Next-to-leading power corrections to event shape variables

[arXiv:2306.17601][1]

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alphas-2024 : Workshop on precision measurement of the QCD coupling constant
ECT* Trento, Italy

[1] Together with : N. Agarwal, M. V. Beekveld, E. Laenen, SM, A. Mukhopadhyay, and A Tripathi

Plan of talk

- Introduction
- Event shape Variables
- Next-to-leading power terms
- Event shape distribution
- Summary and Outlook

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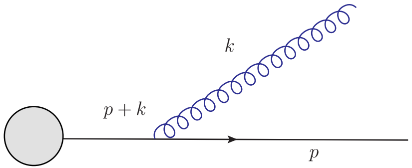
Introduction

Infrared Divergence and eikonal approximation



$$\mathcal{M}_\mu = \mathcal{M}_0\, i g_s\, \bar{u}(p)\gamma_\mu \left(\frac{\not{p} + \not{k} + m}{(p+k)^2 - m^2} \right) T^a$$

$$\propto \frac{1}{2p_0\, k_0 (1 - \cos\theta)}$$

- Emitted radiation have vanishing momenta ($k_0 \to 0$)
- Collinear to the emitting particle ($\theta \to 0$)
- Infrared singularities $\cdots$ now what? $\longrightarrow$ KLN Theorem

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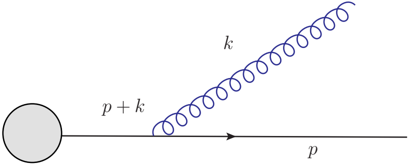

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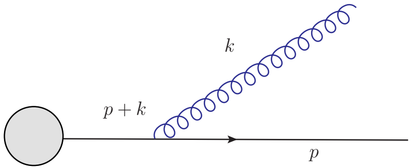

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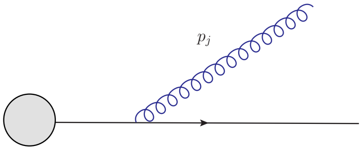
Infrared safe observables

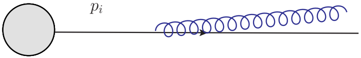
An observable $\mathcal{X}$ is Infrared safe if



➡ Soft emission

$$\mathcal{X}(p_1, ...p_i, p_j, p_k, ...p_n) = \mathcal{X}(p_1, ...p_i, p_k, ...p_n)$$
$$p_j \to 0$$

➡ Collinear splitting



$$\mathcal{X}(p_1, ...p_i, ...p_n) = \mathcal{X}(p_1, ...zp_i, (1-z)p_i, ...p_n)$$

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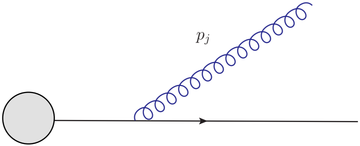

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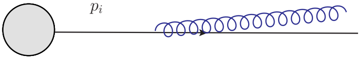

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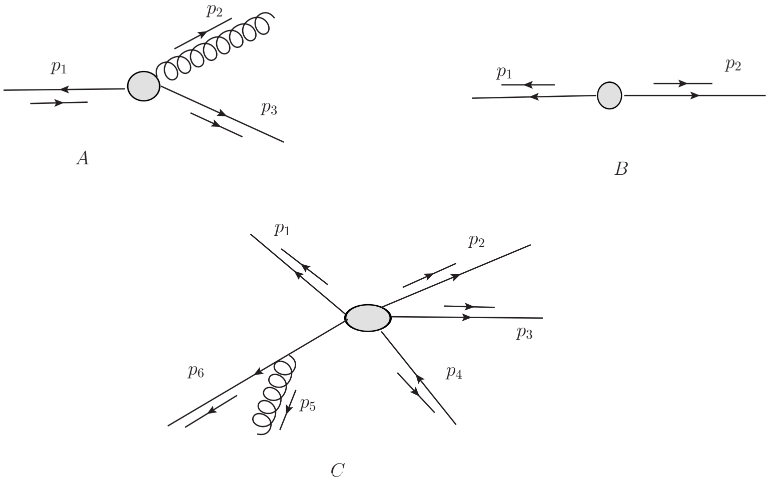
Event Shape Variables

Different *Events* and different *Shapes*



Event shape variables

- Observables to help understand the final state of high energy collisions
- Determine the shape of an event
- Essential tool for precise measurement of strong coupling constant

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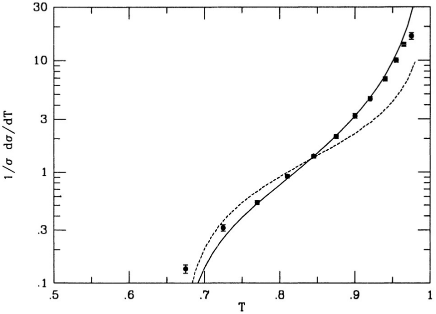
Event shape and α_s

➡ Thrust distribution at [LEP, DELPHI collab.]



➡ Prospects for strong coupling measurement at hadron colliders using soft-drop jet mass [JHEP04(2023)087]

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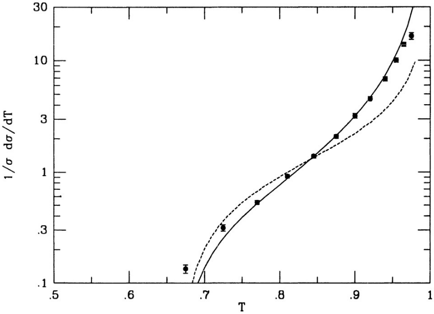

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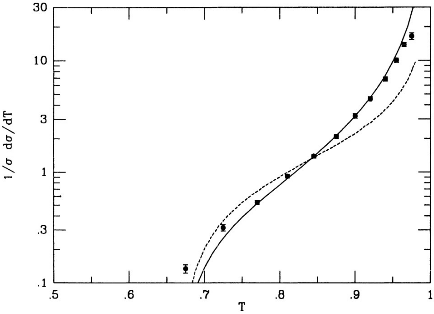

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Event shape variables

Thrust	1977 [E. Farhi]
Spherocity	1977 [H. Georgi and M. Machacek]
C-parameter	1978 [G. C. Fox, S. Wolfram ...]
Jet mass	1981 [L. Clavelli et. al.]
Jet broadening	1992 [S. Catani et. al.]
Angularities	2003 [C.F. Berger et. al.]

Thrust and C-parameter

- Thrust :

$$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p_i} \cdot \mathbf{n}|}{\sum_i |\mathbf{p_i}|} = \max\{x_1, x_2, x_3\}$$

- C-parameter :

$$C = 3 - \frac{3}{2} \sum_{i,j} \frac{\left(p^{(i)} \cdot p^{(j)}\right)^2}{\left(p^{(i)} \cdot q\right)\left(p^{(j)} \cdot q\right)}$$

$$c = \frac{C}{6} = \frac{(1 - x_1)(1 - x_2)(1 - x_3)}{x_1 \; x_2 \; x_3}$$

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Next-to-leading power terms

Next-to-leading Power terms

Distribution in variable ξ

$$\frac{d\sigma}{d\xi} \propto \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} \left[\overbrace{c_{nm}^{(-1)} \left(\frac{\log^m \xi}{\xi}\right)_+ + c_n \delta(\xi)}^{\text{LP terms}} + \underbrace{c_{nm}^{(0)} \log^m \xi}_{\text{NLP terms}} + \dots \right]$$

Thrust distribution

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \frac{2\alpha_s}{3\pi} \left(\overbrace{\frac{-3 - 4\log\tau}{\tau}}^{\text{LP terms}} - \underbrace{2 + 2\log\tau}_{\text{NLP terms}} + \dots \right)$$

LP and NLP terms

LP terms

- Universally process independent form
- Linked to soft and collinear divergences
- Resummation is well understood

NLP terms

- Linked to next-to-soft and collinear divergences
- No general Resummation framework

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Event Shape distribution

$$\frac{d\sigma}{dX} = \frac{1}{2s} \int |\mathcal{M}|^2 \overbrace{\delta(X - f(x_1, x_2, x_3))}^{\textbf{new condition}} d\Phi$$

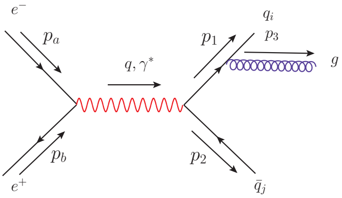
Thrust distribution for $e^+e^- \to q\bar{q}g$

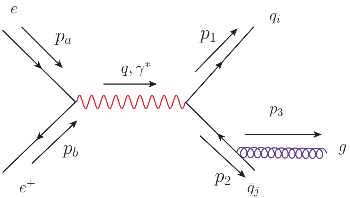
$$\frac{1}{\sigma_0}\frac{d\sigma}{dT} = \frac{2\alpha_s}{3\pi} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \left[\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}\right] \; \delta[T - \max(x_1, x_2, x_3)]$$

The formalism of shifted kinematics

➡ Shifted kinematic approximation[2]

$$\overline{\sum}|\mathcal{M}_{\text{shift}}|^2 = g_s^2 C_F \underbrace{\frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}}_{\text{Eikonal}} |\mathcal{M}_0(p_1 - \delta p_1, p_2 - \delta p_2)|^2$$





➡ Focuses on contribution from next-to-soft gluon emissions

[2] Del Duca et. al. (2017)

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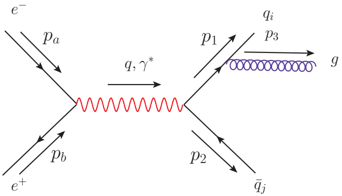

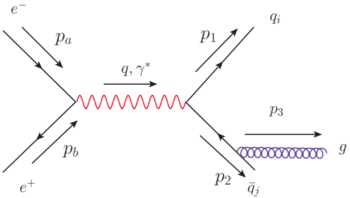

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The Matrix Elements

- Exact approach

$$\overline{\sum}|\mathcal{M}_{\text{exact}}|^2 = 8(e^2 e_q)^2 N_c g_s^2 C_F \frac{1}{3Q^2} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

- Shifted kinematics

$$\overline{\sum}|\mathcal{M}_{\text{shift}}|^2 = 8(e^2 e_q)^2 N_c g_s^2 C_F \frac{1}{3Q^2} \frac{2x_1 + 2x_2 - 2}{(1-x_1)(1-x_2)}$$

- Soft quark approximation

$$\overline{\sum}|\mathcal{M}_{\text{rem}}|^2 = 8(e^2 e_q)^2 N_c g_s^2 C_F \frac{1}{3Q^2} \left(\frac{1-x_1}{1-x_2} + \frac{1-x_2}{1-x_1}\right)$$

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Thrust distribution

Exact approach	$$\frac{2\alpha_s}{3\pi}\left(\frac{-3-4\log\tau}{\tau}-2+2\log\tau\right)$$
Shifted kinematics	$$\frac{2\alpha_s}{3\pi}\left(\frac{-4-4\log\tau}{\tau}+4+4\log\tau\right)$$
Soft quark	$$\frac{2\alpha_s}{3\pi}\left(\frac{1}{\tau}-6+2\log\tau\right)$$

Breakdown of thrust distribution from region-I

➡ Upper limit contributions

$$\frac{1}{\sigma_0(s)}\frac{d\sigma}{d\tau}\bigg|_{\mathrm{I,u}} = \frac{2\alpha_s}{3\pi}\left(\frac{-2\log\tau}{\tau} + 2 + 2\log\tau - \frac{\tau}{2} + \tau\log\tau + \mathcal{O}\left(\tau^2\right)\right),$$

➡ Lower limit contributions

$$\frac{1}{\sigma_0(s)}\frac{d\sigma}{d\tau}\bigg|_{\mathrm{I,l}} = \frac{2\alpha_s}{3\pi}\left(\frac{3}{2\tau} + 2 - 2\tau + \mathcal{O}(\tau^2)\right).$$

➡ LLs at LP and NLP from soft and next-to-soft gluon emissions

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- LLs at LP and NLP from soft and next-to-soft gluon emissions

Breakdown of thrust distribution from region-III

➡ Upper limit contributions

$$\frac{1}{\sigma_0(s)}\frac{d\sigma}{d\tau}\bigg|_{\mathrm{I,u}} = \frac{2\alpha_s}{3\pi}\left(-2-\log\tau+(1-\log\tau)\tau+\mathcal{O}\left(\tau^2\right)\right),$$

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➡ LL at NLP from soft (anti-) quark emissions[3]

[3] JHEP03(2020)106

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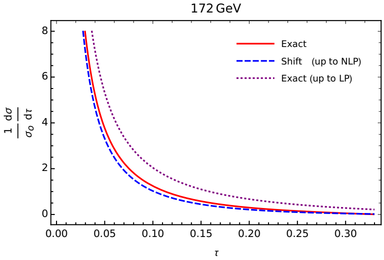
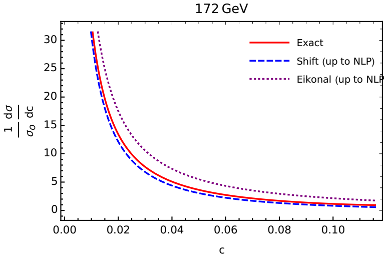
c-parameter distribution

Exact approach	$\frac{2\alpha_s}{3\pi}\left(\frac{-3-4\log c}{c}-3+4\log c\right)$
Shifted kinematics	$\frac{2\alpha_s}{3\pi}\left(\frac{-4-4\log c}{\tau}+4+8\log c\right)$
Soft quark	$\frac{2\alpha_s}{3\pi}\left(\frac{1}{c}-7-4\log c\right)$

Plots for thrust and c-parameter distribution

A comparison graph between Eikonal approximation and Shifted formalism





The shifted formalism provides a better approximation!

Struggles with Elliptic integrals

Elliptic integrals appearing in c-parameter distribution and their quandaries

- Final integrand

$$\frac{1}{\sigma_0(s)}\frac{\mathrm{d}\sigma}{\mathrm{dc}}\bigg|_{\text{NLO}} = \frac{2\alpha_s}{3\pi}\int_{y_1}^{y_2} dy \frac{2(1-y)\big(y\left(c(y-2)^2+(y-3)y+4\right)-2\big)}{c(cy+y-1)\sqrt{y(cy+y-1)\left(c(y-2)^2+(y-1)y\right)}}\,.$$

- The limits

$$y_1 = \frac{1+4c-\sqrt{1-8c}}{2(1+c)}\,, \qquad y_2 = \frac{1+4c+\sqrt{1-8c}}{2(1+c)}\,.$$

- Final form

$$\frac{1}{\sigma_0(s)}\frac{d\sigma}{dc}\bigg|_{\text{NLO}} = \frac{2\alpha_s}{3\pi}\Big(e(c)\ \mathrm{E}[m_1(c)] + p(c)\ \Pi[n_1(c), m_1(c)] + k(c)\ \mathrm{K}[m_1(c)]\Big)\,,$$

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Transformation of elliptic integrals

The three kinds of incomplete elliptic integrals are

$$F[\phi, m] = \int_0^{\sin\phi} \frac{dt}{\sqrt{(1-t^2)(1-mt^2)}} \,,$$

$$E[\phi, m] = \int_0^{\sin\phi} dt \sqrt{\frac{1-mt^2}{1-t^2}} \,,$$

$$\Pi[n, \phi, m] = \int_0^{\sin\phi} \frac{dt}{(1-nt^2)\sqrt{(1-t^2)(1-mt^2)}} \,.$$

ϕ, m and n are called the amplitude, parameter and characteristic respectively

Transformation rules

If the amplitude $\phi = \frac{\pi}{2}$

$$\begin{aligned} F[\phi, m] &= K[m]\,, \\ E[\phi, m] &= E[m]\,, \\ \Pi[n, \phi, m] &= \Pi[n, m]\,. \end{aligned}$$

The incomplete elliptic integral reduces to complete elliptic integral!

Amplitude of elliptic integrals

Non reducible

$$\phi_1\Big|_{y=y_2} \;\&\; \phi_2\Big|_{y=y_1}$$

Reducible

$$\phi_1\Big|_{y=y_1} \;\&\; \phi_2\Big|_{y=y_2}$$

$$\phi_1\Big|_{y=y_2} = \phi_2\Big|_{y=y_1} = 0$$

Trouble with transformations

Elliptic integral and their amplitudes

$$\phi_{1,2}(c,y) = \left(\frac{-1+\sqrt{1-8c}-4c \pm 8c/y}{2\sqrt{1-8c}} \right)^{1/2} .$$

- Non-reducible for $\phi_1\Big|_{y=y_2}$ & $\phi_2\Big|_{y=y_1}$
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Fixing the trouble with non-reducible elliptic integrals

Use an off-shell parameter[4]

$$E\left[\sin^{-1}\left(\frac{\left(-4c+\sqrt{1-8c}-1\right)(c+1)e}{\sqrt{1-8c}\left(2c(e+2)+\sqrt{1-8c}+2e+1\right)}\right)^{1/2},\ m_1(c)\right]$$

Argument $(\phi) \to 0$ as $e \to 0$

$$E[\phi_1(c,e), m_1(c)] = \sqrt{\frac{\left(-4c+\sqrt{1-8c}-1\right)(c+1)}{\sqrt{1-8c}\left(4c+\sqrt{1-8c}+1\right)}}\sqrt{e} + \mathcal{O}(e^{3/2})$$

[4] $y_{(1,2)} \to y_{(1,2)} + e$

The final form

$$\frac{1}{\sigma_0(s)}\frac{d\sigma}{dc}\bigg|_{\text{NLO}} = \frac{2\alpha_s}{3\pi}\Big(e(c)\ \mathrm{E}[m_1(c)] + p(c)\ \Pi[n_1(c), m_1(c)] + k(c)\ \mathrm{K}[m_1(c)]\Big)\,,$$

$$\frac{1}{\sigma_0(s)}\frac{d\sigma}{dc}\bigg|_{\text{NLO}} = \frac{2\alpha_s}{3\pi}\left(\frac{-3-4\log c}{c} - 3 + 4\log c\right).$$

- Modify the limits, by introducing an off-shell parameter.
- Categorize the elliptic integrals into reducible and non-reducible type
- Expand the non-reducible integral around $e = 0$

The final form

$$\left.\frac{1}{\sigma_0(s)}\frac{d\sigma}{dc}\right|_{\text{NLO}} = \frac{2\alpha_s}{3\pi}\Big(e(c)\ \mathrm{E}[m_1(c)] + p(c)\ \Pi[n_1(c), m_1(c)] + k(c)\ \mathrm{K}[m_1(c)]\Big)\,,$$

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- ➡ Modify the limits, by introducing an off-shell parameter.
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The final form

$$\frac{1}{\sigma_0(s)}\frac{d\sigma}{dc}\bigg|_{\text{NLO}} = \frac{2\alpha_s}{3\pi}\Big(e(c)\ \mathrm{E}[m_1(c)] + p(c)\ \Pi[n_1(c), m_1(c)] + k(c)\ \mathrm{K}[m_1(c)]\Big)\,,$$

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Summary and Outlook

- Shifted kinematics together with soft quark approximation captures LL and NLL upto NLP accuracy.
- Non-reducible elliptic integrals can be expanded around an off-shell parameter.
- Application of shifted kinematics to other event shapes such as spherocity, Angularities and Jet broadening

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DANK U
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MERSI
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danke schön
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Back up slides

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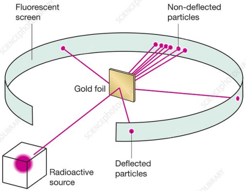
Fluorescent screen
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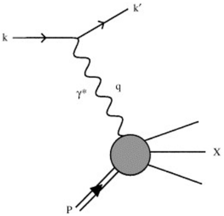
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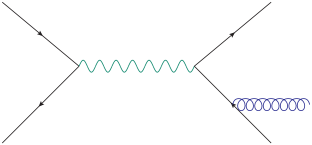
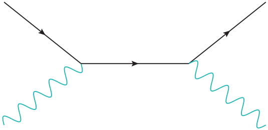
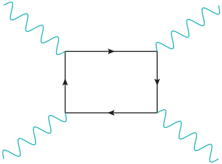
Scattering in QFT

- Computation of physical observables







- $$\sigma = \frac{1}{2s}\int d\Phi \; |\mathcal{M}|^2 \quad , \quad \frac{d\sigma}{d\tau} \quad , \quad \tau$$

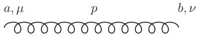
QCD - The Theory for Strong Interaction

- Interaction between quarks and gluons
- Non-abelian gauge theory with gauge group $SU(3)$
- Asymptotically free theory

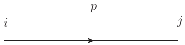
$$\mathcal{L}_{\mathsf{QCD}} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4}(F^a_{\mu\nu})^2 + g\bar{\psi}\gamma^\mu T^a \psi A^a_\mu + \mathcal{L}_{\mathsf{GF}} + \mathcal{L}_{\mathsf{ghost}}$$

- $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$
- $[T^a, T^b] = i f^{abc} T^c$
- $\mathcal{L}_{\mathsf{GF}} = \frac{-1}{2\xi}(\partial^\mu A^a_\mu)(\partial^\nu A^a_\nu)$
- $\mathcal{L}_{\mathsf{ghost}} = -\bar{c}^a \left(\partial^\mu(\delta^{ab}\partial_\mu + g_s f_{abc} A^c_\mu)\right) c^b$

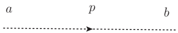
Feynman Rules for QCD



$$\delta_{ab}\frac{-ig_{\mu\nu}+(1-\alpha)\frac{p_\mu p_\nu}{p^2}}{p^2+i\epsilon}$$

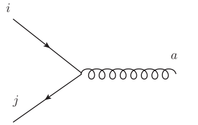


$$\delta_{ij}\frac{i(\not p+m)}{p^2-m^2+i\epsilon}$$

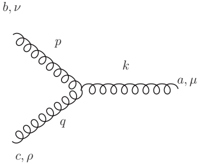


$$\frac{i\delta_{ab}}{p^2+i\epsilon}$$

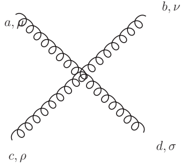
Feynman Rules for QCD



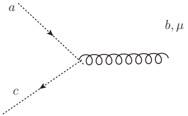
$$ig\gamma^\mu T^a_{ij}$$



$$gf^{abc}[g^{\mu\nu}(k-p)^\rho + CP]$$



$$-ig^2[f^{abc}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + CP]$$

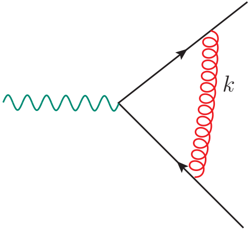


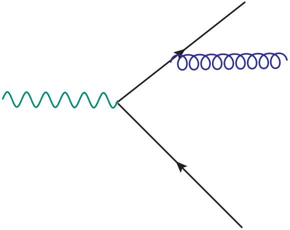
$$-gf^{abc}p^\mu$$

Infrared singularities now what?

KLN Theorem

Singularities from loop integrations will cancel with the singularities from phase space integrations





leaving behind large logs! .... Resummation

$$\delta p_1^\alpha = -\frac{1}{2}\left(\frac{p_2\cdot k}{p_1\cdot p_2}p_1^\alpha - \frac{p_1\cdot k}{p_1\cdot p_2}p_2^\alpha + k^\alpha\right),$$
$$\delta p_2^\alpha = -\frac{1}{2}\left(\frac{p_1\cdot k}{p_1\cdot p_2}p_2^\alpha - \frac{p_2\cdot k}{p_1\cdot p_2}p_1^\alpha + k^\alpha\right).$$