α_s from quark and gluon hadron multiplicities

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Precision measurements of the strong coupling constant
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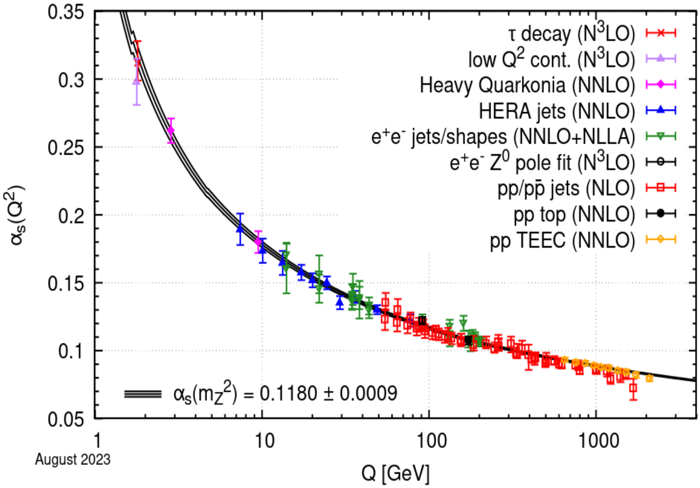
Outline

- **Historical overwiew of 2 approaches: MLLA versus $\overline{\mathrm{MS}}$.**
- **DGLAP evolution of FFs (1st moment = multiplicity).**
- **Resummation of soft and collinear gluon logarithms within NLO+NNLL in the $\overline{\mathrm{MS}}$ scheme.**
- **Novel diagonalisation of DGLAP equations at NLO+NNLL based on SUSY-like relation and inclusion of NNLO $\alpha(Q)$.**
- **Results for the Q-evolution of gluon and quark jet multiplicities and the ratio $N_g/N_q(Q)$**
- Fitted $\alpha_s(m_Z^2)$ from the Q-dependence of N_g^h, N_q^h & N_g^h/N_q^h in e^+e^- over [10,200] GeV.
- **MLLA evolution on the extraction of α_s from the same observables.**
- Fitted $\alpha_s(m_Z^2)$ from the Q-dependence of N_g^h, N_q^h & N_g^h/N_q^h in e^+e^- over [10,200] GeV.

World α_s determination (PDG 2023)



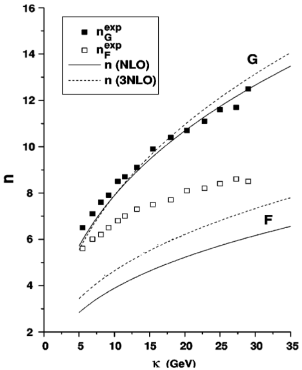
Motivation

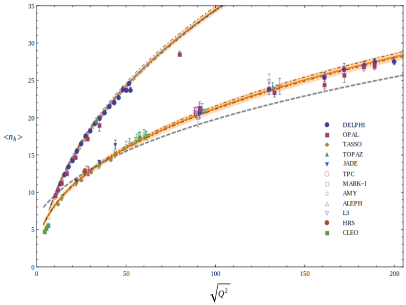
The QCD coupling constant α_s

1. Less precisely determined among the coupling constants of the SM of particle physics
2. Importance: many fundamental SM observables at the LHC and future FCC-ee depend on this key parameter described by QCD
3. Current uncertainty of the strong coupling world-average value: $\alpha_s(m_Z^2) = 0.1180 \pm 0.0009$ is about 0.8%
4. Motivation: reduce the uncertainty by combining current α_s extractions with novel high-precision observables.
5. $\alpha_s(m_Z^2)$ determination from the Q-evolution of N_g^h, N_q^h & N_g^h/N_q^h.

Historical view: 2 theory approaches to parton FFs

Charged hadron multiplicities in gluon and quark jets

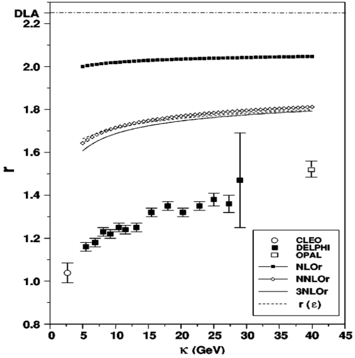


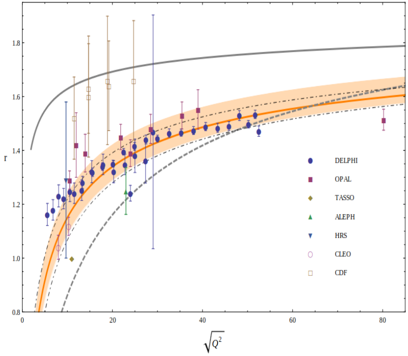


1. MLLA+LPHD: Cappella et al.; PRD 61 (2000) 074009 "Ev. of average mult. of quark and gluon jets" with $\mathcal{K}^{ch}$ & Λ_{QCD} as free par.
2. NLO+NNLL: Bolzoni et al.; Nucl.Phys.B 875 (2013) 18 on the same subject with 4 free pars.

Historical view: 2 theory approaches to parton FFs

Ratio of hadron multiplicities in gluon over quark jets





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Main goals

Goals:

Improve $\overline{\textbf{MS}}$ (Bolzoni et al.) results from a novel diagonalisation method of DGLAP evolution including running NNLO α_s.

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Improve MLLA (Capella et al.) results including running NNLO α_s into the MLLA+LPHD evolution along with higher order corrections.

Part I

DGLAP evolution of FFs

DGLAP evolution of FFs

From Mellin-Laplace transform:

$$D_a^h(\omega, Q^2) = \int_0^1 dx\, x^\omega D_a^h(x, Q^2) \Rightarrow N_a^h(Q^2) \equiv D(0, Q^2) = \int_0^1 dx\, D_a^h(x, Q^2)$$

$\Rightarrow$ FF Mellin moments:

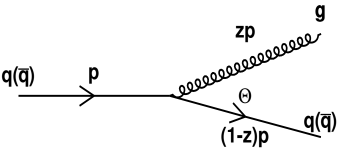
$$\frac{Q^2 d}{dQ^2}\begin{pmatrix} D_s(\omega, Q^2) \\ D_g(\omega, Q^2) \end{pmatrix} = \begin{pmatrix} P_{qq}(\omega) & P_{gq}(\omega) \\ P_{qg}(\omega) & P_{gg}(\omega) \end{pmatrix} \begin{pmatrix} D_s(\omega, Q^2) \\ D_g(\omega, Q^2) \end{pmatrix}$$

$P_{ab}(\omega)$: splitting functions (LO, NLO, NNLO...) &

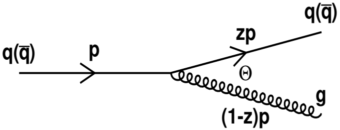
$$D_s = (1/2n_f)\sum_{q=1}^{n_f}(D_q + D_{\bar{q}})$$

is the quark singlet component.

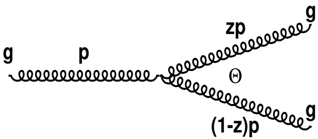
DGLAP LO splitting functions



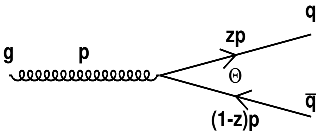
$$P_q^{gq}(z)=C_F\frac{1+(1-z)^2}{z}$$



$$P_q^{qg}(z)=C_F\frac{1+z^2}{1-z}$$



$$P_g^{gg}(z)=2C_A\left(\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right)$$



$$P_g^{q\bar{q}}(z)=T_R\left[z^2+(1-z)^2\right]$$

Part II

Resummation of soft and collinear gluon logarithms within NLO+NNLL in the $\overline{\mathbf{MS}}$ scheme.

Resummation procedure

1. DGLAP implemented with N^nLO splitting functions: FFs at large $x \sim 1$ w.r.t. the evolution time variable $t = \ln Q^2$ of p_t-ordered partons.
2. Resummation of N^nLL SGL terms at small $x \ll 1$ in the $\overline{\mathrm{MS}}$ scheme:
   - Soft FF $x < 0.1$ ($\omega \ll 1$): bulk of hadron production in jets.
3. NLO+NNLL evolution by combining (1) and (2): DGLAP evolution from small to large x.

Reference:

- A. Vogt, *Resummation of small-x double logarithms in QCD: Semi-inclusive electron-positron annihilation*, **JHEP 10 (2011) 025**

Resummation procedure

Example:

$$P_{Ag}(\omega,Q^2) = \frac{A}{4}\left(-\omega+\sqrt{\omega^2+32N_c a_s(Q^2)}\right)$$

$$\approx \overset{\text{(LO)}}{\frac{4N_c}{\omega}} a_s - \overset{\text{(NLO)}}{\frac{32N_c^2}{\omega^3}} a_s^2 + \overset{\text{(NNLO)}}{\frac{512N_c^3}{\omega^5}} a_s^3 + \mathcal{O}(a_s^4)$$

- The anomalous dimension is infra-red safe as $\omega \to 0$.
- Rate of multiplicity growth given by: $P_{gg}(\omega=0,Q^2)=\sqrt{2N_c a_s}$ at LO+LL.

Reference:

- A. Vogt, *Resummation of small-x double logarithms in QCD: Semi-inclusive electron-positron annihilation*, **JHEP 10 (2011) 025**

Resummation

$\overline{\mathrm{MS}}$ NLO+resummed (small-x) splittings functions (based on C.-H. Kom, A. Vogt, K. Yeats **JHEP 1210 (2012) 033**)

It turns out that the resummed splitting functions in Mellin space can be expressed in terms of

$$\mathcal{S} = (1-4\xi)^{1/2} \quad \text{and} \quad \mathcal{L} = \ln\left(\tfrac{1}{2}(1+\mathcal{S})\right) = -\ln\left(\tfrac{1}{2\xi}(1-\mathcal{S})\right) \tag{3.1}$$

with $\xi = -8C_A a_s/\bar{N}^2$ and $\bar{N} \equiv N-1$. At NNLL accuracy, i.e., resumming the contributions $\alpha_s^n x^{-1} \ln^{2n-a} x$ with $a = 2, 3, 4$ the (flavour-singlet) splitting functions in Eq. (1.1) are given by

$$\begin{aligned}
P^T_{qq}(N) = {} & \tfrac{4}{3}\frac{C_F n_f}{C_A} a_s \left\{\tfrac{1}{2\xi}(\mathcal{S}-1)(\mathcal{L}+1)+1\right\} \\
& + \tfrac{1}{18}\frac{C_F n_f}{C_A^3} a_s \bar{N} \Big\{(-11C_A^2+6C_An_f-20C_Fn_f)\tfrac{1}{2\xi}(\mathcal{S}-1+2\xi)+10C_A^2\tfrac{1}{\xi}(\mathcal{S}-1)\mathcal{L} \\
& -(51C_A^2-6C_An_f+12C_Fn_f)\tfrac{1}{2}(\mathcal{S}-1)+(11C_A^2+2C_An_f-4C_Fn_f)\mathcal{S}^{-1}\mathcal{L} \\
& +(5C_A^2-2C_An_f+6C_Fn_f)\tfrac{1}{\xi}(\mathcal{S}-1)\mathcal{L}^2+(51C_A^2-14C_An_f+36C_Fn_f)\mathcal{L}\Big\},
\end{aligned} \tag{3.2}$$

$$P^T_{qg}(N) = \frac{C_A}{C_F}P^T_{qq}(N) - \tfrac{2}{9}\frac{n_f}{C_A^2} a_s\bar{N}(C_A^2+C_An_f-2C_Fn_f)\left\{\tfrac{1}{2\xi}(\mathcal{S}-1)(\mathcal{L}+1)+1\right\}, \tag{3.3}$$

NLL

$$\begin{aligned}
P^T_{gg}(N) = {} & \tfrac{1}{4}\bar{N}(\mathcal{S}-1) - \frac{1}{6C_A}a_s(11C_A^2+2C_An_f-4C_Fn_f)(\mathcal{S}^{-1}-1) - P^T_{qq}(N) \\
& + \frac{1}{576C_A^3}a_s\bar{N}\Big\{([1193-576\zeta_2]C_A^4-140C_A^3n_f+4C_A^2n_f^2-56C_A^2C_Fn_f+16C_AC_Fn_f^2 \\
& -48C_F^2n_f^2)(\mathcal{S}-1)+([830-576\zeta_2]C_A^4+96C_A^3n_f-8C_A^2n_f^2-208C_A^2C_Fn_f \\
& +64C_AC_Fn_f^2-96C_F^2n_f^2)(\mathcal{S}^{-1}-1)+(11C_A^2+2C_An_f-4C_Fn_f)^2(\mathcal{S}^{-3}-1)\Big\},
\end{aligned} \tag{3.4}$$

LL

NNLL

$$\begin{aligned}
P^T_{gq}(N) = {} & \frac{C_F}{C_A}P^T_{gg}(N) - \tfrac{1}{3}\frac{C_F}{C_A^2}a_s(C_A^2+C_An_f-2C_Fn_f)\tfrac{1}{\xi}(\mathcal{S}-1+2\xi) \\
& + \tfrac{1}{36}\frac{C_F}{C_A^4}a_s\bar{N}\Big\{(11C_A^4+13C_A^2n_f(C_A-2C_F)+2C_A^2n_f^2-8(C_A-C_F)C_Fn_f^2)(1-\mathcal{S}^{-1}) \\
& -(48C_A^4-45C_A^3C_F-72\zeta_2C_A^3(C_A-C_F)-33C_A^3n_f+2C_A^2n_f^2+48C_A^2C_Fn_f \\
& -8C_F^2n_f^2)\tfrac{1}{\xi}(\mathcal{S}-1+2\xi)+(-54C_A^4+45C_A^3C_F+72\zeta_2C_A^3(C_A-C_F)+23C_A^3n_f \\
& -28C_A^2n_fC_F-8(C_A-2C_F)C_Fn_f^2)\tfrac{1}{\xi}(\mathcal{S}-1)\mathcal{L}\Big\}
\end{aligned} \tag{3.5}$$

Novel diagonalisation at NLO+NNLL.

Diagonal terms: $a = q, \bar{q}, g$

$$P_{aa}(\omega = 0, Q^2) = \gamma_0(\delta_{ag} + K_a^{(1)}\gamma_0 + K_a^{(2)}\gamma_0^2)$$

$$\gamma_0(Q^2) = \sqrt{2C_A a_s(Q^2)}$$

- Off-diagonal terms:

$$P_{gq}(0, Q^2) = C(P_{gg}(0, Q^2) + A) \qquad P_{qg}(0, Q^2) = C^{-1}(P_{qq}(0, Q^2) + A)$$

$$K_g^{(1)} = -\frac{1}{12}[11 + 2\varphi(1 + 6C)]$$

$$K_g^{(2)} = \frac{1193}{288} - 2\zeta(2) - \frac{5\varphi}{72}(7 - 38C) + \frac{\varphi^2}{72}(1 - 2C)(1 - 18C)$$

with $\varphi = n_f/N_c$ and $C = C_F/C_A$.

Novel diagonalisation at NLO+NNLL.

Diagonal terms: $a = q, \bar{q}, g$

$$P_{aa}(\omega = 0, Q^2) = \gamma_0(\delta_{ag} + K_a^{(1)}\gamma_0 + K_a^{(2)}\gamma_0^2)$$

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$$P_{gq}(0, Q^2) = C(P_{gg}(0, Q^2) + A) \qquad P_{qg}(0, Q^2) = C^{-1}(P_{qq}(0, Q^2) + A)$$

SUSY-like relation at NLO+NNLL:

$$P_{qq}(0, Q^2) + C^{-1}P_{gq}(0, Q^2) = P_{gg}(0, Q^2) + C\,P_{qg}(0, Q^2)$$

Diagonalisation of DGLAP equations

- Transition matrix:

$$\begin{pmatrix} D_s \\ D_g \end{pmatrix} = U \begin{pmatrix} D_- \\ D_+ \end{pmatrix} \qquad U = \begin{pmatrix} 1 & -1 \\ \frac{1-\alpha}{\epsilon} & \frac{\alpha}{\epsilon} \end{pmatrix}$$

$$U^{-1} \begin{pmatrix} P_{qq} & P_{gq} \\ P_{qg} & P_{gg} \end{pmatrix} U = \begin{pmatrix} P_{--} & 0 \\ 0 & P_{++} \end{pmatrix}$$

where

$$\alpha = \frac{P_{qq} - P_{++}}{P_{--} - P_{++}}, \qquad \epsilon = \frac{P_{gq}}{P_{--} - P_{++}}$$

$$P_{\pm\pm} = \frac{1}{2} \left[P_{qq} + P_{gg} \pm \sqrt{(P_{qq} - P_{gg})^2 + 4 P_{qg} P_{gq}} \right]$$

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$$P_{--} = -K_q^{(1)}\gamma_0^2, \qquad P_{++} = P_{qq} + P_{gg} - K_q^{(1)}\gamma_0^2$$

Diagonalisation of DGLAP equations

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$$D_{\pm}(0, Q^2) \simeq \hat{T}_{\pm}(\gamma_0(Q^2)) D_{\pm}(0, Q_0^2) \Rightarrow N_s(Q^2) \;\; \& \;\; N_g(Q^2)$$

Diagonalisation of DGLAP equations

- Transition matrix:

$$\begin{pmatrix} D_s \\ D_g \end{pmatrix} = U \begin{pmatrix} D_- \\ D_+ \end{pmatrix} \qquad U = \begin{pmatrix} 1 & -1 \\ \frac{1-\alpha}{\epsilon} & \frac{\alpha}{\epsilon} \end{pmatrix}$$

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where

$$\alpha = \frac{P_{qq} - P_{++}}{P_{--} - P_{++}}, \qquad \epsilon = \frac{P_{gq}}{P_{--} - P_{++}}$$

$$D_\pm(0, Q^2) \simeq \hat{T}_\pm(\gamma_0(Q^2)) D_\pm(0, Q_0^2) \Rightarrow r(Q^2) = N_g(Q^2)/N_q(Q^2)$$

Predictions of $N_g(Q)$, $N_q(Q)$ and $r(Q)$ at NNLO*+NNLL

- Quark and gluon multiplicities:

$$N_s(Q^2) = N_1(Q_0^2)H_+(Q^2,Q_0^2)\hat{T}_+(\gamma_0(Q^2)) + N_2(Q_0^2)\hat{T}_-(\gamma_0(Q^2))$$

$$N_g(Q^2) = N_1(Q_0^2)r_+(Q^2)H_+(Q^2,Q_0^2)\hat{T}_+(\gamma_0(Q^2)) + N_2(Q_0^2)r_-(Q^2)\hat{T}_-(\gamma_0(Q^2))$$

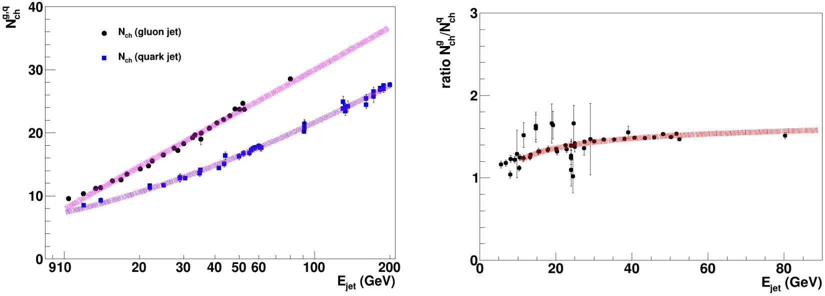
where ($\gamma_0 \propto \sqrt{\alpha_s}$)

$$\begin{aligned} T_+(\gamma_0) \quad = \quad & \gamma_0^{d_+} \exp\left[\frac{4C_A}{\beta_0\gamma_0} - \frac{4C_A}{\beta_0}\left(K_q^{(2)} + K_g^{(2)} - b_1\right)\gamma_0\right. \\ + \quad & \frac{2C_A}{\beta_0} b_1 \left(2K_q^{(1)} + K_g^{(1)}\right)\gamma_0^2 \\ + \quad & \left.\frac{4C_A}{3\beta_0}\left(b_1^2 - b_2 + b_1\left(K_g^{(2)} + K_q^{(2)}\right)\right)\gamma_0^3\right] \end{aligned}$$

- Ratio of gluon over quark jet multiplicities:

$$r(Q^2) = \frac{N_g(Q^2)}{N_q(Q^2)}$$

Preliminary α_s fit to the e^+e^- data



Results:

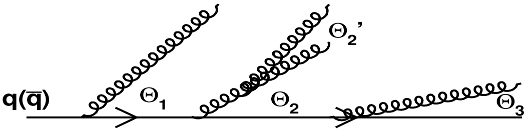
Fit result: $\alpha_s(\mathrm{m_Z}) \approx 0.121$ at NNLO*+NNLL with caveats:

- Using fraction of the data (uncerts. underestimated in some cases) and without accounting for correlations.
- Fit uncertainties $\approx 4\%$. Theoretical uncertainties to be determined.
- Just a proof-of-principle for now. Upcoming improvements.

Part III

MLLA evolution on the extraction of α_s from the same observables.

Angular Ordering (MLLA)



AO: $\Theta_1 \geq \Theta_2 (\geq \Theta_2') \geq \Theta_3$

Successive parton decays (soft/collinear and/or hard/collinear) ruled by:

- QCD coherence $\rightarrow$ Angular Ordering (AO) $\rightarrow$ MLLA evolution equations for FFs at small $x \ll 1$: ev. time variable $t = \ln \Theta$
  - Soft FF $x < 0.1$: bulk of hadron production in jets

Resummation schemes at small x and Θ

- DLA: $\alpha_s \log(1/x) \log\Theta$ ($\alpha_s \log^2 \sim 1 \Rightarrow \log \sim \alpha_s^{-1/2}$): resummation of soft and collinear gluons:
  - main ingredient to the estimation of inclusive observables in jets,
  - neglects the energy balance.
- Single Logs (SL): $\alpha_s \log\Theta$:
  - collinear splittings (i.e. DGLAP FO approach or LLA of FFs, PDFs at large $x \sim 1$).
- MLLA: $\underbrace{\alpha_s \log\log}_{\mathcal{O}(1)} + \underbrace{\alpha_s \log}_{\mathcal{O}(\sqrt{\alpha_s})}$: the SL corrections to DLA:
  - "restore" the energy balance,
  - take into account the running of $\alpha_s(k_\perp) \Rightarrow (\beta_0, \beta_1, \beta_2) \times \alpha_s^n \log^n \Theta$".
- Next. . .-to-. . . MLLA: $\underbrace{\alpha_s \log\log}_{\mathcal{O}(1)} + \underbrace{\alpha_s \log}_{\mathcal{O}(\sqrt{\alpha_s})} + \underbrace{\alpha_s \log\log^{-1}}_{\mathcal{O}(\alpha_s)} + \dots$
  - improve the restoration of the energy balance and running coupling constant effects

The anomalous dimension

Anomalous dimension at $\mathcal{O}(\alpha_s^2)$

$$\gamma_g(Q) = \gamma_0(Q) - a_1\gamma_0^2(Q) - a_2\gamma_0^3(Q) - a_3\gamma_0^4(Q) + \mathcal{O}(\gamma_0^5)$$

with revisited a_3.

The anomalous dimension

Anomalous dimension at $\mathcal{O}(\alpha_s^{5/2})$

$$\gamma_g(Q) = \gamma_0(Q) - a_1\gamma_0^2(Q) - a_2\gamma_0^3(Q) - a_3\gamma_0^4(Q) - a_4\gamma_0^5(Q) + \mathcal{O}(\gamma_0^6)$$

with revisited a_3 and new coefficient a_4.

The anomalous dimension

Mean Multiplicities at $\mathcal{O}(\alpha_s^{5/2})$

$$N_g\left(y\right) \propto \exp\left[\int dy \gamma_g(y) dy\right] \quad \& \quad N_q\left(y\right) \propto \exp\left[\int dy \gamma_q(y) dy\right]$$

with revisited a_3 and new coefficient a_4.

Predictions of $N_g(Q)$, $N_q(Q)$ at "NNLO*+NNLL"

$$N_g(y) = K^{ch} \exp\left[2.50217\sqrt{y} - 0.491546\ln y - (0.06889 - 0.41151\ln y)\frac{1}{\sqrt{y}} - (0.220519 + 0.161681\ln y)\frac{1}{y} - (1.04739 + 0.0860755\ln y + 0.169194\ln^2 y)\frac{1}{y^{3/2}}\right]$$

$$N_q(y) = K^{ch}\frac{C_F}{N_c} \exp\left[2.50217\sqrt{y} - 0.491546\ln y + (0.178238 + 0.41151\ln y)\frac{1}{\sqrt{y}} + (0.608827 - 0.161681\ln y)\frac{1}{y} - (1.46095 + 0.0860755\ln y + 0.169194\ln^2 y)\frac{1}{y^{3/2}}\right]$$

with

2 free parameters:

$$y = \ln\left(\frac{Q}{\Lambda_{\rm QCD}}\right)$$

& the non-perturbative $\mathcal{K}^{ch}$ normalization.

Predictions of $r(Q)$ at "NNLO*+NNLL"

Ratio

Ratio at $\mathcal{O}(\alpha_s^{3/2})$:

$$r = \frac{N_c}{C_F}(1 - r_1\gamma_0 - r_2\gamma_0^2 - r_3\gamma_0^3)$$

with r_3 revisited.

Predictions of $r(Q)$ at "NNLO*+NNLL"

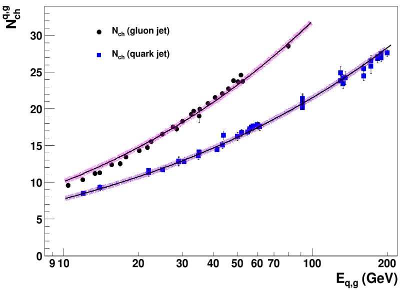
Ratio

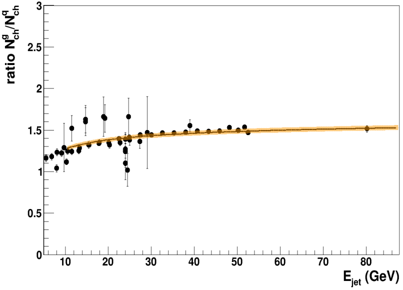
Ratio at $\mathcal{O}(\alpha_s^2)$:

$$r = \frac{N_c}{C_F}(1 - r_1\gamma_0 - r_2\gamma_0^2 - r_3\gamma_0^3 - r_4\gamma_0^4)$$

with r_3 revisited and new term r_4.

Preliminary $\alpha_s(m_Z)$ results:





Results:

Fit result: $\alpha_s(m_Z) \approx 0.117$ at "NNLO*+NNLL" with caveats:

- Using fraction of the data (uncerts. underestimated in some cases) and without accounting for correlations.
- Fit uncertainties $\approx 5\%$. Theoretical uncertainties to be determined.
- Just a proof-of-principle for now. Upcoming fit improvements.

Conclusions

- Computed hadron multiplicities in gluon and quark jets (and their ratio) at NNLO*+NNLL accuracy in two formalisms: $\overline{\mathrm{MS}}$ and MLLA
- $\overline{\mathrm{MS}}$: Novel diagonalisation of DGLAP evolution eqs. for (first moment of) parton FFs, and inclusion of NNLO $\alpha_s(Q)$ running.
- MLLA: Revised calculations and inclusion of higher-order terms via NNLO $\alpha_s(Q)$ running.
- First attempt to fit the e^+e^- gluon and quark jet data vs. E_{jet}: $\alpha_s(m_Z) \approx 0.121$ ($\overline{\mathrm{MS}}$) and $\alpha_s(m_Z) \approx 0.117$ (MLLA)
- Outlook: Proper data global fit with uncertainties. Inclusion of NNLO splitting functions to extract alphas at full-NNLO in $\overline{\mathrm{MS}}$ scheme.