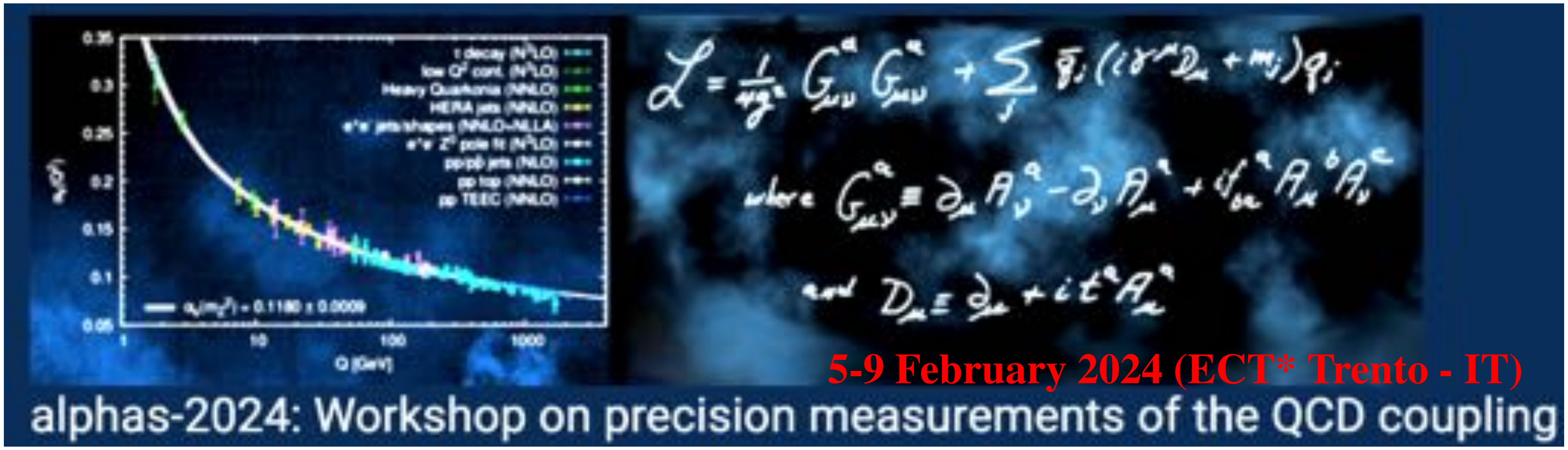

Power corrections and α_s from $e^+e^- \to$ Hadrons

Stephan Narison

Preambnle

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  - QCD condensates are used as inputs in some other approaches.

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- ♠ Data inputs: $e^+e^- \to$ I=1 Hadrons below 2 GeV from

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- Bridge between High AND Low energy QCD regions

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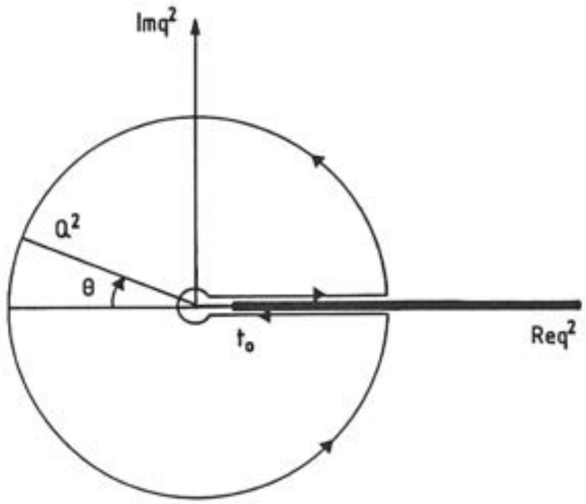

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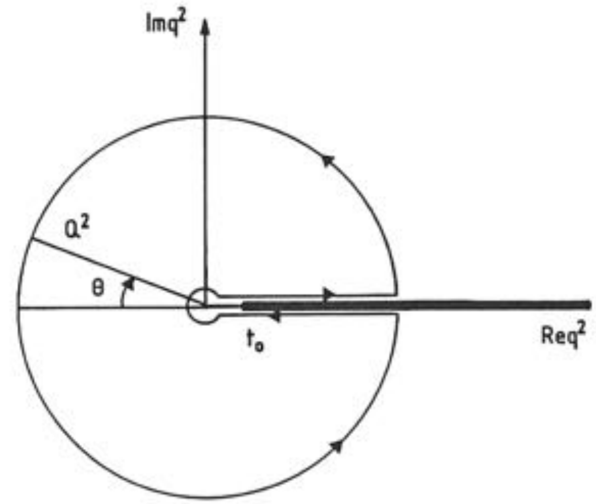

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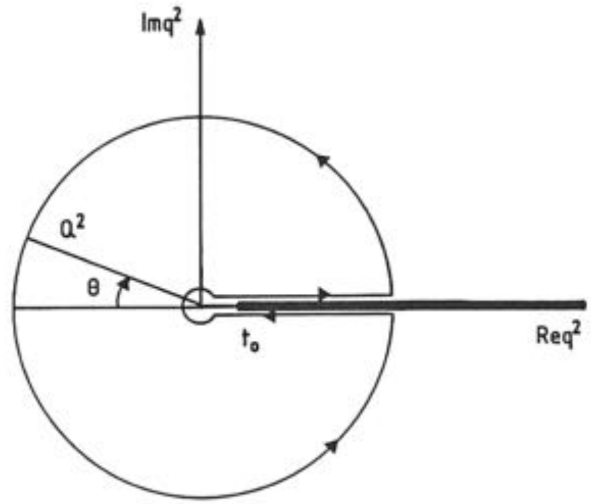

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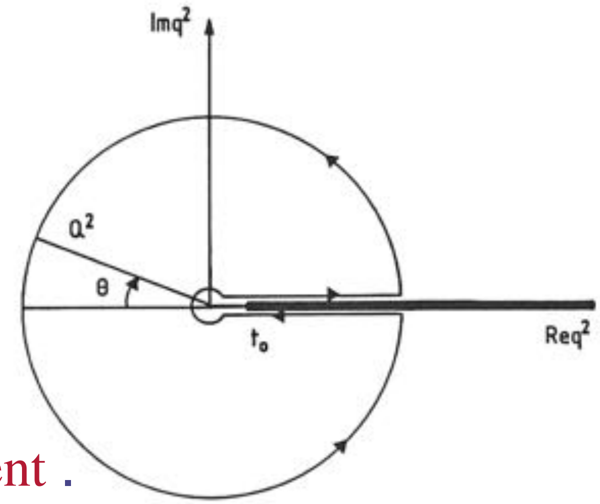

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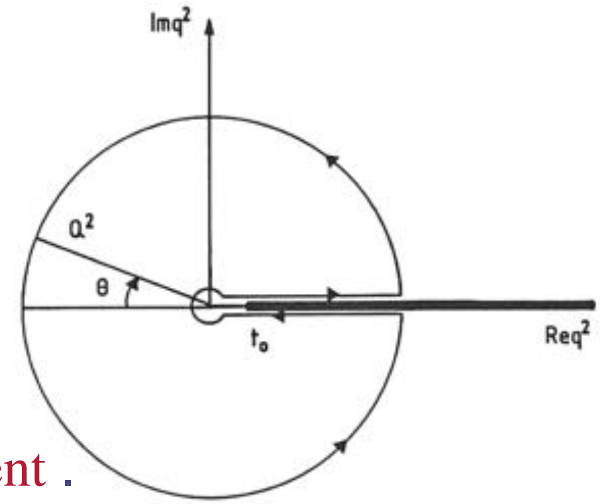

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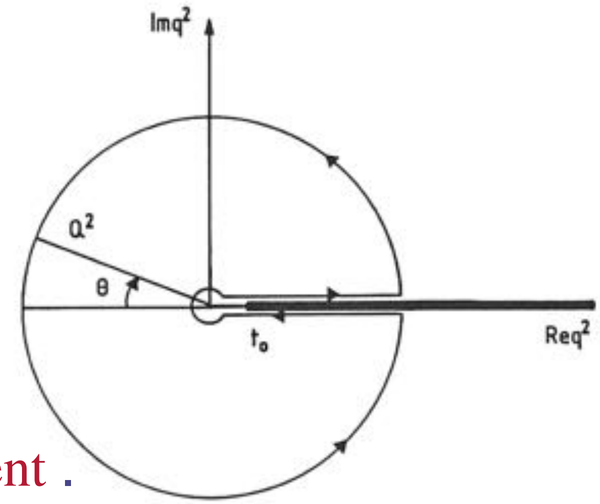

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  - A truncation of the OPE up to $D=6$ is enough for Phenomenology !

The SVZ-OPE Anatomy

- ♣ Vector two-point function

$$8\pi^2\Pi(Q^2) = \sum_{D=0,2,4,\ldots} \frac{C_D\langle 0|O_D|0\rangle}{Q^D} \quad : \qquad d_D \equiv C_D\langle 0|O_D|0\rangle$$

- ♢ In terms of the QCD PT and NP parameters
  - $D=0$: $\Pi^{I=1}(Q^2)|_{LO} = -\log(\frac{Q^2}{\mu^2})$ usual PT series $(a_s \equiv \alpha_s/\pi)$
  - $D=2$: $d_2|m_q = -3(\bar{m}_u^2+\bar{m}_d^2)\left(1+\frac{2}{3}a_s\right)$.: quark mass corrections

    $d_2|_{tach} \equiv C_2\langle O_2\rangle|_{tach} = \left(\frac{32}{2} - 8\zeta_3\right)\alpha_s\lambda^2 \log\frac{Q^2}{\mu^2}$. : tachyonic gluon CNZ 98

    $\equiv \sum$ Large Order PT corrections NZ 09
  - $D=4$: $d_4|_{\langle\bar\psi\psi\rangle} = 4\pi^2\left(1+\frac{a_s}{3}\right)\langle m_u\bar\psi_u\psi_u + m_d\bar\psi_d\psi_d\rangle$ quark condensate

    $d_4|_{\langle G^2\rangle} = \frac{\pi}{3}\langle\alpha_s G^2\rangle\left(1+\frac{7}{6}a_s\right)$ gluon condensate
  - $D=6$: $d_6 = -\frac{896}{81}\pi^3\rho\alpha_s\langle\bar\psi_q\psi_q\rangle^2$: four-quark condensates : $\rho = 1$: factorization.
  - $D=8$: $d_8 = \langle GGGG\rangle$: 4-gluon condensate $\oplus\cdots$

- ♡ Truncation of the OPE
  - A truncation of the OPE up to $D=6$ is enough for Phenomenology !
  - No good control of condensates beyond $D=6$: violation of factorization, mixing under renormalization, often some classes of diagrams are only computed,...

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- ♣ Improvement of the dispersion relation

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  - $\mathcal{R}(\tau)$: less senstive to α_s-corrections $\Longrightarrow$ Good tools for extracting the QCD condensates

    Launer-SN-Tarrach 84

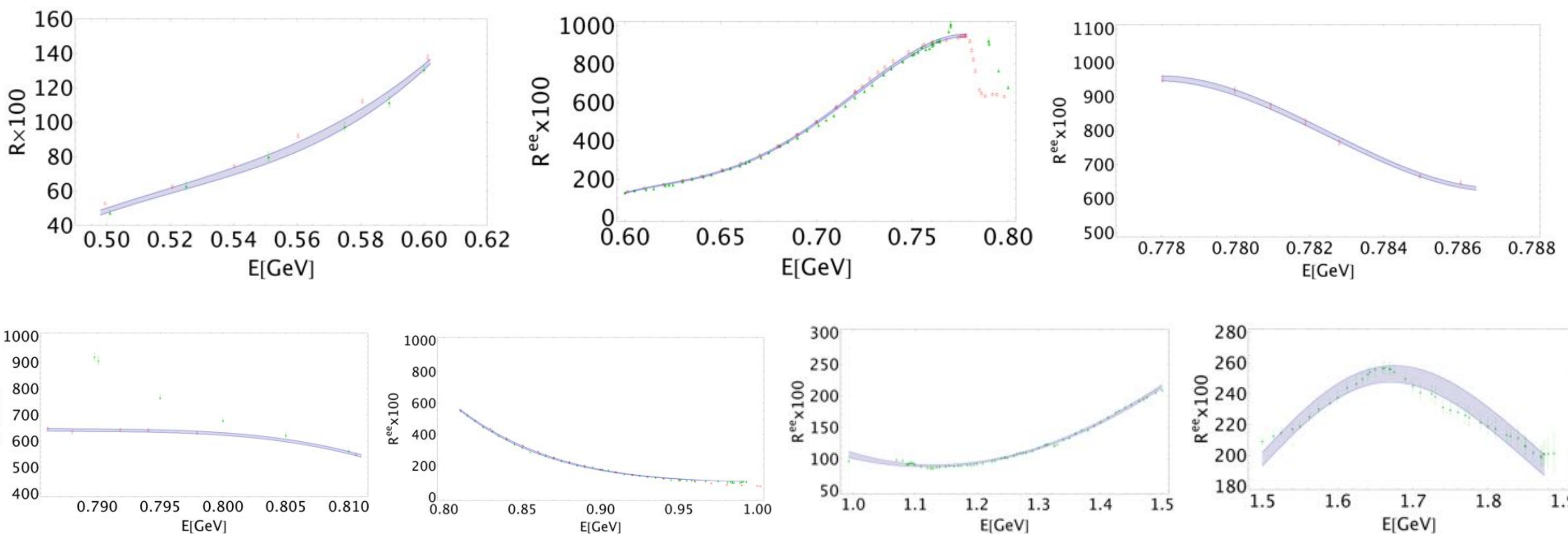
Spectral Function from $e^+e^- \to$ Hadrons

- ♣ Data compiled by PDG22 $\oplus$ New CMD3 ($E \leq 1.875$ GeV)

$$R^{ee} \equiv \frac{\sigma(e^+e^- \to \text{ Hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 12\pi \,\mathrm{Im}\Pi(t) \;\; : \quad \text{Optical theorem}$$

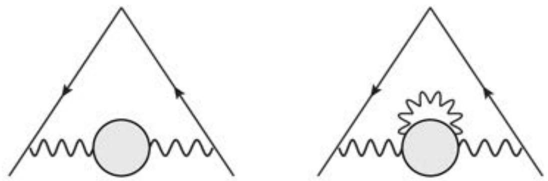
We divide the region $0.5 \leq \sqrt{t} \leq 1.9$ GeV into 7 subregions

and use a Mathematica Interpolating Fitting program.



Test / Calibration of the Fit

- ♢ Hadronic Vacuum Polarisation of $\frac{1}{2}(g-2)_\mu$



$$a_\mu|_{l.o}^{hvp} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} dt\, K_\mu(t)\, \sigma(e^+e^- \to \text{hadrons}) \;:\; K_\mu(t) = \int_0^1 dx \frac{x^2(1-x)}{x^2+(t/m_\mu^2)(1-x)}.$$

$$a_\mu|_{l.o}^{hvp}[2m_\pi \to 1.875] = (6492.3 \pm 8.8) \times 10^{-11}$$

$+(100-130)\times 10^{-11}$: Larger than Davier et al, Nomura et al using KLOE data (2020) BUT agrees with CMD3 (2023) $\Longrightarrow$

$$a_\mu|_{l.o}^{hvp} = (7036.5 \pm 36.9) \times 10^{-11} \Longrightarrow \; a_\mu^{exp} - a_\mu^{th} = (142 \pm 42_{th} \pm 41_{exp}) \times 10^{-11}$$

SN23

Less tension with SM !

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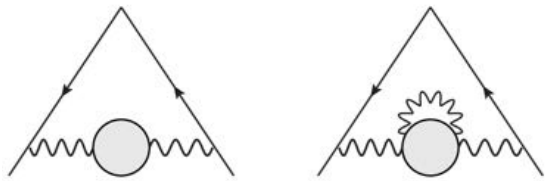

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- ♡ Hadronic contribution to $\Delta\alpha^{(5)}(M_Z)\times 10^5 = 2766.3 \pm 4.5$

QCD condensates from LSR

- Choice of the ratio of LSR Launer-SN-Tarrach 84

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- Relative contributions of the condensates increased in the OPE !

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- 3-parameter fit (d_4, d_6, d_8) : not conclusive ! Like τ-decay Moments !

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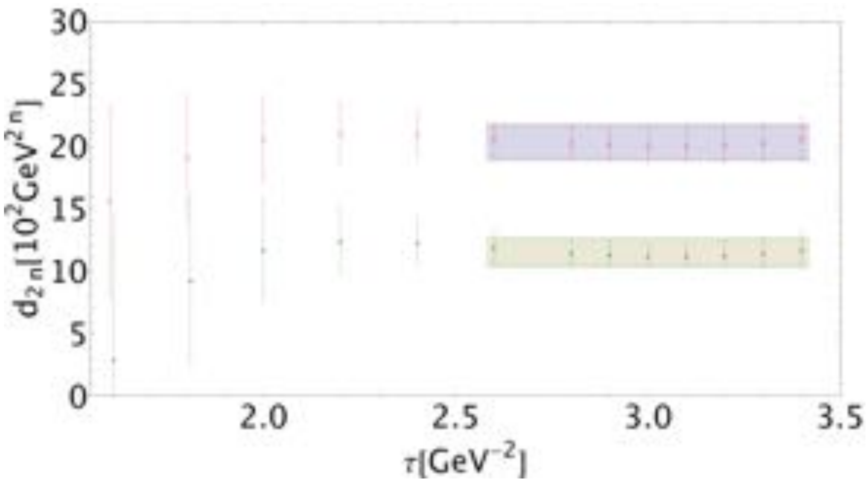
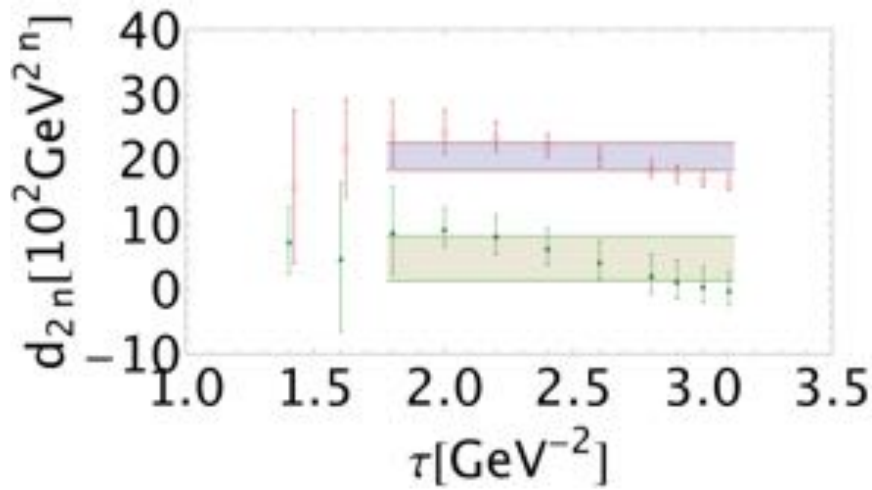
- 3-parameter fit (d_4, d_6, d_8) : not conclusive ! Like τ-decay Moments !
- $\langle \alpha_s G^2 \rangle$ from heavy quarkonia $\oplus$ 2-parameter fit of $(d_6,\ d_8)$:

$O(\alpha_s^2)$ $\qquad\qquad$ $O(\alpha_s^4)$





d_6 stable for $\alpha_s^2 \to \alpha_s^4$ but not d_8. To order α_s^4 :

$$d_6 = -(20.5 \pm 2.0) \times 10^{-2}\,\mathrm{GeV}^6, \quad \mathrm{d}_8 = (4.7 \pm 3.5) \times 10^{-2}\ \mathrm{GeV}^8.$$

NB : $\tau \simeq (2 \sim 3)$ GeV^{-2} relatively big ! Results to be improved later on !

τ-like decay moments

$$R_n^{ee} = \int_0^1 dx_0\,(1-3x_0^2+2x_0^3)x_0^n\,2R_{ee}^{I=1}(x_0)\ :\ x_0 \equiv (t/M_0^2)$$

- Perturbative corrections $\delta_n^{(0)}$ to order α_s^4 (FO) for $n \geq 1$, :

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- Perturbative corrections $\delta_n^{(0)}$ to order α_s^4 (FO) for $n \geq 1$, :
- Condensates of the standard OPE to lowest order of α_s:

$$\begin{aligned}
R_0^{ee} &= f(\delta_0^{(0)}, d_6, d_8), & d_{n\geq 10} = 0\\
R_1^{ee} &= f(\delta_1^{(0)}, d_4, d_8, d_{10}), & d_{n\geq 12} = 0\\
R_2^{ee} &= f(\delta_2^{(0)}, d_6, d_{10}, d_{12}), & d_{n\geq 14} = 0\\
R_3^{ee} &= f(\delta_3^{(0)}, d_8, d_{12}, d_{14}), & d_{n\geq 16} = 0\\
R_4^{ee} &= f(\delta_4^{(0)}, d_{10}, d_{14}, d_{16}), & d_{n\geq 18} = 0
\end{aligned}$$

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- Initial Inputs : α_s, and $\langle \alpha_s G^2 \rangle$ $(J/\psi, \Upsilon)$, d_6 (LSR) condensates $\Longrightarrow$ R_0 gives d_8 $\Longrightarrow$ R_1 gives d_{10} $\Longrightarrow$ R_2 gives d_{12} $\Longrightarrow$ R_3 gives d_{14} $\Longrightarrow$ R_4 gives d_{16}

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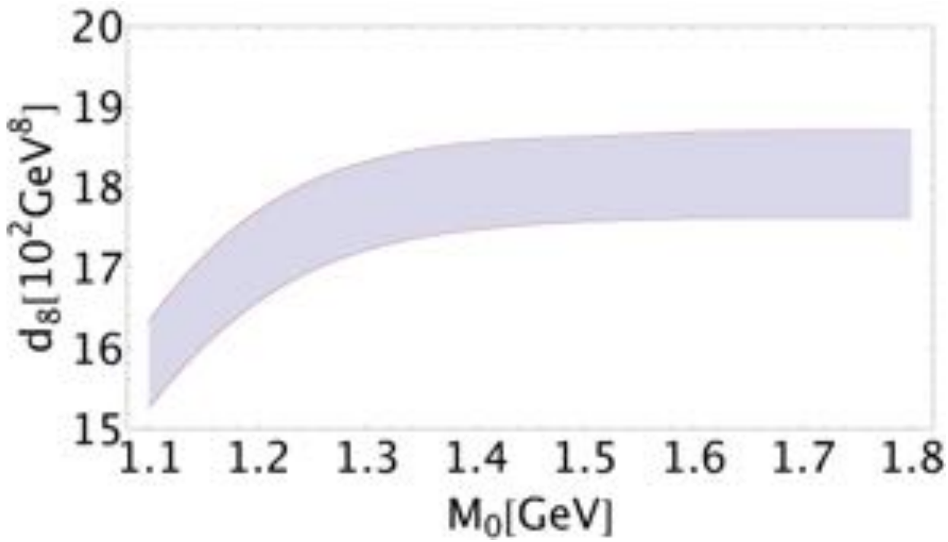
- We proceed iteratively to improve the results

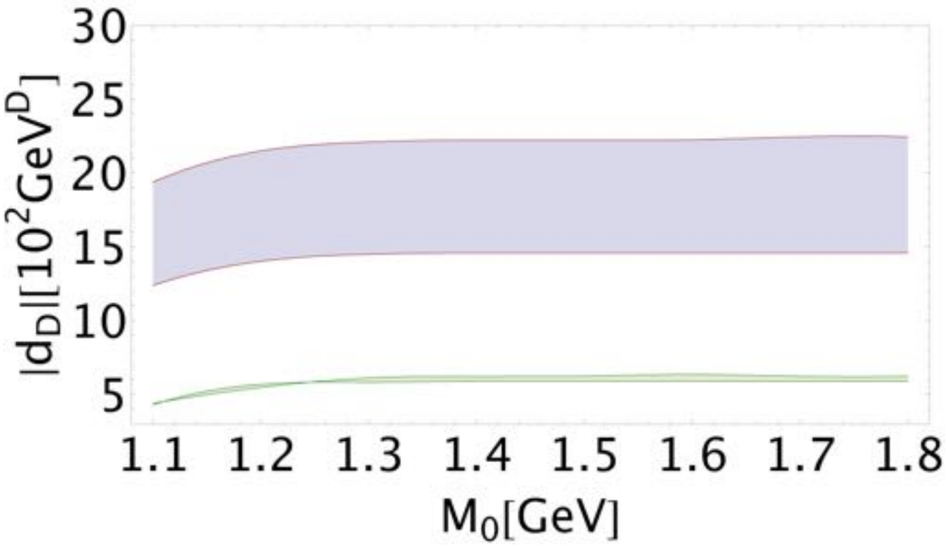
$$
\begin{array}{ccccccccccccc}
d_4 & \longleftrightarrow & d_6 & \longleftrightarrow & d_8 & \overset{(2)}{\longleftrightarrow} & d_{10} & \overset{(6)}{\longleftrightarrow} & d_{12} & \longrightarrow & d_{14} & \longrightarrow & d_{16} \\
J/\psi,\Upsilon & LSR & & R_0 & & R_1 & & R_2 & & R_3 & & R_4 &
\end{array}
$$

where (2) and (6) indicate the number of iterations.

Analysis from τ-like decay moments

- d_8 from R_1 : inputs α_s, d_4, d_{10} — d_{10}, d_{12} from R_2 : inputs α_s, d_6





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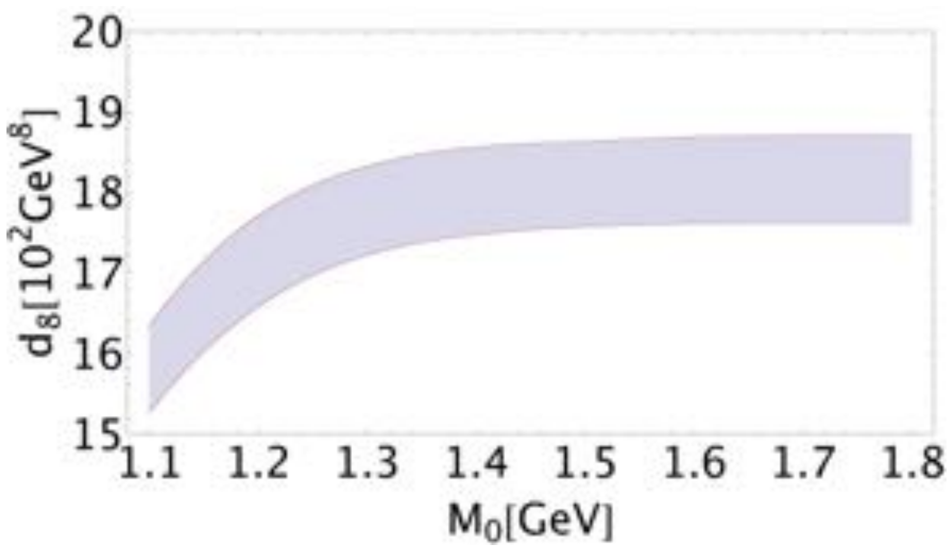
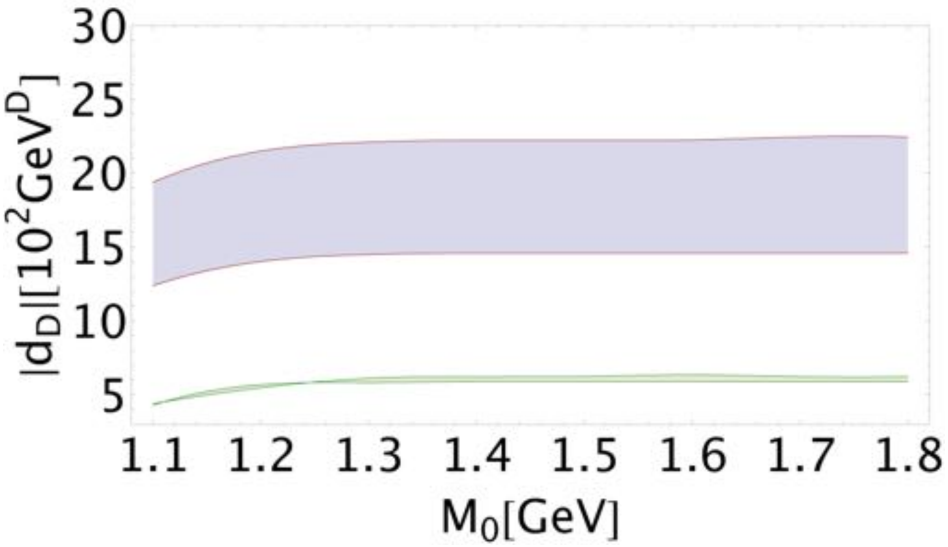
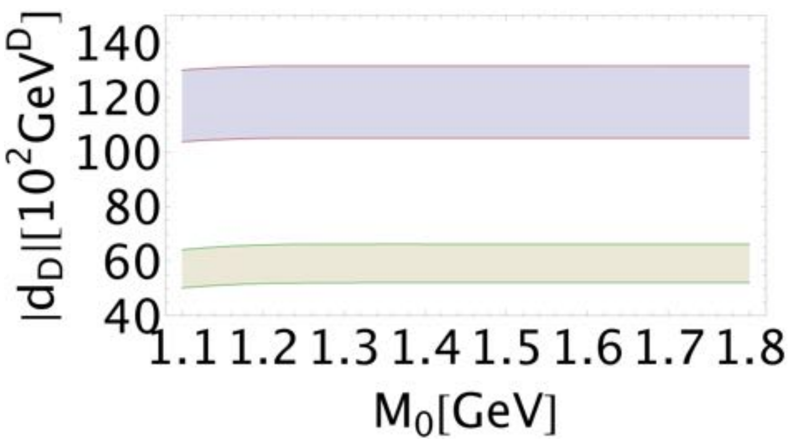




- d_{14}, d_{16} from R_3 and R_4 : inputs $\alpha_s, d_8, d_{10}, d_{12} \oplus d_{14}$ for d_{16}.



Re-analysis of the ratio of LSR $R_{10}(\tau)$

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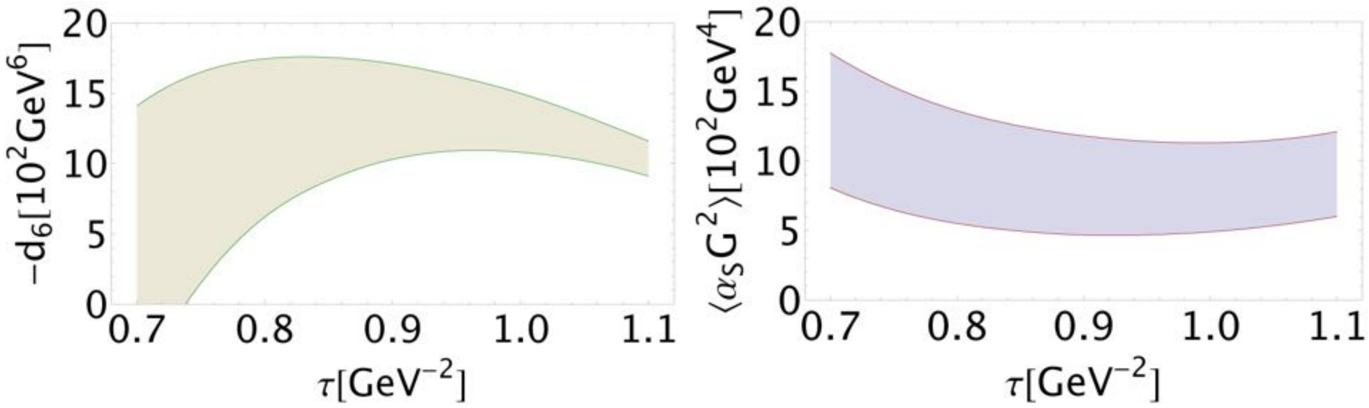

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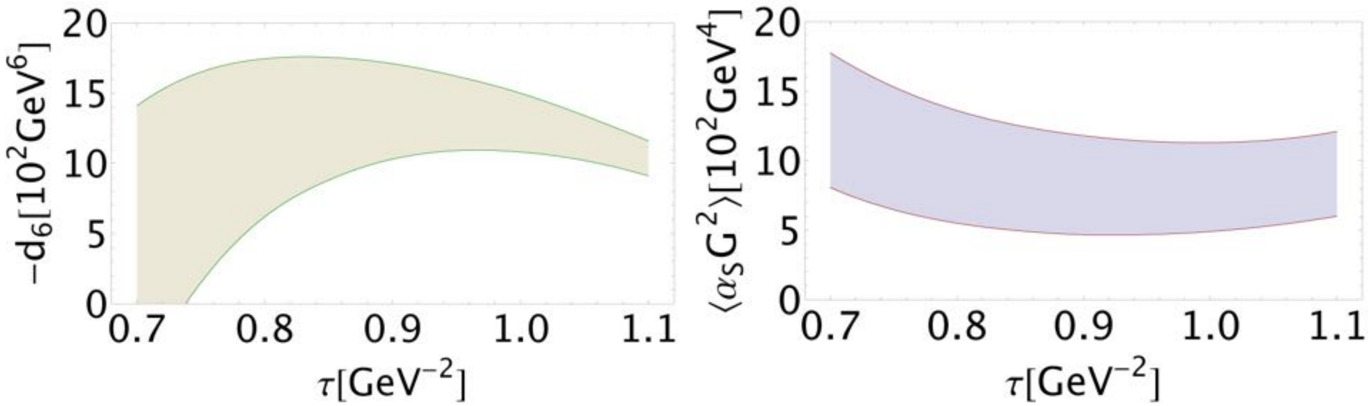

- $d_6 = -(14.2 \pm 3.3) \times 10^{-2}$ GeV^6 : for $\tau \simeq (0.85 \sim 0.95)$ GeV^{-2}
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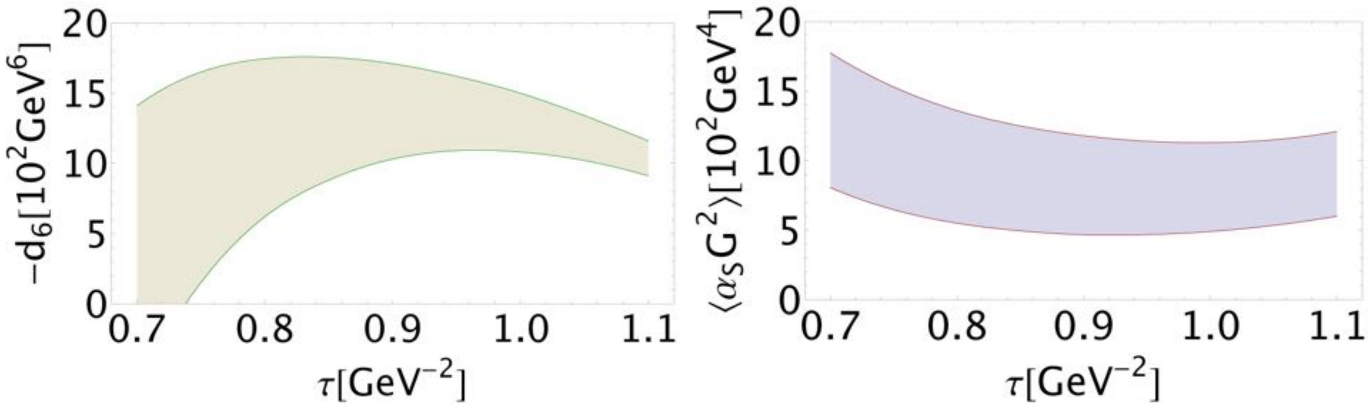

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  – Agrees with $(6.49 \pm 0.35) \times 10^{-2}$ GeV^4 from Heavy quarks but less accurate !

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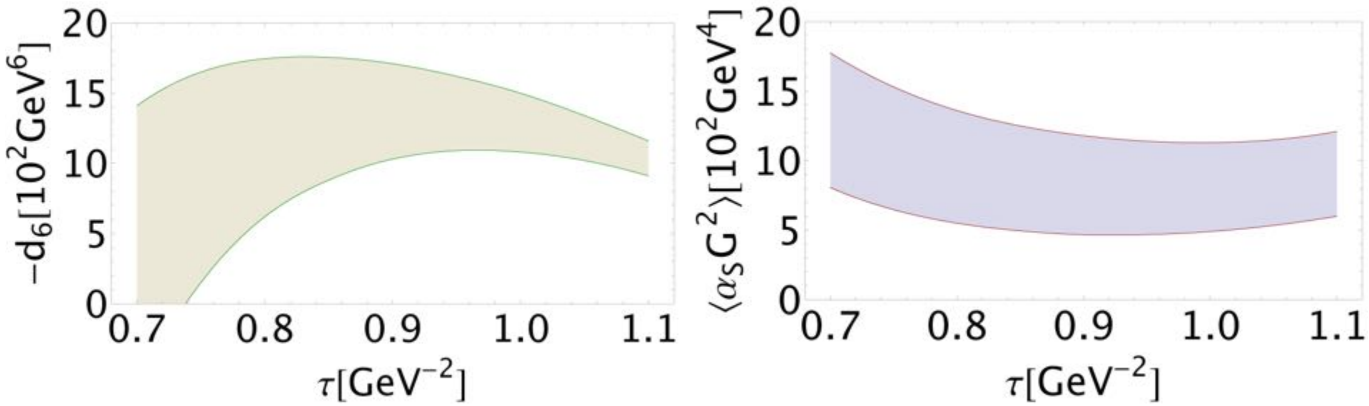

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- Good convergence of the OPE at the optimization scale :

$$\tau R_{10}(\tau) = 1 + \beta_1 a_s^2 + \cdots - 0.123\tau^2 + 0.164\tau^3 - .140\tau^4 - 0.015\tau^5 + 0.050\tau^6 - 0.034\tau^7 + 0.193\tau^8$$

$$= 1 + 10^{-3} + \sum_{n=2}^{3} d_{2n}\tau^n = 0.127(0.091) + \sum_{n=4}^{8} d_{2n}\tau^n = 0.017(-0.023) \text{ for } \tau = 0.95\ (0.85)\ \mathrm{GeV}^{-2}$$

Summary : QCD condensates from $e^+e^- \to$ Hadro

- This work at (FO) : $\langle\alpha_s G^2\rangle = (6.49 \pm 0.35) \times 10^{-2}$ GeV^4 input from $J/\psi, \Upsilon$ for $d_n \geq 6$. Units : 10^{-2} GeV^D : D dimension of the condensates

$\langle\alpha_s G^2\rangle$	$-d_6$	d_8	$-d_{10}$	$-d_{12}$	d_{14}	d_{16}
8.0 ± 3.3	14.2 ± 3.3	18.2 ± 0.6	6.0 ± 0.2	18.4 ± 3.8	59.2 ± 7.1	118.3 ± 3.2

– d_6 confirms the violation of factorization Launer-SN-Tarrach 84, SN96, 23,

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5.34 ± 3.64	14.2 ± 3.5	21.3 ± 2.5			OPAL 99
$3.5^{+2.2}_{-3.8}$	$19.7^{+11.8}_{-7.9}$	$23.7^{+11.8}_{-15.8}$	11.8 ± 19.7	7.9 ± 19.7	Pich-Rodriguez 16 ($d_{14,16} = 0$)

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$\alpha_s(M_\tau)$ *from* τ*-like decay BNP lowest moment*

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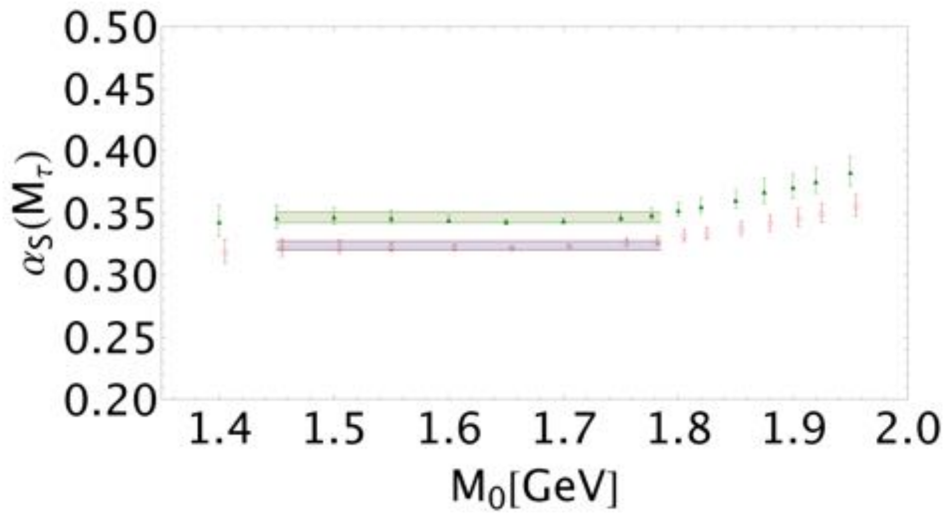

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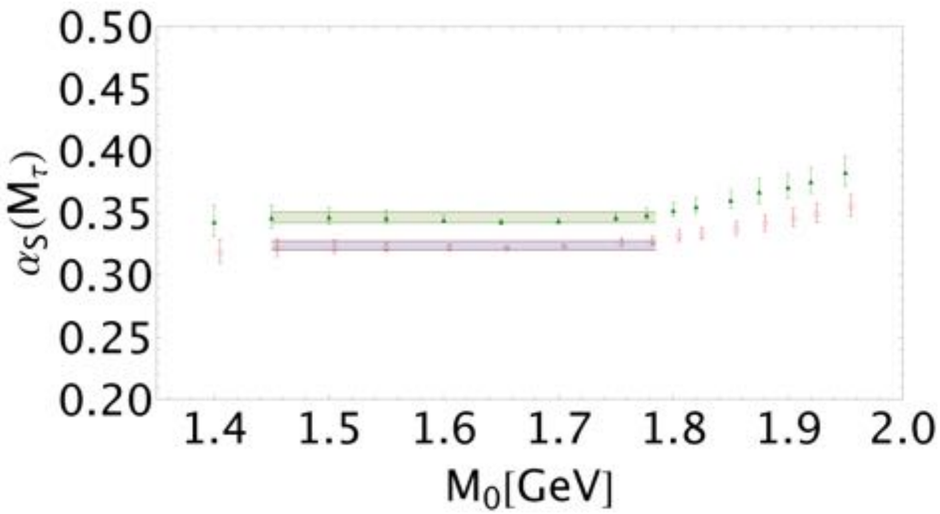

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$$\alpha_s(M_\tau) = 0.3238(36) \Longrightarrow \alpha_s(M_Z) = 0.1190(2) \quad \text{FO}$$

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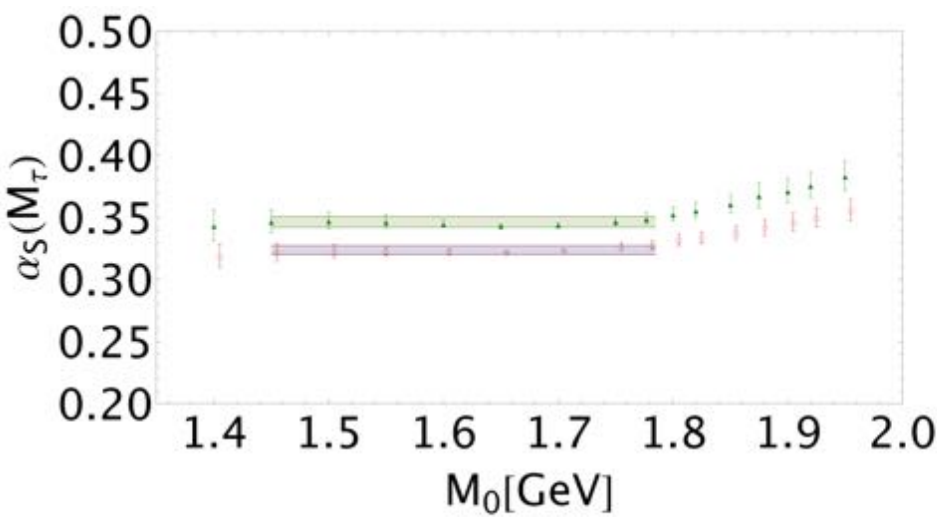

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- Sum of NP terms :

$$\sum \delta_{NP}(1.675) = (2.8 \pm 0.9) \times 10^{-2},$$
$$\sum \delta_{NP}(M_\tau) = (2.0 \pm 0.6) \times 10^{-2} \qquad \approx \alpha_s^3 \;\; (FO), \quad \alpha_s^2 \;\; (CI).$$

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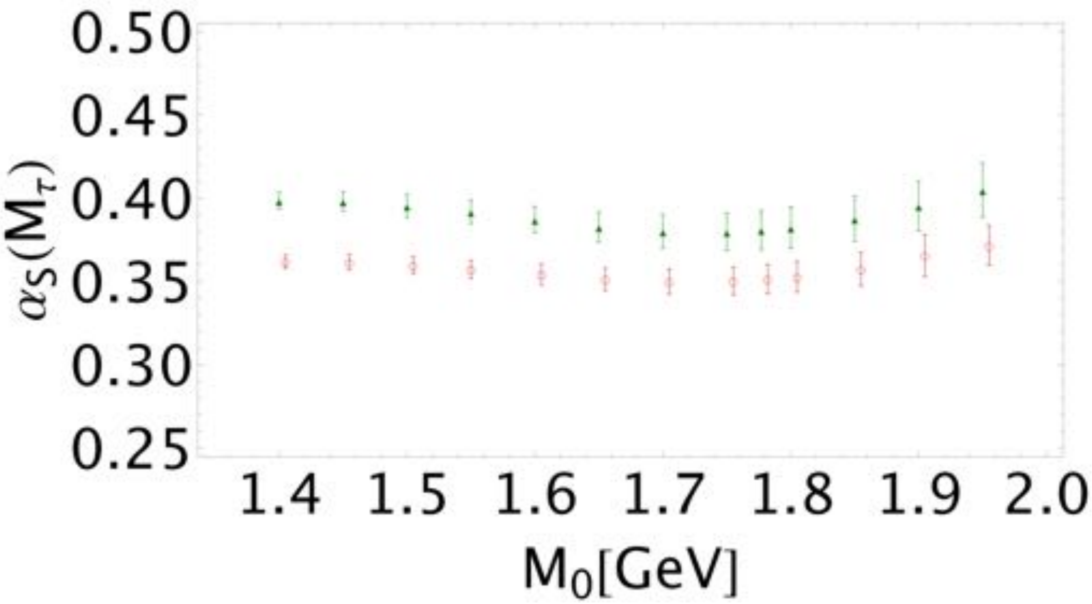

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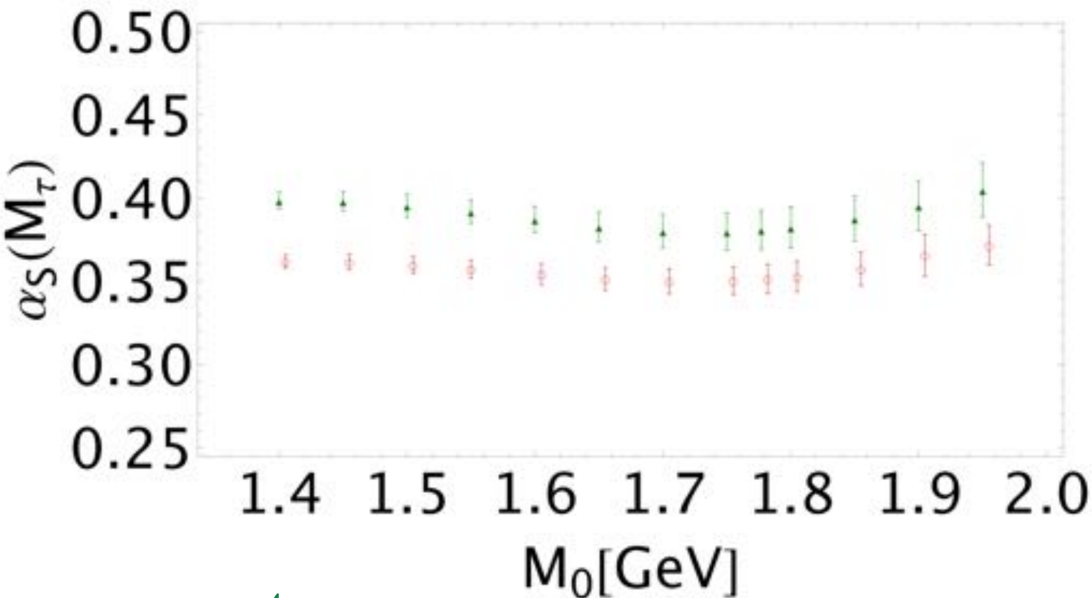

- One obtains to order $O(\alpha_s^4)$:

$$\alpha_s(M_\tau) = 0.3514(68) \Longrightarrow \alpha_s(M_Z) = 0.1221(7) \quad \text{FO}$$
$$= 0.3827(90) \Longrightarrow \alpha_s(M_Z) = 0.1252(9) \quad \text{CI}$$

Relatively High compared to the PDG 23 average 0.117 !

Estimate of the error due to α_s^5

- Geometric growth of different PT series SN-Zakharov 09 :

$$D(Q^2) = \sum_n a_s^n c_n \ : \ c_0 = c_1 = 1, \ \ c_2 = 1.656, \ c_3 = 6.37, \ c_4 = 49.09$$

$$\Longrightarrow \quad c_4 \approx c_3^2 \Longrightarrow c_5 \simeq (c_3/c_2)\, c_4^2 \approx (228 \pm 114).$$

$$R_0 = \sum_n a_s^n (g_n + c_n) : \quad g_n \ \mathrm{from\ RG-resummation}$$

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- Mean : assume that $\Delta\alpha_s^5$ absorbs the $\neq$ between FO and CI at H.O :

$$\alpha_s(M_\tau) = 0.3358(55)(107)_{syst} \Longrightarrow \alpha_s(M_Z) = 0.1204(7)(3)_{evol}$$

where syst comes from the distance of the mean central value to the ones of FO or CI.

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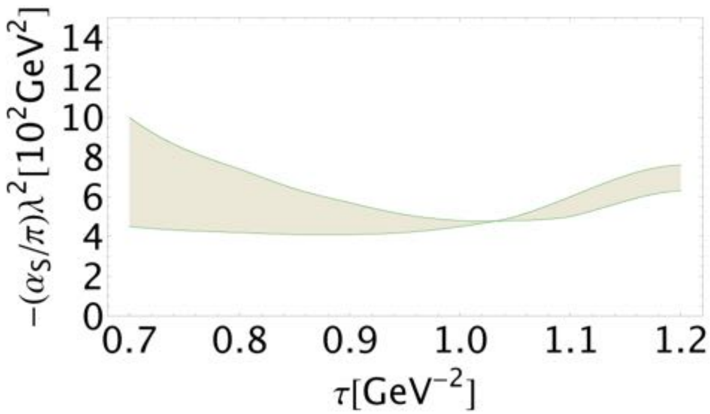

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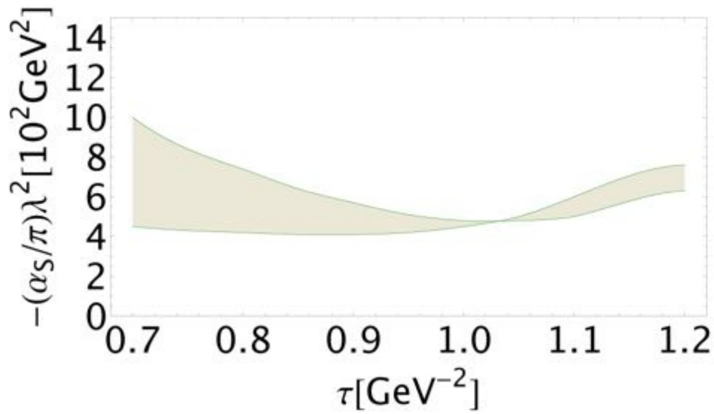

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- Small size instantons

  Contributes as operators of dimension 9 :

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  - On the value of $\hat{s}_0$from FESR (local duality) BLR85, BDLPR88:

$$\int_0^{\hat{s}_0} ds R_{ee}^{I=1} = \frac{3}{2}\hat{s}_0 \Big[1 + a_s + a_s^2(1.6398 - \frac{\beta_1}{2} + \cdots\Big]$$

– $\rho-$meson only : $\hat{s}_0 = 1.5$ GeV^2.

– Complete $e^+e^- \to$ Hadrons data : $\hat{s}_0 = (4.5 \sim 5)$ GeV^2

(also checked from LSR) SN23 $\Longrightarrow$ huge exponential suppression ?

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    Good agreement as expected from Isospin Symmetry !

- $1/Q^2$ Tachyonic gluon $\oplus$ SVZ OPE $\oplus$ PT $O(\alpha_s^4)$

$$\alpha_s(M_\tau) = 0.3047(32)_{fit}(61)_{\alpha_s^5} \Longrightarrow \alpha_s(M_Z) = 0.1165(4)(3)_{evol} \quad \text{FO}$$
$$= 0.3209(42)_{fit}(61)_{\alpha_s^5} \Longrightarrow \alpha_s(M_Z) = 0.1188(5)(3)_{evol} \quad \text{CI}$$
$$\Longrightarrow \text{Mean} = 0.3122(51)(87)_{syst} \Longrightarrow \alpha_s(M_Z) = 0.1175(13)(3)_{evol}$$

  - Compare withPDG 23 average : $\alpha_s(M_Z) = 0.1178(5)$

Misaotra Anareo @ ny Faharetana !

Merci pour Votre Patience !

Thanks for Your Patience !

https://www.lupm.in2p3.fr/users/qcd/agmm