α_s FROM τ DECAY

Diogo Boito, Maarten Golterman, Kim Maltman, Santiago Peris
(& Marcus Rodrigues, Wilder Schaaf)

- arXiv: 2012.10440 (PRD)
- new arXiv to appear.

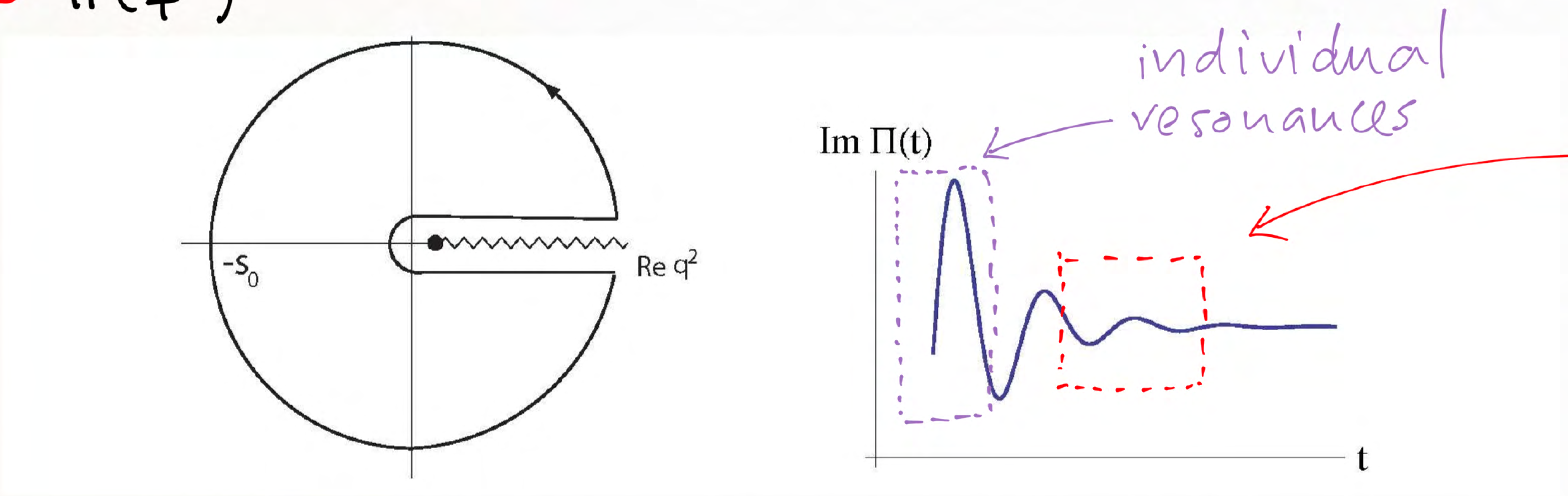
(see also arXiv:1611.03457 (PRD), 1907.03360 (PRD))

ALPHA_S (2024) @ TRENTO

FESRs, OPE & DVs

Shankar 77; Braaten, Narison, Pich '92

- $\Pi(q^2)$



wiggles!
(resonances overlap)
Poggio, Quinn, Weinberg '76

- "Cauchy's Theorem" ($z = q^2$, $w(z) =$ polynomial):

$$\int_0^{s_0} dt\, w(t) \underbrace{\frac{1}{\pi} \mathrm{Im}\,\Pi(t)}_{\text{exp.}} = -\frac{1}{2\pi} \oint_{|z|=s_0} dz\, w(z) \underbrace{\Pi(z)}_{\Pi_{OPE} + \Pi_{DV}}$$

$$= -\frac{1}{2\pi} \oint_{|z|=s_0} dz\, w(z) \underbrace{\Pi_{OPE}(z)}_{\substack{O(\alpha_s^5) \\ + \text{Condensates}}} - \int_{s_0}^{\infty} dt\, w(t) \frac{1}{\pi} \mathrm{Im}\,\Pi_{DV}(t)$$

Cata, Golterman, S.P. '05, '08, '09

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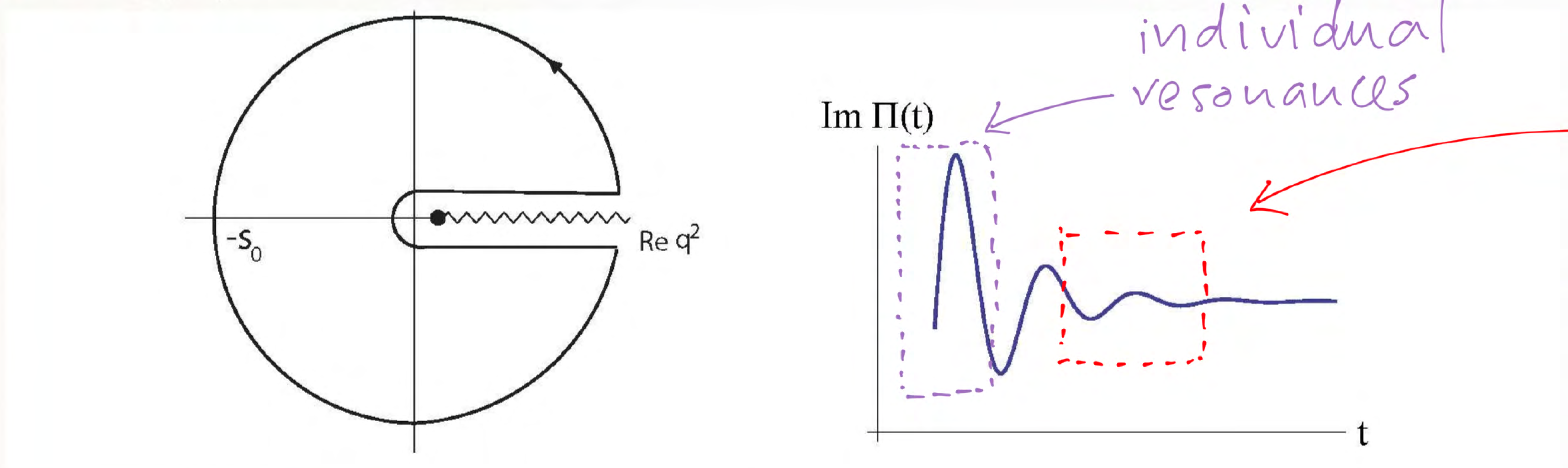

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Cata, Golterman, S.P. '05, '08, '09

OPE ASYMPTOTIC

A TALE OF TWO WORLDS

- **Pert. Theory** $\to$ CIPT ✗
  $\to$ FOPT($\overline{MS}$) ✓ (Hoang's talk)

- **Non Perturbative**

* **DV Approach:**

$\Pi_{OPE} \sim N! \left[\frac{\Lambda^2_{QCD}}{m^2_\tau}\right]^N$ (asymptotic)

$$\mathrm{Im}\,\Pi_{DV}(s) = e^{-\delta-\gamma s}\sin(\alpha+\beta s)\left(1+\mathcal{O}\left(\frac{1}{s},\frac{1}{N_c},\frac{1}{\log s}\right)\right)$$

(Asymptotic Regge spectrum @ large N_c)

$\to$ $\alpha_s, \delta, \gamma, \alpha, \beta$ extracted from fit to data in $s_0 \in [s_{min}, m^2_\tau]$

* **tOPE:**

$\Pi_{OPE} \sim \left[\frac{\Lambda^2_{QCD}}{m^2_\tau}\right]^N << 1$ $(N \gg 1)$

(convergent) FAPP

$\to$ "pinch": use polynomial w zero @ highest $s_0 \simeq m^2_\tau$

$\to$ truncate OPE to lowest orders that allow a fit @ single $s_0 \simeq m^2_\tau$

$\to$ set DV = 0

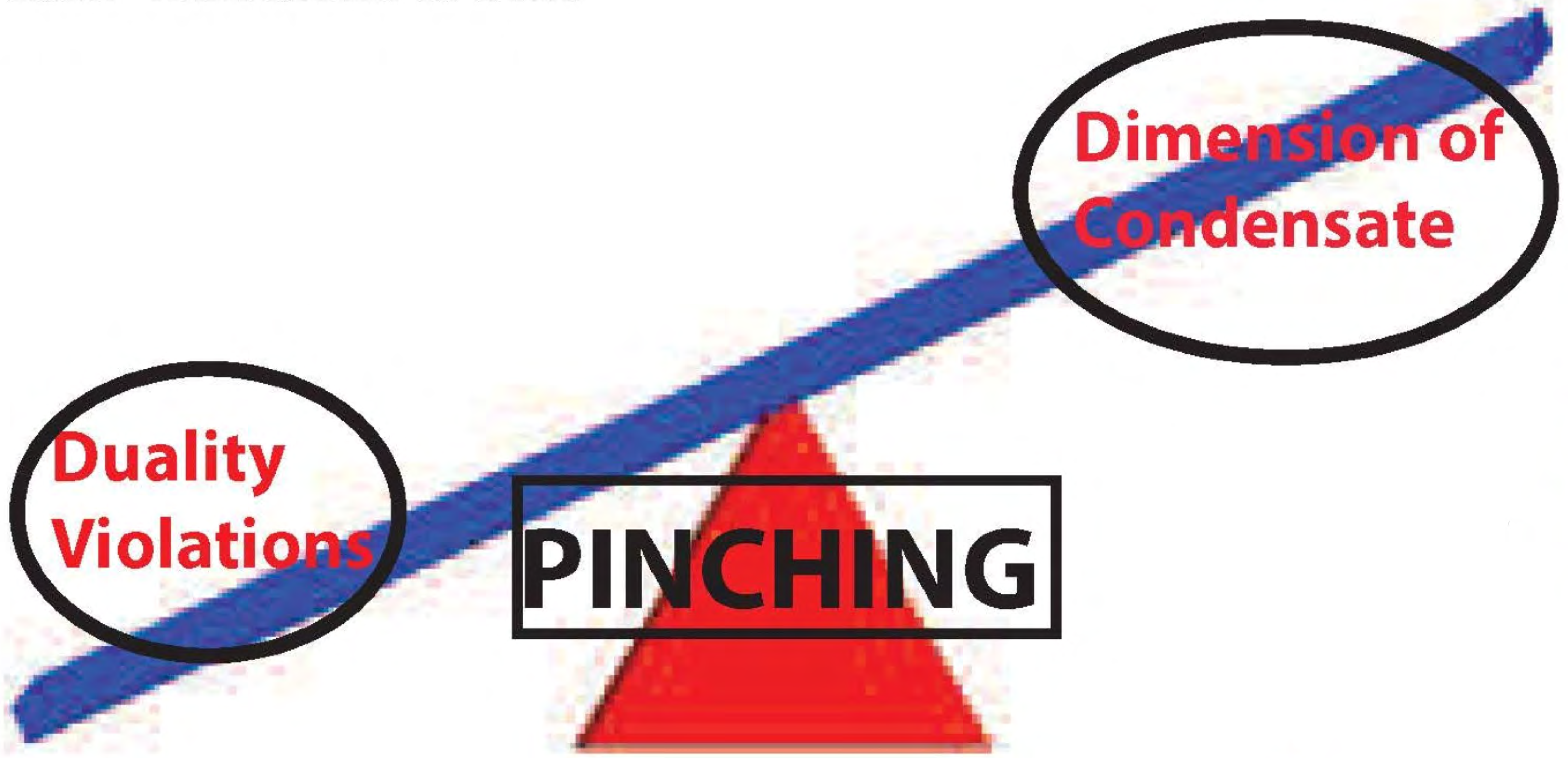
BUT...

Main Theoretical Message

- with pinching $\exists$ price to pay:

It's NOT possible to simultaneously suppress DVs and condensates

★ "**Seesaw**" mechanism at work:



OPE Asymptotic

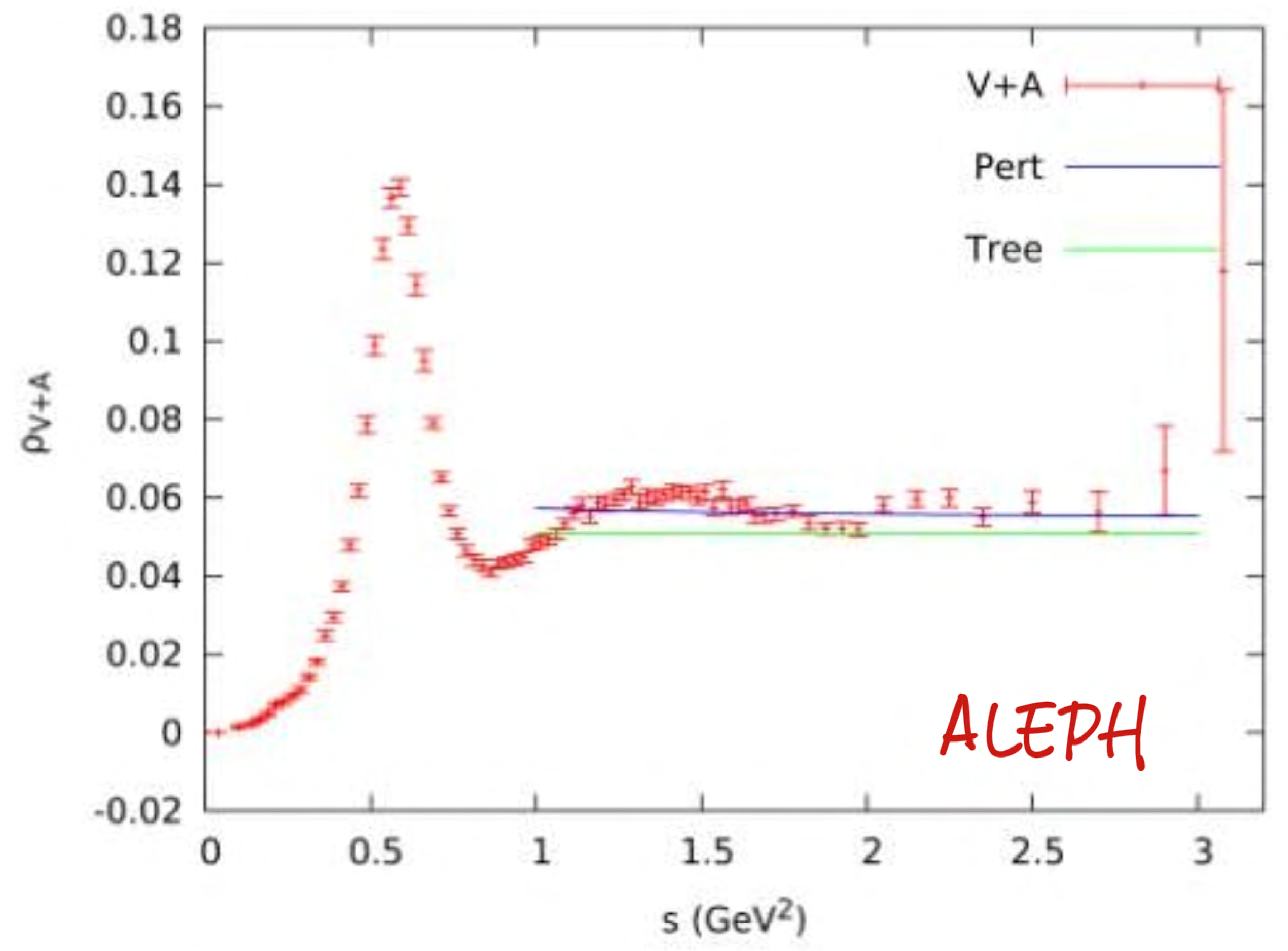
$\langle O_N \rangle \sim N!$

$\Downarrow$

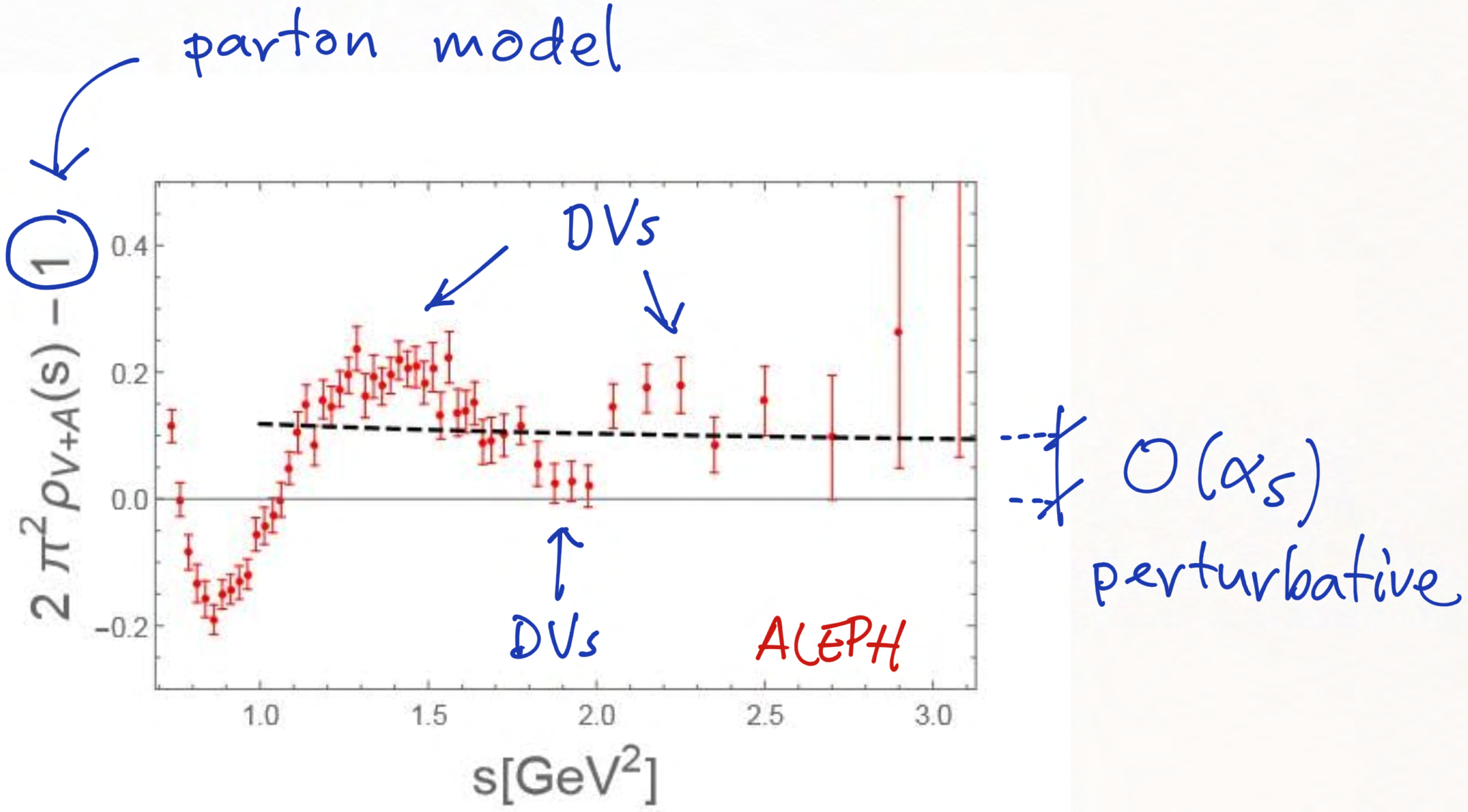
Restrict OPE to low orders consistently*

$^{*}\; w(t) = t^N \longleftrightarrow \text{OPE} = \dfrac{C_{2N+2}}{s^{N+1}}$

BEWARE!

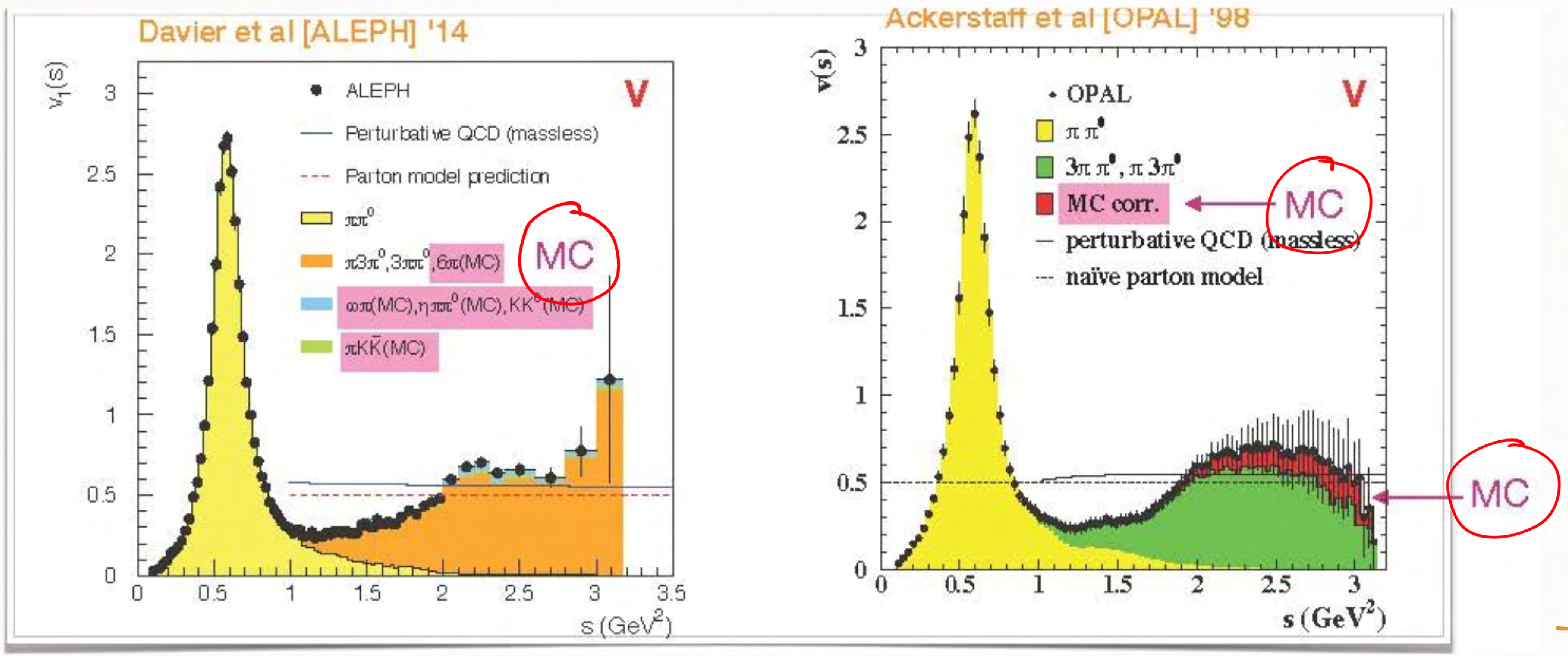


Pich, Rodriguez S. '22



ALEPH & OPAL Vector Channel Data Sets

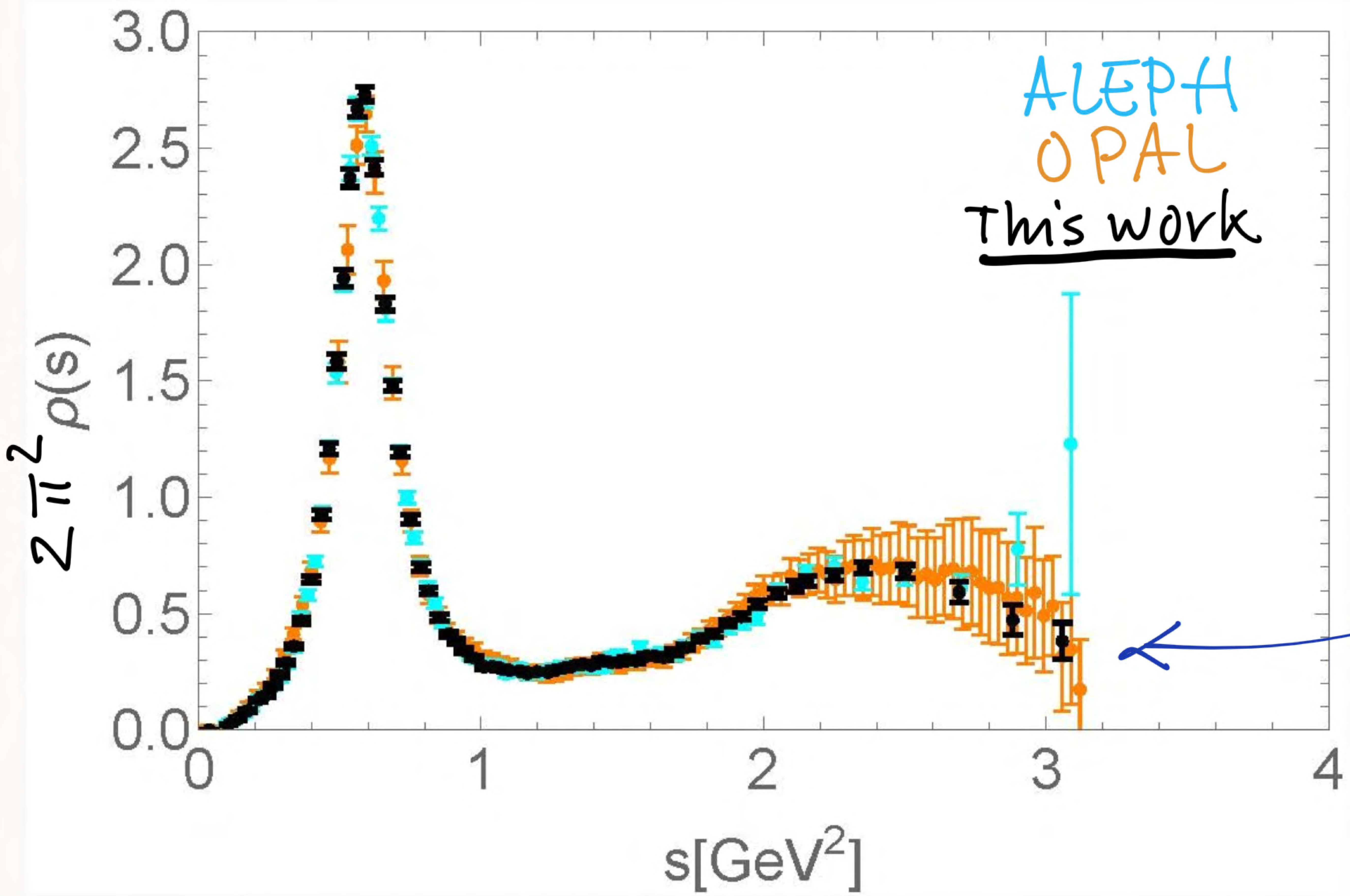
- V channel dominated by $\tau \to \nu_\tau \begin{smallmatrix} 2\pi \\ 4\pi \end{smallmatrix}$
- Residual channels subdominant (but important for α_s!)
- $\exists$ Monte Carlo (MC) input for several channels.



Recent measurements in e^+e^- allow to replace MC.

New Vector spectral function

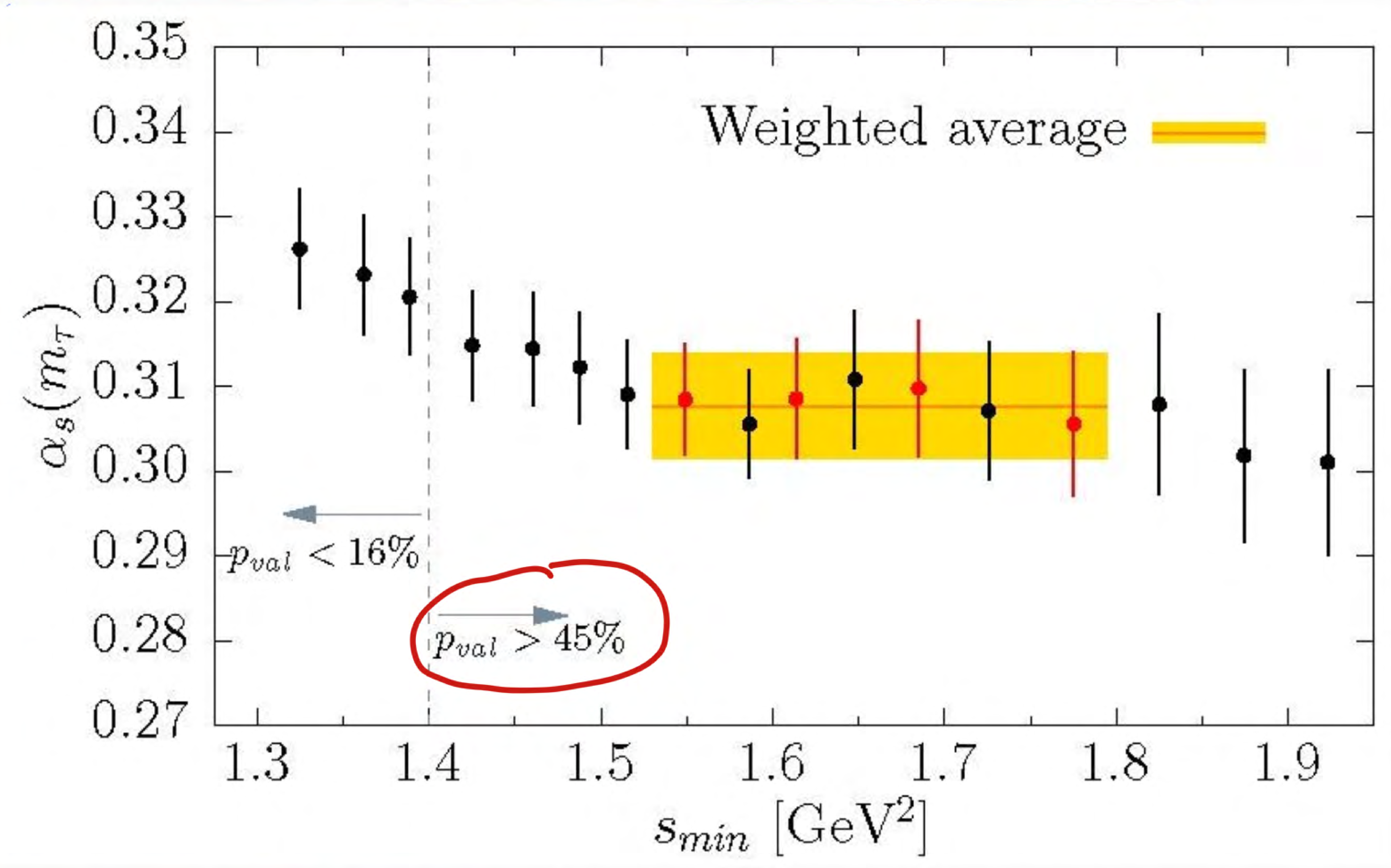
- Combined 2π & 4π channels from ALEPH & OPAL (à la Keshavarzi et al. '18 for $(g-2)_\mu$)
- No more Monte Carlo: 7 residual channels from e^+e^- using CVC (IB corr's small)
  and BABAR for $\tau \to K K_s \nu_\tau$
  + updated BRs.
  (all concentrated near end point)



- Dramatic error improvement near end point!

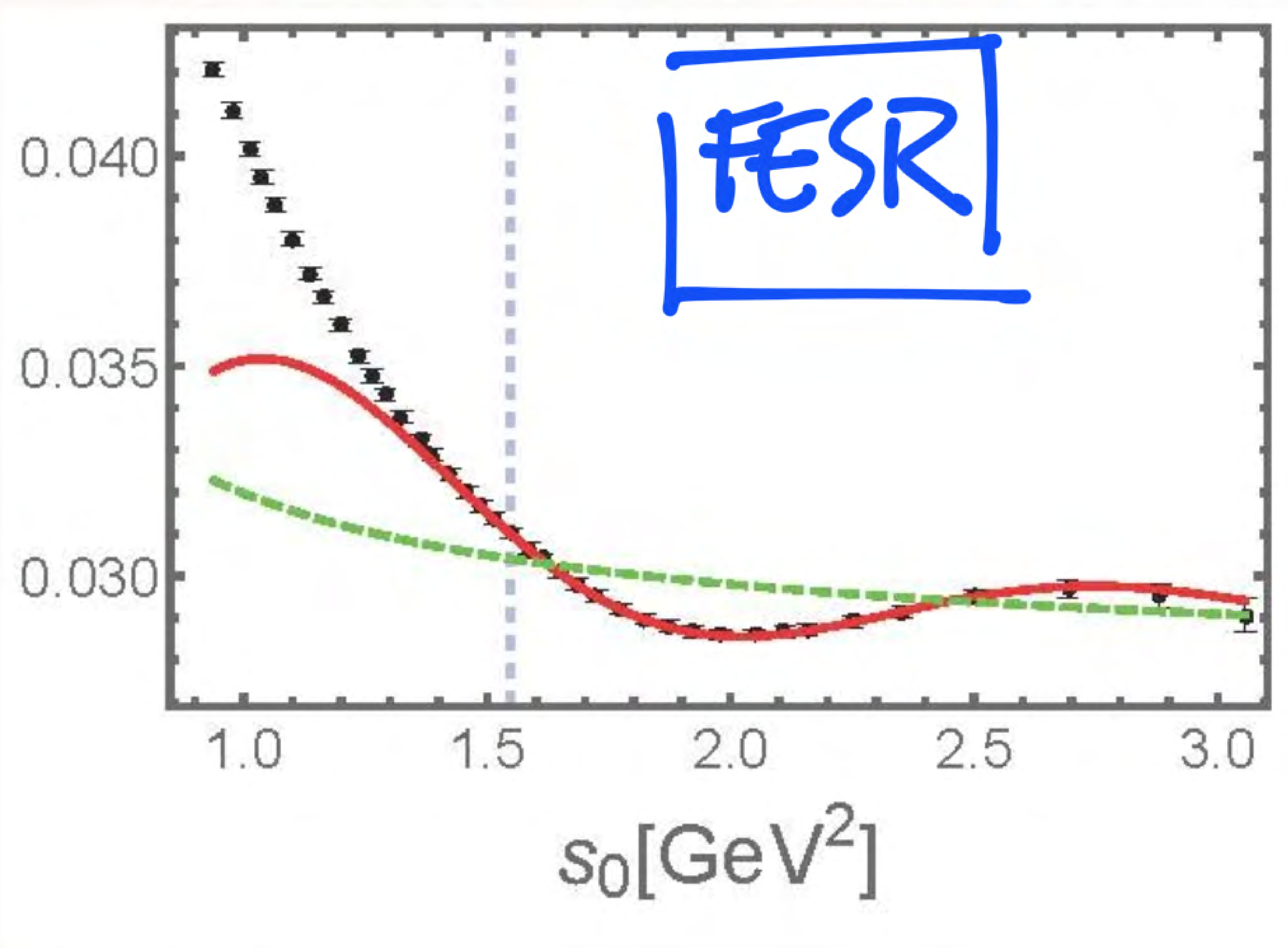
Extraction of α_s (I)

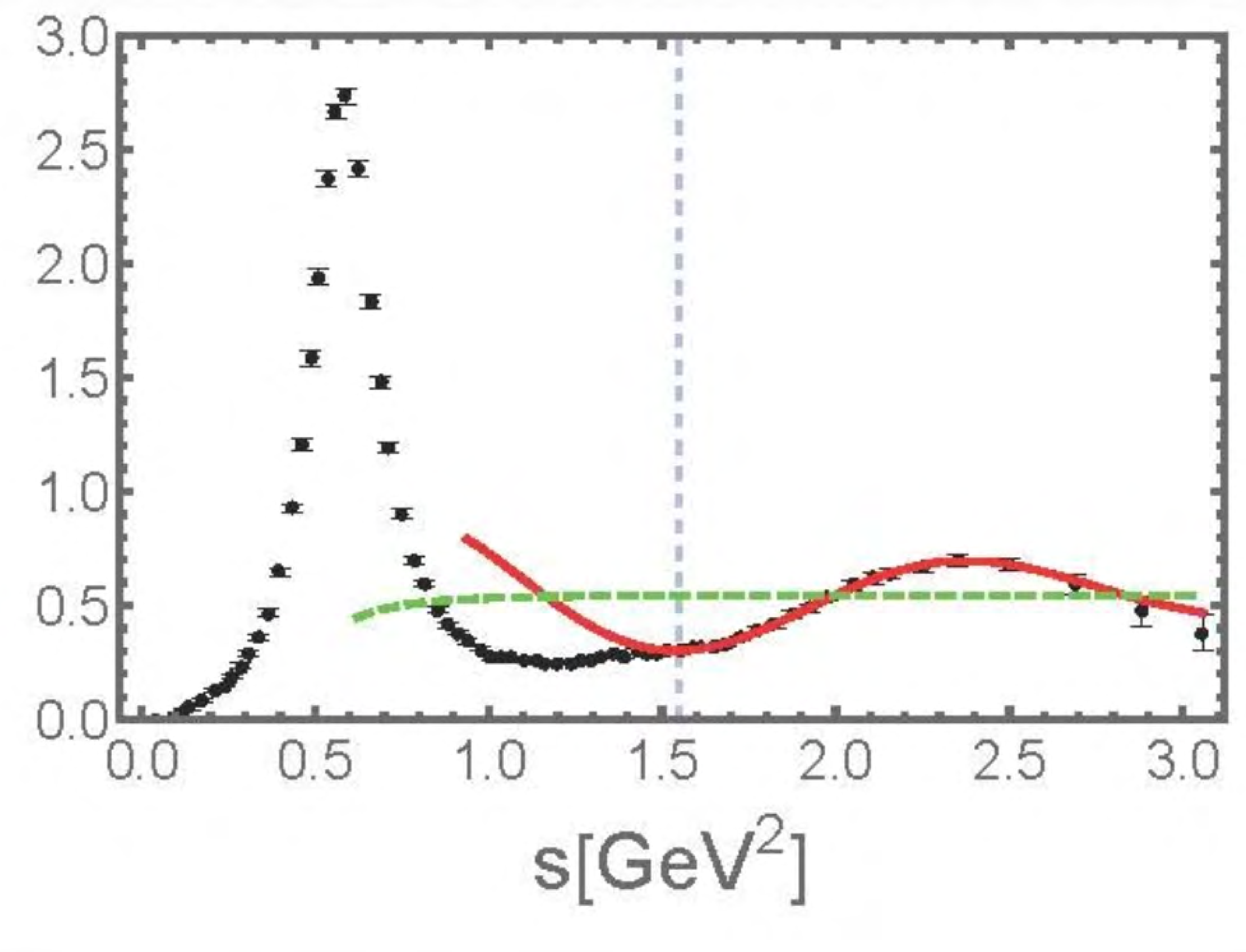
- $w_0 = 1$



$s_0 \in [s_{min}, m_\tau^2]$ $\boxed{\text{FESR}}$

$$\int_0^{s_0} ds\, \frac{1}{\pi}\mathrm{Im}\Pi_V(s) = -\frac{1}{2i\pi}\oint_{|z|=s_0} dz\, \Pi_V(z) - \int_{s_0}^{\infty} ds\, \frac{1}{\pi}\mathrm{Im}\Pi_{DV}(s)$$





Data

— OPE + DVs

- - - OPE w/o DVs

$w_0 = 1 \leftrightarrow$ OPE = Pert. Theory

Extraction of α_s (II)

mom.	α_s	$c_6\,[\mathrm{GeV}^6]$
w_0	0.3077(65)	$--$
$w_0\&w_2$	0.3091(69)	−0.0059(13)
$w_0\&w_3$	0.3080(70)	−0.0070(12)
$w_0\&w_4$	0.3079(70)	−0.0068(12)

$w_0 = 1$ $\quad w_3 = (1-y)^2(1+2y)$

$w_2 = 1-y^2$ $\quad w_4 = (1-y^2)^2$

- Several fits, single moments & combined
- Many fit windows $[s_{min}, m_\tau^2]$
- Consistency: α_s, condensates & DV parameters.

$$\boxed{\begin{aligned} \alpha_s(m_\tau) &= 0.3077 \pm \boxed{0.0075} \quad (n_f=3, \mathrm{FOPT}) \\ \alpha_s(M_Z) &= 0.1171 \pm 0.0010 \quad (n_f=5, \overline{MS}) \end{aligned}}$$

pert. series truncation & scale variation & DVs

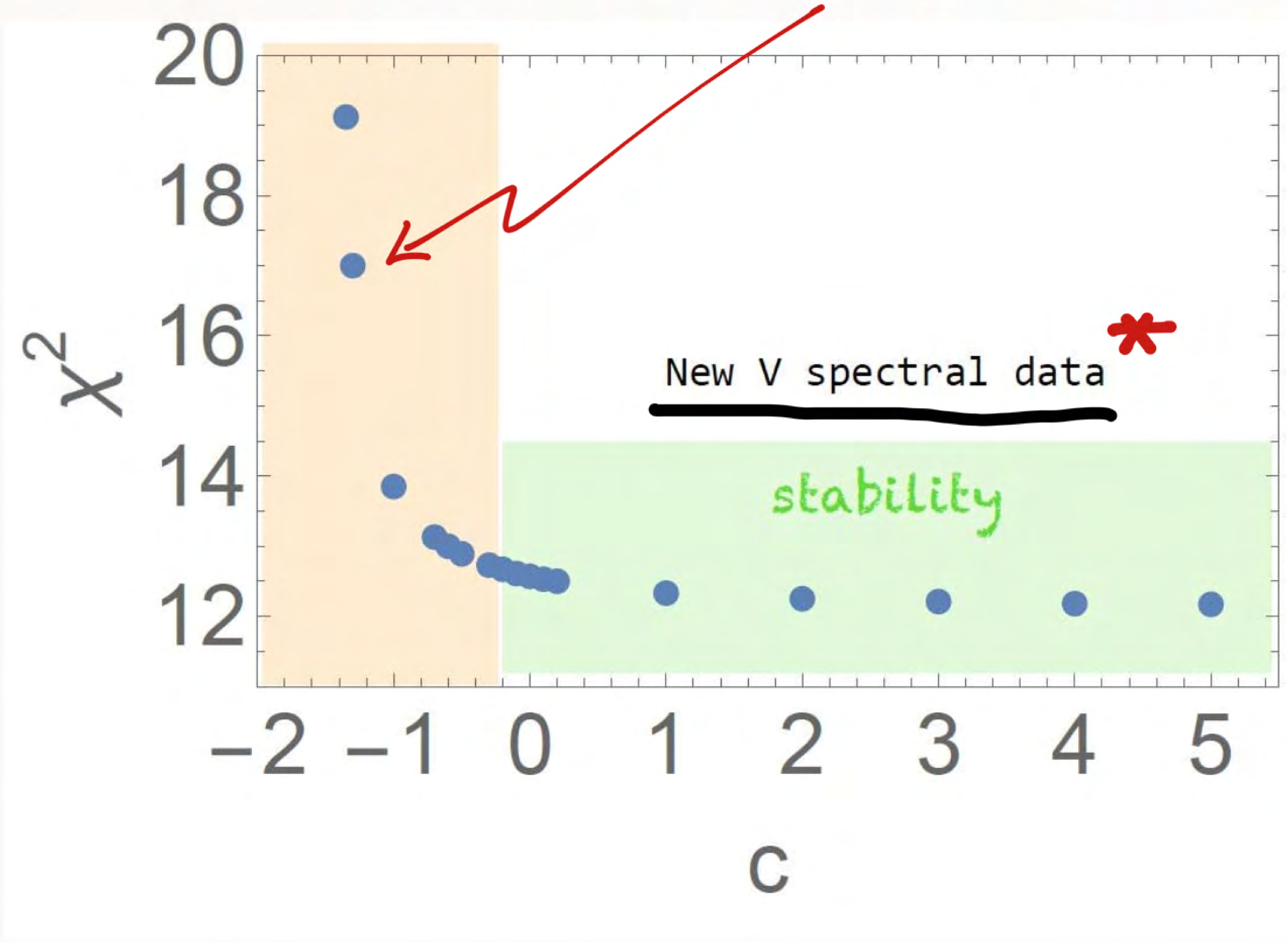
Response to Criticisms from Pich & Rodriguez Sanchez (JHEP '22)

- "DVs unstable against variations"

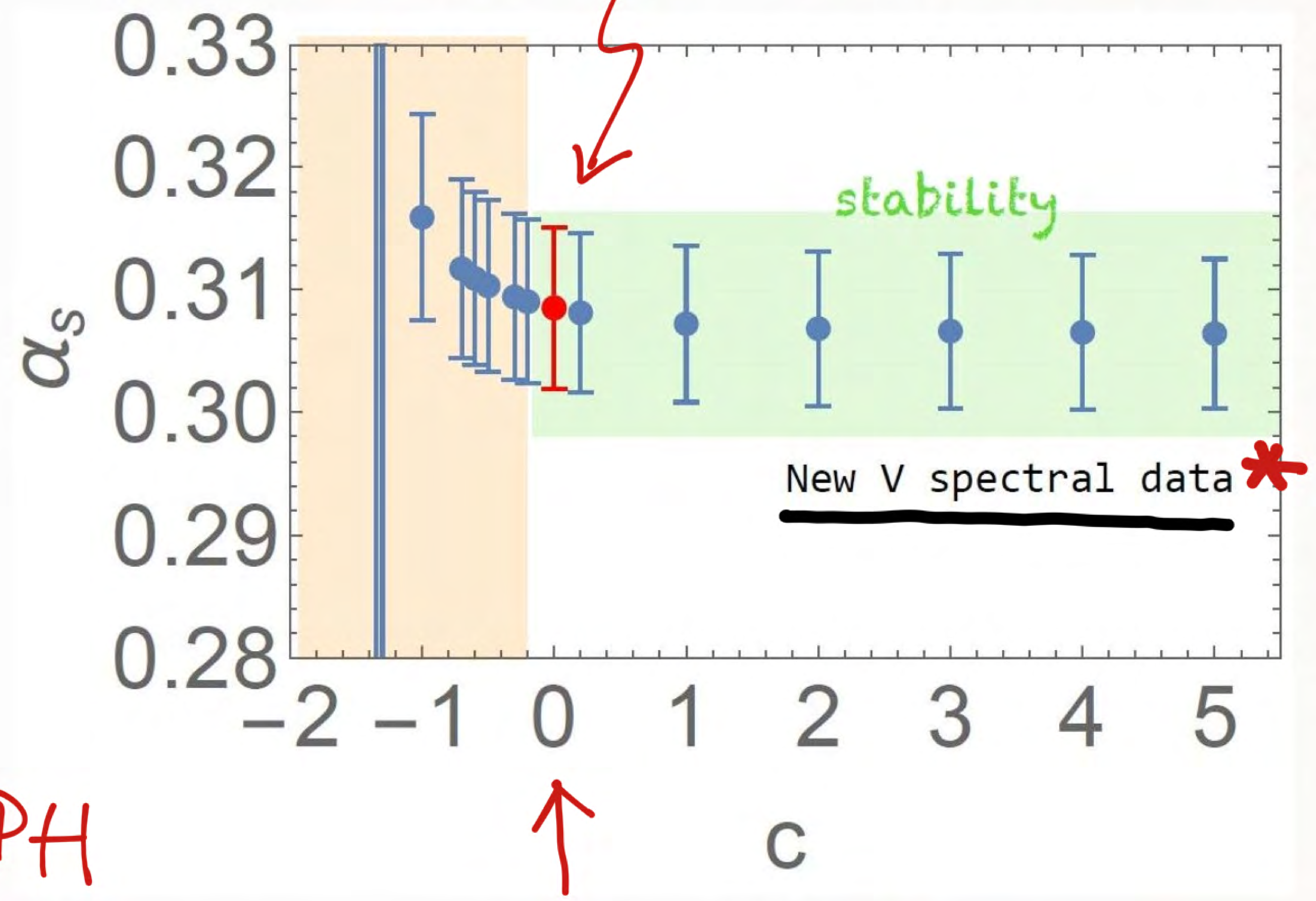
$$\rho_{DV}(s) = e^{-\delta-\gamma s}\sin(\alpha+\beta s)\left(1+\frac{c}{s}\right) \; ; \quad 1.55\,\mathrm{GeV}^2 \le s_0 \le 3.06\,\mathrm{GeV}^2$$

choice of P-RS (w ALEPH data)

α_s stable!



* better than ALEPH



PRS also consider: $\rho_{DV}(s) = s^{\lambda}\, e^{-\delta-\gamma s} \sin(\alpha+\beta s)$, $\lambda = 0, \dots, 8$

i) and the difference between $\lambda=0$ and $\lambda=8$ to claim an error in α_s of 0.016
However, a discrepancy in α_s of 0.034 (Table 2, PRS'22) is "amazing stability".

ii) totally ad-hoc: why not $\lambda=20$? Regge/large N_c analysis leads to $\lambda=0$
+ possible $1/s$ corrections. Can you justify $\lambda=8$?

iii) Our improved V spectrum shows that p-value ↓ when λ ↑.
So, $\lambda=8$ also disfavored by data.

we'll stick to $\lambda=0$.

- "Logarithms in condensates invalidate analysis"

$$C_D^L(s) = C_D(\mu^2)\left(1 + L_D \log\frac{-s}{\mu^2}\right)$$ PRS choose $L_D \sim 0.2$

However $L_4 = \frac{11}{8}\left(\frac{\alpha_s}{\pi}\right)^2 \simeq 0.012$ & $L_6 = -\frac{19}{63}\left(\frac{\alpha_s}{\pi}\right) \simeq 0.03 \ll 0.2$

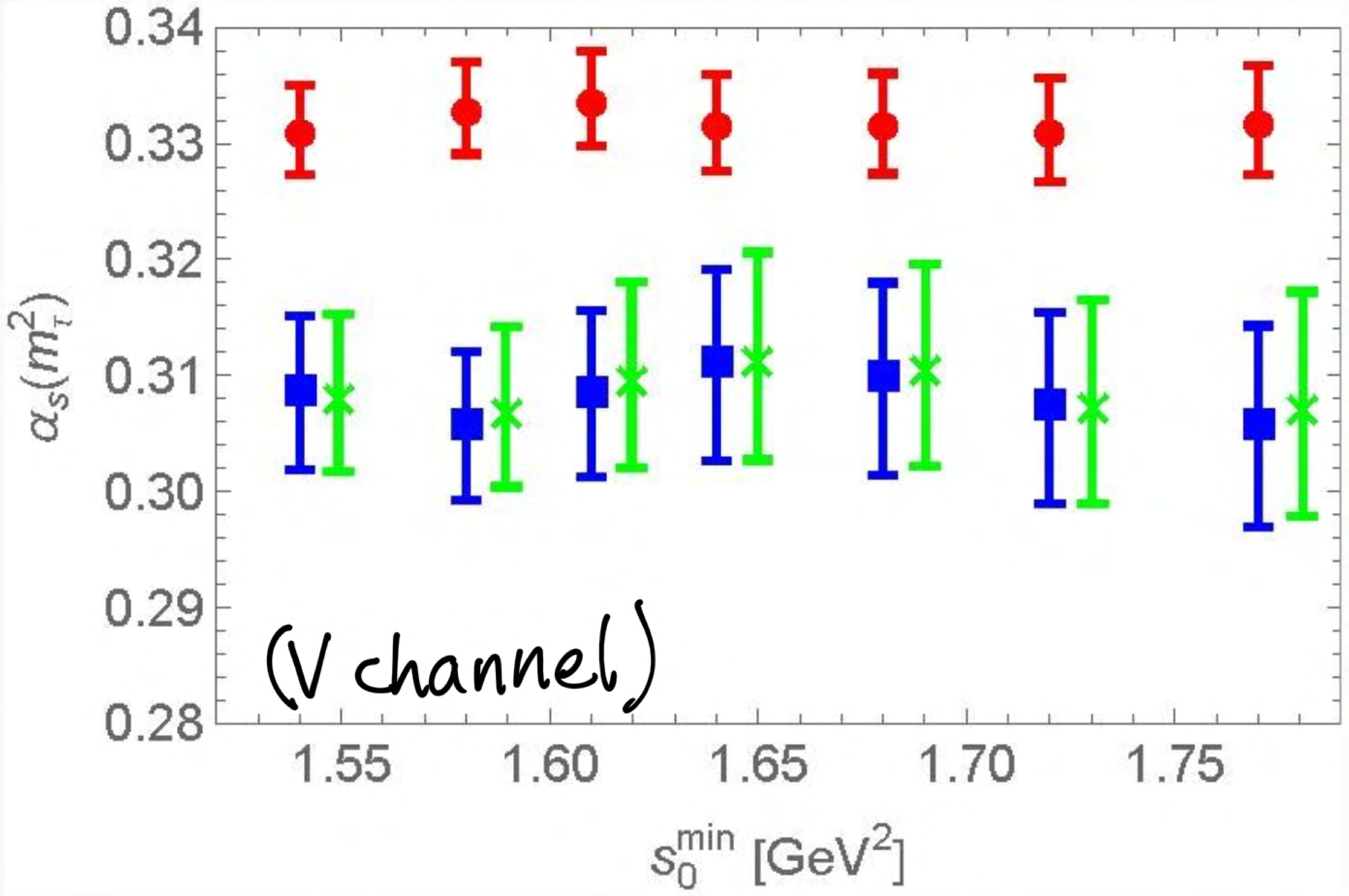
(large N_c factorization)

These values of C_4^L & C_6^L do not affect our analysis.

In fact $L_D \neq 0$ would affect FESR s_0-scaling: not seen in our fits.

(And recall OPE asymptotic: unreliable at large condensate dimension.)

- Tautology: "α_s determined by $w_0 = 1$, other weights redundant"



- Fit to $w_0 = 1$
- Fit to $w_0 = 1$, $w_2 = 1 - x^2$, OPE $\sim 1/s_0^3$ (right)
- Fit " " " but OPE $\sim 1/s_0^5$ (wrong!)

$\Downarrow$

NO REDUNDANCY! (math error in PRS)

cf. Backup slides

True Redundancy in τOPE

and, in general, in single-s_0 analyses.

- Take, e.g., the optimal weights: $w_{2m}(x) = 1-(m+2)\,x^{m+1} + (m+1)\,x^{m+2}$
  $(m=1,\dots,5)$ (Pich, Rodríguez S. '16)

OPE balance:
$w_{21} \to C_6, C_8$ $w_{23} \to C_{10}, C_{12}$ $w_{25} \to C_{14}, C_{16}$
$w_{22} \to C_8, C_{10}$ $w_{24} \to C_{12}, C_{14}$

Following ALEPH (OPAL) analysis, set $\boxed{C_{10}=C_{12}=C_{14}=C_{16}=0}$ (*) V+A $s_0 = 2.8\,\mathrm{GeV}^2$

Fits:
$w_{23}, w_{24}, w_{25} \longrightarrow \alpha_s = 0.3125(23)$, NO OPE AT ALL! $(\chi^2 = 11.6/2)$
add $w_{22} \longrightarrow \alpha_s = 0.3125(23)$, determines C_8 $(\chi^2 = 11.6/2)$
add $w_{21} \longrightarrow \alpha_s = 0.3125(23)$, determines C_6 (same C_8) $(\chi^2 = 11.6/2)$

These are not good fits & C_8, C_6 very unstable wrt (*)

Therefore

i) value of α_s determined by Pert. Theory only

would get same value even if, e.g., "$\Pi_{OPE}(q^2) \sim (\log q^2)^{100}$"

(or any other behavior)

ii) i.e. same as in QED

Might as well calculate the integral of the spectral function directly.

Then, why bother about FESRs, Cauchy contours, etc... ?

$$\int_0^{s_0} dt\, w(t)\, \rho(t)_{EXP} = \int_0^{s_0} dt\, w(t)\, \rho(t)_{\text{Pert. theory}}$$

let's focus on this last example on V+A (ALEPH):

$s_0 = 2.8\,\mathrm{GeV}^2$ $\quad \alpha_s(m_\tau)_{①} - \alpha_s(m_\tau)_{②} = 0.0103\,(38)_{\text{exp \& th}}\,,\ \underline{\underline{2.7\,\sigma}}$ $\quad$ ① w_{23}, w_{24}, w_{25}

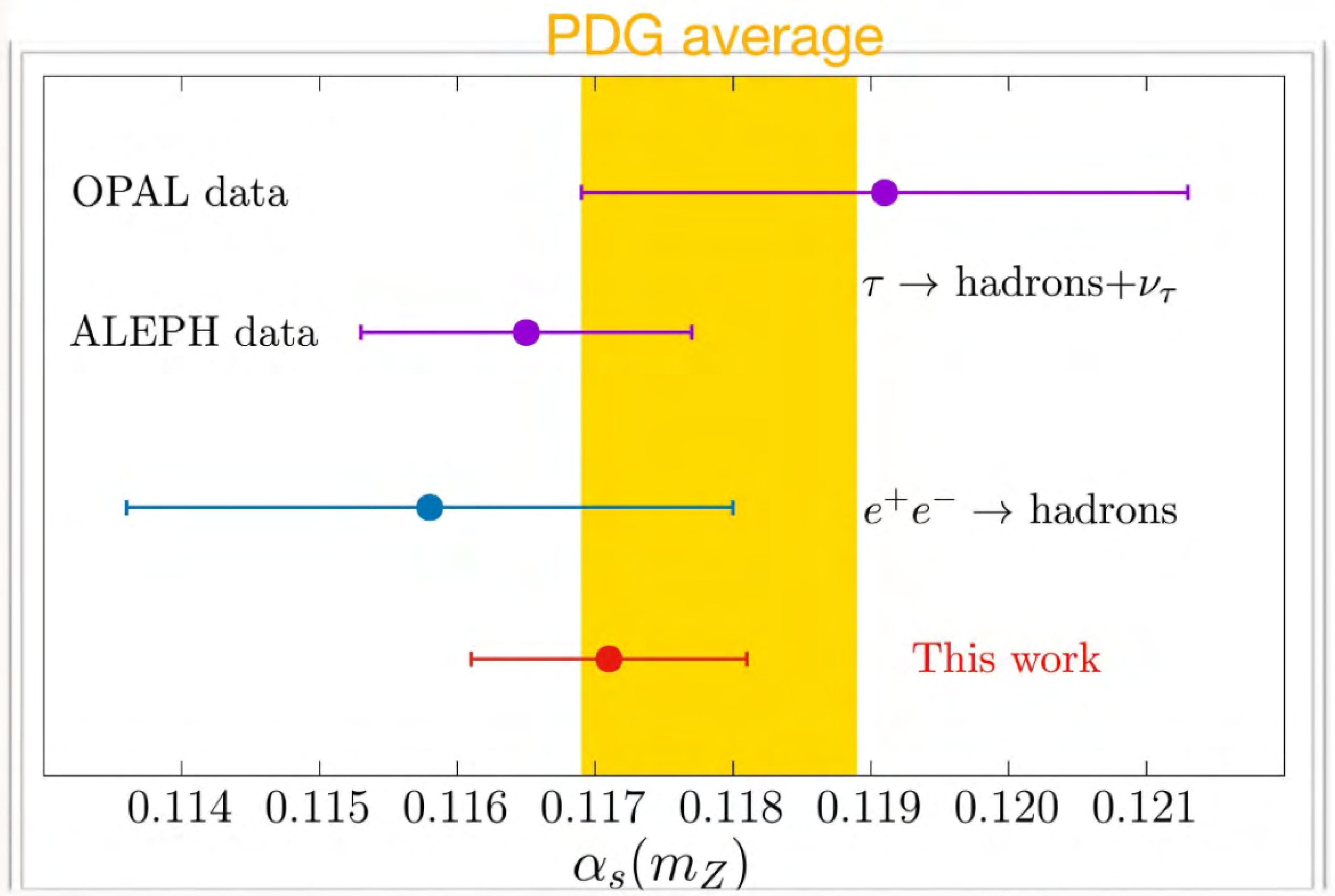
$(-1\text{ bin})\ \big\downarrow\ (-60\,\mathrm{MeV})$ $\qquad$ (includes correlations)

$s_0 = 2.6\,\mathrm{GeV}^2$ $\quad \alpha_s(m_\tau)_{①} - \alpha_s(m_\tau)_{②} = 0.0102\,(29)_{\text{exp \& th}}\,,\ \underline{\underline{3.6\,\sigma}}$ $\quad$ ② w_{25}

* SIGNIFICANT DISCREPANCY *

(Even worse problems using our improved V spectral function: $\sim \underline{\underline{8\text{–}10\,\sigma}}$)

CONCLUSIONS



DB, Golterman, Jamin, Mahdavi, Maltman, Osborne, Peris, '12

DB, Golterman, Maltman, Osborne, Peris, '15

DB, Golterman, Keshavarzi, Maltman, Nomura, Peris, Teubner '18

DB, Golterman, Maltman, Peris, Rodrigues, Schaaf, '21

$$\boxed{\alpha_s(m_\tau) = 0.3077 \pm 0.0075}$$

$$\Downarrow$$

$$\boxed{\alpha_s(M_Z) = 0.1171 \pm 0.0010} \quad (n_f=5)$$

$$\boxed{\alpha_s(M_Z) = 0.1180 \pm 0.0009}$$

PDG

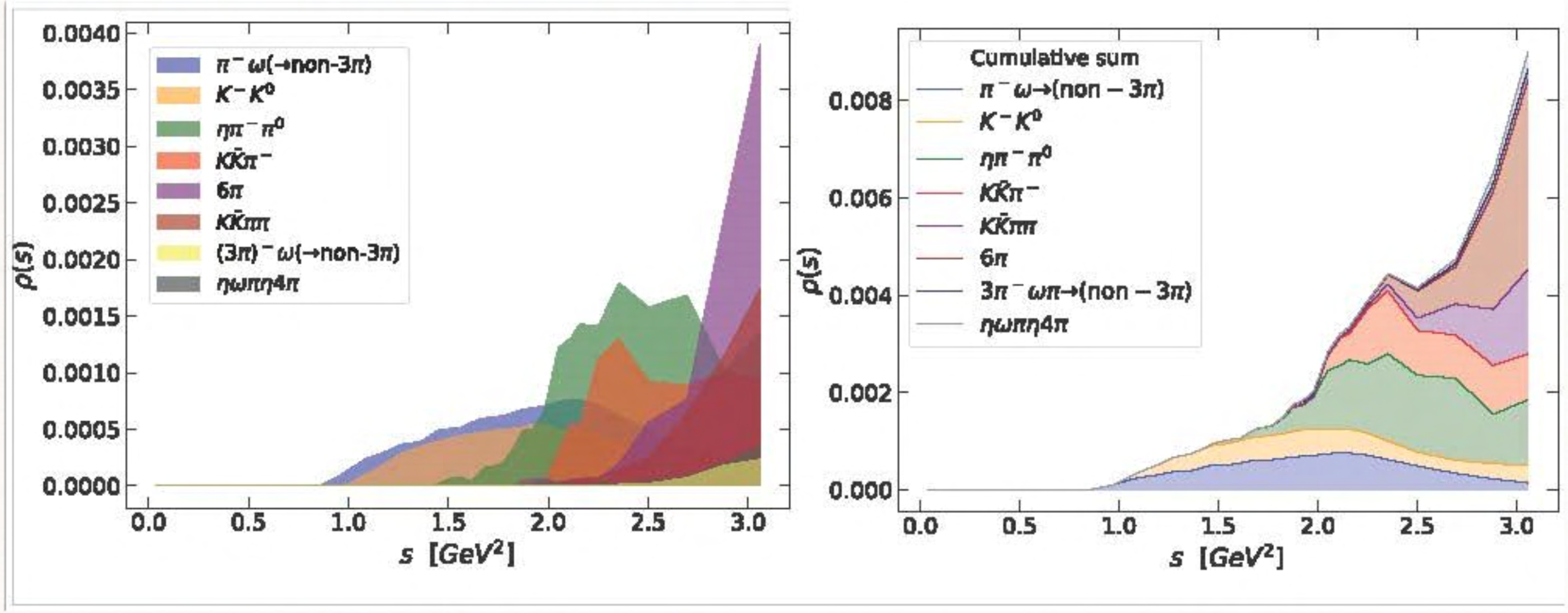
- Based on new V isovector spect. function w/o Monte Carlo input.
- PRS criticisms mistaken or unsupported by data.
- single-s_0 analysis of α_s in tOPE purely perturbative, redundant across set of weights and showing inconsistency in α_s values: Unreliable

BACK-UP

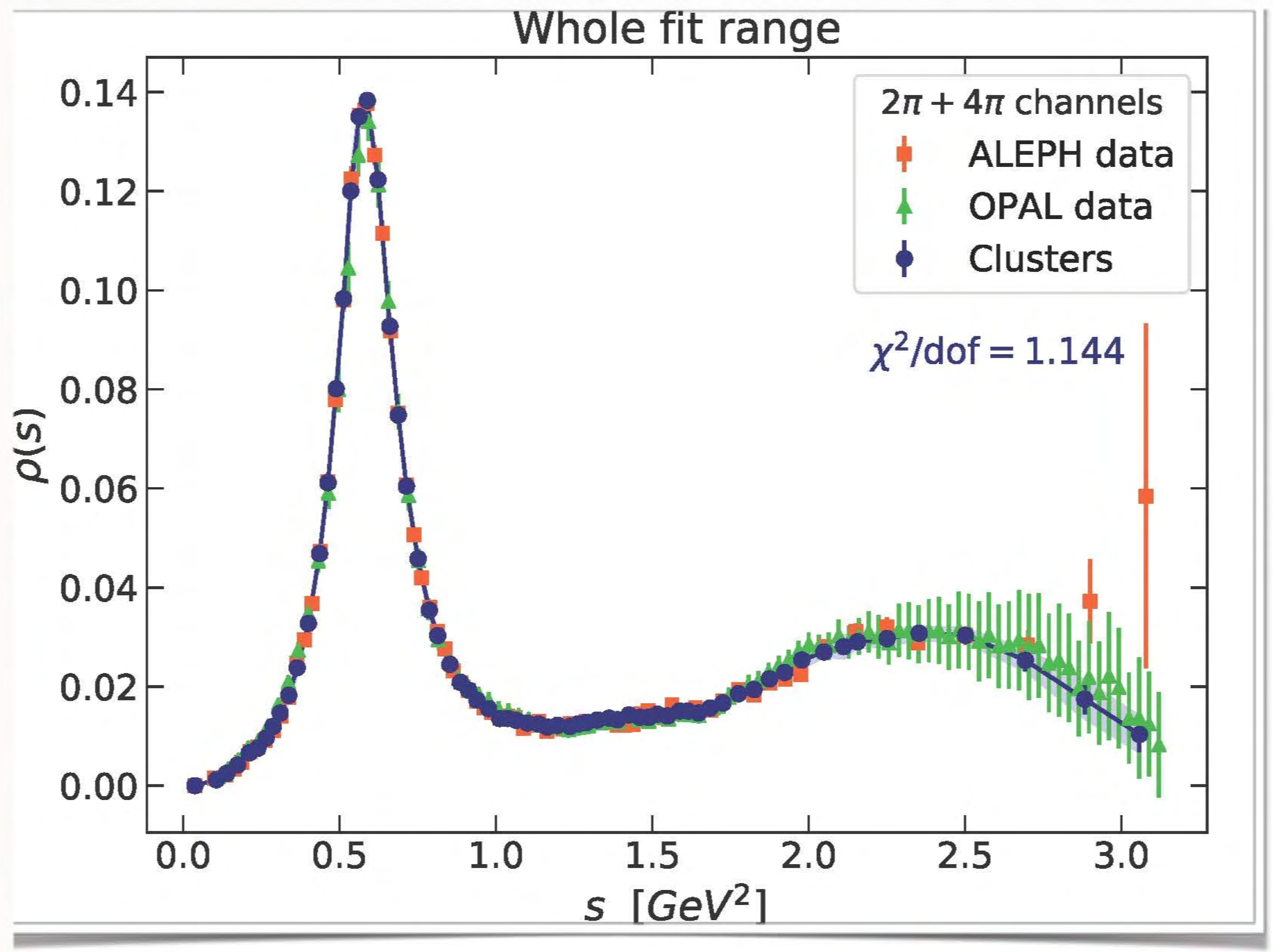
SLIDES

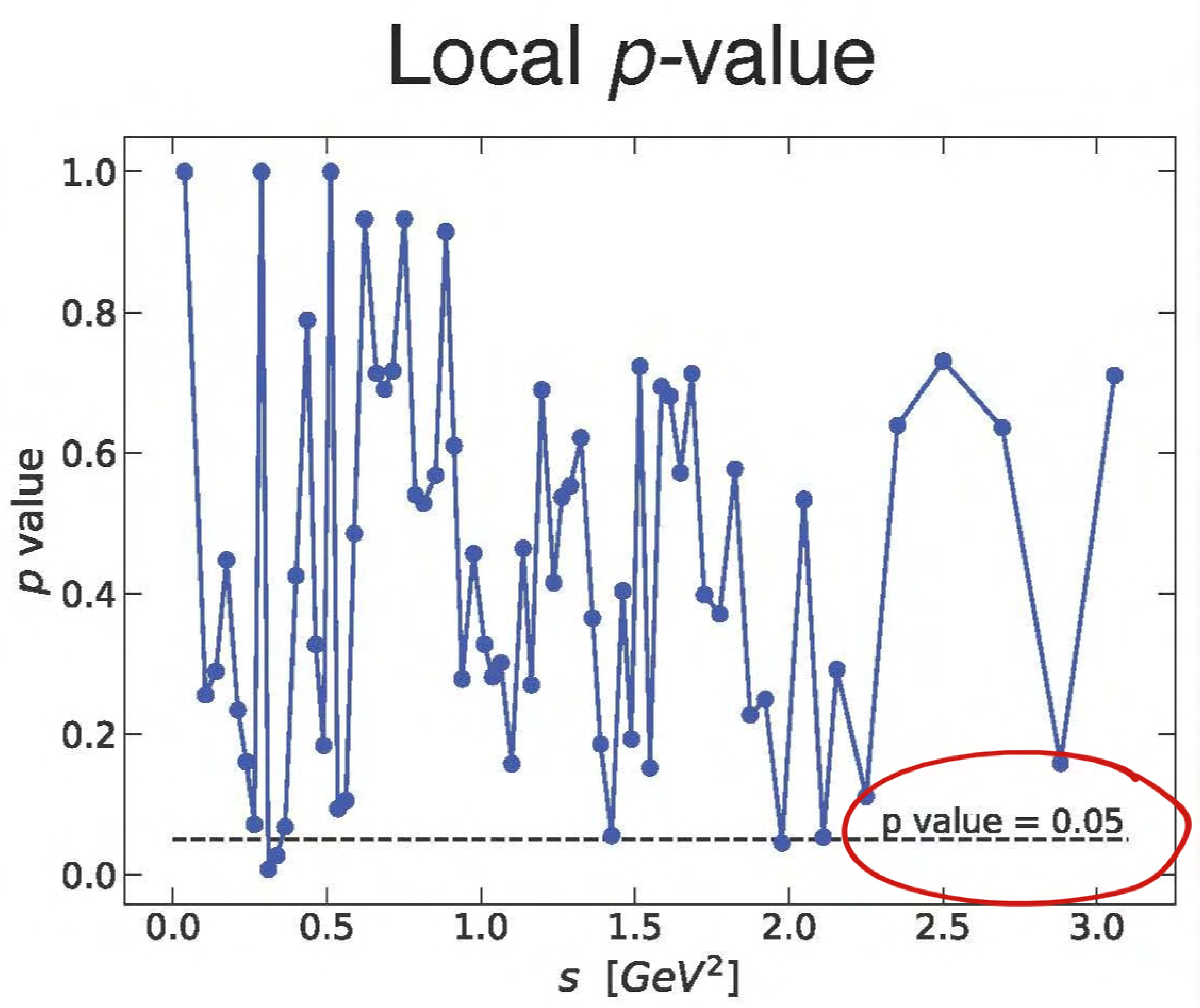
Improved V spectrum: residual channels (~ 2% of total).

Original data sets from: BABAR, SND and CMD-3



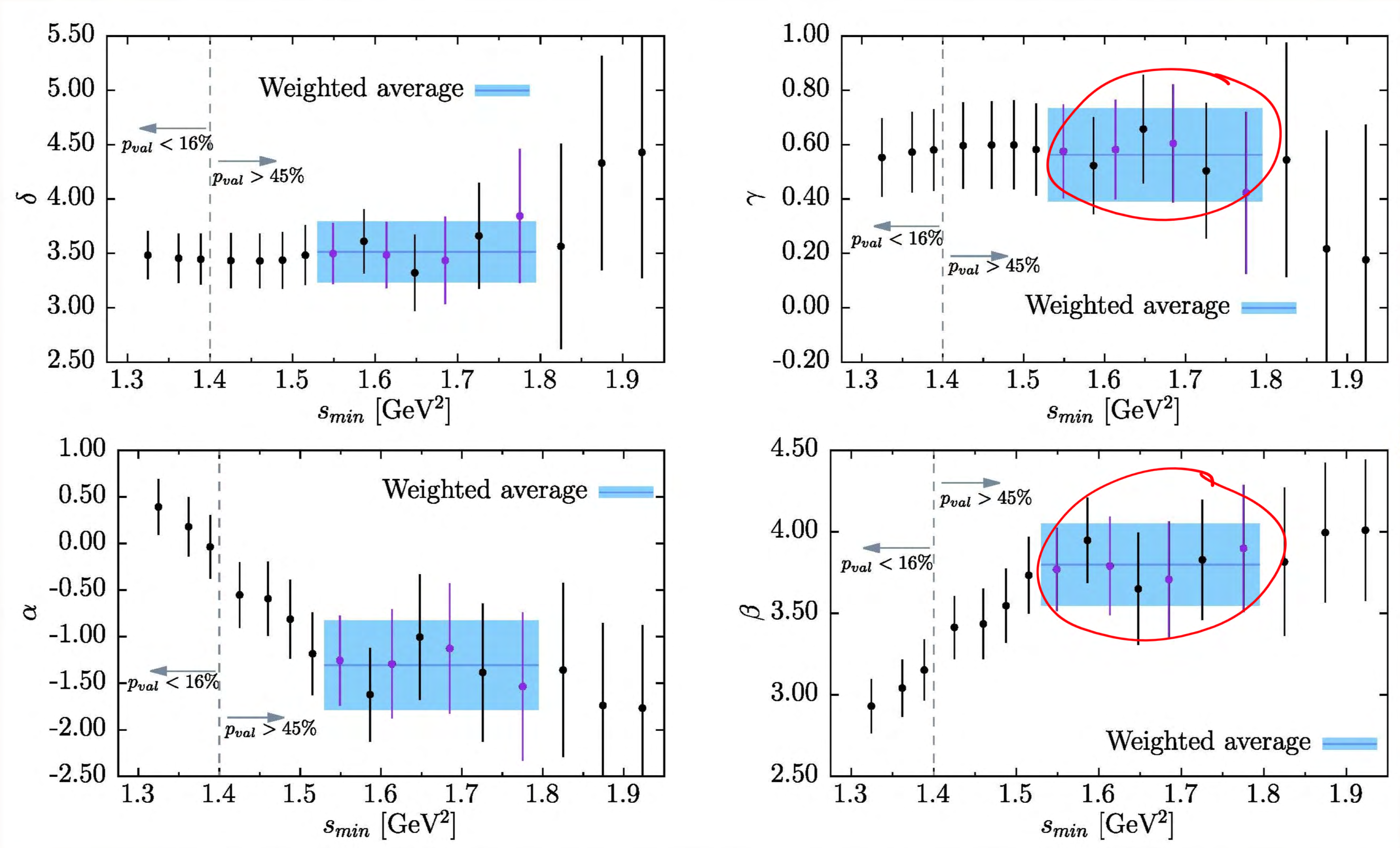
Improved V spectrum





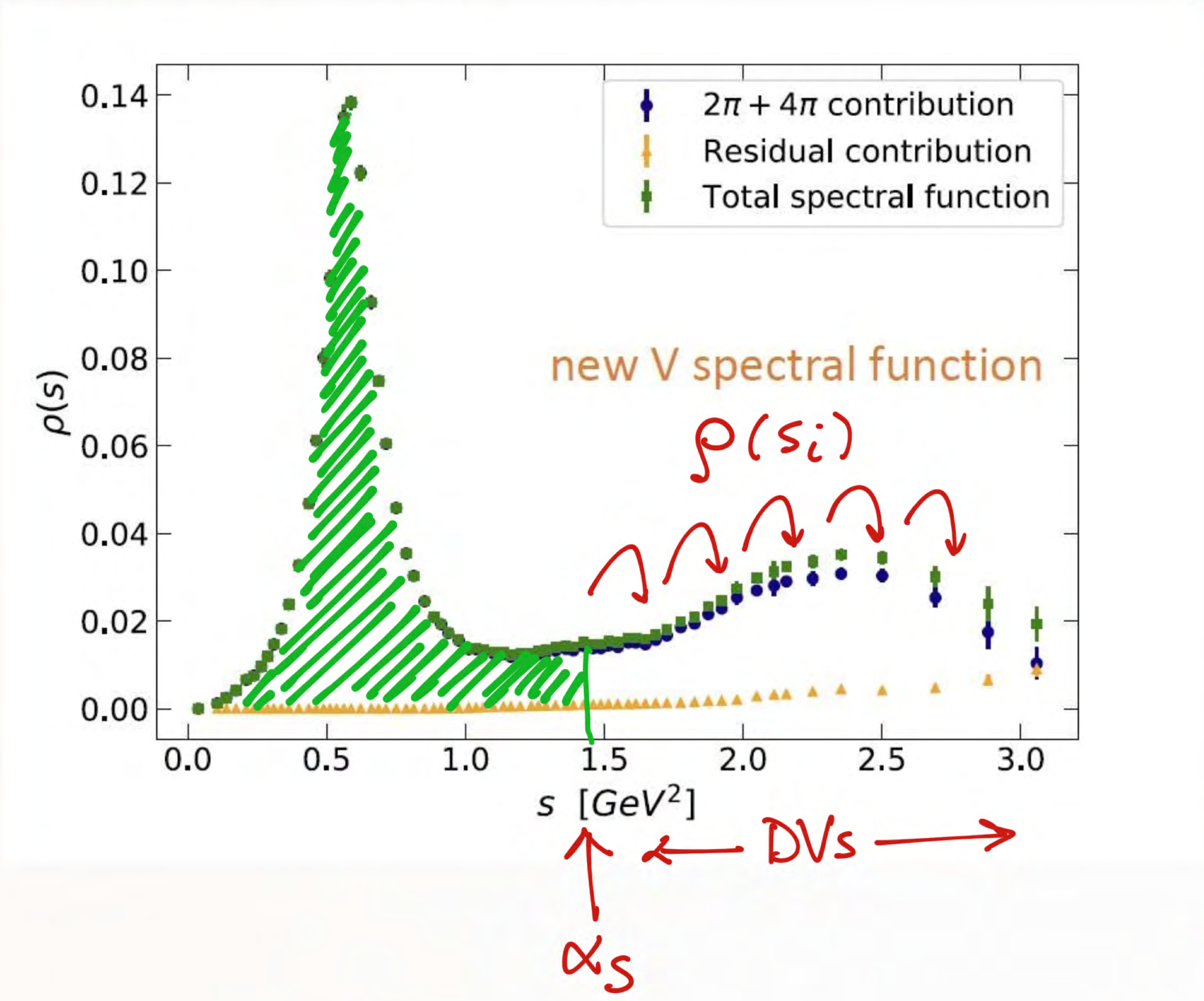
DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, arXiv:2012.10440

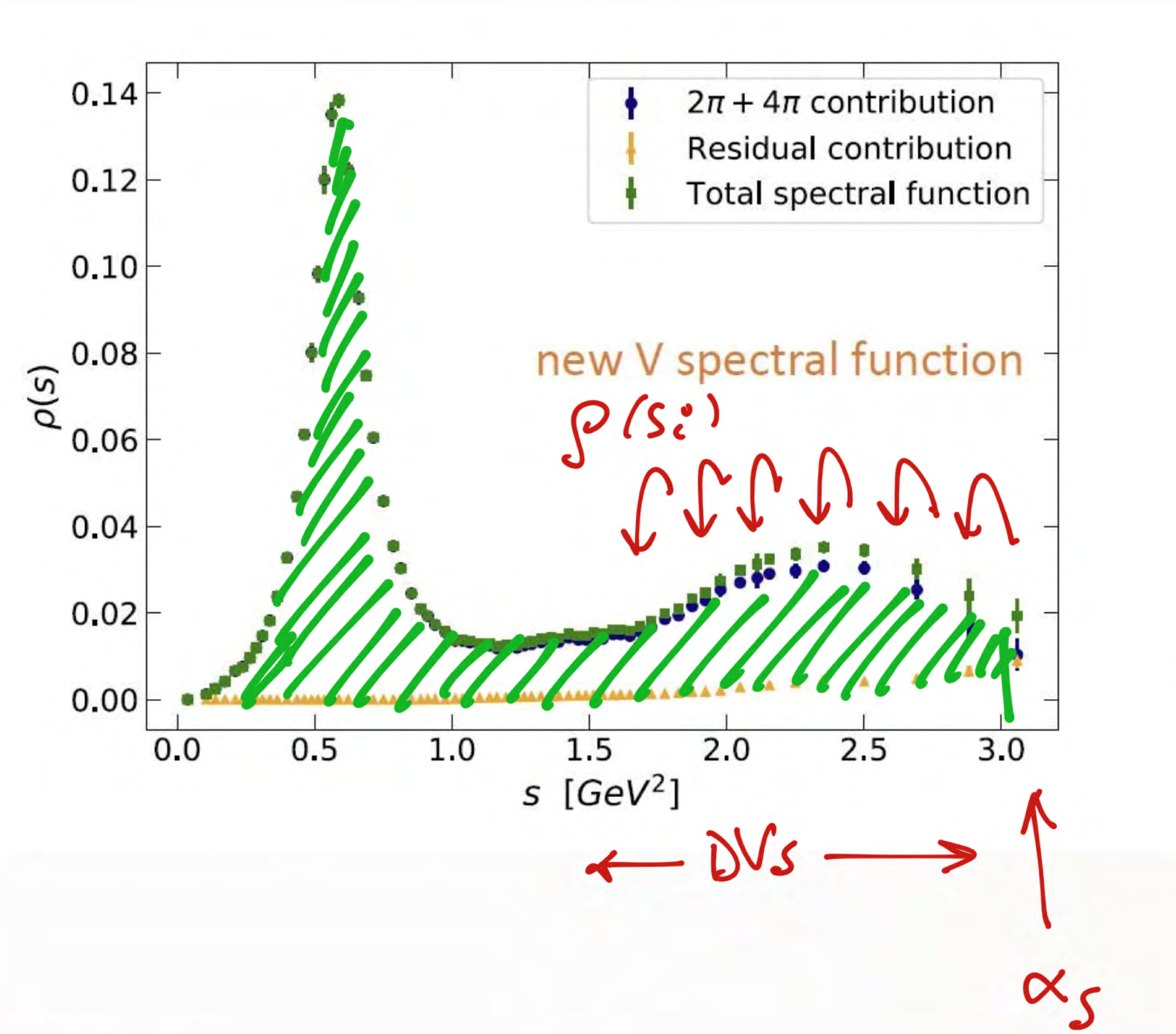
DV parameters



agreement with Regge/largeN_c analysis (not obvious)

Is α_s fixed at $s_0 = 1.55\ GeV^2$ OR α_s fixed at $s_0 = m_\tau^2$





$\Downarrow$

NEITHER!

(truth: both α_s & DVs fixed in window $s_0 \in [1.55, m_\tau^2]\ GeV^2$)

Leading dim-6 contribution to OPE are 4-quark condensates:

In the large-N_c approximation, these operators can be expressed, after Fierzing, in terms of $\langle \bar{q}q\rangle$ condensates with the result:

$$\begin{aligned}\langle O_1\rangle &= \frac{1}{2}\langle\bar{u}u\rangle\langle\bar{d}d\rangle\ ,\ \langle O_2\rangle = -\frac{1}{2}\langle\bar{u}u\rangle\langle\bar{d}d\rangle\ ,\ \langle O_4\rangle = -\frac{1}{2}\langle\bar{u}u\rangle^2 - \frac{1}{2}\langle\bar{d}d\rangle^2\\ \langle O_5\rangle &= \frac{1}{2}\langle\bar{u}u\rangle^2 + \frac{1}{2}\langle\bar{d}d\rangle^2\ ,\ \langle O_7\rangle = -\frac{1}{2}\langle\bar{u}u\rangle^2 - \frac{1}{2}\langle\bar{d}d\rangle^2 - \frac{1}{2}\langle\bar{s}s\rangle^2,\end{aligned}\tag{44}$$

with $\langle O_3\rangle = \langle O_6\rangle = 0$. These condensates give the following contribution to the vector two-point correlator

$$\begin{aligned}(-s)^3\Pi^{(0+1)}_{V;ud}(s) &= -8\pi^2\left[1-\frac{9}{8}\left(\frac{\alpha_s}{\pi}\right)L\right]\left(\frac{\alpha_s}{\pi}\right)\langle O_1\rangle + 5\pi^2 L\left(\frac{\alpha_s}{\pi}\right)^2\langle O_2\rangle\\ &\quad -\frac{8\pi^2}{9}\left[1-\frac{95}{72}L\left(\frac{\alpha_s}{\pi}\right)\right]\left(\frac{\alpha_s}{\pi}\right)\langle O_4\rangle + \frac{5\pi^2}{9}L\left(\frac{\alpha_s}{\pi}\right)^2\langle O_5\rangle\\ &\quad +\frac{24\pi^2}{81}L\left(\frac{\alpha_s}{\pi}\right)^2\langle O_7\rangle,\end{aligned}\tag{45}$$

with $L = \log(-s/s_0)\,, \alpha_s \equiv \alpha_s(s_0)$ and $\langle O_i\rangle \equiv \langle O_i\rangle(s_0)$. A further simplification may be obtained in the $SU(3)$ limit where $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \langle\bar{s}s\rangle \equiv \langle\bar{q}q\rangle$,

$$(-s)^3\Pi^{(0+1)}_{V;ud}(s) = -\frac{28}{9}\pi\,\rho\,\alpha_s\langle\bar{q}q\rangle^2\left(1-\frac{19}{63}\alpha_s L\right),\tag{46}$$

where ρ is a factor parametrizing the possible deviations from the large-N_c and $SU(3)$ limits.

spectral moments: $I_n(s_k) = \sum_{i=1}^{k} \rho(s_i)\, s_i^{n}\, \Delta$, $s_k \in \{s_{N_0}, \dots, s_{N_0+N}\}$

bin width

theory representation: $t_n(s_k)$

weight

$$\begin{pmatrix} I_n(s_{N_0}) - t_n(s_{N_0}) \\ I_n(s_{N_0}+\Delta) - t_n(s_{N_0}+\Delta) \\ I_n(s_{N_0}+2\Delta) - t_n(s_{N_0}+2\Delta) \\ \vdots \\ I_n(s_{N_0+N}) - t_n(s_{N_0+N}) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots \\ 1 & (s_{N_0}+\Delta)^n\Delta & 0 & 0 & \dots \\ 1 & (s_{N_0}+\Delta)^n\Delta & (s_{N_0}+2\Delta)^n\Delta & 0 & \dots \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & (s_{N_0}+\Delta)^n\Delta & \dots & & s_N^n\Delta \end{pmatrix} \begin{pmatrix} I_n(s_{N_0}) - t_n(s_{N_0}) \\ \rho(s_{N_0}+\Delta) - \frac{t_n(s_{N_0}+\Delta)-t_n(s_{N_0})}{(s_{N_0}+\Delta)^n\Delta} \\ \rho(s_{N_0}+2\Delta) - \frac{t_n(s_{N_0}+2\Delta)-t_n(s_{N_0}+\Delta)}{(s_{N_0}+2\Delta)^n\Delta} \\ \vdots \\ \rho(s_{N_0+N}) - \frac{t_n(s_{N_0+N})-t_n(s_{N_0+N}-\Delta)}{s_{N_0+N}^n\Delta} \end{pmatrix}$$

If $t_n(s_k)$ neglected, looks like a fit to $\rho(s_k)$ independent of weight "n".
However, when $t_n(s_k)$ included, there is nontrivial constraint on t_n & t_m.

$$\frac{t_n(s_{N_0}+k\Delta) - t_n(s_{N_0}+(k-1)\Delta)}{(s_{N_0}+k\Delta)^n} = \frac{t_m(s_{N_0}+k\Delta) - t_m(s_{N_0}+(k-1)\Delta)}{(s_{N_0}+k\Delta)^m}$$

ALEPH V+A $s_0 = 2.8\ \text{GeV}^2$

Redundancy in tOPE strategy: Assumption $C_{12} = C_{14} = C_{16} = 0$

"Optimal weights" (PRS '16):

$w_{21} = 1 - 3y^2 + 2y^3$; $w_{22} = 1 - 4y^3 + 3y^4$

$w_{23} = 1 - 5y^4 + 4y^5$

$$\boxed{\begin{aligned} w_{24} &= 1 - 6y^5 + 5y^6 \\ w_{25} &= 1 - 7y^6 + 6y^7 \end{aligned}} \Rightarrow \text{fix } \alpha_s \text{ w/o OPE!}$$

Results $\Rightarrow$ **REDUNDANCY**

$w_{24} \oplus w_{25}$: $\alpha_s = 0.3168(27)$, $\chi^2 = 3.06933$

$\oplus\, w_{23}$: $\alpha_s = 0.3168(27)$, $\chi^2 = 3.06933$, $C_{10} = -41(25) \times 10^{-4}\ \text{GeV}^{10}$

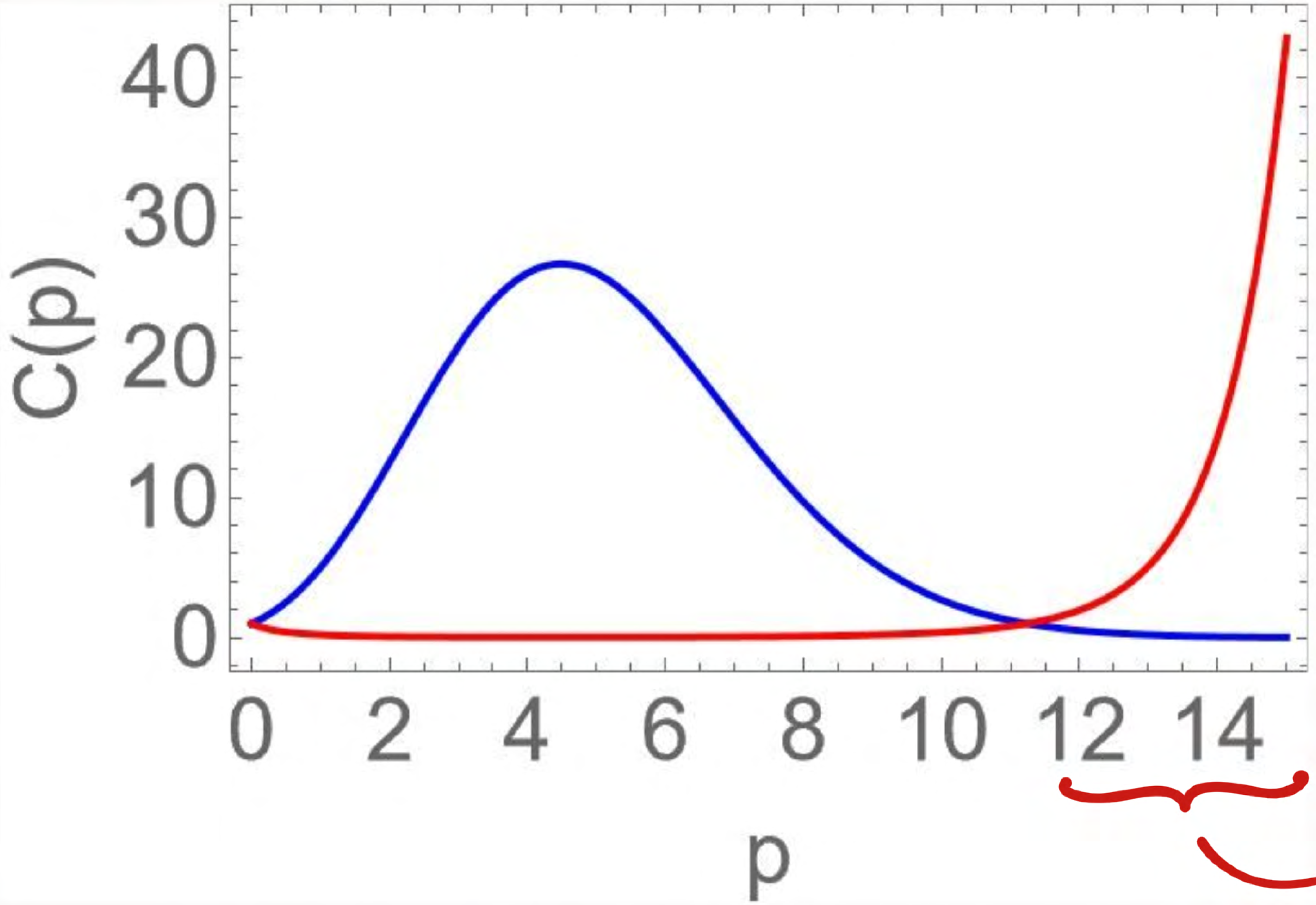
$\oplus\, w_{22}$: $\alpha_s = 0.3168(27)$, $\chi^2 = 3.06933$, $C_{10} =$ " , $C_8 = 16(14) \times 10^{-4}\ \text{GeV}^8$

$\oplus\, w_{21}$: $\alpha_s = 0.3168(27)$, $\chi^2 = 3.06933$, $C_{10} =$ " , $C_8 =$ " , $C_6 = 5(5) \times 10^{-4}\ \text{GeV}^6$

w_{25} only: $\alpha_s = 0.3202(24)$ Test Pert. Theory, not the OPE!

Asymptotic vs. Convergent

$$f(z) = \sum_{p=1}^{N \overset{?}{\to} \infty} C(p)\, z^p$$



$C(p) = \left(\frac{5^p}{p!}\right)$, convergent

$C(p) = \left(\frac{p!}{5^p}\right)$, Asymptotic

Must avoid being sensitive to high-order contributions

[Poincare (Circa 1893): discussion Geometers vs. Astronomers]