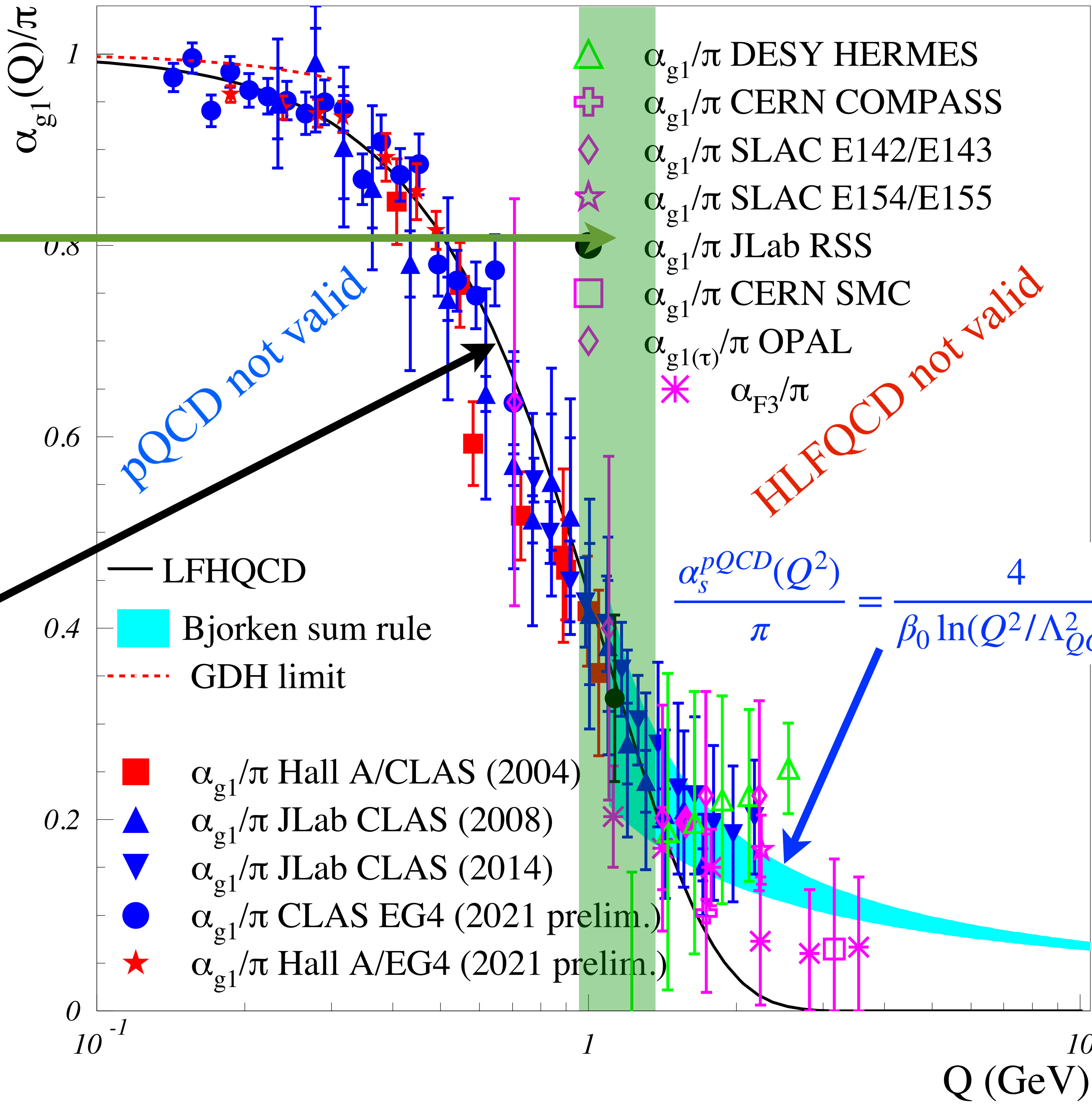

$$\frac{\alpha_s^{pQCD}(Q^2)}{\pi} = \frac{4}{\beta_0 \ln(Q^2/\Lambda_{QCD}^2)}(1+\text{higher orders})$$

$\kappa = \Lambda_{QCD}\, e^{(a+1)}(a/2)^{1/2}$ At LO

$a=4(\sqrt{\ln(2)^2-1+\beta_0/4}-\ln(2))/\beta_0$

$\kappa = 1.607\Lambda_{\overline{MS}}$ At N³LO

Deur, Brodsky, de Téramond, PLB 750, 528 (2015)

Predictions of the hadronic mass spectrum

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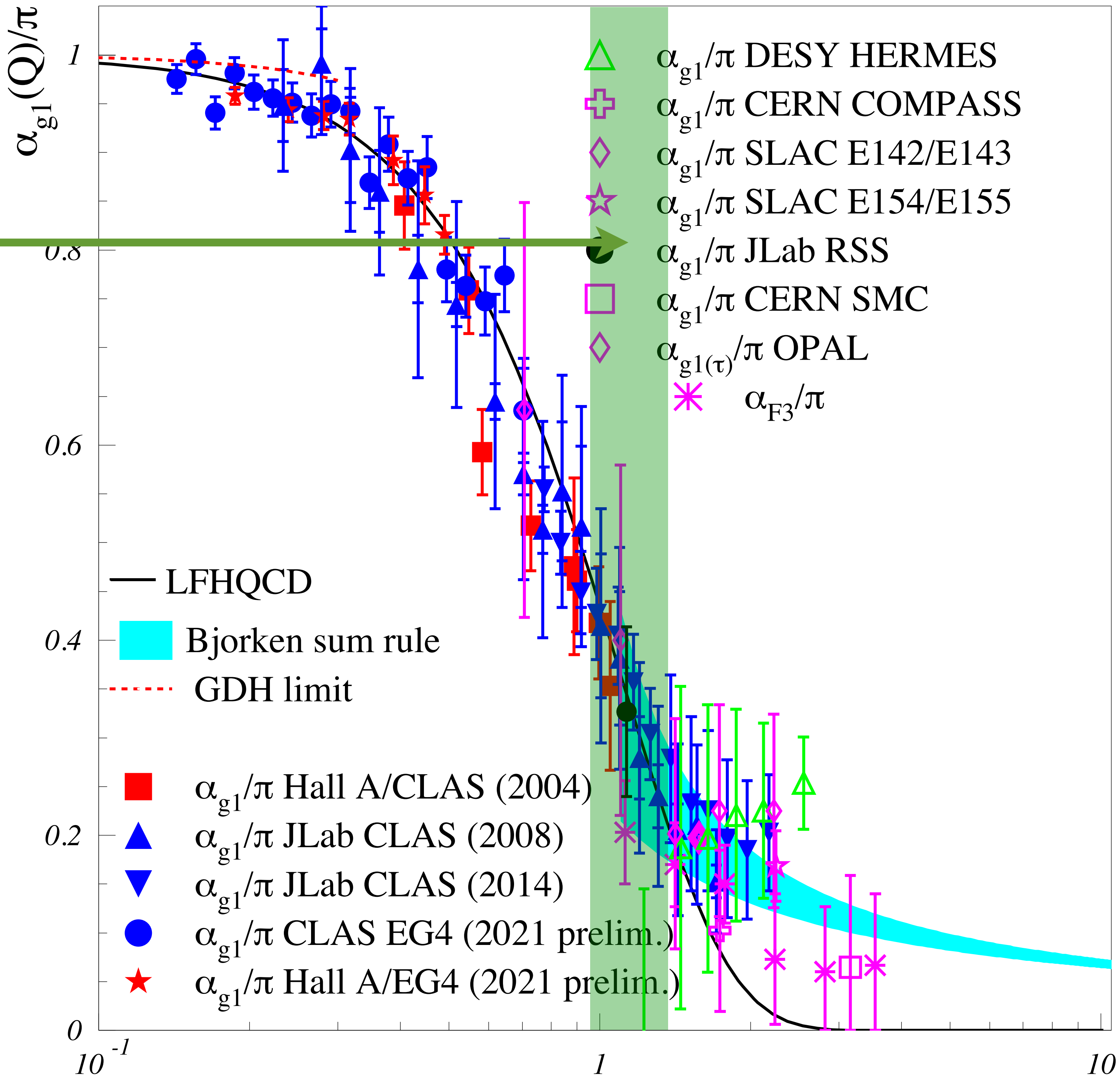

$\Rightarrow$ One can use κ, known from hadron masses or form factors, to predict Λ_{QCD}.

$\Lambda_{QCD}^{AdS} = 0.339(19)$ GeV

AD, Brodsky, de Téramond, J.Phys.G 44,10, 105005 (2017)

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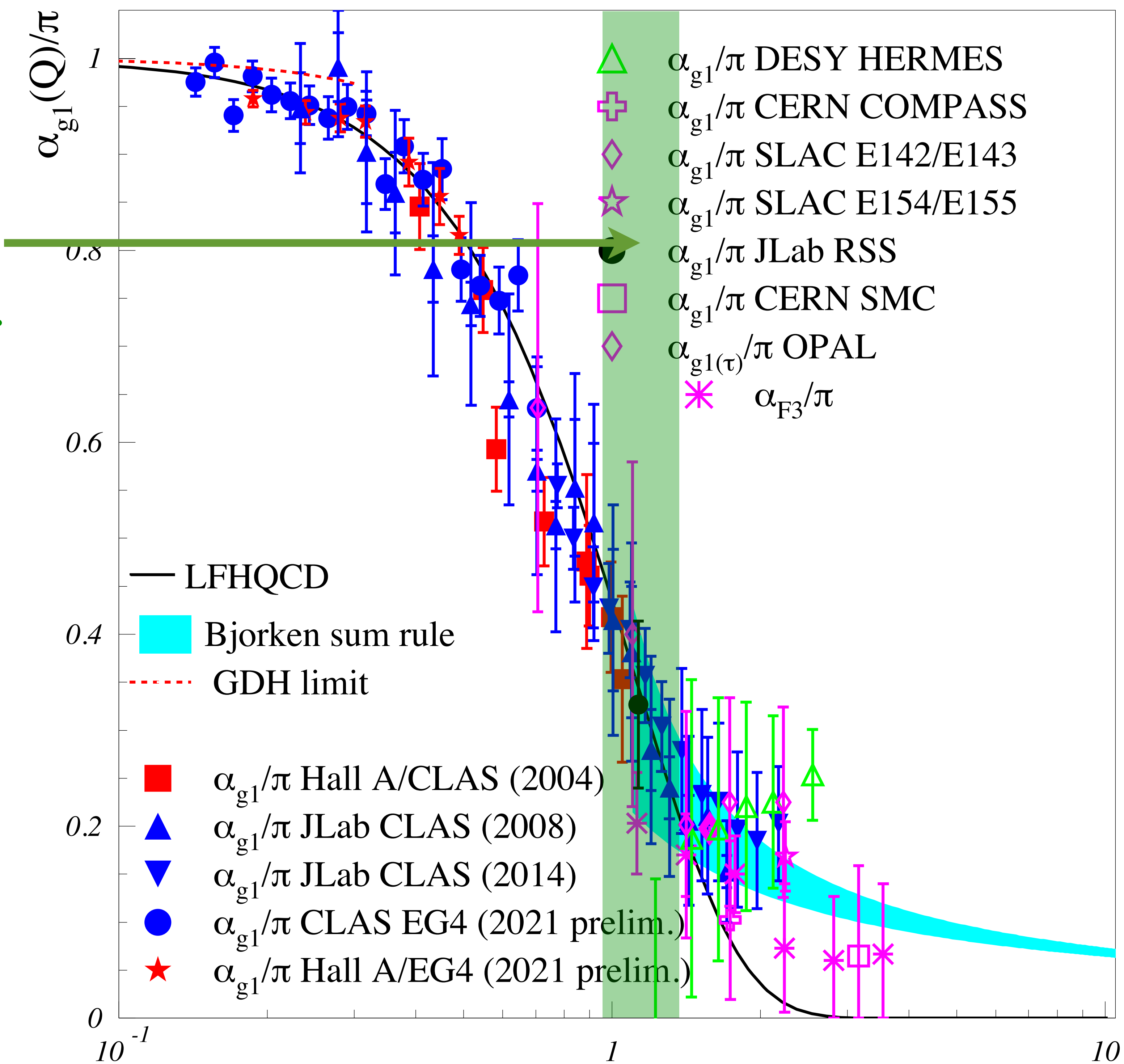

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Compare well with PDG: $\Lambda_{QCD}^{PDG} = 0.332(19)$ GeV

PDG, 2015

On-going work

HLFQCD+Matching procedure yields compelling results but it has limitations:

- Assume overlap of validity domains of pQCD & HLFQCD (verified phenomenologically);
- Approximate matching at a single point Q_0, (transition between pQCD & Strong QCD occurs over a range);
- Neglect Higher-Twists (For the Bjorken SR not a bad approximation if $Q_0 \simeq$ GeV);
- Matches only $\alpha_s(Q_0)$ and $\frac{d\alpha_s(Q)}{dQ}|_{Q_0}$;
- Not entirely satisfactory to have two distinct analytical expressions of $\alpha_s(Q)$.

$\Longrightarrow$ Unified description of $\alpha_s(Q)$ in the nonperturbative and perturbative transition domain

S. J. Brosky, G. F. de Téramond, A. Deur, H. G. Dosch, T. Liu, A. Paul and R. S. Sufian (HLFHS Collaboration)

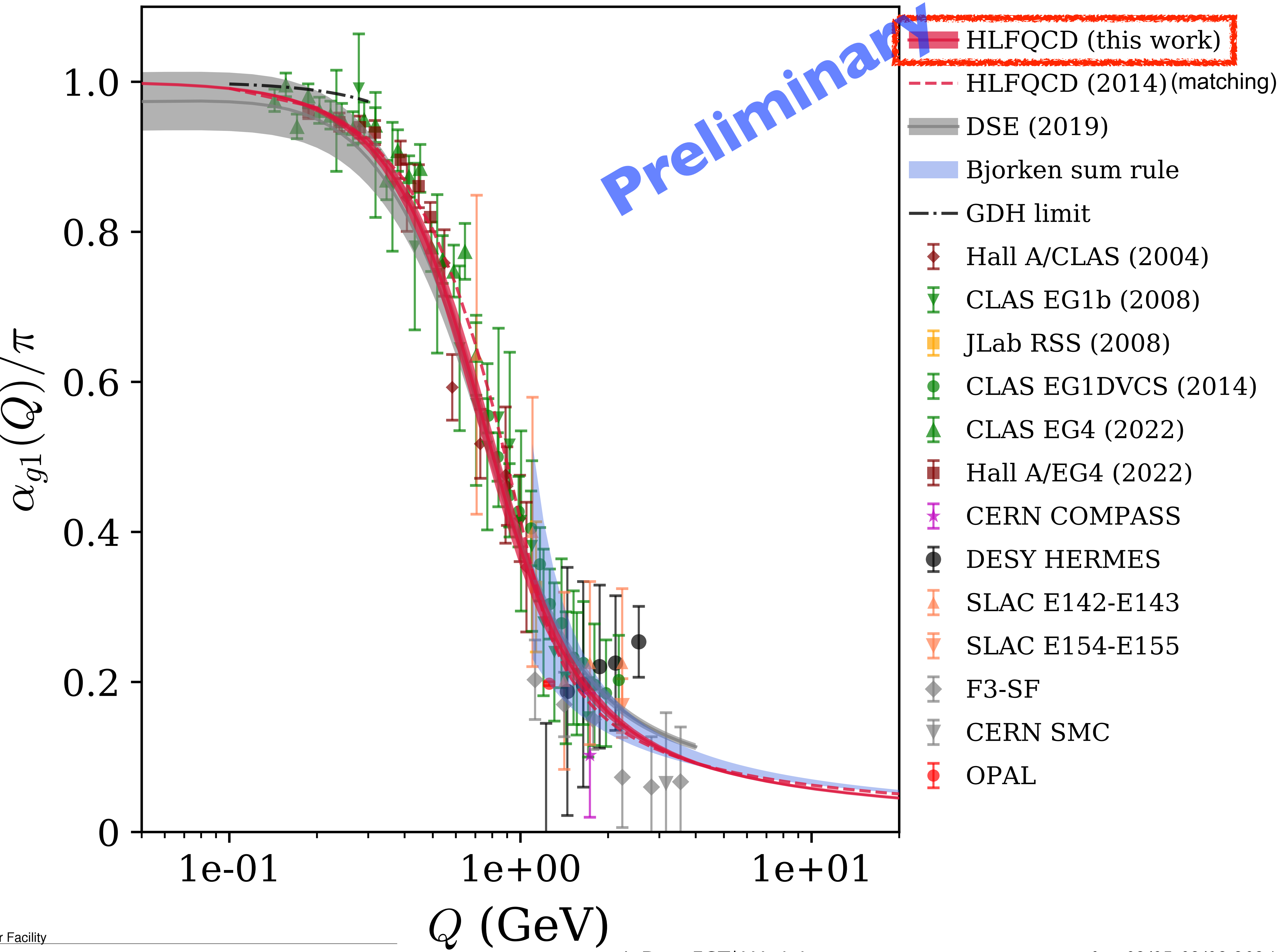
On-going work

Extension of HLFQCD description of α_s into the transition domain where pQCD effects become important.

- New approach based on the underlying conformality of QCD and analytic continuation.
- Single analytic expression provides a continuous transition for the β function and any higher derivative.
- Incorporates a confinement mechanism. Suppress the unphysical Landau pole.
- The formalism is valid in the nonperturbative and transition domains: Not the full perturbation theory (no quark mass thresholds in conformal limit)
- Flow of singularities in the complex plane leads to specific relation between the κ and $\Lambda_{QCD.}$ $\Rightarrow$ determination of α_s

On-going work

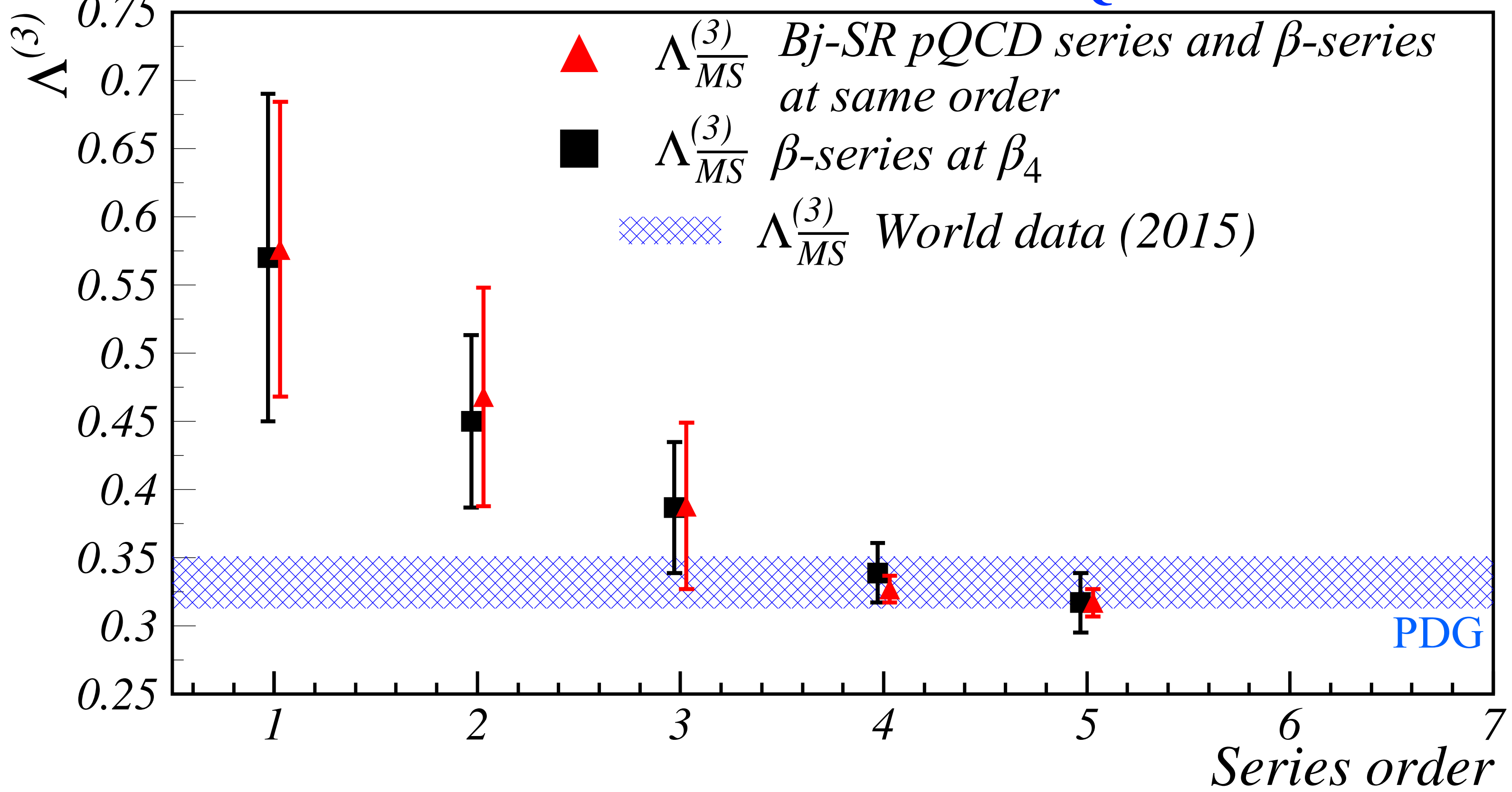
Resulting unified coupling model describes JLab data in the nonperturbative and the perturbative transition domain:



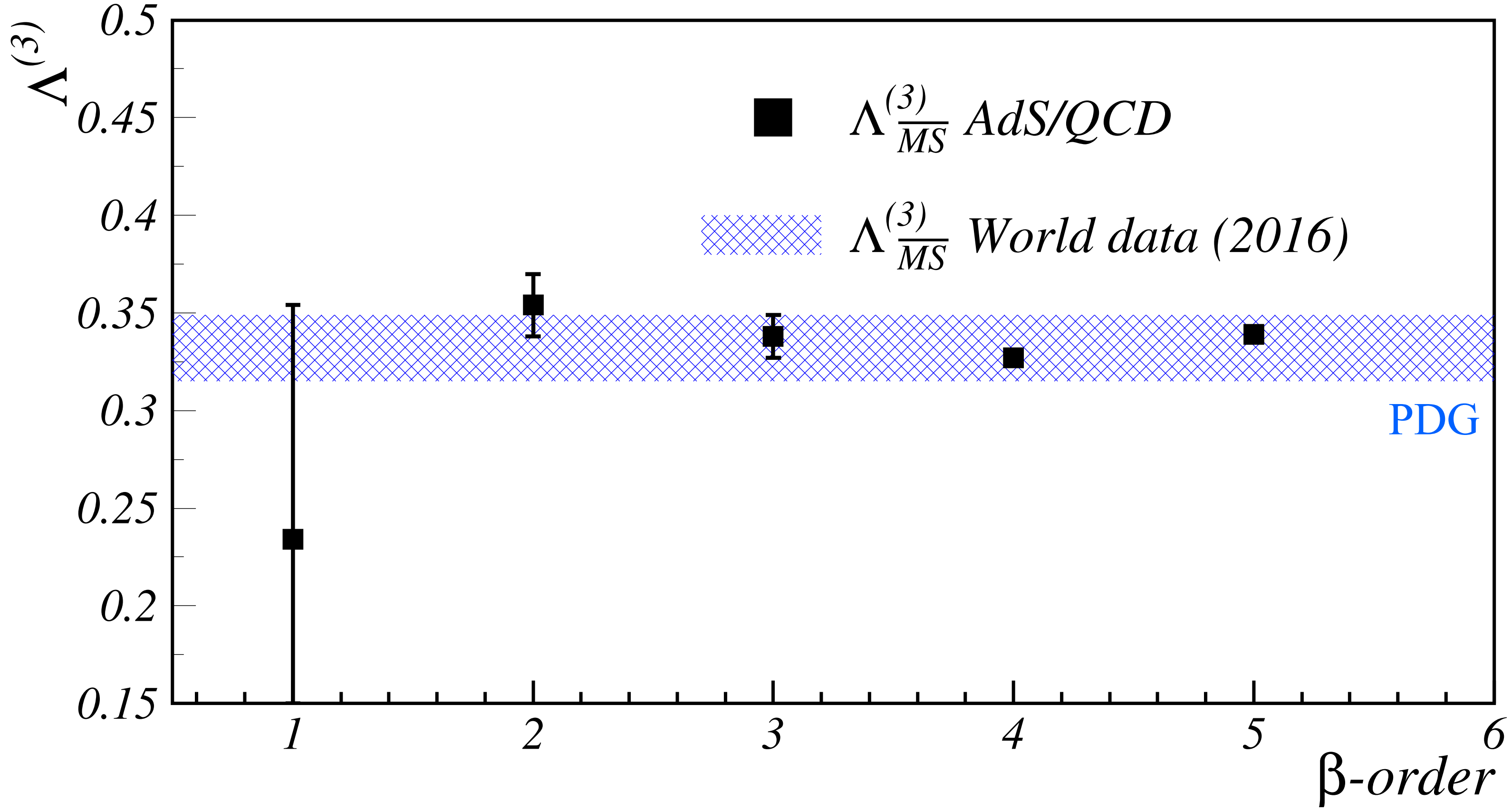
Summary

- Bjorken Sum Rule is advantageous to define an effective charge α_{g1}.
- Data and sum rules allow us to know α_{g1} at all Q^2.
- α_{g1} plateaus at low Q^2 $\Rightarrow$ Application of AdS/CFT to non-perturbative QCD.
- α_s obtained with HLFQCD.
  - Its form is imposed by QCD's basic (approximate) symmetries: either conformal symmetry of QCD Lagrangian (mass scale emerging in QCD's Action: dAFF mechanism), or chiral symmetry (massless pion).
  - No free parameters (uses only one parameter, κ, known from very different phenomenology).
  - Agrees remarkably with experimental data on α_{g1} and with Schwinger-Dyson Eqs. prediction.
  - Matching between HLFQCD and pQCD: high precision prediction of Λ_{QCD}.
  - On-going work on obtaining unique α_{g1} expression in both Strong QCD and transition domain.

Prediction of Λ_{QCD} from hadronic observable



Convergence with Bjorken sum rule pQCD approximant order.



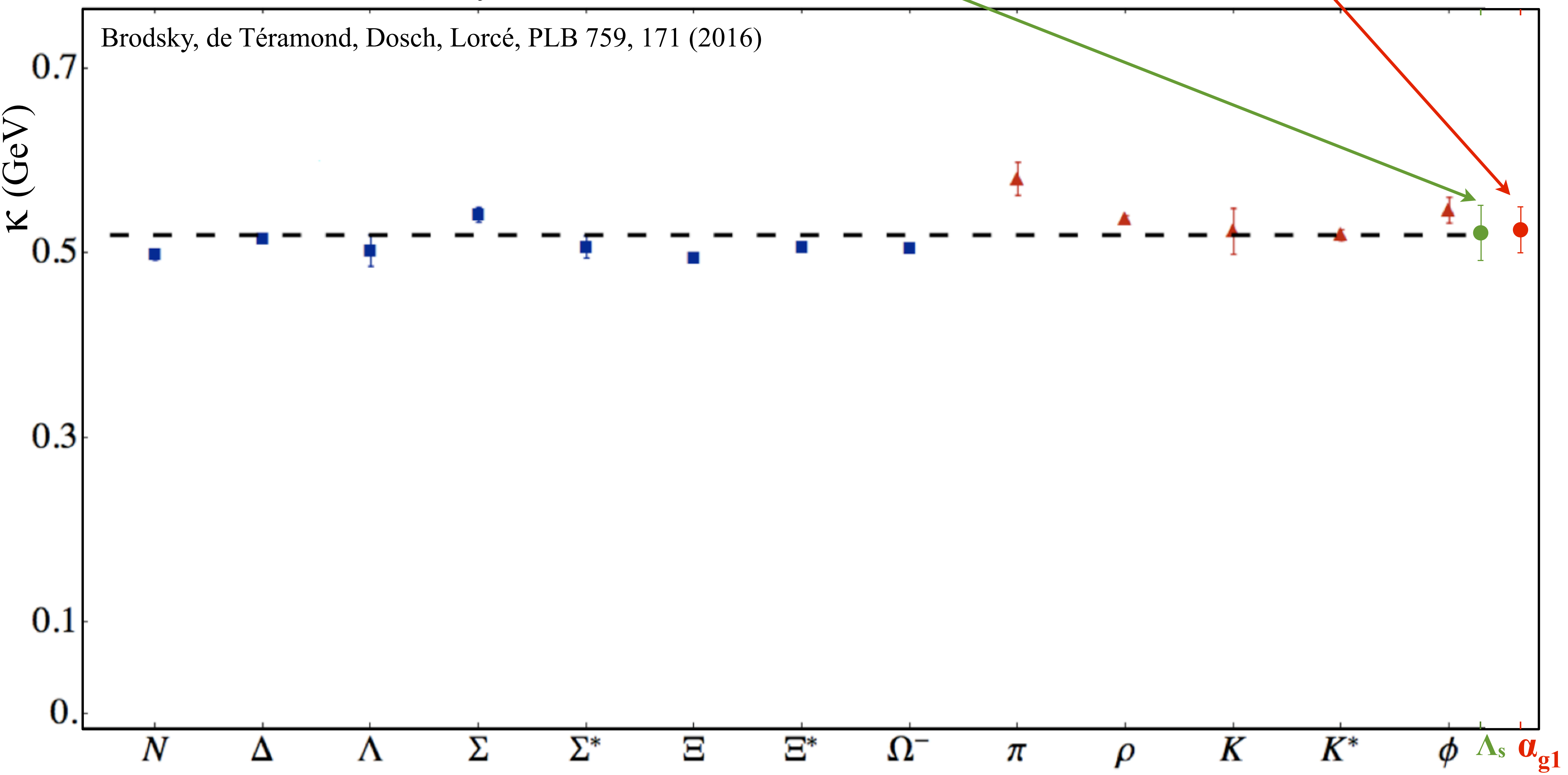
Convergence with β-series order.

α_s and HLFQCD: Comparison with data

One can also fit the $\alpha_{g1}(Q^2)$ data to get κ: κ=0.513±0.025 GeV

Or use the relation between κ and Λ_{QCD} (latter slides):

PDG value for Λ_{QCD} yields κ=0.512±0.030 GeV



Agree with other determinations of κ. ~10% universality of κ confirms that HLFQCD is a good model for QCD. (Also, nucleon or pion Form Factors provide compatible κ.)