

# Determination of $\alpha_s$ from the Z-boson transverse-momentum distribution at the Tevatron

**Giancarlo Ferrera**  
Milan University & INFN, Milan

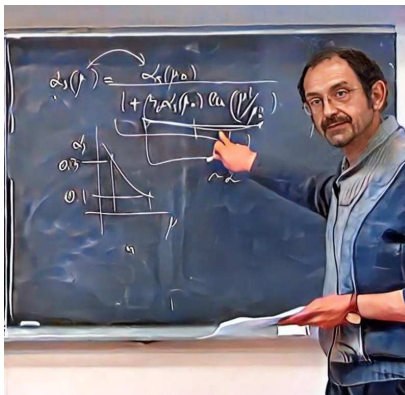


Based on:

**Stefano Camarda, G. F., Matthias Schott**  
Eur.Phys.J.C 84 (2024), e-Print: 2203.05394

**Workshop  $\alpha_s$ -2024**  
**ECT\* – Trento – 7/2/2024**

## Stefano Catani (1958-2024)



*Wonderful person, outstanding physicist*

# The idea: $\alpha_S$ from semi-inclusive processes

## QCD COHERENT BRANCHING AND SEMI-INCLUSIVE PROCESSES AT LARGE $x^*$

S. CATANI\*\* and B.R. WEBBER

*Cambridge Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 0H3, UK*

G. MARCHESINI

*Dipartimento di Fisica, Università di Parma, INFN, Gruppo Collegato di Parma, Italy*

Received 22 June 1990

$$\alpha_s^{(\text{MC})} = \alpha_s^{(\overline{\text{MS}})} \left( 1 + K \frac{\alpha_s^{(\overline{\text{MS}})}}{2\pi} \right),$$

$$\begin{aligned} \Lambda_{\text{MC}} &= \Lambda_{\overline{\text{MS}}} \exp(K/4\pi\beta_0) \\ &\approx 1.569 \Lambda_{\overline{\text{MS}}} \quad \text{for } N_f = 5. \end{aligned}$$

*In this paper we have studied [...] the next-to-leading logarithmic terms in semi-inclusive hard processes such as the DIS and DY processes at large  $x$ . Since the Monte Carlo algorithm with these improvements is accurate to next-to-leading order in the large- $x$  region, it can be used to determine the fundamental QCD scale  $\Lambda_{\overline{\text{MS}}}$*

# The idea: $\alpha_S$ from semi-inclusive processes

## Advantages:

- **higher sensitivity to  $\alpha_S$**  w.r.t. *inclusive* observables;
- calculable at **higher theoretical accuracy** w.r.t. *exclusive* observables.

## Challenges:

- sensitivity to **infrared (Sudakov) logs**;
- sensitivity **non perturbative QCD** effects.

Classical semi-inclusive obs. at hadron colliders:  
**high invariant-mass Drell–Yan lepton pair  
at small transverse-momentum ( $q_T$ ).**

# $\alpha_S$ from Z-boson $q_T$ distribution

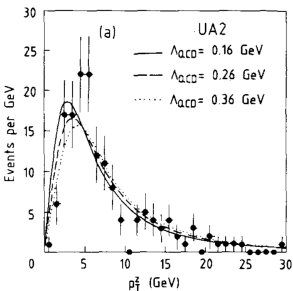
→ this talk

→ M. Schott talk

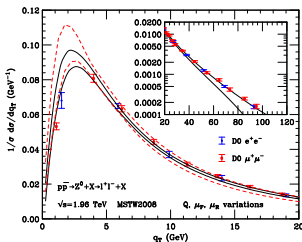
Sp $\bar{p}$ S ( $\sqrt{s} = 0.63$  TeV)

Tevatron ( $\sqrt{s} = 1.96$  TeV)

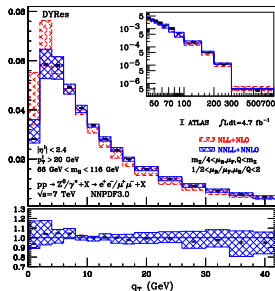
LHC ( $\sqrt{s} = 7 - 8$  TeV)



[UA2 Coll. ('92)]  
 compared with  
 [Altarelli et al. ('84)]



[D0 Coll. ('08, '10)]  
 compared with  
 [Catani et al. ('10)]

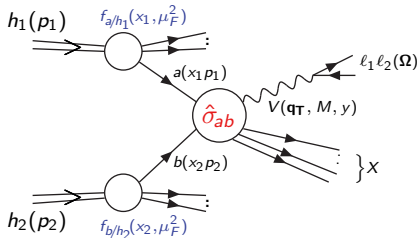


[ATLAS Coll. ('14)]  
 compared with  
 [Catani et al. ('15)]

# Drell-Yan $q_T$ distribution

$$h_1(p_1) + h_2(p_2) \rightarrow \mathbf{V} + \mathbf{X} \rightarrow \ell_1 + \ell_2 + \mathbf{X}$$

where  $V = Z^0/\gamma^*, W^\pm$



QCD factorization formula:

$$\frac{d\sigma}{dq_T^2} = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \frac{d\hat{\sigma}_{ab}}{dq_T^2}(\alpha_S(\mu_R^2), \mu_R^2, \mu_F^2).$$

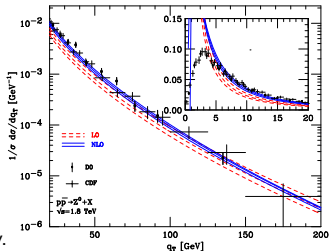
Fixed-order perturbative expansion reliable

only for  $q_T \sim M$ . When  $q_T \ll M$ :

$$\int_0^{q_T^2} d\bar{q}_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{d\bar{q}_T^2} \sim 1 + \alpha_S \left[ c_{12} L_{q_T}^2 + c_{11} L_{q_T} + \dots \right] + \alpha_S^2 \left[ c_{24} L_{q_T}^4 + \dots + c_{21} L_{q_T} + \dots \right] + \mathcal{O}(\alpha_S^3)$$

with  $\alpha_S^n L_{q_T}^m \equiv \alpha_S^n \log^m(M^2/q_T^2) \gtrsim 1$ .

Resummation of logarithmic corrections mandatory.



# $q_T$ resummation in QCD

[Catani, de Florian, Grazzini ('01)]  
[Bozzi, Catani, de Florian, Grazzini ('03, '06)]

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

In the impact parameter space:  $q_T \ll M \Leftrightarrow Mb \gg 1$ ,  $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} \mathcal{W}(b, M),$$

In the Mellin space (with respect to  $z = M^2/\hat{s}$ ) we have:

$$\mathcal{W}_N(b, M) = \mathcal{H}_N(\alpha_S) \times \exp\{\mathcal{G}_N(\alpha_S, L)\}$$

with  $L \equiv \log(M^2 b^2)$  and  $\alpha_S L \sim 1$

$$\mathcal{G}(\alpha_S, L) = L g^{(1)}(\alpha_S L) + g^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}(\alpha_S L) + \dots \quad \mathcal{H}(\alpha_S) = \hat{\sigma}^{(0)} \left( 1 + \frac{\alpha_S}{\pi} \mathcal{H}^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)} + \dots \right)$$

LL ( $\sim \alpha_S^n L^{n+1}$ ):  $g^{(1)}$ ,  $(\hat{\sigma}^{(0)})$ ; NLL ( $\sim \alpha_S^n L^n$ ):  $g^{(2)}$ ,  $\mathcal{H}^{(1)}$ ; ... N<sup>k</sup>LL ( $\sim \alpha_S^n L^{n+k-1}$ ):  $g^{(k+1)}$ ,  $\mathcal{H}^{(k)}$ ;

Resummed result at small  $q_T$  *matched* with corresponding fixed “finite” part at large  $q_T$ : *uniform accuracy* for  $q_T \ll M$  and  $q_T \sim M$ .

- Resummed effects exponentiated in a **universal** of Sudakov form factor, process-dependence factorized in the hard-virtual factor  $H_c^F(\alpha_S)$  via all-order formula [Catani, Cieri, de Florian, G.F., Grazzini('14)].
- Resummation performed at partonic cross section level: (collinear) PDF evaluated at  $\mu_F \sim M$ ,  $f_N(b_0^2/b^2) = \exp\left\{-\int_{b_0^2/b^2}^{\mu_F^2} \frac{dq^2}{q^2} \gamma_N(\alpha_S(q^2))\right\} f_N(\mu_F^2)$ : no PDF extrapolation in the non perturbative region, study of  $\mu_R$  and  $\mu_F$  dependence as in fixed-order calculations.
- No need for NP models: Landau singularity of  $\alpha_S$  regularized using a *Minimal Prescription* without power-suppressed corrections [Laenen et al.('00)], [Catani et al.('96)].
- Introduction of **resummation scale**  $Q \sim M$ : variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln(M^2 b^2) \rightarrow \ln(Q^2 b^2) + \ln(M^2/Q^2)$$

- Perturbative **unitarity constraint**: recover *exactly* the total cross-section (upon integration on  $q_T$ )

$$\ln(Q^2 b^2) \rightarrow \tilde{L} \equiv \ln(Q^2 b^2 + 1) \Rightarrow \exp\{\alpha_S^n \tilde{L}^k\}|_{b=0} = 1 \Rightarrow \int_0^\infty dq_T^2 \left(\frac{d\hat{\sigma}}{dq_T^2}\right) = \hat{\sigma}^{(tot)};$$

- General procedure to treat the  $q_T$  recoil [Catani, de Florian, G.F., Grazzini('15)]:

$$\frac{d\hat{\sigma}^{(0)}}{d\Omega} = \hat{\sigma}^{(0)}(M^2) F(\mathbf{q}_T; M^2, \Omega) \text{ with } F(\mathbf{q}_T; M^2, \Omega) = F(\mathbf{0}; M^2, \Omega) + \mathcal{O}(q_T^2/M^2)$$



## $q_T$ resummation: perturbative accuracy

- Formalism implemented in **numerically efficient** and **publicly available** code:

**DYTurbo**: computes resummed and fixed-order fiducial cross section and related distributions it retains full kinematics of the vector boson and of its leptonic decay products [Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vinster, Schott('20)]

<https://dyturbo.hepforge.org>.

- We have explicitly included in **DYTurbo** up to:
  - **N<sup>4</sup>LL** logarithmic contributions to **all orders** (i.e. up to  $\exp(\sim \alpha_S^n L^{n-3})$ );
  - Approximated **N<sup>4</sup>LO** corrections (i.e. up to  $\mathcal{O}(\alpha_S^4)$ ) at **small**  $q_T$ ;
  - **NLO** corrections (i.e. up to  $\mathcal{O}(\alpha_S^2)$ ) at **large**  $q_T$ ;
- Matching with **NNLO** corrections (i.e. up to  $\mathcal{O}(\alpha_S^3)$ ) at **large**  $q_T$  from results in [Boughezal et al.('16)], [Gehrmann-DeRidder et al.('16)], [MCFM ('23)];
- Results up to **N<sup>3</sup>LO** (i.e. up to  $\mathcal{O}(\alpha_S^3)$ ) recovered for the **total cross section** (from unitarity).

## $q_T$ resummation: perturbative accuracy

- Formalism implemented in **numerically efficient** and **publicly available** code:

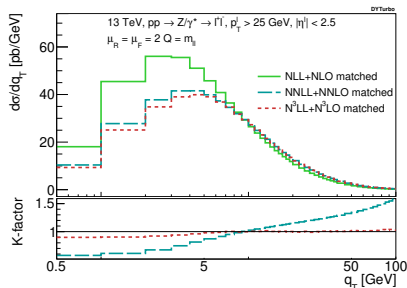
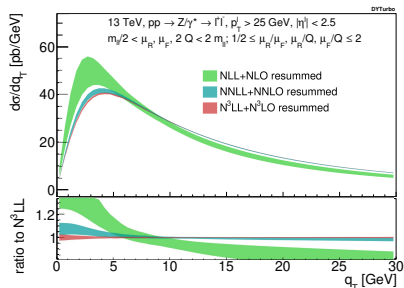
**DYTurbo**: computes resummed and fixed-order fiducial cross section and related distributions it retains full kinematics of the vector boson and of its leptonic decay products [Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vinciter, Schott('20)]

<https://dyturbo.hepforge.org>.

- We have explicitly included in **DYTurbo** up to:
  - **N<sup>4</sup>LL** logarithmic contributions to **all orders** (i.e. up to  $\exp(\sim \alpha_S^n L^{n-3})$ );
  - Approximated **N<sup>4</sup>LO** corrections (i.e. up to  $\mathcal{O}(\alpha_S^4)$ ) at **small  $q_T$** ;
  - **NLO** corrections (i.e. up to  $\mathcal{O}(\alpha_S^2)$ ) at **large  $q_T$** ;
- Matching with **NNLO** corrections (i.e. up to  $\mathcal{O}(\alpha_S^3)$ ) at **large  $q_T$**  from results in [Boughezal et al.('16)], [Gehrmann-De Ridder et al.('16)], [MCFM ('23)];
- Results up to **N<sup>3</sup>LO** (i.e. up to  $\mathcal{O}(\alpha_S^3)$ ) recovered for the **total cross section** (from unitarity).

# $Z/\gamma^*$ production at $N^3LL+N^3LO$ (resummed and matched)

[Camarda, Cieri, G.F. ('21)]

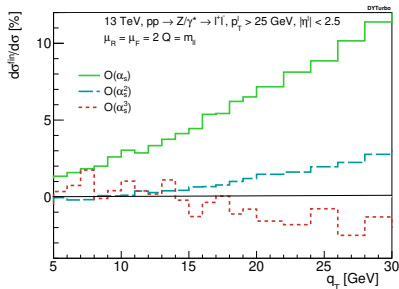
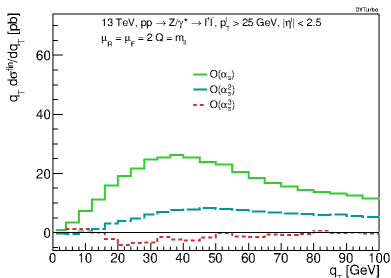


DYTurbo results. Resummed (left) and matched (right) NLL, NNLL and  $N^3LL$  bands for  $Z/\gamma^*$   $q_T$  spectrum.

Lower panel: ratio with respect to the  $N^3LL$  central value.

# $Z/\gamma^*$ production: finite part

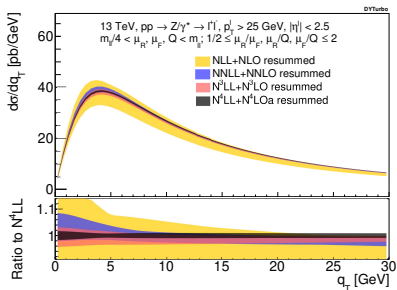
[Camarda, Cieri, G.F. ('21)]



Finite part at  $\mathcal{O}(\alpha_s)$ ,  $\mathcal{O}(\alpha_s^2)$  and  $\mathcal{O}(\alpha_s^3)$  (left) and ratio wrt matched results (right).

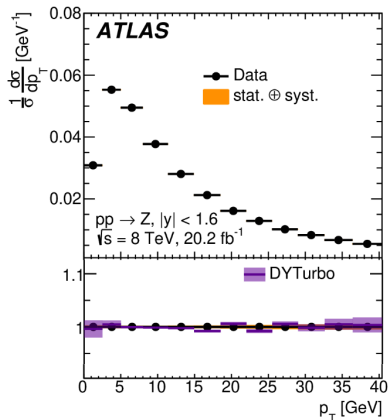
# $Z/\gamma^*$ production at $N^4\text{LL}+N^4\text{LOa}$ resummed

[Camarda, Cieri, G.F. ('23)]



**DYTurbo** results. Left: Resummed NLL, NNLL,  $N^3\text{LL}$  and  $N^4\text{LLa}$  bands for  $Z/\gamma^*$  (left). Right: Uncertainties from approximations of the perturbative coefficients at  $N^4\text{LL}+N^4\text{LOa}$  compared to scale variations.

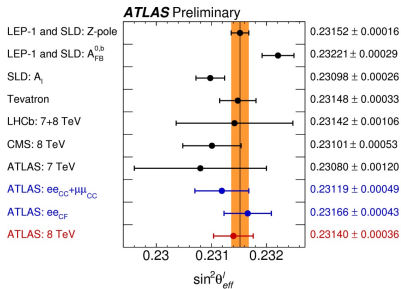
# $Z/\gamma^*$ production theory vs data



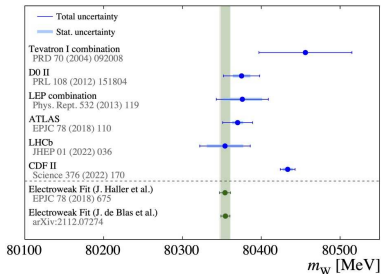
DYTurbo results at N<sup>4</sup>LLa accuracy compared with data [ATLAS Co11.('23)].

**Time performance of  $\mathcal{O}$  (seconds):** (with exception of V+jet term with fiducial lepton cuts).

# Modelling Z (and W) production for $\sin^2\theta_{eff}^l$ and $M_W$ determinations



Comparison of the measurements of the  $\sin^2\theta_{eff}^l$ .



Measured values of  $M_W$  compared with the prediction of from the global electroweak fit

# Combining QED and QCD $q_T$ resummation

[Cieri, G.F., Sborlini ('18)]

We start considering QED contributions to the  $q_T$  spectrum in the case of colourless and **neutral** high mass systems, e.g. on-shell  $Z$  boson production

$$h_1 + h_2 \rightarrow Z^0 + X$$

In the impact parameter and Mellin spaces resummed partonic cross section reads:

$$\mathcal{W}_N(b, M) = \hat{\sigma}^{(0)} \mathcal{H}'_N(\alpha_S, \alpha) \times \exp \{ \mathcal{G}'_N(\alpha_S, \alpha, L) \}$$

$$\mathcal{G}'(\alpha_S, \alpha, L) = \mathcal{G}(\alpha_S, L) + L g'^{(1)}(\alpha L) + g'^{(2)}(\alpha L) + \sum_{n=3}^{\infty} \left(\frac{\alpha}{\pi}\right)^{n-2} g'^{(n)}(\alpha L)$$

$$+ g'^{(1,1)}(\alpha_S L, \alpha L) + \sum_{\substack{n,m=1 \\ n+m \neq 2}}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n-1} \left(\frac{\alpha}{\pi}\right)^{m-1} g_N'^{(n,m)}(\alpha_S L, \alpha L)$$

$$\mathcal{H}'(\alpha_S, \alpha) = \mathcal{H}(\alpha_S) + \frac{\alpha}{\pi} \mathcal{H}'^{(1)} + \sum_{n=2}^{\infty} \left(\frac{\alpha}{\pi}\right)^n \mathcal{H}_N''^{(n)} + \sum_{n,m=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \left(\frac{\alpha}{\pi}\right)^m \mathcal{H}_N'^{F(n,m)}$$

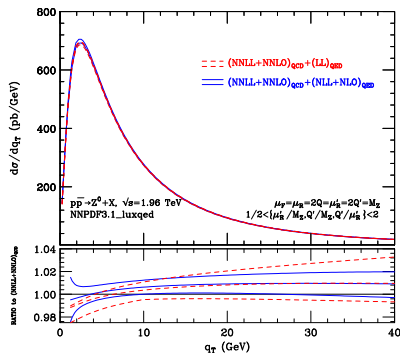
LL QED ( $\sim \alpha^n L^{n+1}$ ):  $g'^{(1)}$ ; NLL QED ( $\sim \alpha^n L^n$ ):  $g'^{(2)}$ ,  $\mathcal{H}'^{(1)}$ ;

LL mixed QCD-QED ( $\sim \alpha_S^n \alpha^n L^{2n}$ ):  $g'^{(1,1)}$ ;

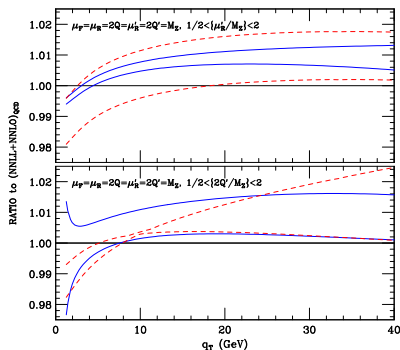


# Combined QED and QCD $q_T$ resummation for Z production at the Tevatron

[Cieri, G.F., Sborlini ('18)]

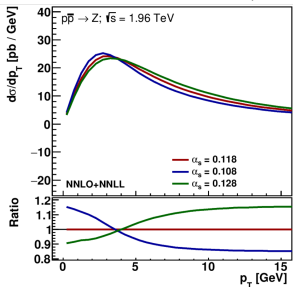
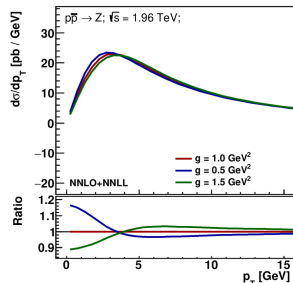


Z  $q_T$  spectrum at the LHC.  
 NNLL+NNLO QCD combined with the LL (red dashed) and NLL+NLO (blue solid) QED with corresponding QED uncertainty bands.



Ratio of the resummation (upper panel) and renormalization (lower panel) QED scale-dependent results with respect to the central value NNLL+NNLO QCD result.

# Non perturbative effects



- Up to now discussed result in a complete perturbative framework (except for PDFs).

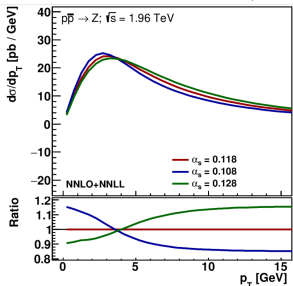
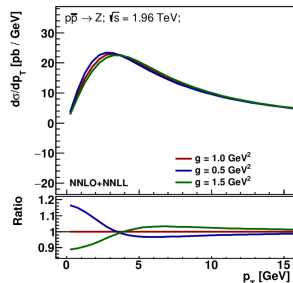
- Non perturbative *intrinsic*  $k_T$  effects parametrized by a NP form factor

$$S_{NP} = \exp\{-gb^2\} \text{ with } 0 < g < 1.2 \text{ GeV}^2:$$

$$\exp\{\mathcal{G}_N(\alpha_S, \tilde{L})\} \rightarrow \exp\{\mathcal{G}_N(\alpha_S, \tilde{L})\} S_{NP}$$

- NP effects increase the hardness of the  $q_T$  spectrum at small values of  $q_T$ . **Non trivial interplay of perturbative and NP effects.**
- However possible to disentangle the effects: scale of the NP effects is  $\langle q_T \rangle \sim 1 \text{ GeV}$  ( $g \sim 0.5 \text{ GeV}^2$ ), scale of "soft gluon" recoil is  $\langle q_T \rangle \sim 10 \text{ GeV}$ .

# Non perturbative effects



- Up to now discussed result in a complete perturbative framework (except for PDFs).

- Non perturbative *intrinsic*  $k_T$  effects parametrized by a NP form factor

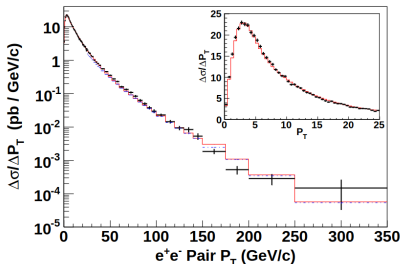
$$S_{NP} = \exp\{-gb^2\} \text{ with } 0 < g < 1.2 \text{ GeV}^2:$$

$$\exp\{\mathcal{G}_N(\alpha_S, \tilde{L})\} \rightarrow \exp\{\mathcal{G}_N(\alpha_S, \tilde{L})\} S_{NP}$$

- NP effects increase the hardness of the  $q_T$  spectrum at small values of  $q_T$ . **Non trivial interplay of perturbative and NP effects.**
- However possible to disentangle the effects: scale of the NP effects is  $\langle q_T \rangle \sim 1$  GeV ( $g \sim 0.5 \text{ GeV}^2$ ), scale of “soft gluon” recoil is  $\langle q_T \rangle \sim 10$  GeV.

# Z-boson $q_T$ measurement at CDF

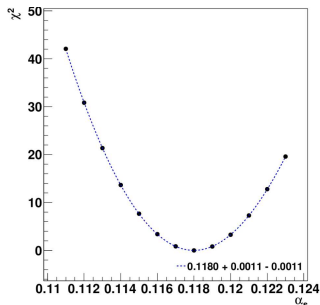
The CDF measurement of  $Z/\gamma^* \rightarrow e^+e^-$  ( $\sqrt{s} = 1.96 \text{ TeV}$  with  $\int \mathcal{L} = 2.1 \text{ fb}^{-1}$ ) [CDF Coll. ('10)] is ideal for  $\alpha_S(m_Z)$  determination.



[CDF Coll. ('10)]

- Measurement in full-lepton phase space with small extrapolation using angular coefficients method  $\Rightarrow$  allows fast analytic predictions with **DYTurbo**.
- $p\bar{p}$  collisions: small contribution from heavy-flavour in initial state (0.4%  $b\bar{b} \rightarrow Z$ , 1.3%  $c\bar{c} \rightarrow Z$ ). Quark mass effects negligible.
- Low pile-up and good electron resolution. Fine  $q_T$  bins (0.5 GeV) with relatively small bin-to-bin correlations.

# Methodology



- **DYTurbo** interfaced to **xFitter**.  
Fit region:  $Z q_T < 30 \text{ GeV}$ , predictions at  $N^3\text{LL} + \mathcal{O}(\alpha_s^3)$  (i.e.  $N^3\text{LL} + N^3\text{LO}$  at low  $q_T$ ) with NNPDF4.0 PDF at NNLO.
- Defined  $\chi^2$  with experimental ( $\beta_{\text{exp}}$ ) and PDFs ( $\beta_{\text{th}}$ ) uncertainties (equivalent to including the new dataset in the PDF using profiling/reweighting).
- The non-perturbative form factor is  $S_{NP} = \exp\{-gb^2\}$  with  $g$  left free in the fit.

$$\chi^2(\beta_{\text{exp}}, \beta_{\text{th}}) = \sum_{i=1}^{N_{\text{data}}} \frac{\left( \sigma_i^{\text{exp}} + \sum_j \Gamma_{ij}^{\text{exp}} \beta_{j,\text{exp}} - \sigma_i^{\text{th}} - \sum_k \Gamma_{ik}^{\text{th}} \beta_{k,\text{th}} \right)^2}{\Delta_i^2} + \sum_j \beta_{j,\text{exp}}^2 + \sum_k \beta_{k,\text{th}}^2.$$

# Theory uncertainties

	PDF fit	Hessian profiling
$\alpha_S(m_Z)$	$0.1188 \pm 0.0008$	$0.1184 \pm 0.0006$
$g$ [GeV <sup>2</sup> ]	$0.69 \pm 0.05$	$0.71 \pm 0.05$
Dataset	$\chi^2/\text{points}$	$\chi^2/\text{points}$
NC DIS H1-ZEUS $e^+p$	955/905	
CC DIS H1-ZEUS $e^+p$	46/39	
NC DIS H1-ZEUS $e^-p$	219/159	
CC DIS H1-ZEUS $e^-p$	53/42	
H1-ZEUS correlated $\chi^2$	91	
CDF Z $p_T$	41/55	40/55
Total	1405 / 1184	

- Bias from  $\alpha_S$ -PDFs correlations [Forte, Kassabov('20)] → PDFs refitted.
- Other PDF sets considered. CT18, CT18Z, MSHT20, HERAPDF2.0, ABMP16 (NNLO) and MSHT20an3lo. The midpoint value is the nominal result and the PDF envelope as an additional uncertainty.
- Uncertainty from missing higher orders:  $m_{||}/2 < \{\mu_R, \mu_F, Q\} < 2m_{||}$  with  $0.5 < \{\mu_R/\mu_F, \mu_R/Q, \mu_F/Q\} < 2$ .
- NP effects:  $b_*$ -pr.  $b_* = b/\sqrt{1 + b^2/b_{lim}^2}$  ( $b_{lim} = 2 - 3 \text{ GeV}^{-1}$ ) and *minimal* pr. ( $b_{lim} \rightarrow \infty$ ); quartic term  $\exp(-qb^4)$  and different parametrization of  $S_{NP}$  [Collins, Rogers('15)].
- Uncertainty from finite component at  $\mathcal{O}(\alpha_S^3)$ .
- Check with D0 data and fit boundaries.

# Theory uncertainties

	$\alpha_S(m_Z)$	$g$ [GeV <sup>2</sup> ]	$\chi^2/\text{dof}$
NNPDF4.0	$0.1192 \pm 0.0008$	$0.66 \pm 0.05$	41/53
CT18	$0.1189 \pm 0.0010$	$0.67 \pm 0.05$	40/53
CT18Z	$0.1198 \pm 0.0009$	$0.62 \pm 0.05$	41/53
MSHT20	$0.1185 \pm 0.0009$	$0.72 \pm 0.05$	40/53
HERAPDF2.0	$0.1188 \pm 0.0008$	$0.69 \pm 0.05$	40/53
ABMP16	$0.1185 \pm 0.0007$	$0.62 \pm 0.05$	42/53
MSHT20an3lo (N <sup>3</sup> LL)	$0.1184 \pm 0.0009$	$0.73 \pm 0.05$	40/53
PDF fit	$0.1184 \pm 0.0006$	$0.71 \pm 0.05$	1405/1184

- Bias from  $\alpha_S$ -PDFs correlations [Forte, Kassabov('20)] → PDFs refitted.
- Other PDF sets considered. CT18, CT18Z, MSHT20, HERAPDF2.0, ABMP16 (NNLO) and MSHT20an3lo. The midpoint value is the nominal result and the PDF envelope as an additional uncertainty.
- Uncertainty from missing higher orders:  $m_{||}/2 < \{\mu_R, \mu_F, Q\} < 2m_{||}$  with  $0.5 < \{\mu_R/\mu_F, \mu_R/Q, \mu_F/Q\} < 2$ .
- NP effects:  $b_*$ -pr.  $b_* = b/\sqrt{1 + b^2/b_{lim}^2}$  ( $b_{lim} = 2 - 3 \text{ GeV}^{-1}$ ) and *minimal pr.* ( $b_{lim} \rightarrow \infty$ ); quartic term  $\exp(-qb^4)$  and different parametrization of  $S_{NP}$  [Collins, Rogers('15)].
- Uncertainty from finite component at  $\mathcal{O}(\alpha_S^3)$ .
- Check with D0 data and fit boundaries.

# Theory uncertainties

$\mu_R/m_{\ell\ell}$	$\mu_F/m_{\ell\ell}$	$Q/m_{\ell\ell}$	$\alpha_S(m_Z)$	$g$ [GeV <sup>2</sup> ]	$\chi^2/\text{dof}$
1	1	1	0.1192 ± 0.0008	0.66 ± 0.05	41/53
1	1	2	0.1183 ± 0.0007	0.77 ± 0.05	40/53
1	1	0.5	0.1196 ± 0.0008	0.57 ± 0.05	42/53
1	2	1	0.1194 ± 0.0008	0.66 ± 0.05	41/53
1	2	2	0.1183 ± 0.0007	0.77 ± 0.05	41/53
1	0.5	1	0.1193 ± 0.0008	0.68 ± 0.05	42/53
1	0.5	0.5	0.1196 ± 0.0008	0.59 ± 0.05	42/53
2	1	1	0.1193 ± 0.0008	0.67 ± 0.05	42/53
2	1	2	0.1194 ± 0.0008	0.70 ± 0.05	41/53
2	2	1	0.1192 ± 0.0008	0.65 ± 0.05	42/53
2	2	2	0.1192 ± 0.0008	0.67 ± 0.05	41/53
0.5	1	1	0.1184 ± 0.0007	0.75 ± 0.05	42/53
0.5	1	0.5	0.1192 ± 0.0007	0.64 ± 0.05	41/53
0.5	0.5	1	0.1183 ± 0.0007	0.75 ± 0.05	42/53
0.5	0.5	0.5	0.1192 ± 0.0007	0.64 ± 0.05	42/53

- Bias from  $\alpha_S$ -PDFs correlations  
[Forte, Kassabov ('20)] → PDFs refitted.
- Other PDF sets considered. CT18, CT18Z, MSHT20, HERAPDF2.0, ABMP16 (NNLO) and MSHT20an3lo. The midpoint value is the nominal result and the PDF envelope as an additional uncertainty.
- Uncertainty from missing higher orders:  
 $m_{ll}/2 < \{\mu_R, \mu_F, Q\} < 2m_{ll}$  with  
 $0.5 < \{\mu_R/\mu_F, \mu_R/Q, \mu_F/Q\} < 2$ .
- NP effects:  $b_*$ -pr.  $b_* = b/\sqrt{1 + b^2/b_{lim}^2}$   
( $b_{lim} = 2 - 3 \text{ GeV}^{-1}$ ) and *minimal* pr.  
( $b_{lim} \rightarrow \infty$ ); quartic term  $\exp(-qb^4)$  and  
different parametrization of  $S_{NP}$   
[Collins, Rogers ('15)].
- Uncertainty from finite component at  $\mathcal{O}(\alpha_S^3)$ .
- Check with D0 data and fit boundaries.

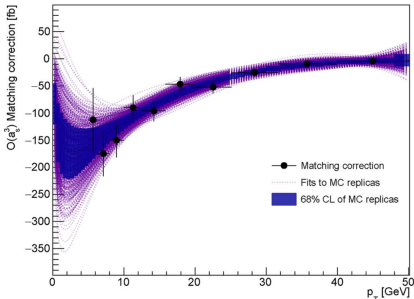


# Theory uncertainties

	$\alpha_S(m_Z)$	$g$ [GeV <sup>2</sup> ]
$b_{lim} = 2 \text{ GeV}^{-1}$	$0.1187 \pm 0.0007$	$0.83 \pm 0.05$
$b_{lim} \rightarrow \infty$	$0.1199 \pm 0.0008$	$0.42 \pm 0.05$
$g_k$	$0.1186 \pm 0.0008$	$0.65 \pm 0.05$
$q = 0.1 \text{ GeV}^4$	$0.1197 \pm 0.0008$	$0.51 \pm 0.05$
VFN PDF evolution	$0.1190 \pm 0.0007$	$0.71 \pm 0.05$

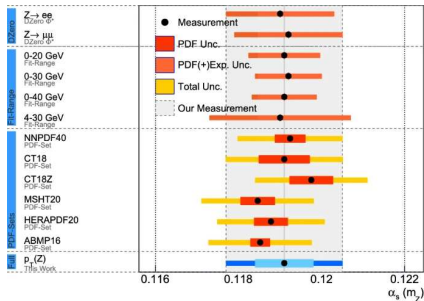
- Bias from  $\alpha_S$ -PDFs correlations  
[Forte, Kassabov ('20)]  $\rightarrow$  PDFs refitted.
- Other PDF sets considered. CT18, CT18Z, MSHT20, HERAPDF2.0, ABMP16 (NNLO) and MSHT20an3lo. The midpoint value is the nominal result and the PDF envelope as an additional uncertainty.
- Uncertainty from missing higher orders:  
 $m_{ll}/2 < \{\mu_R, \mu_F, Q\} < 2m_{ll}$  with  
 $0.5 < \{\mu_R/\mu_F, \mu_R/Q, \mu_F/Q\} < 2$ .
- NP effects:  $b_*$ -pr.  $b_* = b/\sqrt{1 + b^2/b_{lim}^2}$   
( $b_{lim} = 2 - 3 \text{ GeV}^{-1}$ ) and *minimal pr.*  
( $b_{lim} \rightarrow \infty$ ); quartic term  $\exp(-qb^4)$  and  
different parametrization of  $S_{NP}$   
[Collins, Rogers ('15)].
- Uncertainty from finite component at  $\mathcal{O}(\alpha_S^3)$ .
- Check with D0 data and fit boundaries.

# Theory uncertainties



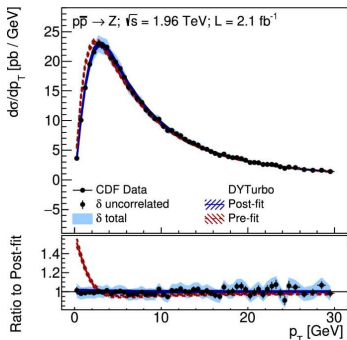
- Bias from  $\alpha_S$ -PDFs correlations  
[Forte, Kassabov('20)]  $\rightarrow$  PDFs refitted.
- Other PDF sets considered. CT18, CT18Z, MSHT20, HERAPDF2.0, ABMP16 (NNLO) and MSHT20an3lo. The midpoint value is the nominal result and the PDF envelope as an additional uncertainty.
- Uncertainty from missing higher orders:  
 $m_{||}/2 < \{\mu_R, \mu_F, Q\} < 2m_{||}$  with  
 $0.5 < \{\mu_R/\mu_F, \mu_R/Q, \mu_F/Q\} < 2$ .
- NP effects:  $b_*$ -pr.  $b_* = b/\sqrt{1 + b^2/b_{lim}^2}$   
( $b_{lim} = 2 - 3 \text{ GeV}^{-1}$ ) and *minimal* pr.  
( $b_{lim} \rightarrow \infty$ ); quartic term  $\exp(-qb^4)$  and  
different parametrization of  $S_{NP}$   
[Collins, Rogers('15)].
- Uncertainty from finite component at  $\mathcal{O}(\alpha_S^3)$ .
- Check with D0 data and fit boundaries.

# Theory uncertainties



- Bias from  $\alpha_S$ -PDFs correlations [Forte, Kassabov('20)]  $\rightarrow$  PDFs refitted.
- Other PDF sets considered. CT18, CT18Z, MSHT20, HERAPDF2.0, ABMP16 (NNLO) and MSHT20an3lo. The midpoint value is the nominal result and the PDF envelope as an additional uncertainty.
- Uncertainty from missing higher orders:  $m_{||}/2 < \{\mu_R, \mu_F, Q\} < 2m_{||}$  with  $0.5 < \{\mu_R/\mu_F, \mu_R/Q, \mu_F/Q\} < 2$ .
- NP effects:  $b_*$ -pr.  $b_* = b/\sqrt{1 + b^2/b_{lim}^2}$  ( $b_{lim} = 2 - 3 \text{ GeV}^{-1}$ ) and *minimal* pr. ( $b_{lim} \rightarrow \infty$ ); quartic term  $\exp(-qb^4)$  and different parametrization of  $S_{NP}$  [Collins, Rogers('15)].
- Uncertainty from finite component at  $\mathcal{O}(\alpha_S^3)$ .
- Check with D0 data and fit boundaries.

# Fit results



Statistical uncertainty		$\pm 0.7$
Experimental systematic uncertainty		$\pm 0.1$
PDF uncertainty (NNPDF4.0)		$\pm 0.4$
PDF uncertainty (envelope of PDFs)		$\pm 0.7$
Scale variations uncertainties	+0.4	-0.9
Matching at $\mathcal{O}(\alpha_S^3)$		$\pm 0.1$
Non-perturbative model		$\pm 0.7$
Flavour model	0	-0.3
QED ISR		$< \pm 0.1$
Lower limit of fit range		$\pm 0.2$
Total	+1.3	-1.6

Simultaneous fit of  $\alpha_S(m_Z)$  and  $g$  at  $N^3\text{LL} + \mathcal{O}(\alpha_S^3)$  ( $N^3\text{LL} + N^3\text{LO}$ ):

$$\alpha_S(m_Z) = 0.1191^{+0.0013}_{-0.0016}$$

$$g = 0.66 \pm 0.05 \text{ GeV}^2$$

# Conclusions

- Novel methodology for determination of  $\alpha_S(m_Z)$  based on Z-boson small- $q_T$  distribution.
- Based on N<sup>3</sup>LL+N<sup>3</sup>LO resummed QCD predictions.
- Result in agreement with the world average. Uncertainty comparable to other determinations.
- Precise collider determination: 1.2% relative uncertainty.
- Crucial development of DYTurbo program to compute fast and accurate theoretical predictions:  
<https://dyturbo.hepforge.org>