α_s in (2+1+1)-Flavor QCD from the Static Energy

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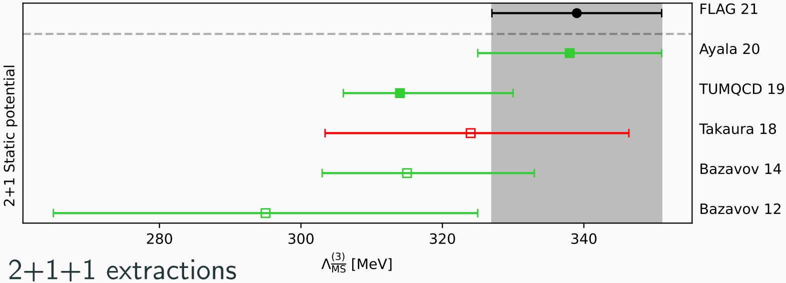
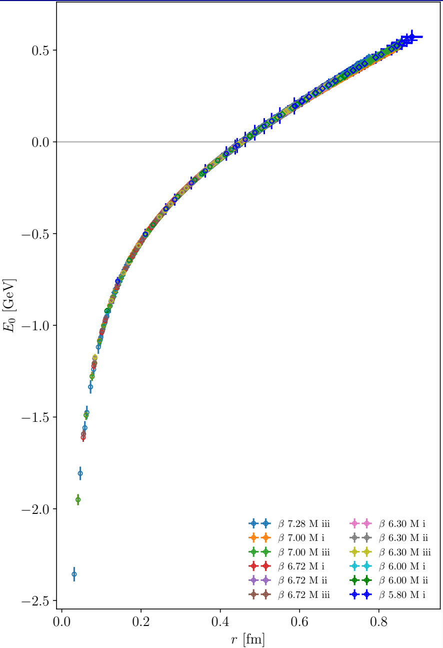
Motivation

- Static energy $E_0(r)$ between a static quark and antiquark
- Defined as a ground state of Wilson loop
- Of major importance for scale setting
- Very well known in perturbation theory
- Physical observable: No scheme change required between lattice and continuum
- Detailed previous studies in $N_f = 0$ and $N_f = 2+1 \Rightarrow \Lambda_{\overline{\mathrm{MS}}}^{N_f=0,3}$



- Very few 2+1+1 extractions

Static energy



- We calculate the static energy in 2+1+1-flavor QCD
- Data ranging from $r \approx 0.03 - 0.9$fm.
- Self-consistently set the scale with

$$r_i^2 F(r_i) = \begin{cases} 1.65, & i = 0^{\,1} \\ 1.0\ , & i = 1^{\,2} \\ 0.5\ , & i = 2^{\,3} \end{cases}, \quad \begin{matrix} r_0 \sim 0.475\,\mathrm{fm} \\ r_1 \sim 0.3106\,\mathrm{fm} \\ r_2 \sim 0.145\,\mathrm{fm} \end{matrix}$$

- Take massive charm quark into account $1/m_c \sim 0.15 fm$
- Aim to extract $\Lambda_{\overline{\mathrm{MS}}}$ from the small distance behavior
- Scale setting and charm effect done in: TUMQCD, PRD107 (2023)

Static energy in perturbation theory

- Determined from the large-time behavior of Wilson loops

$$E(r) = -\lim_{T\to\infty} \frac{\ln\langle \mathrm{Tr}(W_{r\times T})\rangle}{T}\,, \qquad W_{r\times T} = P\left\{ \exp\left(i \oint_{r\times T} dz_\mu \, gA_\mu \right) \right\}$$

- Perturbatively known to $\mathrm{N^3LL}$ [1]:

$$E_0(r) = \Lambda_\mathrm{s} - \frac{C_\mathrm{F}\alpha_\mathrm{s}}{r}\left(1 + \#\alpha_\mathrm{s} + \#\alpha_\mathrm{s}^2 + \#\alpha_\mathrm{s}^3 + \#\alpha_\mathrm{s}^3 \ln \alpha_\mathrm{s} + \#\alpha_\mathrm{s}^4 \ln^2 \alpha_\mathrm{s} + \#\alpha_\mathrm{s}^4 \ln \alpha_\mathrm{s} \ldots\right)$$

- The ultra-soft scale μ_us gives rise to $\ln \alpha_\mathrm{s}(1/r)$ term
- Ultrasoft logs can be resummed
- Set scale as $\mu = 1/r$

[1] For review of perturbative results, see: X. Tormo Mod. Phys. Lett. A28 (2013)

Static energy in perturbation theory

- In the dimensional regularization requires a renormalon subtraction
- In the lattice regularization diverges as $1/a$ towards continuum limit
- Both regularization problems can be absorbed into a constant term
- Integrating the static force eliminates the leading renormalon

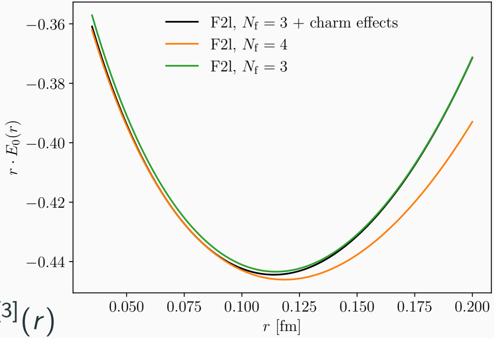
$$E_0(r) = \int_{r^*}^{r} \mathrm{d}r' F(r') + \mathrm{const}$$

Charm quark mass effects in perturbation theory

- Effects due to finite mass of a heavy quark give correction $\delta V_{\mathrm{m}}^{(N_{\mathrm{f}})}(r)$

$$E_{0,m}^{(N_{\mathrm{f}})}(r) = \int_{r^*}^{r} \mathrm{d}r' F^{(N_{\mathrm{f}})}(r') + \delta V_{\mathrm{m}}^{(N_{\mathrm{f}})}(r) + \mathrm{const}$$

$$E_{0,m}^{(N_{\mathrm{f}})} \to E_0^{(N_{\mathrm{f}})} \quad \text{for } r \gg 1/m$$
$$E_{0,m}^{(N_{\mathrm{f}})} \to E_0^{(N_{\mathrm{f}}+1)} \quad \text{for } r \ll 1/m$$



- Finite mass corrections known up to two loops[1]
  $\delta V_{\mathrm{m}}^{(N_{\mathrm{f}})}(r) = \delta V_{\mathrm{m}}^{(N_{\mathrm{f}}),[2]}(r) + \delta V_{\mathrm{m}}^{(N_{\mathrm{f}}),[3]}(r)$

[1] D. Eiras, J. Soto, PRD61 (2000); M. Melles PRD62 (2000); A. H. Hoang hep-ph/0008102 (2000)

Lattice simulations

our naming	$N_\sigma^3 \times N_\tau$	β	$a_{f_{p4s}}$ (fm)	u_0	am_l	am_s	am_c	m_l/m_s	$(am_s)_{\text{tuned}}$	M_π (MeV)	#conf.
β 5.80 M i	$32^3 \times 48$	5.80	0.15294	0.85535	0.00235	0.0647	0.831	phys	0.06852	131	1041
β 6.00 M ii	$32^3 \times 64$	6.00	0.12224	0.86372	0.00507	0.0507	0.628	1/10	0.05296	217	1000
β 6.00 M i	$48^3 \times 64$				0.00184			phys		132	709
β 6.30 M iii	$32^3 \times 96$	6.30	0.08786	0.874164	0.0074	0.037	0.44	1/5	0.03627	316	1008
β 6.30 M ii	$48^3 \times 96$				0.00363	0.0363	0.43	1/10		221	1031
β 6.30 M i	$64^3 \times 96$				0.0012		0.432	phys		129	1074
β 6.72 M iii	$48^3 \times 144$	6.72	0.05662	0.885773	0.0048	0.024	0.286	1/5	0.02176	329	1017
β 6.72 M ii	$64^3 \times 144$				0.0024			1/10		234	1103
β 6.72 M i	$96^3 \times 192$				0.0008	0.022	0.26	phys		135	1268
β 7.00 M iii	$64^3 \times 192$	7.00	0.0426	0.892186	0.00316	0.0158	0.188	1/5	0.01564	315	1165
β 7.00 M i	$144^3 \times 288$				0.000569	0.01555	0.1827	phys		134	478
β 7.28 M iii	$96^3 \times 288$	7.28	0.03216	0.89779	0.00223	0.01115	0.1316	1/5	0.01129	309	821

- Measure static energy as the ground state of the Wilson line correlation function in Coulomb gauge
- Use state of the art 2+1+1 HISQ[1] ensembles from MILC[2]
- Three different light quark masses, physical strange and sea
- Six lattice spacings via f_{p4s} scale

[1] E. Follana, *et.al.*, PRD75 (200/); [2] A. Bazavov, *et.al.*, PRD98 7 (2018)

Static energy on the lattice

- We compute E_0 from Wilson line correlators in Coulomb gauge $W(\mathrm{r},\tau,a) = \prod_{u=0}^{\tau/a-1} U_4(\mathrm{r},ua,a)$

$$C(\mathrm{r},\tau,a) = \left\langle \frac{1}{N_\sigma^3} \sum_x \sum_{y=R(r)} \frac{1}{N_c N_\mathrm{r}} \mathrm{Tr}\, W^\dagger(x+y,\tau,a) W(x,\tau,a) \right\rangle$$

$$C(\mathrm{r},\tau,a) = \mathrm{e}^{-\tau E_0(\mathrm{r},a)} \left(C_0(\mathrm{r},a) + \sum_{n=1}^{N_\mathrm{st}-1} C_n(\mathrm{r},a) \prod_{m=1}^{n} \mathrm{e}^{-\tau \Delta_m(\mathrm{r},a)} \right) + \cdots$$

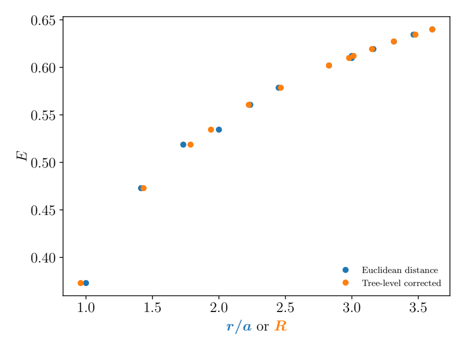
- Vary the fit range with N_st and $|\mathrm{r}|$

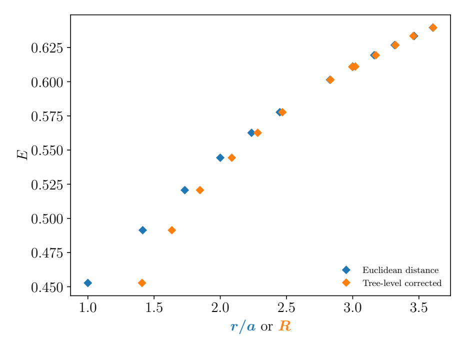
$$\begin{aligned} |\mathrm{r}| + 0.2\text{ fm} &\leq \tau_{\mathrm{min},1} \leq 0.3\text{ fm} && \text{for } N_\mathrm{st} = 1, \quad \Rightarrow \text{ prior values} \\ \tfrac{2}{3}|\mathrm{r}| + 0.1\text{ fm} &\leq \tau_{\mathrm{min},2} \leq \tau_{\mathrm{min},1} - 2a && \text{for } N_\mathrm{st} = 2, \quad \Rightarrow \text{ our pick} \\ \tfrac{1}{3}|\mathrm{r}| &\leq \tau_{\mathrm{min},3} \leq \tau_{\mathrm{min},2} - 2a && \text{for } N_\mathrm{st} = 3, \quad \Rightarrow \text{ cross-check} \end{aligned}$$

- We use Bayesian fits with loose linear priors
- Priors for $E_1 - E_0$ come from pure gauge [1]
- Measurements with and without one step of HYP-smearing[2]

[1] K. Juge, *et.al.*, PRL90 (2003) [2] A. Hasenfratz, *et.al.*, PRD64 (2001);

Discretization effects: Tree-level

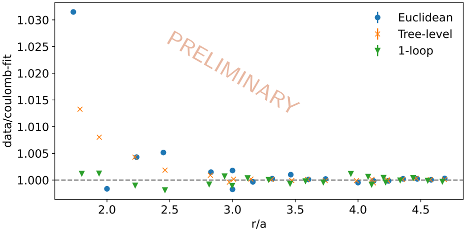


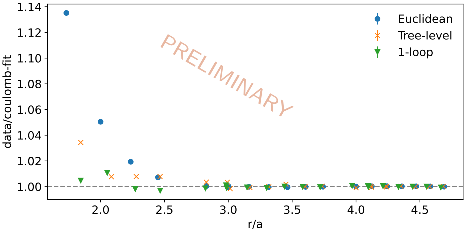


- $E_0(r, a)$ is available only at discrete distances and is direction dependent
- Tree-level improved distance defined with lattice gluon propagator $D_{\mu\nu}(k)$

$$E(r, a) = -C_F g_0^2 \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{ik\cdot r} D_{44}(k) = -\frac{C_F g_0^2}{4\pi} \frac{1}{r_I}$$

Discretization effects: 1-loop





- Ongoing effort to calculate the 1-loop improvement
- Lattice perturbation theory can be too complicated to do by hand
- Use HPsrc and HiPPy programs to numerically calculate the diagrams
- Promising results, but needs fine tuning before prime time in analysis

Lattice scales

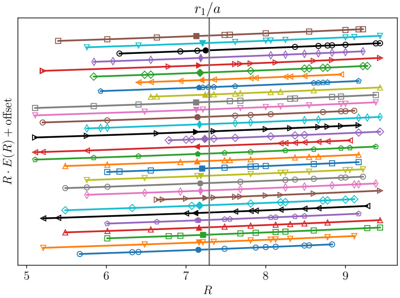
- Static energy allows for determination of lattice scales r_i and the string tension σ

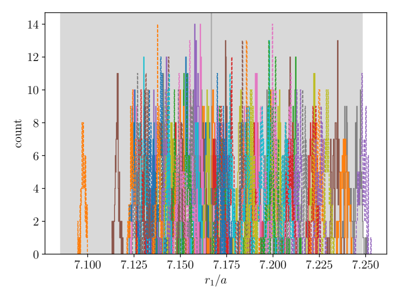
$$r_i^2 F(r_i) = \begin{cases} 1.65, & i = 0\,^{1} \\ 1.0\;, & i = 1\,^{2} \\ 0.5\;, & i = 2\,^{3} \end{cases}, \quad \begin{matrix} r_0 \sim 0.475\,\mathrm{fm} \\ r_1 \sim 0.3106\,\mathrm{fm} \\ r_2 \sim 0.145\,\mathrm{fm} \end{matrix}$$

- $r_2 \sim 1/m_c$, scales r_i expected to be affected by charm differently
- Locally fit the data using Cornell ansatz

$$E(R,a) = -\frac{A}{R} + B + \sigma R$$

- Asymmetric random picking for systematics





[1] R. Sommer, NPB411 (1994); [2] C. Bernard, *et.al.*, PRD62 (2000); [3] A. Bazavov, *et.al.*, PRD97 (2018)

Continuum limit

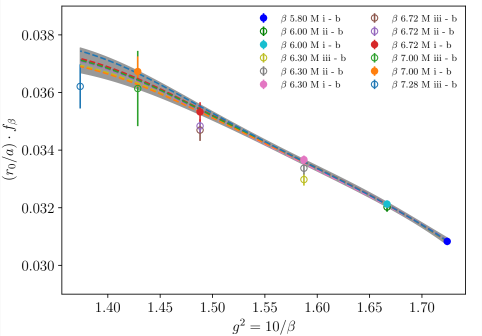
$$\frac{a}{r_i} = \frac{C_0 f_\beta + C_2 g_0^2 f_\beta^3 + C_4 g_0^4 f_\beta^3}{1 + D_2 g_0^2 f_\beta^2}$$

$$f_\beta = (b_0 g_0^2)^{-b_1/(2b_0^2)} e^{-1/(2b_0 g_0^2)},$$

$$C_0 = C_{00} + C_{01l}\frac{am_l}{f_\beta} + C_{01s}\frac{am_s}{f_\beta} + C_{01}\frac{am_{tot}}{f_\beta} + C_{02}\frac{(am_{tot})^2}{f_\beta},$$

$$C_2 = C_{20} + C_{21}\frac{am_{tot}}{f_\beta},$$

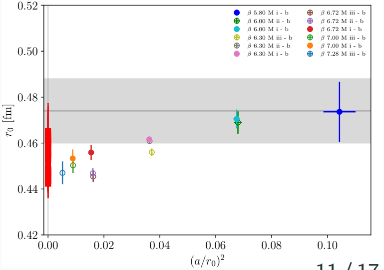
$$am_{tot} = 2am_l + am_s + am_c,$$



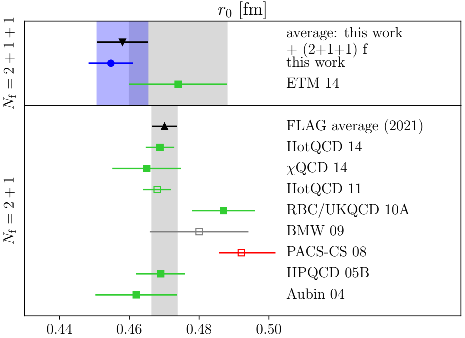
- Smooth the data with Allton ansatz
- Leading discretization effects $\alpha_s^2 a^2$ and a^4
- Lattice spacing dependence: $x = (a/r_{0,1})^2$
- Light quark mass dependence: $y = (am_l)/(am_s)$

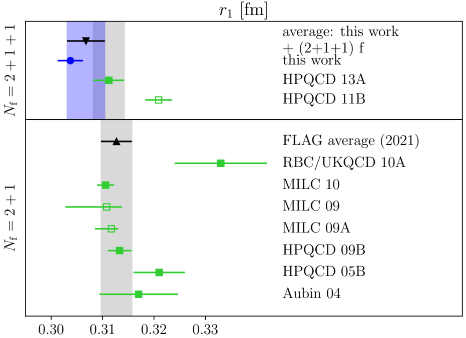
$$\xi = \xi_0 + \alpha^2[\xi_1 x + \xi_2 x y^{(1,2)}] + \xi_3 x^2 + \xi_4 y\,,$$

- Where $\alpha = 1$ or $\alpha = g_0^2/(4\pi u_0^2)$



Extracted scales compared to literature





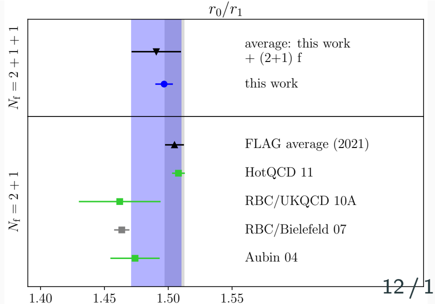
$$r_0 = 0.4547(64) \text{ fm},$$

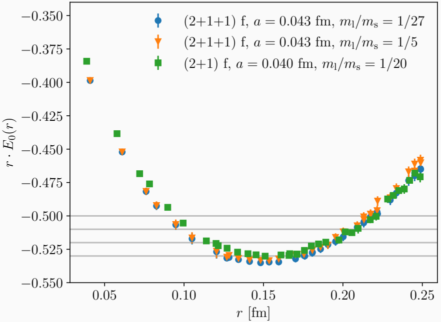
$$r_1 = 0.3037(25) \text{ fm}, \quad r_0/r_1 = 1.4968(69),$$

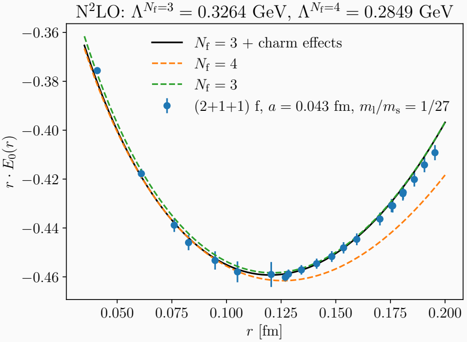
$$r_2 = 0.1313(41) \text{ fm}, \quad r_1/r_2 = 2.313(69).$$

$$\sqrt{\sigma r_0^2} = 1.077 \pm 0.016 \quad (A = A_{r_0}),$$

$$\sqrt{\sigma r_0^2} = 1.110 \pm 0.016 \quad (A = \pi/12).$$



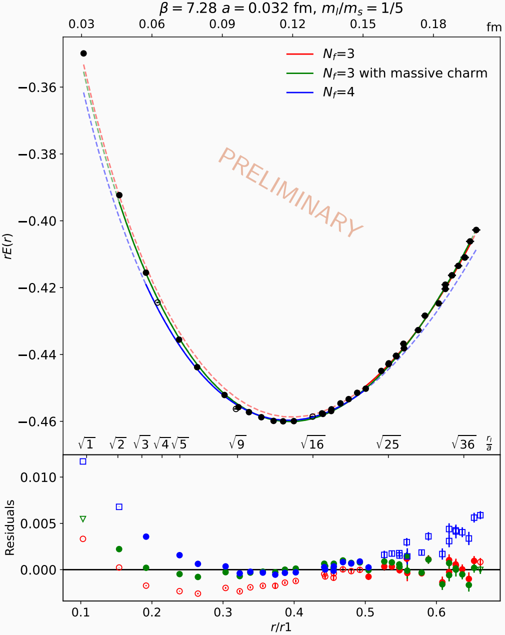




- Clearly visible difference between the behaviors of 2+1[1] and 2+1+1
- Curve with charm effects follows the data better than curves without

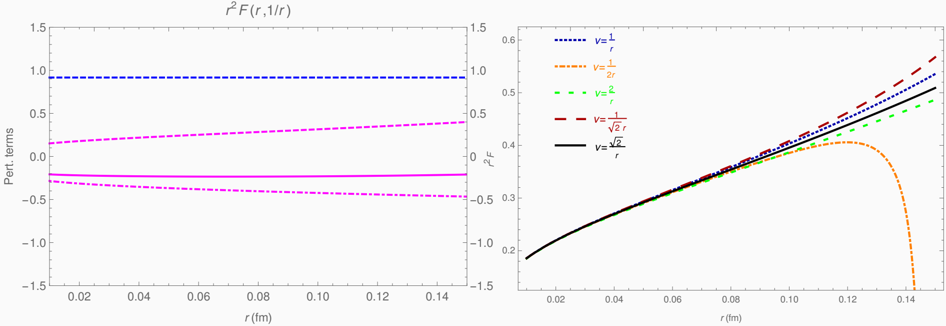
[1] A. Bazavov, *et.al.*, PRD100 (2019)

Fitting lambda



- Fit $N_f = 4$ static potential to $r < 1/m_c$, $N_f = 3$ to $r > 1/m_c$
- Fit $N_f = 3$ plus massive charm to whole range
- Choose fit range with these limits using AIC
- Limits for this talk
  - Limit to 2-loops for consistency
  - Tree-level improvement
  - Limit to off-axis points
  - Focus on finest ensemble

Truncation of perturbation theory



- Variation of ultra soft resummations
- Variation of the soft scale by some factor $\sqrt{2}$ or 2
- Estimated truncation error shrinks for smaller values of max(1/r)

figures from A. Bazavov, *et.al.*, PRD100 (2019)

Some (preliminary) results

- Reminder on scales:

  2+1+1 $r_1 = 0.3037(25)$

  2+1 $r_1 = 0.3106(17)$

- We get very preliminary results:

N_f	$r_1\Lambda^{(N_f)}$	$\Lambda^{(N_f)}$ [MeV]	decouple
$3+1$	$0.501(19)_{\mathrm{lat}}(43)_{\mathrm{soft-scale}}$	$0.325(13)_{\mathrm{lat}}(28)_{\mathrm{soft-scale}}$	0.283(31)
4	$0.457(22)_{\mathrm{lat}}(32)_{\mathrm{soft-scale}}$	$0.297(14)_{\mathrm{lat}}(21)_{\mathrm{soft-scale}}$	0.339(25)

- Comparing to literature results

N_f	TUMQCD19 r_1	TUMQCD19 MeV	FLAG21
3	0.494^{+24}_{-13}	314^{+15}_{-8}	339(12)
4			297(10)

Conclusions

- We have computed the static energy $E_0(r)$ with 2+1+1 flavors
- Determined scales r_0, r_1, and r_2, their ratios, and string tension σ
- All these scales have been measured simultaneously
- We can see charm decoupling well in the data
  - Perturbative charm effects give better description of the data
  - Scales closer to inverse charm mass differ more from their 2+1 flavor counterparts
- Initial, promising results on the $\Lambda_{\overline{\mathrm{MS}}}$
- One loop improvement coming soon

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Thank you for your attention!