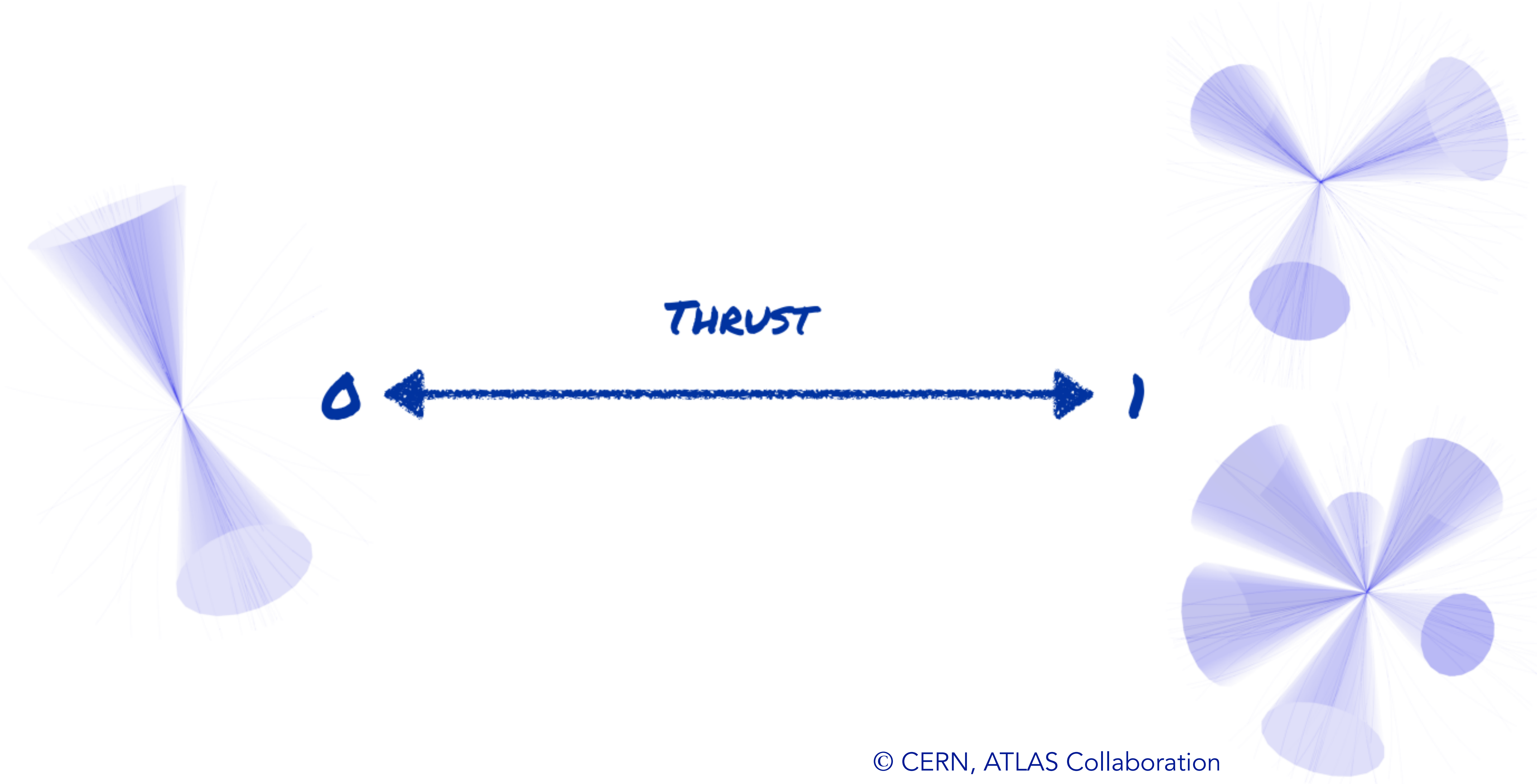
Effect of renormalon scheme and perturbative scale choices on α_s from event shapes

Christopher Lee (LANL)



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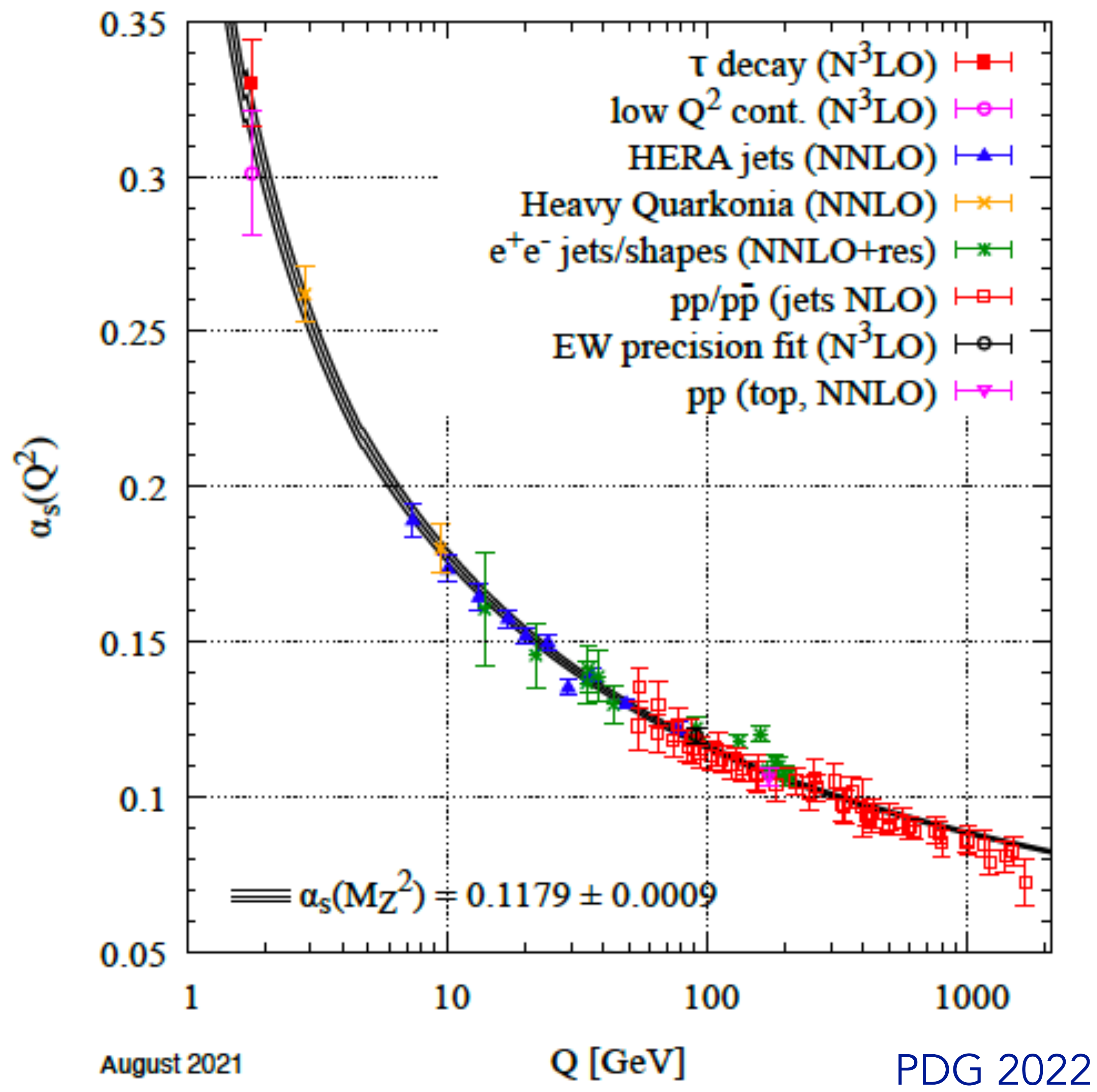

PDG 2022

FONDAZIONE BRUNO KESSLER
ECT* EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

ECT* Workshop: alphas-2024

07 Feb 2024

Los Alamos NATIONAL LABORATORY

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Collaborators

- G. Bell, CL, Y. Makris, J. Talbert, B. Yan, [arXiv:2311.03990]
- See also G. Bell, A. Hornig, CL, J. Talbert, [arXiv:1808.07867]

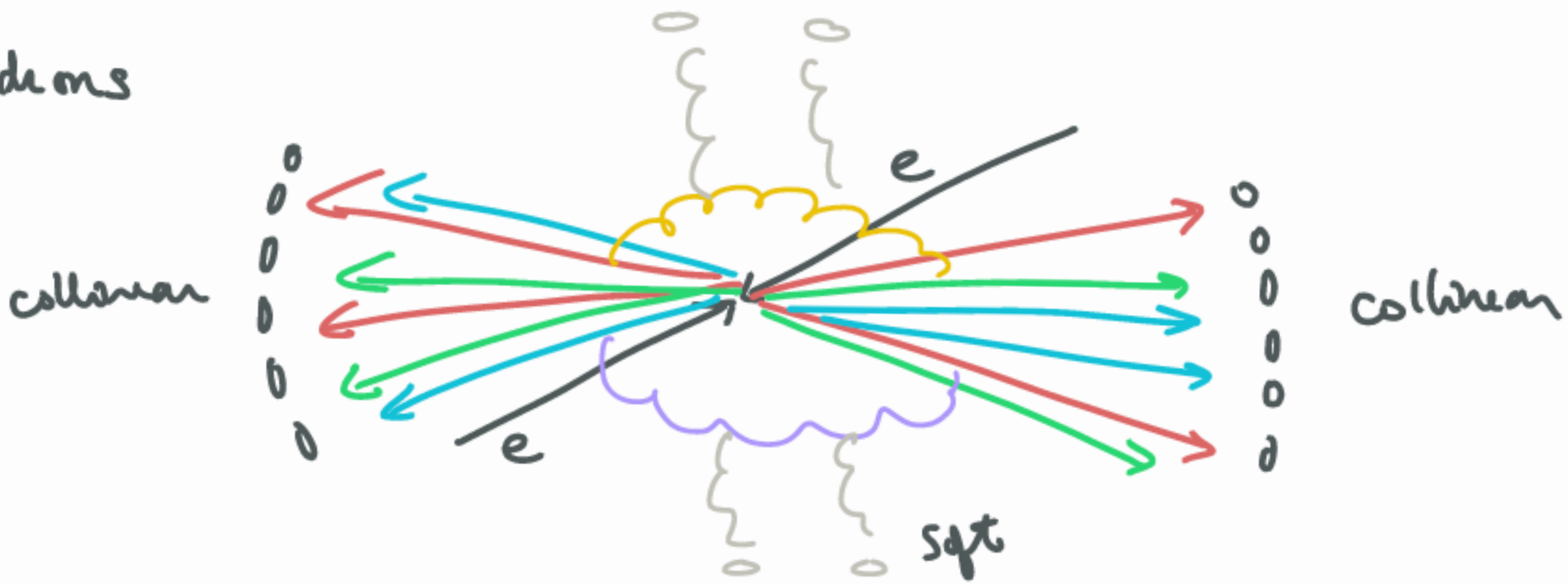
Outline of the talk

- Event shapes, EFT, factorization, resummation of perturbative logs
- Nonperturbative corrections and renormalon subtraction schemes
- Effects of perturbative and nonperturbative scale & scheme choices on fits for α_s
    - In a nutshell: some of these choices *may* have a few % effect on the tails of event shape distributions and the values of α_s, or on the uncertainties thereupon, obtained by comparing them to data
- Motivations for additional data and calculations for event shapes

Event Shapes

HADRONIC EVENT SHAPES: Global measures of "jetty" structure

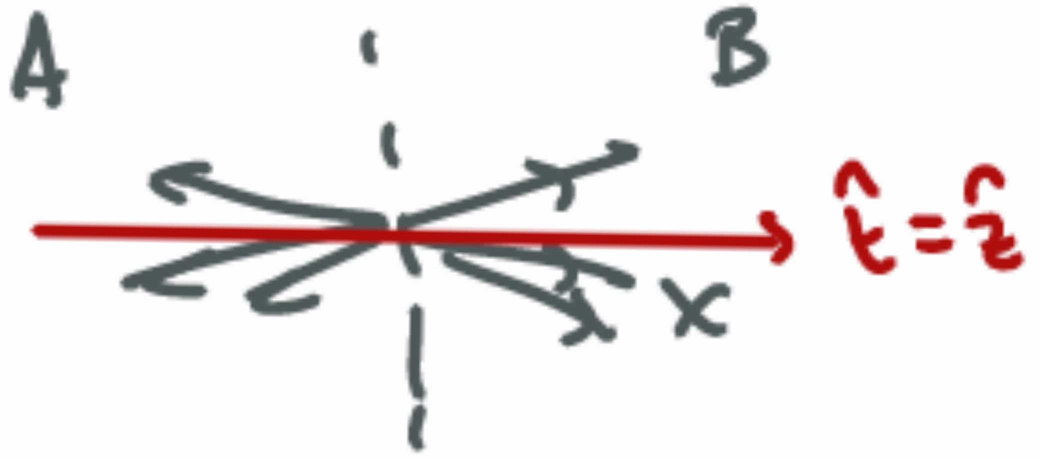
$e^+e^- \to$ hadrons



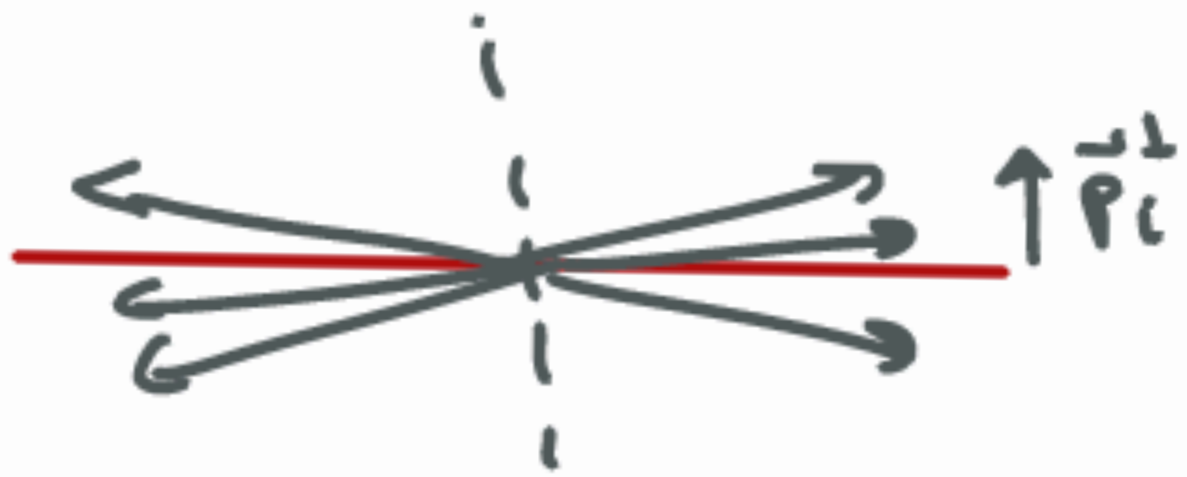
e.g.

THRUST: $T = \frac{1}{Q} \max_{\hat{t}} \sum_{i \in X} |\vec{p}_i \cdot \hat{t}\,|$

$= \frac{2}{Q} |\vec{p}_z^{\,A}|$ or $\tau = 1 - T$



BROADENING: $B = \frac{1}{Q} \sum_{i \in X} |\vec{p}_i^{\,\perp}|$

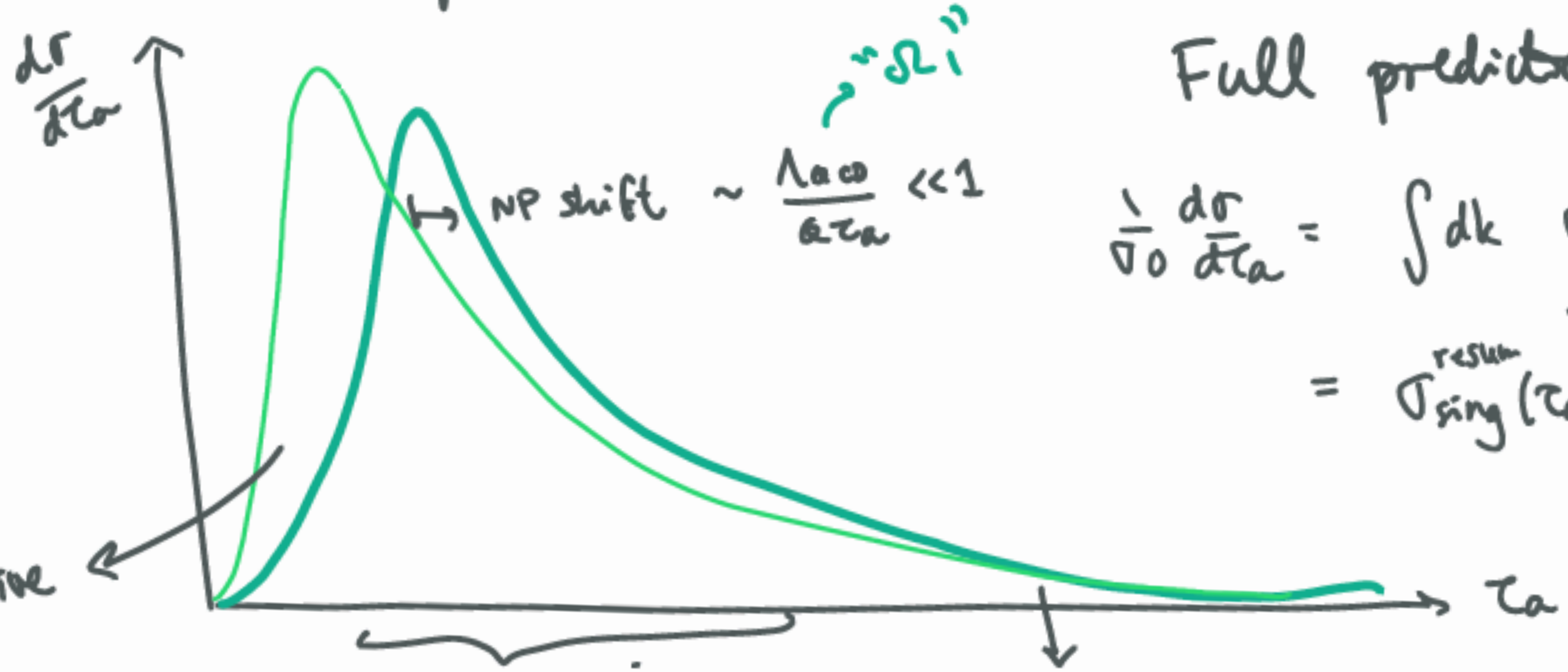


Angularities:

$$\tau_a = \frac{1}{Q} \sum_i |\mathbf{p}_\perp^i|\, e^{-|\eta_i|(1-a)}$$

EVENT SHAPES & SENSITIVITY TO α_s

τ_a's and similar event shapes probe QCD effects over wide range of scales, perturbative and nonperturbative:



Full prediction:

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_a} = \int dk \; \sigma_{PT}\left(\tau_a - \frac{k}{Q}\right) S_{NP}(k)$$

$$= \sigma_{sing}^{resum}(\tau_a; \mu_{H,J,S}) + \sigma_{non\text{-}sing}^{F.O.}(\tau_a; \mu_{ns})$$

$\sigma_{PT}(\tau - \frac{k}{Q}) \otimes S_{NP}(k)$

$\frac{\Lambda_{QCD}}{Q\tau} \sim 1$

↓

SHAPE FUNCTION

expansion in $\alpha_s(Q\tau)$, $\alpha_s(Q\tau^{\frac{1}{2-a}})$

↓

RESUMMATION

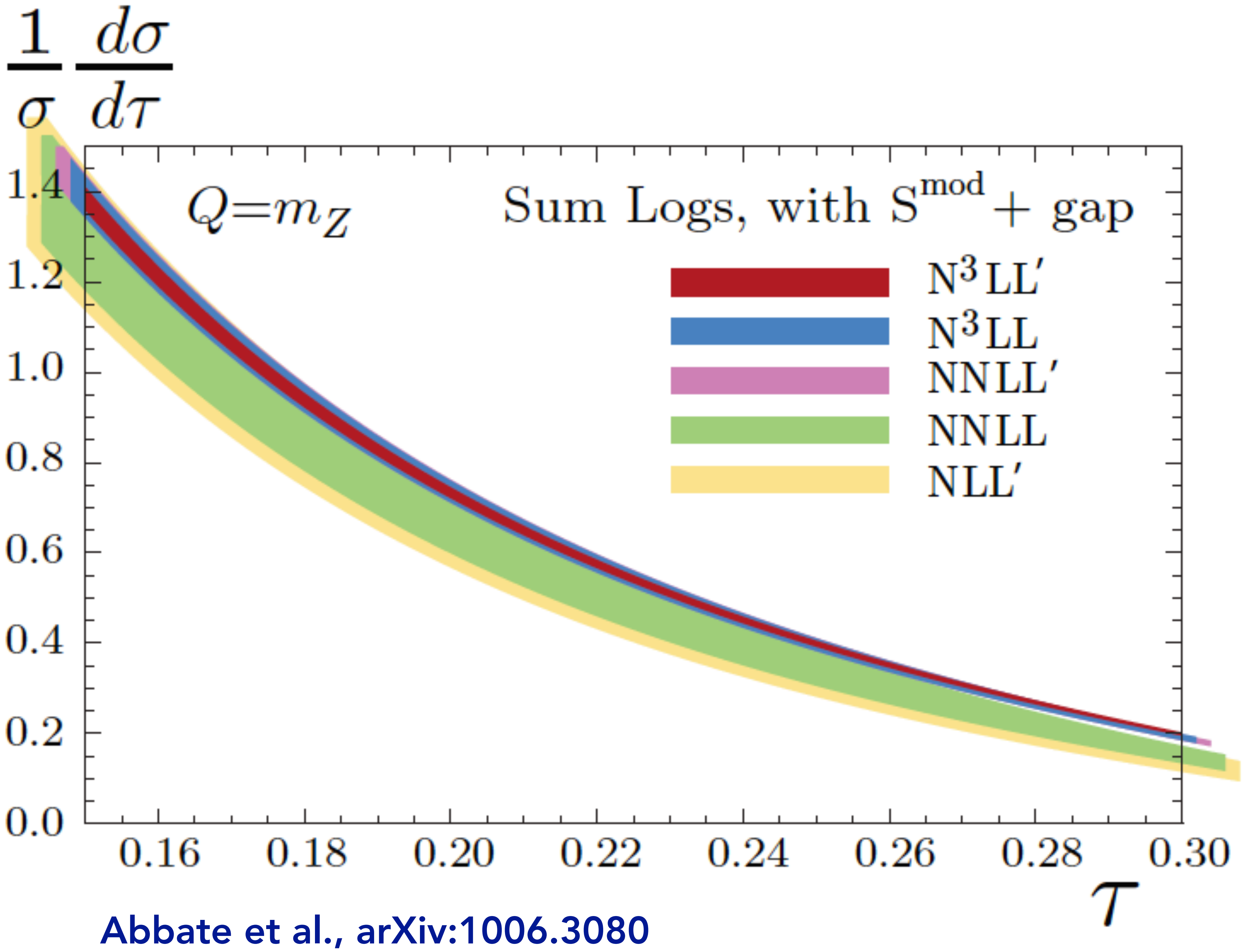
expansion in $\alpha_s(Q)$

↓

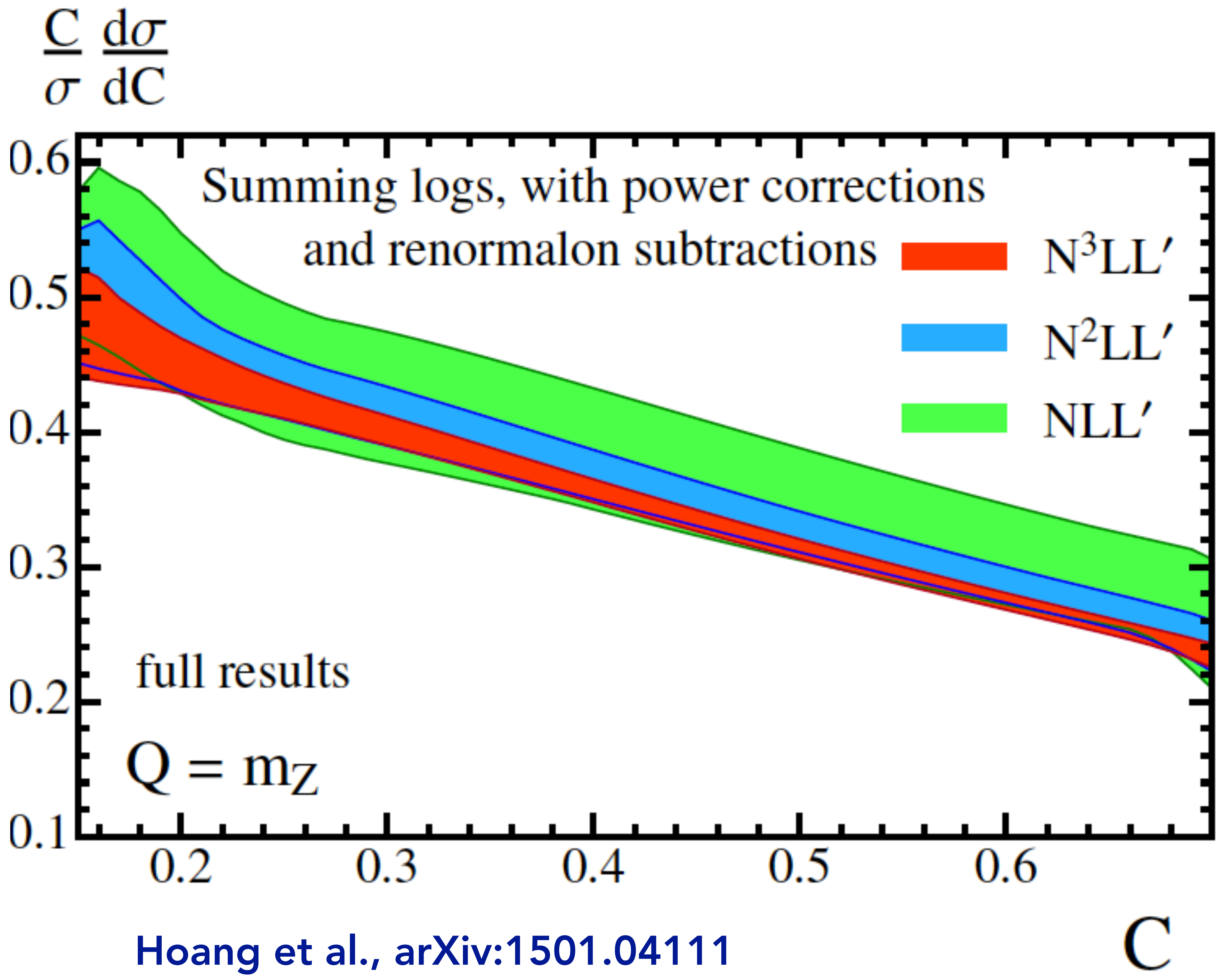
FIXED ORDER

Event shapes to high precision

- **Up to N³LL′ resummed event shape distributions with analytic treatment of nonperturbative corrections, e.g.:**



Abbate et al., arXiv:1006.3080



Hoang et al., arXiv:1501.04111

Makes e+e- event shapes one of the most precise ways, in principle, to determine α_s

Event shapes and the strong coupling

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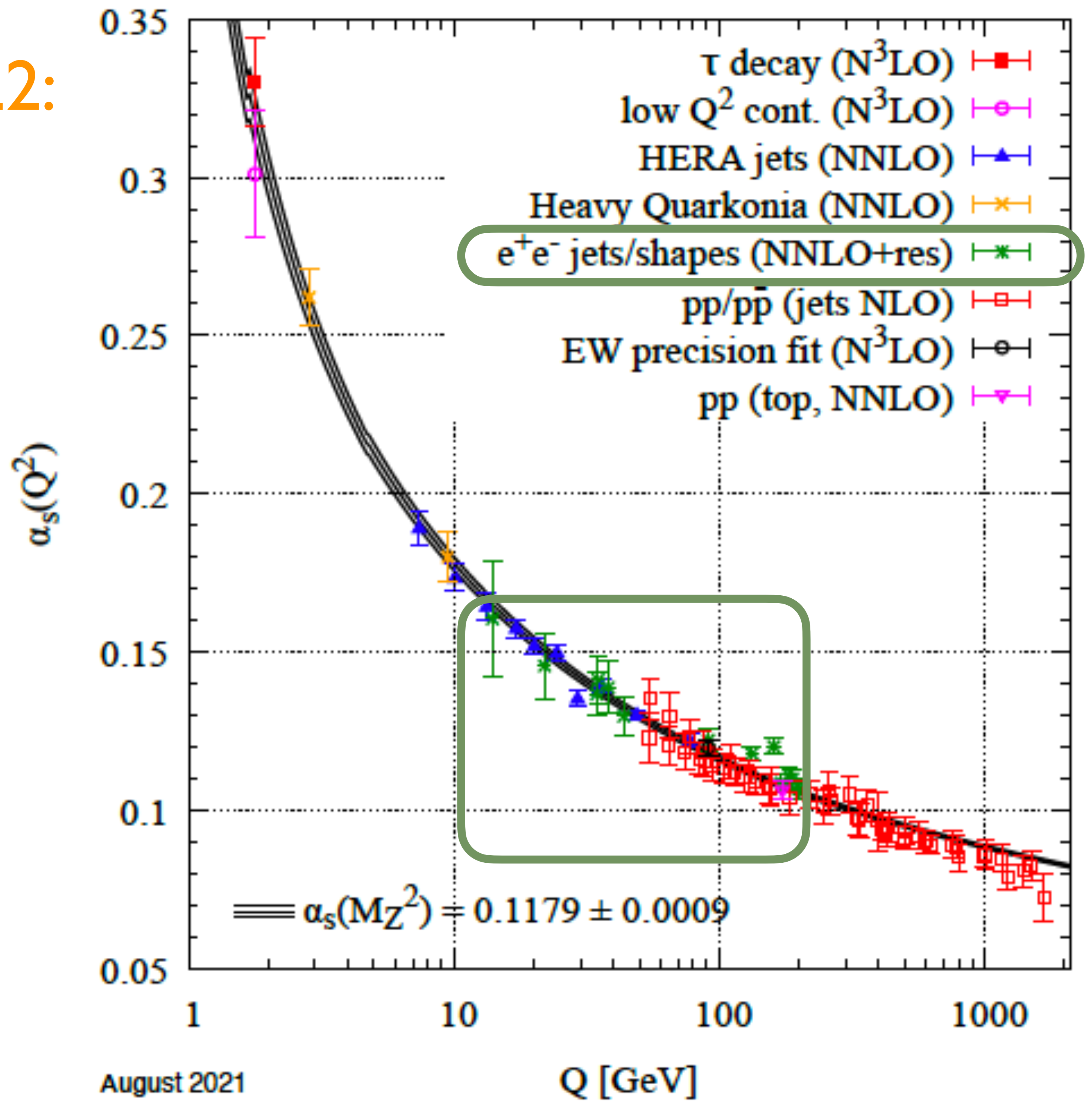
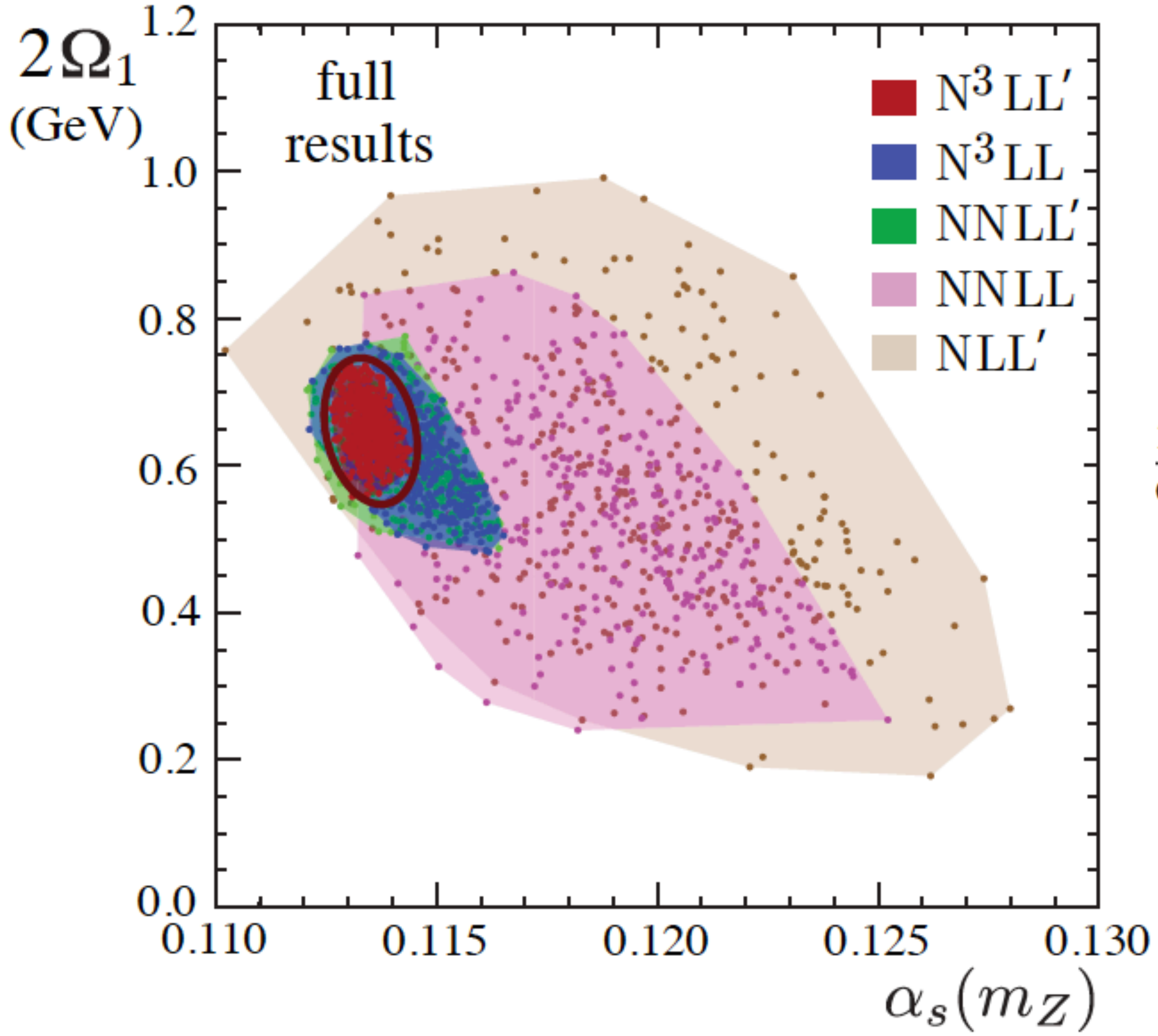
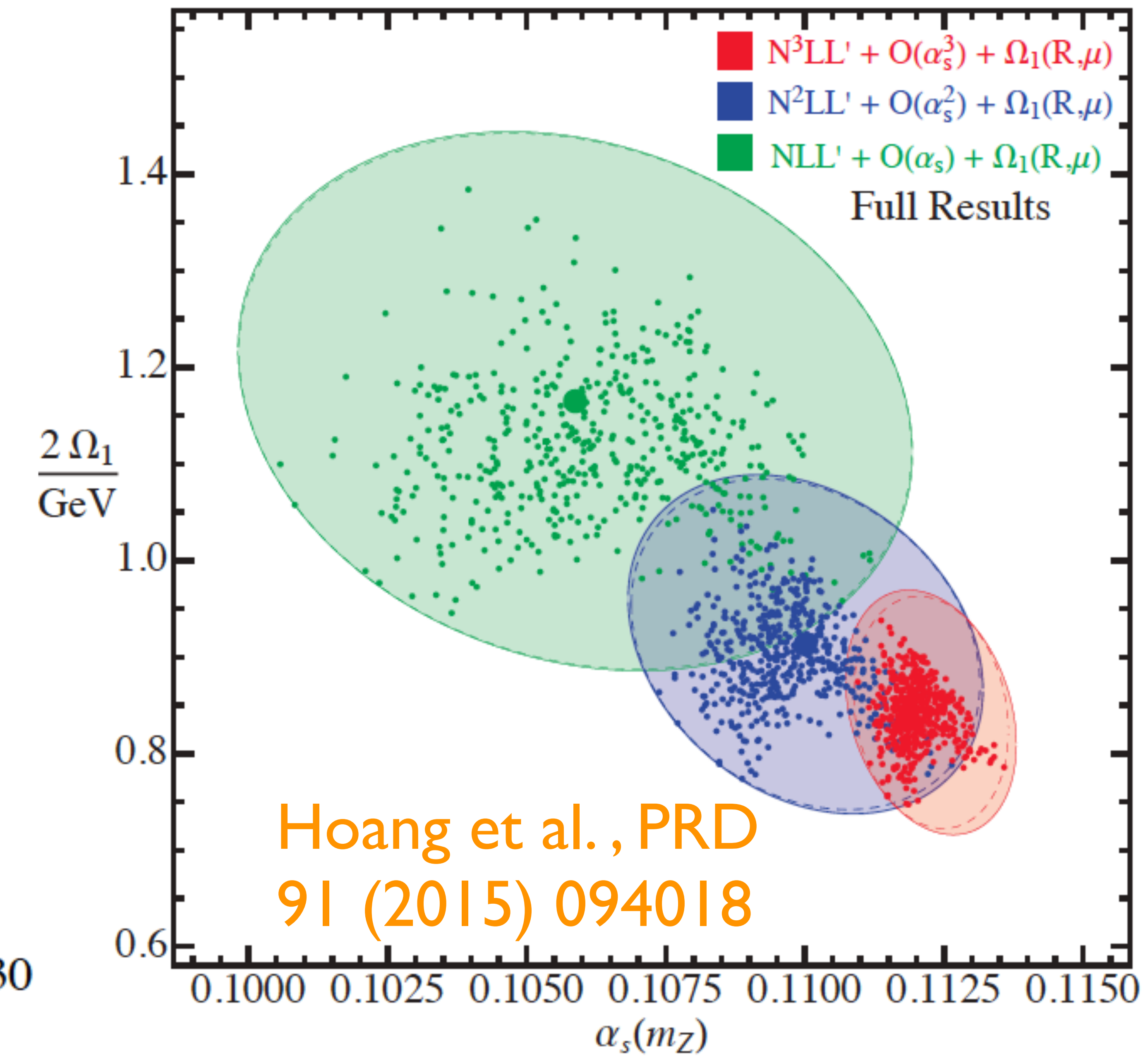


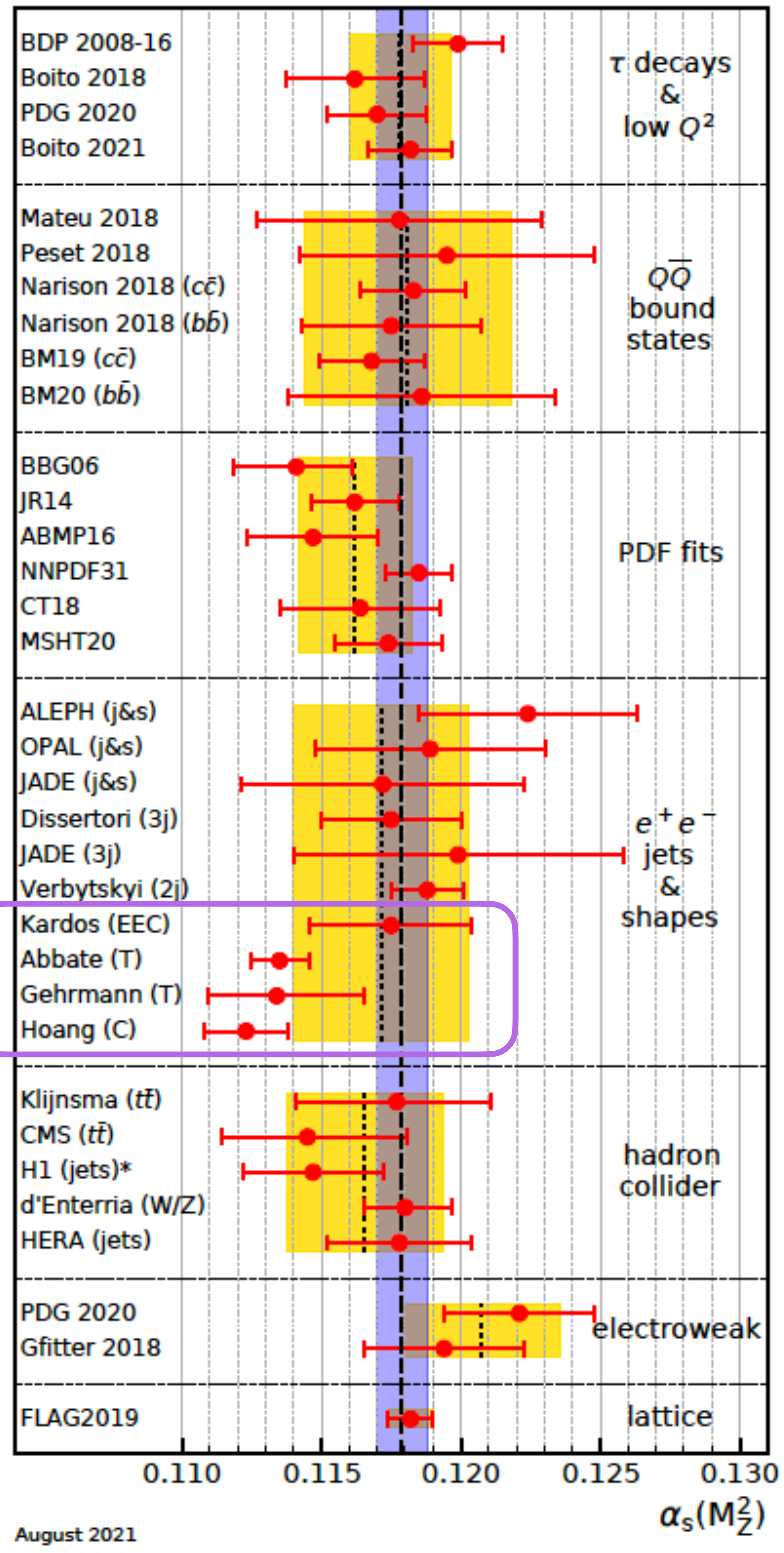
Abbate et al., PRD 83 (2011) 074021





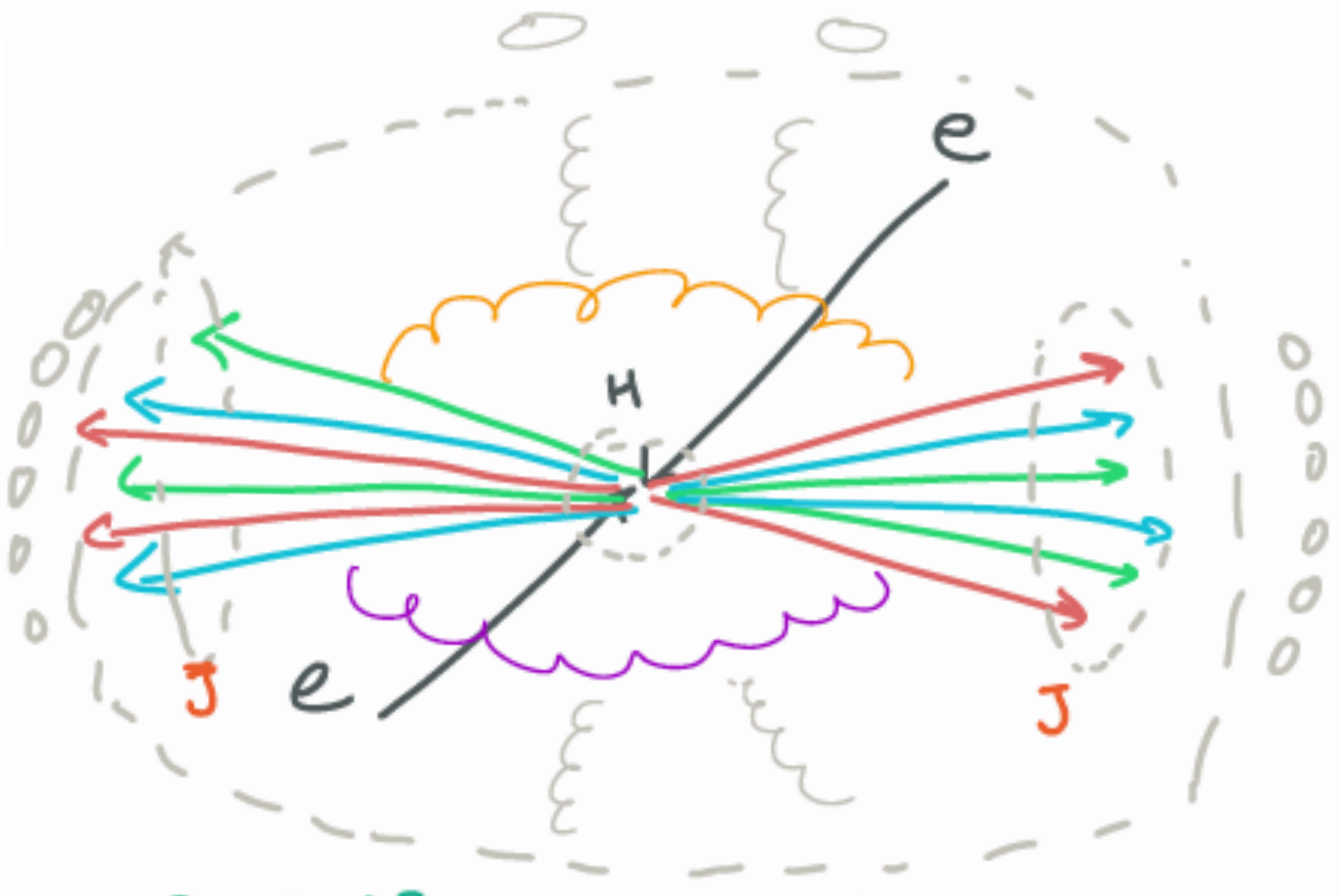
Hoang et al., PRD 91 (2015) 094018

Event shapes



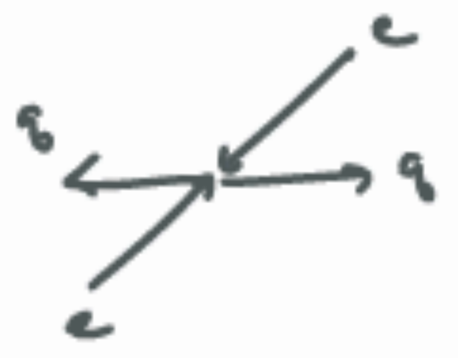
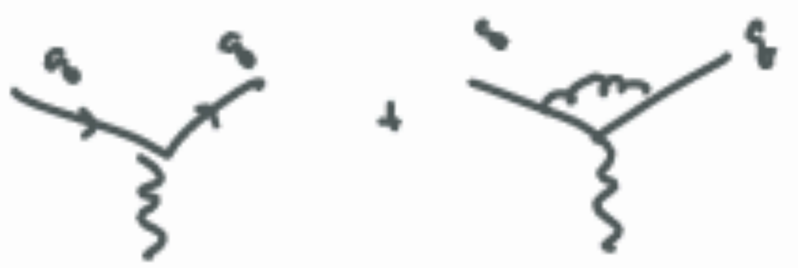
Factorization, Resummation and Nonperturbative Effects in EFT

Factorization in a Nutshell



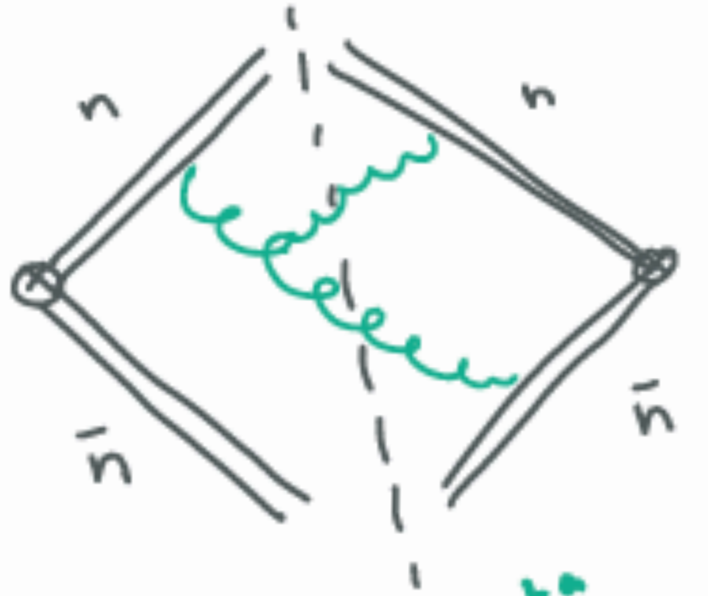
$S = S_{PT} \otimes S_{NP}$

For $\tau_a \ll 1$:

$\frac{d\sigma}{d\tau_a} =$  $\times \Big\{$  $+ \cdots \Big\}$

σ_0 $\qquad$ $H(Q^2,\mu)$

$$\times \int dt_J\, dt_{\bar{J}}\, dk_s\; \delta\left(\tau_a - \frac{t_J + t_{\bar{J}}}{Q^{2-a}} - \frac{k_s}{Q}\right)$$

$\times \Big\{$  $\Big)$ $t_J = \sum_i |p_i^\perp|^{2-a}$ $= t_{\bar{J}}$ $\Big($  $\Big\}$ $\times$ 

$J_n^a(t_J,\mu)\, J_{\bar{n}}^a(t_{\bar{J}},\mu)$

$k_s = \sum_i (n\cdot k_i)^{1-\frac{a}{2}} (\bar{n}\cdot k_i)^{a/2}$

defined as matrix elements of operators in SCET $\longleftarrow$ $S_a(k_s,\mu)$

$$= \sigma_0\, H(Q^2,\mu) \int dt_J\, dt_{\bar{J}}\, dk_s\, \delta\left(\tau_a - \frac{t_J + t_{\bar{J}}}{Q^{2-a}} - \frac{k_s}{Q}\right) J(t_J,\mu)\, J(t_{\bar{J}},\mu)\, S(k_s,\mu)$$

Evolution and resummation of logs

- An all-order dijet factorization theorem for the observable is easily derived in SCET:

$$d\sigma \sim H \cdot \mathcal{J} \otimes \mathcal{J} \otimes \mathcal{S} \quad \xleftrightarrow{\text{RGE}} \quad \frac{dH(Q^2,\mu)}{d\ln\mu} = \left[2\Gamma_{cusp}\ln\left(\frac{Q^2}{\mu^2}\right) + 4\gamma_H(\alpha_s)\right] H(Q^2,\mu)$$

hep-ph/0303051
hep-ph/1401.4460

- Evolving all scales to/from their 'natural' settings, one arrives at the resummed cross section:

$$\frac{\sigma_{\rm sing}(\tau_a)}{\sigma_0} = e^{K(\mu,\mu_H,\mu_J,\mu_S)}\left(\frac{\mu_H}{Q}\right)^{\omega_H(\mu,\mu_H)}\left(\frac{\mu_J^{2-a}}{Q^{2-a}\tau_a}\right)^{2\omega_J(\mu,\mu_J)}\left(\frac{\mu_S}{Q\tau_a}\right)^{\omega_S(\mu,\mu_S)} H(Q^2,\mu_H)$$

$$\times\, \widetilde{J}\left(\partial_\Omega + \ln\frac{\mu_J^{2-a}}{Q^{2-a}\tau_a},\mu_J\right)^2 \widetilde{S}\left(\partial_\Omega + \ln\frac{\mu_S}{Q\tau_a},\mu_S\right) \times \begin{cases} \frac{1}{\tau_a}\mathcal{F}(\Omega) & \sigma = \frac{d\sigma}{d\tau_a} \\ \mathcal{G}(\Omega) & \sigma = \sigma_c \end{cases}$$

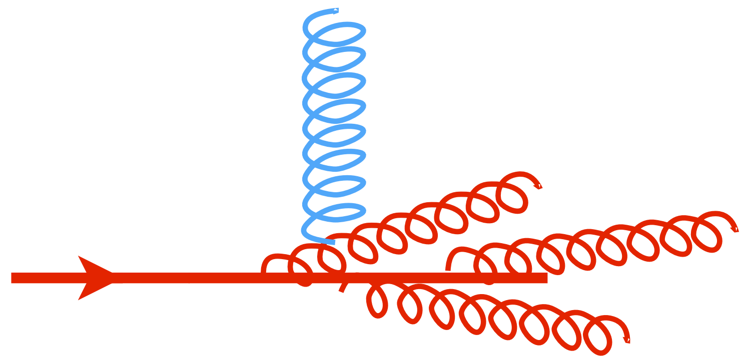
$$\mathcal{F}(\Omega) = \frac{e^{\gamma_E \Omega}}{\Gamma(-\Omega)} \qquad \mathcal{G}(\Omega) = \frac{e^{\gamma_E \Omega}}{\Gamma(1-\Omega)}$$

Hard $\mu_H = Q$

Jet $\mu_J = Q\tau^{1/2}$

Soft $\mu_S = Q\tau$

$p_S \sim Q(\tau,\tau,\tau)$



$p_c \sim Q(1,\tau,\tau^{1/2})$

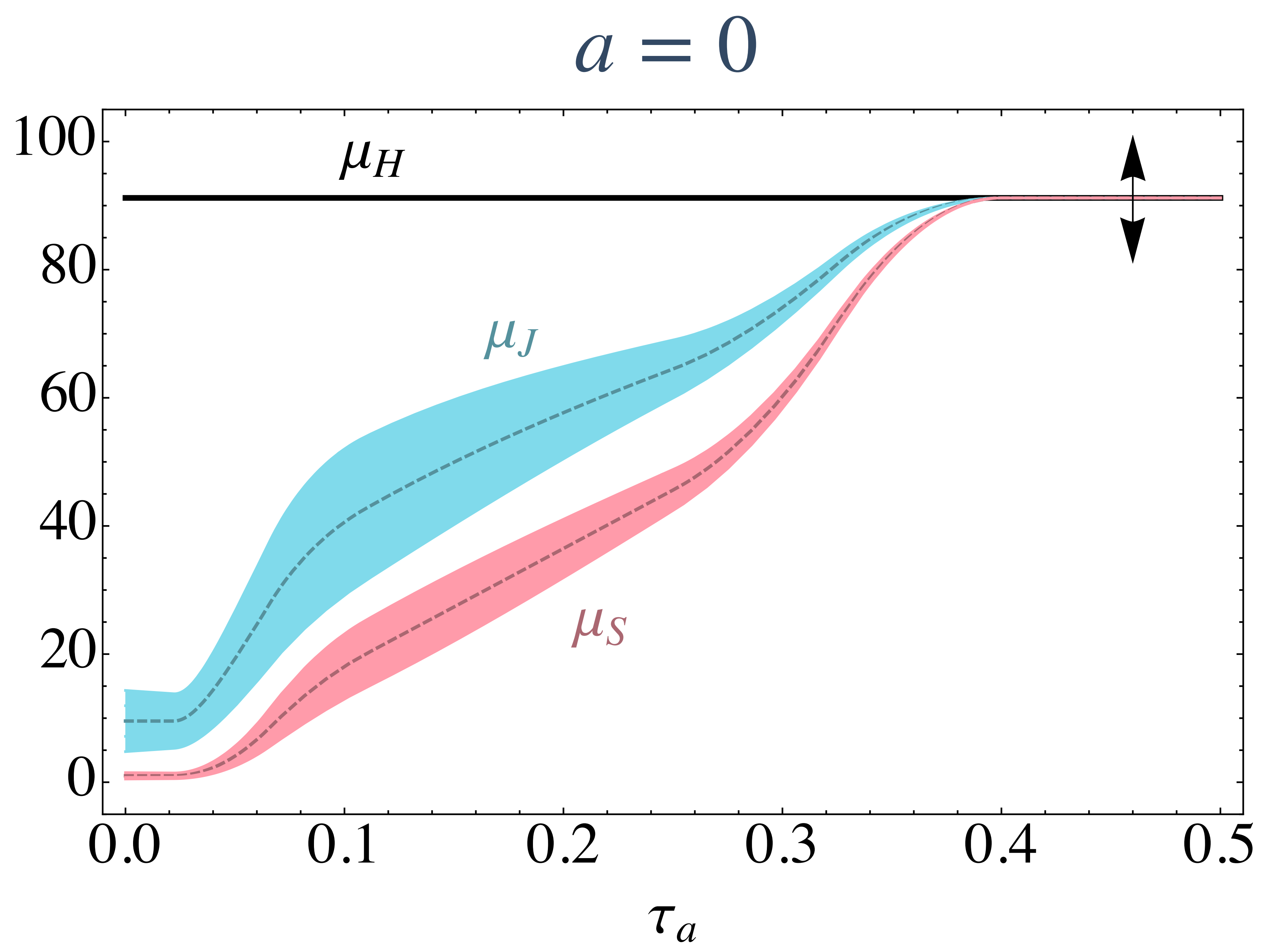
$$\gamma_F(\mu) = -j_F\kappa_F\Gamma_{\rm cusp}[\alpha_s(\mu)]\ln\frac{Q_F}{\mu} + \gamma_F[\alpha_s(\mu)]$$

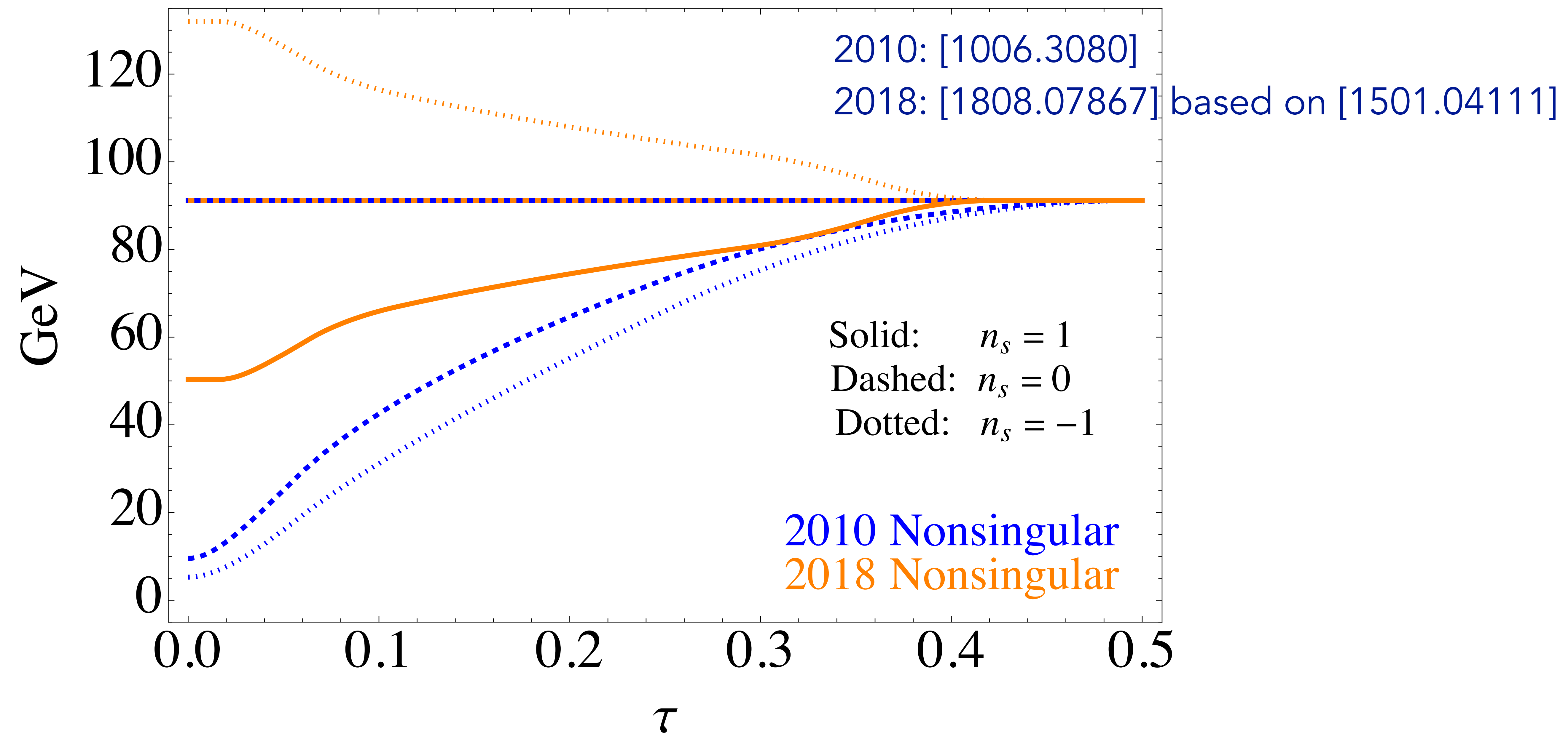
$$\Omega = 2\omega_J + \omega_S$$

$$\omega_F = -2\kappa_F\int_{\mu_F}^{\mu}\frac{d\mu'}{\mu'}\Gamma_{\rm cusp}[\alpha_s(\mu')]$$

$$K = \sum_{F=H,J,S}\int_{\mu_F}^{\mu}\frac{d\mu'}{\mu'}\gamma_F(\mu')$$

Perturbative scale profiles





Full prediction:

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_a} = \int dk\ \sigma_{PT}\left(\tau_a - \frac{k}{Q}\right) S_{NP}(k)$$

$$= \sigma^{\text{resum}}_{\text{sing}}(\tau_a; \mu_{H,J,S}) + \sigma^{\text{F.O.}}_{\text{non-sing}}(\tau_a; \mu_{ns})$$

Will consider two non-singular scale choices "2010" and "2018":

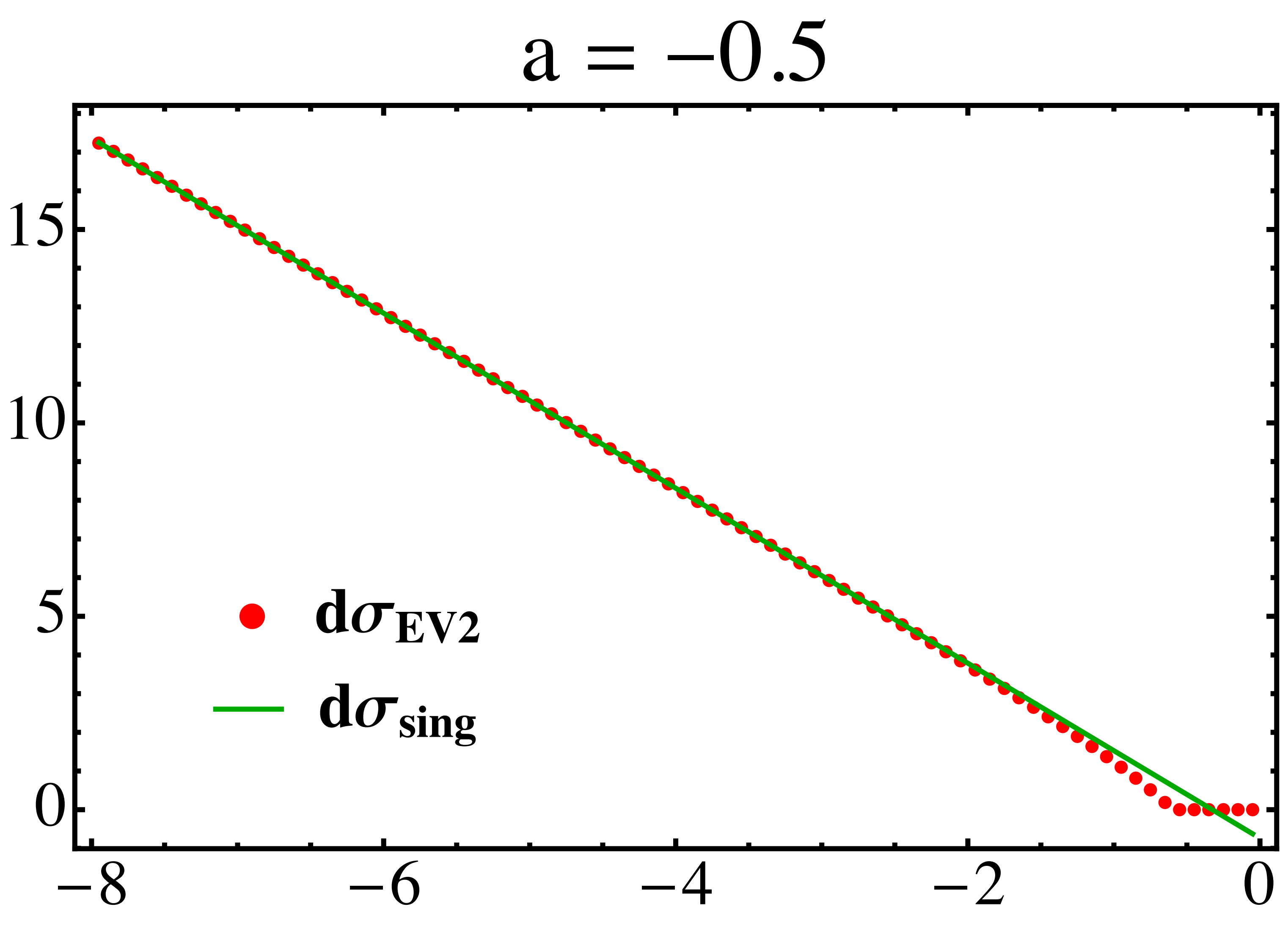
$$\mu_{\text{ns}} = \begin{cases} \mu_J & \text{default} \\ (\mu_J + \mu_S)/2 & \text{lo} \\ \mu_H & \text{hi} \end{cases} \qquad \mu_{\text{ns}} = \begin{cases} \mu_H & \text{default} \\ (\mu_H + \mu_J)/2 & \text{lo} \\ (3\mu_H - \mu_J)/2 & \text{hi} \end{cases}$$

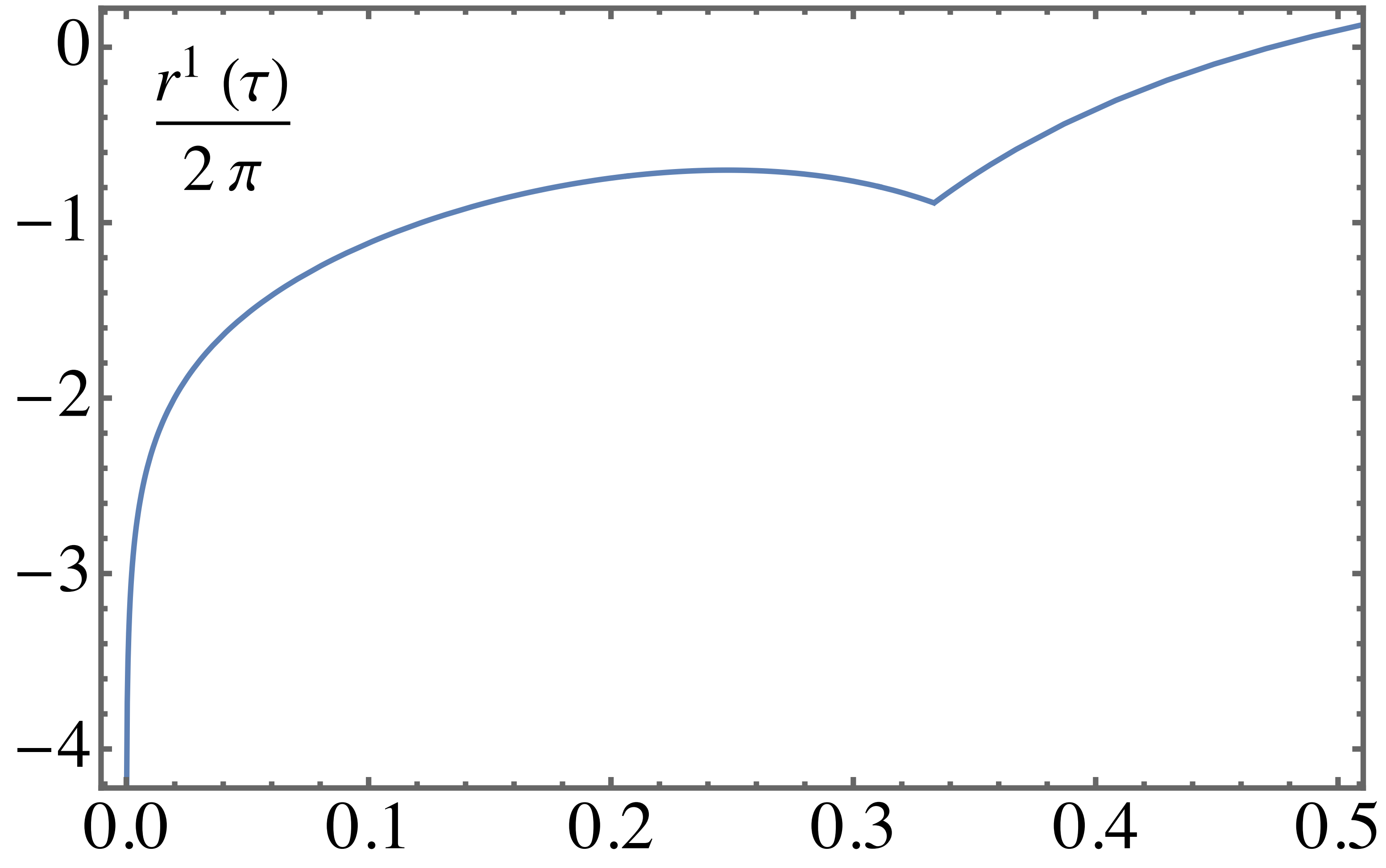
variations try to account for uncertainty due to *subleading* logs that are *not* resummed

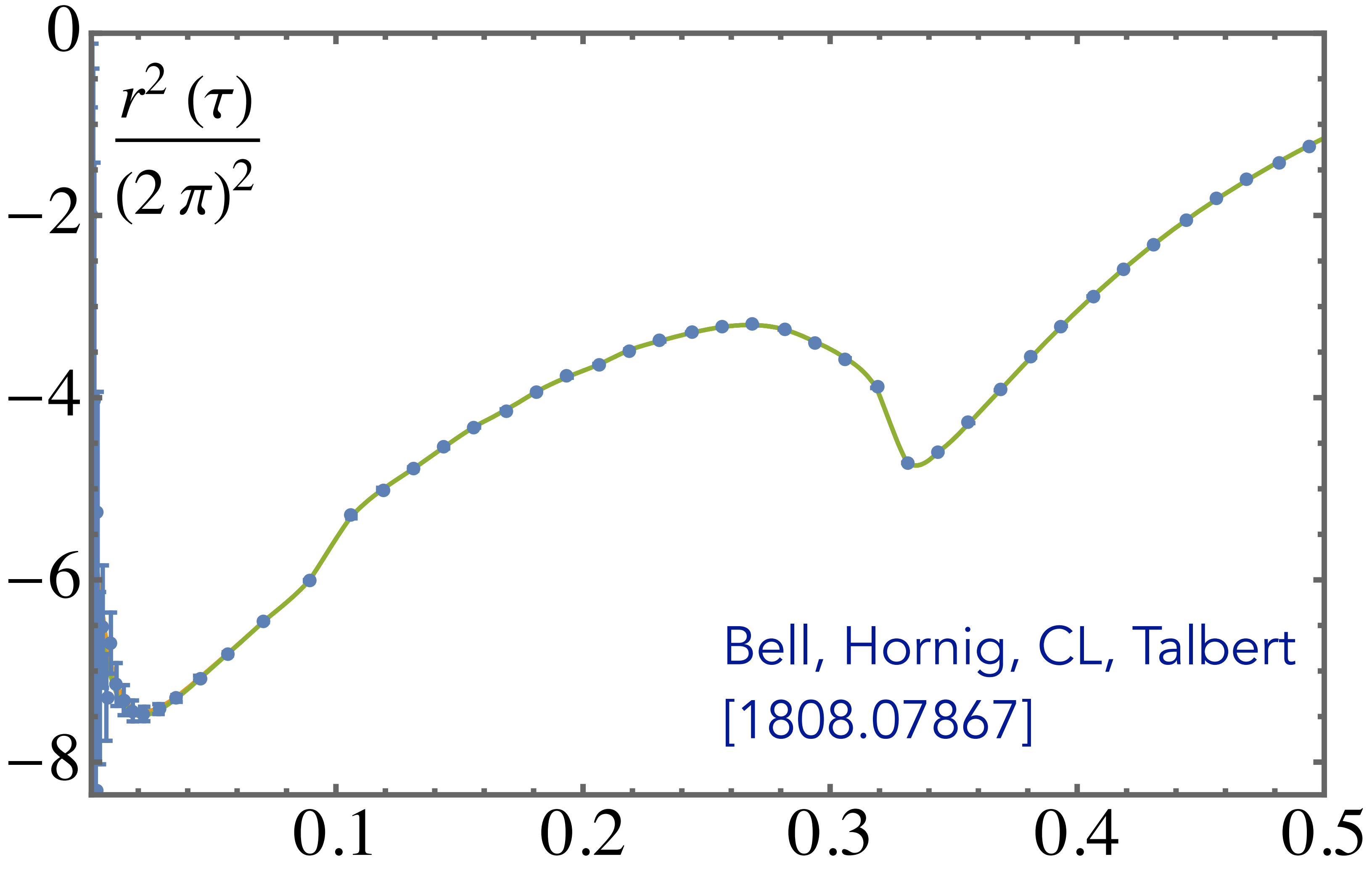
Fixed-order tails

- The above predicts the (resummed) singular component of the cross section.
  One must then match to fixed-order QCD for large τ:

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_a} - \frac{1}{\sigma_0}\frac{d\sigma_{\rm sing}}{d\tau_a} = r(\tau_a) = \frac{\alpha_s(Q)}{2\pi} r^1(\tau_a) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 r^2(\tau_a) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^3 r^3(\tau_a) + \cdots$$







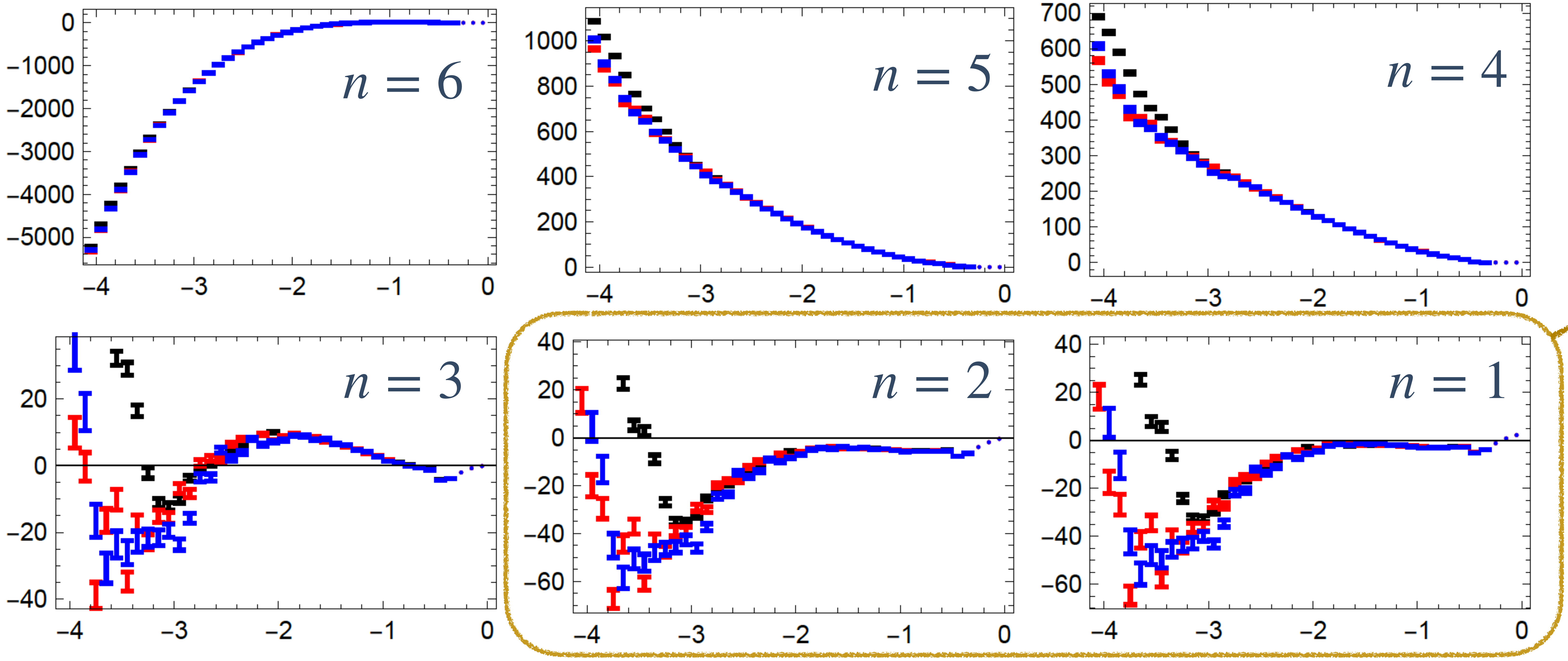
Singular behavior in EERAD3

LANL Institutional Computing clusters

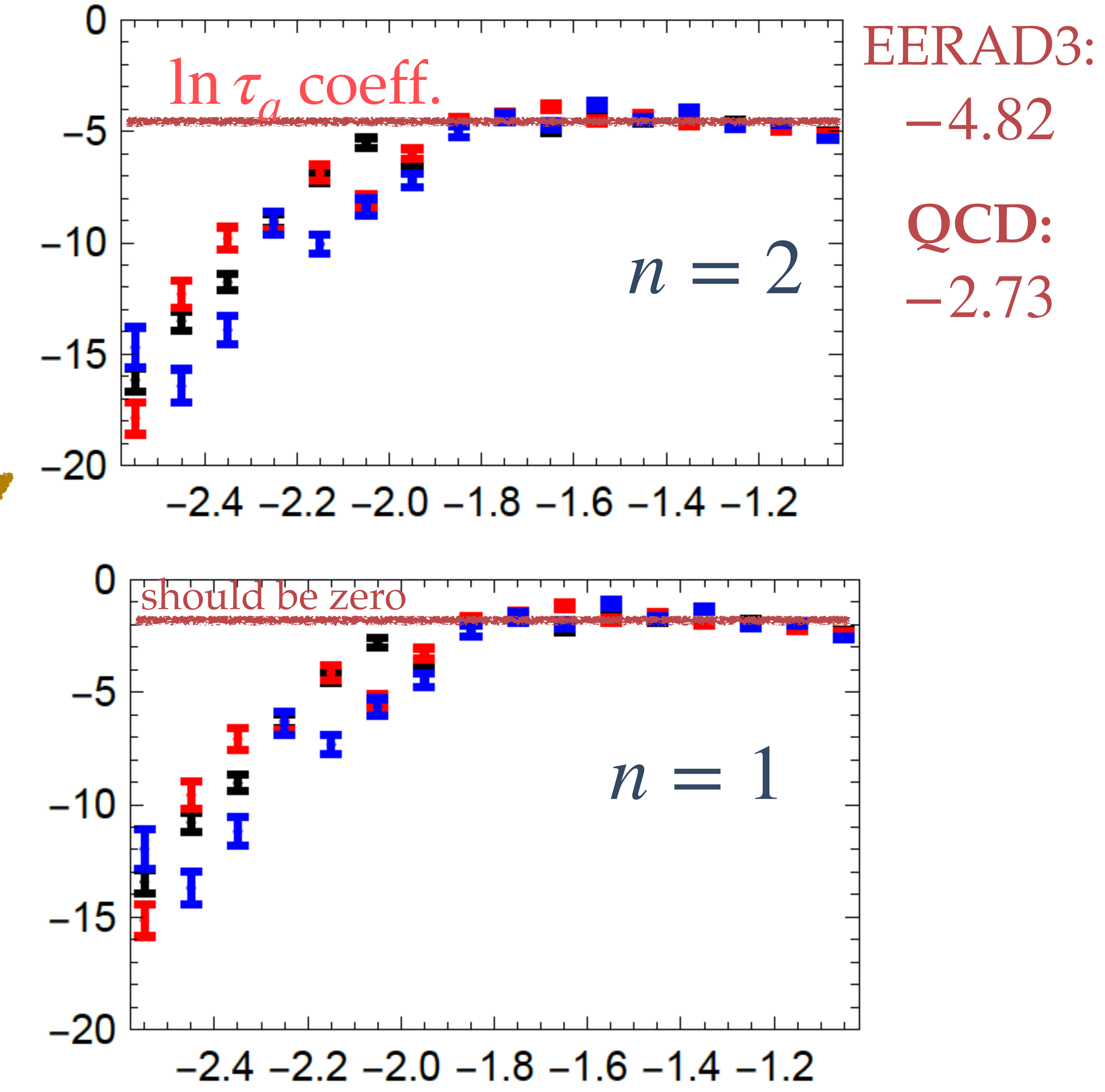
- Results for 3-loop fixed-order angularity distributions from EERAD3 (IR cutoff 10^{-7}, 1.5×10^{10} events)

$\frac{1}{\sigma}\frac{d\sigma}{d\ln\tau}$ minus $\ln^n \tau_a$ terms: for $a = 0$:

- IR cutoffs 10^{-6}, 10^{-7}, 10^{-8}



- Zoom in n=2, 1:



- After subtracting all logs, should go to zero as $\tau \to 0$ if agree with QCD, doesn't quite do so. Calls into question accuracy of nonsingular remainder function itself…

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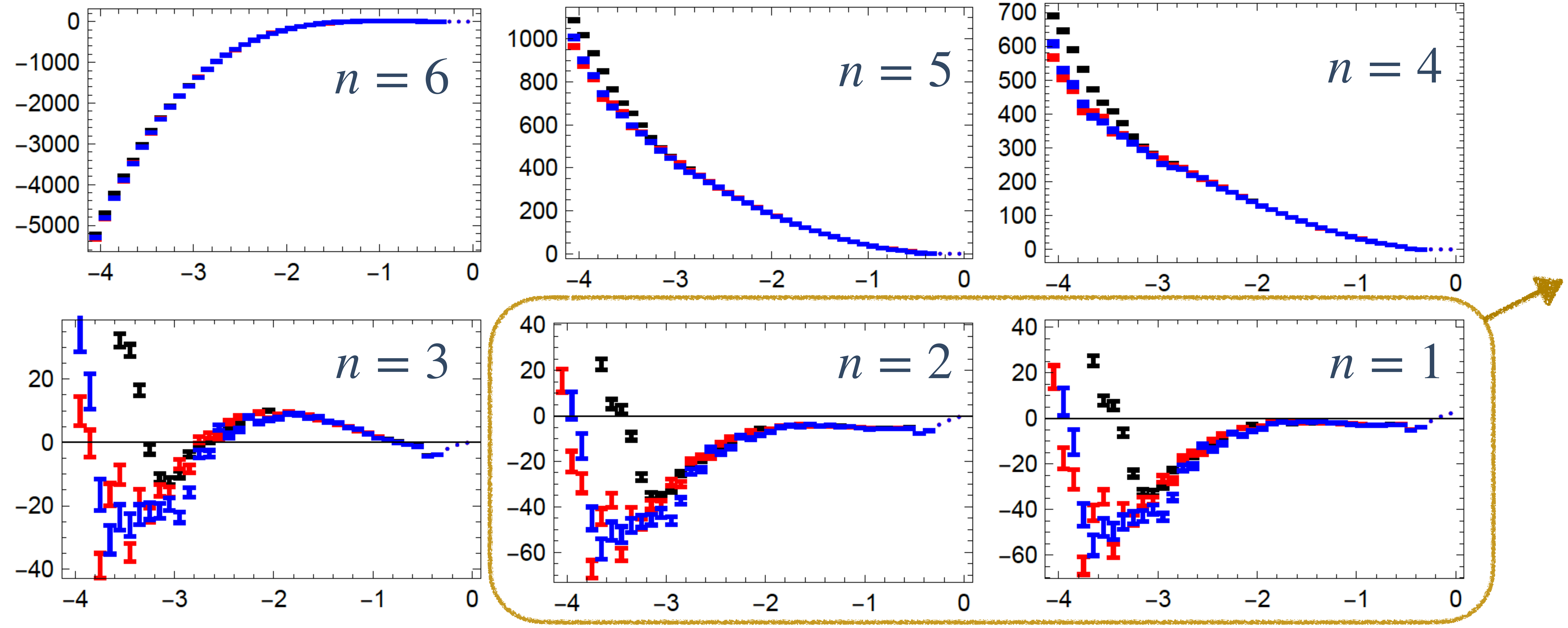
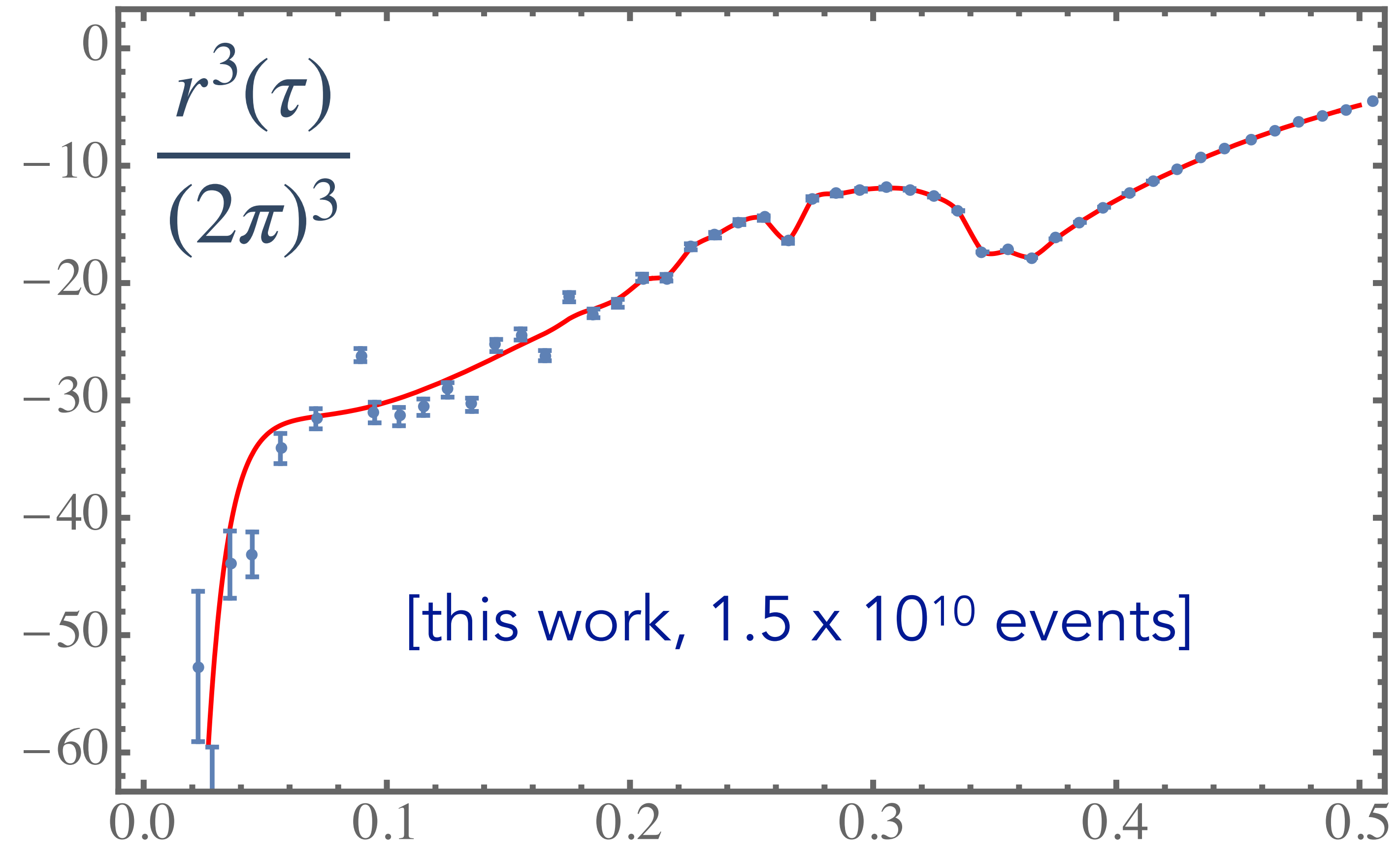
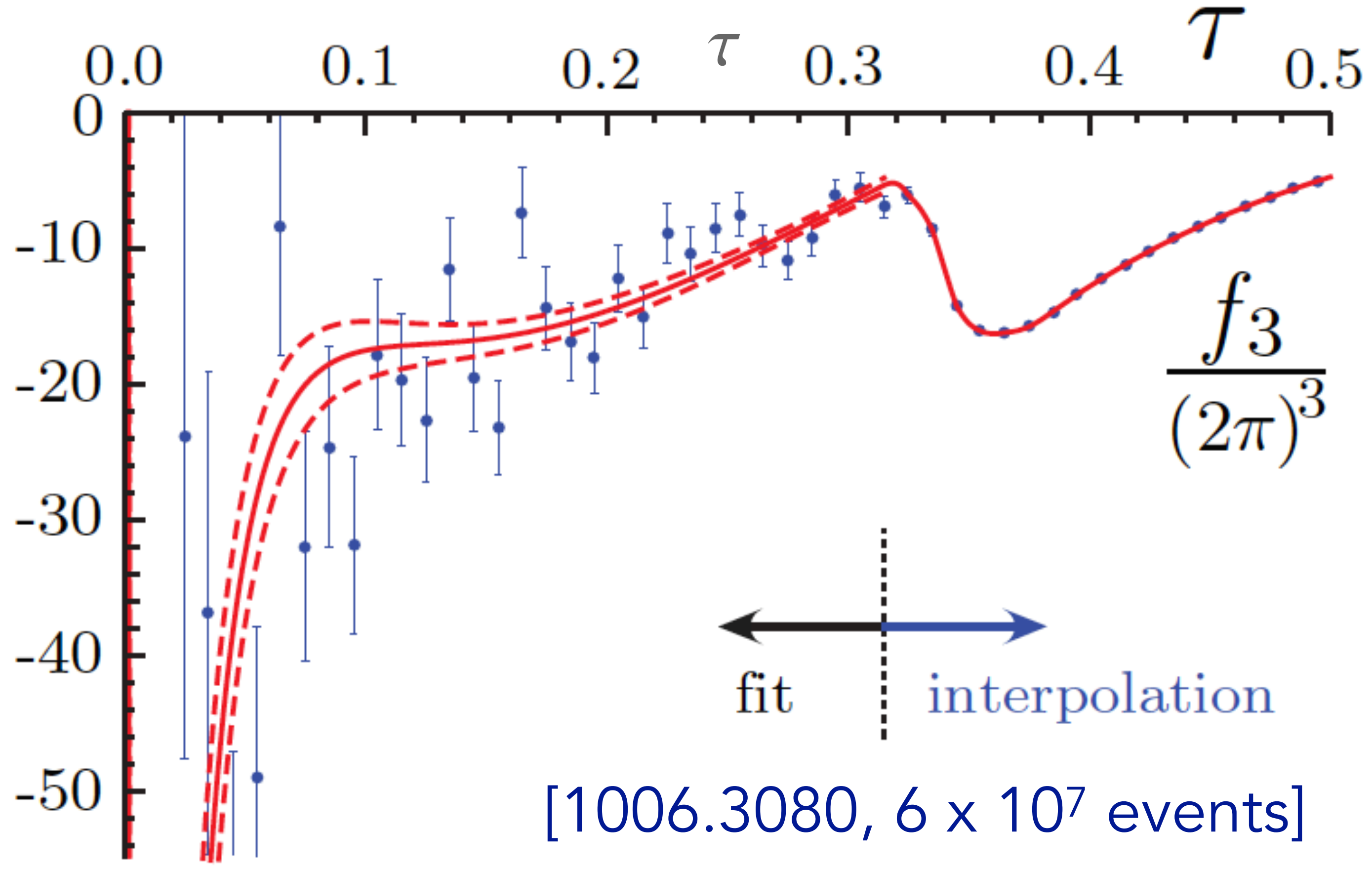


- "Finite" remainder functions, $a=0$:

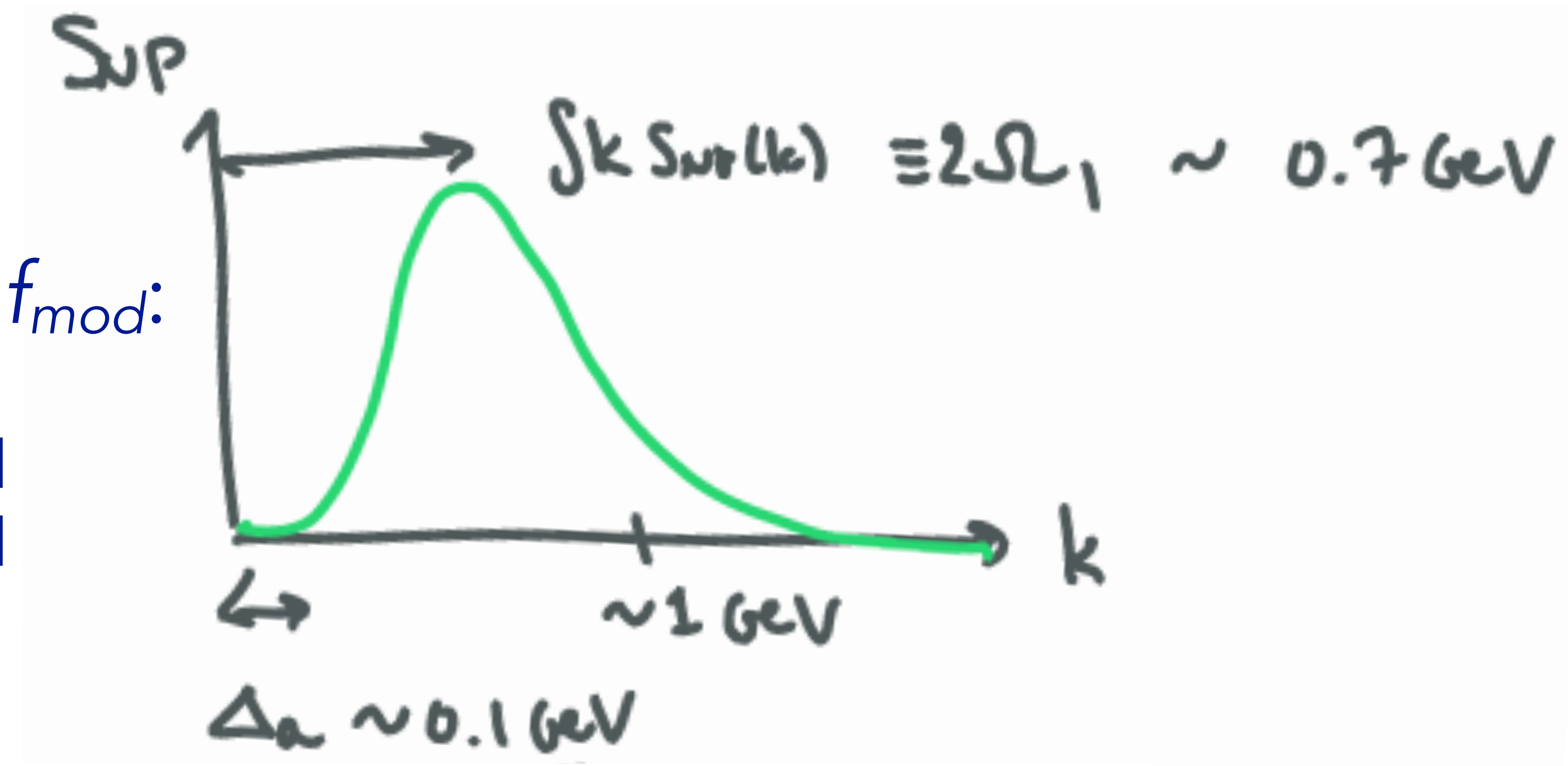




- After subtracting all logs, should go to zero as $\tau \to 0$ if agree with QCD, doesn't quite do so. Calls into question accuracy of nonsingular remainder function itself…

Non-perturbative effects and gapped soft function

- Non-perturbative effects described through the soft function:



- Perturbative matching coefficient and a low-energy model shape function f_{mod}:

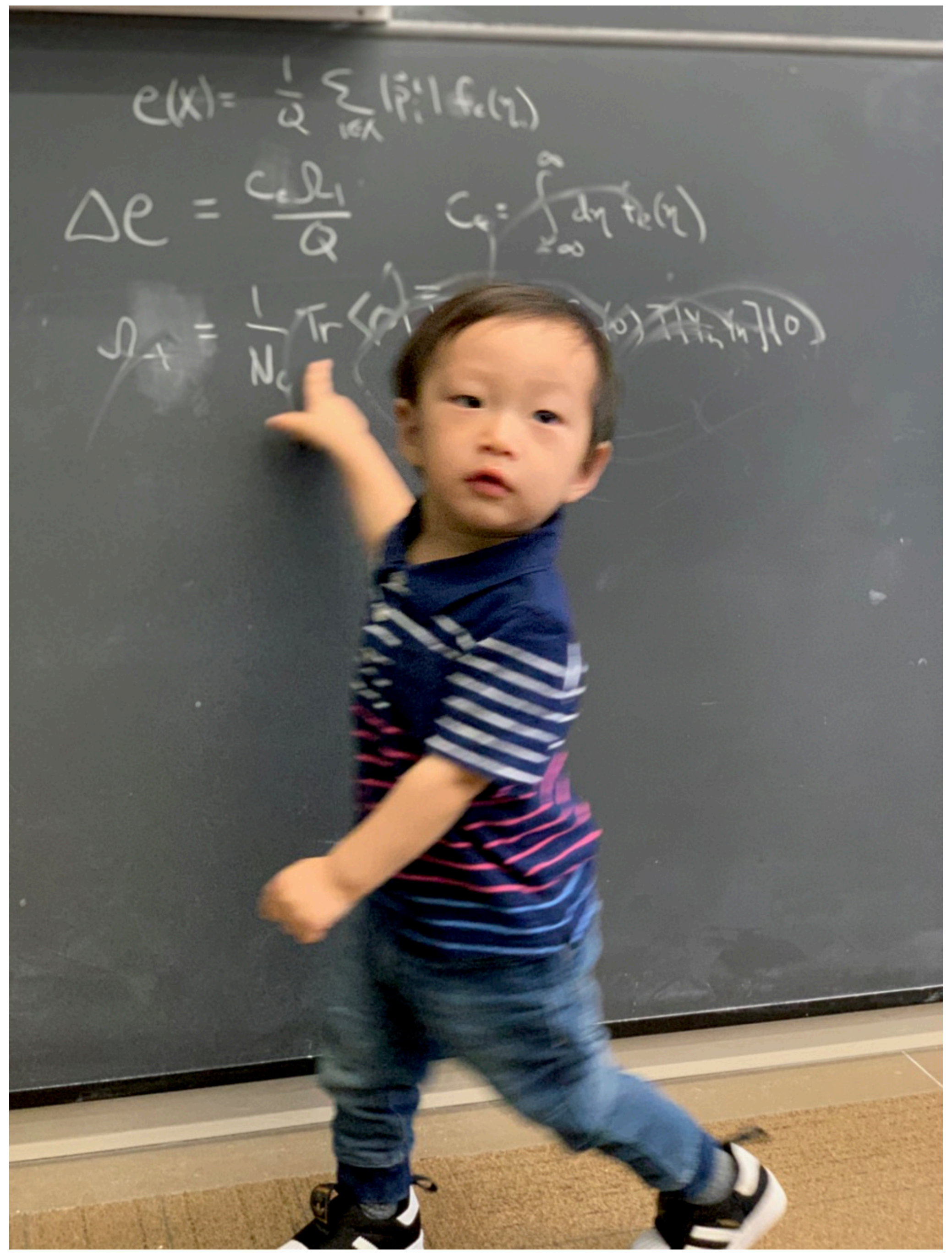
$$S(k,\mu) = \int dk' \; S_{\rm PT}(k-k',\mu)\, f_{\rm mod}(k'-2\overline{\Delta}_a)$$

[0709.3519]
[0807.1926]

'Gap' parameter accounting for parton → hadron transition

- The effect of $f_{\rm mod}$ is to shift the first moment of the perturbative distribution:

$$\langle \tau_a \rangle = \langle \tau_a \rangle_{\rm PT} + \frac{2\Omega_1}{Q(1-a)} \qquad\qquad \frac{2\Omega_1}{1-a} = 2\overline{\Delta}_a + \int dk\, k\, f_{\rm mod}(k)$$



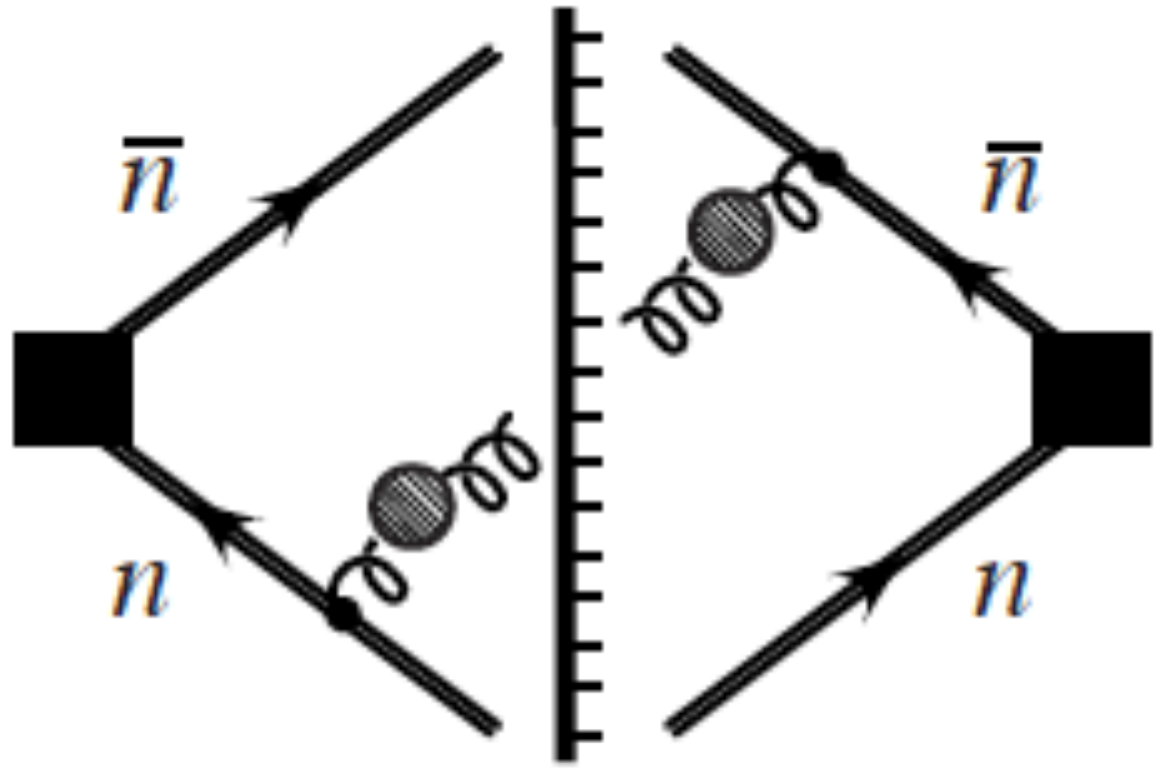
- This scaling and the *universality* of Ω_1 can be proven from QCD / SCET factorization:

Dokshitzer, Webber [hep-ph/9504219, hep-ph/9704298], Berger, Sterman [hep-ph/0307394]
C. Lee & G. Sterman [hep-ph/0611061]

Caltech, March 2019

Non-perturbative effects and gapped soft function

- However, both the perturbative soft function and gap parameter suffer renormalon ambiguities.



$$S(k,\mu) = \int dk'\, S_{\rm PT}(k-k',\mu)\, f_{\rm mod}(k'-2\overline{\Delta}_a)$$

- $\mathcal{O}(\Lambda_{\rm QCD})$ ambiguity in gap $\overline{\Delta}_a$

- Subtract a series with the same/canceling ambiguity from both PT and NP pieces:

$$\overline{\Delta}_a = \Delta_a(\mu) + \delta_a(\mu) \xrightarrow{\text{Laplace space}} \widetilde{S}(\nu,\mu) = \left[e^{-2\nu\Delta_a(\mu)}\,\widetilde{f}_{\rm mod}(\nu)\right]\left[e^{-2\nu\delta_a(\mu)}\,\widetilde{S}_{\rm PT}(\nu,\mu)\right]$$

renormalon free   *renormalon free*

R_{gap} scheme

[0803.4214]
[0806.3852]

- Choosing the $\boldsymbol{R_{gap}}$ **scheme** to cancel the leading renormalon,

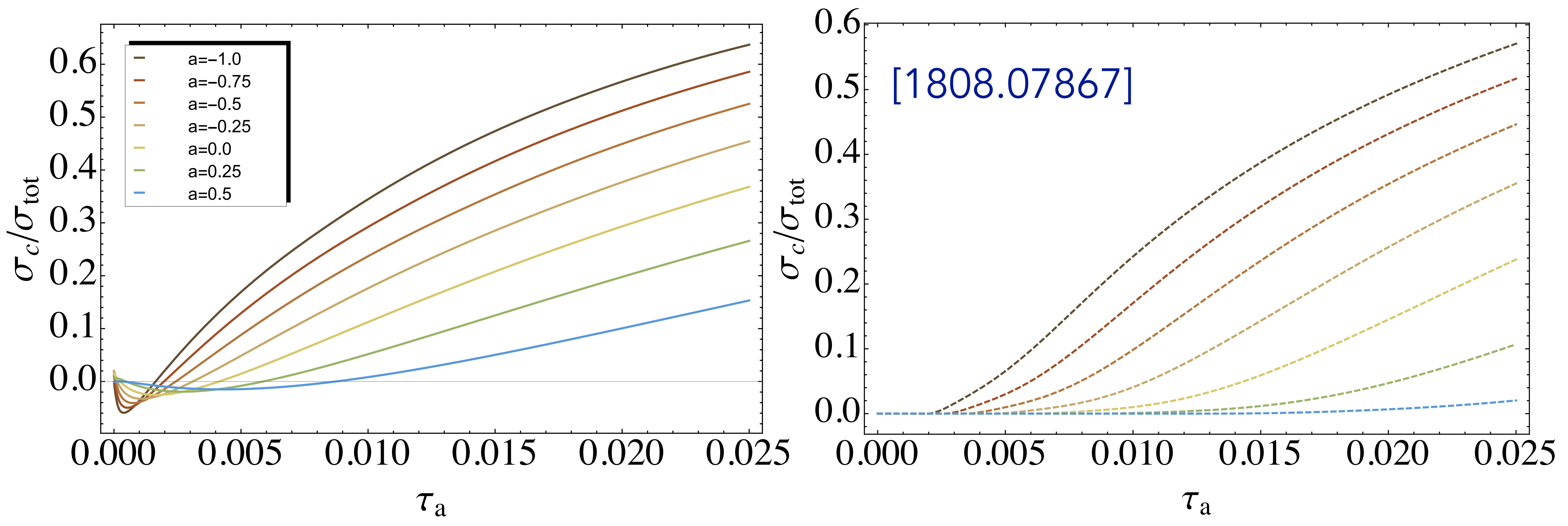
$$R e^{\gamma_E} \frac{d}{d\ln\nu}\left[\ln \widehat{S}_{\rm PT}(\nu,\mu)\right]_{\nu=1/(Re^{\gamma_E})} = 0 \longrightarrow \delta_a(\mu,R) = \frac{1}{2} R e^{\gamma_E} \frac{d}{d\ln\nu}\left[\ln \widetilde{S}_{\rm PT}(\nu,\mu)\right]_{\nu=1/(Re^{\gamma_E})},$$

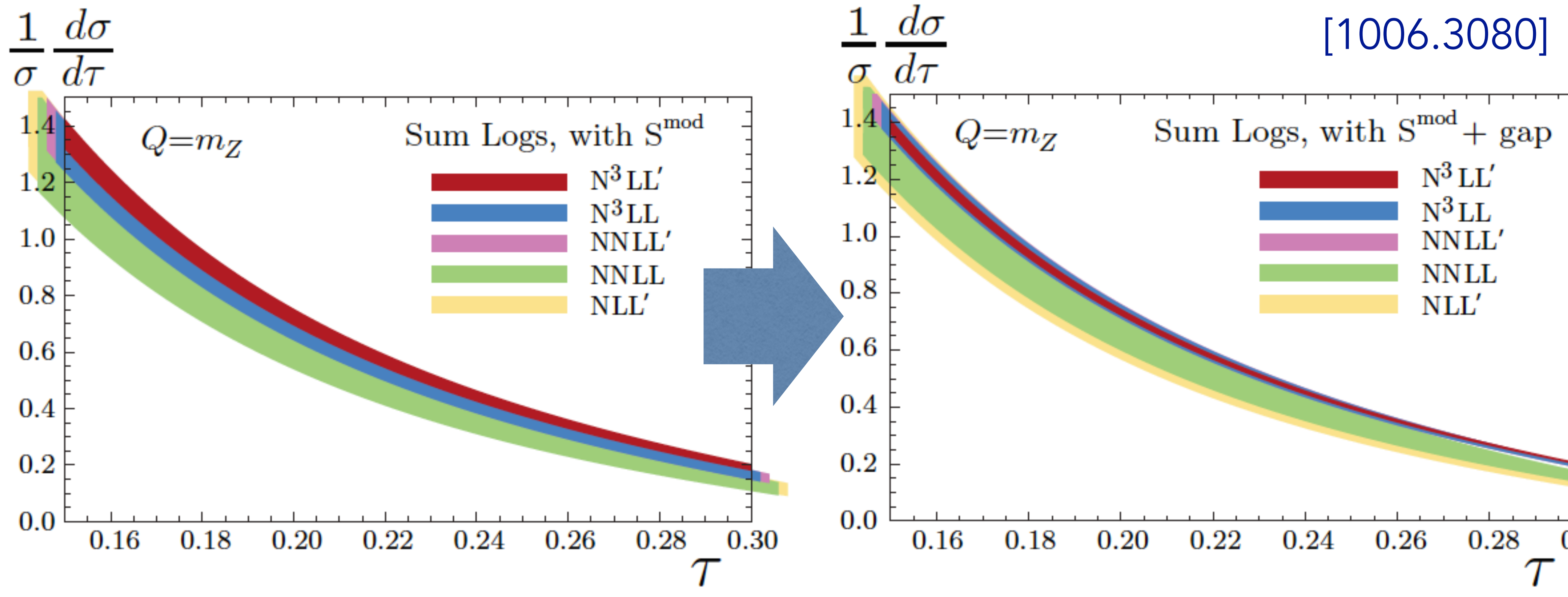
$$\widehat{S}_{\rm PT}(\nu,\mu) = e^{-2\nu\delta_a(\mu)} \widetilde{S}_{\rm PT}(\nu,\mu)$$

Gapped and renormalon free soft function
$$S(k,\mu) = \int dk'\, S_{\rm PT}(k-k',\mu)\left[e^{-2\delta_a(\mu,R)\frac{d}{dk'}} f_{\rm mod}(k'-2\Delta_a(\mu,R))\right]$$

Final cross section is expanded order-by-order in bracketed term
$$\frac{1}{\sigma_0}\sigma(\tau_a) = \int dk\, \sigma_{\rm PT}\left(\tau_a - \frac{k}{Q}\right)\left[e^{-2\delta_a(\mu_S,R)\frac{d}{dk}} f_{\rm mod}(k-2\Delta_a(\mu_S,R))\right]$$

- Improves small τ_a behavior and perturbative convergence:





R_{gap} scheme

[0803.4214]
[0806.3852]

- Choosing the ***R***

$$Re^{\gamma_E}\frac{d}{d\ln\nu}\Big[\ln\widehat{S}_{\rm PT}(\nu,\mu)\Big]_{\nu=1/(Re^{\gamma_E})}=0 \longrightarrow \delta_a(\mu,R)=\frac{1}{2}Re^{\gamma_E}\frac{d}{d\ln\nu}\Big[\ln\widetilde{S}_{\rm PT}(\nu,\mu)\Big]_{\nu=1/(Re^{\gamma_E})},$$

$$\delta(\mu,R)=\frac{Re^{\gamma_E}}{2}\sum_{n=1}^{\infty}\left(\frac{\alpha_S(\mu)}{4\pi}\right)^n\delta^n(\mu_S,R)$$

$$\delta^1(\mu_S,R)=2\Gamma_s^0\ln\frac{\mu_S}{R}$$

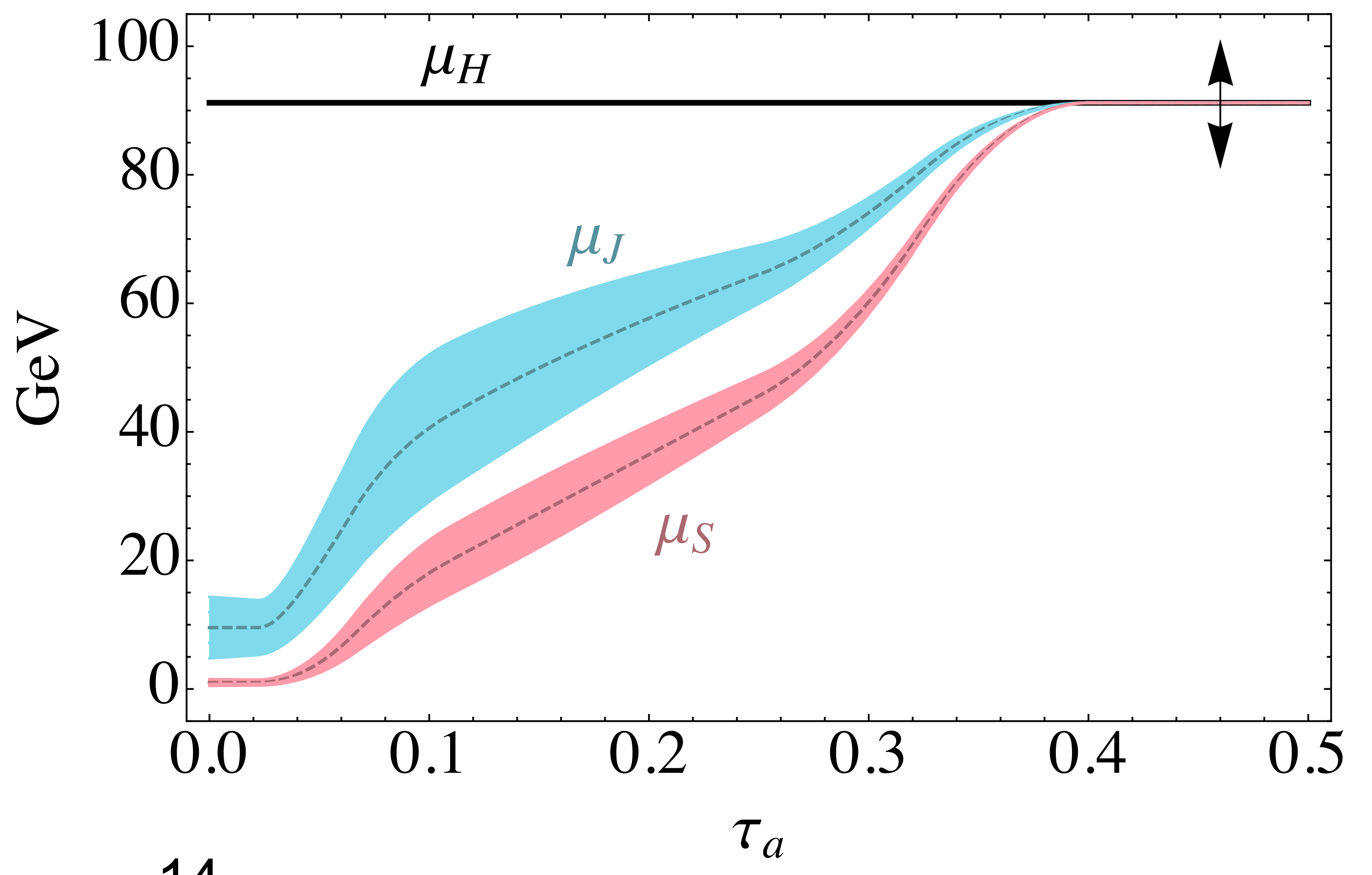
$$\delta^2(\mu_S,R)=2\Gamma_s^0\beta_0\ln^2\frac{\mu_S}{R}+2\Gamma_s^1\ln\frac{\mu_S}{R}+\gamma_s^1+2c_{\tilde{S}}^1\beta_0$$

$$\delta^3(\mu_S,R)=\frac{8}{3}\Gamma_s^0\beta_0^2\ln^3\frac{\mu_S}{R}+2(2\Gamma_s^1\beta_0+\Gamma_s^0\beta_1)\ln^2\frac{\mu_S}{R}+2(\Gamma_s^2+2\gamma_s^1\beta_0+4c_{\tilde{S}}^1\beta_0^2)\ln\frac{\mu_S}{R}$$
$$+\gamma_s^2+2c_{\tilde{S}}^1\beta_1+4c_{\tilde{S}}^2\beta_0-2(c_{\tilde{S}}^1)^2\beta_0$$

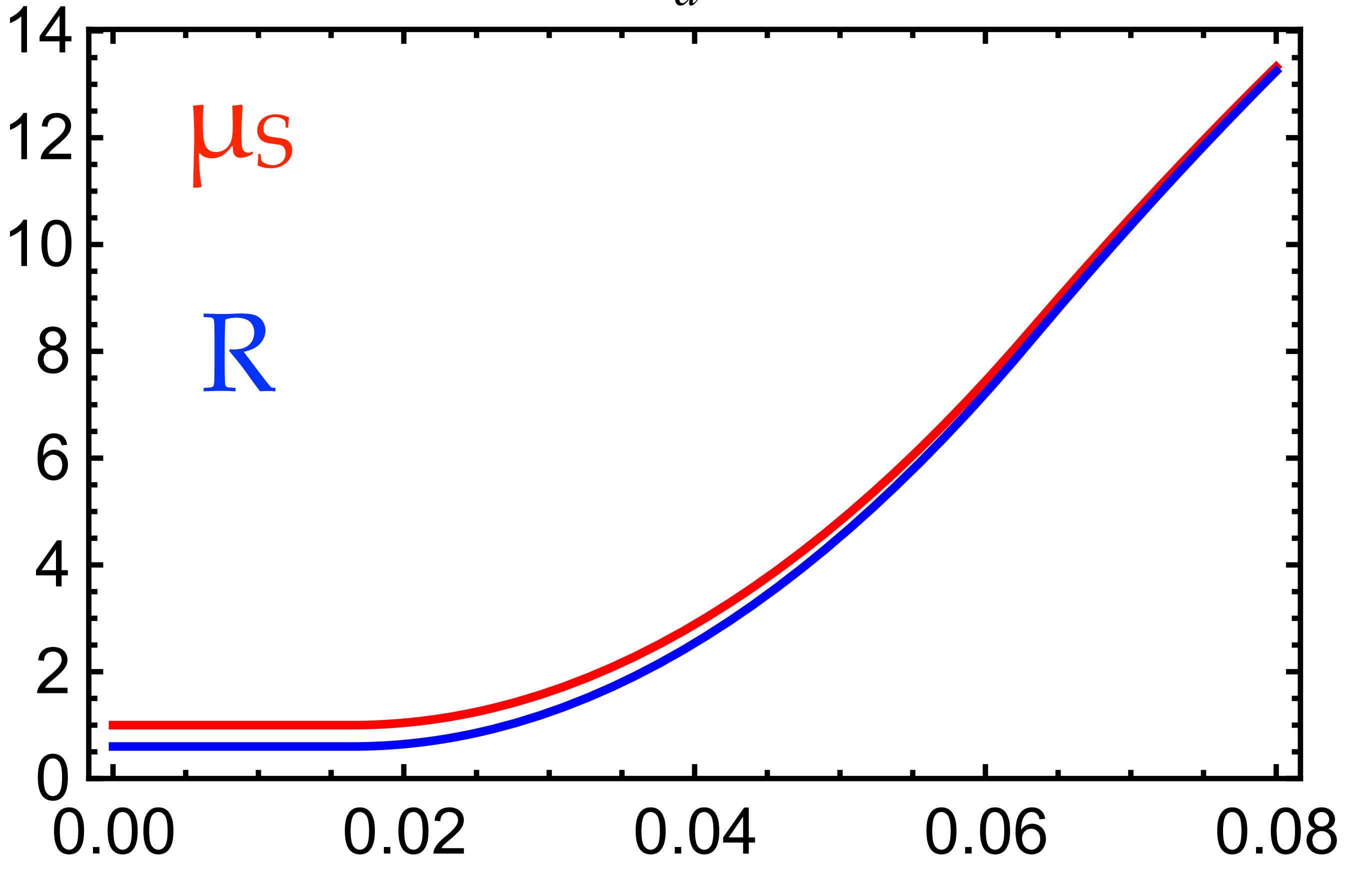
R-evolution

[0801.0743]
[0908.3189]

- Want to keep R near IR scales, but also avoid large logs $\ln \frac{\mu_S}{R}$ in subtraction terms



- but μ_S grows to be as large as Q:



τ_0.

- Sum logs by μ and R evolution: $\mu \frac{d}{d\mu} \Delta_a(\mu, R) = -\mu \frac{d}{d\mu} \delta_a(\mu, R) \equiv \gamma_\Delta^\mu[\alpha_s(\mu)]$

$$\frac{d}{dR} \Delta_a(R, R) = -\frac{d}{dR} \delta_a(R, R) \equiv -\gamma_R[\alpha_s(R)]$$

- Anomalous dimensions:

$$\gamma_\Delta^\mu[\alpha_s(\mu)] = -Re^{\gamma_E} \Gamma_S[\alpha_s(\mu)]$$

$$\gamma_R[\alpha_s(R)] = \sum_{n=0}^{\infty} \left(\frac{\alpha_s(R)}{4\pi} \right)^{n+1} \gamma_R^n \qquad \gamma_R^0 = 0 , \quad \gamma_R^1 = \frac{e^{\gamma_E}}{2} [\gamma_S^1(a) + 2c_{\tilde{S}}^1 \beta_0]$$

$$\gamma_R^2 = \frac{e^{\gamma_E}}{2} \left[\gamma_s^2 + 2c_{\tilde{S}}^1 \beta_1 + 4c_{\tilde{S}}^2 \beta_0 - 2(c_{\tilde{S}}^1)^2 \beta_0 - 4\gamma_s^1 \beta_0 - 8c_{\tilde{S}}^1 \beta_0^2 \right]$$

Effective non-perturbative shifts

- Before considering gapped renormalons, the leading-order NP effect is a constant shift:

$$\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow[\text{NP}]{} \frac{d\sigma}{d\tau_a}\left(\tau_a - c_{\tau_a}\frac{\Omega_1}{Q}\right) \qquad c_{\tau_a} = \frac{2}{1-a} \qquad \Omega_1 = \frac{1}{N_C}\text{Tr}\,\langle 0|\,\overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(0)\, Y_n \overline{Y}_{\bar{n}}\,|0\rangle$$

Note: this is only valid in the tail region!

- Define an 'effective shift' of the distribution in the R_{gap} scheme:

$$\int dk\, k\, e^{-2\delta_a(\mu_S,R)\frac{d}{dk}} f_{\text{mod}}\left(k - 2\Delta_a(\mu_S,R)\right) = \int dk\, k\left[\sum_i f_{\text{mod}}^{(i)}\left(k - 2\Delta_a(\mu_S,R)\right)\right] \equiv \frac{2}{1-a}\Omega_1^{\text{eff}}$$

- Shape function expanded order-by-order depending on logarithmic accuracy:

$$\bar{f}_{\text{mod}}^{(0)}(k-2\Delta) = f_{\text{mod}}(k-2\Delta) \qquad f_{\text{mod}}^{(i)} = \left(\frac{\alpha_s(\mu_S)}{4\pi}\right)^i \bar{f}_{\text{mod}}^{(i)}$$

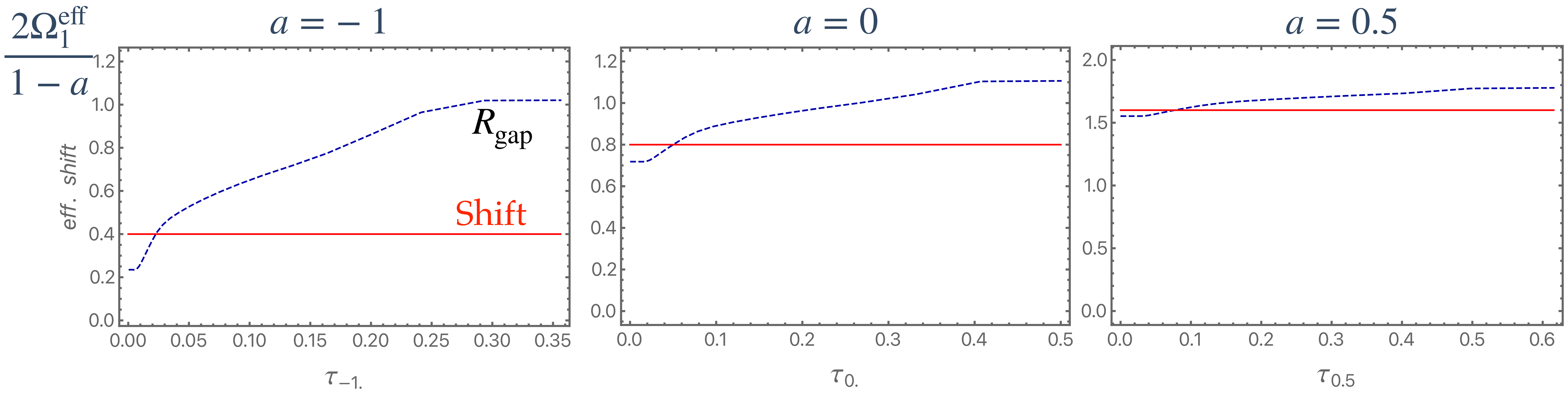
$$\bar{f}_{\text{mod}}^{(1)}(k-2\Delta) = -2(Re^{\gamma_E})\delta^1 f'_{\text{mod}}(k-2\Delta)$$

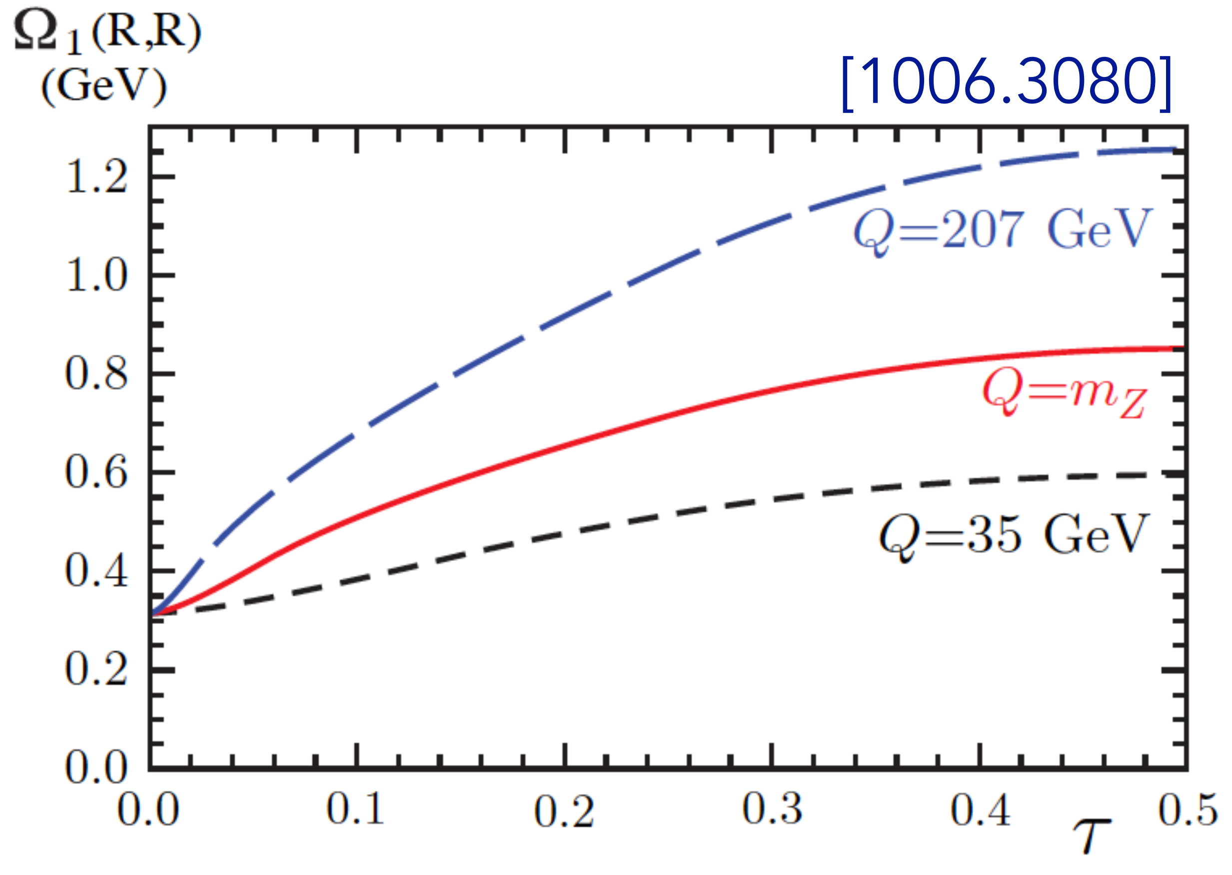
$$\bar{f}_{\text{mod}}^{(2)}(k-2\Delta) = -2(Re^{\gamma_E})\delta^2 f'_{\text{mod}}(k-2\Delta) + 2(Re^{\gamma_E}\delta^1)^2 f''_{\text{mod}}(k-2\Delta)$$

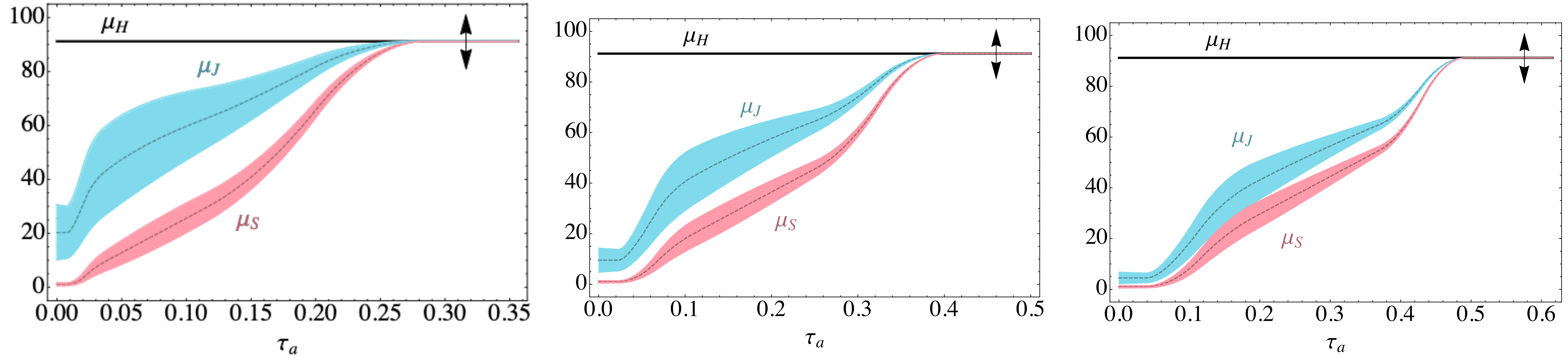
$$\bar{f}_{\text{mod}}^{(3)}(k-2\Delta) = -2(Re^{\gamma_E})\delta^3 f'_{\text{mod}}(k-2\Delta) + 4\delta^1\delta^2(Re^{\gamma_E})^2 f''_{\text{mod}}(k-2\Delta) - \frac{4}{3}(Re^{\gamma_E}\delta_1)^3 f'''_{\text{mod}}(k-2\Delta)$$

Growing shifts in event shape tails

- Distributional shifts at NNLL′ accuracy (central profile scales):







- Effectively, we shift the distribution to the right by *larger* amounts as we move from the 2-jet region out to the multi-jet tail. Is this reasonable? What might be the effect on extracting α_s? Can we find a way to vary it?

Limiting (or varying) the growth of the shift

- Can we find a way to cut off the growth of this shift? i.e. turn off R-evolution above some $\tau = \tau_{\max}$:

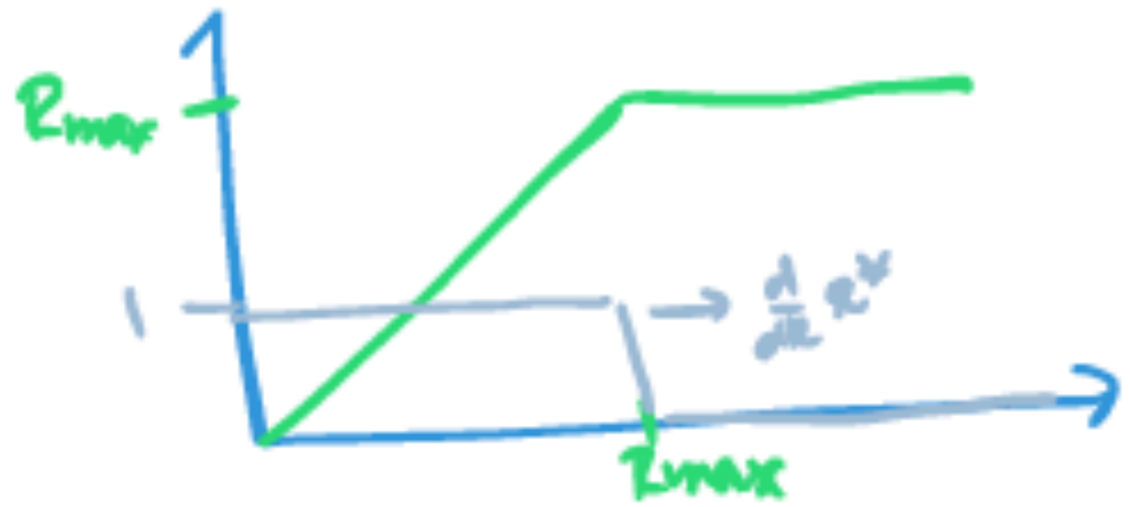
$$\gamma_R \to \theta(R_{\max} - R)\gamma_R \qquad\qquad R = R(\tau)$$

need: $$\frac{d}{dR}\delta_a(R,R) = \gamma_R[\alpha_s(R)]\theta(R_{\max} - R)$$

recall: $$\delta_a(R,R) = Re^{\gamma_E}\Big[\frac{\alpha_s(R)}{4\pi}\delta_a^1(R,R) + \Big(\frac{\alpha_s(R)}{4\pi}\Big)^2\delta_a^2(R,R) + \cdots\Big]$$

to the order we need, just change R to:

$$R^* \equiv \begin{cases} R & R < R_{\max} \\ R_{\max} & R \geq R_{\max} \end{cases}$$



however this can reintroduce large logs of $\mu_S/R_{\max}$ …

in R_{gap}:

$$\delta^1(\mu_S, R) = 2\Gamma_s^0 \ln\frac{\mu_S}{R}$$

$$\delta^2(\mu_S, R) = 2\Gamma_s^0\beta_0 \ln^2\frac{\mu_S}{R} + 2\Gamma_s^1 \ln\frac{\mu_S}{R} + \gamma_s^1 + 2c_{\tilde{S}}^1\beta_0$$

Another scheme

"R^* scheme"

$$\delta_a^*(R) = \frac{1}{2} R^* e^{\gamma_E} \frac{d}{d\ln\nu} \left[\ln S_{\rm PT}(\nu, \mu = R^*)\right]_{\nu = 1/(R^* e^{\gamma_E})}$$

Bell et al. [this work]

we are not forced to set $\mu = \mu_S$ in the subtraction series, we can pick $\mu = R$

Bachu, Hoang, Mateu, Pathak, Stewart [2012.12304]

To the order we work:

$$\delta_a^*(R) = \frac{R e^{\gamma_E}}{2} \left[\frac{\alpha_s(R)}{4\pi} \cdot 0 + \left(\frac{\alpha_S(R)}{4\pi}\right)^2 (\gamma_S^1 + 2c_{\tilde S}^1 \beta_0) + \left(\frac{\alpha_S(R)}{4\pi}\right)^3 \left(\gamma_S^2 + 2c_{\tilde S}^1 \beta_1 + 4c_{\tilde S}^2 \beta_0 - 2(c_{\tilde S}^1)^2 \beta_0 - 4\gamma_s^1 \beta_0 - 8c_{\tilde S}^1 \beta_0^2\right) + \mathcal{O}(\alpha_s^4) \right]$$

R-evolution:

$$\gamma_R^* = e^{\gamma_E} \left[\frac{\alpha_s(R)}{4\pi} \cdot 0 + \left(\frac{\alpha_S(R)}{4\pi}\right)^2 (\gamma_S^1 + 2c_{\tilde S}^1 \beta_0) + \left(\frac{\alpha_S(R)}{4\pi}\right)^3 \left(\gamma_S^2 + 2c_{\tilde S}^1 \beta_1 + 4c_{\tilde S}^2 \beta_0 - 2(c_{\tilde S}^1)^2 \beta_0 - 4\gamma_s^1 \beta_0 - 8c_{\tilde S}^1 \beta_0^2\right) + \mathcal{O}(\alpha_s^4) \right]$$

μ-evolution: $\gamma_\Delta[\alpha_s(\mu)] = 0$

no large logs of μ_S/R, yet…

- Nothing special about this scheme, just a way to test the impact of changing the effective shift in event shapes.

Another scheme

"R^* scheme"

$$\delta_a^*(R) = \frac{1}{2} R^* e^{\gamma_E} \frac{d}{d\ln\nu}\left[\ln S_{\rm PT}(\nu, \mu = R^*)\right]_{\nu = 1/(R^* e^{\gamma_E})}$$

Bell et al. [2311.03990]

we are not forced to set $\mu = \mu_S$ in the subtraction series, we can pick $\mu = R$

Bachu, Hoang, Mateu, Pathak, Stewart [2012.12304]

To the order we work:

$$\delta_a^*(R) = \frac{Re^{\gamma_E}}{2}\left[\frac{\alpha_s(R)}{4\pi}\cdot 0 + \left(\frac{\alpha_S(R)}{4\pi}\right)^2 (\gamma_S^1 + 2c_{\tilde{S}}^1\beta_0) + \left(\frac{\alpha_S(R)}{4\pi}\right)^3 \left(\gamma_S^2 + 2c_{\tilde{S}}^1\beta_1 + 4c_{\tilde{S}}^2\beta_0 - 2(c_{\tilde{S}}^1)^2\beta_0 - 4\gamma_s^1\beta_0 - 8c_{\tilde{S}}^1\beta_0^2\right) + \mathcal{O}(\alpha_s^4)\right]$$

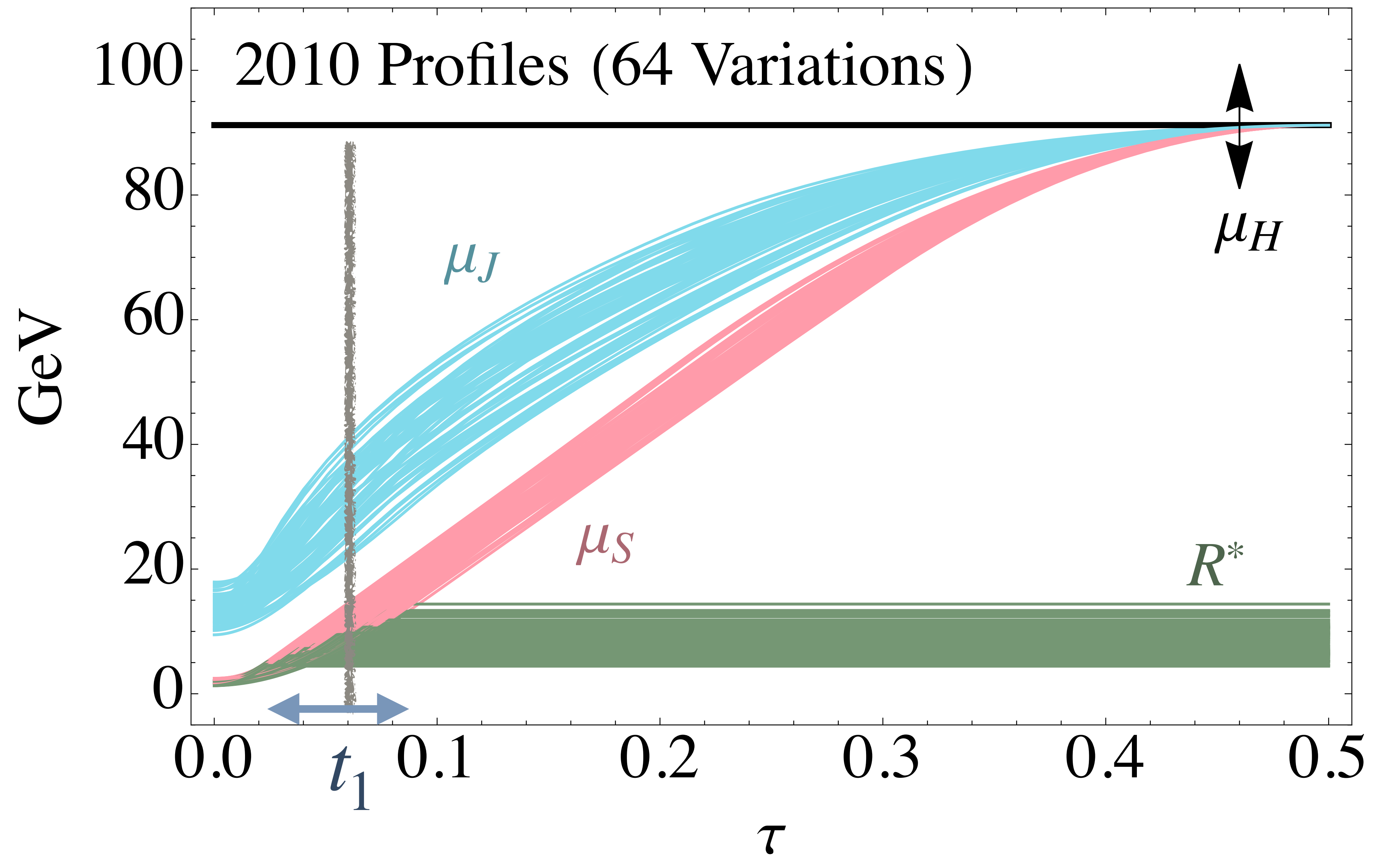
$$\delta_a^*(R) = \frac{Re^{\gamma_E}}{2}\left[\frac{\alpha_s(\mu_S)}{4\pi}\cdot 0 + \left(\frac{\alpha_S(\mu_S)}{4\pi}\right)^2 (\gamma_S^1 + 2c_{\tilde{S}}^1\beta_0) + \left(\frac{\alpha_S(\mu_S)}{4\pi}\right)^3 \left(\gamma_S^2 + 2c_{\tilde{S}}^1\beta_1 + 4c_{\tilde{S}}^2\beta_0 - 2(c_{\tilde{S}}^1)^2\beta_0 - 4\gamma_s^1\beta_0 - 8c_{\tilde{S}}^1\beta_0^2 + 4\beta_0 \ln\frac{\mu_S}{R}(\gamma_S^1 + 2c_{\tilde{S}}^1\beta_0)\right) + \mathcal{O}(\alpha_s^4)\right]$$

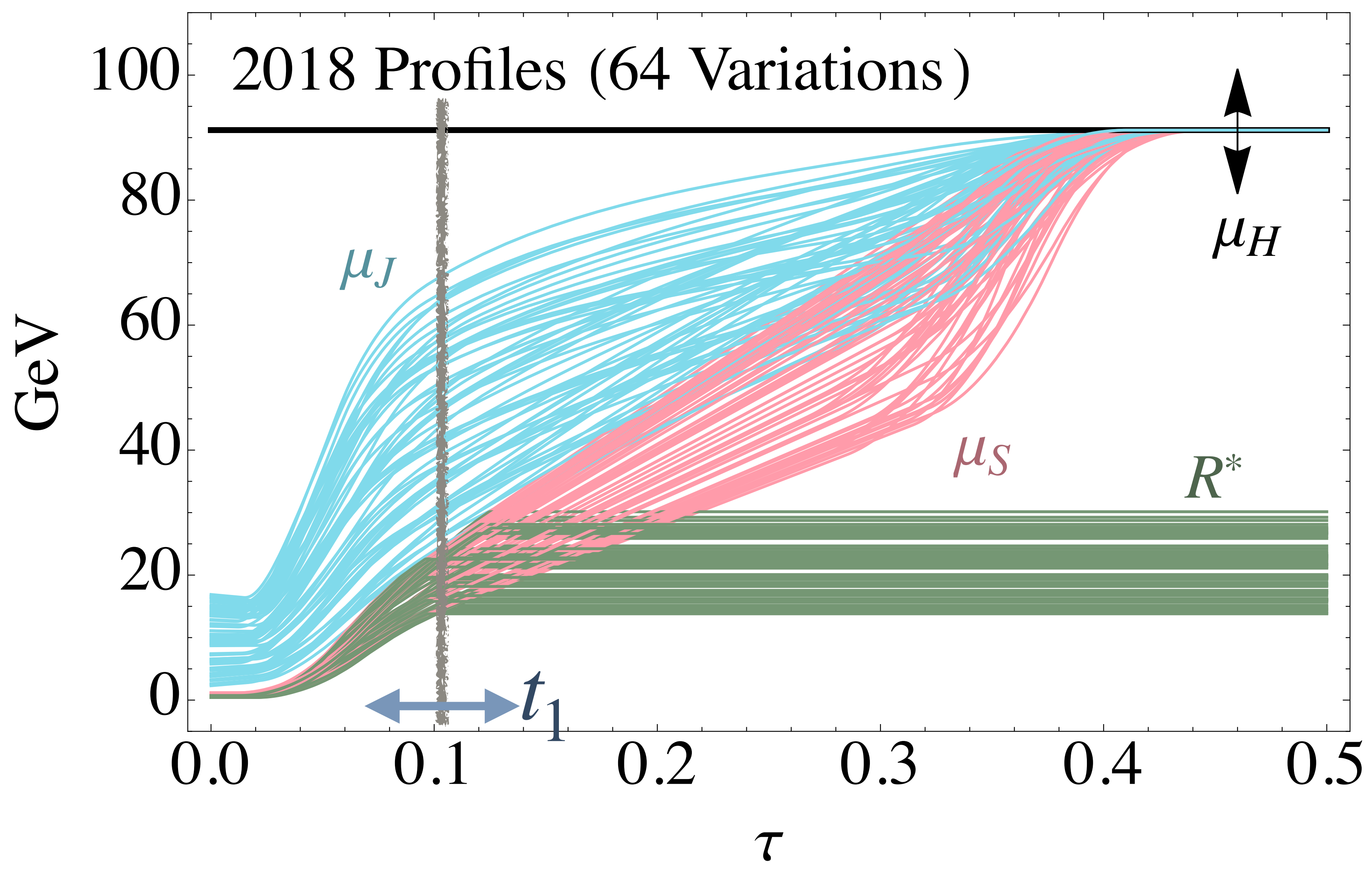
- So, not perfect, but modest effect at $\mathcal{O}(\alpha_s^3)$, simply gives us a handle to study change of "effective shift"

Scale variations and R vs R* profiles

- In our results, we let R^* grow until we hit $\tau_a = t_1(a)$, where we finish transitioning from "shape function" region to "resummation region" in profile functions:
- Random scans over profile function parameters:

2010: [1006.3080]
2018: [1808.07867] based on [1501.04111]



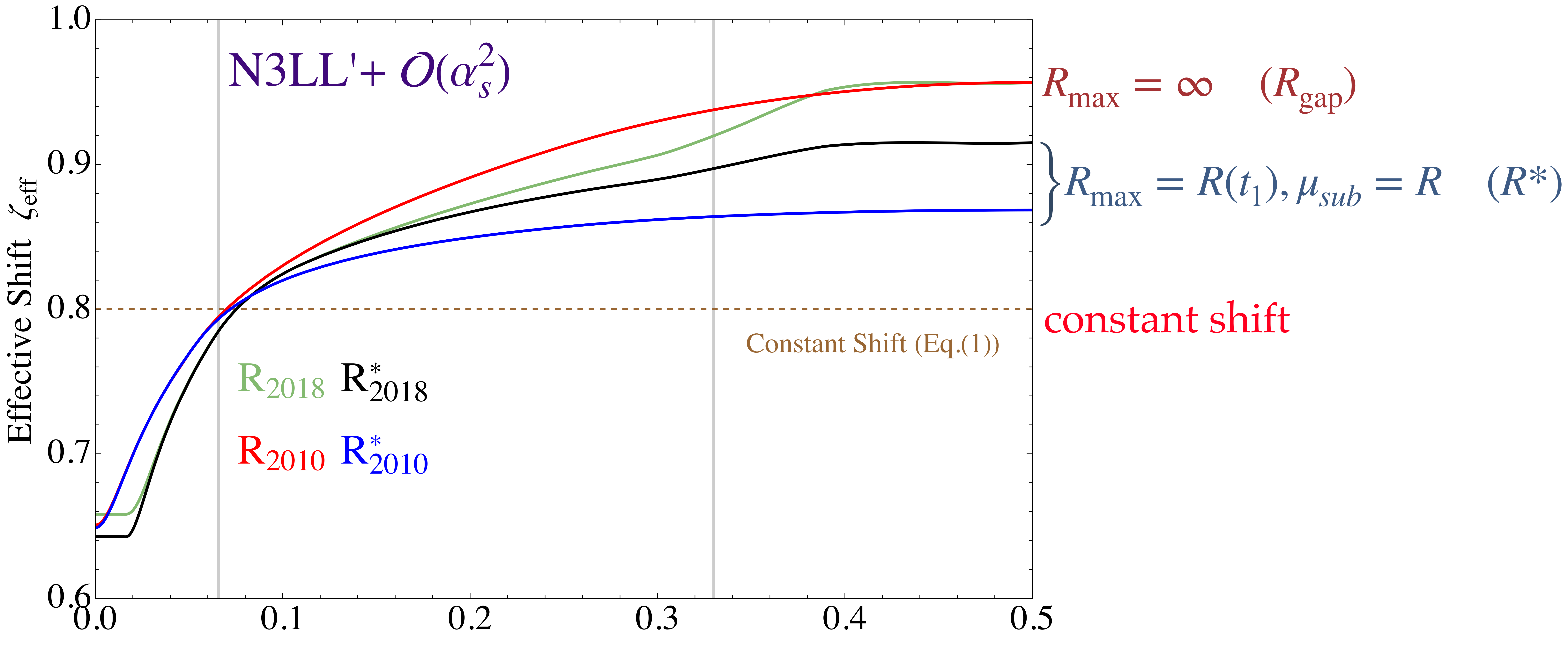


$$\sigma_{PT}(\tau) = \sigma_{sing}(\tau; \mu_H, \mu_J, \mu_S) + \sigma_{ns}(\tau; \mu_{ns})$$

$$\sigma(\tau) = \sigma_{PT}(\tau; \mu_i, R) \otimes f_{mod}(\tau, \Delta(R))$$

- Up to ~1000 variations considered in our final $\{\alpha_s, \Omega_1\}$ fits
- 2018 variations more conservative than 2010/2015 due to motivation of achieving perturbative convergence across wide range of angularities in 1808.07867.

Flattened shifts in tails



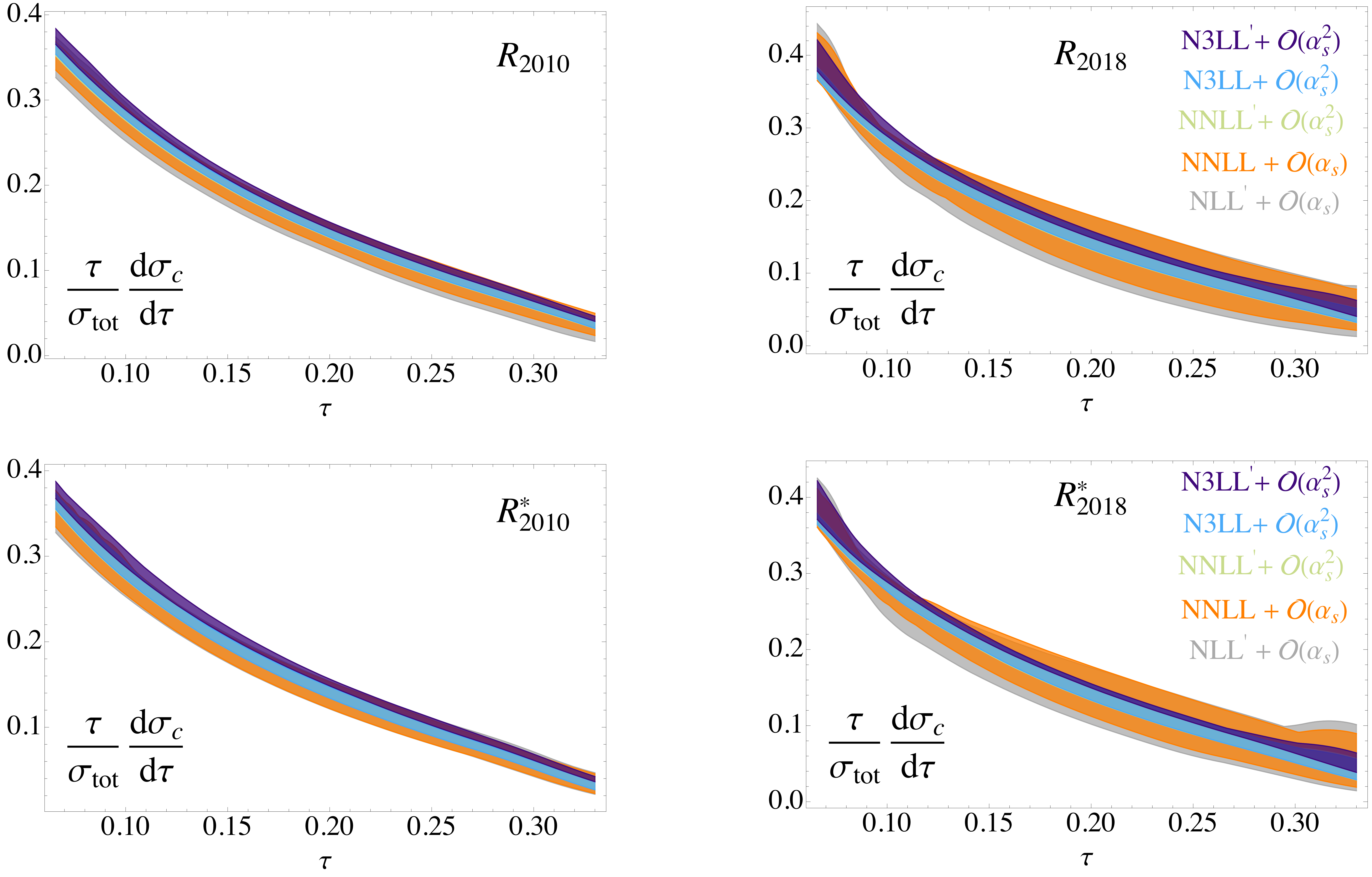
- The numerical effect can be loosely compared to the effect of 3-jet power corrections (e.g. $\zeta(e)$) modeled in, e.g. Luisoni, Monni, Salam [2012.00622]; Caola *et al.* [2108.08897, 2204.02247]; Nason & Zanderighi [2301.03607]
- Our method is a way to study variations of how we treat power corrections that exist within a 2-jet factorization framework

Convergence in R vs R* schemes

$Q = M_Z,\ a = 0$

R_{gap} scheme:

R^* scheme:



Comparison with data and determination of α_s

Data sets

▪ For thrust:

- ALEPH-2004: 133. GeV (7)
- ALEPH-2004: 161. GeV (7)
- ALEPH-2004: 172. GeV (7)
- ALEPH-2004: 183. GeV (7)
- ALEPH-2004: 189. GeV (7)
- ALEPH-2004: 200. GeV (6)
- ALEPH-2004: 206. GeV (8)
- ALEPH-2004: 91.2 GeV (26)
- AMY-1990: 55.2 GeV (5)
- DELPHI-1999: 133. GeV (7)
- DELPHI-1999: 161. GeV (7)
- DELPHI-1999: 172. GeV (7)
- DELPHI-1999: 89.5 GeV (11)
- DELPHI-1999: 93. GeV (12)
- DELPHI-2000: 91.2 GeV (12)
- DELPHI-2003: 183. GeV (14)
- DELPHI-2003: 189. GeV (15)
- DELPHI-2003: 192. GeV (15)
- DELPHI-2003: 196. GeV (14)
- DELPHI-2003: 200. GeV (15)
- DELPHI-2003: 202. GeV (15)
- DELPHI-2003: 205. GeV (15)
- DELPHI-2003: 207. GeV (15)
- DELPHI-2003: 45. GeV (5)
- DELPHI-2003: 66. GeV (8)
- DELPHI-2003: 76. GeV (9)
- JADE-1998: 35. GeV (5)
- JADE-1998: 44. GeV (7)
- L3-2004: 130.1 GeV (11)
- L3-2004: 136.1 GeV (10)
- L3-2004: 161.3 GeV (12)
- L3-2004: 172.3 GeV (12)
- L3-2004: 182.8 GeV (12)
- L3-2004: 188.6 GeV (12)
- L3-2004: 194.4 GeV (12)
- L3-2004: 200. GeV (11)
- L3-2004: 206.2 GeV (12)
- L3-2004: 41.4 GeV (5)
- L3-2004: 55.3 GeV (6)
- L3-2004: 65.4 GeV (7)
- L3-2004: 75.7 GeV (7)
- L3-2004: 82.3 GeV (8)
- L3-2004: 85.1 GeV (8)
- L3-2004: 91.2 GeV (10)
- OPAL-1997: 161. GeV (7)
- OPAL-2000: 172. GeV (8)
- OPAL-2000: 183. GeV (8)
- OPAL-2000: 189. GeV (8)
- OPAL-2005: 133. GeV (6)
- OPAL-2005: 177. GeV (8)
- OPAL-2005: 197. GeV (8)
- OPAL-2005: 91. GeV (5)
- SLD-1995: 91.2 GeV (6)
- TASSO-1998: 35. GeV (4)
- TASSO-1998: 44. GeV (5)

------ Summary ------

Totlal: 516

Q > 95 : 345

Q < 88 : 89

Q ~ MZ : 82

▪ For angularities:

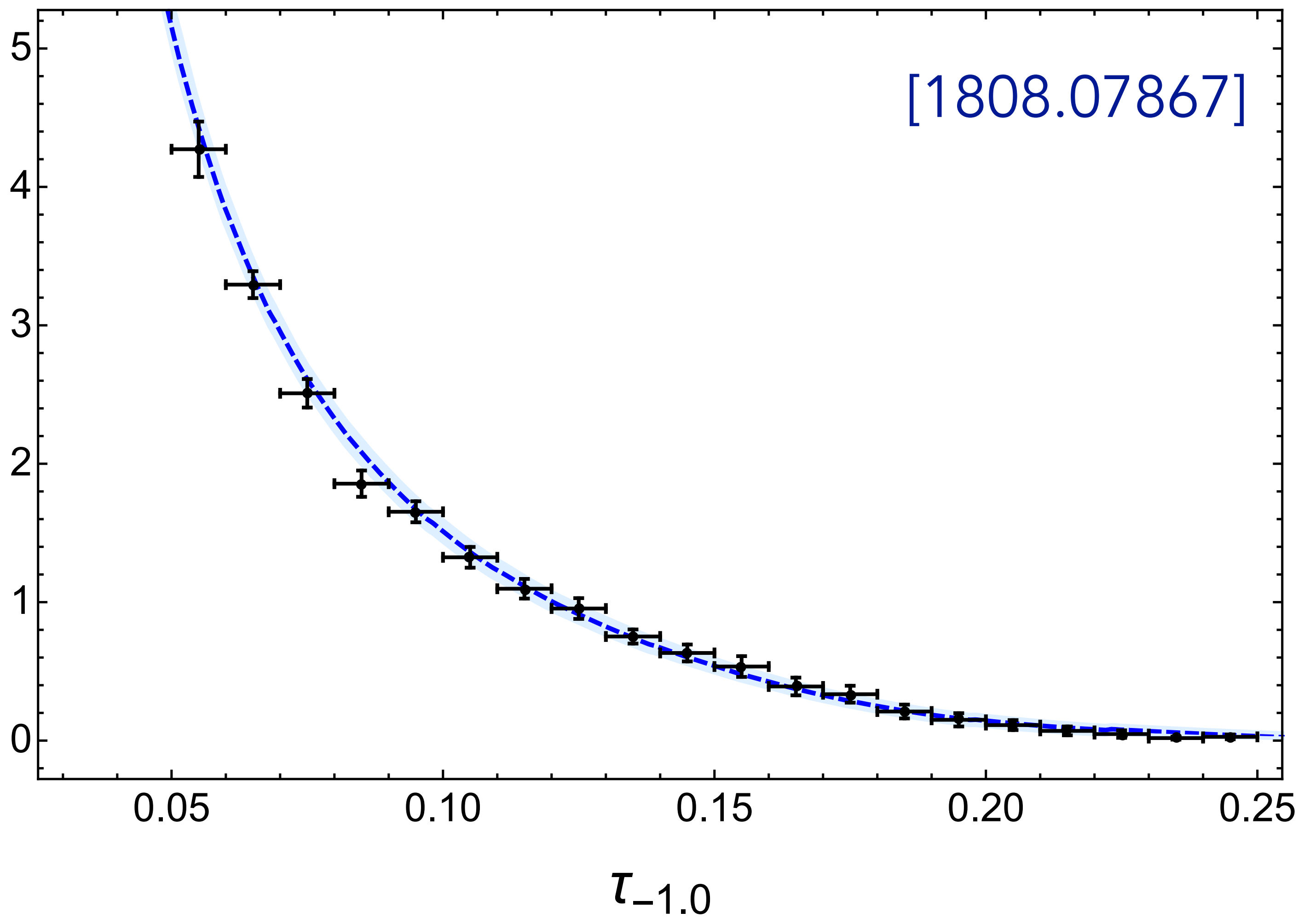
Generalized event shape and energy flow studies in e^+e^- annihilation at $\sqrt{s} = 91.2$-208.0 GeV

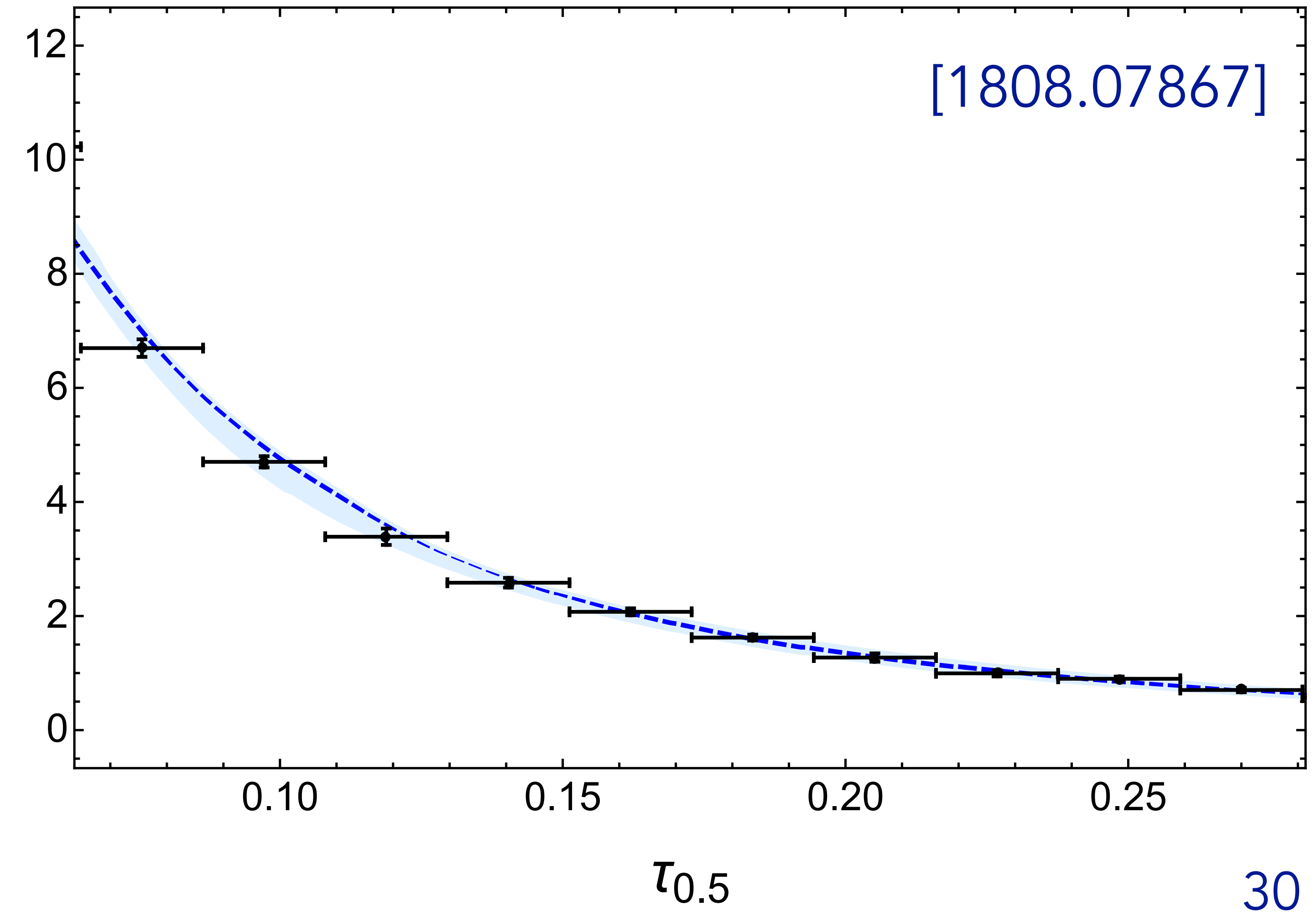
L3 Collaboration

JHEP 10 (2011) 143

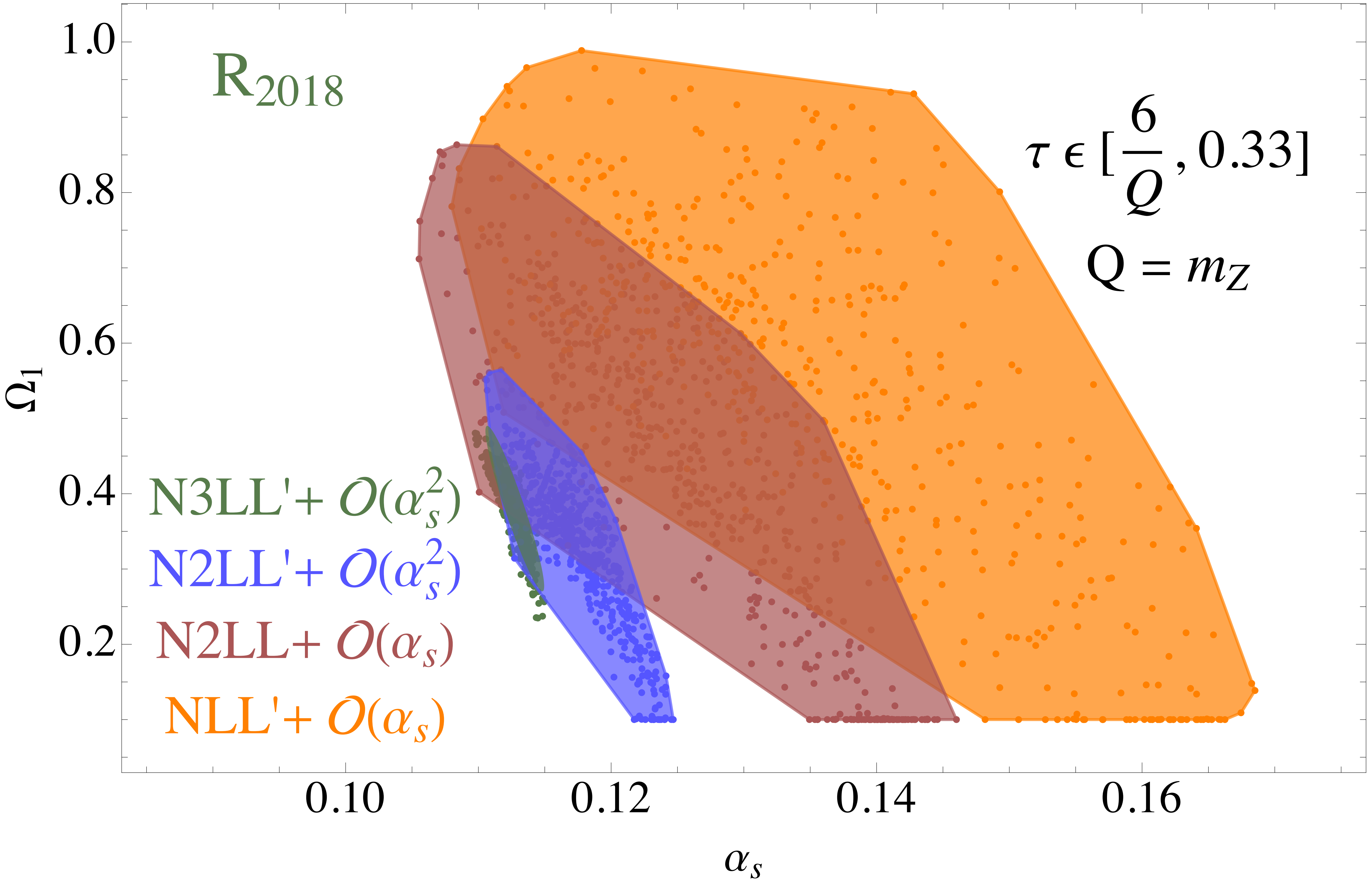
Also see PhD thesis by P. Jindal, Panjab University

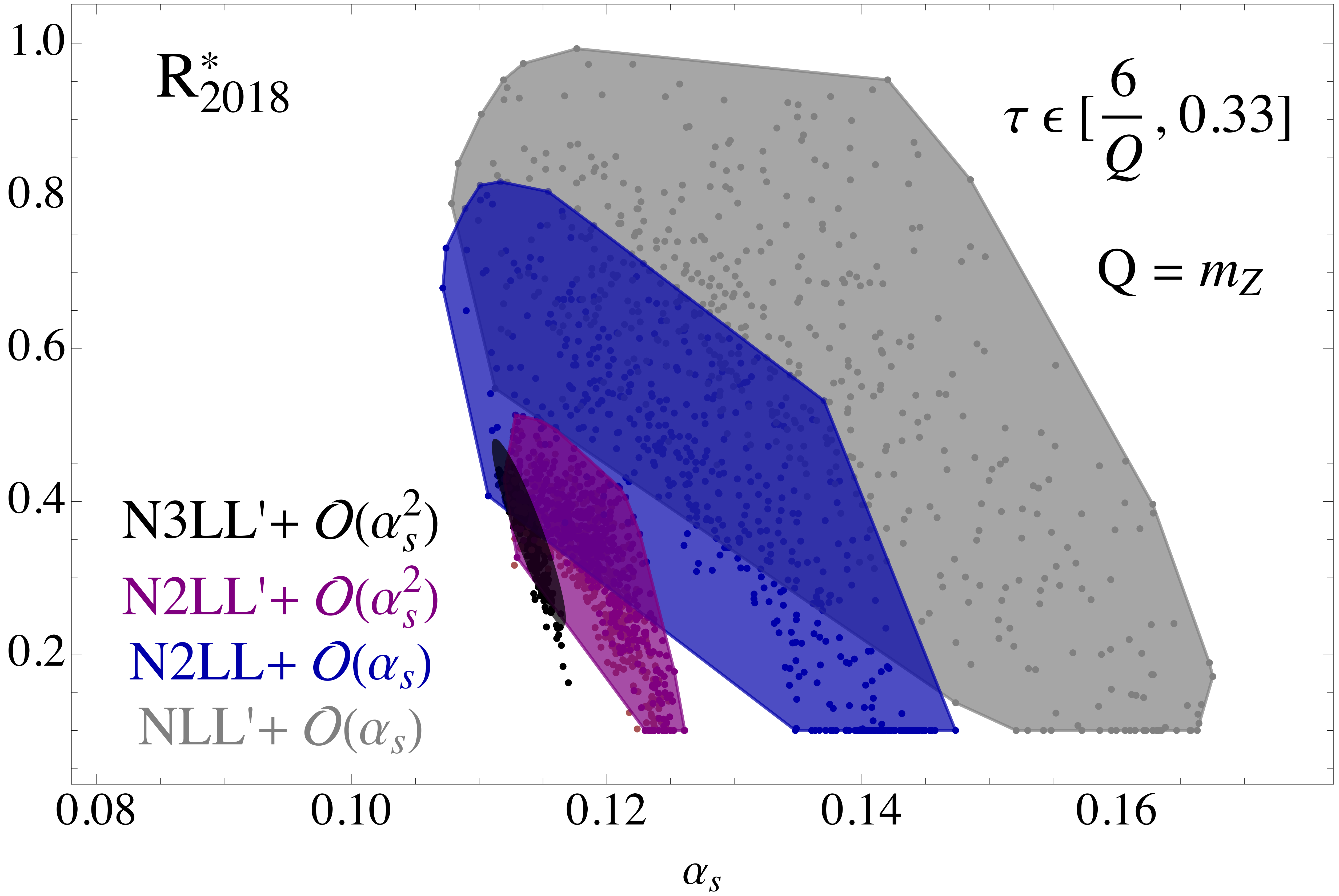
- Data for a = {**-1.0**, **-0.75**. **-0.5**, **-0.25**, **0.0**, **0.25**, **0.5**, 0.75} at **91.2** and 197 GeV
- Total number of bins = (bins per a) x (number of a) = 25 x 7 = 175 bins @ Q = 91.2 GeV
- e.g. a = -1 and 0.5, Q = 91.2 GeV, compared to our NNLL' prediction:





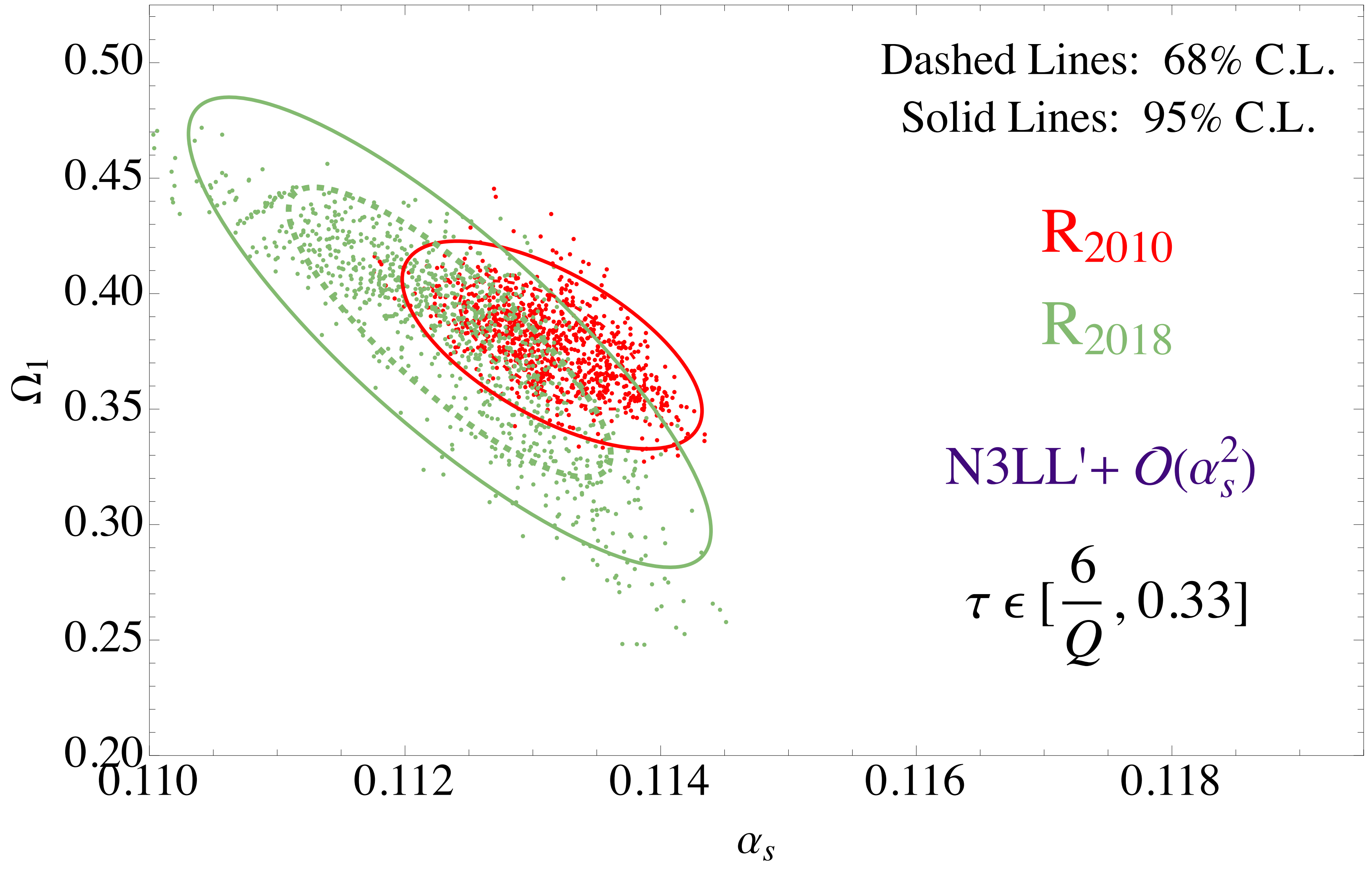
Effect on thrust fits [N³LL′+$\mathcal{O}(\alpha_s^2)$]

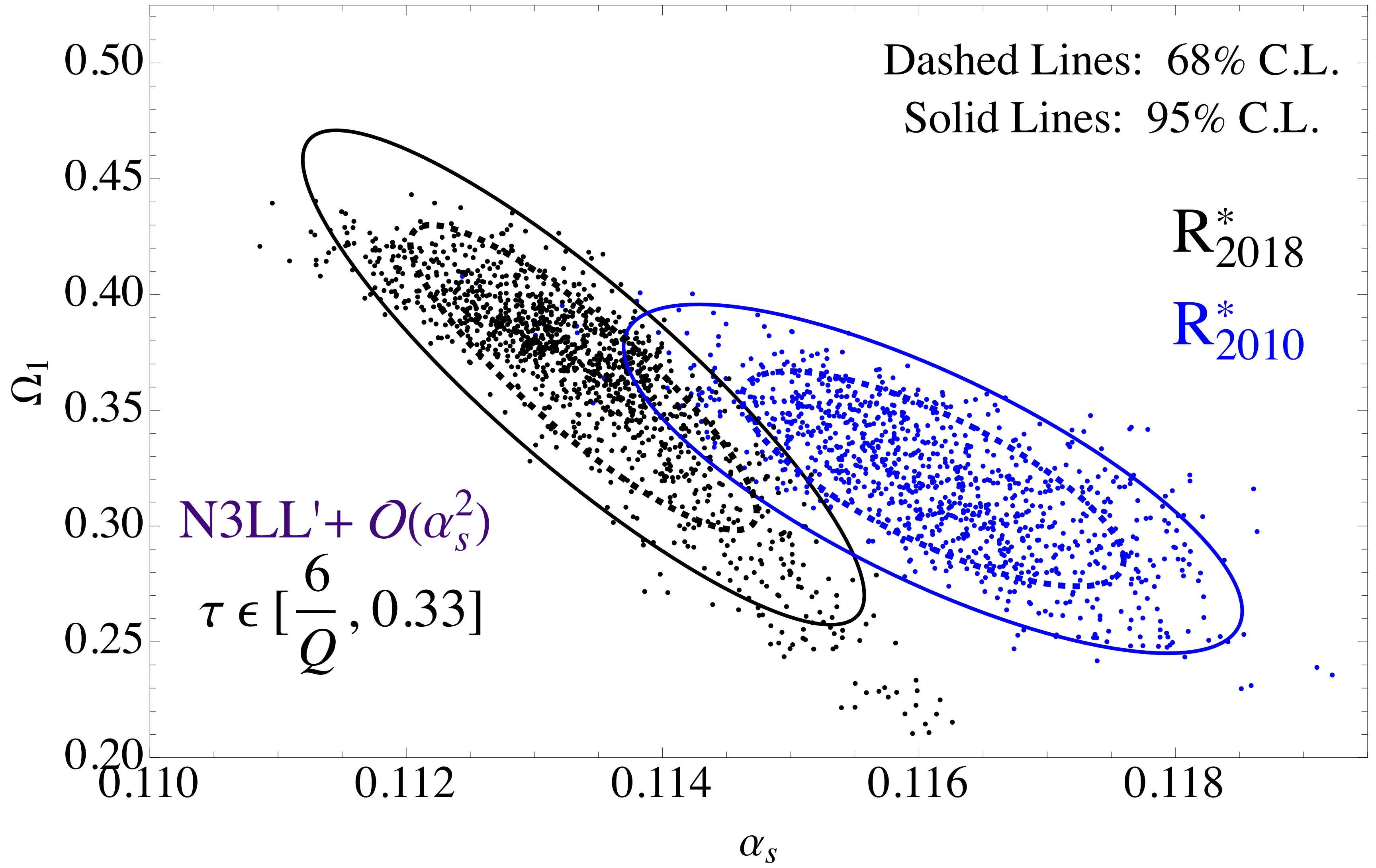




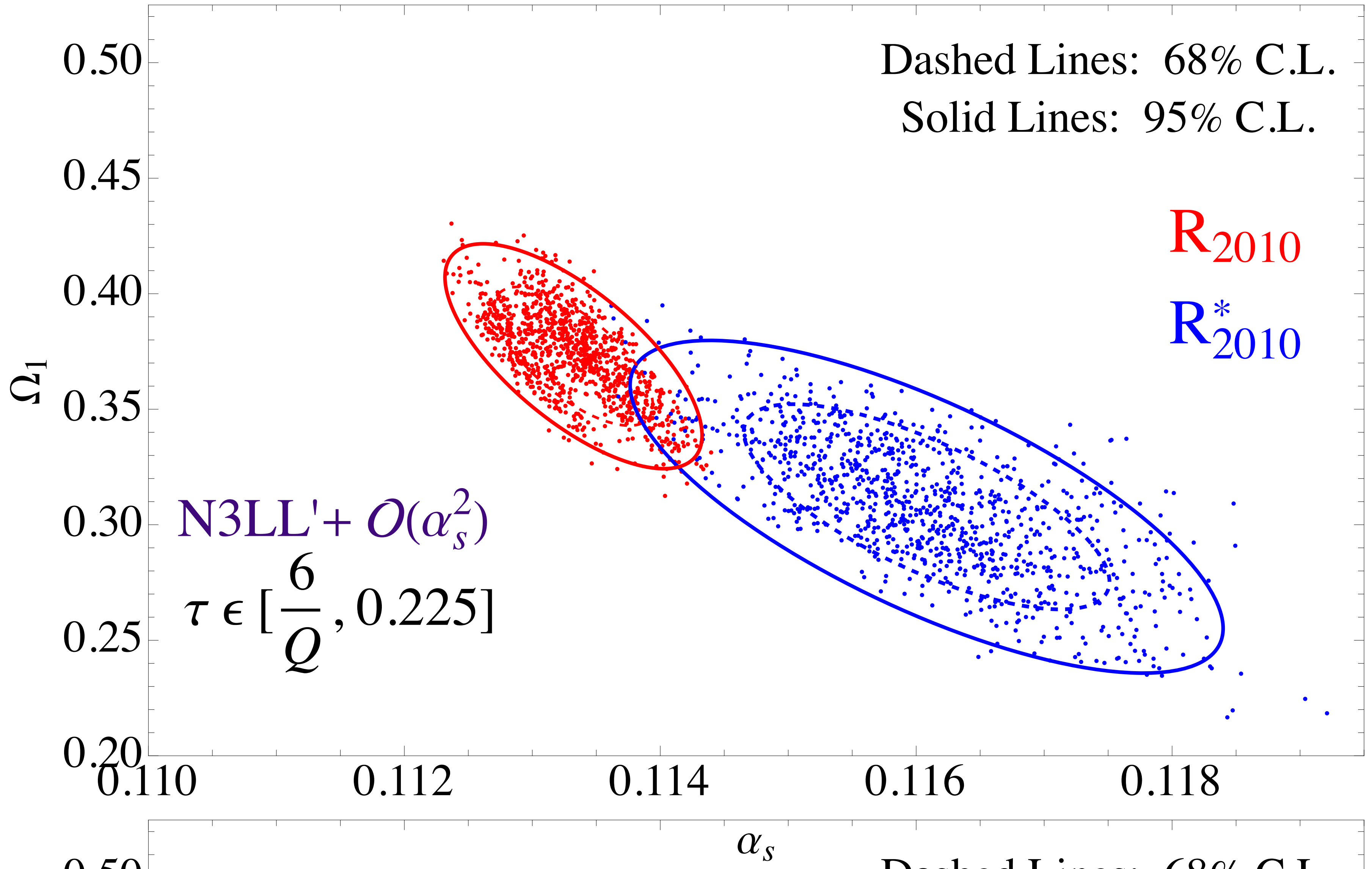
Effect on thrust fits [N³LL′+$\mathcal{O}(\alpha_s^2)$]

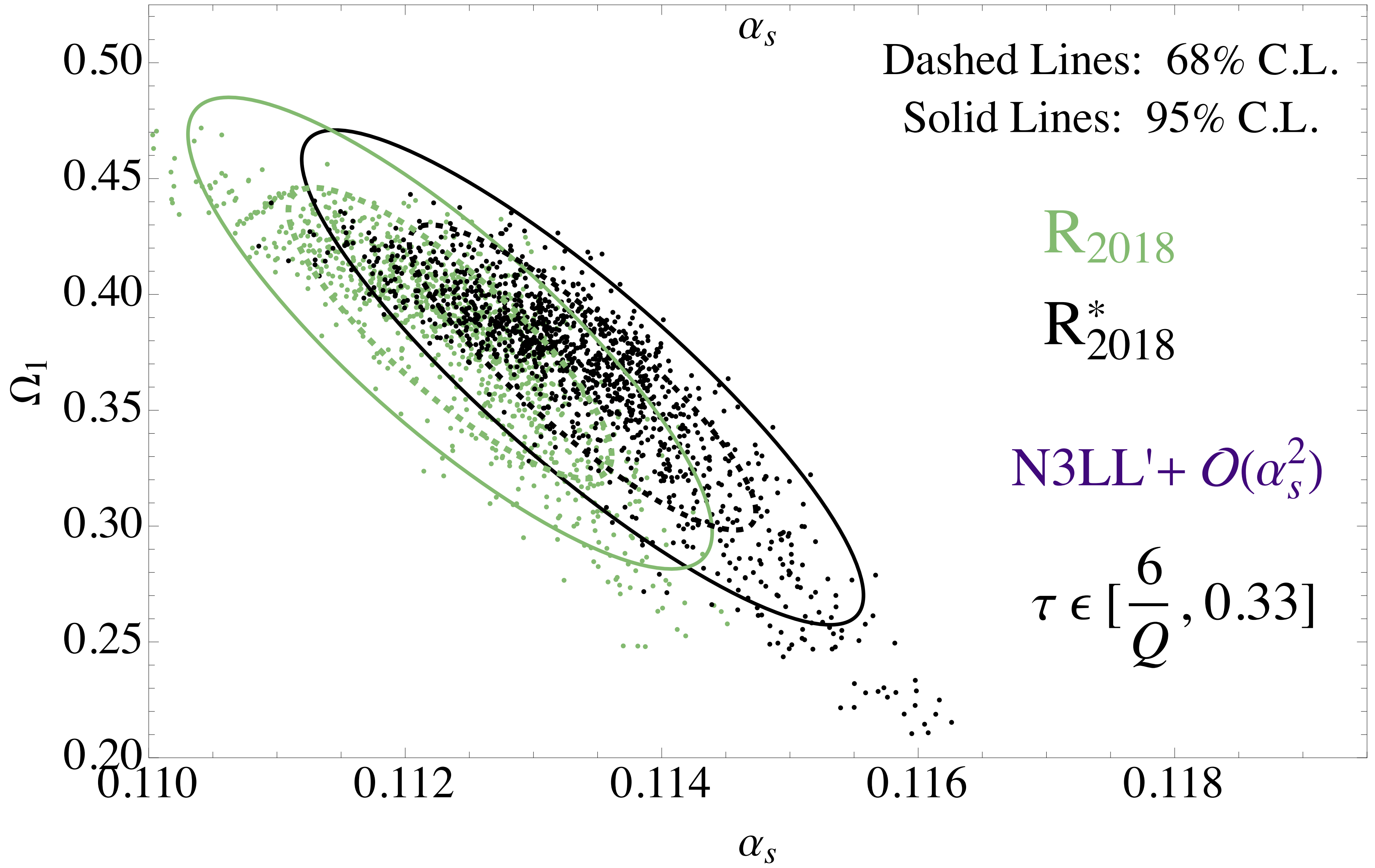
Vary profiles:





Vary renormalon schemes:

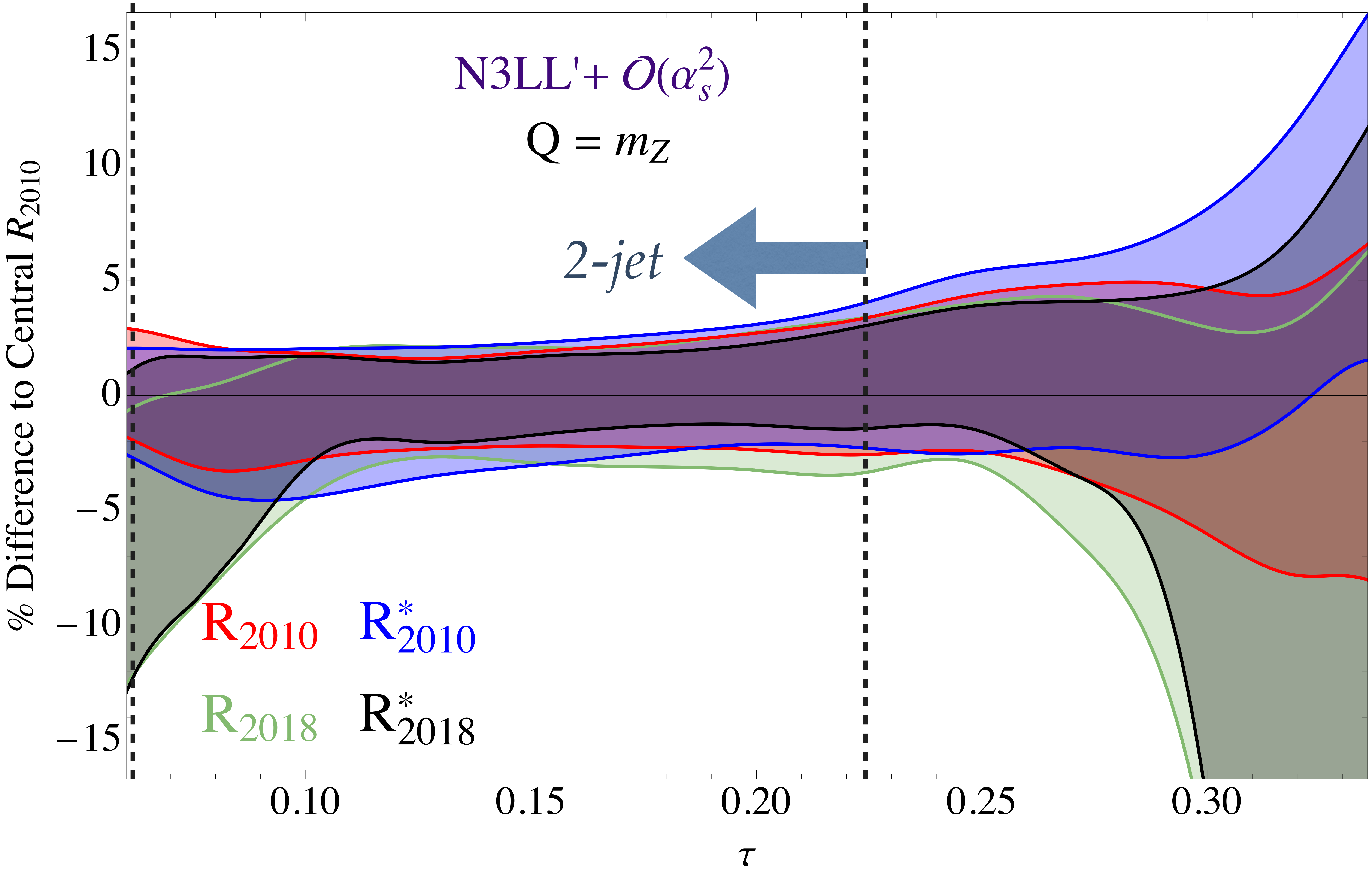




Fit in a narrower 2-jet region

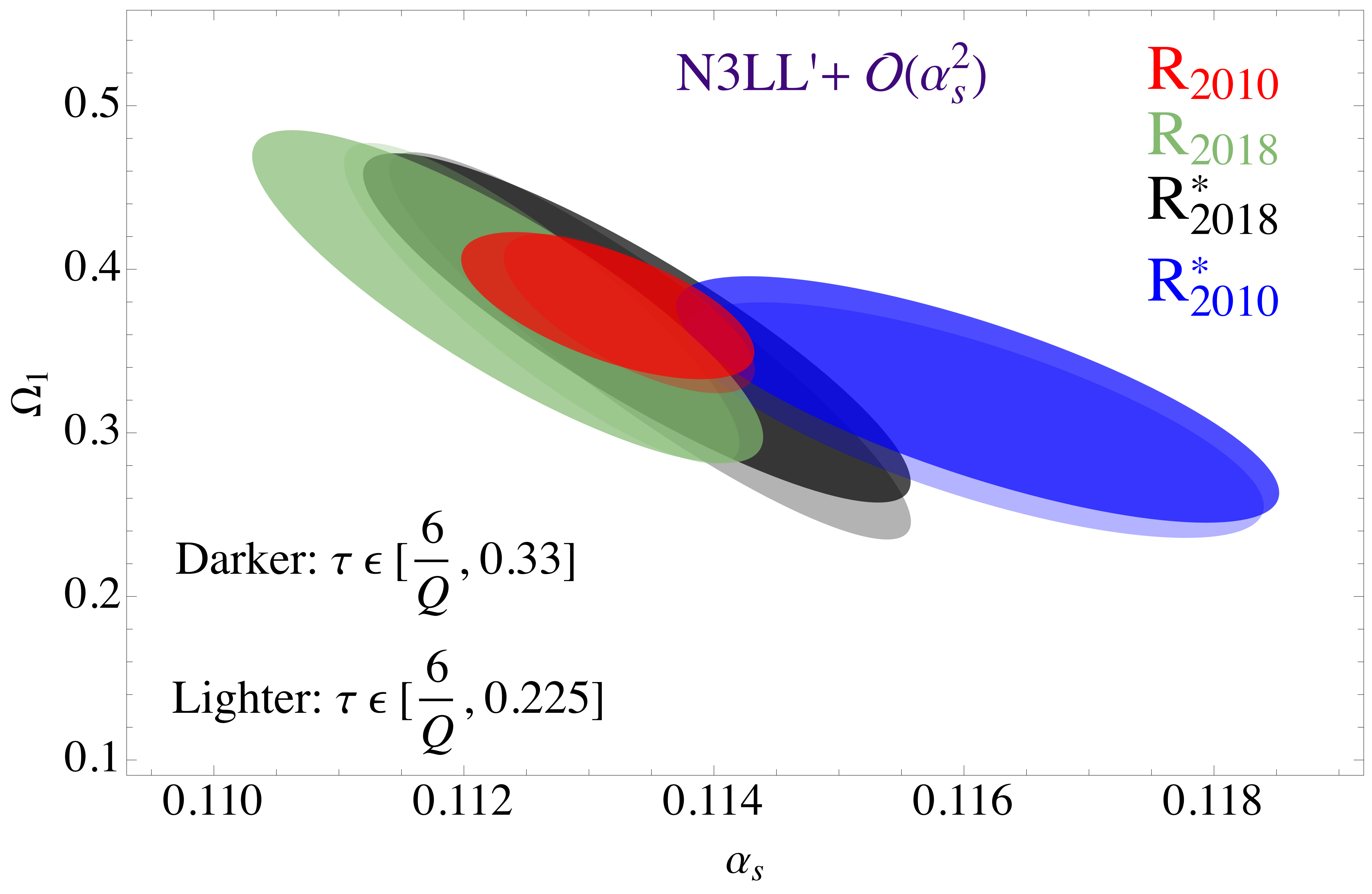
$Q = M_Z,\ a = 0$

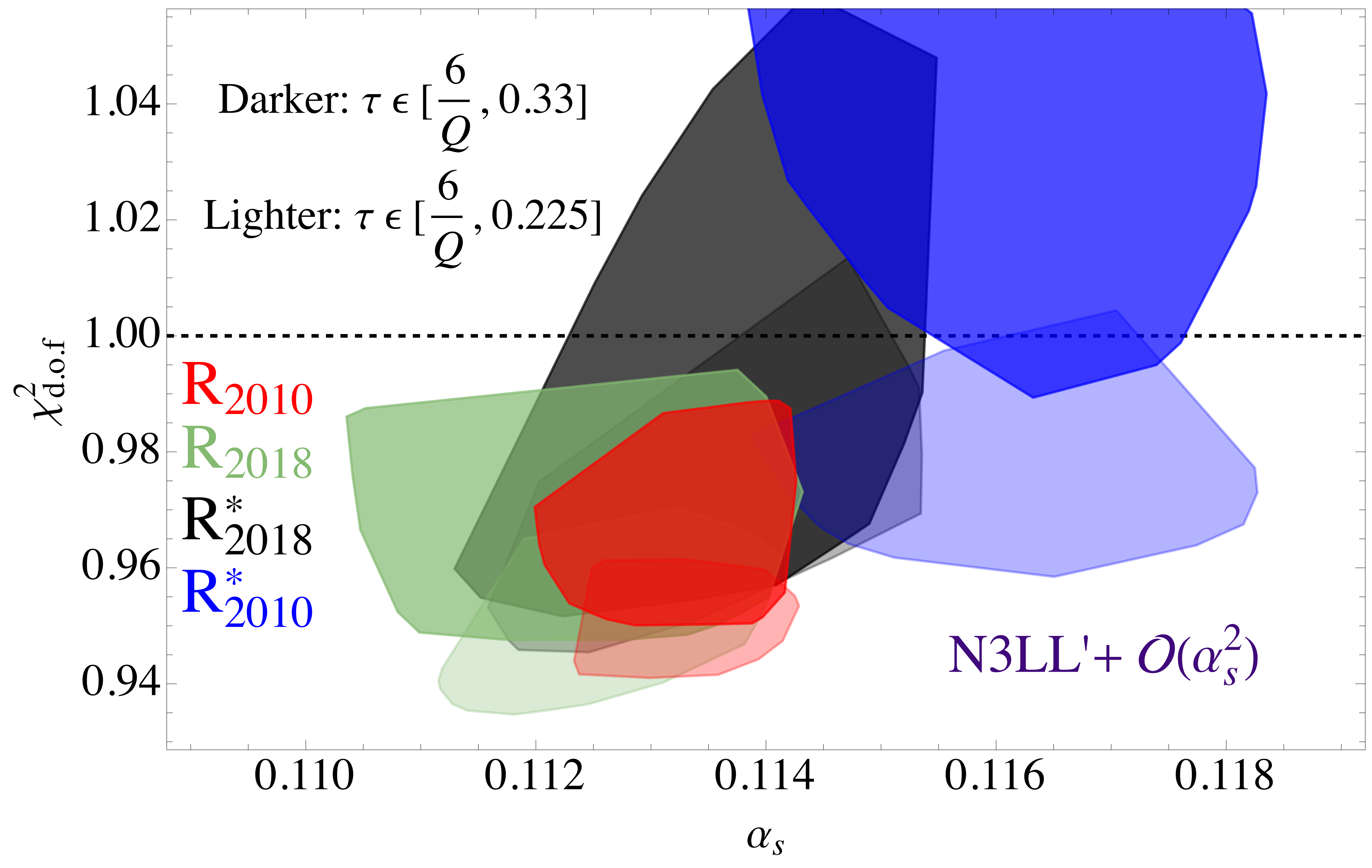
- Variability by scheme lessened in more 2-jet like region vs multi jet tail
- Try limiting fit window to, e.g, $\tau < 0.225$:



Fit in a narrower 2-jet region

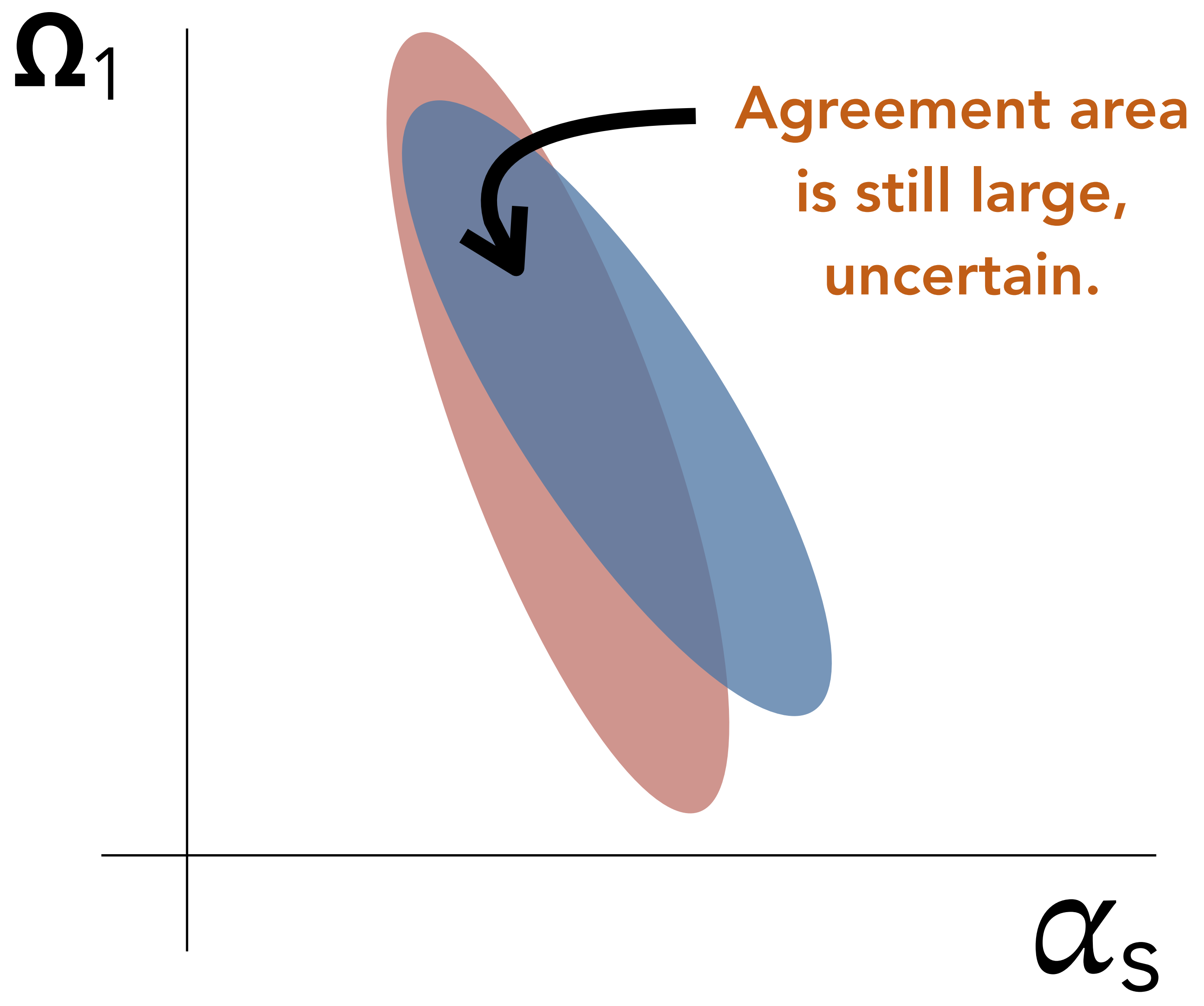
- Not too much shift in the fit ellipses, but improved *quality* of fit:

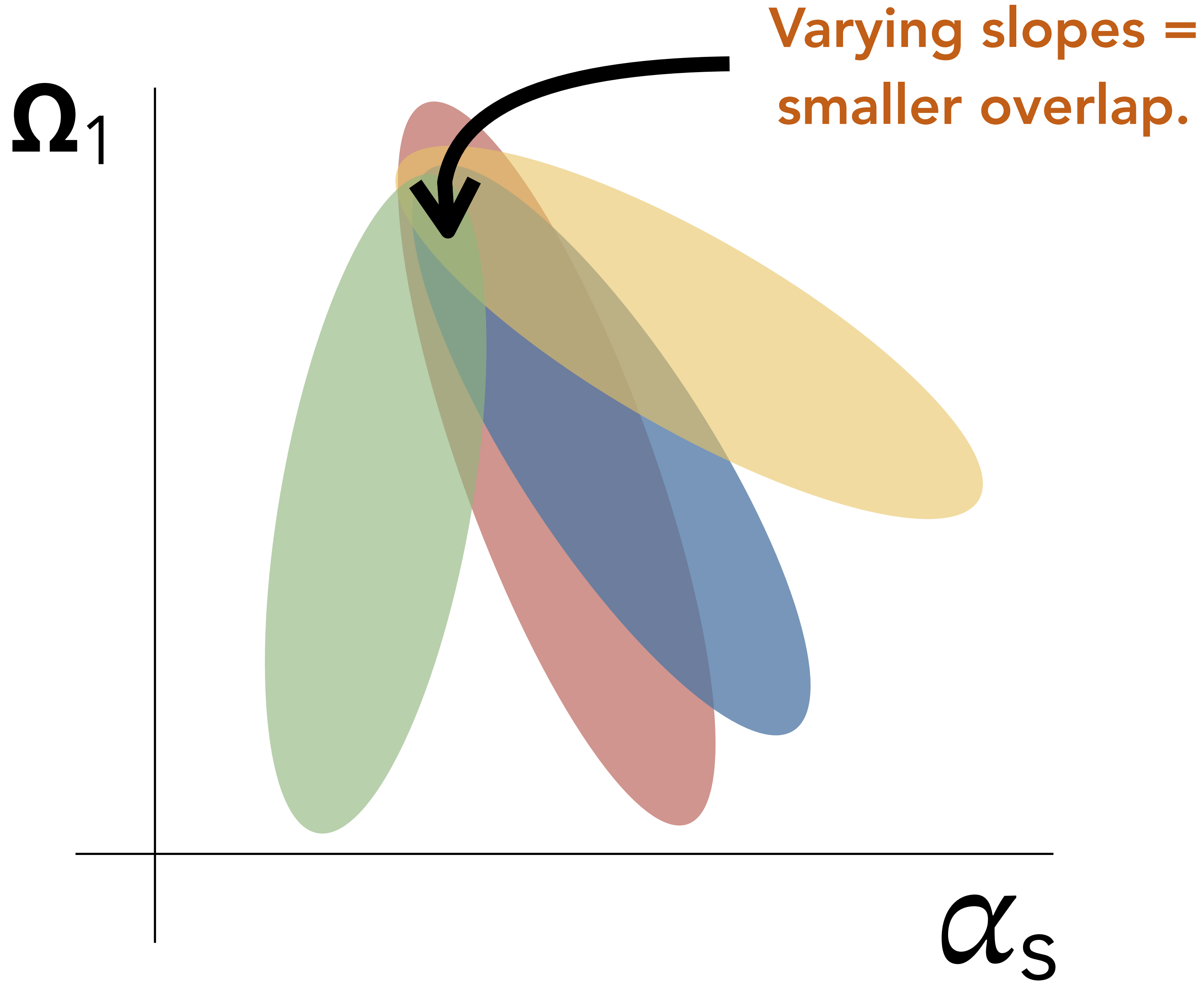




Future outlook: angularities break degeneracies

- In tail region, leading nonperturbative effect is a shift by $c_e\Omega_1/Q$





Use different Q's.
Or different event shapes.

- Angularities:
  Leading nonperturbative shift is $\dfrac{2\Omega_1}{Q(1-a)}$: changing *a* is like changing *Q*.

- We have preliminary fits based on angularities, but with quite a small amount of data. *More would be welcome!*

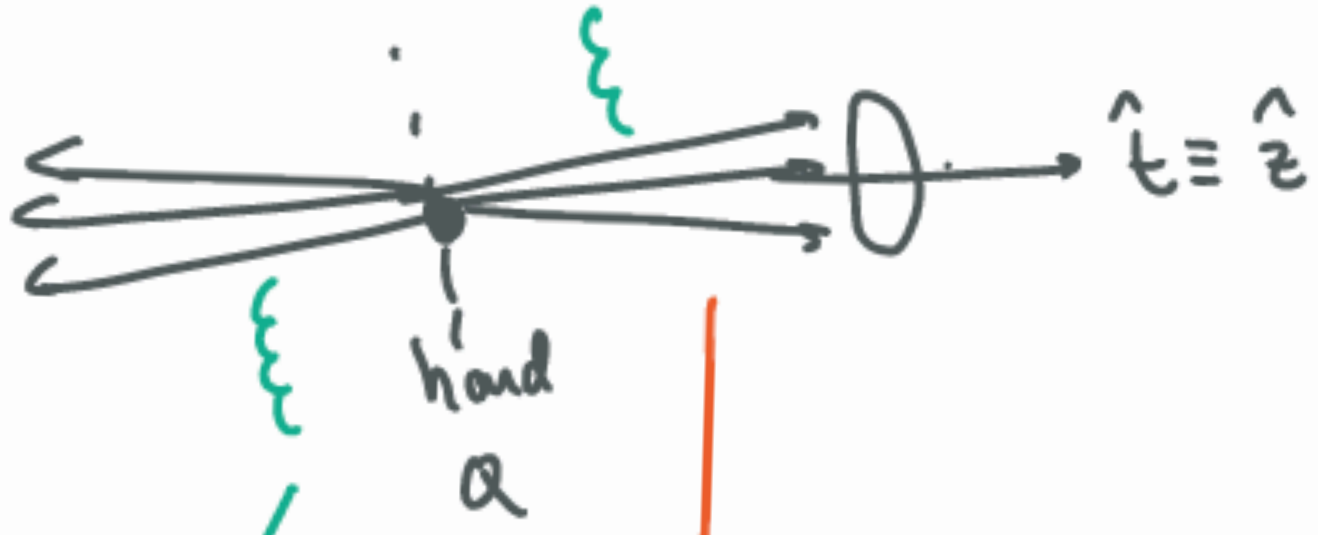
Looking ahead

- Welcome more work to understand robust estimation of theoretical uncertainty due to renormalon schemes
- Encouraging signs pointing to the purely 2-jet-like region for fitting
- Better computation of 3-loop fixed-order thrust distribution also welcome, extracting small contributions out of large singular background challenging
- **We do not yet propose a new value of α_s or Ω_1** [our results limited to N^{3}LL′+$\mathcal{O}(\alpha_s^2)$]
- **We do observe an upward shift in α_s or of its uncertainties when switching from standard R_{gap} to R* scheme and/or between some perturbative scale choices.**
- Dedicated new analyses or measurements of data in *true* two-jet region may yield the best results for fits from two-jet event shapes, complementing more rigorous understanding of nonperturbative effects on 3-jet tail to reduce uncertainties in that region
- **May be able to reduce uncertainty from fixed-order nonsingular scale by resumming subleading logs**

Backups

Relevant Physical Scales

Thrust: $M^2 = M_A^2 + M_B^2 = Q^2 \tau$ (if $\ll Q^2$)



$n = (1, +\hat{z})$
$\bar{n} = (1, -\hat{z})$

light-cone coordinates:
$p^\mu = (\bar{n}\cdot p, n\cdot p, \vec{p}_\perp)$

(Angularities:)
$p_c \sim Q(1, \tau_a^{\frac{2}{2-a}}, \tau_a^{\frac{1}{2-a}})$

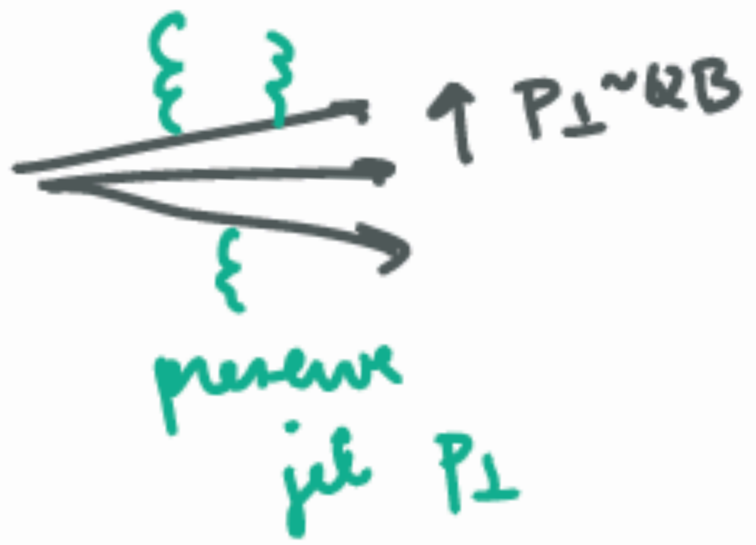
preserve jet mass

collinear $p_c \sim (Q, \frac{M^2}{Q}, M) \sim Q(1, \tau, \sqrt{\tau})$

↕ same

soft $k_s \sim (\frac{M^2}{Q}, \frac{M^2}{Q}, \frac{M^2}{Q}) \sim Q(\tau, \tau, \tau)$

Broadening:

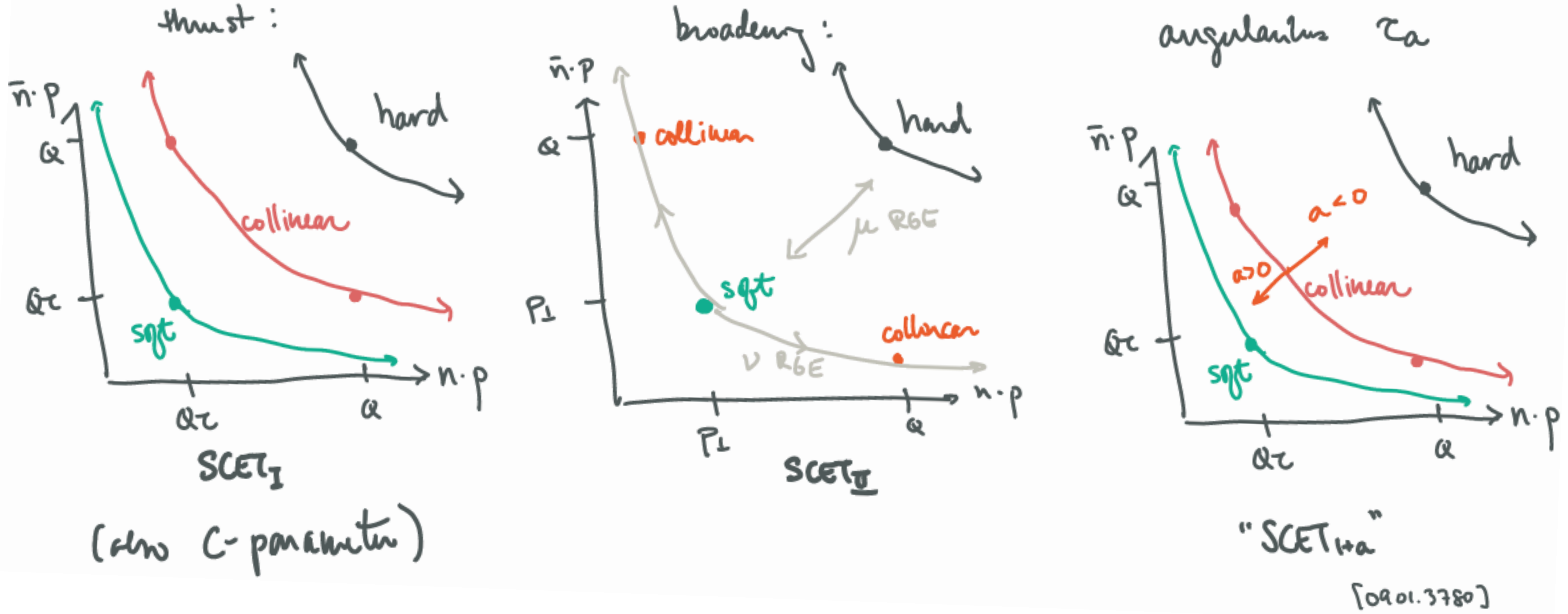


coll $p_c \sim (Q, QB^2, QB)$

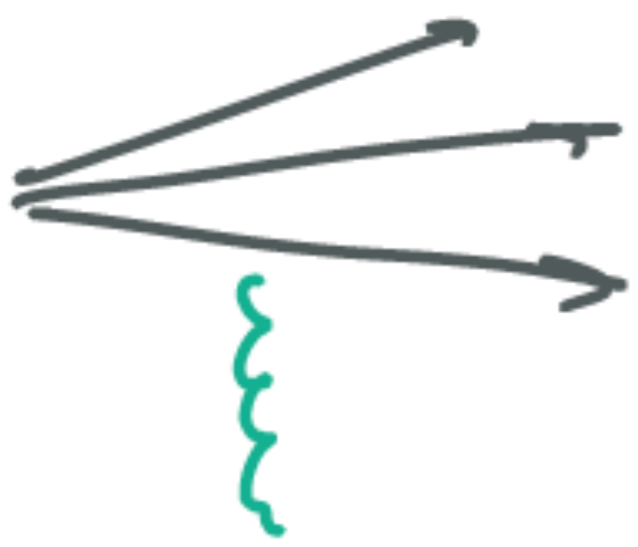
↕ same

soft $k_s \sim Q(B, B, B)$

SCALES & MODES :



Angularities:



$$\tau_a \sim \frac{p_\perp}{Q}\left(\frac{p^+}{p^-}\right)^{\frac{1-a}{2}}$$

$$\sim \frac{1}{Q}\,(p^+)^{1-\frac{a}{2}}\,(p^-)^{\frac{a}{2}}$$

$\Rightarrow$ coll $\quad \tau_a \sim \left(\frac{p^+}{Q}\right)^{1-\frac{a}{2}} \quad \Rightarrow \quad p^+ \sim Q\,\tau_a^{\frac{2}{2-a}}$

$$p_\perp \sim Q\,\tau_a^{\frac{1}{2-a}}$$

$$p_c \sim Q\left(1,\ \tau_a^{\frac{2}{2-a}},\ \tau_a^{\frac{1}{2-a}}\right)$$

soft $\quad \tau_a \sim \frac{k_s}{Q} \quad \Rightarrow \quad p_s \sim Q(\tau_a, \tau_a, \tau_a)$

Angularity event shapes in e^+e^- collisions

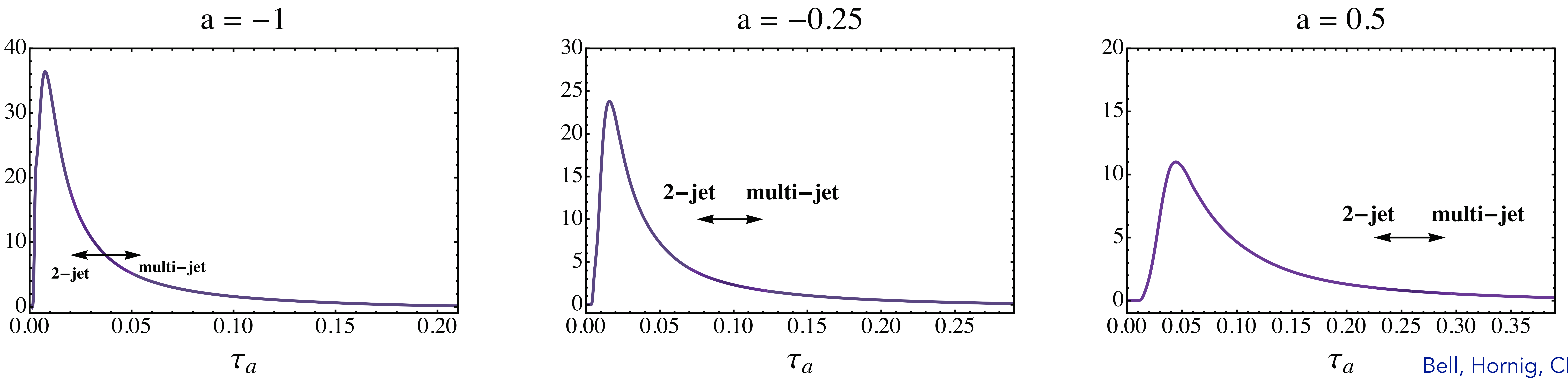
- Consider *Angularities*, which can be defined in terms of the rapidity and p_T of a final state particle 'i', with respect to the thrust axis:

Berger, Kucs, Sterman [hep-ph/0303051]

IR safe for a ∈ {-∞, 2}

$$\tau_a = \frac{1}{Q} \sum_i |\mathbf{p}_\perp^i| \, e^{-|\eta_i|(1-a)}$$

a = 0 <-> `Thrust'
a = 1 <-> `Jet Broadening' (for us a<1)



Bell, Hornig, CL, Talbert [1808.07867]

(for some arbitrary, but uniform, definition of "2-jet")

PROOF OF UNIVERSAL SHIFT

[Cl, Sterman 2006]

$$\Delta\langle e\rangle_s = \frac{1}{a}\int_{-\infty}^{\infty} d\eta\, f_e(\eta)\, \frac{1}{N_c}\mathrm{Tr}\langle 0|\bar T[Y_{\bar n}^\dagger Y_n^\dagger]\,\hat{\mathcal{E}}_T(\eta)\, T[Y_n Y_{\bar n}]|0\rangle$$

$\hat{\mathcal{E}}_T(\eta)|X\rangle = \sum_{i\in X}|p_i^\perp|\,\delta(\eta-\eta_i)|X\rangle$

Lorentz boosts: $\Lambda_{\eta'}^{-1}\Lambda_{\eta'}$ $\Lambda_{\eta'}^{-1}\Lambda_{\eta'}$

$Y_n = P\exp\left[ig\int_0^\infty ds\, n\cdot A_s(ns)\right] \longrightarrow Y_n$

$|0\rangle \longrightarrow |0\rangle$

$\hat{\mathcal{E}}_T(\eta) \longrightarrow \hat{\mathcal{E}}_T(\eta+\eta')$

Pick η' to be anything!

$$\Rightarrow \Delta\langle e\rangle_s = \underbrace{\frac{1}{a}\int_{-\infty}^{\infty} d\eta\, f_e(\eta)}_{=\,c_e}\ \underbrace{\frac{1}{N_c}\mathrm{Tr}\langle 0|\bar T[Y_{\bar n}^\dagger Y_n^\dagger]\,\hat{\mathcal{E}}_T(0)\, T[Y_n Y_{\bar n}]|0\rangle}_{\Omega_1}$$

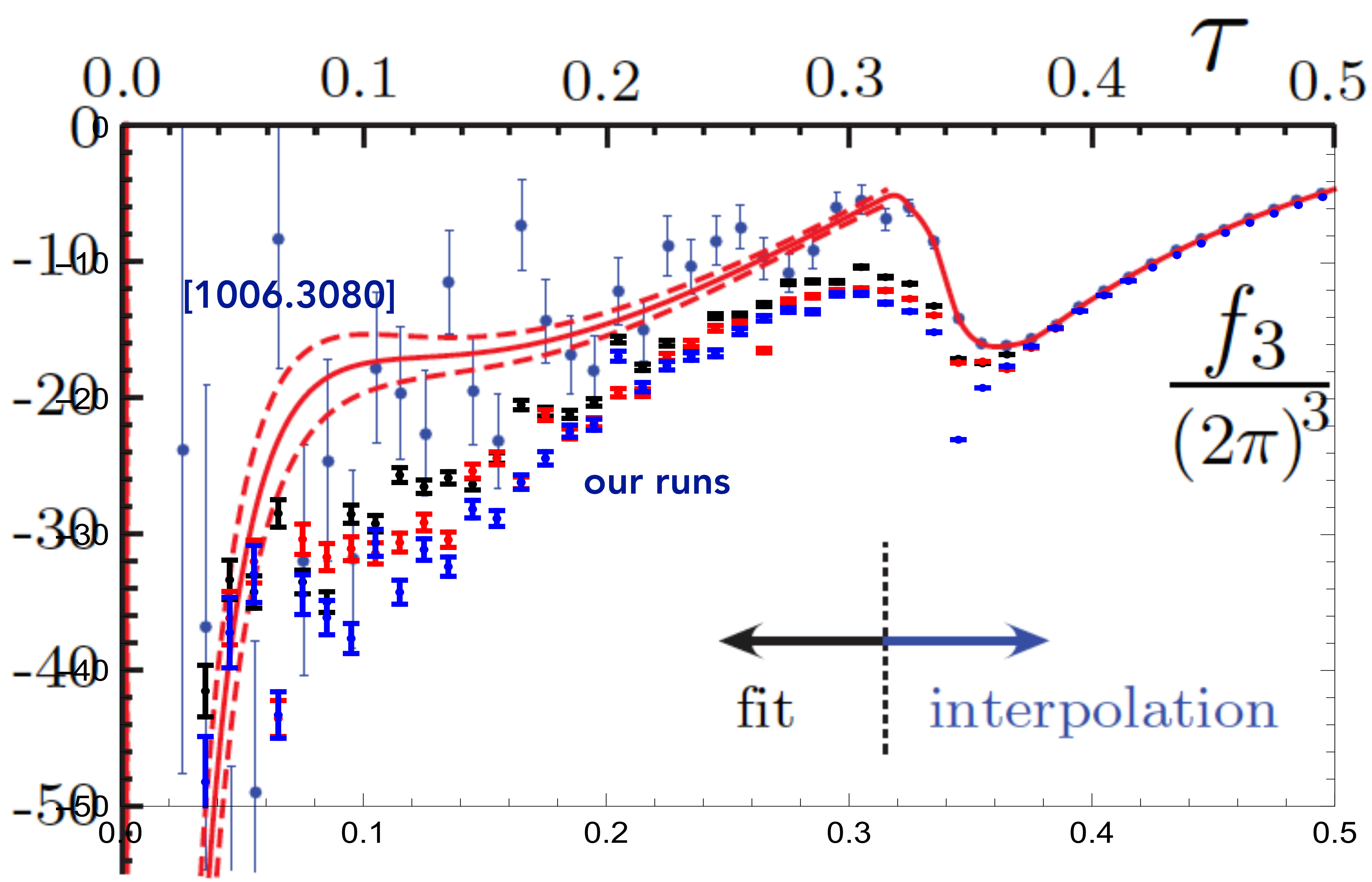
(massless parton case)

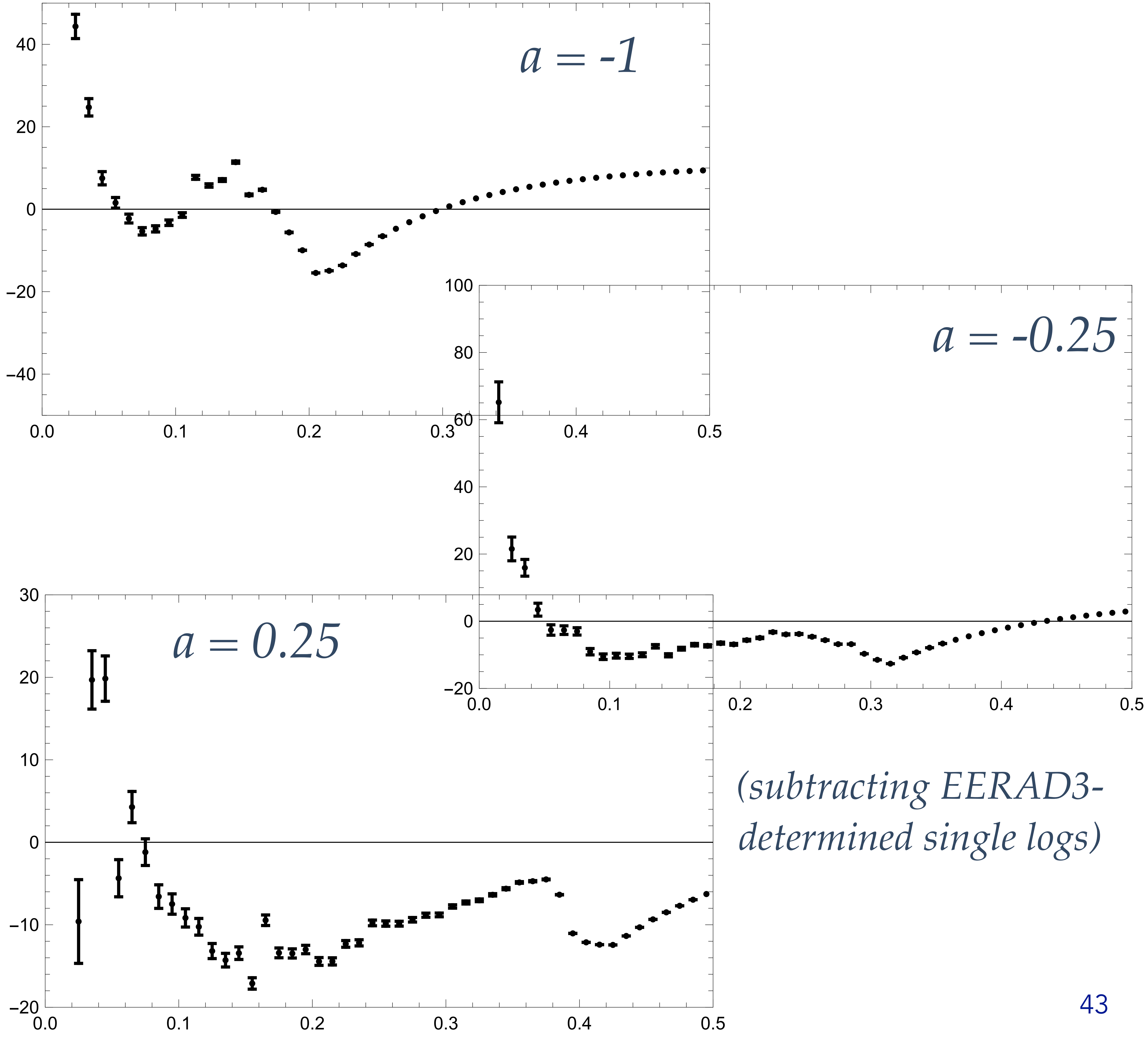
generalizes single emission models e.g. Dokshitzer-Webber 95-96

New remainder functions from EERAD3

using LANL Institutional Computing

- a=0:

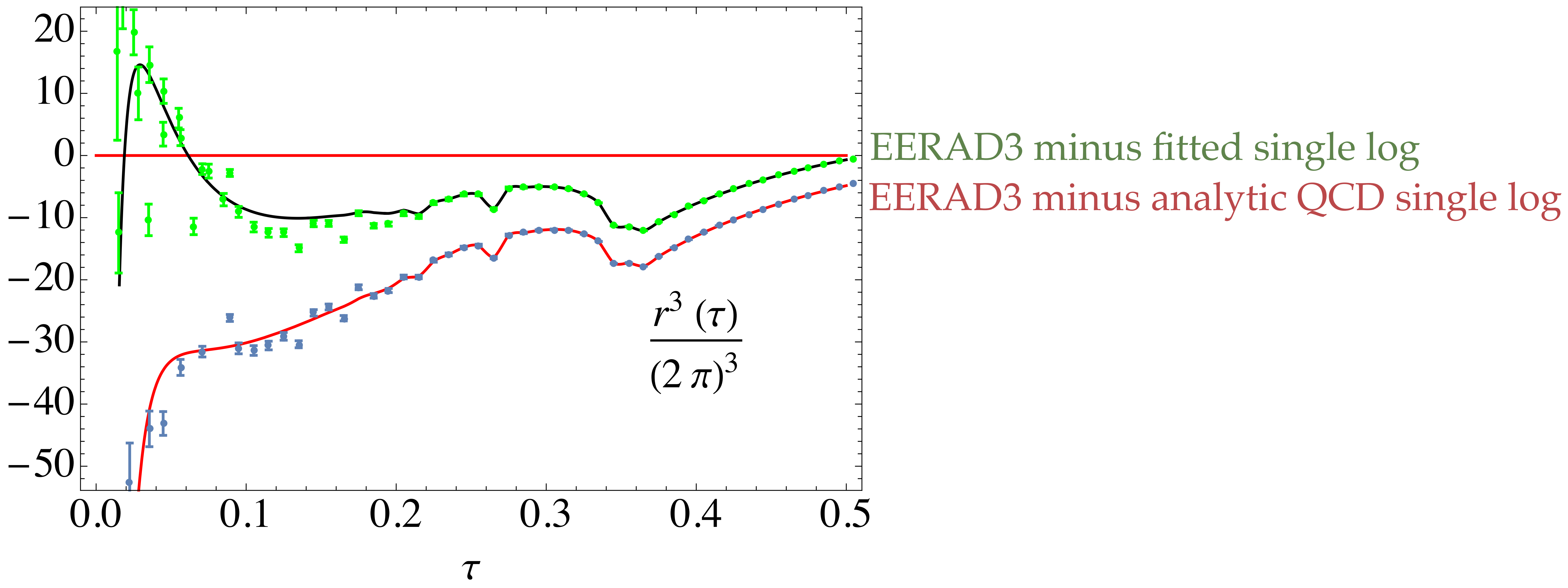




(subtracting EERAD3-determined single logs)

Uncertainty in nonsingular remainder from EERAD3

- Nonsingular remainder $r^3(\tau)$ with different values for the subtracted single log term:



- Instability in single log coefficient leads to large uncertainty in fixed-order nonsingular remainder
  - For this reason in [2311.03990] we omitted the $\mathcal{O}(\alpha_s^3)$ fixed-order matching, it did not measurably affect our conclusions about scheme/profile dependence of α_s, Ω_1
  - We are eager to see alternative calculations of r^3 !

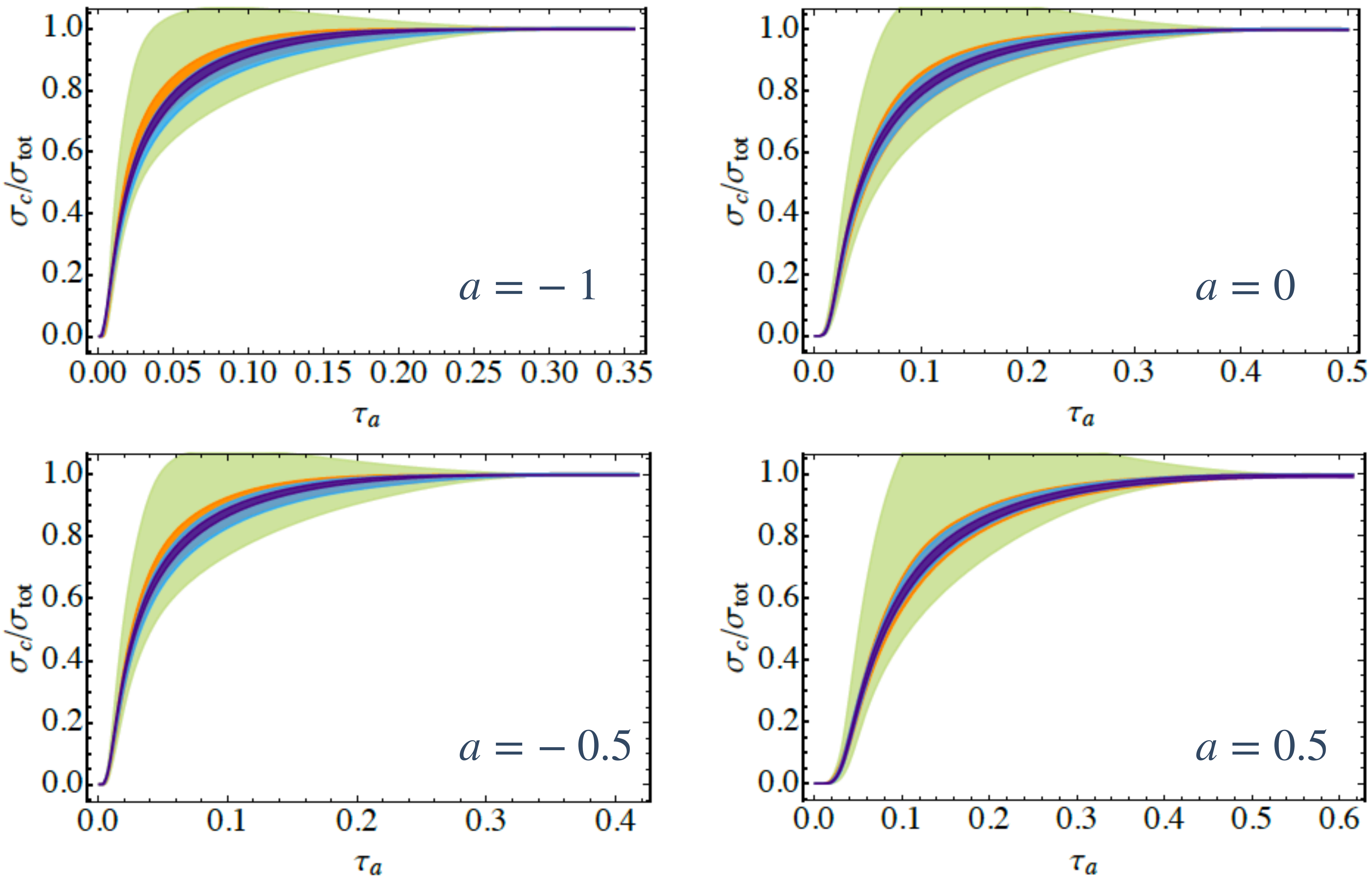
Fit in a narrower 2-jet region

- Variability by scheme lessened in more 2-jet like region vs multi jet tail
- Try limiting fit window to, e.g, $\tau < 0.225$:
- Not too much shift in the fit ellipses, but improved *quality* of fit:

Angularity distributions

[NNLL′+$\mathcal{O}(\alpha_s^2)$]
1808.07867

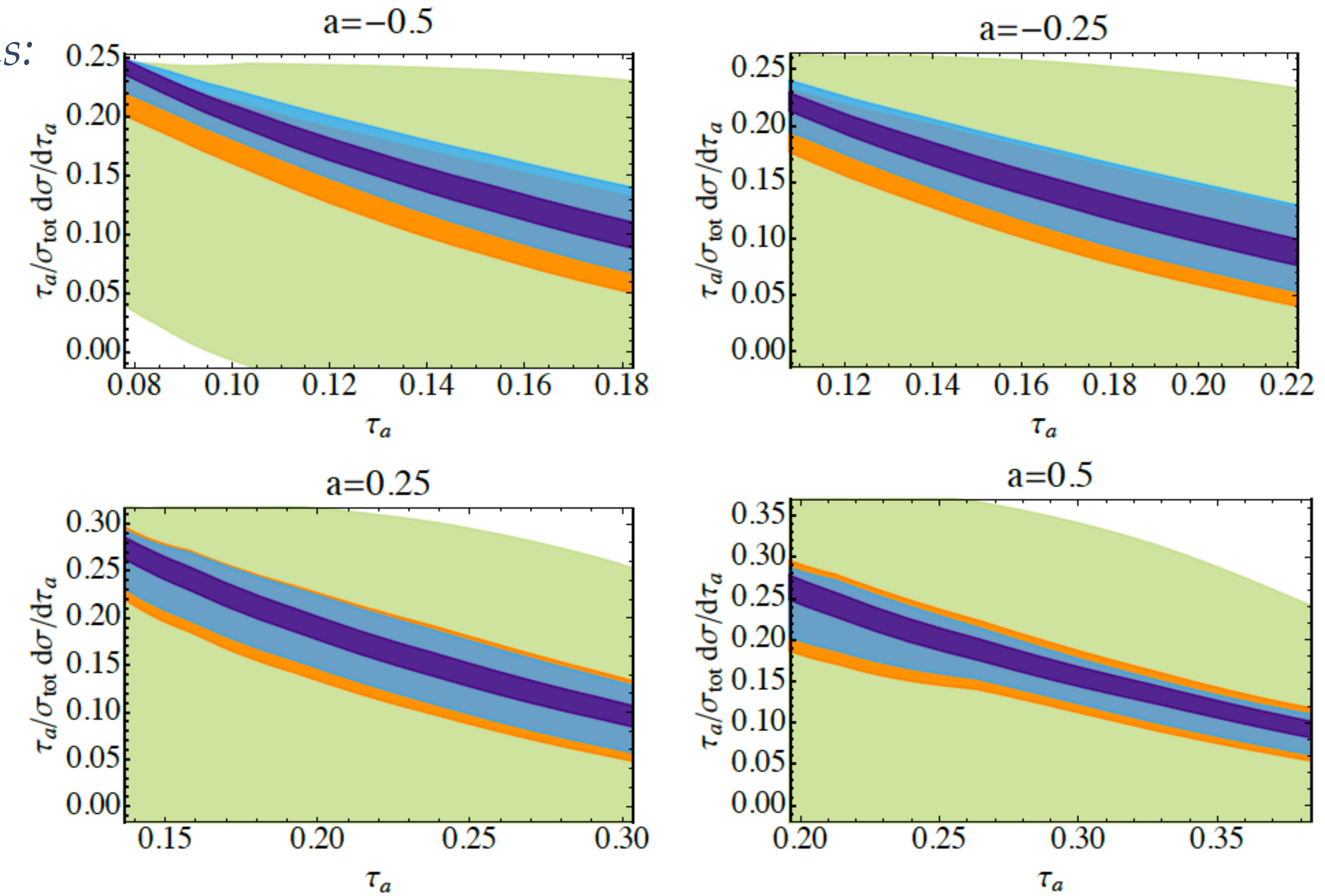
Integrated distributions:



Angularity distributions

[NNLL′+$\mathcal{O}(\alpha_s^2)$]
1808.07867

Differential distributions:



Hybrid 2010/2018 scales

2010 scales with 2018 t_1 parameter: