

# Power corrections in event-shape distributions



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G. Vita & I. Stewart

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- Dijet power corrections [PRD 75 (2007) 014022]
- Soft renormalon subtraction [PLB 660 (2008) 483-493]

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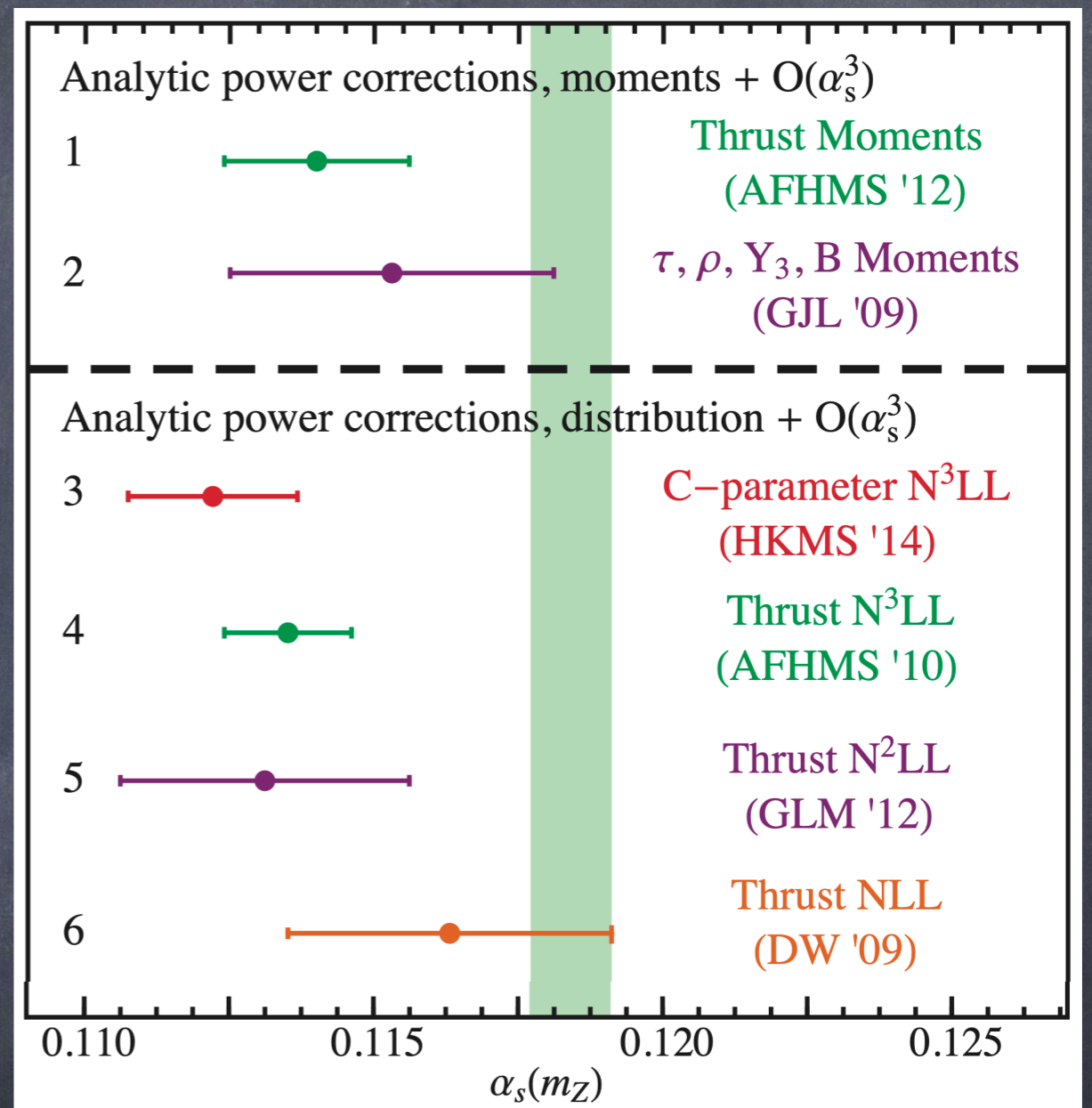
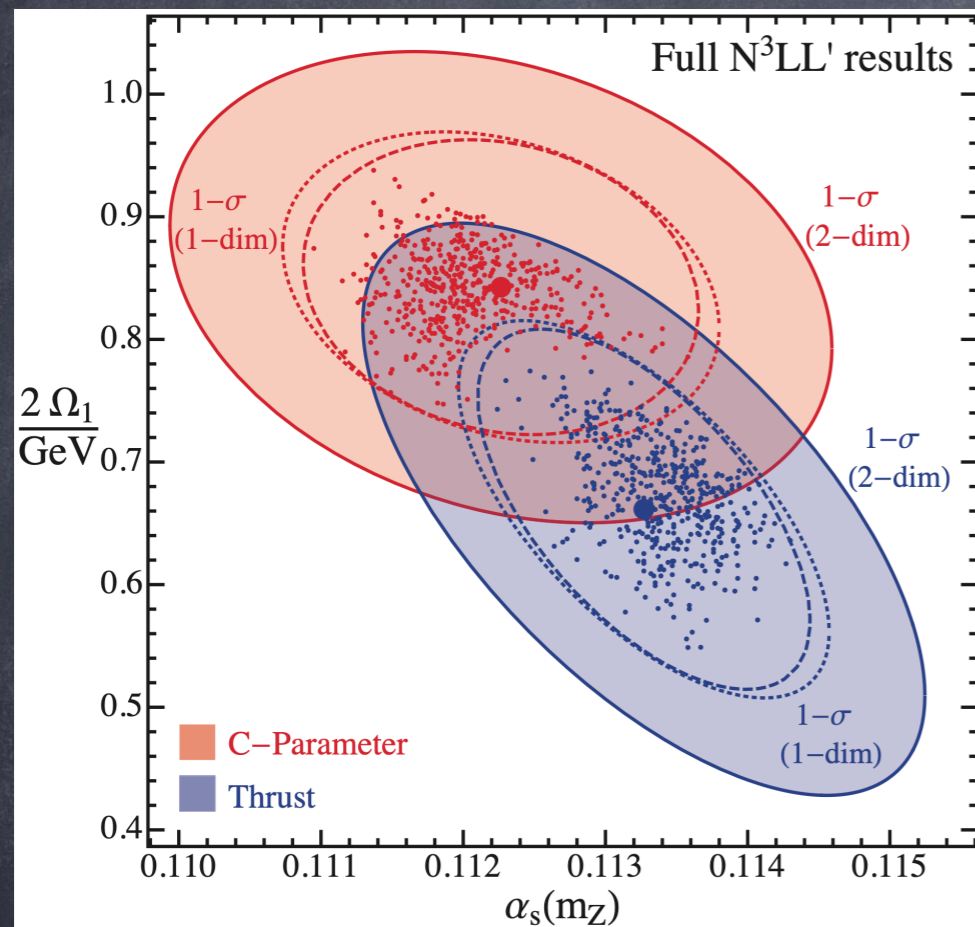
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Experimental data on event shapes also very precise

Motivated fits to data to obtain strong coupling

- Thrust [PRD 83 (2011) 074021, EPJC 73 (2013) 1, 2265]
- Thrust moments [PRD 86 (2012) 094002]
- C-parameter [PRD 91 (2015) 9, 094017-18]
- Moments to many event-shapes [EPJC 67 (2010) 57-72]

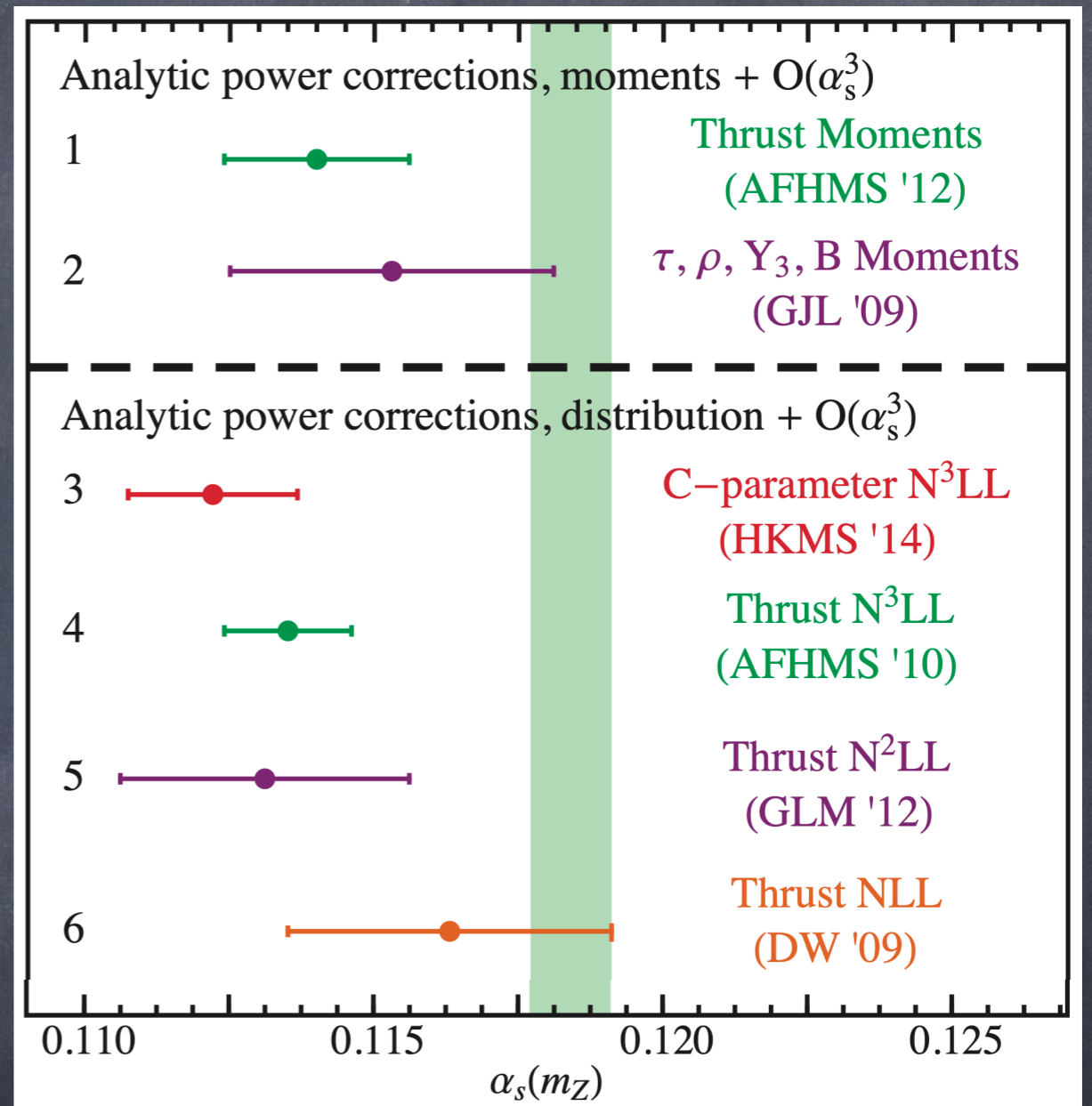
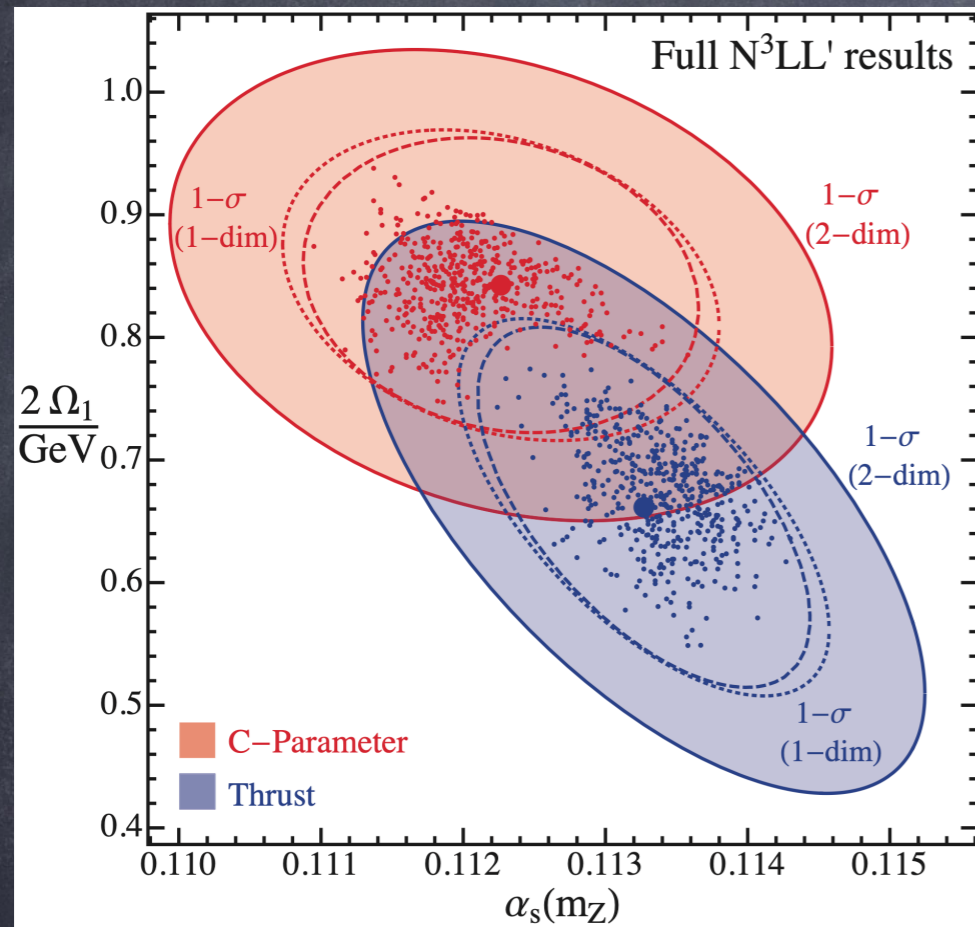
# Introduction



Although  $\alpha_s$  was obtained with high accuracy, the **central value** is **uncomfortably small** (compared to world average)

This has raised **controversy** in recent years

# Introduction



Low values confirmed by recent analysis

[Bell, Lee, Makris, Talbert, Yan, 2311.03990]

Review on strong coupling  
fits from event-shape  
distributions



Theoretical overview

# Partonic cross section

Partonic cross section  $\frac{d\hat{\sigma}}{d\tau} = \frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau}$

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Singular dominates in peak and tail

Large Sudakov logs, need resummation

factorisation achieved in SCET, resummation through RG evolution

$$\frac{1}{\sigma_0} \frac{d\hat{\sigma}_s}{d\tau} = H(Q, \mu_H) U_H(Q, \mu_H, \mu_J) \int ds J_\tau(s, \mu_J) \int dk U_s^\tau(k', \mu_J, \mu_S) \hat{S}_\tau\left(Q\tau - \frac{s}{Q}, \mu_S\right)$$

[Schwartz PRD 77 (2008) 014026]

[Becher & Schwartz JHEP 07 (2008) 034]

[Fleming, Hoang, Mantry, Stewart PRD 77 (2008) 074010]

[Bauer, Fleming, Lee, Sterman PRD 78 (2008) 034027]

# Partonic cross section

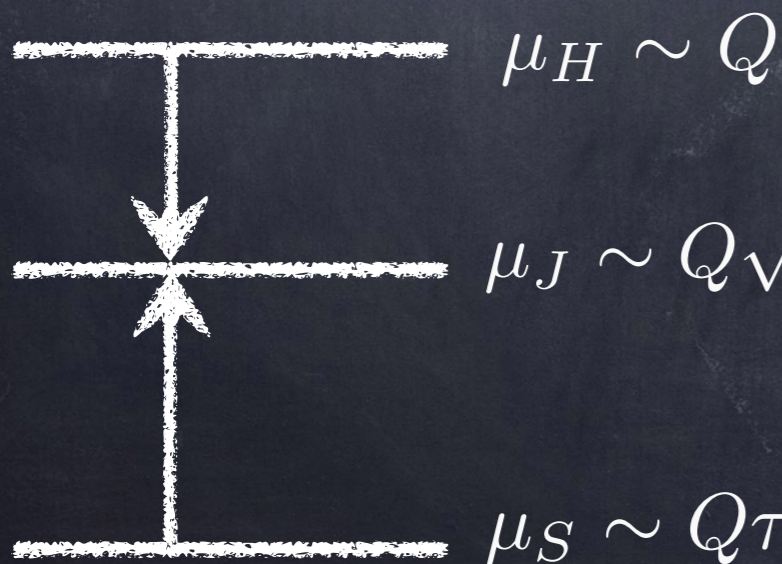
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"canonical"  
scale setting

if  $\mu_H = \mu_H = \mu_S$   
resummation is  
switched off

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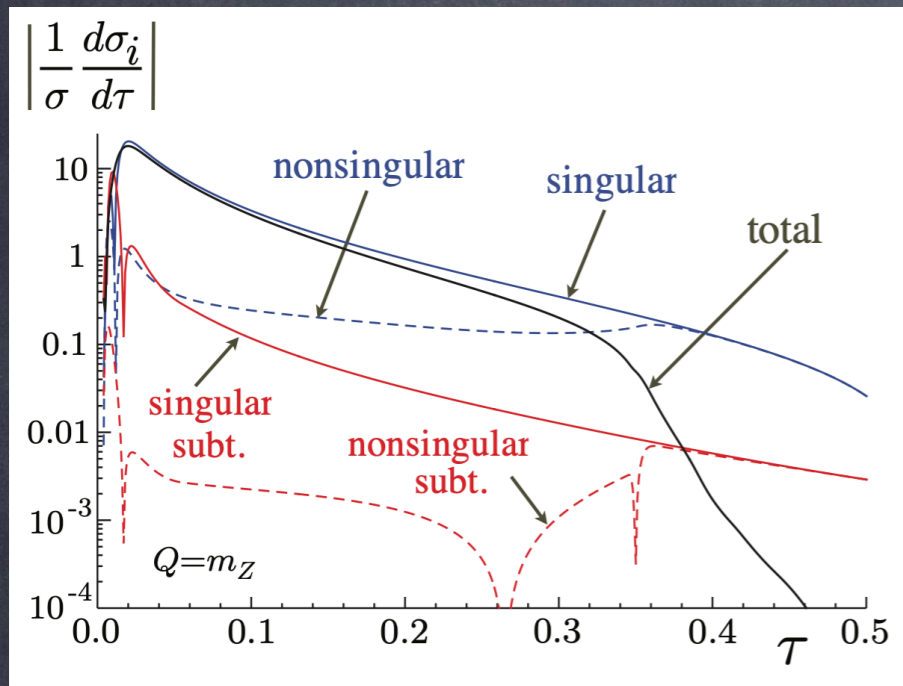
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Non-singular is a power correction in peak and tail

resummation in non-singular is work in progress (known @ LL)

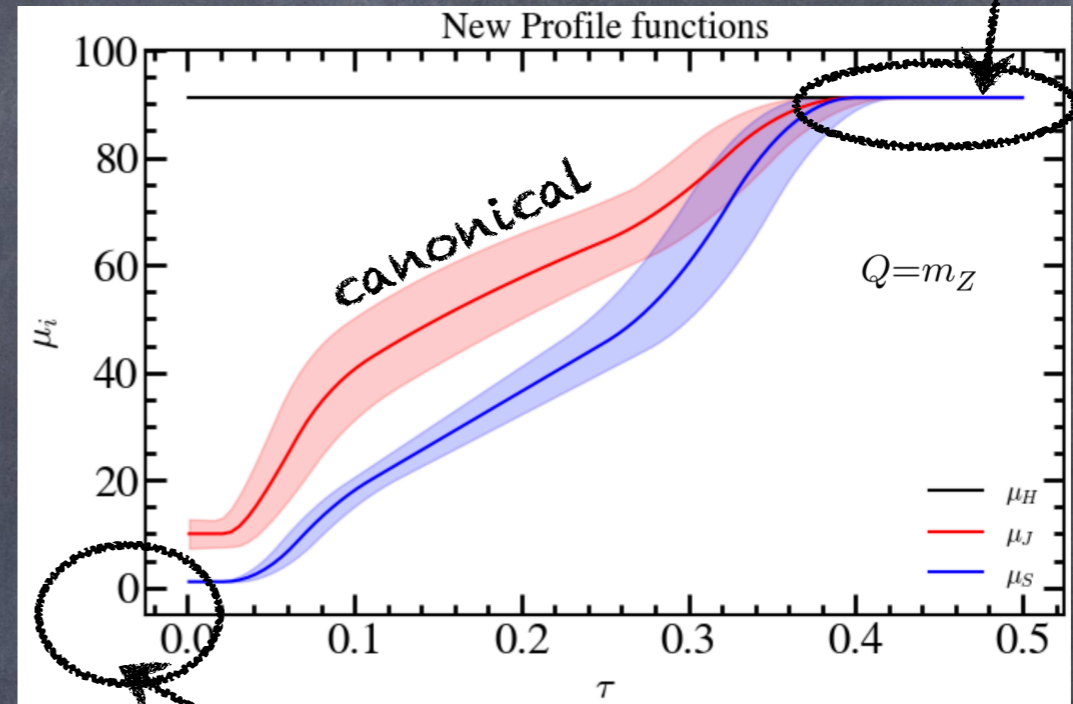
as important as singular in far-tail region

# Partonic cross section



cross section totally dominated by dijet for  $\tau < 0.11$

no resummation  
[PRD 91 (2015) 9, 094017]



non-perturbative

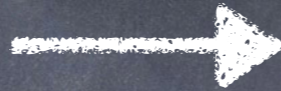
The soft function has an  $u = 1/2$  renormalon that must be removed  
[PLB 660 (2008) 483-493]

Use soft gap subtractions [see talk by M. Benitez]

Other subtractions & profile for subtraction scale: [see talk by C. Lee]

# Hadronization corrections

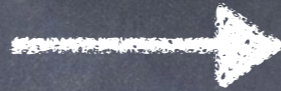
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modes with lowest virtuality  
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$$S_\tau(\ell) = \int d\ell' \hat{S}_\tau(\ell - \ell') F(\ell') \simeq \hat{S}_\tau(\ell) - \Omega_1^\tau \frac{d\hat{S}_\tau(\ell)}{d\ell} \simeq \hat{S}_\tau(\ell - \Omega_1^\tau)$$

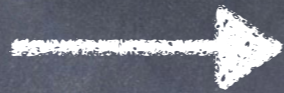
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OPE: valid in the tail  $Q_\tau \gg \Lambda_{\text{QCD}}$



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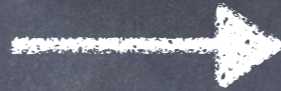
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Operator definition: [Lee, Sterman] [VM, Thaler, Stewart]

$\Omega_1^e = \langle 0 | \bar{Y}_{\bar{n}}^\dagger Y_n^\dagger (Q\hat{e}) Y_n \bar{Y}_{\bar{n}} | 0 \rangle$  involves 4 Wilson lines in fixed directions because 2-jet configuration is "unique"

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Implications for the cross section

$$\frac{d\sigma_s}{d\tau} = \frac{d\hat{\sigma}_s}{d\tau} \left( \tau - \frac{\Omega_1^\tau}{Q} \right) + \mathcal{O} \left( \frac{\Lambda_{\text{QCD}}^2}{Q^2 \tau^2} \right) \longrightarrow \frac{d\sigma}{d\tau} = \frac{d\hat{\sigma}}{d\tau} \left( \tau - \frac{\Omega_1^\tau}{Q} \right) + \mathcal{O} \left( \frac{\Lambda_{\text{QCD}}^2}{Q^2 \tau^2} \right)$$

is this justified?

Fit results

# Results

Fit to thrust data at many c.o.m. energies  $\tau \in \left[ \frac{6}{Q}, 0.33 \right]$

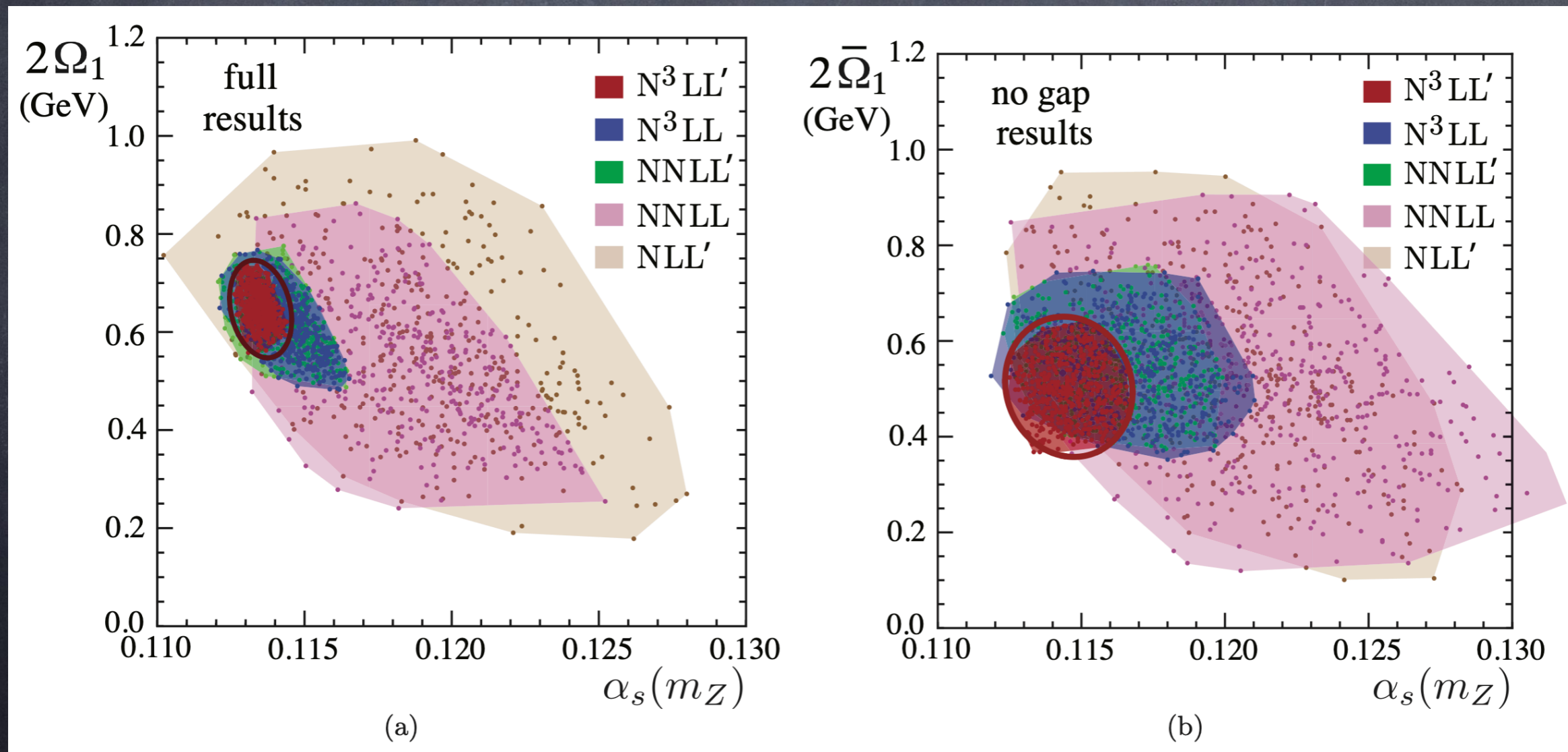
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# Results

Fit to thrust data at many c.o.m. energies  $\tau \in \left[ \frac{6}{Q}, 0.33 \right]$

Use minimal overlap model for experimental correlations

Perform random scan on perturbative parameters to account for theory correlations



Updated results in [talk by M. Benitez]

# Caveats on fit results

Main point of concern: dataset in fits includes 3-jet region  
assuming the 2-jet power correction is valid there

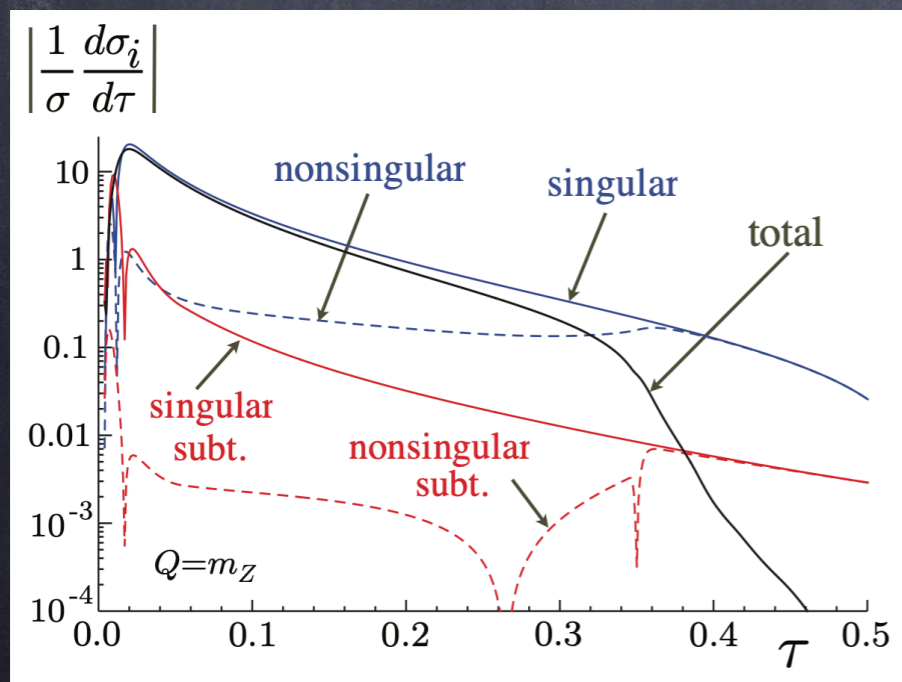
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Motivations for such assumption:

- Ensures singular-vs non-singular cancellation

if singular and ns shifted  
differently, cancellation ruined

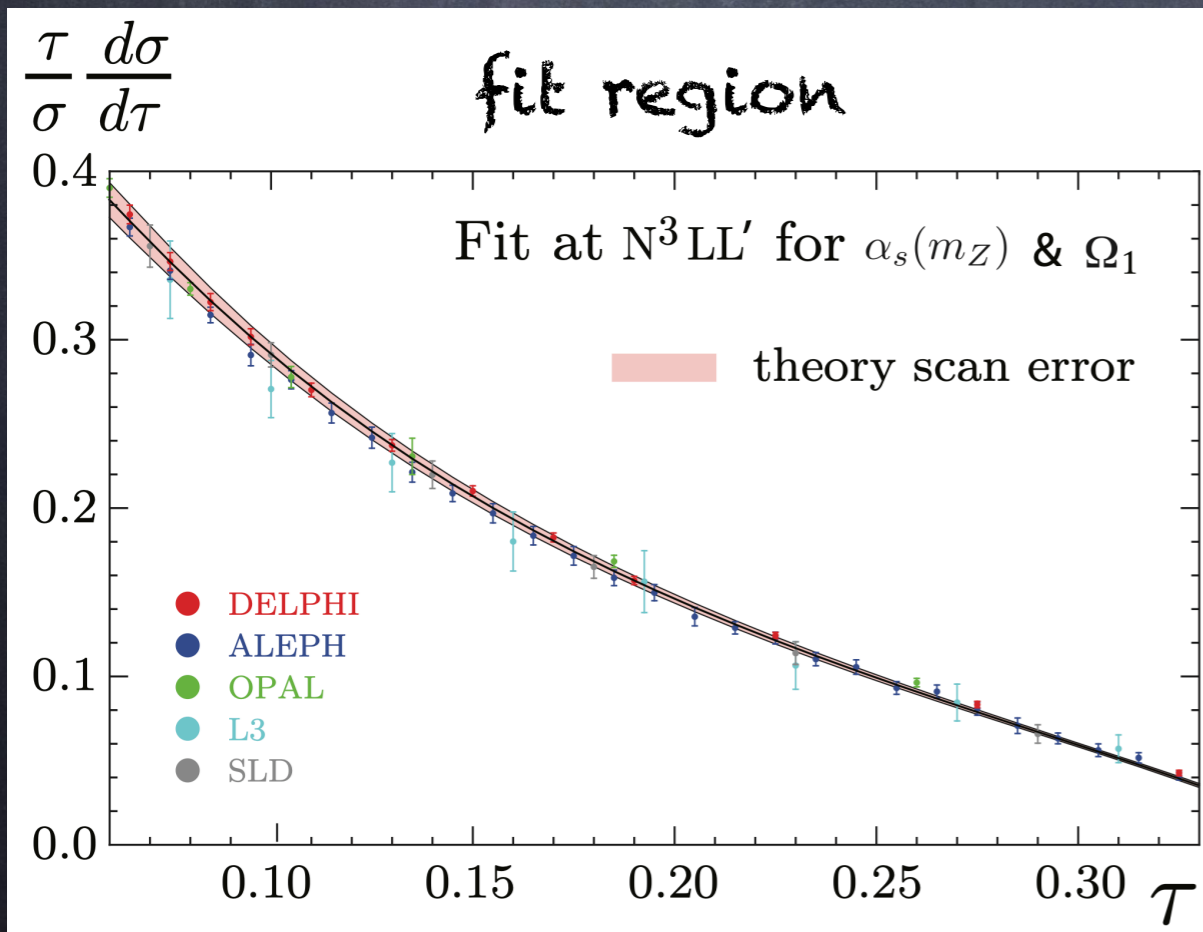


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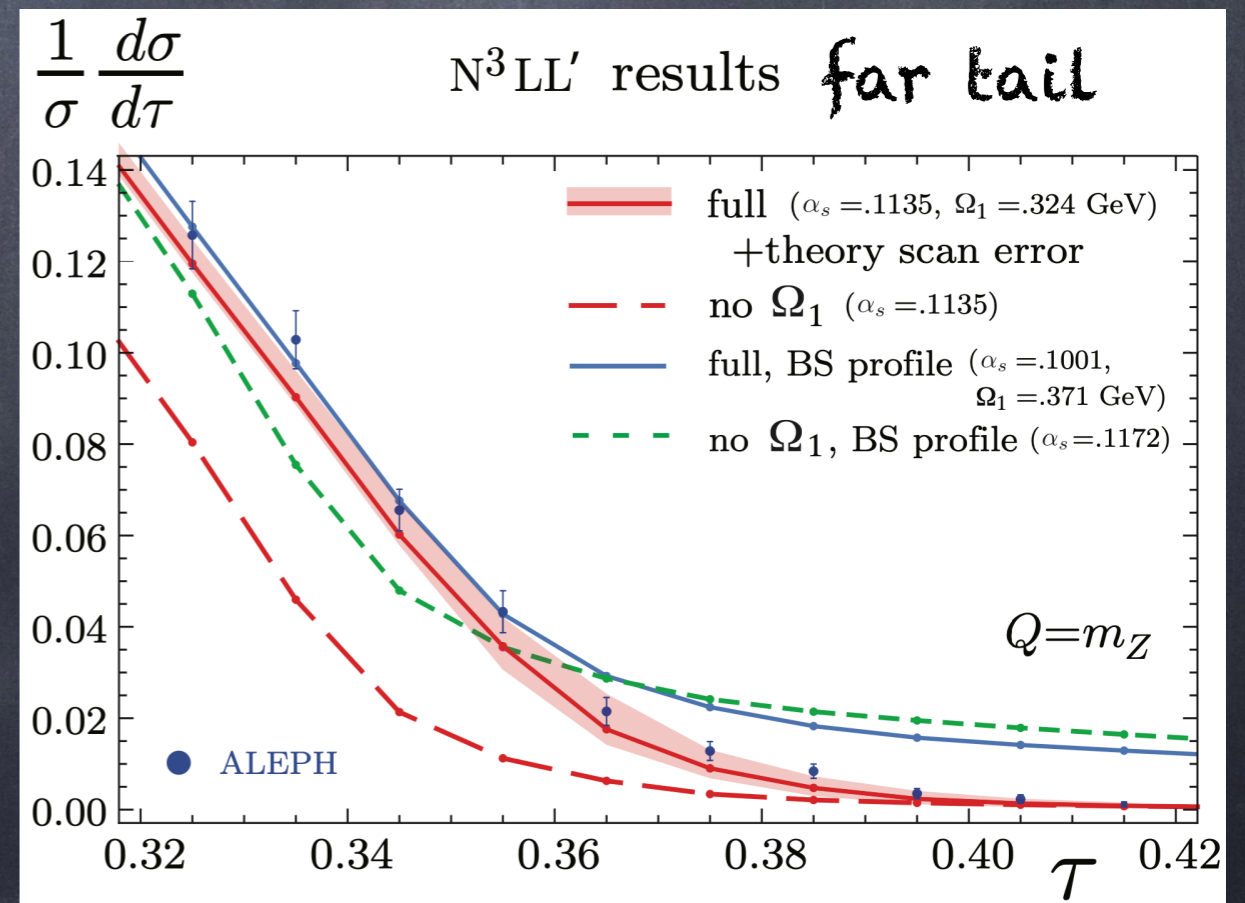
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not a fit



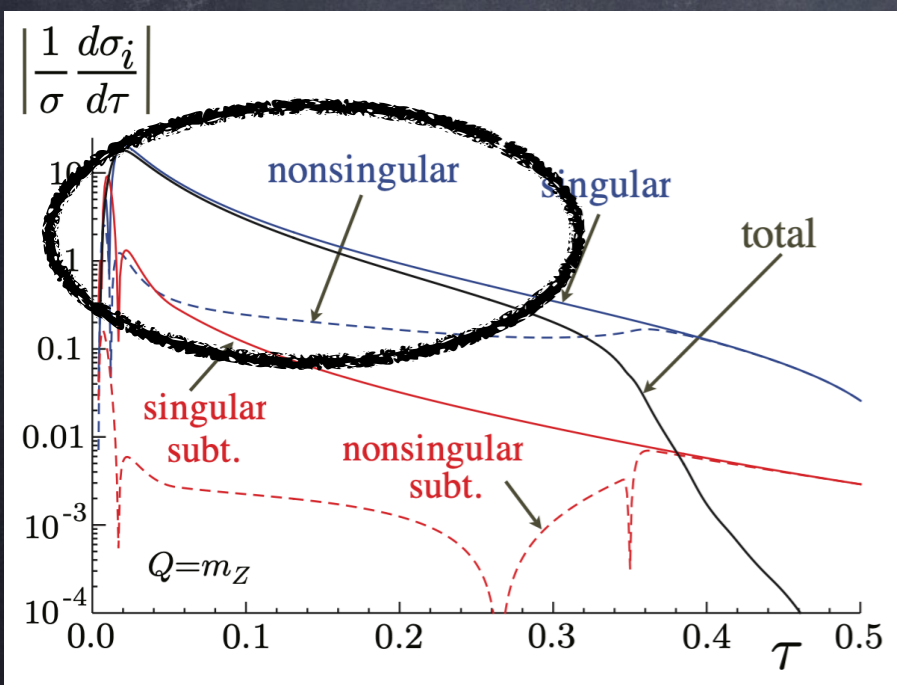


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Motivations for such assumption:

- Ensures singular-vs non-singular cancellation
- Data suggests such behaviour
- If singular terms dominate, this is justified



in most of the fit region, singular  
clearly dominates

dijet region obviously exists!!!

more conservative analysis:

restrict fit region [see talk by M. Benitez]

Recent Progress in power corrections

# Recent studies of power corrections

Luisoni, Monni, Salam [EPJC 81 (2021) 2, 158]

Use dispersive model to "compute" power correction at  $C = 3/4$

Find that, within that model  $\Omega_1^{2\text{-jet}} \simeq 2 \Omega_1^{3\text{-jet}}$

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Real-life computation: needs an operator definition

2-jet: in terms of 4 Wilson lines }  
3-jet: in terms of 6 Wilson lines } can't be related

Non-perturbative vacuum matrix element of different operators

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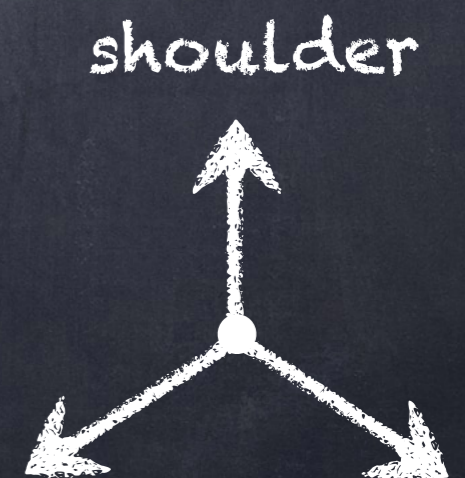
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In certain limits, direction of WLs are fixed



# Recent studies of power corrections

Caola et al. [JHEP 12 (2022) 062] [see talk by P. Nason]

Use large- $\beta_0$  limit, giving the gluon a small mass

Power correction given by term linear in  $m_g$

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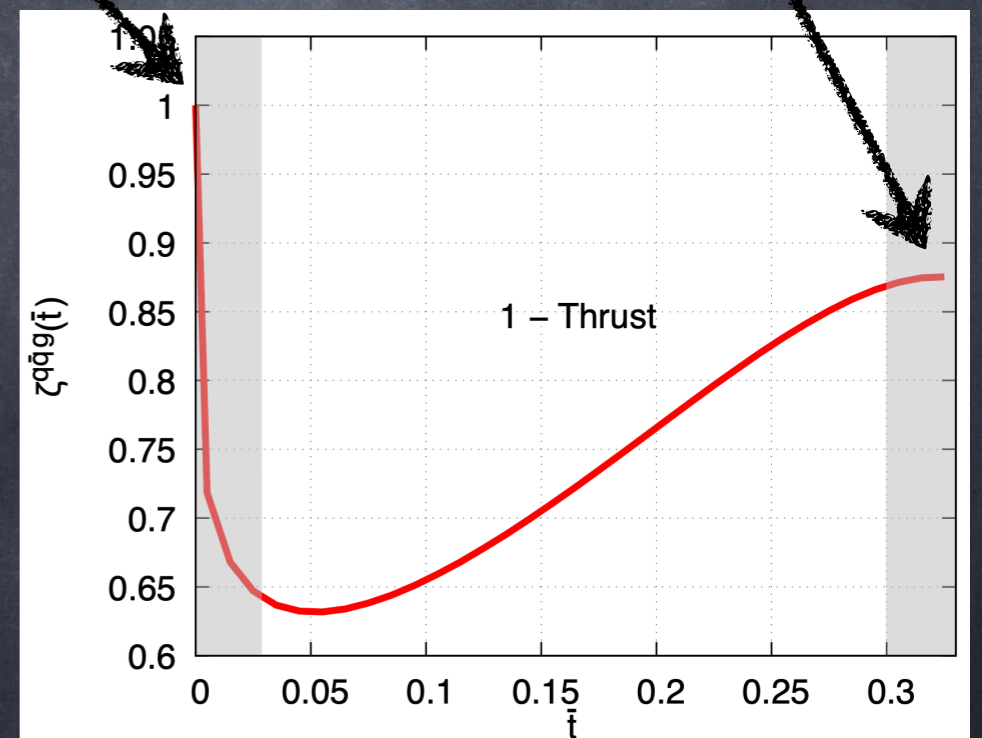
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$$\frac{d\sigma}{d\tau} = \frac{d\hat{\sigma}}{d\tau} \left[ \tau - \frac{\Omega_1^T \zeta_\tau(\tau)}{Q} \right]$$

Power correction is a shift,  
but the shift is  $\tau$  dependent

2-jet p.c.

3-jet p.c.





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## Caveats

Assume gluon mass is the smallest scale in the problem

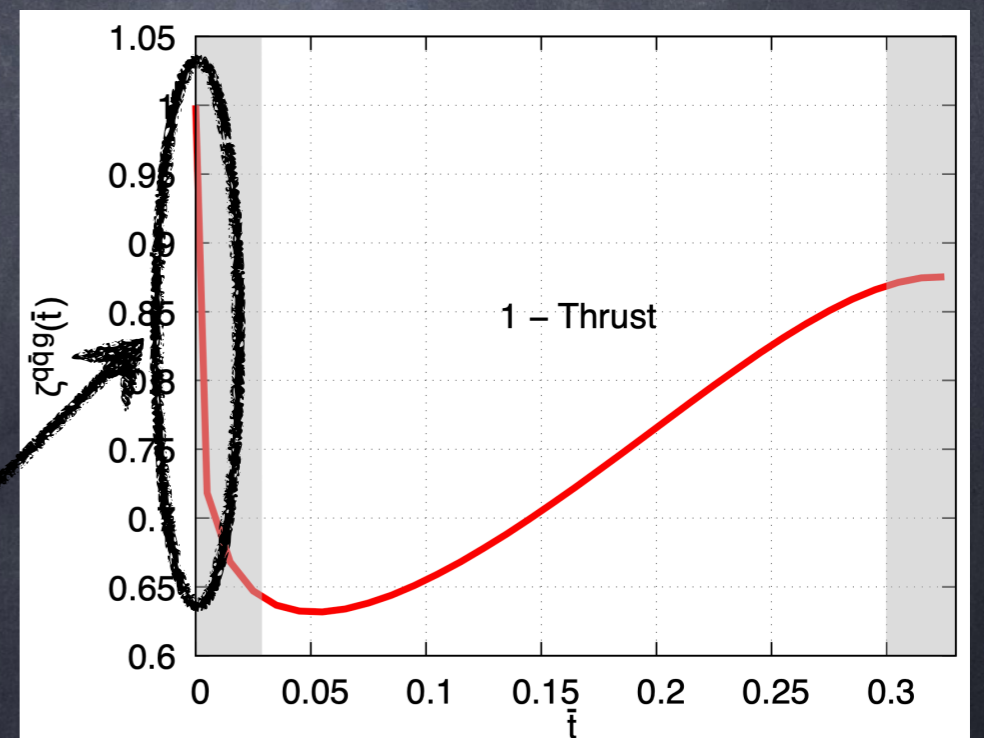
Assume 3 jets can be resolved in di-jet regions

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very steep falloff

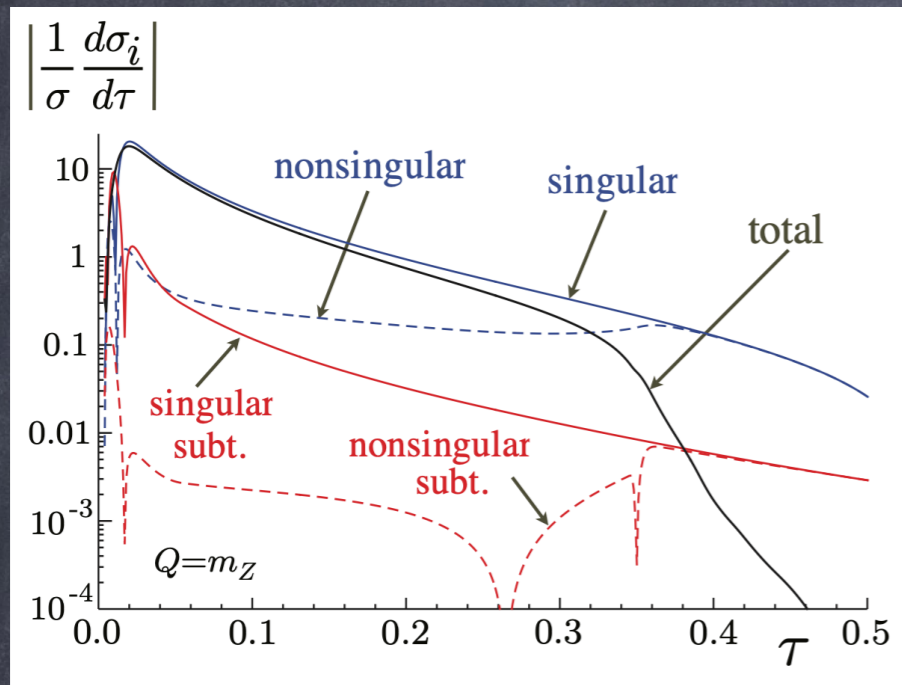
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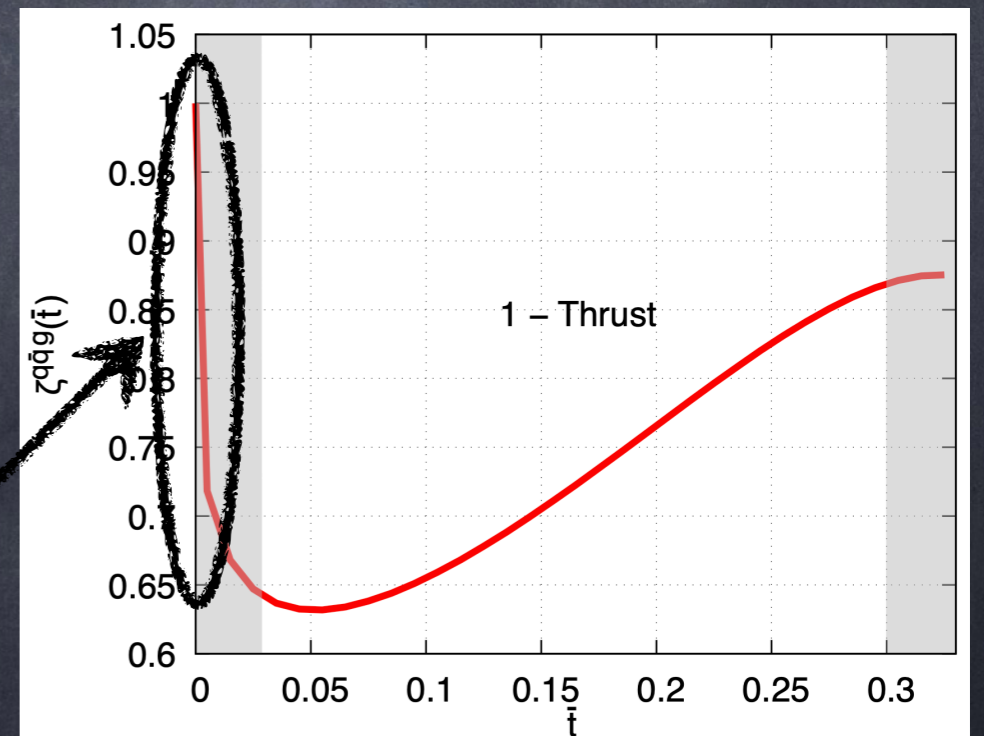
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but it does... and **overly dominates!**

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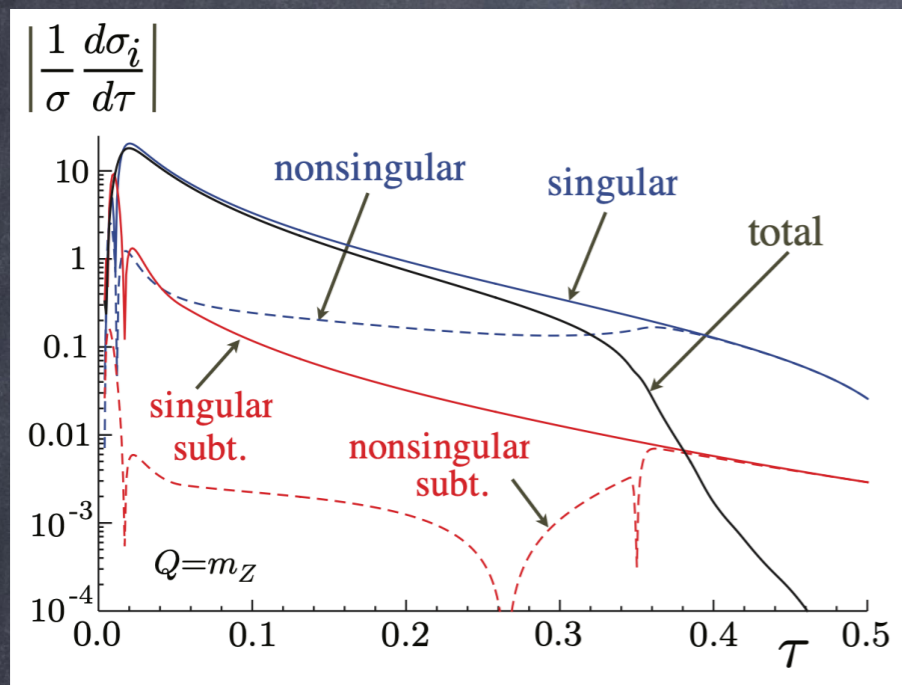
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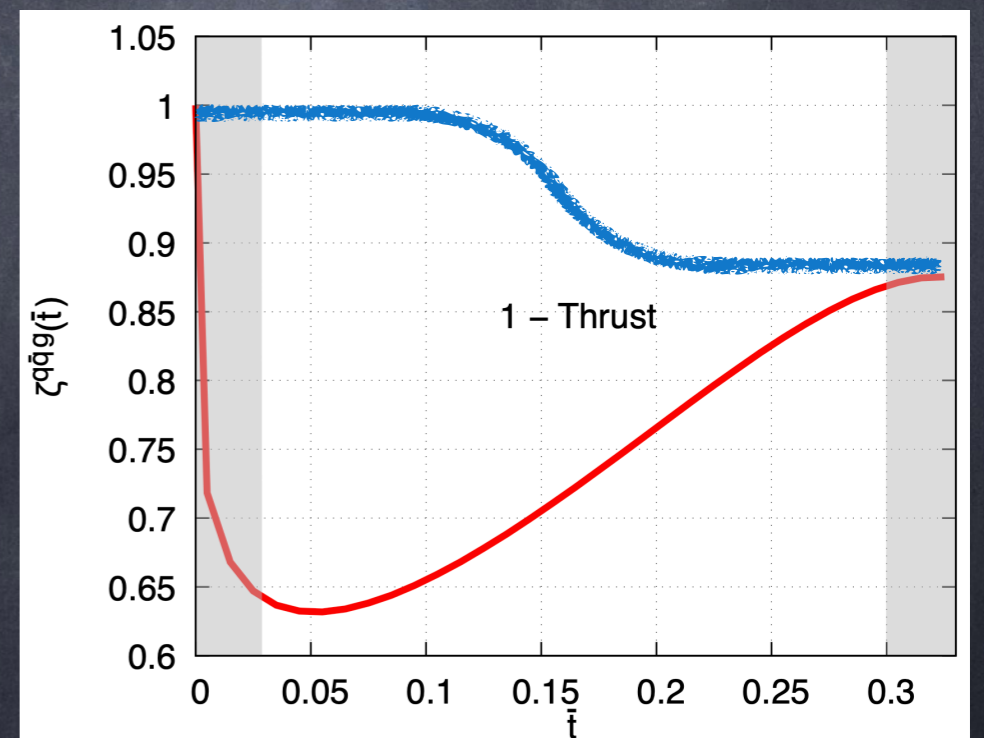


but it does... and **overly dominates!**

seems more **natural**

di-jet dominates for a while and then smoothly becomes the 3-jet power correction

exact form not known: one must investigate the effect of varying its functional form on uncertainty

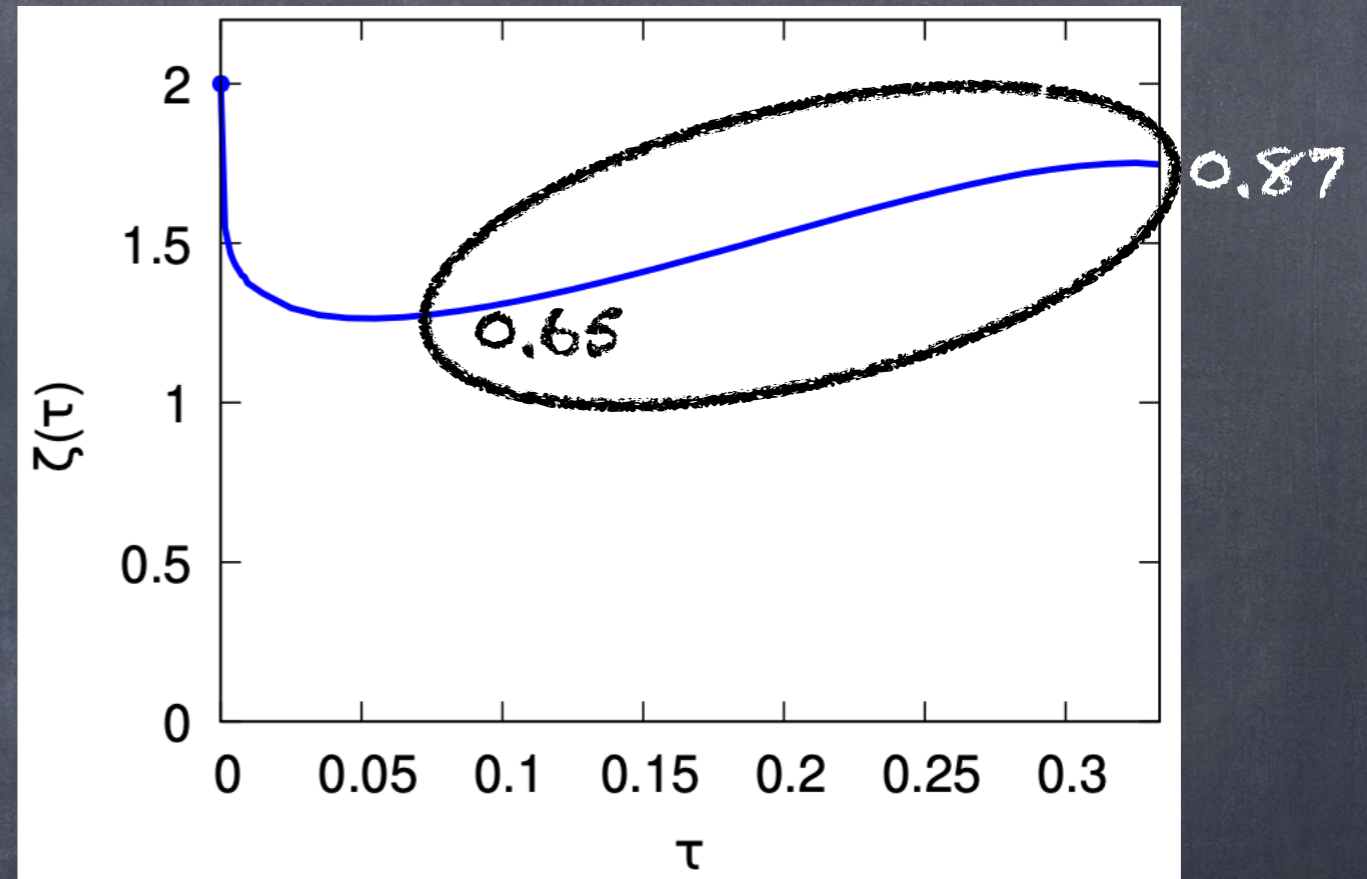


# Recent studies of power corrections

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variation quite mild  
in fit region (30%)

expect mild effect  
on  $\alpha_s$  fits



since power correction **grows** in  
the fit region, one expects  $\alpha_s$  will  
come out **smaller** when fitting

# CONCERNS ON Caola et al.

Same problems as Luisoni et al.

Gluon mass "universal" non-perturbative parameter

No operator definition was given

One would expect operator with 6 WLS with variable directions  
a single thrust value requires varying the WLS direction

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Conceptual issue: 3 jets cannot be resolved for small tau values

scaling of jet momentum:

$$p^\mu = (p^+, p^-, \vec{p}_\perp)$$

$$\left. \begin{array}{l} p^- \sim Q \\ p^+ \sim \Delta \end{array} \right\} \longrightarrow |\vec{p}_\perp| \sim \sqrt{\Delta Q}$$

$\Delta$  can be estimated from  
peak position or size  
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$$\Delta \in (1.3, 2.3) \text{ GeV}$$

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opening angle of jet  $\theta_J \sim 4\sqrt{\frac{2\Delta}{Q}}$

to resolve 3-jets: angle between jets larger than opening angle  
jet energy larger than  $p_\perp$  of sub-jet

# Resolving 3 jets

To be definite,  $a \gg b$  means  $a > 3b$

2 jets collinear, back to back with the hardest jet





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$$\theta_{23} \simeq 4\sqrt{\tau}$$

from the condition  $\theta_{23} \gg \theta_J$   
one gets jets are resolve for

$$\tau > 0.18 \text{ at } Q = m_Z$$

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1 subject soft, other subject collinear to the hardest jet



$$E_1 = E_2 = \frac{Q}{2}(1 - \tau)$$

$$E_3 = Q\tau \ll E_{1,2}$$

$$|\vec{p}_2^\perp| \sim \sqrt{Q\Delta(1 - \tau)} \simeq \sqrt{Q\Delta}$$

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To be definite,  $a \gg b$  means  $a > 3b$  or  $a > \sqrt{2}b$  more conservative

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from the condition  $E_3 \gg |\vec{p}_2^\perp|$  one gets  $\tau > 0.2$  at  $Q = m_Z$

No 3-jet consideration is of any relevance for  $\tau < 0.2$  at  $Q = m_Z$

Recent analysis using new developments

# Analysis of Nason-Zanderighi

New results on power corrections have triggered a new analysis

[JHEP 06 (2023) 058]

- Uses data only at the  $Z$  pole (exclude low  $Q$  data)
- Combines various variables: thrust,  $C$ -param,  $y_3$
- Includes event-shape dependent power correction
- Neglects resummation in the entire spectrum
- Ignores theory correlation among observables and bins

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[JHEP 06 (2023) 058]

- Uses only ALEPH data at the Z pole (exclude low Q data)
- Combines various variables: thrust, C-param,  $y_3$
- Includes event-shape dependent power correction
- Neglects resummation in the entire spectrum
- Ignores theory correlation among observables and bins

No info on outcome {  
including resummation  
including low energy data  
including theory correlations  
of fits at lower orders (convergence?)

Reduced  $\chi^2$  is astonishingly small (below 0.2)

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Resummation crucial to have order-by-order convergence, & essential for dataset independence [see talk by M. Benitez]

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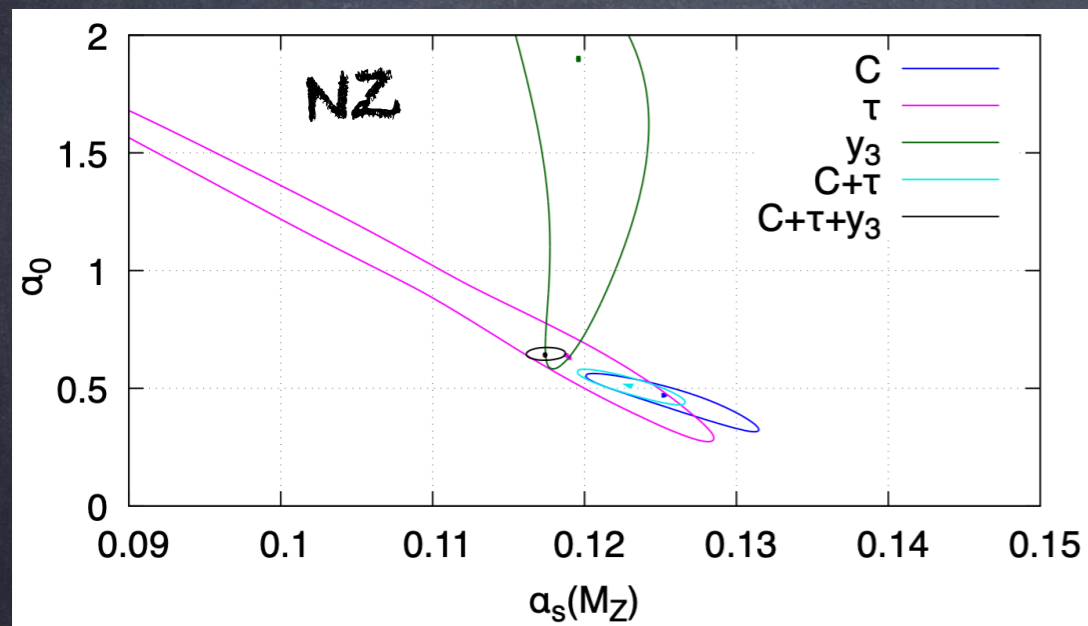


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in particular since it is crucial to break degeneracy between strong coupling and power correction

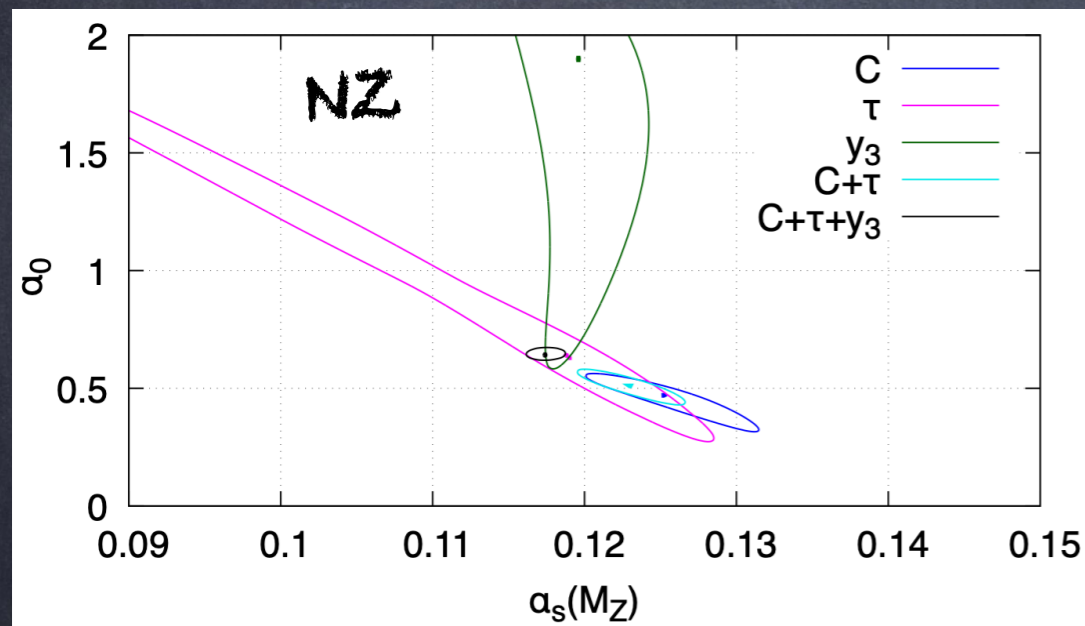
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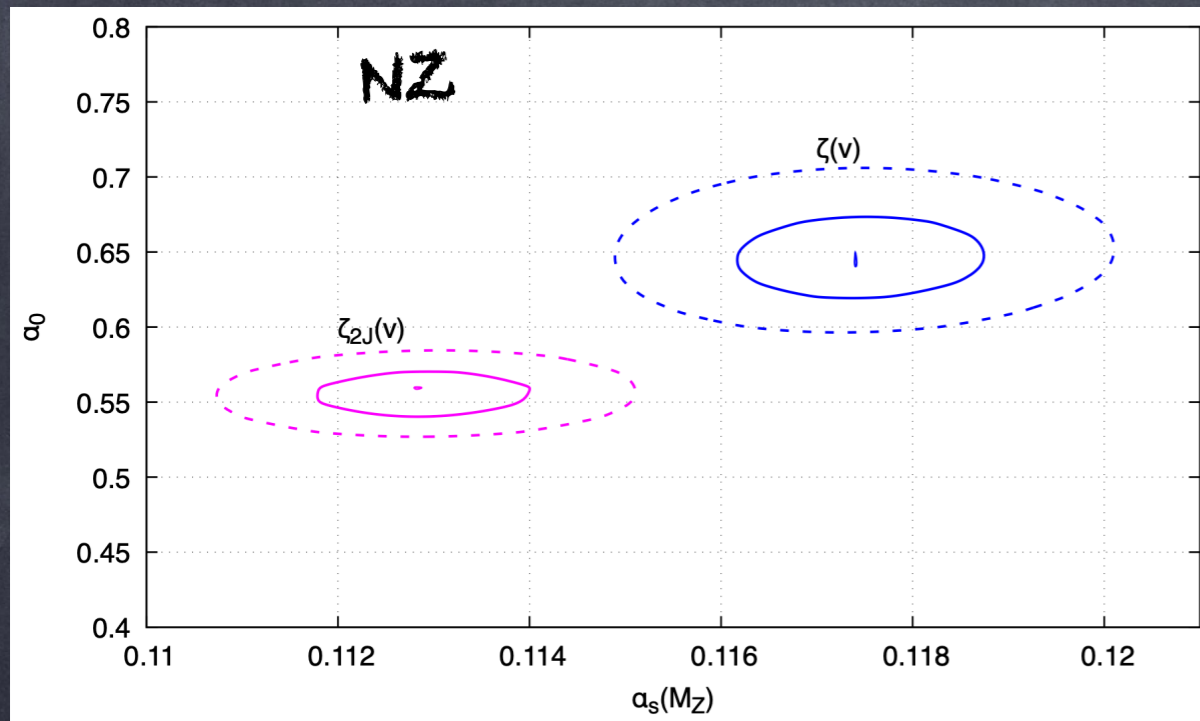
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Using low-energy data could help clarify these issues

Since we do not understand some points, we did a reanalysis

[see talk by M. Benitez]

# CONCERNS ON NASON-ZANDERIGHI

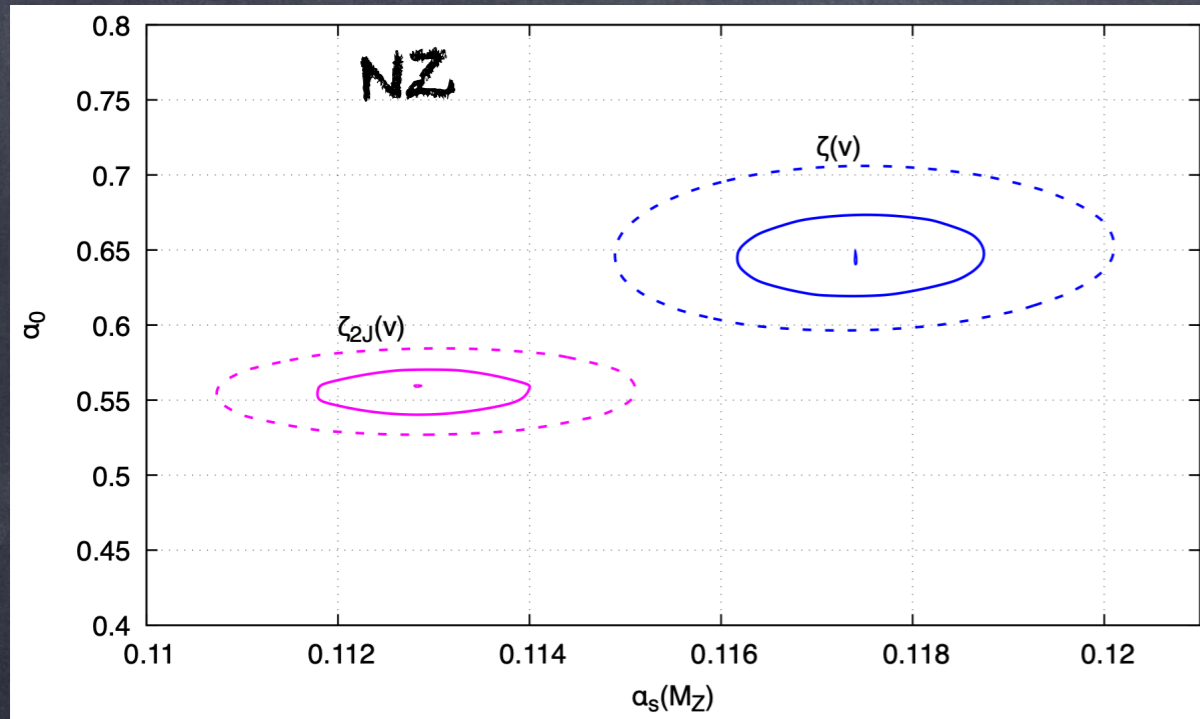


surprising outcome given the mild dependence of power correction in fit region

Our reanalysis looks quite different! [see talk by M. Benitez]

Could this mean NZ's fits are not robust?

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Could this mean NZ's fits are not robust?

Looking at thrust by itself, indeed mild change, but pushes  $\alpha_s$  down

	Variation	$\alpha_s(M_Z)$	$\alpha_0$	$\chi^2$	$\chi^2/N_{\text{deg}}$
NZ	$\tau$	0.1188	0.64	0.7	0.03
	$\tau$	0.1194	0.51	1.0	0.05

$\tau$ -dependent power correction  
constant power correction

Large values of  $\alpha_s$  possibly caused by excluding resummation

# Conclusions

Very relevant point raised by recent publications:

Can one trust 2-jet power corrections in the 3-jet region?

Most likely: **no**

This has triggered new computations and a new analysis

But these are based on some questionable assumptions

- Outcome of models not necessarily correct
- 3-jets cannot be resolved for small event-shape values
- Dijet region exists!
- Resummation is important
- Low energy data is important
- Correlations are important

Motivated to make a reanalysis

More details in next talk [by M. Benitez-Rathgeb]